

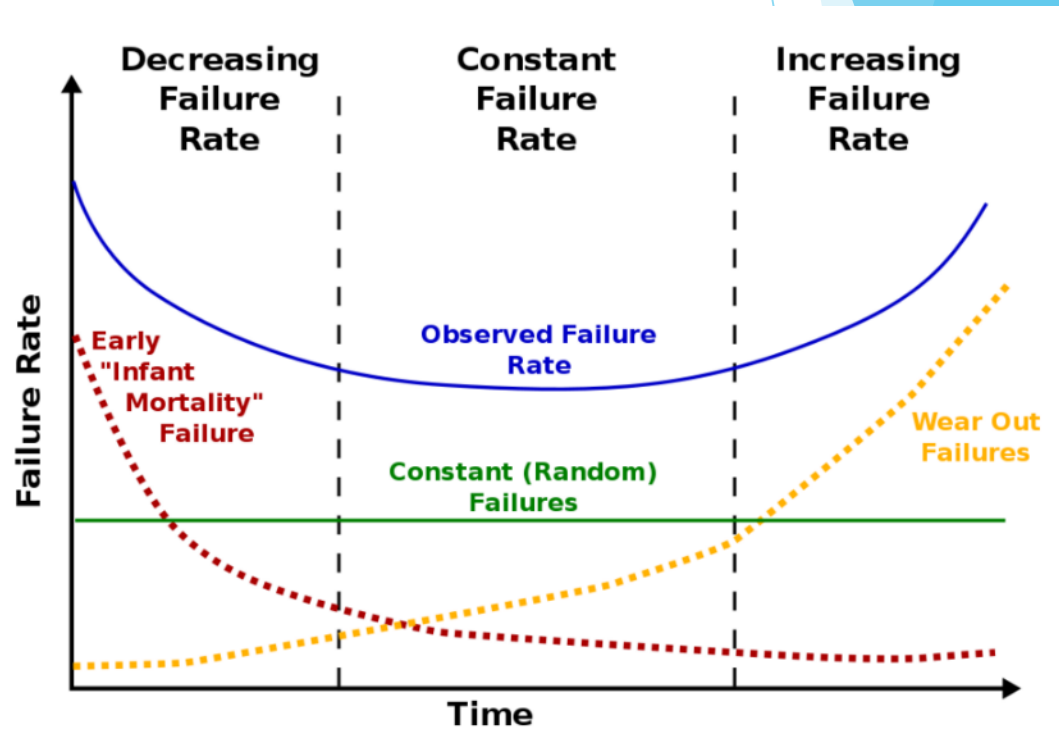
Overview of Reliability Functions for Life Testing in Minitab

Agenda

- ▶ Intro to Reliability
- ▶ Selecting a distribution
- ▶ Testing to failure - Right censoring
- ▶ Testing to failure - Arbitrary censoring
- ▶ Accelerated testing - Single Factor
- ▶ Accelerated testing - Multiple factors
- ▶ Summary
- ▶ Questions

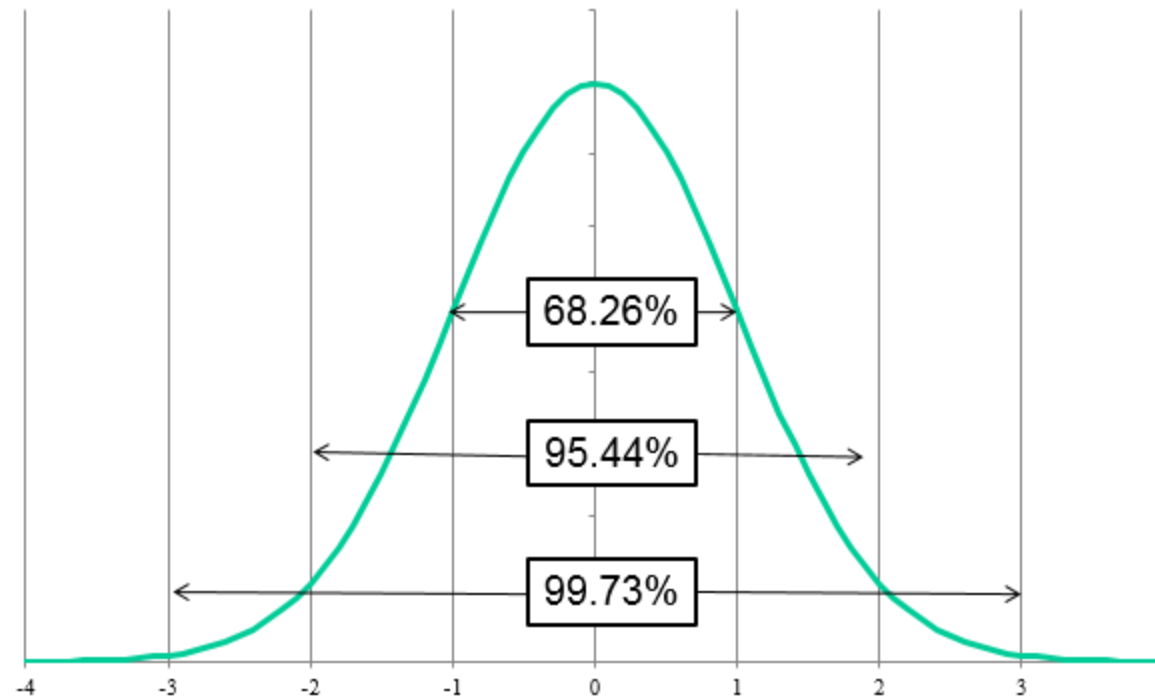
Quick Intro to Reliability

- ▶ Reliability = quality over time
 - ▶ Field failures
 - ▶ Failure modes
 - ▶ Testing to replicate failure modes
 - ▶ Measuring time to failure
 - ▶ Attempt to predict failure rate



Probability Distribution Function (PDF)

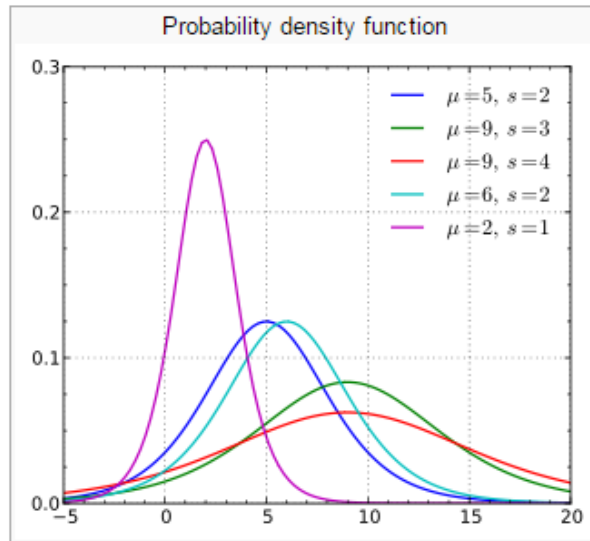
- ▶ A function that describes the relative likelihood for this random variable to take on a given value
- ▶ Integral over a range = probability of the variable falling within that range



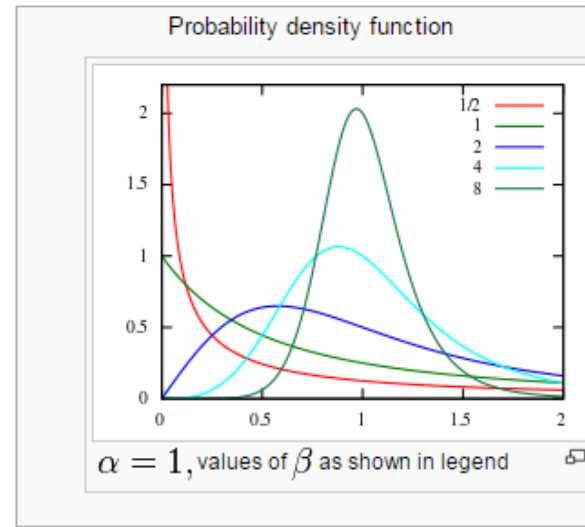
Many Distributions

Quick Intro to Reliability

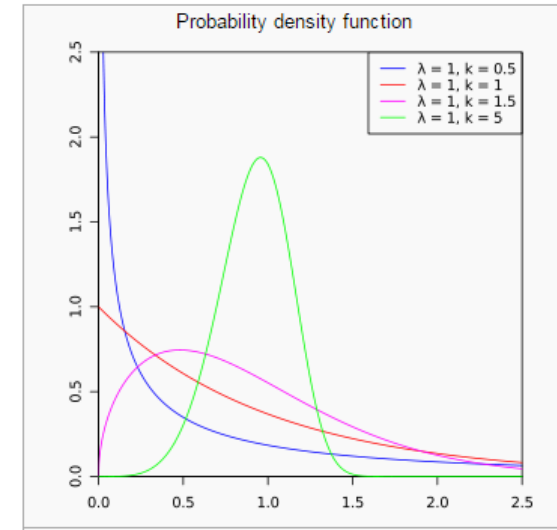
Logistic



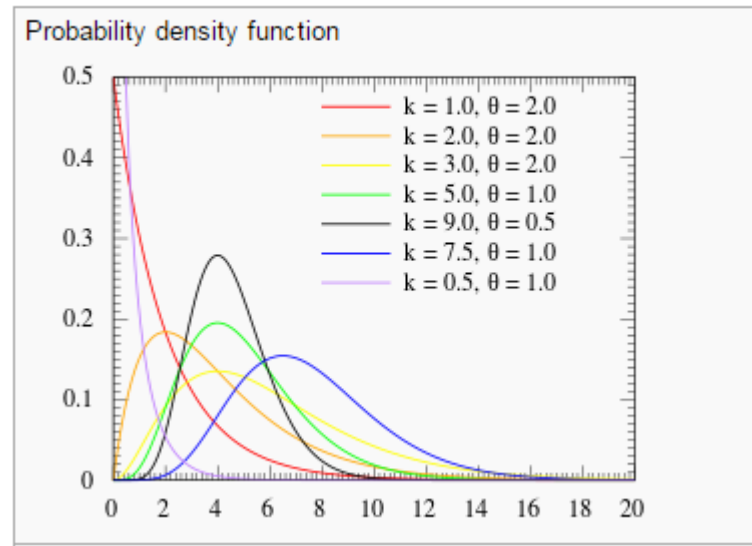
Log-logistic



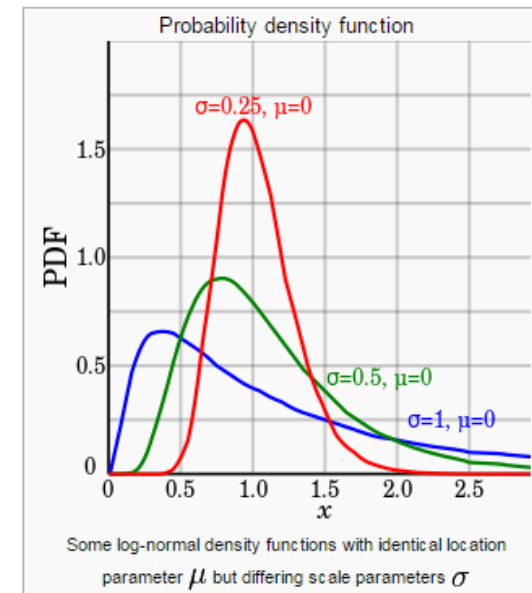
Weibull (2-Parameter)



