

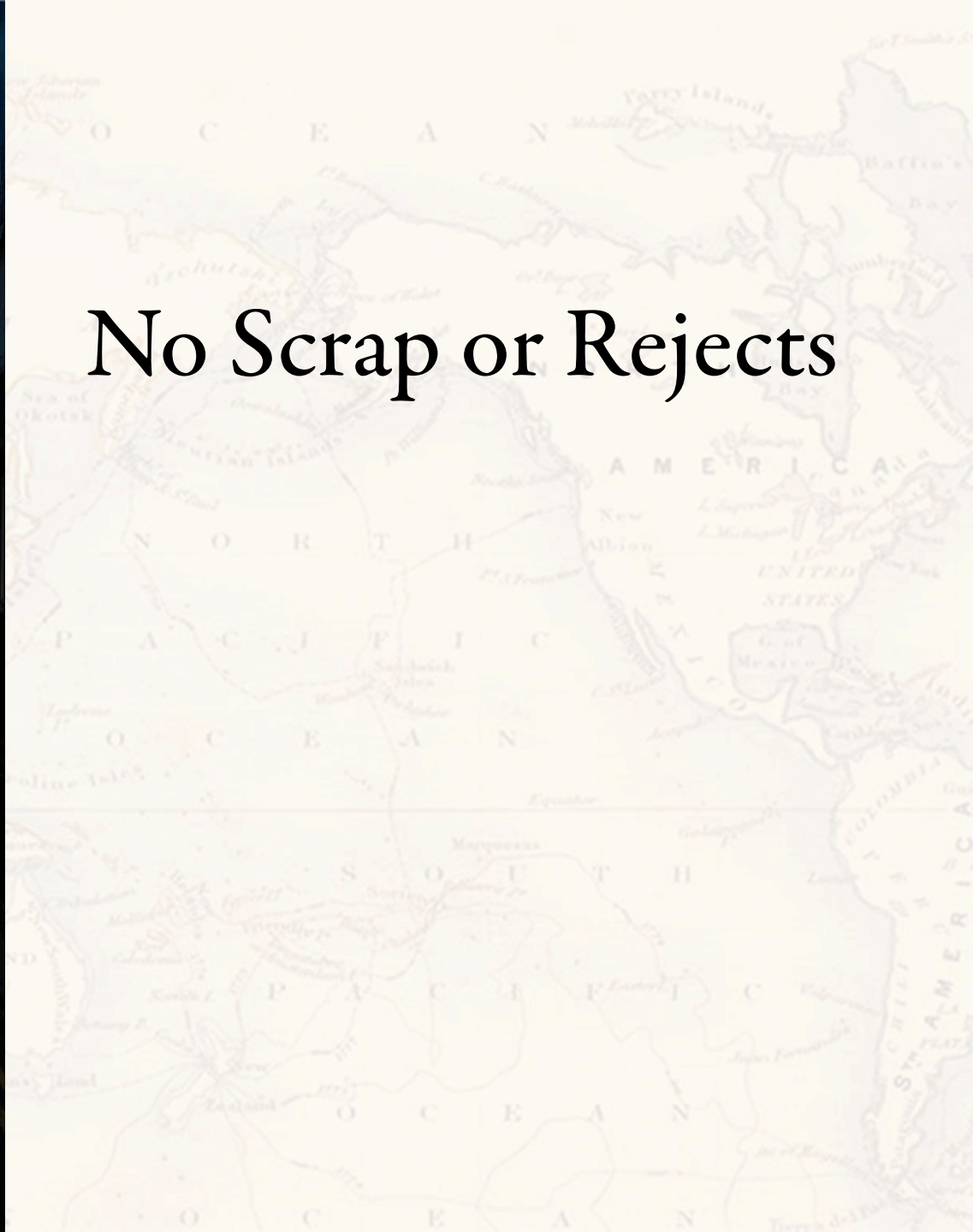


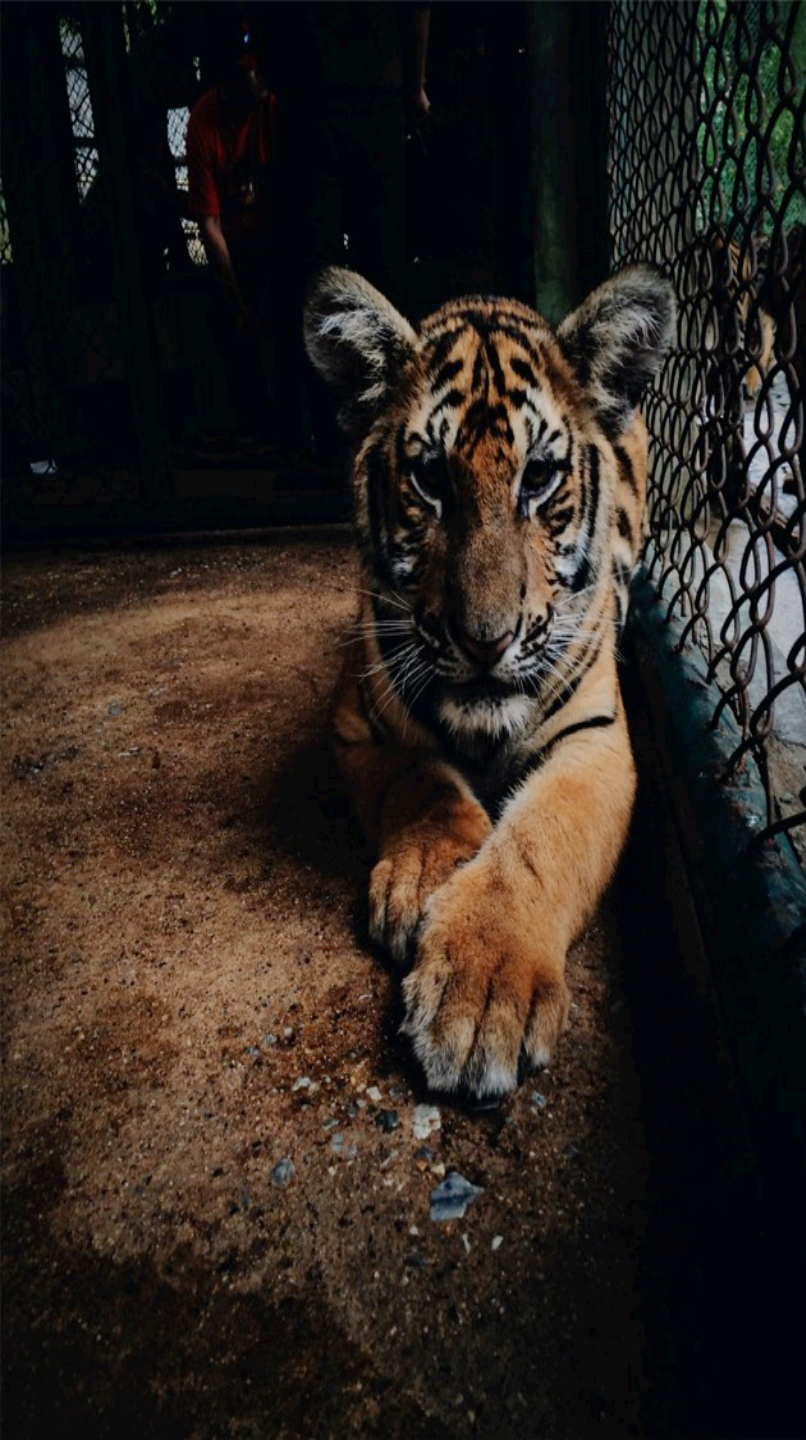
Statistical Tolerance Analysis

Fred Schenkelberg

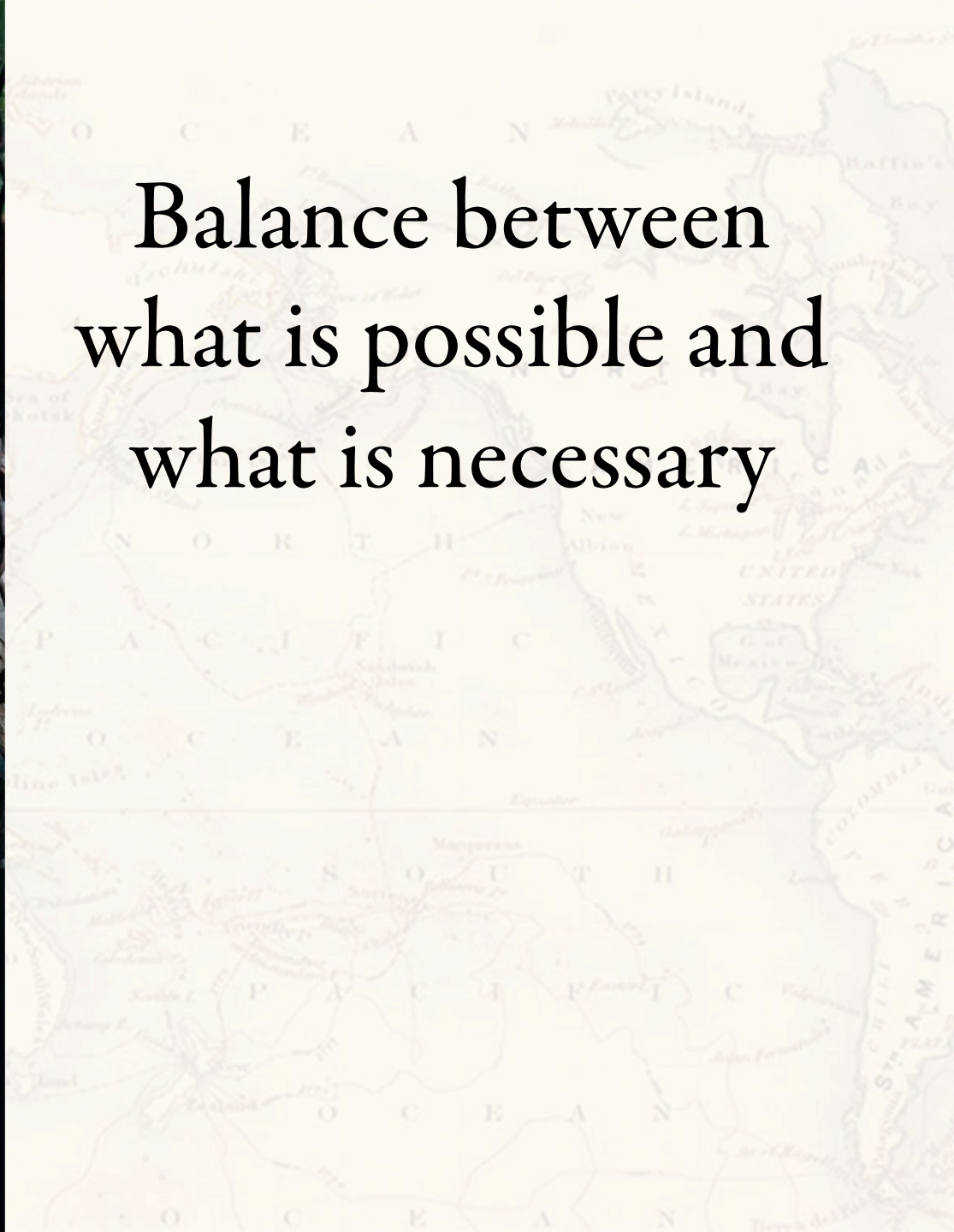
Perfect Fit







Balance between
what is possible and
what is necessary





The communication of variation

PURPOSE OF TOLERANCES

Communicate Acceptable Variation



What We Want



What They Provide





The impact of variation

TOLERANCES AND RELIABILITY

