



# Fundamentals of Monte Carlo Analysis

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Variation

# Modeling





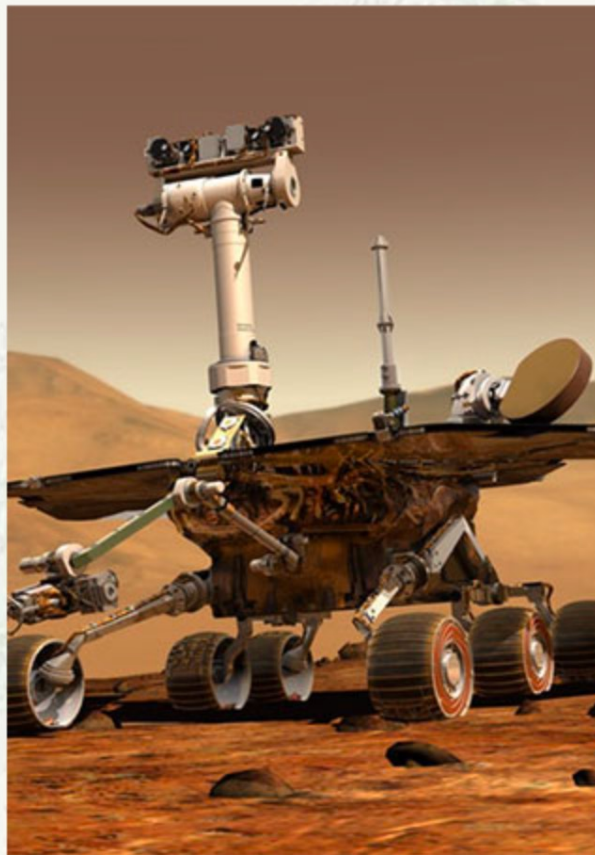
# Tolerance Analysis

Product Life  
Estimation

# Defining Monte Carlo



Definition



Concept



Benefits

# Definition



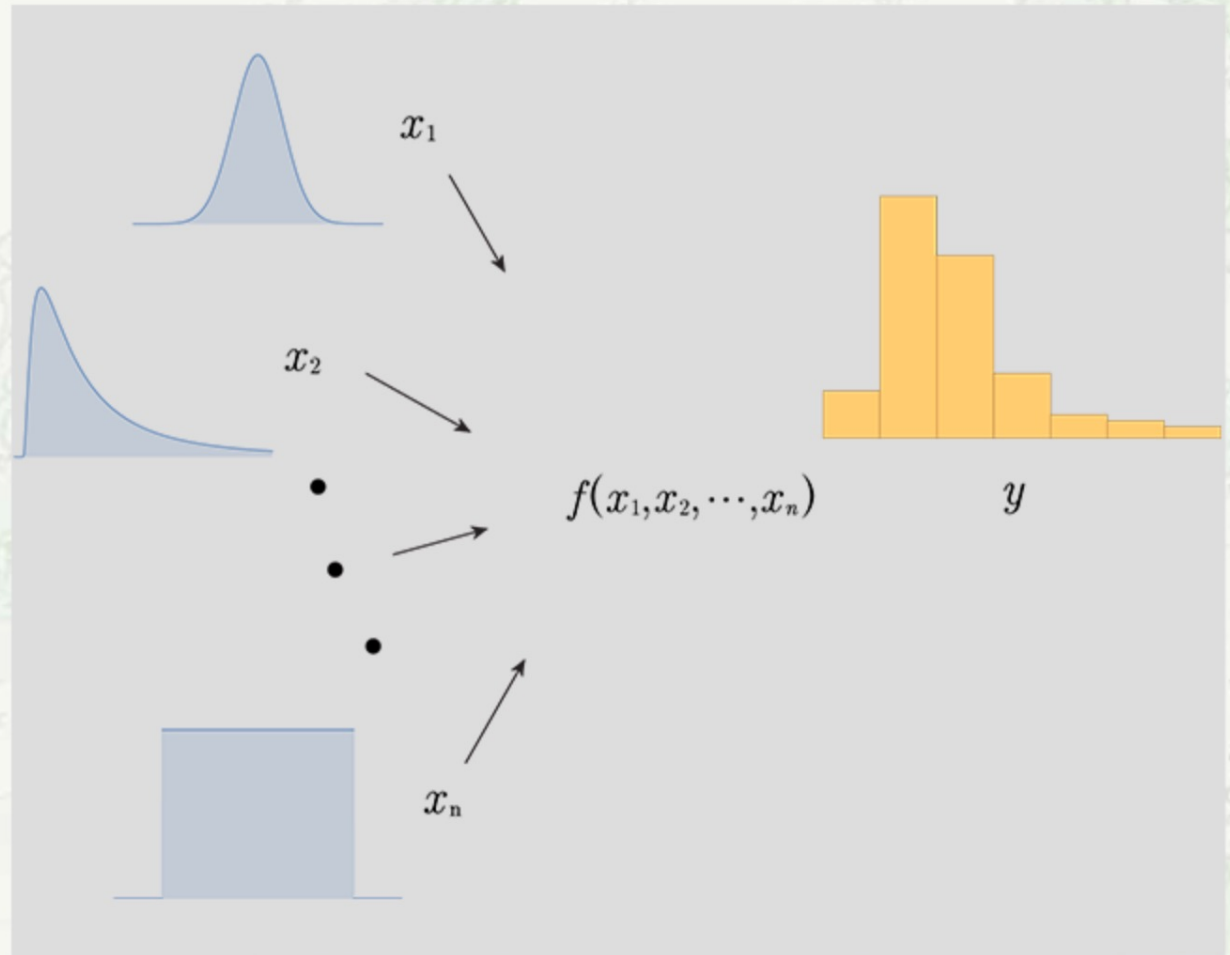
computer power makes this feasible

The Monte Carlo Method is  
a broad class of computational algorithms  
that rely on repeated random sampling  
to obtain numerical results.