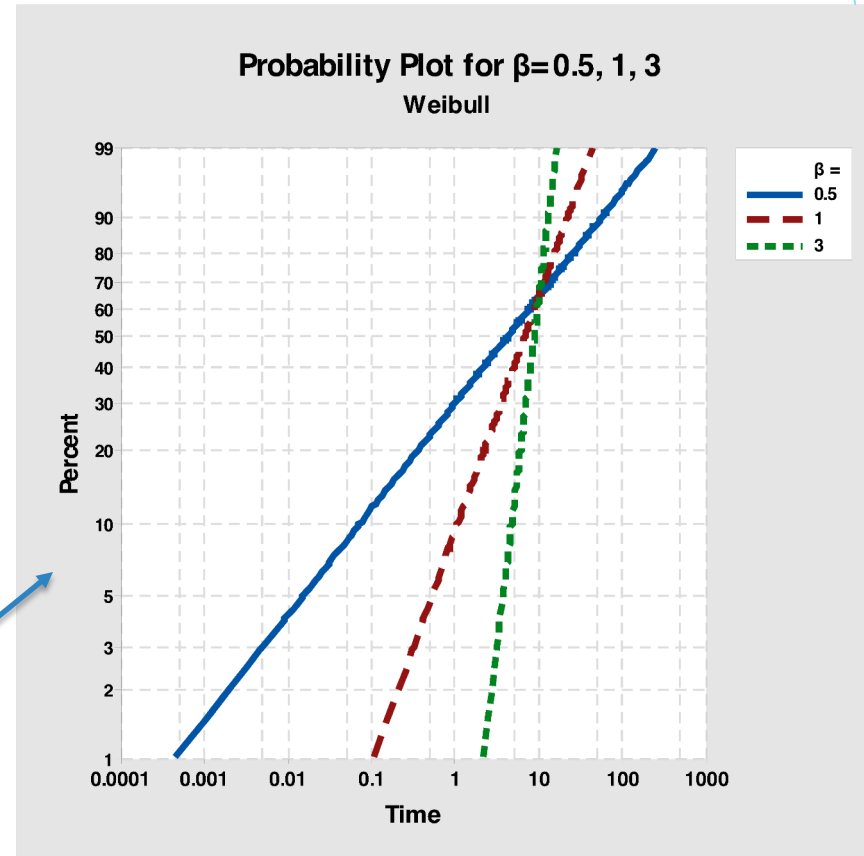
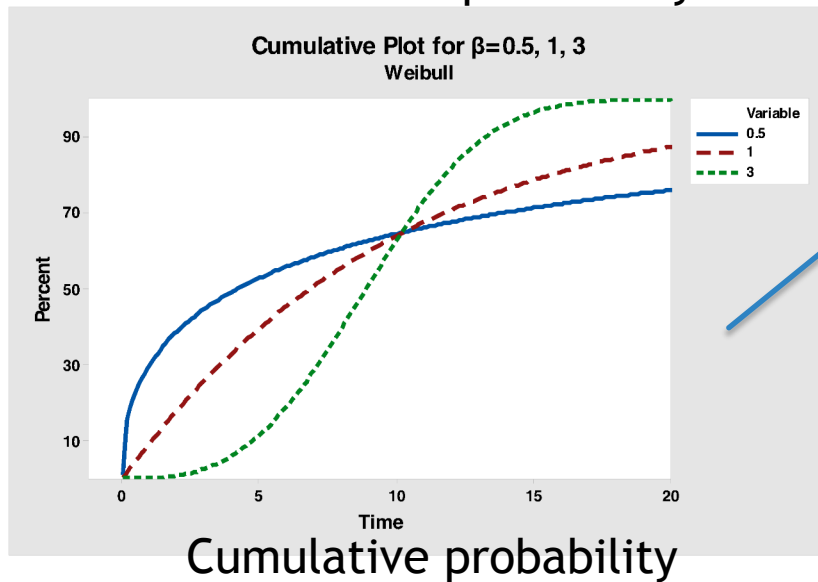
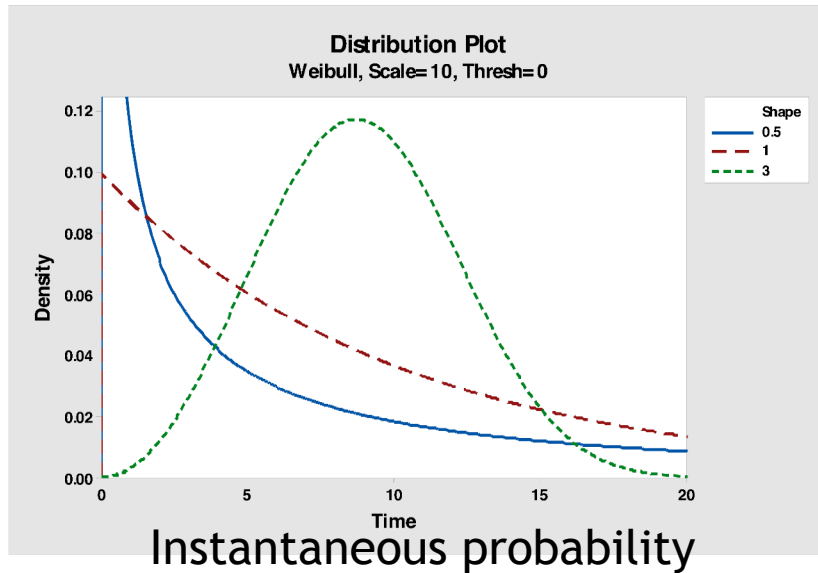
Gamma



Log-normal



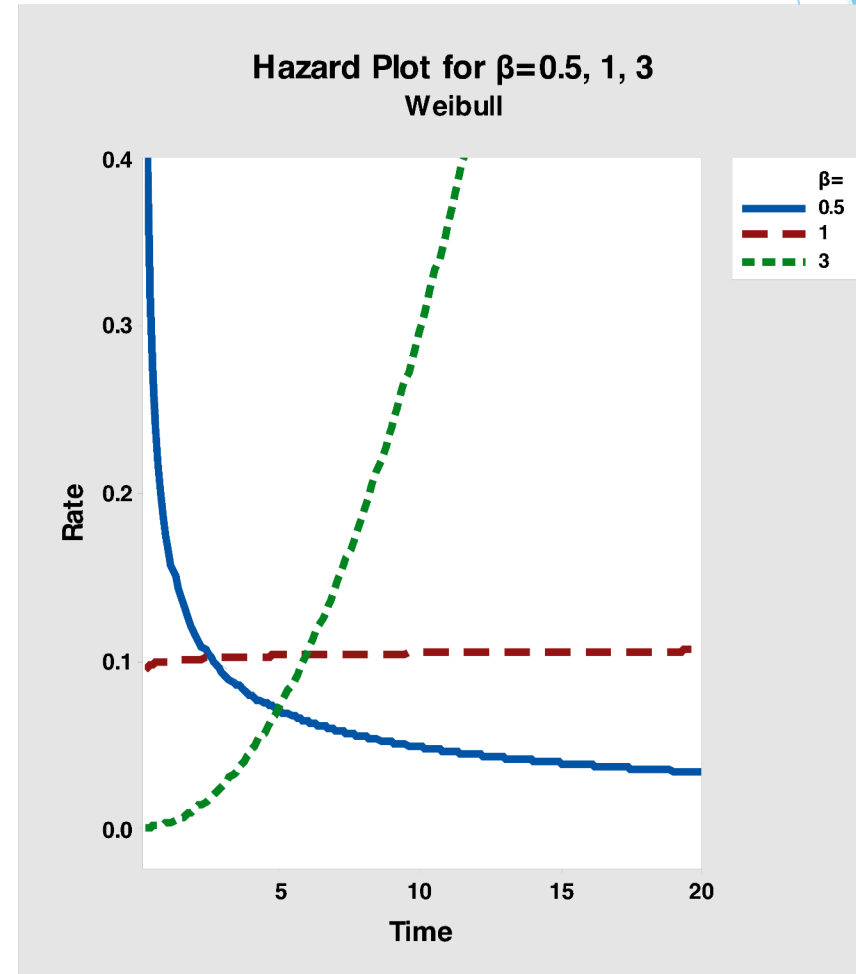
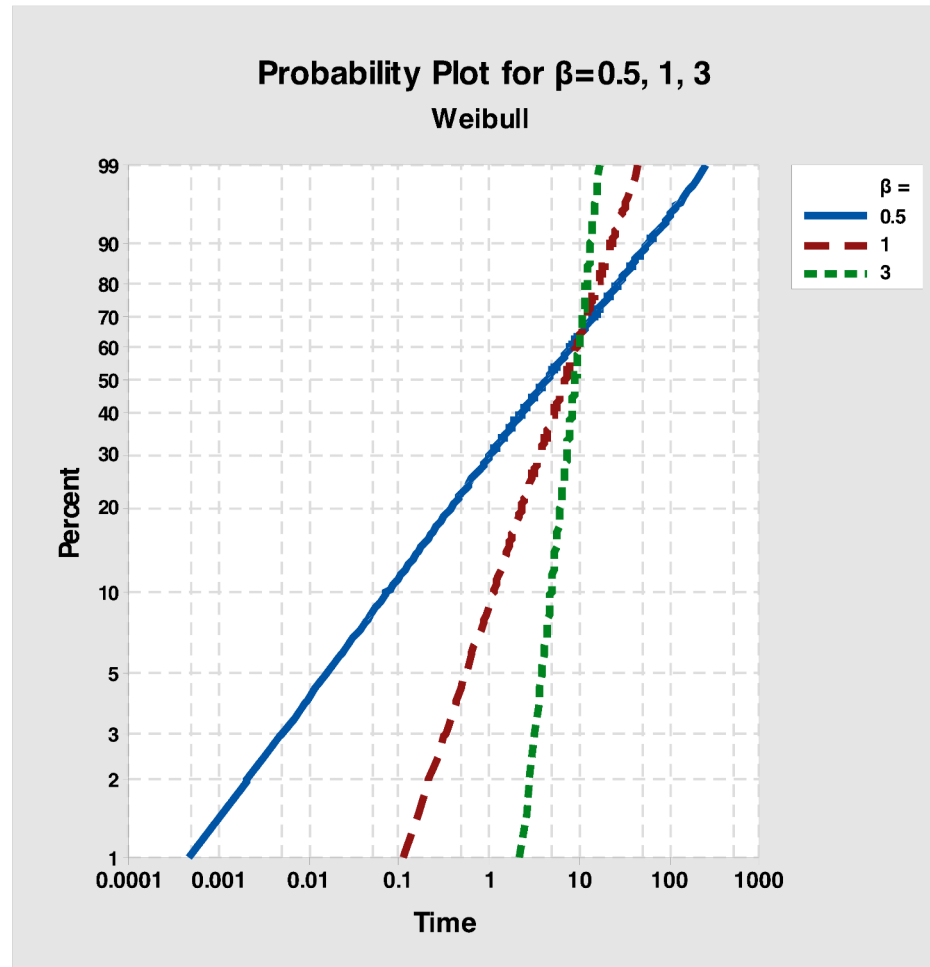
Different ways to visualize the data



Straight line cumulative probability
transformed so that the fitted distribution
line forms a straight line

Parameters

Shape = Slope: slope of the line in a cumulative straight line probability plot



Parameters

Scale = Shifts the data “out”

In a failure plot that means more time passes before each failure



Parameters

Threshold / Location:

The 3rd parameter - shifts the zero point

Forced to zero in a:

2-Parameter Lognormal

Normally called just “Lognormal”

1-Parameter Exponential

Normally called just “Exponential”

2-Parameter Weibull

Normally called just “Weibull”

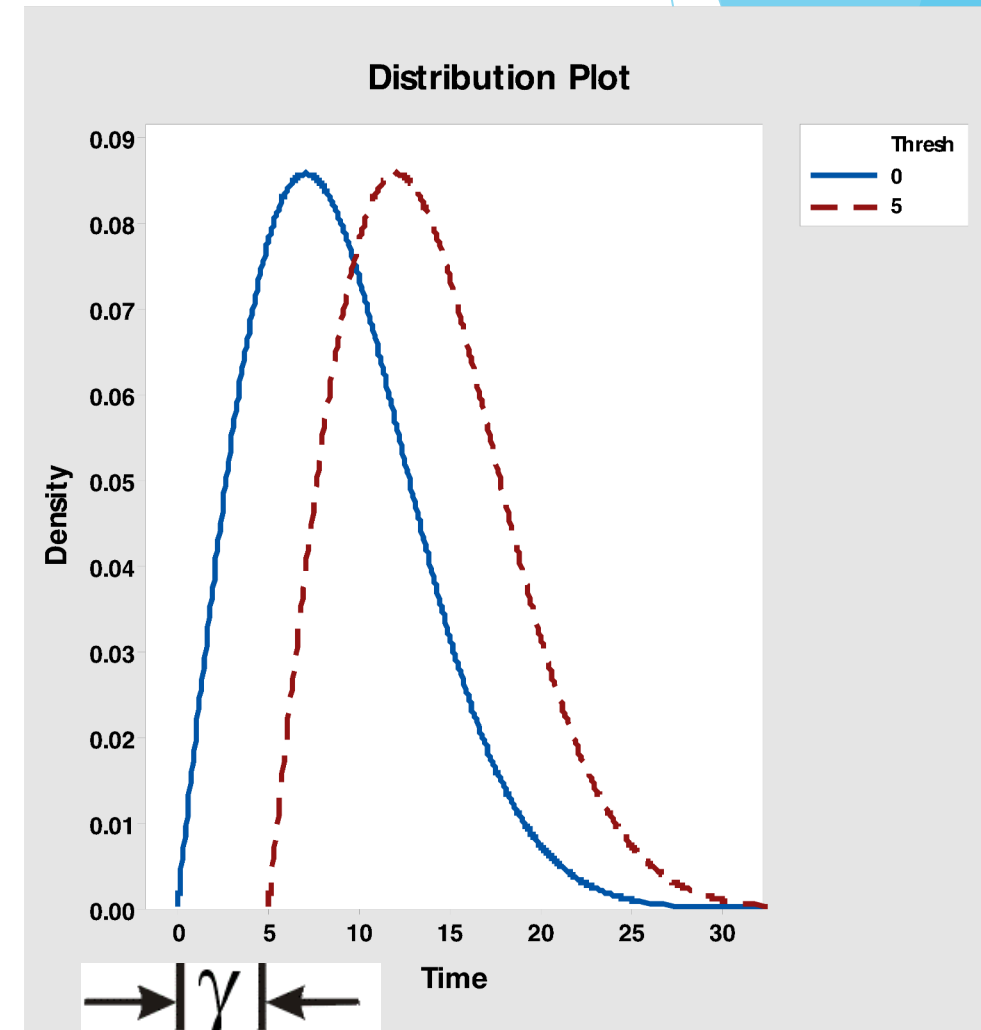
2-Parameter Gamma

Normally called just “gamma”

2-Parameter Loglogistic

Normally called just “Loglogistic”

(not represented on straight line cumulative probability plot)



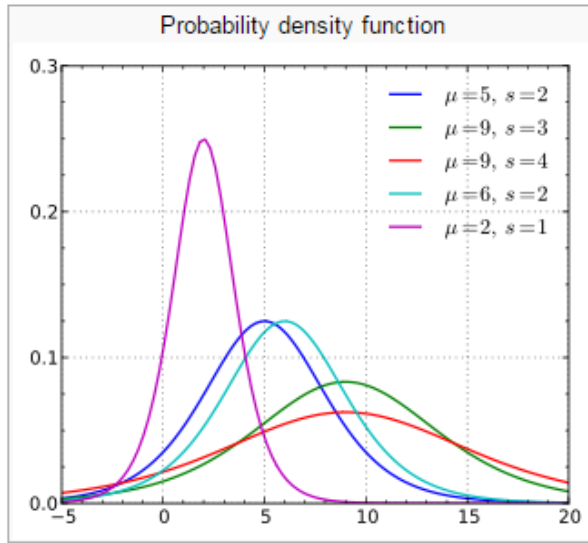
Testing to Failure: What is censoring?

