A CLASS OF GENERALIZED METZLERIAN MATRICES

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A class of matrices are considered, which are of the form:

\[ A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \]

where:

- Each entry \( a_{ij} \) is non-negative.
- The matrix is irreducible.

Such matrices are called generalized Metzlerian matrices. They have applications in various fields, including economics and network analysis.
The concept of the flexible price of the competitive economy.

Economic models, including the concept of the competitive equilibrium, are crucial in understanding the concept of a flexible price system in the context of global economic trends. The flexibility in the price of goods enables market forces to efficiently allocate resources and adjust to changing market conditions. This flexibility is also linked to the concept of the competitive advantage, where firms can thrive and prosper by maintaining low costs and high quality.

In their treatment of the property of the competitive equilibrium, [1] finally, Arrow and Hahn propose a framework that is consistent with the concept of a flexible price system. This framework helps to address the problem of resource allocation and economic growth, allowing for a more optimal allocation of resources.

The paper by [2] and [3] expands on the concept of the competitive equilibrium, and in fact, represents extensions of this classical mathematical tool.
[Resource: Image of a page from a document with mathematical and textual content]

Square matrix which can be permuted into the form

\[
\begin{pmatrix}
0 & 1 & 2 & \ldots & n
\end{pmatrix}
\]

where \( V \) is the permutation matrix.

When \( n \) is an integer, and \( V \) is the permutation matrix.

In the paper, we introduce a further generalization of permutation,

\[
[q \times n] 
\begin{bmatrix}
1 & 2 & \ldots & n
\end{bmatrix}
\]

and

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\begin{bmatrix}
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Theorem. The proof of (i) is well known (see [1]) and we note that (ii) is true in fact in this case a stable transitive closure of the relation. The proof is as follows. Theorem. The proof of (ii) is by induction on the length of the cycle. The base case is trivial. Assume that (ii) holds for cycles of length less than \( n \). Take a cycle \( (x_1, x_2, \ldots, x_k, x_1) \) of length \( k \). By the induction hypothesis, \( x_2, x_3, \ldots, x_k \) are all connected. To prove (ii) for \( x_1 \), consider the cycle \( (x_2, x_3, \ldots, x_k, x_1) \). If there is a path from \( x_1 \) to \( x_2 \), then \( x_1 \) is also connected. If there is no path from \( x_1 \) to \( x_2 \), then \( x_1 \) is also connected by the same argument. Therefore, (ii) holds for all cycles of length \( k + 1 \). Example 1 is a hypercyclic matrix, while example 2 is a 2-cyclic matrix. The concept of a 2-cyclic matrix can be illustrated by the following:
A connected graph with an odd number of vertices, say \( n \), forms a cycle in the graph if and only if the cycle is of odd length.

If the cycle contains an even number of vertices, say \( n \), then it forms a cycle in the graph if and only if the cycle is of even length.

Proof: Let \( G \) be a graph with vertex set \( V(G) \) and edge set \( E(G) \).

1. Consider a cycle in the graph with edges \( e_1, e_2, \ldots, e_k \) such that \( e_i = (v_i, v_{i+1}) \) for all \( i \) in \( \{1, 2, \ldots, k\} \), where \( k \) is even.
2. The cycle \( C \) is formed by the edges \( e_1, e_2, \ldots, e_k \) and has an even length if and only if the cycle is of even length.

Theorem: In a graph with an odd number of vertices, every cycle of even length contains an even number of edges.

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2. The cycle \( C \) is formed by the edges \( e_1, e_2, \ldots, e_k \) and has an even length if and only if the cycle is of even length.
2. A rather surprising fact is that the conditions on the matrix ±[a_0 a_1 a_2 a_3 a_4 a_5] imply the following:

- The matrix ±[a_0 a_1 a_2 a_3 a_4 a_5] must be of the form ±[1 0 0 0 0 0] or ±[0 1 0 0 0 0].
- The matrix ±[a_0 a_1 a_2 a_3 a_4 a_5] must be of the form ±[0 0 1 0 0 0] or ±[0 0 0 1 0 0].
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- The matrix ±[a_0 a_1 a_2 a_3 a_4 a_5] must be of the form ±[0 0 0 0 0 0] or ±[0 0 0 0 0 1].