# Concept



Really just making a lot of systems virtually





# Benefits



It's a simulation  
compare options  
identify critical elements

More accurate than  
worst case analysis  
root sum squared analysis

Relatively quick

It's fun, too

What are some

applications you see  
for this approach?

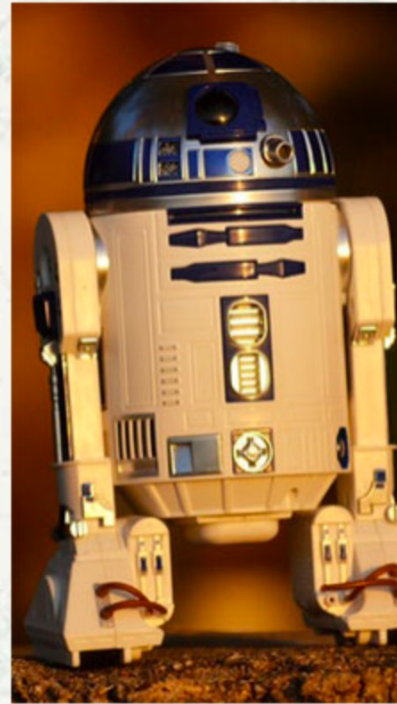
# Analysis Steps



Transfer  
Function



Collect  
Data



Estimate  
Runs



Analyze

# Select a Transfer Function



This is the formula that determines the tolerance stack up or product life estimate.

It's a formula when some or all of inputs have the potential to vary

What is being calculated?

# Collect Data



**Better data makes for a better analysis (most of the time)**

Like the weather data in the last webinar - find the

- best data
- or description
- or estimate

of how the variable varies

# Estimate Number of Runs



confidence precision etc

$$m = \left( \frac{Z_{\alpha/2} \times \sigma / \mu}{Er(\mu) / \mu} \right)^2$$

$m$  is the number of runs to hit the precision with stated confidence

$Er(\mu)$  is the standard error of the mean

# Do the Analysis



I always plot the data,  
always.

Histogram  
Confidence interval  
Best fit distribution

Calculate percentiles of interest

Life data analysis on time to failure  
results

Make decisions, ask better questions

# Questions?

what do you see is  
the hardest part?



# Tolerance Analysis Example

# Define the Problem

What is the appropriate tolerance for individual plates used in a stack of five to achieve a combined thickness of  $125 \pm 3$  mm?

Given the variation of the plate thicknesses, how many assemblies will have thicknesses outside the desired tolerance range?



# Define the Transfer Function

for a stack of plate we add the plate thicknesses.

# Collect the data

What is the variation of plate thickness?

let's say it is  $25 \pm 0.99$ mm

# Estimate number of runs

The standard deviation of the combined five plates is  $\sigma = 0.7379$

We want 95% confidence thus  $Z_{\alpha/2} = 1.645$

The mean plate thickness is 25mm

The desired precision  $Er(\mu)$  is 1%

We calculate we need 14,735 runs...



# Generate Random Samples

Virtually pull 5 plates from the distribution of plate sizes

One example might be:

25.540, 25.008, 24.565, 24.878, and  
24.248 mm.



# Evaluate the transfer function

Here we simply add the virtual plate thicknesses

In one example we find:

25.540, 25.008, 24.565, 24.878, and  
24.248 mm equals 124.238 mm.



Repeat for number of  
runs

create a virtual stack of five plates

add up their thicknesses

record





# Analyze the data

In this simple example we have normal distribution with a mean of 125mm and standard deviation of 0.7379

How does

tolerance relate to  
reliability?

Product  
Life  
Estimate  
Example

# Electro-migration

## Black's equation

$$TTF = A(J^{-n})e^{\frac{Ea}{kT}}$$

A is a constant based on cross-sectional area

J is the current density

Ea is the activation energy

k is Boltzmann constant

T is temperature

n is a scaling factor

A historical map of the world, likely from the 17th or 18th century, showing the continents of Asia, Europe, Africa, and America. The map is rendered in a light green and tan color scheme. Overlaid on the map is the text "many failure mechanisms" in a large, orange, serif font. The text is centered horizontally and spans across the top half of the image, partially covering the northern parts of the continents.

many failure mechanisms



Variation  
in use  
conditions

# Dealing with discreet data



Questions?

Want to give it a go?



# In Summary

Transfer function



Collect data



Generate Samples



Conduct Analysis



# More Information

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Let me know if you have any questions.

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