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6 Essential Reliability Engineering Formulas

Purpose

$$\int_0^L r^2 dm = \int_0^L r^2 dm$$

Limitations



we have

$$\frac{1}{x^2 + 4x + 5} = \frac{A}{x} + \frac{B}{x^2} + \frac{C(2x + 4) + D}{x^2 + 4x + 5};$$

$$R(x^2 + 4x + 5) + Cx^2(2x + 4) + Dx^2.$$

$$\begin{aligned} B &= -2, \\ A &= 2, \\ C &= -1, \\ D &= -1. \end{aligned}$$

$$\frac{1}{x^2 + 4x + 5} = \frac{2}{x} - \frac{2}{x^2} - \frac{2}{x+2} + \frac{1}{x+5} - \tan^{-1}(x+2) + C$$

Integrate the following integrals

Ans. $\frac{1}{2} \ln \frac{x-1}{x+1}$

$$\int \frac{x^2 + x + 2}{x^2 - 1} dx.$$

$$\int \frac{y^2 - y + 1}{y^2 - y^2} dy$$

Assumptions

+50% $\sqrt{2}$



What is your

favorite formula?

Bayes Theorem

$$= \frac{2\pi a c \epsilon_0}{k a} \left(\frac{k a}{2} \right) \left[1 - \frac{1}{2} \left(\frac{k a}{2} \right)^2 \right]$$

$$\approx \hat{z} \frac{I_0}{\pi a c \epsilon_0} \frac{\sin(\omega t)}{k a} \left[1 - \left(\frac{k a}{2} \right)^2 \right]$$

and

$$\vec{B} \approx \hat{\phi} \frac{I_0}{2\pi a c^2 \epsilon_0} \sin \omega t \left[1 - \frac{1}{2} \left(\frac{k a}{2} \right)^2 \right]$$

$$\approx \hat{\phi} \frac{I_0}{2\pi a c^2 \epsilon_0} \sin \omega t \left[1 - \frac{1}{2} \left(\frac{k a}{2} \right)^2 \right]$$

b)

$$W_e = \frac{1}{2} \int \vec{E} \cdot \vec{D} dV = \frac{1}{2} \epsilon_0 \int_0^{2\pi} d\phi \int_0^d dz$$

Using

$$\langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$W_e = \frac{d I_0^2}{2\pi \epsilon_0 c^2 a^2} \frac{1}{k^2 a^2} = \frac{d}{4\pi \epsilon_0 c^2 a^2}$$



Bayes Theorem

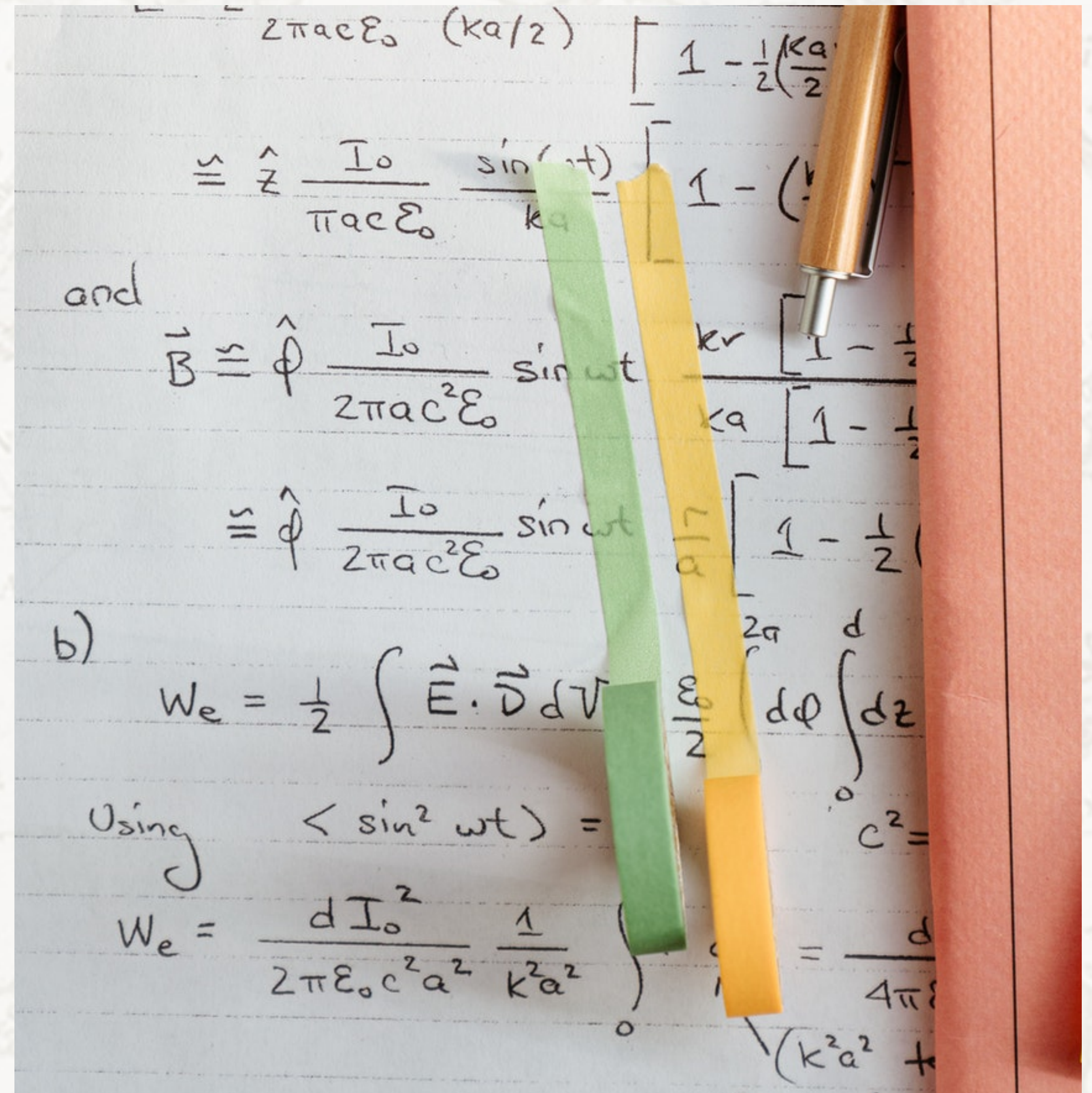
$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Purpose

To describe the probability of an event, based on prior knowledge of conditions that might be related to the event.

To account for or incorporate what you know as it influences the calculation of the probability given that information.

To check for statistical independence.



Handwritten physics equations on lined paper, featuring a pen and two sticky notes (green and yellow).