Variation Leads to Failures

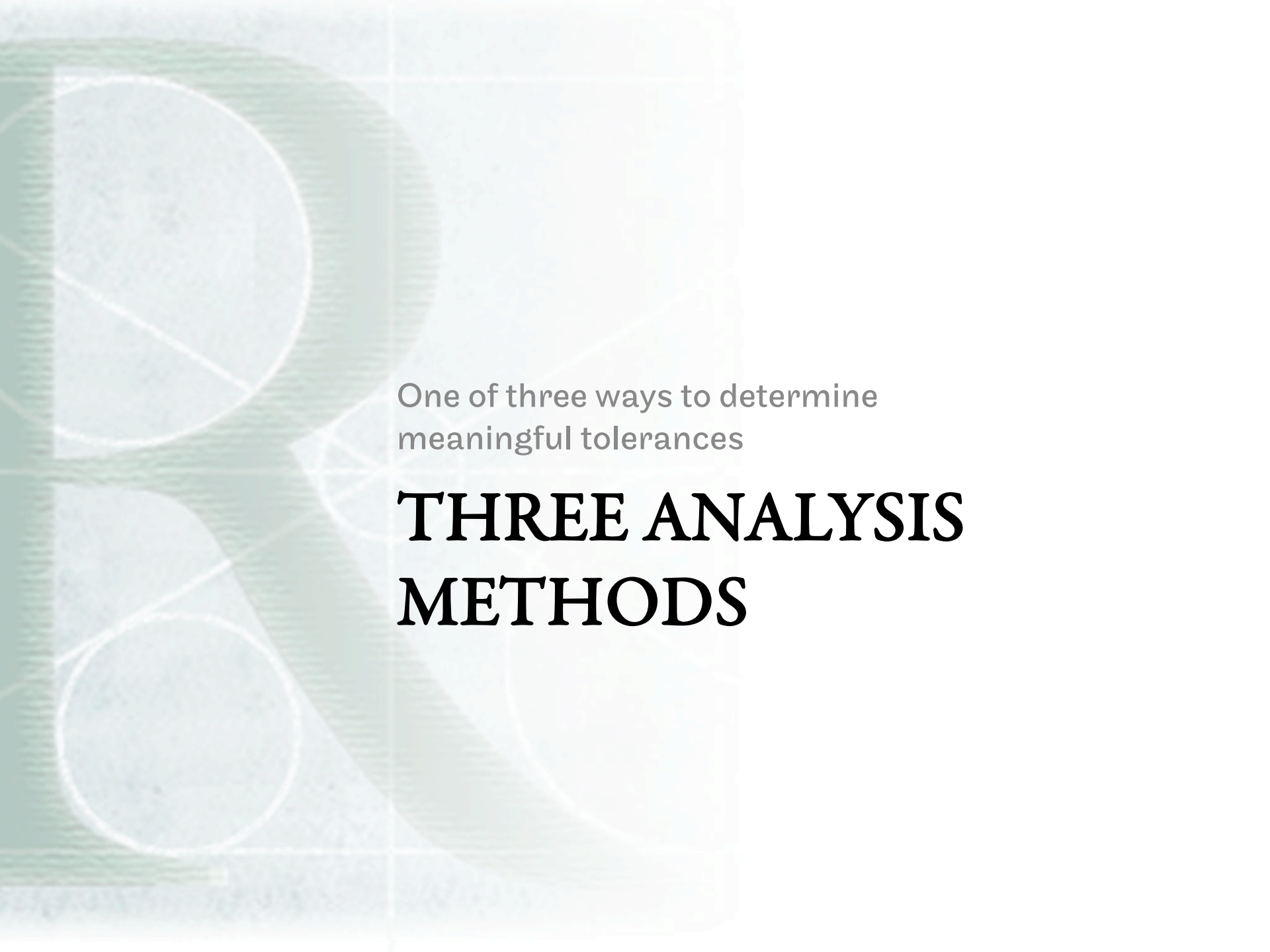




Match Capability to Requirements

Parts/systems within
design expectations last longer





One of three ways to determine
meaningful tolerances

THREE ANALYSIS METHODS

Worst Case Method

- Tally the stack up at maximums
- Simple math
- Only need range of variation

Root Sum Square Method

- Use standard deviations to estimate combined stack up standard deviation
- Able to estimate fall out or failures
- Assumes normal distribution

Monte Carlo Method

- Simulation technique to estimate stack up standard deviation
- Use estimates for distributions of each tolerance in the stack up.
- Most accurate, assuming good data input



Walk though example

LET'S SAY ...

Stack of Plates

- 5 plates
- Overall thickness of interest
- Each plate thickness is $25\text{mm} \pm 0.1\text{mm}$



Worst Case Method

- Add nominal values
- Add minimum values
- Add maximum values

Assumptions & Benefits

- Nothing outside tolerance limits
- Easy and quick
- Conservative
- No distributions

Root Sum Squared Method

- Estimate standard deviation for each part
 - Sum means
 - Combine standard deviations
- ← Hard Part (next slide)

Estimating Standard Deviation

- Tolerance divided by 3
or total tolerance
range divided by 6
- Make measurements
and calculate
standard deviation