- ▶ Some tests will not be able to run to the end due to time constraints
 - = Right censored data or Suspended data
- ▶ Some tests cannot be continuously monitored
 - = Arbitrary censoring

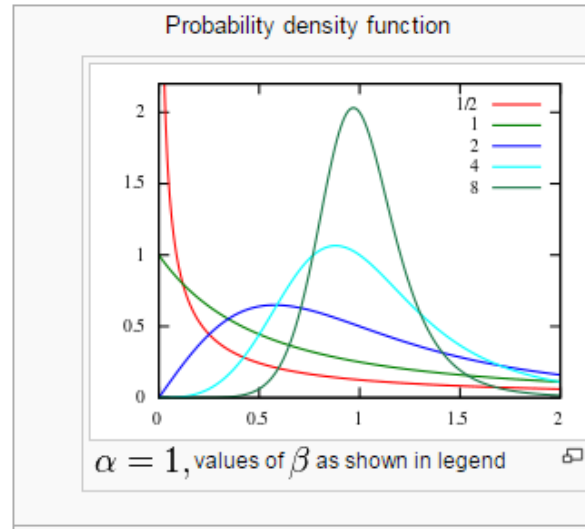
Distribution Identification

Many Distributions - and more: which one?

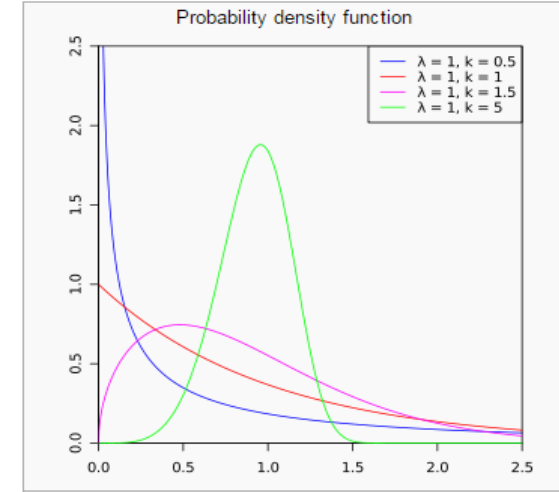
Logistic



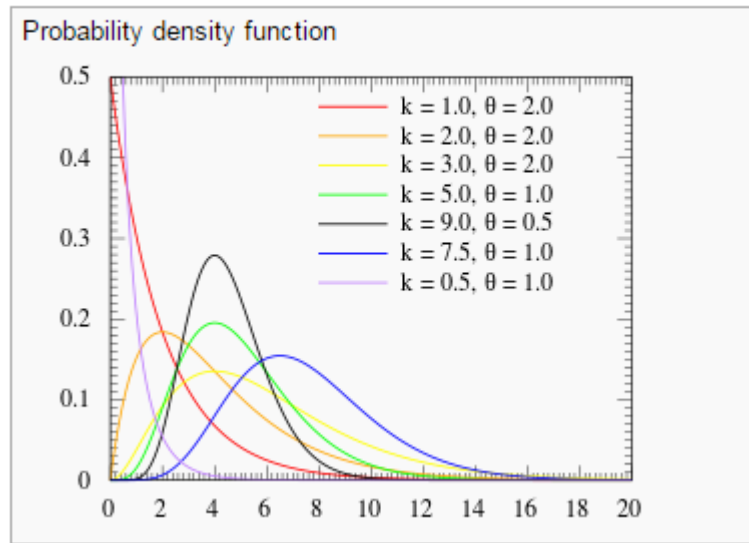
Log-logistic



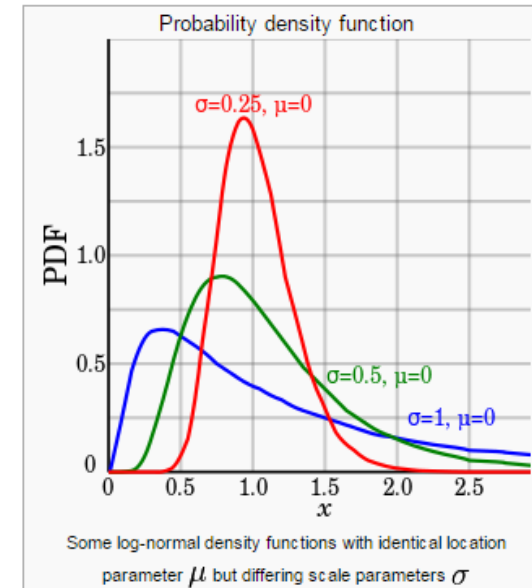
Weibull (2-Parameter)



Gamma



Log-normal



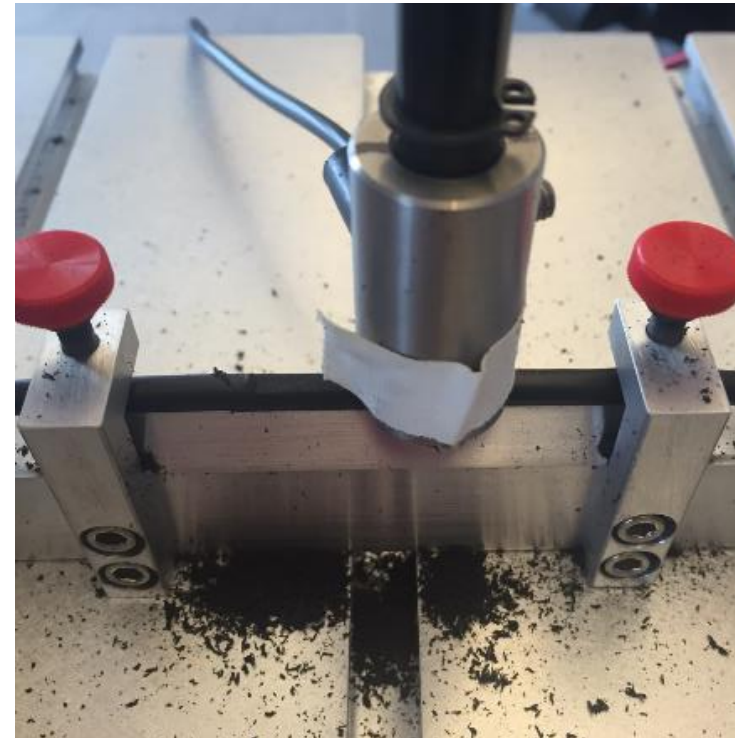
- What do you know about your data?
Reliability data starts at time=0
- Is it bound?
Bound by zero (or something greater)
- Any historical references?
Weibull, Lognormal, Gumbel (extreme value)
- Finally Does it look like it fits?

Statistics to look at:

- ▶ Anderson-Darling test: How well your data fits the PDF
 - ▶ Smaller fits better
- ▶ p value: Student's t-test - does your data fit the PDF?
 - ▶ H_0 = data fits the distribution
 - ▶ p is low, the null must go! i.e. if $p < 0.05$, it doesn't fit the distribution
- ▶ Likelihood ratio test (p value) - does 3 parameter fit better?
 - ▶ H_0 = 2 parameter fits better
 - ▶ i.e. if < 0.05 , the 3 parameter fits better

Example: Wire Scrape Test

- ▶ Cable gets frayed in the field
- ▶ Need to replicate
- ▶ Scrape at several points
- ▶ Measure cycles until breakthrough
- ▶ ISO 6722

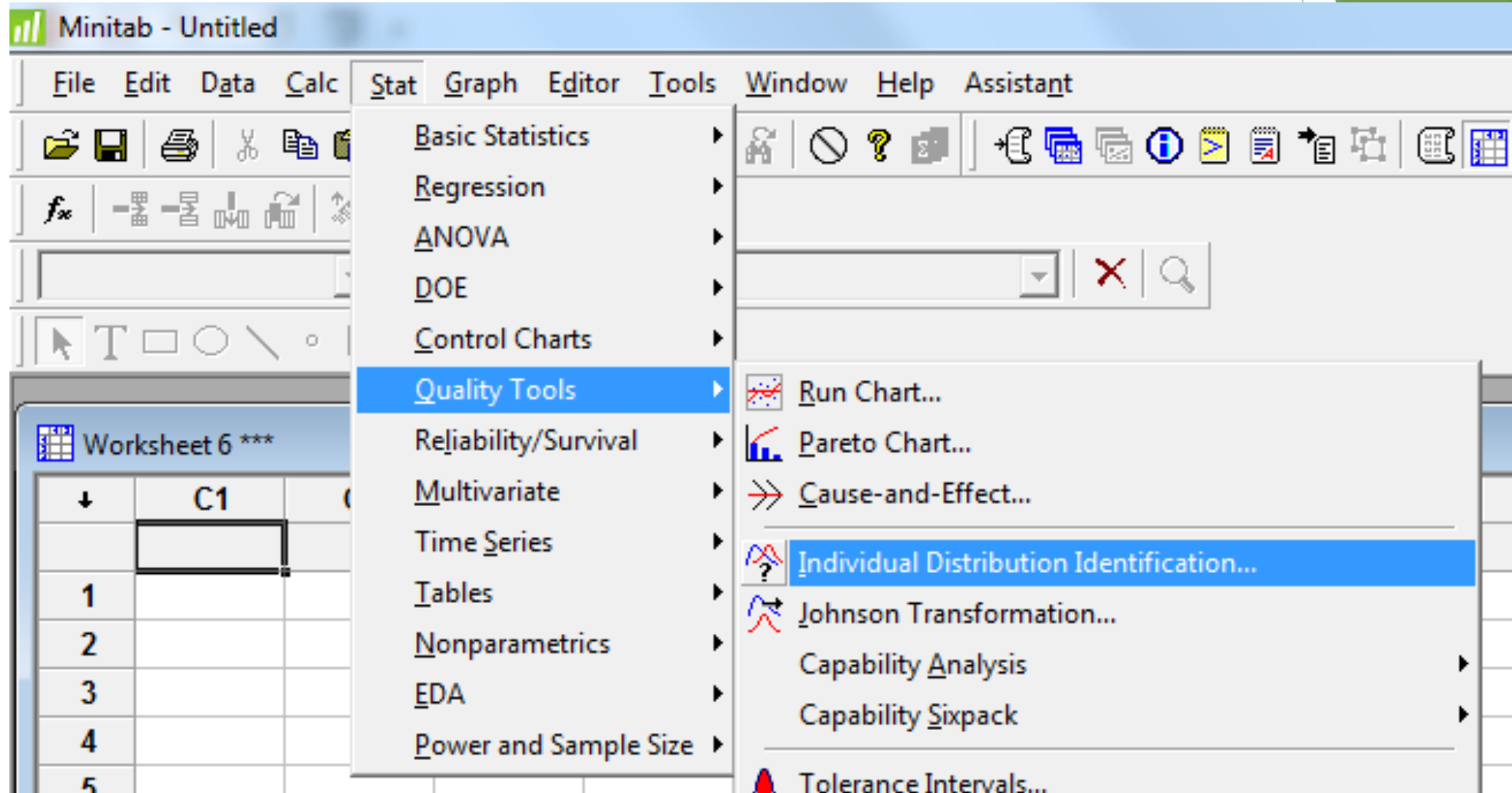


3 steps to identify the distribution

1. Look at graphs
2. Look at warnings
3. Look at Anderson darling numbers

Wire Scrape

Cable scrape 1kg	
	1046
	1199
	1249
	874
	842
	957
	1181
	1134
	933
	1145
	802
	988
	949
	874
	728
	651
	818
	414
	596
	610
	968
	684



Wire Scrape

Cable scrape 1kg
1046
1199
1249
874
842
957
1181
1134
933
1145
802
988
949
874
728
651
818
414
596
610
968
684

Individual Distribution Identification

Data are arranged as

☒ Single column: Cable scrape 1kg'

Subgroup size: 1

(use a constant or an ID column)

☐ Subgroups across rows of:

☒ Use all distributions and transformations

☐ Specify

☒ Distribution 1: Normal

☒ Distribution 2: Exponential

☒ Distribution 3: Weibull

☒ Distribution 4: Gamma

Box-Cox...

Johnson...

Options...