Equations shown:

$$2\pi a c \epsilon_0 (ka/2) \left[1 - \frac{1}{2} \left(\frac{ka}{2} \right)^2 \right]$$

$$\approx \hat{z} \frac{I_0}{\pi a c \epsilon_0} \frac{\sin(kt)}{ka} \left[1 - \left(\frac{ka}{2} \right)^2 \right]$$

and

$$\vec{B} \approx \hat{\phi} \frac{I_0}{2\pi a c^2 \epsilon_0} \sin \omega t \frac{kr}{ka} \left[1 - \frac{1}{2} \left(\frac{ka}{2} \right)^2 \right]$$

$$\approx \hat{\phi} \frac{I_0}{2\pi a c^2 \epsilon_0} \sin \omega t \frac{a}{r} \left[1 - \frac{1}{2} \left(\frac{ka}{2} \right)^2 \right]$$

b)

$$W_e = \frac{1}{2} \int \vec{E} \cdot \vec{D} dV = \frac{\epsilon_0}{2} \int d\phi \int dz \int dr$$

Using $\langle \sin^2 \omega t \rangle = \frac{1}{2}$

$$W_e = \frac{d I_0^2}{2\pi \epsilon_0 c^2 a^2} \frac{1}{k^2 a^2} \left(\frac{1}{2} \right) = \frac{d}{4\pi \epsilon_0 c^2 a^2} \left(\frac{1}{k^2 a^2} + \dots \right)$$

Bayes Theorem

$$P(A | B) = \frac{P(B | A) \cdot P(A)}{P(B)}$$

Limitations

Knowledge of probability of individual and conditional probabilities (at least three)

Difference between causation and coincidence

Getting accurate data

Counting all that should be counted



Bayes Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Assumptions

Probability of A and B do not equal zero

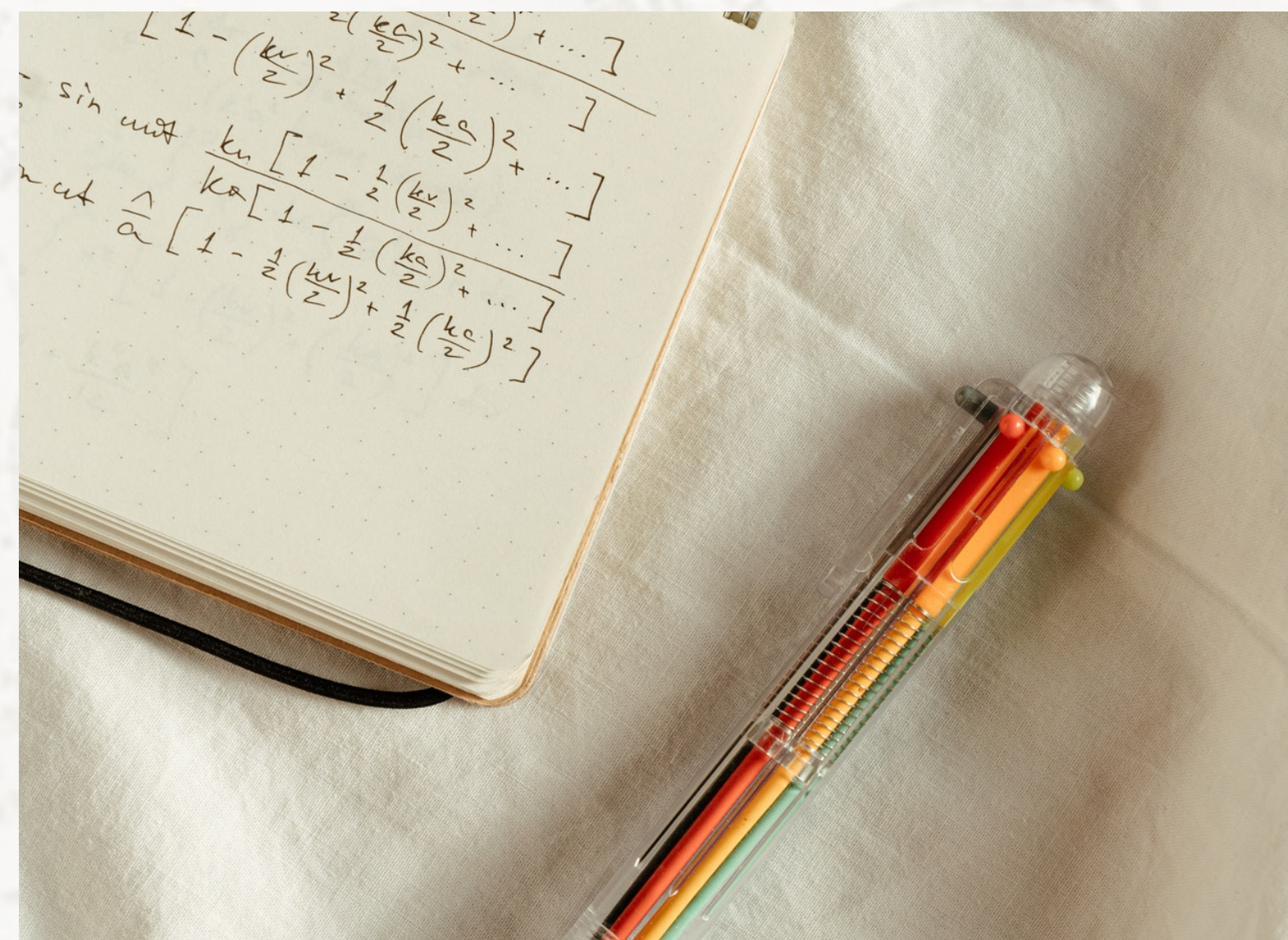
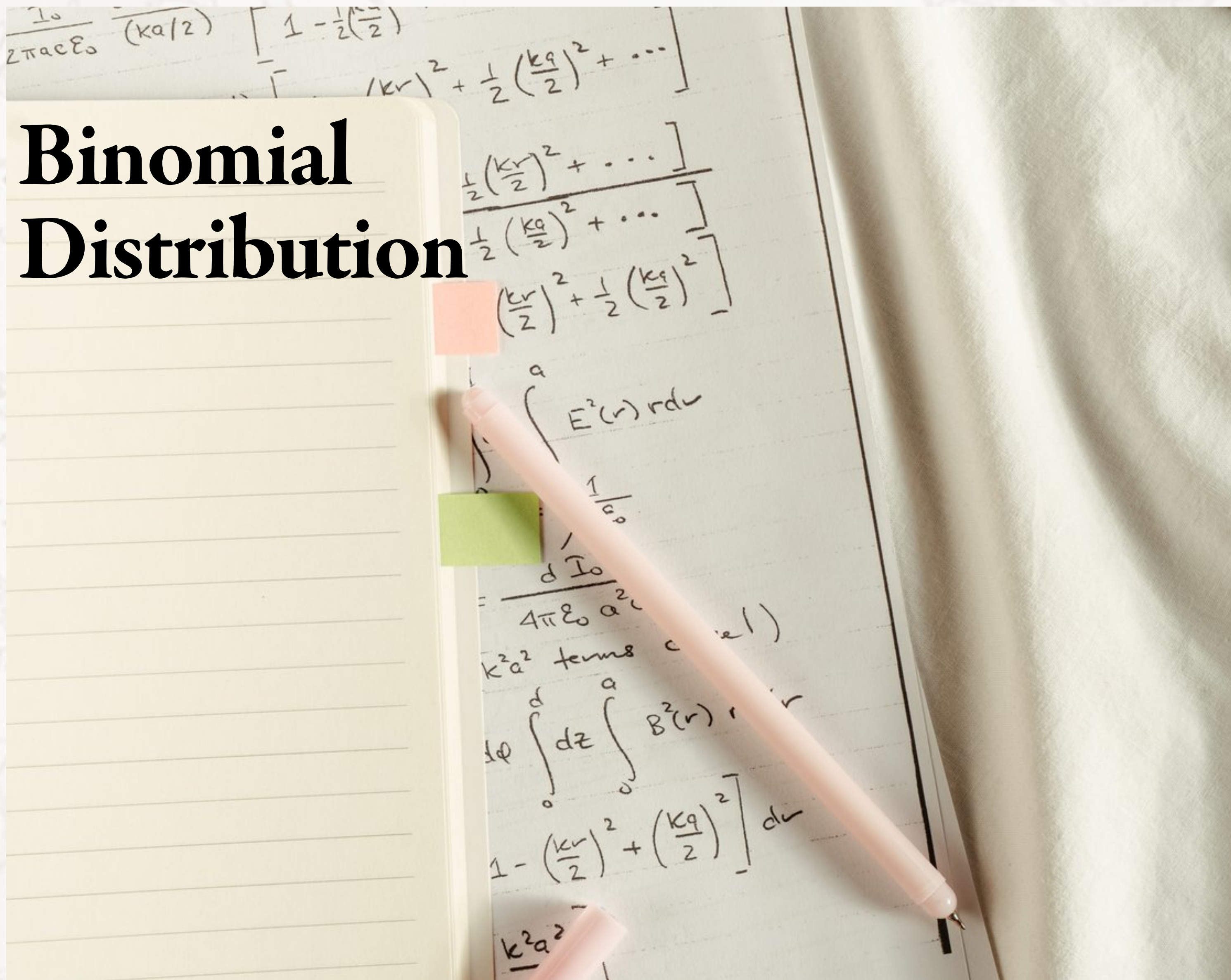
We're ignoring the probability of C which may have a role with the probabilities of A, B, A|B, and B|A