Assumptions & Benefits

- Normal Distribution
- Stable Process
- Minimal Measurement Error
- Accounts for statistical spread of data
- Allows estimate of fraction out of tolerance
- Allows cost/benefit tradeoff analysis

Monte Carlo Method

- Estimate distributions ← Hard Part
(more later)
- Estimate number of simulations
- Run simulations & analyze

Assumptions & Benefits

- Reasonable estimates for distributions
- Stable processes
- Accounts for statistical spread of data
- Allows estimate of fraction out of tolerance
- Allows cost/benefit tradeoff analysis
- Any set of distributions

Comparison of Results

- Worst Case – quick & conservative
- Root Sum Squared – easy & normal?
- Monte Carlo – accurate & flexible

Economic Tradeoffs

- Relax precision/tolerance for lower cost manufacturing
- Estimate defect rate costs (RSS & Monte Carlo methods)



Let's get some data

DATA SOURCES

Engineering Judgment



Industry Standards or Guidelines



A close-up photograph of a glass carafe with a wooden stopper and a leather strap. The carafe is partially filled with a dark liquid and has condensation droplets on its surface. In the background, a potted plant with green leaves is visible, slightly out of focus.

Measurements





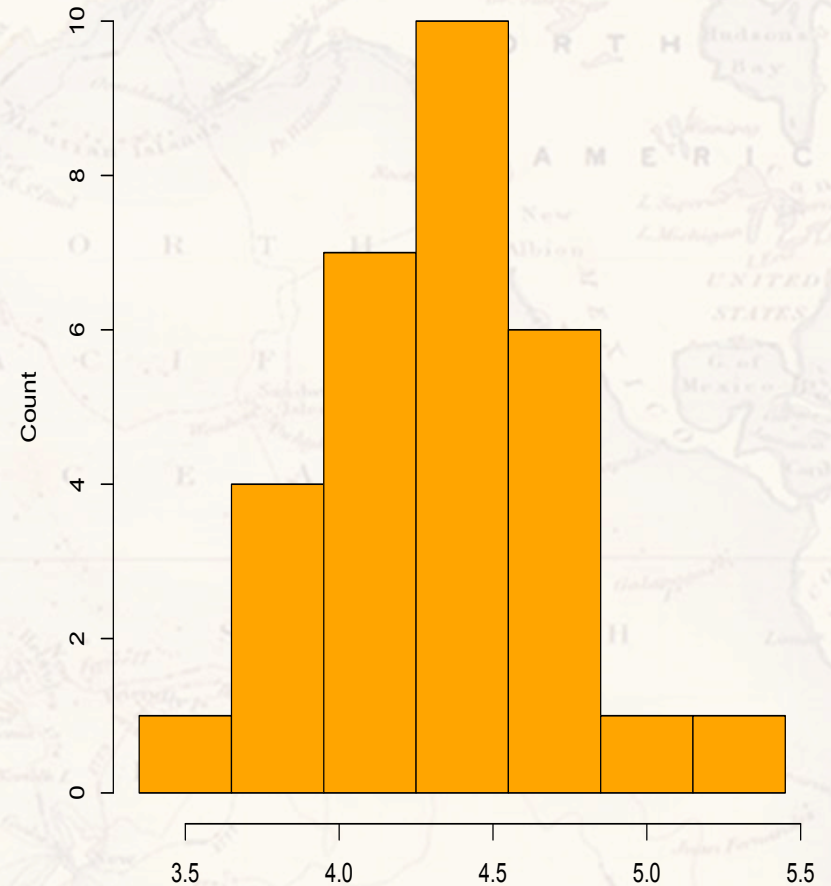
Summarize the Data

DISTRIBUTIONS

Histogram

Lengths

4.5 3.4 4.3 4.6 3.8
4.5 4.3 4.5 4.0 4.5
3.9 4.2 4.6 4.4 4.0
4.6 4.3 4.8 4.4 4.0
4.1 4.7 3.9 4.5 3.7
4.2 4.6 5.2 4.2 5.0