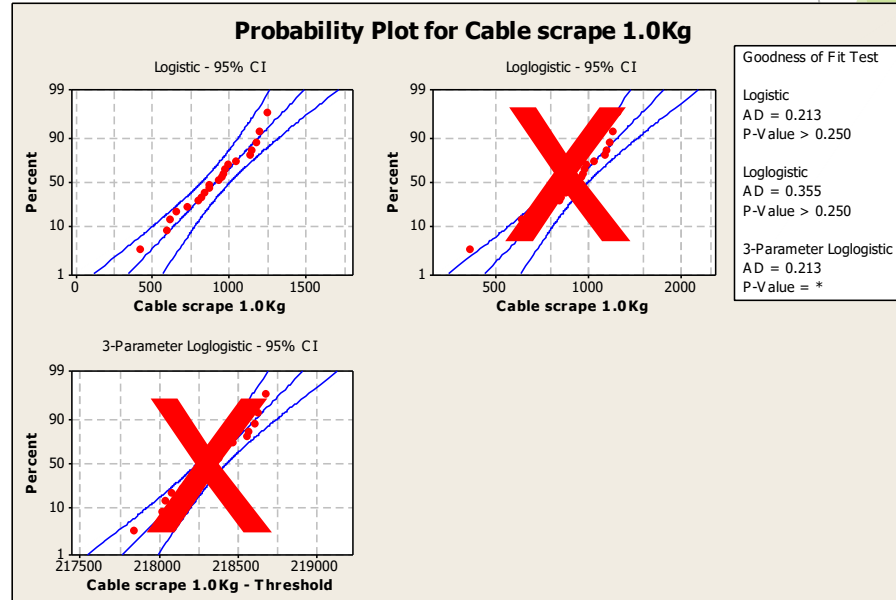
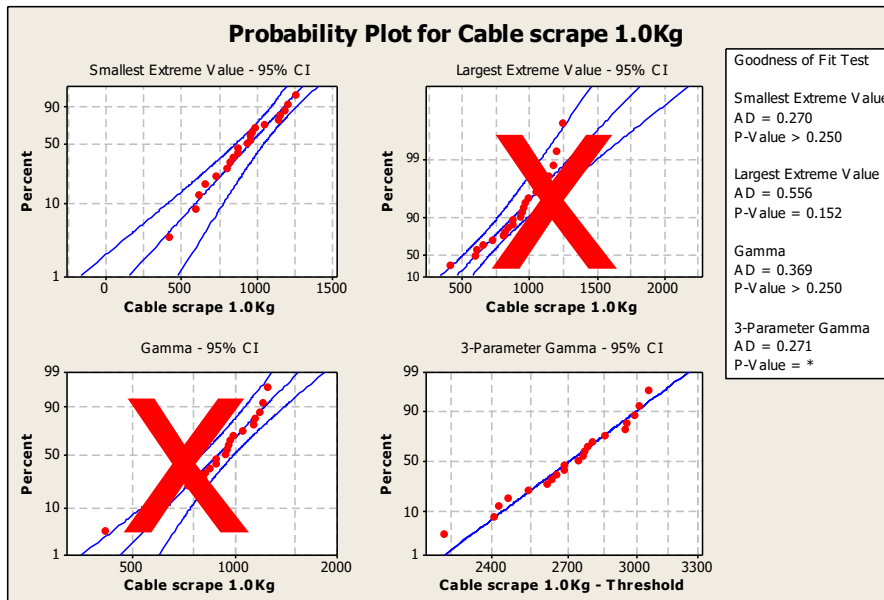
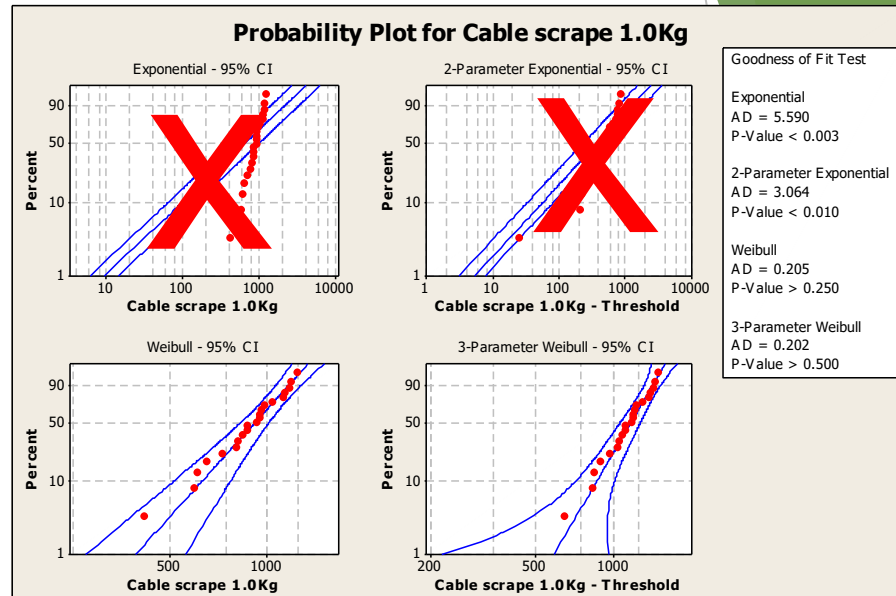
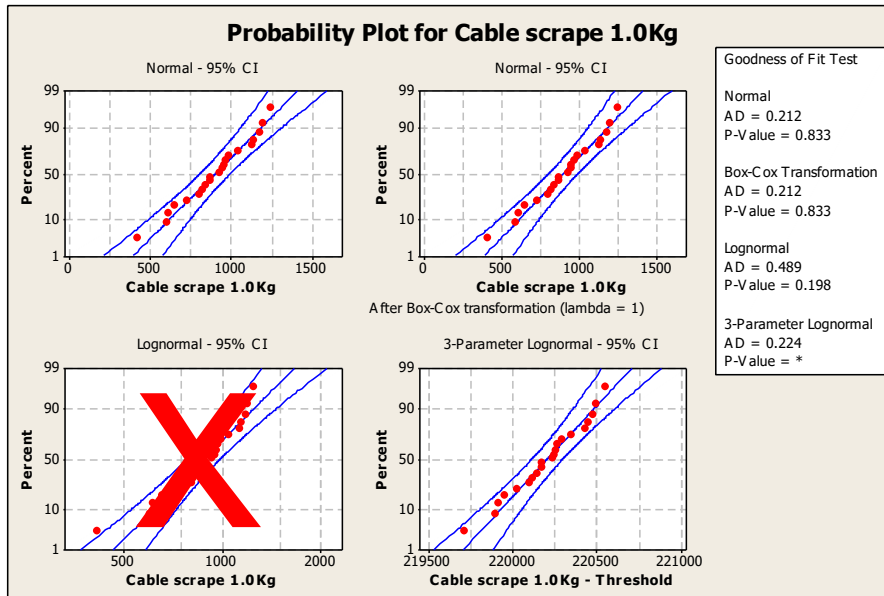
Results...

Select

Help

OK

Cancel



Note the Warnings!

3-Parameter Lognormal

```
* WARNING * Newton-Raphson algorithm has not converged after 50 iterations.  
* WARNING * Convergence has not been reached for the parameter estimates  
              criterion.
```

2-Parameter Exponential

```
* WARNING * Variance/Covariance matrix of estimated parameters does not exist.  
              The threshold parameter is assumed fixed when calculating  
              confidence intervals.
```

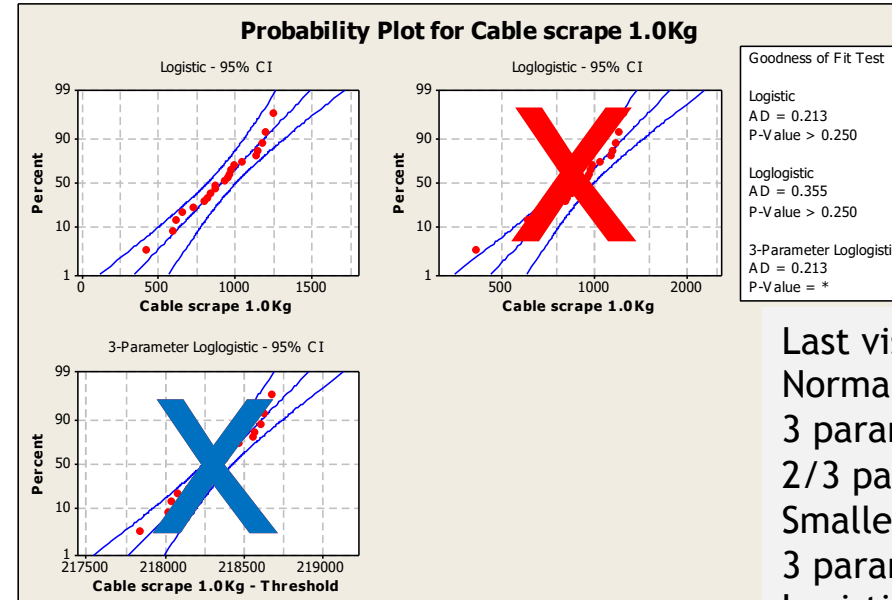
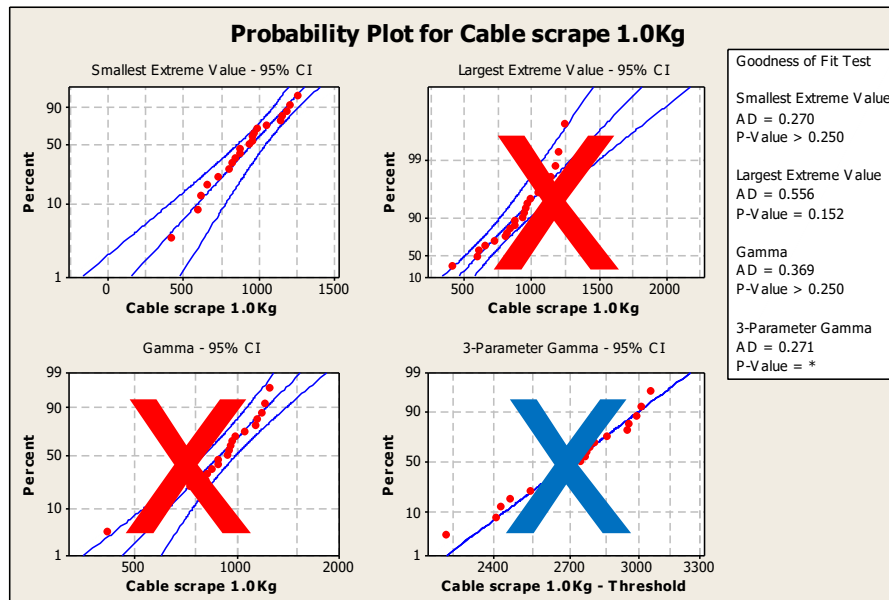
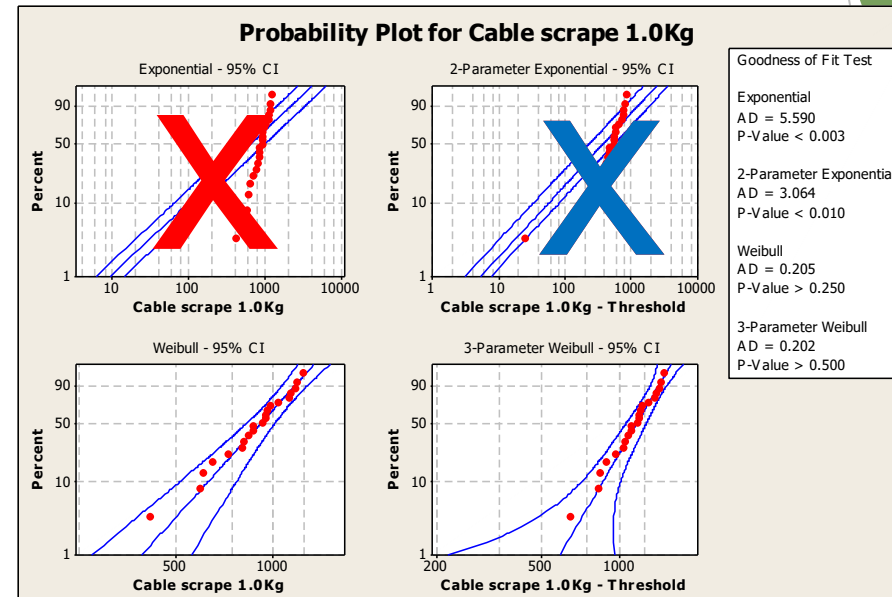
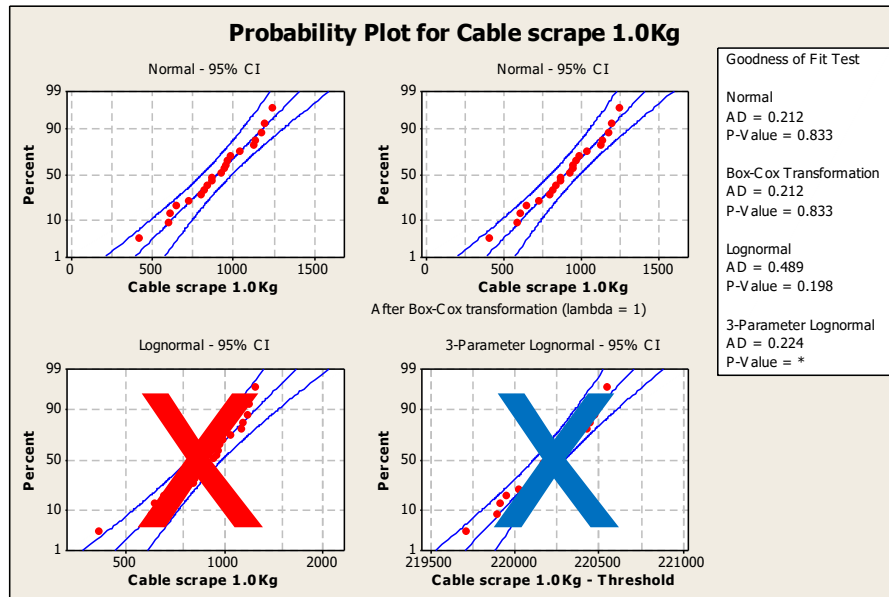
3-Parameter Gamma

```
* WARNING * Newton-Raphson algorithm has not converged after 50 iterations.  
* WARNING * Convergence has not been reached for the parameter estimates  
              criterion.
```

3-Parameter Loglogistic

```
* WARNING * Newton-Raphson algorithm has not converged after 50 iterations.  
* WARNING * Convergence has not been reached for the parameter estimates  
              criterion.
```

These are probably poor choices



Last visual inspection kept:
Normal
3 param. lognormal
2/3 param. Weibull
Smallest Extreme Val
3 param. gamma
Logistic