What formula deals

with pass/fail data?

Binomial Distribution



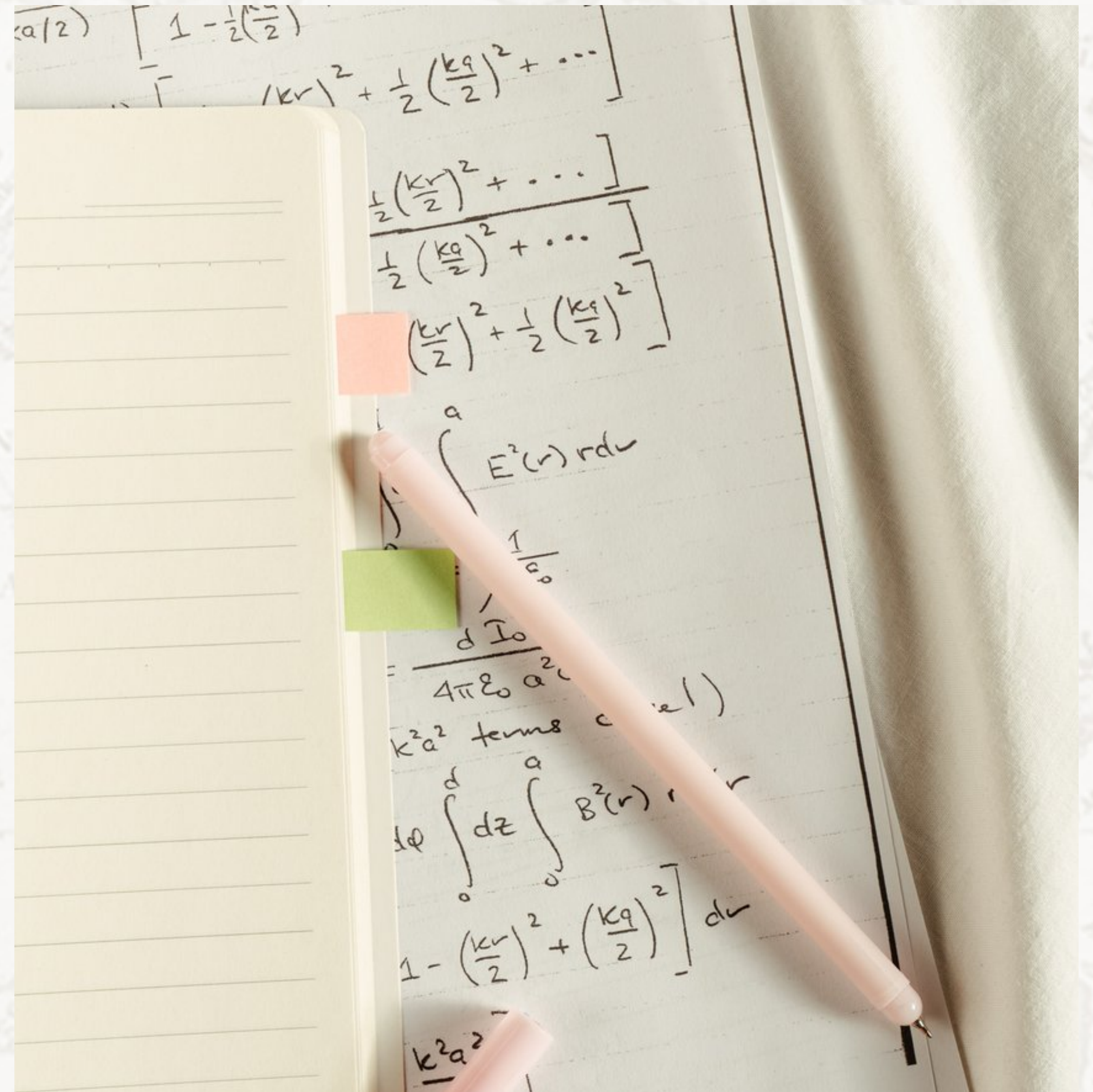
Binomial Distribution

$$f(k, n, p) = \binom{n}{k} p (1-p)^{n-k}$$

Purpose

The probability of getting exactly k successes in n independent Bernoulli trials

Dealing with pass/fail data



Binomial Distribution

$$f(k, n, p) = \binom{n}{k} p (1 - p)^{n-k}$$

Limitations

Independent trails

Discreet data

Less informative then using
continuous data



Binomial Distribution

$$f(k, n, p) = \binom{n}{k} p (1 - p)^{n-k}$$

Assumptions

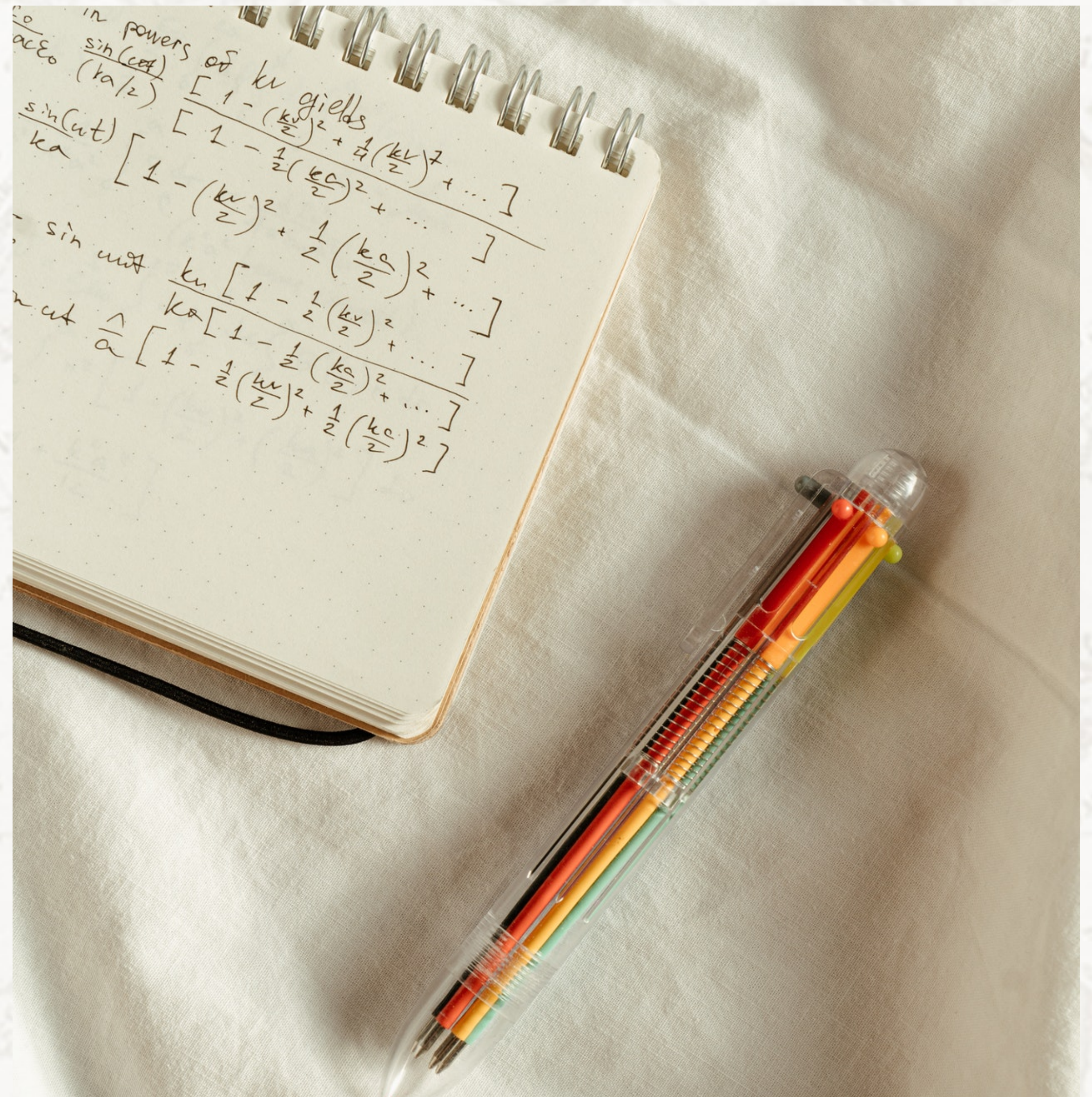
Independent Bernoulli trials

Probability of success remains constant over entire experiment

Random sampling

Test to pass strategy is valid

Measurement error is small



How to do you

make comparisons?

Hypothesis Testing

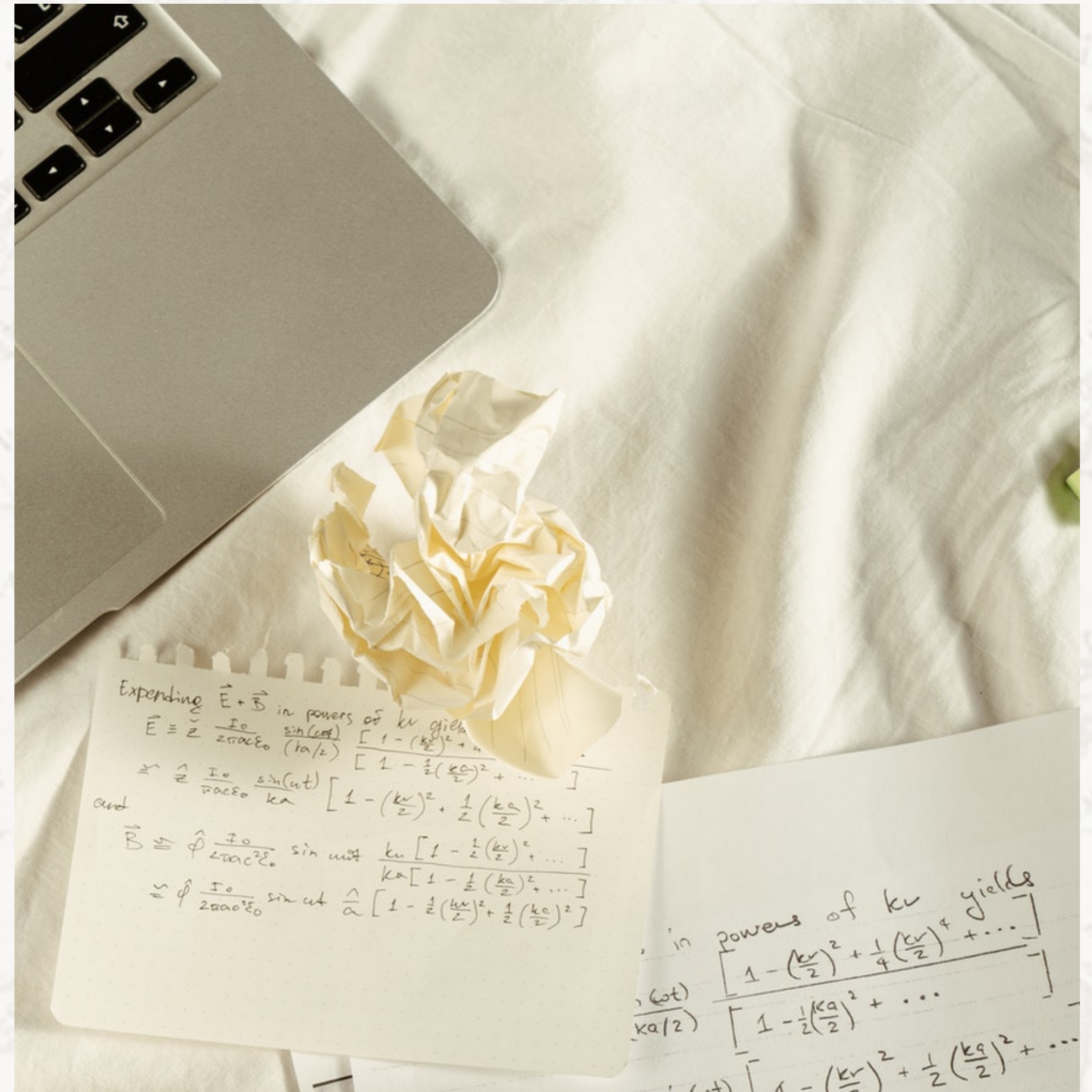


Hypothesis Testing

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{Purpose}$$

To test if a population via a sample mean is similar to a fix value

To test if two populations via samples have similar means.



Hypothesis Testing

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{Limitations}$$

Not useful for comparing more than two populations

Random sampling

Correct establishment of null and alternative hypothesis statements

Selecting the correct hypotheses test for the given situation and data



Hypothesis Testing

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{Assumptions}$$

The sample mean follows a normal distribution

The variances come from independent and identical normal distributions.

Z and s are independent



How to do you

measure capability?

Process Capability

Check ladies' c... at the... over... flock, funderheath... bucket o...