Number of Bins

- Sturge's rule

$$\text{bins} = 1 + 3.3 \log_{10}(N)$$

- Rice rule

$$\text{bins} = 2\sqrt[3]{N}$$

- Adjust number of bins to best view the data

Probability Density Function

Discrete Data

$$P[X = x] = p(x)$$

$p(x)$ is nonnegative for all real x

$$\sum_j p_j = 1$$

$$0 \leq p(x) \leq 1$$

Continuous Data

$$p[a \leq x \leq b] = \int_a^b f(x) dx$$

it is nonnegative for all real x

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

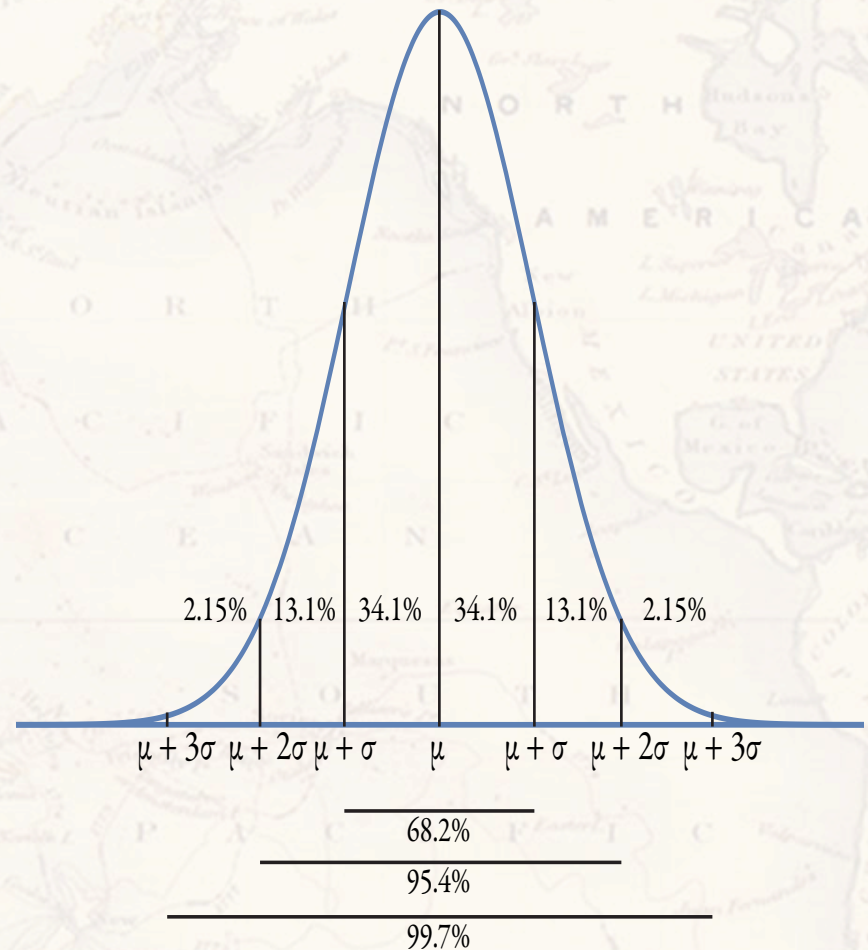
$$0 \leq p(x) \leq 1$$

Normal Distribution

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

when $\mu = 0$ and $\sigma = 1$

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$



Normal Distribution Parameters

- Mean, μ

$$\mu \simeq \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

- Standard Deviation, σ

$$\sigma \simeq s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

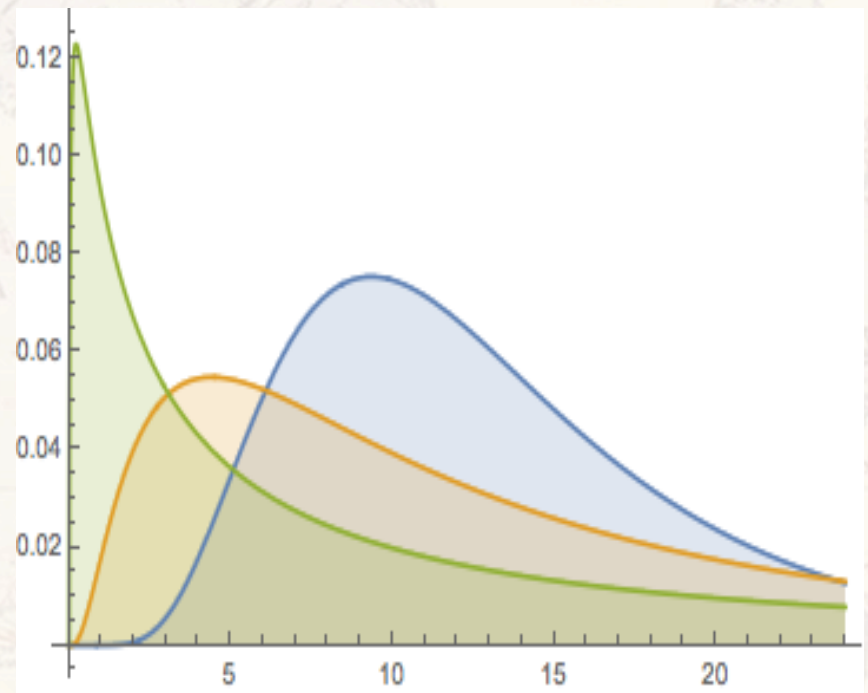
LogNormal Distribution

$$f(x) = \frac{1}{(x-\theta)\sigma\sqrt{2\pi}} e^{-\left(\frac{((\ln(x-\theta))/m)^2}{2\sigma^2}\right)}$$

$$x > \theta; m, \sigma > 0$$

when $\theta = 0$ and $m = 1$

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left(\frac{(\ln x)^2}{2\sigma^2}\right)}$$