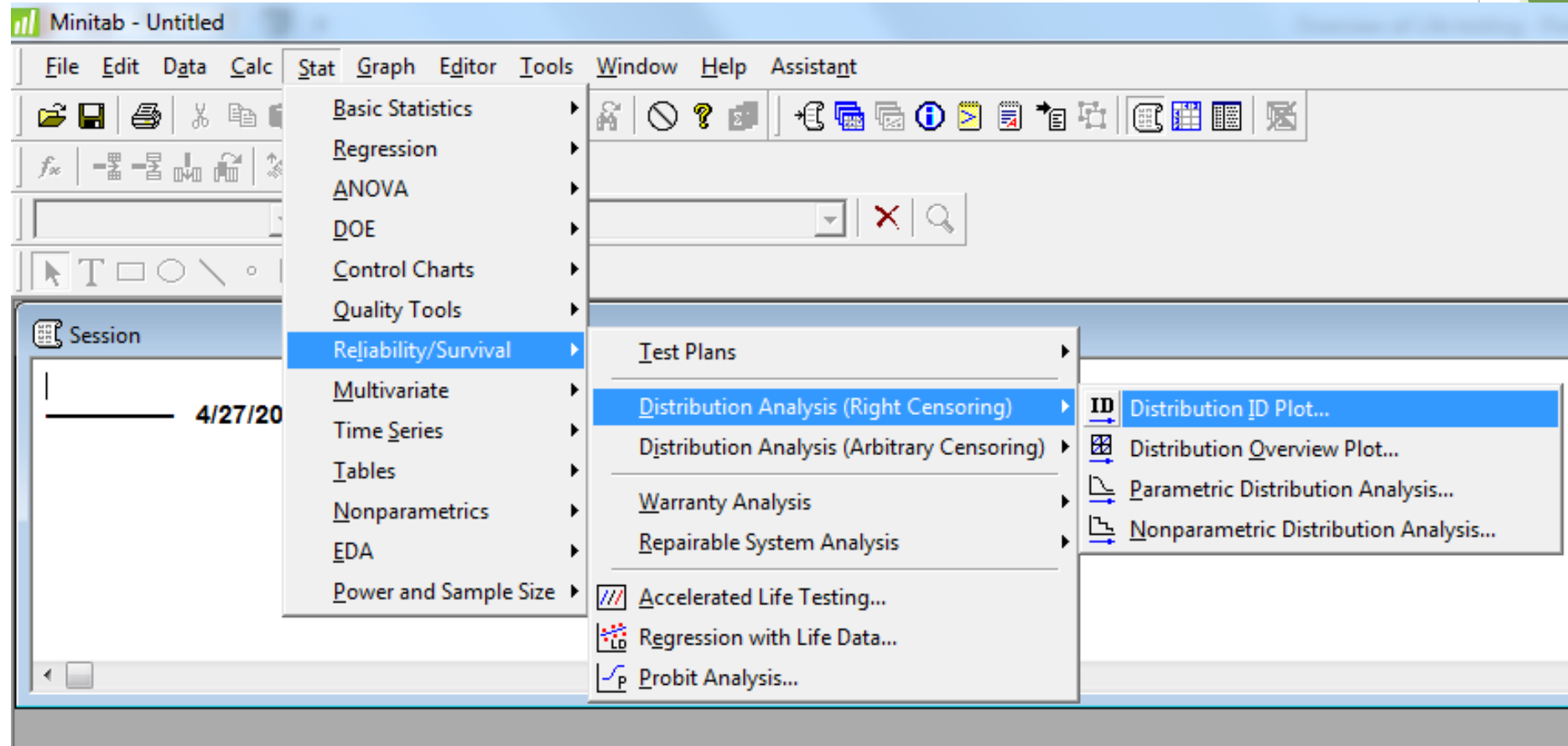
Goodness of Fit Test

Distribution	AD	P	LRT	P
Normal	0.212	0.833		
Box-Cox Transformation	0.212	0.833		
Lognormal	0.489	0.198		
3-Parameter Lognormal	0.224	*	0.061	
Exponential	5.590	<0.003		
2-Parameter Exponential	3.064	<0.010	0.000	
Weibull	0.205	>0.250		
3-Parameter Weibull	0.202	>0.500	0.808	
Smallest Extreme Value	0.270	>0.250		
Largest Extreme Value	0.556	0.152		
Gamma	0.369	>0.250		
3-Parameter Gamma	0.271	*	0.229	
Logistic	0.213	>0.250		
Loglogistic	0.355	>0.250		
3-Parameter Loglogistic	0.213	*	0.147	

Likelihood ratio test = if small (<.05), 3 parameter fits better

Wire Scrape: Alternatively...

Cable scrape 1kg
1046
1199
1249
874
842
957
1181
1134
933
1145
802
988
949
874
728
651
818
414
596
610
968
684



Wire Scrape

Distribution ID via
Reliability/Survival

Distribution ID Plot-Right Censoring

Variables:
'Cable scrape 1kg'

Frequency columns (optional):

☐ By variable:

☒ Use all distributions

☐ Specify

☒ Distribution 1: Weibull

☒ Distribution 2: Lognormal

☒ Distribution 3: Exponential

☒ Distribution 4: Normal

Select

Help

OK

Cancel

Censor...

Options...

Distribution ID Plot - Options

Estimation Method

☐ Least Squares (failure time(X) on rank(Y))

☒ Maximum Likelihood

Estimate percentiles for these percents:

Handle tied failure times by plotting

☒ All points

☐ Maximum of tied points

☐ Average (median) of tied points

Show different variables or by levels:

Minimum and Maximum X Scale

☒ Use default values

☐ Use: Minimum X scale: Maximum X scale:

Title:

Help

OK

Cancel

Least Squares vs Maximum Likelihood

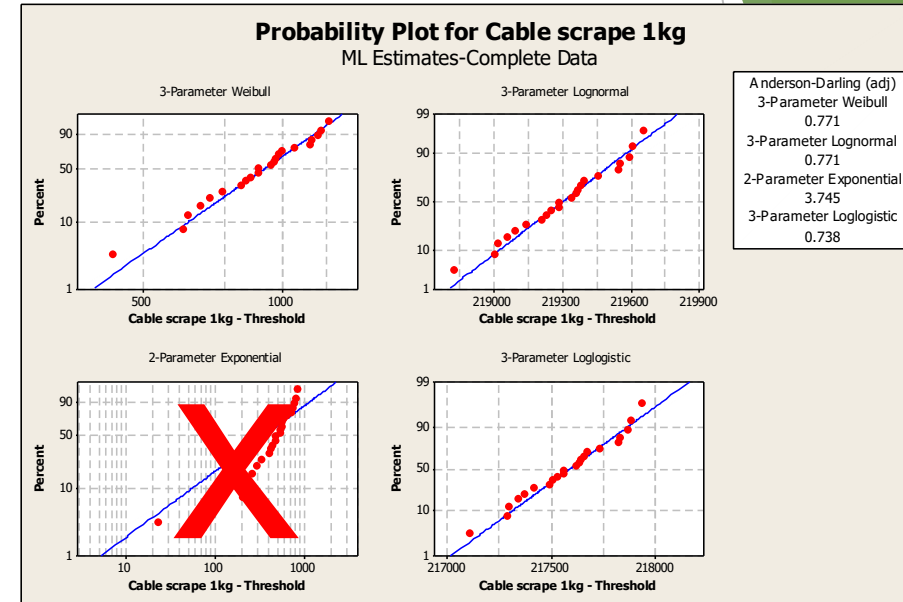
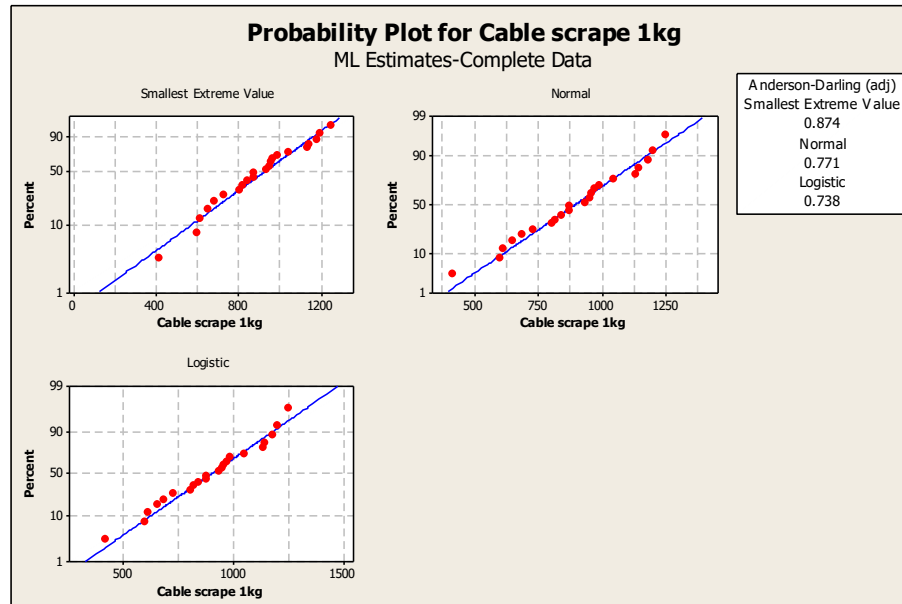
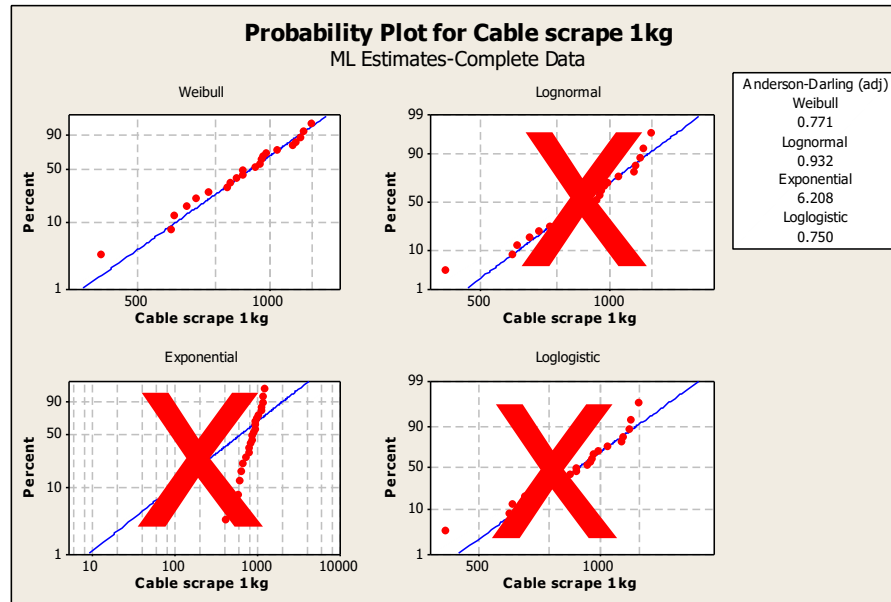
Least squares (LSXY)

- ▶ Better graphical display
- ▶ Better for small samples without censoring

Maximum likelihood (MLE)

- ▶ More precise than least squares (XY)
- ▶ Better for heavy censoring
- ▶ MLE allows you to perform an analysis when there are no failures

Distribution ID via Reliability/Survival



Review the Warnings Again

3-Parameter Lognormal

```
* WARNING * Newton-Raphson algorithm has not converged after 50 iterations.  
* WARNING * Convergence has not been reached for the parameter estimates  
             criterion.
```

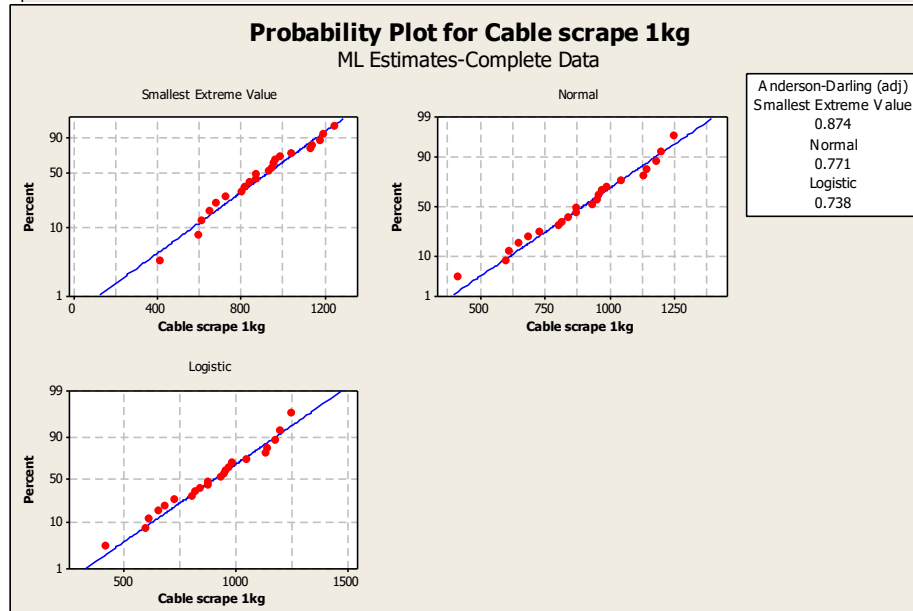
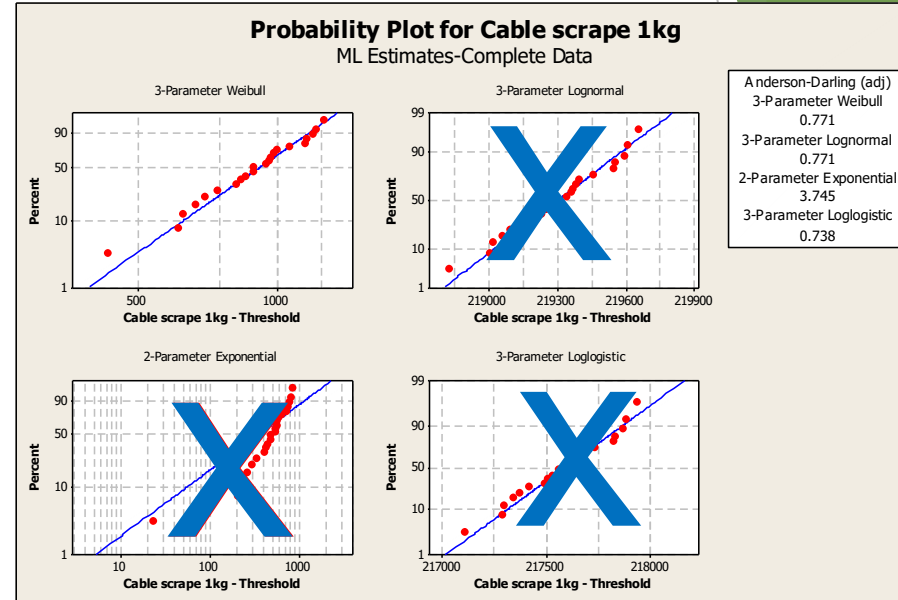
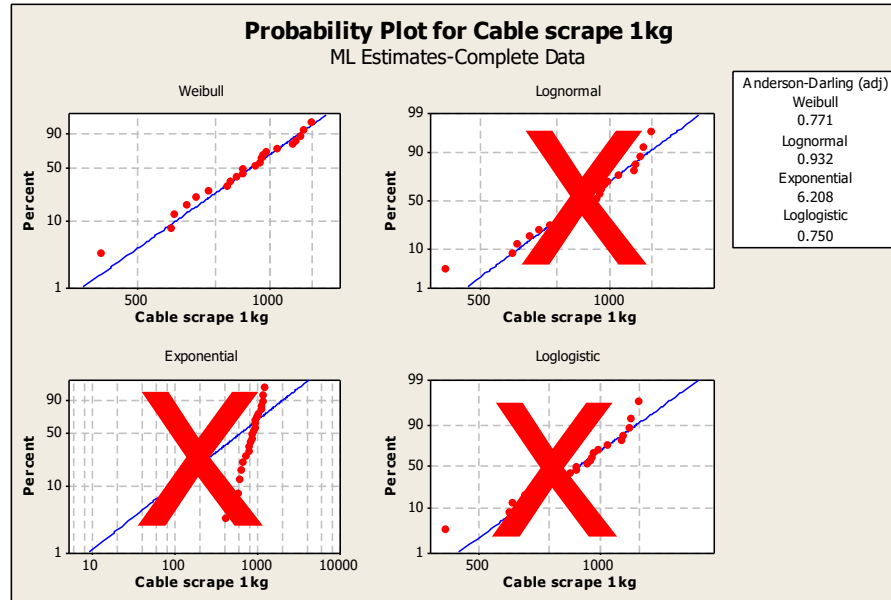
2-Parameter Exponential

```
* WARNING * Variance/Covariance matrix of estimated parameters does not exist.  
             The threshold parameter is assumed fixed when calculating  
             confidence intervals.
```