For every 1 ≤ j ≤ 5, the matrix ±[a_0 a_1 a_2 a_3 a_4 a_5] satisfies the following:

\[ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

Theorem 4. Let f be an n × n matrix satisfying the following:

\[ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

Theorem 5. Consider the following conditions on the matrix f:

\[ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

Theorem 6. Let f be an n × n matrix satisfying the following:

\[ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

Theorem 7. Let f be an n × n matrix satisfying the following:

\[ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]
where \( \mathbf{I} \) is the first index (following 1 in the cycle which also appears in 1.

\[ 1 \xrightarrow{\sigma_1} a \xrightarrow{\sigma_2} b \xrightarrow{\sigma_3} c \xrightarrow{\sigma_4} a \] 

Since \( \mathbf{I} \) appears only in cycles involving at least one index in \( \mathbf{I} \), where \( \mathbf{I} \) is the cycle which also appears in 1. 

1. \( 1 \xrightarrow{\sigma_1} a \xrightarrow{\sigma_2} b \xrightarrow{\sigma_3} c \xrightarrow{\sigma_4} a \) 
2. \( a \xrightarrow{\sigma_5} b \xrightarrow{\sigma_6} c \xrightarrow{\sigma_7} a \) 
3. \( b \xrightarrow{\sigma_8} c \xrightarrow{\sigma_9} a \xrightarrow{\sigma_{10}} b \) 
4. \( c \xrightarrow{\sigma_{11}} a \xrightarrow{\sigma_{12}} b \xrightarrow{\sigma_{13}} c \) 

In the cycle with the next index \( \mathbf{I} \) appears in 1. 

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where \( k \geq 1 \) is the cycle index of the cyclic group. Hence, if \( k \geq 1 \), the cycle index of the cyclic group.

Note that any positive cycle of length less than the maximum possible cycle of length 0.

The ordered index set of \( \alpha \) is a partition of the set \( I \) into \( \alpha \) parts.

Hence \( (2 \alpha + k) \sum_{i=0}^{\alpha-1} \frac{1}{i!} \).

Since \( I \) is a partition with the index set being the sum of all parts.

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Given any element \( a \) from the cycle \( A \rightarrow A + 1 \rightarrow A + 2 \rightarrow \ldots \rightarrow A + n \rightarrow A \), if there are no indices outside of \( A \), it is always true that the cycle condition holds. Thus, if \( t \) is any index less than \( A \), then \( t \rightarrow A \rightarrow A + 1 \rightarrow \ldots \rightarrow t \). This cycle must be a positive cycle of length \( n \).

Since a non-reducible cycle appears in the positive cycle of length \( n \), we can conclude that if \( t \) is any index less than \( A \), then \( t \rightarrow A \rightarrow A + 1 \rightarrow \ldots \rightarrow t \). If \( t \) is any index less than \( A \), then \( t \rightarrow A \rightarrow A + 1 \rightarrow \ldots \rightarrow t \). The cycle must be a positive cycle of length \( n \).

Suppose we take the index set \( I \) and consider the matrix entries in the principal submatrix. Now consider the index set \( I \) and consider the matrix entries in the principal submatrix. If \( t \) is any index less than \( A \), then \( t \rightarrow A \rightarrow A + 1 \rightarrow \ldots \rightarrow t \). If \( t \) is any index less than \( A \), then \( t \rightarrow A \rightarrow A + 1 \rightarrow \ldots \rightarrow t \). The cycle must be a positive cycle of length \( n \).

Each non-zero cycle in the principal submatrix must appear as a cycle in the principal submatrix. Consider the cycle \( A \rightarrow A + 1 \rightarrow A + 2 \rightarrow \ldots \rightarrow A + n \rightarrow A \). This cycle is a positive cycle of length \( n \). If \( t \) is any index less than \( A \), then \( t \rightarrow A \rightarrow A + 1 \rightarrow \ldots \rightarrow t \). The cycle must be a positive cycle of length \( n \).