Charge on small shell to get

to cancel out $-2Q$ charge from outside of R_1 inside of R_2 must be $+2Q$

To get total of $+Q$ on R_2 outside of R_2 must be $-Q$

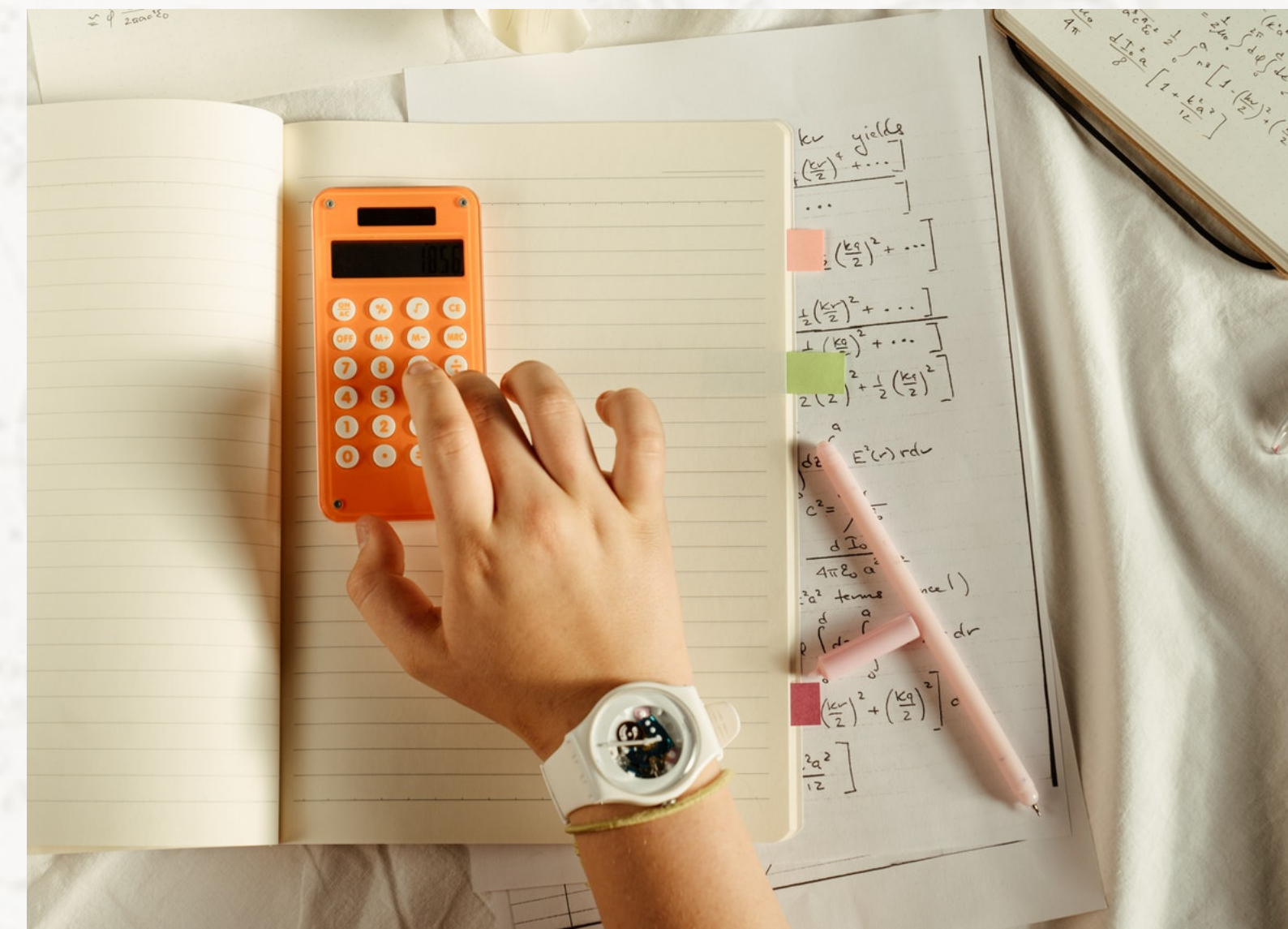
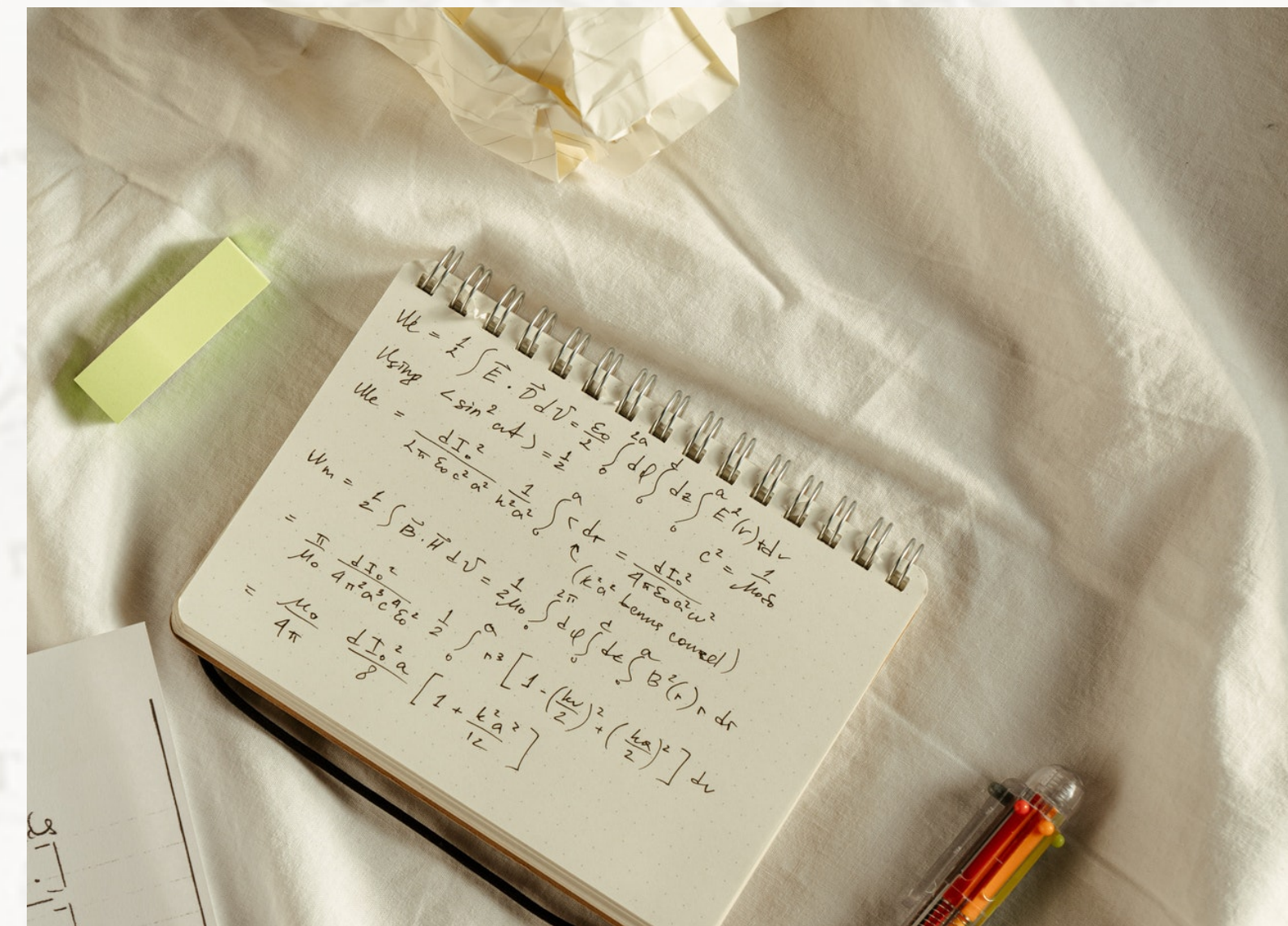
as inside of R_3 must cancel out $-Q$ so it is $+Q$

f $R_3 = +Q$

Expanding $\vec{E} + \vec{B}$ in powers of

$$\vec{E} \approx \hat{z} \frac{I_0}{2\pi a \epsilon_0} \frac{\sin(\omega t)}{(ka/2)} \left[1 - \left(\frac{kr}{2}\right)^2 \right]$$
$$\approx \hat{z} \frac{I_0}{\pi a \epsilon_0} \frac{\sin(\omega t)}{ka} \left[1 - \left(\frac{kr}{2}\right)^2 \right]$$