3-Parameter Loglogistic

```
* WARNING * Newton-Raphson algorithm has not converged after 50 iterations.  
* WARNING * Convergence has not been reached for the parameter estimates  
             criterion.
```

Still poor choices



Last visual inspection kept:
Weibull
3 parameter Weibull
Smallest extreme value
Logistic
Normal

Anderson-Darling

Distribution	(adj)
Weibull	0.771
Lognormal	0.932
Exponential	6.208
Loglogistic	0.750
3-Parameter Weibull	0.771
3-Parameter Lognormal	0.771
2-Parameter Exponential	3.745
3-Parameter Loglogistic	0.738
Smallest Extreme Value	0.874
Normal	0.771
Logistic	0.738

Note:

- Using Adjusted Anderson-Darling statistic*
- No P value provided
- no Likelihood ratio test

BUT! Can use censored data

*Even when the data are uncensored, the adjusted Anderson-Darling statistic will not necessarily yield the same result as the non-adjusted Anderson-Darling statistic for small samples. However, for large sample sizes, the disparity between the two approaches vanishes.

Other Output:

Distribution ID via
Reliability/Survival

Table of Percentiles

Standard	95% Normal CI				
Distribution	Percent	Percentile	Error	Lower	Upper
Weibull	1	376.505	68.5763	263.471	538.032
Lognormal	1	464.956	50.8249	375.289	576.047
Exponential	1	8.97312	1.91308	5.90836	13.6276
Loglogistic	1	445.882	60.9477	341.090	582.869
3-Parameter Weibull	1	373.325*	155.365	68.8161	677.835
3-Parameter Lognormal	1	393.301*	88.0037	220.817	565.785
2-Parameter Exponential	1	396.241	1.07483	394.140	398.353
3-Parameter Loglogistic	1	320.697	112.297	100.599	540.795
Smallest Extreme Value	1	123.762	162.753	-195.228	442.752
Normal	1	392.831	88.2110	219.940	565.721
Logistic	1	320.018	112.575	99.3745	540.661

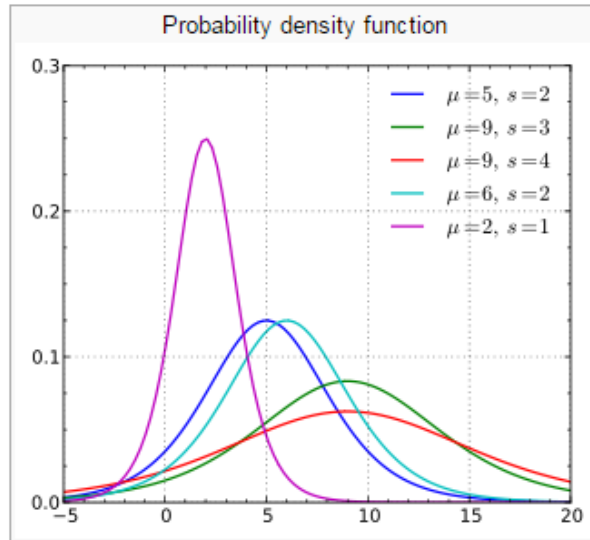
Table of MTTF

		Standard	95% Normal CI	
Distribution	Mean	Error	Lower	Upper
Weibull	894.250	45.098	810.087	987.16
Lognormal	895.154	51.723	799.309	1002.49
Exponential	892.818	190.349	587.877	1355.94
Loglogistic	917.307	51.467	821.783	1023.94
3-Parameter Weibull	894.247*	45.156	805.742	982.75
3-Parameter Lognormal	892.885*	45.828	803.065	982.71
2-Parameter Exponential	892.816	106.945	705.995	1129.07
3-Parameter Loglogistic	898.124	47.193	805.628	990.62
Smallest Extreme Value	887.984	51.629	786.794	989.18
Normal	892.818	45.822	803.009	982.63
Logistic	898.053	47.191	805.560	990.55

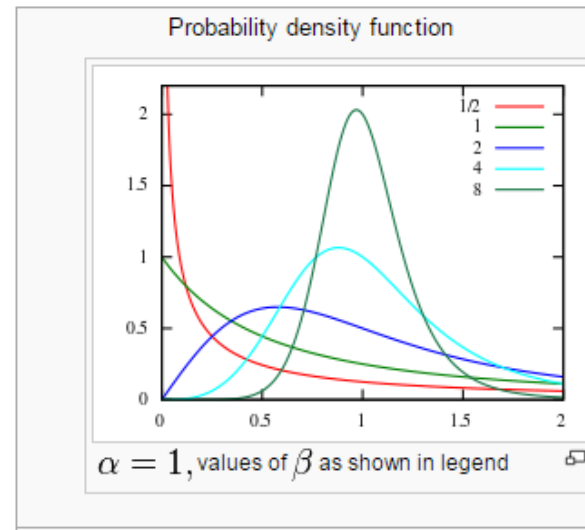
*note: the 3rd parameter (threshold) is not commonly accurate for sample sizes below 10, 20 or greater is recommended

Many Distributions - notice the similarity

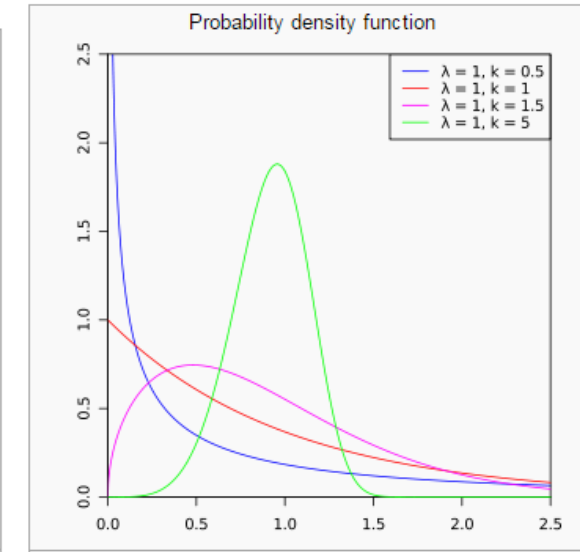
Logistic



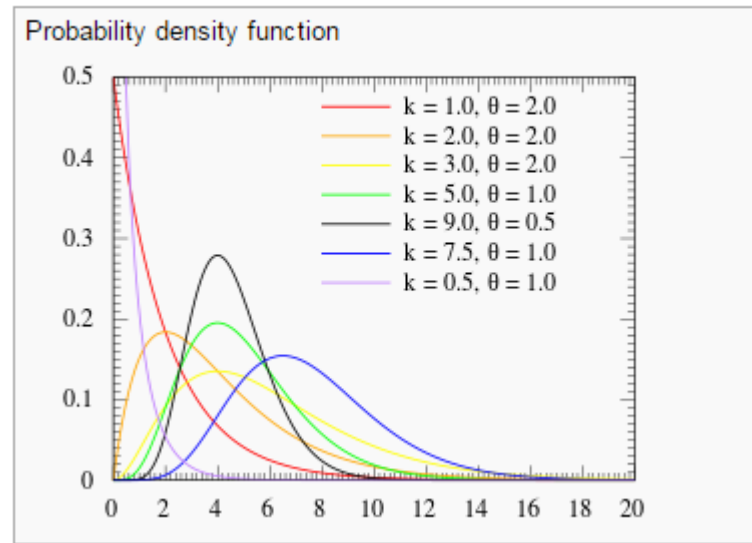
Log-logistic



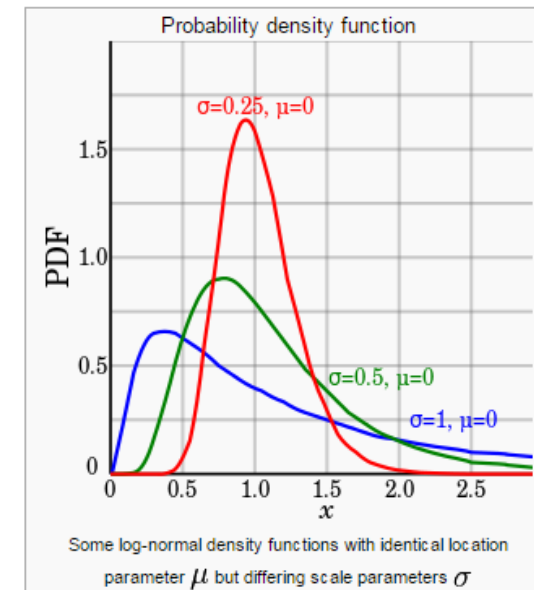
Weibull (2-Parameter)



Gamma

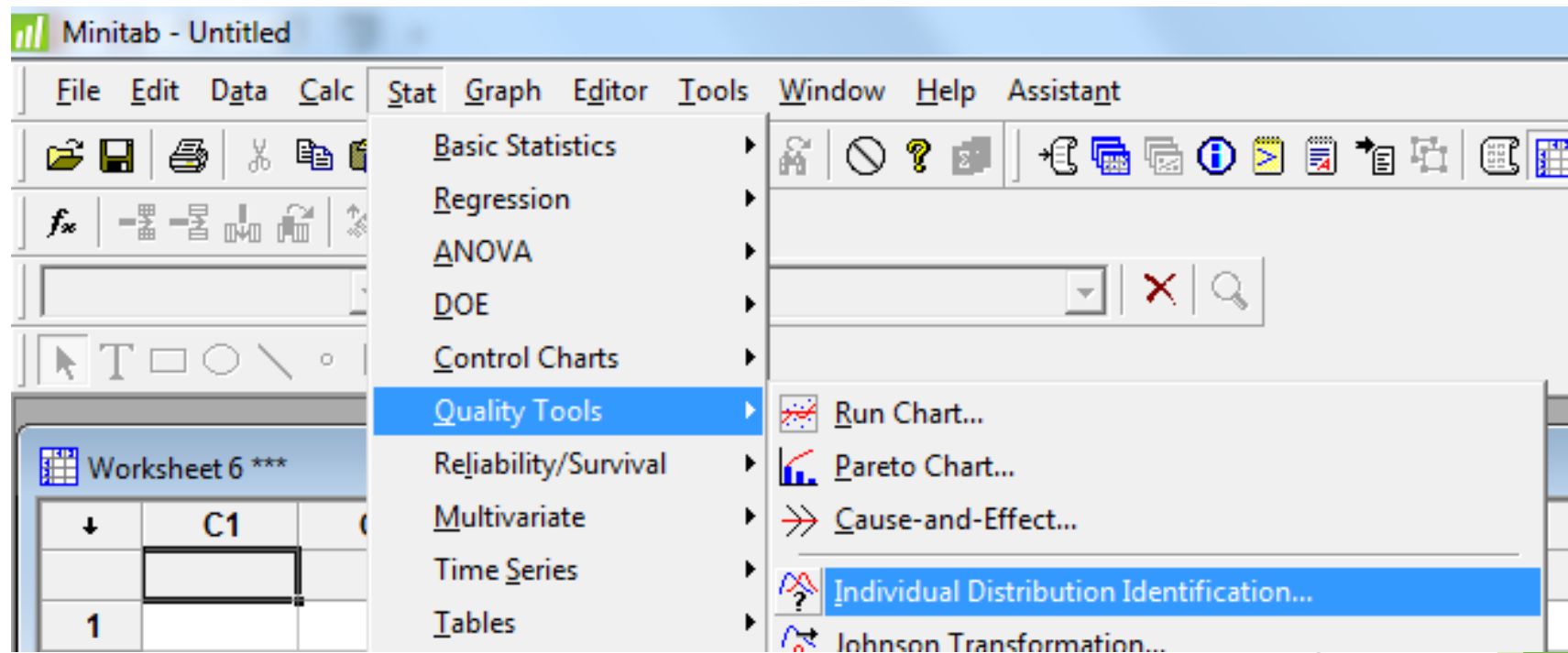


Log-normal



Which distribution ID to use?