Uniform Distribution

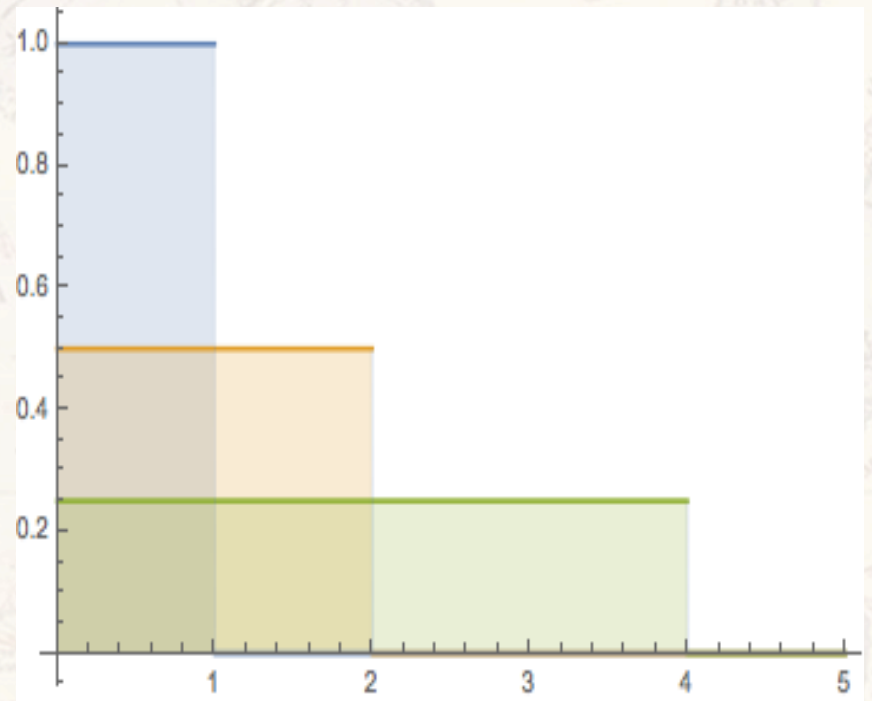
$$f(x) = \frac{1}{B - A}$$

for $A \leq x \leq B$

when $A = 0$ and $B = 1$

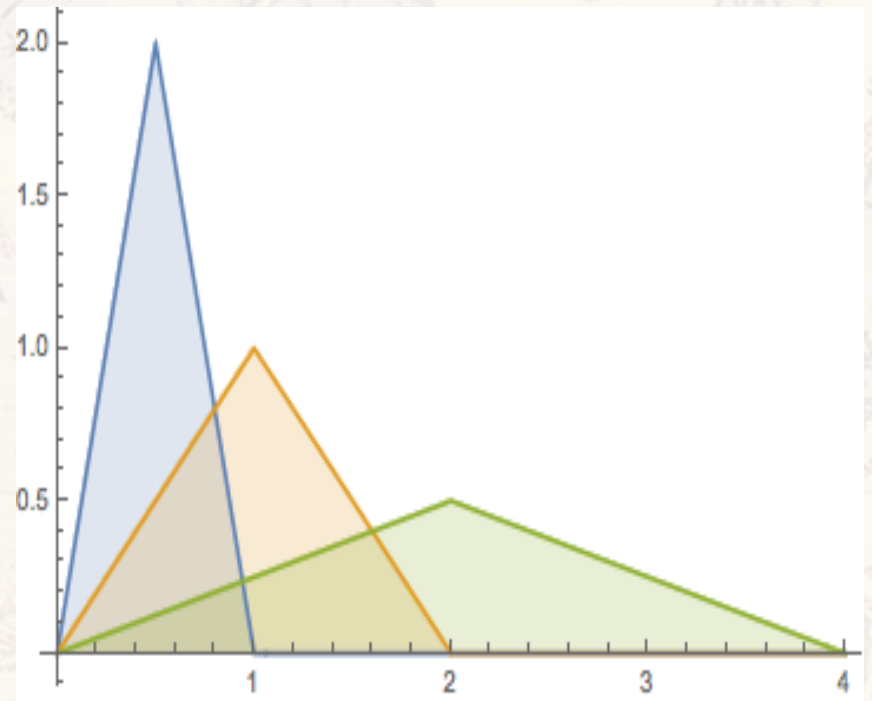
$$f(x) = 1$$

for $0 \leq x \leq 1$



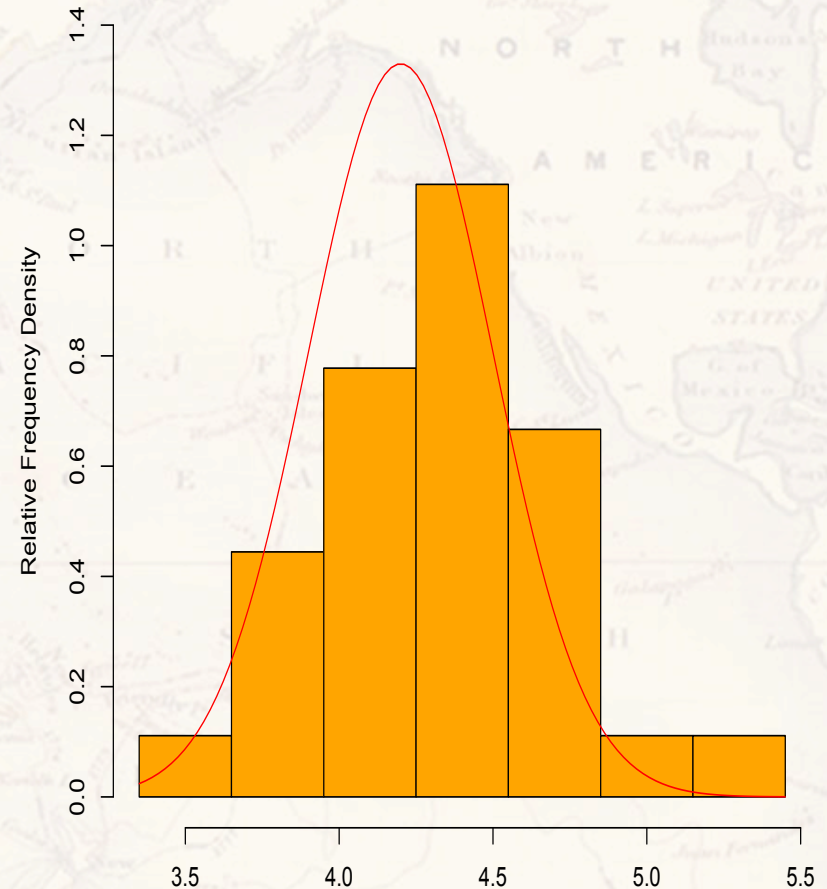
Triangle Distribution

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$



Test the Distribution Fit

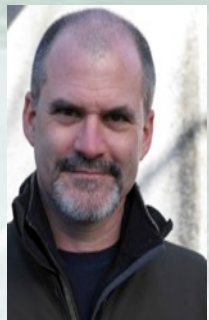
- Visual
- Probability Plotting
- Goodness of Fit





Homework

SELECT A TOLERANCE STACK UP TO STUDY



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