START
15 MONTHS



Process Capability

$$C_{pk} = \min \left(\frac{USL - \bar{x}}{3\hat{\sigma}}, \frac{\bar{x} - LSL}{3\hat{\sigma}} \right) \quad \text{Purpose}$$

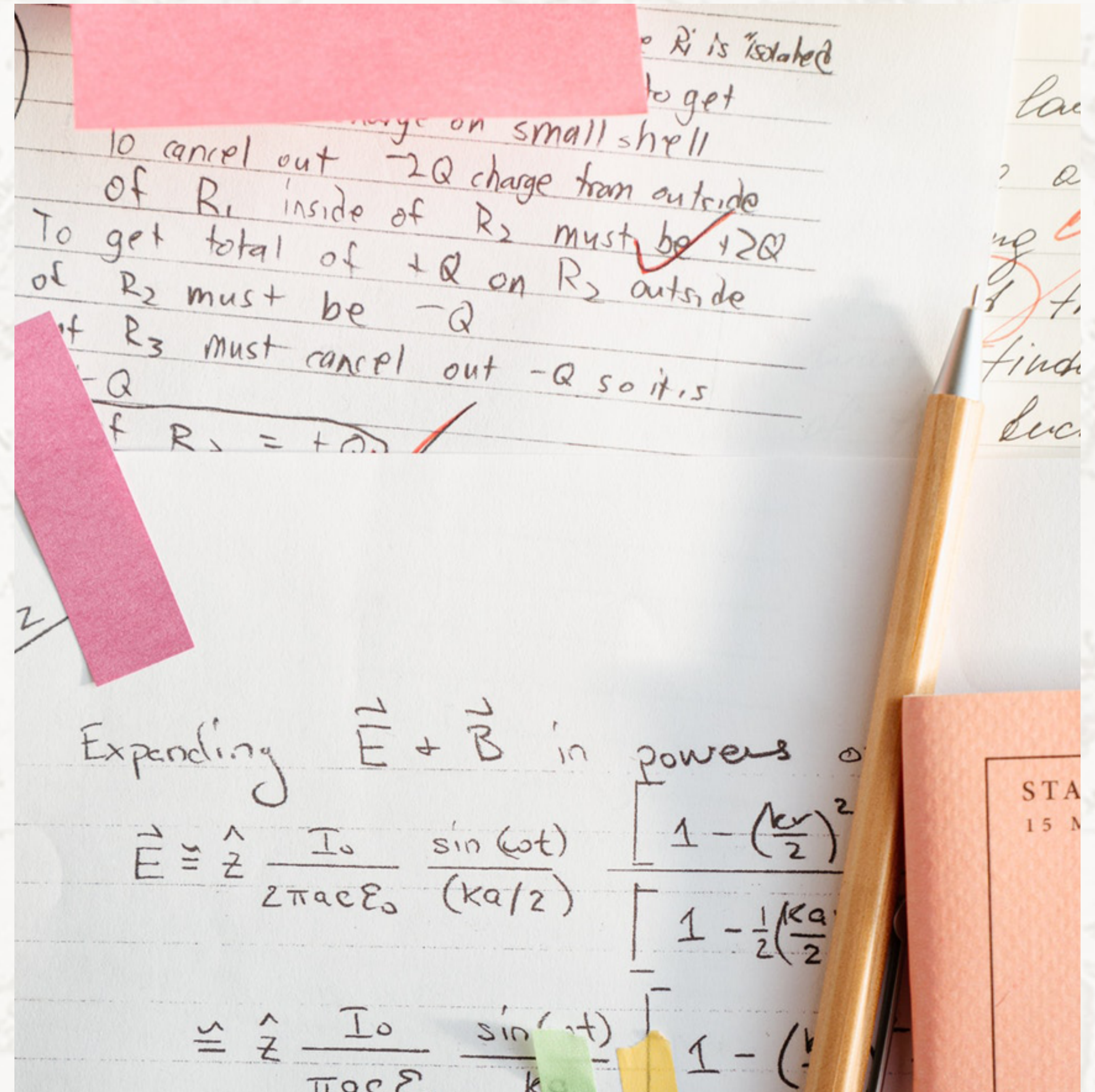
To compare an item's variability to a desired specification (tolerance)

Useful when:

evaluating a process

reviewing tolerances

estimating process yield



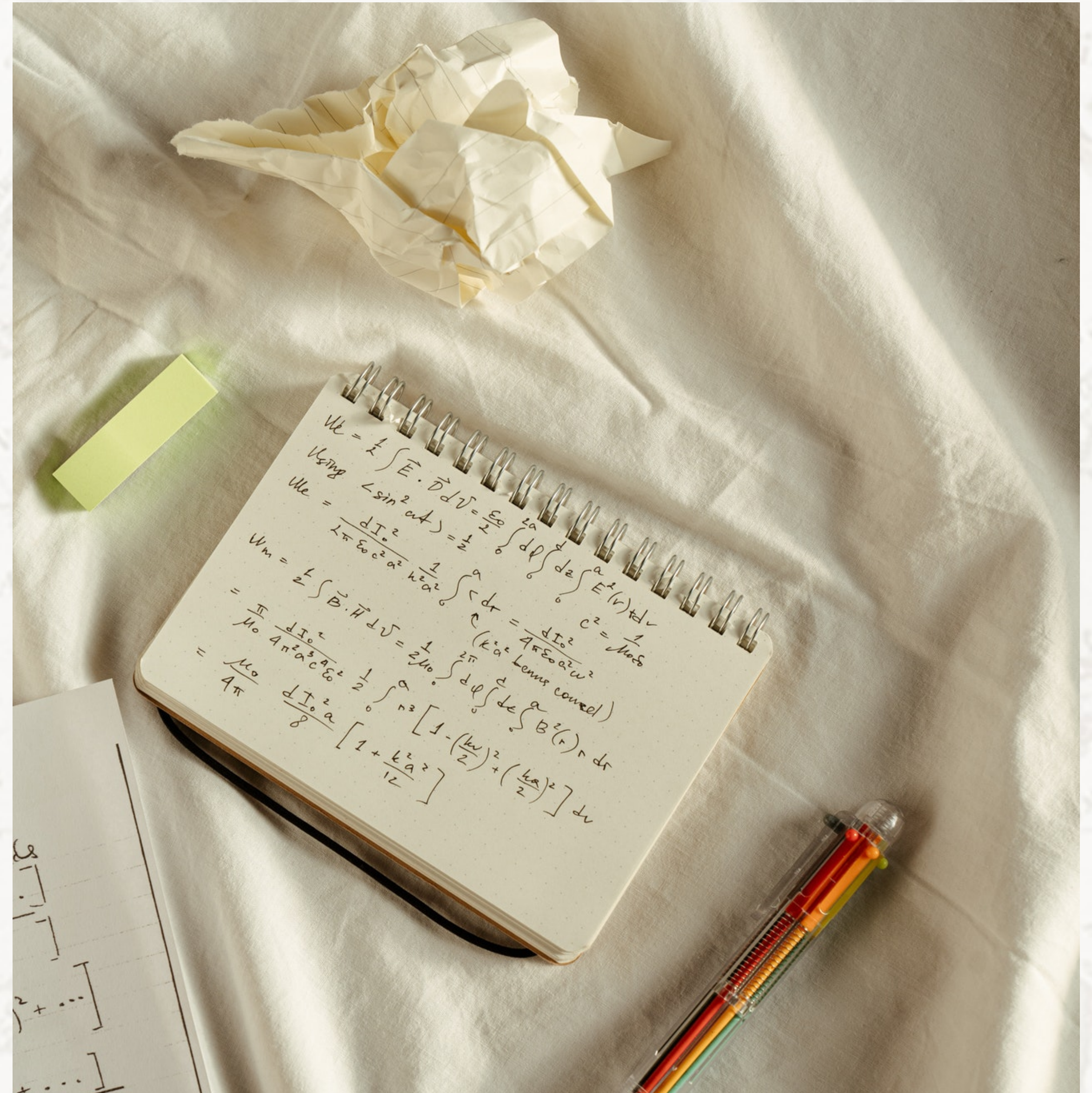
Process Capability

$$C_{pk} = \min \left(\frac{USL - \bar{x}}{3\hat{\sigma}}, \frac{\bar{x} - LSL}{3\hat{\sigma}} \right) \quad \text{Limitations}$$