- Recommend using Stat > Quality Tools > Individual Distribution Identification, Since it includes P values and Maximum Likelihood Ratio
- Unless you have censored data



Testing to failure

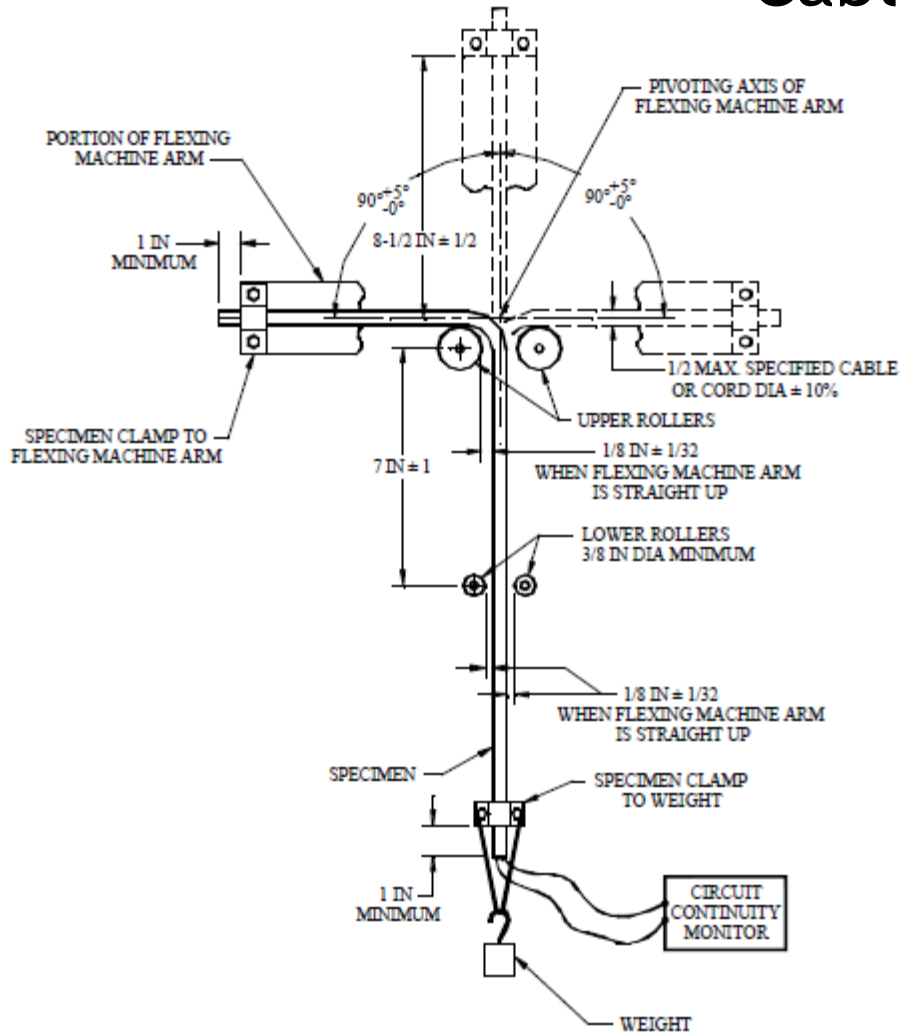
(Right censoring)

Testing to Failure - Remember censoring?

- ▶ Some tests will not be able to run to the end due to time constraints
 - = Right censored data or Suspended data
- ▶ Some tests cannot be continuously monitored
 - = Arbitrary censoring

Testing to Failure

Cable Flex -Right Censored

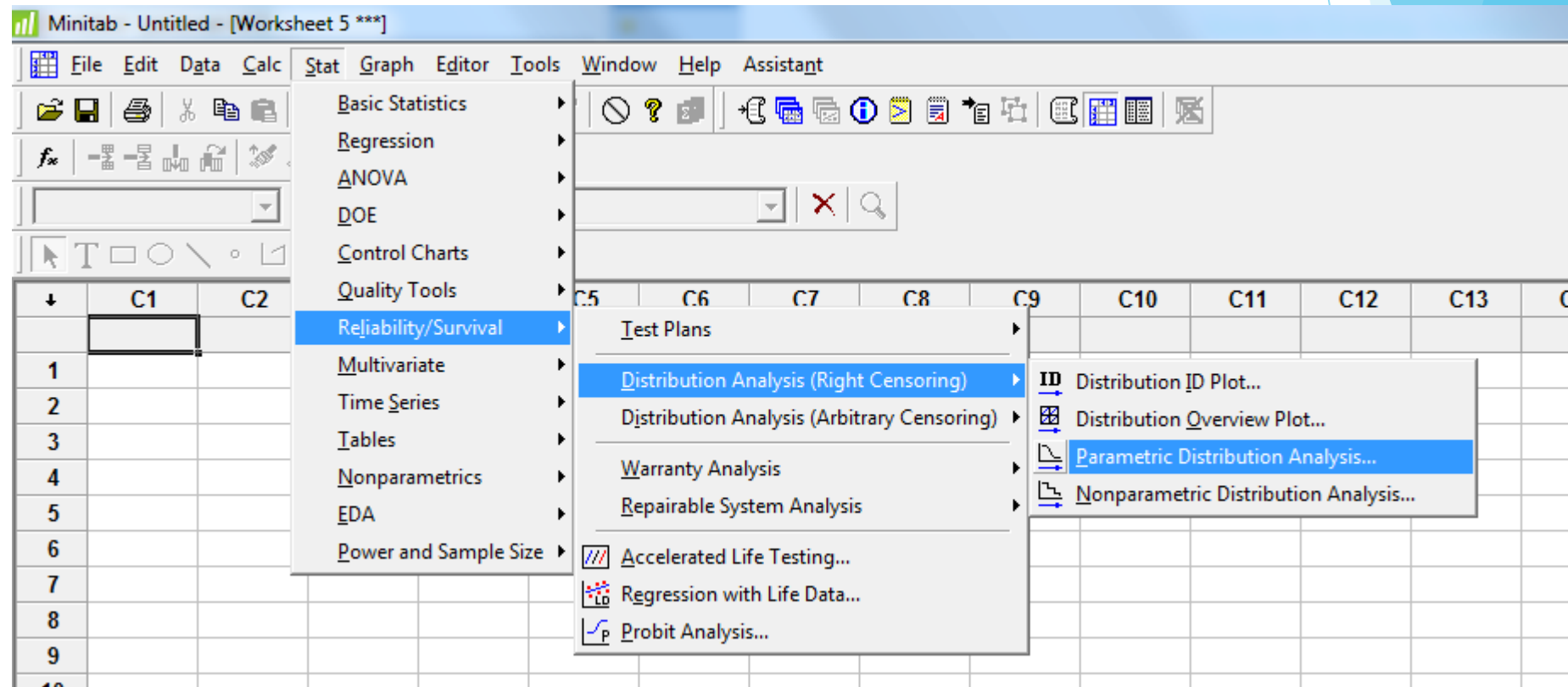


- ▶ MIL DTL -915G
- ▶ Checking for continuity
- ▶ Failure mode - open on any conductor
- ▶ Each unit is independently measured every millisecond and cycles are recorded

cycles	susp	qty
218457	s	3
78014	n	1
124859	n	1
36657	n	1
109032	n	1
58111	n	1
169316	n	1
204050	n	1

Testing to Failure

Cable Flex - Right Censored



cycles	susp	qty
218457	s	3
78014	n	1
124859	n	1
36657	n	1
109032	n	1
58111		
169316		
204050		

Testing to Failure

Cable Flex - Right Censored

Parametric Distribution Analysis-Right Censoring

Variables:
cycles

Frequency columns (optional):
qty

☐ By variable: weight

Assumed distribution: Weibull

Select

Help

Censor...
FMode...
Estimate...
Test...
Graphs...
Results...
Options...
Storage...
OK
Cancel

Parametric Distribution Analysis - Censor

Censoring Options
☒ Use censoring columns:
susp

Censoring value: "s"

☐ Time censor at:
☐ Failure censor at:

Select

Help

OK

Cancel

Testing to Failure

Cable Flex - Right Censored

Parametric Distribution Analysis - Estimate

Estimation Method

- ☐ Least Squares (failure time(X) on rank(Y))
- ☒ Maximum Likelihood

☐ Assume common shape (slope-Weibull) or scale (1/slope-other dists)

Bayes Analysis

Set shape (slope-Weibull) or scale (1/slope-other dists) at:

Set threshold at:

Estimate percentiles for these additional percents:

Estimate probabilities for these times (values):

☒ Estimate survival probabilities

☐ Estimate cumulative failure probabilities

Select

Confidence level: 95.0

Confidence intervals: Two-sided

Help OK Cancel

Testing to Failure

Least Squares vs Maximum Likelihood

Least squares (LSXY)

- ▶ Better graphical display
- ▶ Better for small samples without censoring

Maximum likelihood (MLE)

- ▶ More precise than least squares (XY)
- ▶ Better for heavy censoring
- ▶ MLE allows you to perform an analysis when there are no failures

Testing to Failure

Cable Flex -Right Censored

Probability Plot for cycles
Weibull - 95% CI
Censoring Column in susp - ML Estimates

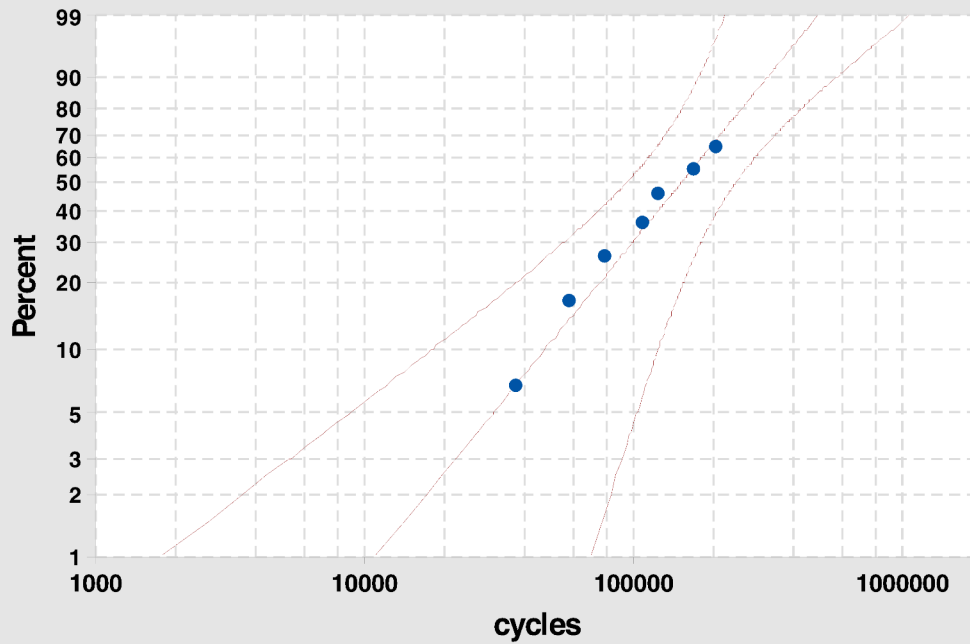


Table of Statistics	
Shape	1.62135
Scale	191314
Mean	171324
StDev	108311
Median	152607
IQR	145293
Failure	7
Censor	3
AD*	22.416

Cumulative Failure Plot for cycles
Weibull - 95% CI
Censoring Column in susp - ML Estimates

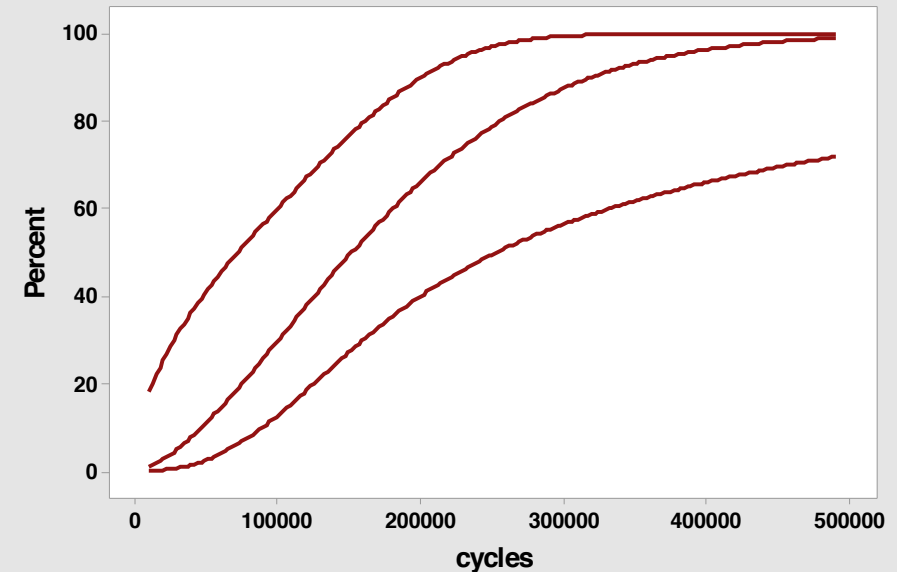


Table of Statistics	
Shape	1.62135
Scale	191314
Mean	171324
StDev	108311
Median	152607
IQR	145293
Failure	7
Censor	3
AD*	22.416