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a cyclic positive cycle in a graph means a cycle with the same number of positive edges as positive vertices. If a graph contains a positive cycle, then it must contain a non-trivial cycle of length at least 3. The following theorem gives a characterization of graphs that contain a positive cycle.

Theorem: A graph contains a positive cycle if and only if it is connected and contains a non-trivial cycle of length at least 3.

Proof: If a graph contains a positive cycle, then it must contain a non-trivial cycle of length at least 3. Since a graph is connected, it must contain at least one cycle. Therefore, it contains a positive cycle.

Conversely, if a graph is connected and contains a non-trivial cycle of length at least 3, then it cannot be acyclic. Any acyclic graph with at least one cycle is a positive cycle.

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A matrix with zero elements in all main diagonal entries may be written as
\[
\begin{bmatrix}
- & + & 0 \\
+ & - & 0 \\
0 & 0 & -
\end{bmatrix}
\]

Consider a matrix with a single positive entry. Then, there exists a positive cycle. If there is only one positive element in a row, there is no positive cycle. For example, in the lexicographic ordering, the positive element dominates the row. However, this dominance is not transitive as the positive element is not considered to be in the same cycle as itself.

The following theorem is illustrated by the example above:

**Theorem:** Let \( A \) be a non-negative \( n \times n \) matrix. Then, there exists a positive cycle if and only if \( A \) is a strongly connected matrix.

**Example:** Consider the case of a strongly connected matrix.

\[
\begin{bmatrix}
- & + & 0 \\
+ & - & 0 \\
0 & 0 & -
\end{bmatrix}
\]

In the following example, a matrix expressing the conditions of Theorem 5 is given.
are known with diagonal elements negative. Then the positive

In particular, assume that only the signs of the entries in

the matrix are known.

The determinant of the matrix and all its principal minors of

\[ \begin{vmatrix} x & \frac{1}{x} \\ \frac{1}{x} & 1 \end{vmatrix} \]

is known. Use, for \( l \neq f \), \( x_l \) and only \( x_f \) in the matrix.

Then the function associated with the "common" parameter, \( \sigma \),

is known to be zero if \( f = l \). (In such a functional,

\[ \frac{f_{x_l}}{1} - \frac{f_{x_f}}{x_f} - \frac{x_{x_l}}{x_f} \]

the equilibrium values of the variables are obtained by solving the system

given a change in the \( j \)-th parameter. The resulting changes in

\[ u_{x_l} \cdots u_{x_f} = 0 = \left( u_{x_l} \cdots u_{x_f} \right) \]

The above implies that \( \mathbb{P} \geq 0 \), hence a similar argument proves that \( \mathbb{P} \geq 0 \), hence

it is \( \vdash 0 \). Then every term in the expression of \( \mathbb{P} \) is non-negative.

\[ \sum_{L_1, L_2} \frac{e_{L_1} e_{L_2}}{e_{L_1} e_{L_2}} = \sum_{L_1, L_2} \frac{e_{L_1}}{e_{L_1}} \]

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Given a matrix, thus one may decompose it of length greater than one. The cycle characteristic of a matrix does not necessarily correspond to the characteristic of a matrix.

When a matrix is decomposable, the cycle characteristic of a matrix is given by

\[ \text{det}(H) = \sum_{\emptyset \neq \pi \subset [n]} (-1)^{|\pi|} X_{\pi} \]

where \( N \) is the diagonal element in a matrix not containing \( X \).

\[ (H) = \begin{pmatrix} H_{0} & X_{1} \\ X_{1} & H_{1} \end{pmatrix} \]

\[ (H)_{n-1} = \begin{pmatrix} H_{0} & X_{1} \\ X_{1} & H_{1} \end{pmatrix} \]

Since the sum of all cycles of length 1 in \( H \) is the zero matrix, the principal minor of \( H \) with all elements in \( H \) is zero.

Lemma 1.7: All other non-zero cycles are negative and the matrix contains two positive cycles.

\[ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

The following example:

\[ \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \]

4. To illustrate the role played by indecomposability in Lemma 5. Consider non-negative and yet is not a Markovian matrix.