The process must first be stable
else the capability index is
inaccurate

The process may shift or change
thus invalidating the index value

Need to use the appropriate index
for the data and specifications



Process Capability

$$C_{pk} = \min \left(\frac{USL - \bar{x}}{3\hat{\sigma}}, \frac{\bar{x} - LSL}{3\hat{\sigma}} \right) \quad \text{Assumptions}$$

The process is stable

The process remains stable

The measurement system is stable

Fixed specifications



How to do you model

time-to-failure data?

Weibull Distribution

$$\left[\frac{1 - (kx)^2}{2} + \frac{1 - (kx)^2}{2} + \dots \right]$$



Weibull Distribution

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}$$

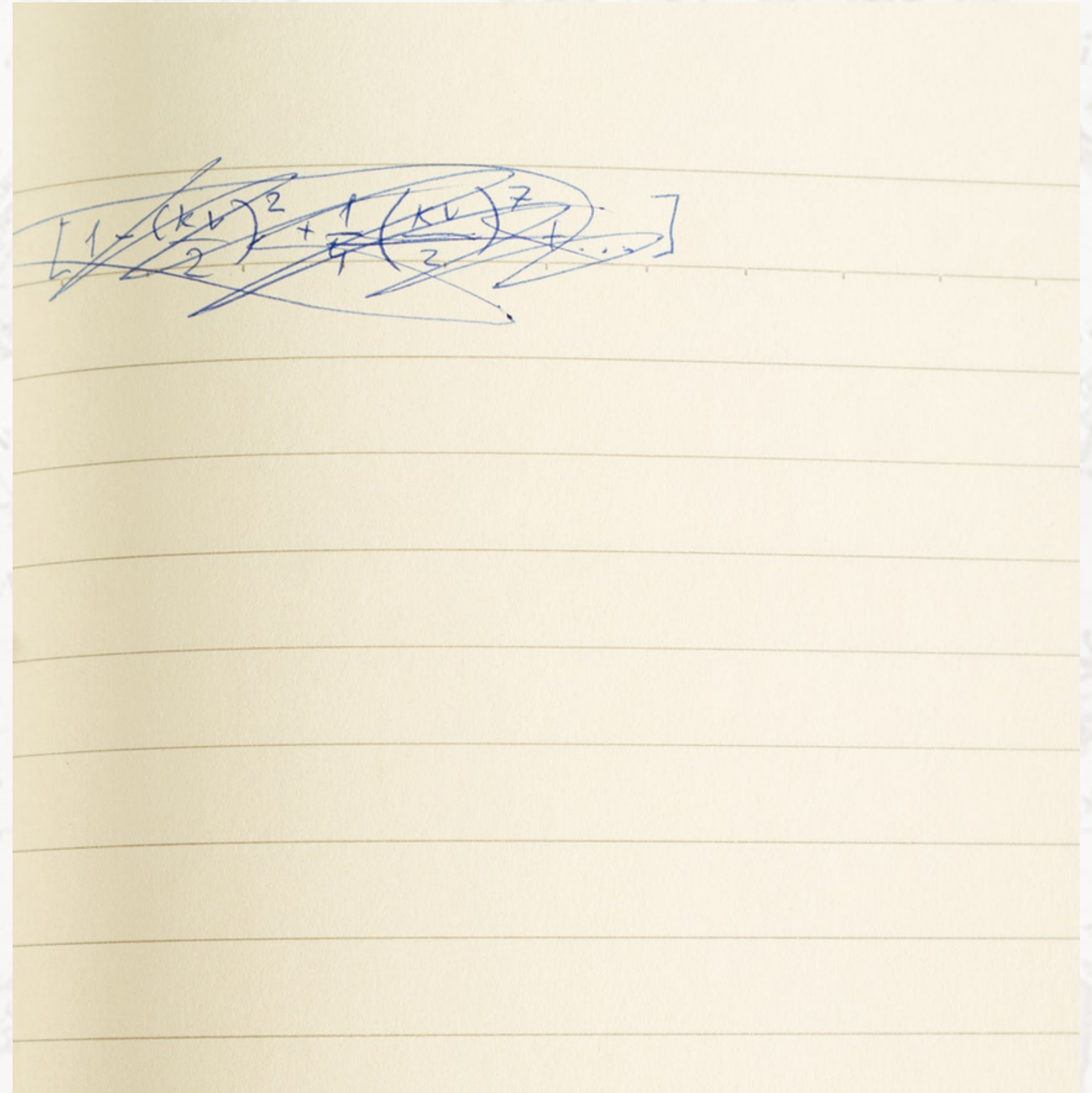
Purpose

To describe the cumulative probability of failure from time zero till time t.

To describe time to failure data

To detect changes in rate of failure

To estimate future failure behavior



Weibull Distribution

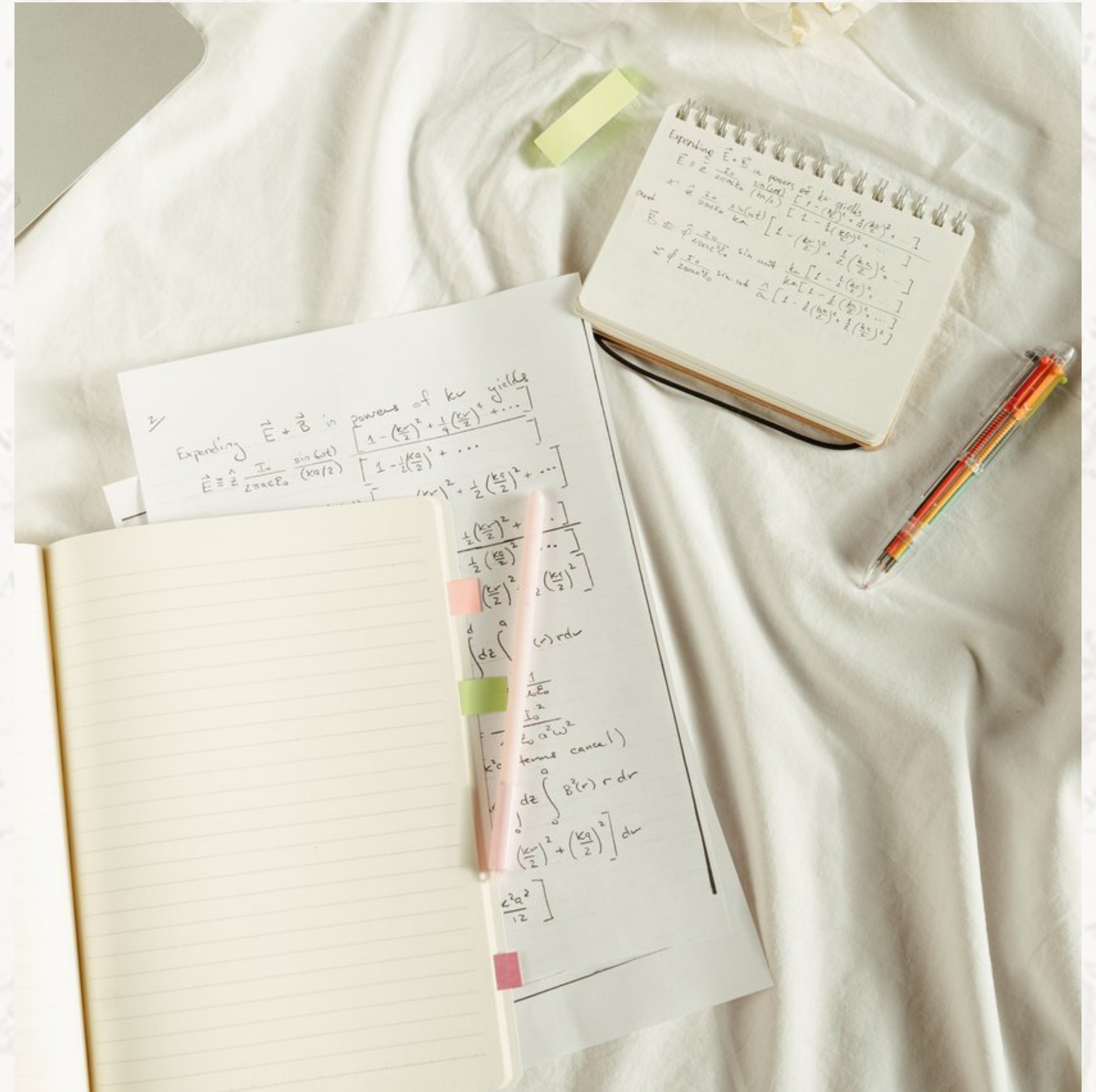
$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}$$

Limitations

It's just a model based on regression

No useful for repaired systems

Selecting the right distribution affects ability to forecast



Weibull Distribution

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}$$

Assumptions

The data is complete and censored data is accurate

The time to failure for individual items occurs independently of other items



How to do you model

**real world
relationships?**

Arrhenius equation

Home work

$$\frac{\left[1 - \left(\frac{kV}{z}\right)^2 + \frac{1}{4} \left(\frac{kV}{z}\right)^4 + \dots\right]}{\left[1 - \frac{1}{2} \left(\frac{kC}{z}\right)^2 + \dots\right]}$$



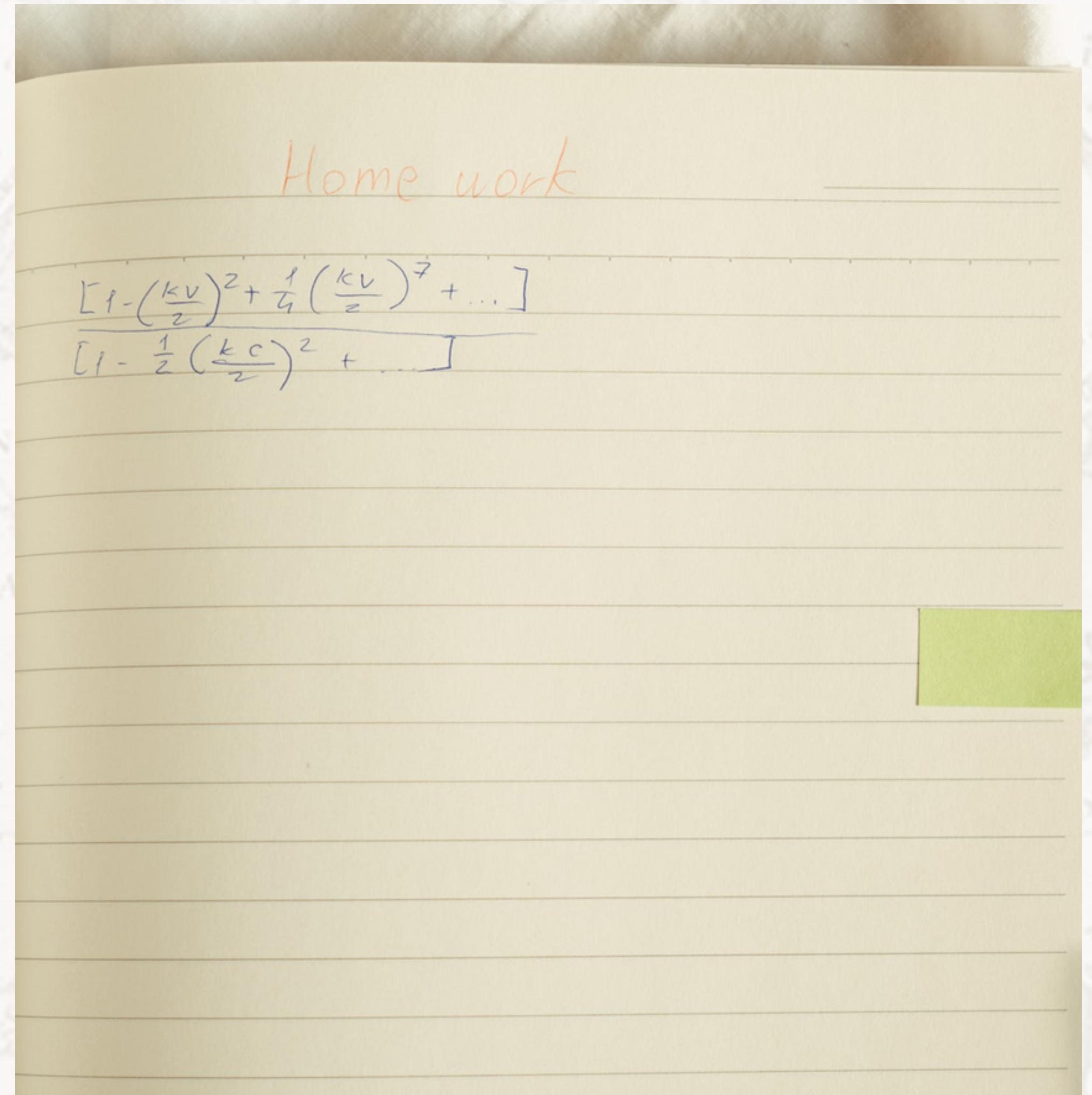
Arrhenius equation

$$k = Ae^{\frac{-e_a}{k_B T}}$$

Purpose

To describe the temperature dependence of reaction rates

As part of an acceleration factor equation to translate time to failure information from test to use conditions



Arrhenius equation

$$k = Ae^{\frac{-e_a}{k_B T}} \quad \text{Limitations}$$

The activation energy is often unknown

Not valid across material phase changes

Not appropriate for all thermal related failure mechanism relationships



Arrhenius equation

$$k = Ae^{\frac{-e_a}{k_B T}} \quad \text{Assumptions}$$

The activation energy (empirically determined) adequately describes threshold energies within chemical processes

There is a linear relationship between temperature and reaction rate

Temperature is dominant factor for failure causing mechanisms



Check Assumptions

something to remember



What are your questions?

Remember to visit accendoreliability.com