Testing to Failure

Cable Flex -Right Censored

Parameter Estimates

Parameter	Estimate	Standard	95.0% Normal CI	
		Error	Lower	Upper
Shape	1.62135	0.533550	0.850680	3.09021
Scale	191314	44798.7	120900	302738

Log-Likelihood = -91.724

Goodness-of-Fit

Anderson-Darling (adjusted) = 22.416

Characteristics of Distribution

	Estimate	Standard	95.0% Normal CI	
		Error	Lower	Upper
Mean (MTTF)	171324	40868.3	107342	273444
Standard Deviation	108311	45660.1	47406.2	247462
Median	152607	36459.0	95546.4	243743
First Quartile(Q1)	88719.9	29101.9	46645.5	168746
Third Quartile(Q3)	234013	58343.0	143556	381467
Interquartile Range(IQR)	145293	53304.7	70787.5	298216

Testing to Failure

Cable Flex -Right Censored

Table of Percentiles

Percent	Percentile	Standard	95.0% Normal CI	
		Error	Lower	Upper
1	11208.6	10546.2	1772.79	70867.5
2	17241.5	13868.6	3563.63	83417.6
3	22210.1	16102.7	5363.08	91978.8
4	26606.6	17800.1	7170.08	98731.0
5	30630.3	19167.9	8984.34	104428
6	34386.8	20309.6	10805.7	109429
7	37940.2	21285.8	12634.1	113934
8	41332.9	22134.9	14469.6	118069
9	44595.1	22883.4	16311.9	121918
10	47749.0	23550.3	18161.2	125541
20	75853.2	27772.4	37010.2	155463
30	101298	30281.9	56382.8	181995
40	126421	32818.9	76005.8	210277
50	152607	36459.0	95546.4	243743
60	181272	42265.0	114780	286283
70	214521	51626.6	133850	343811
80	256577	67119.2	153655	428437
90	320001	96517.1	177180	577945
91	328954	101139	180064	600957
92	338787	106333	183133	626741
93	349725	112247	186434	656036
94	362093	119102	190037	689926
95	376392	127234	194045	730092
96	393445	137211	198625	779351
97	414776	150087	204083	842987
98	443734	168222	211070	932863
99	490701	199090	221545	1086856

The background features abstract, overlapping green geometric shapes, primarily triangles and polygons, in various shades of green, creating a modern and dynamic visual effect.

Testing to failure

(Arbitrary censoring)

Testing to Failure

Stepped Drop Test - Arbitrary Censoring



Drop on the connector

10 drops at 40" then
10 drops at 50" then
10 drops at 60" etc.
until failure

Testing to Failure

Stepped Drop Test - Arbitrary Censoring

Tabulated Data

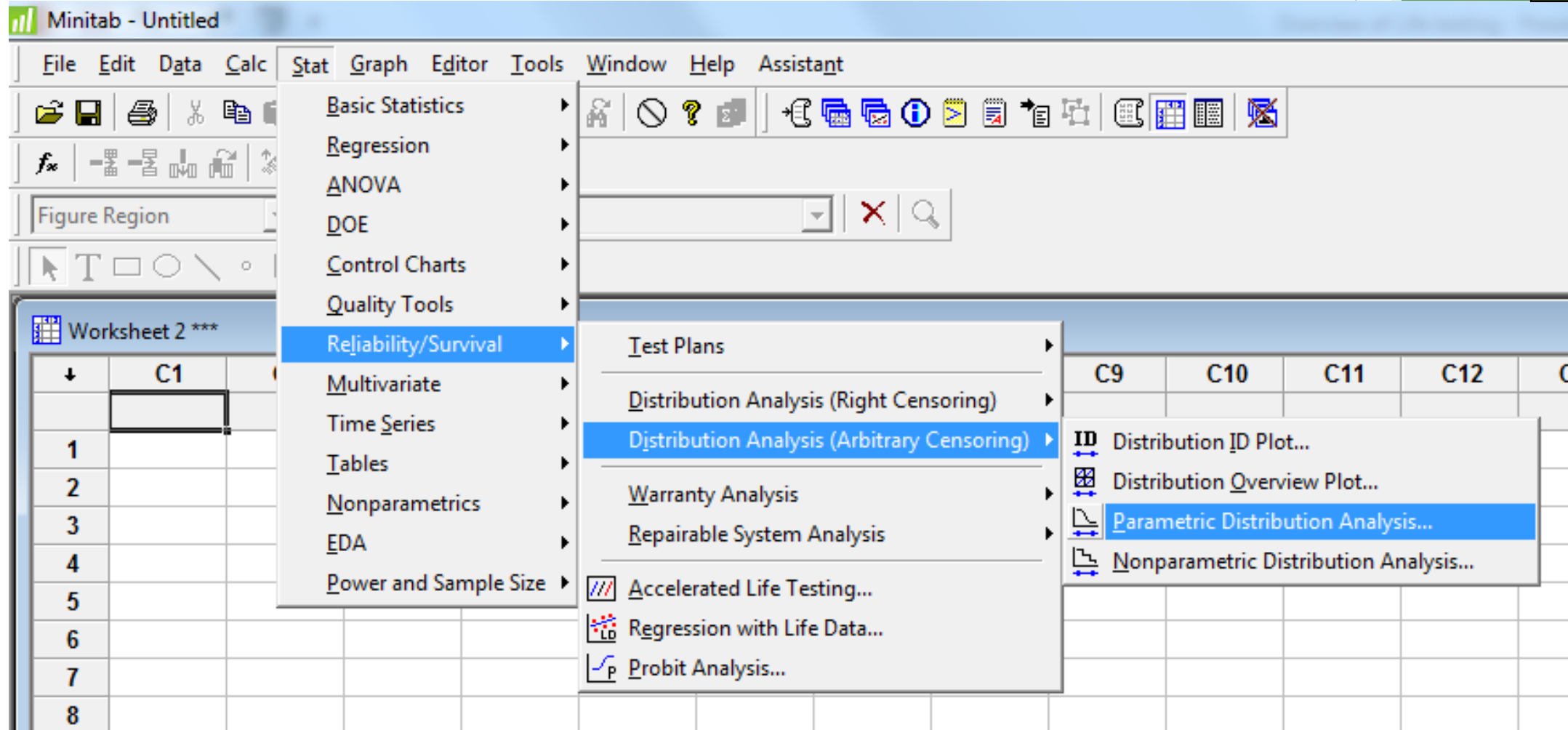
	Number of drops before failure @ 40 inch(if <10)	Number of drops before failure @ 50 inch(if <10)	Number of drops before failure @ 60 inch(if <10)	Total inches Passed	Total inches Failed
1	10	10	6	1260	1320
2	10	4		600	650
3	10	10	2	1020	1080
4	10	8		800	850
5	10	7		750	800
6	10	10	2	1020	1080

Miner's rule, cumulative energy

Testing to Failure

Stepped Drop Test - Arbitrary Censoring

Total inches Passed	Total inches Failed
1260	1320
600	650
1020	1080
800	850
750	800
1020	1080



Testing to Failure

Stepped Drop Test - Arbitrary Censoring

Total inches Passed	Total inches Failed
1260	1320
600	650
1020	1080
800	850
750	800
1020	1080

Parametric Distribution Analysis-Arbitrary Censoring

Start variables:
'Total inches Passed'

End variables:
'Total inches Failed'

Frequency columns (optional):

☐ By variable:

Assumed distribution: Weibull

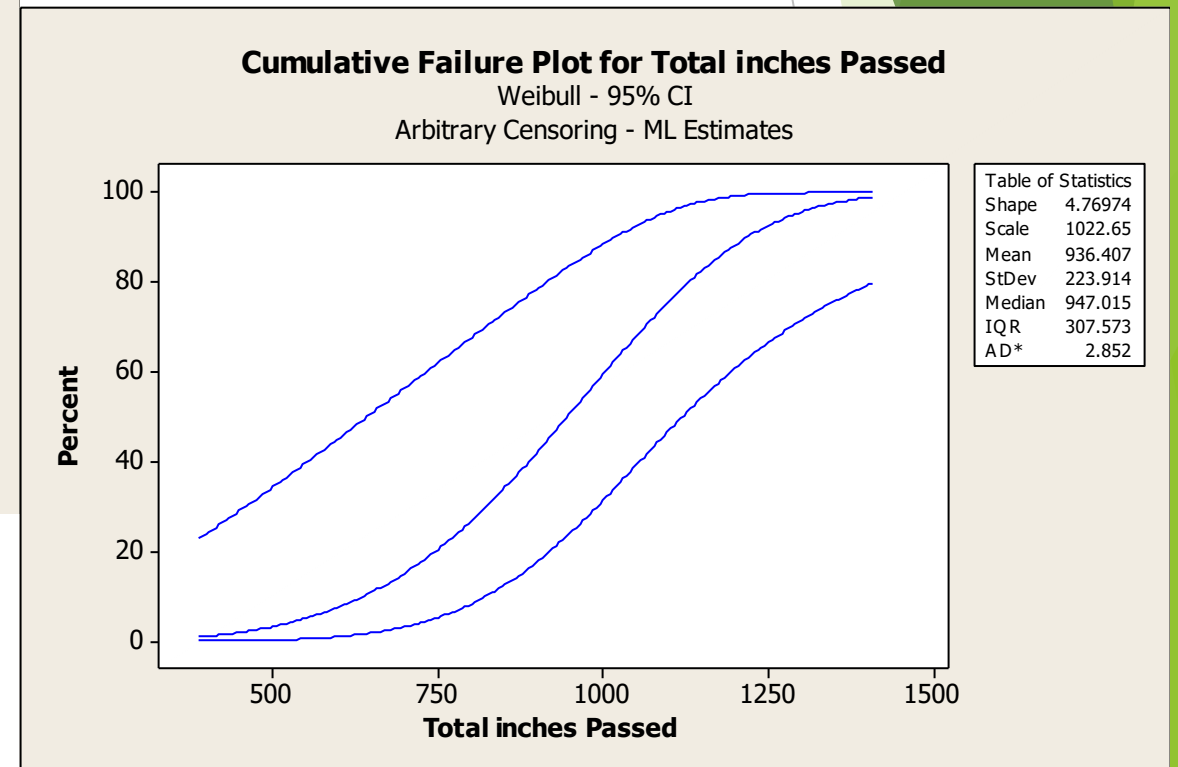
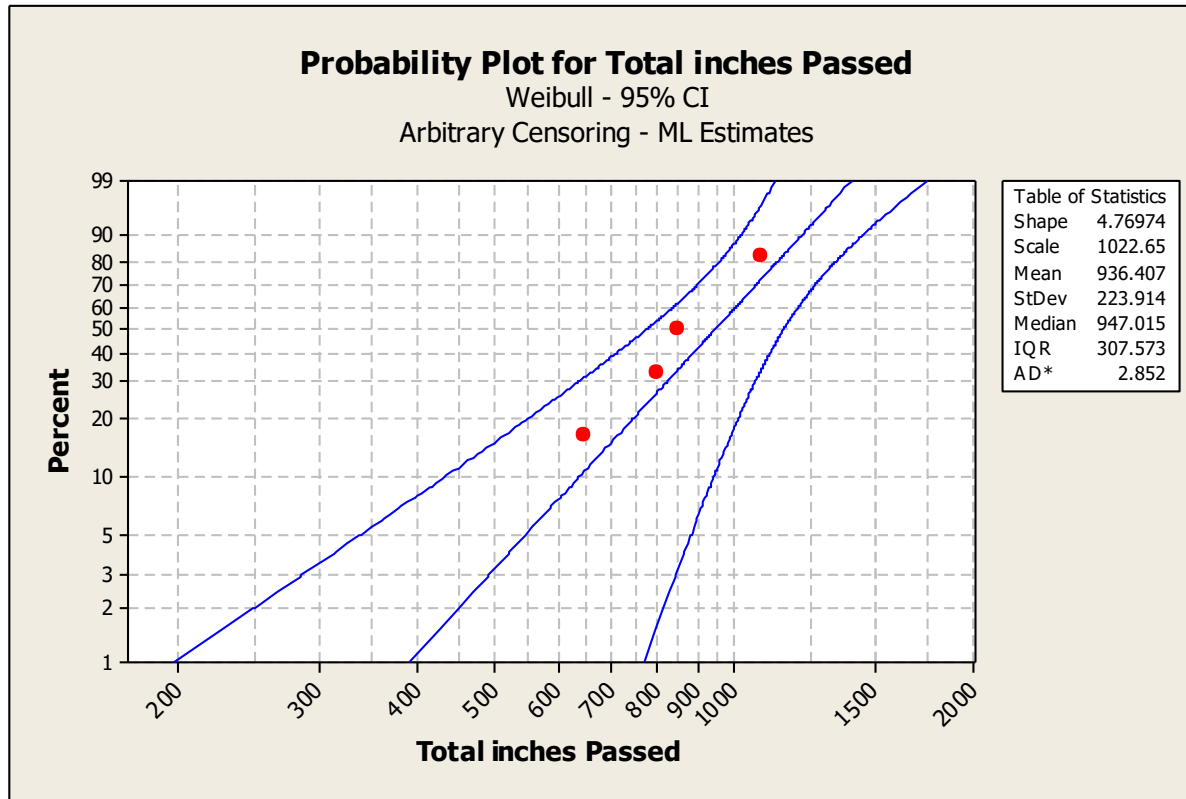
Select

Help

FMMode...
Estimate...
Test...
Graphs...
Results...
Options...
Storage...
OK
Cancel

Testing to Failure

Stepped Drop Test - Arbitrary Censoring



Testing to Failure

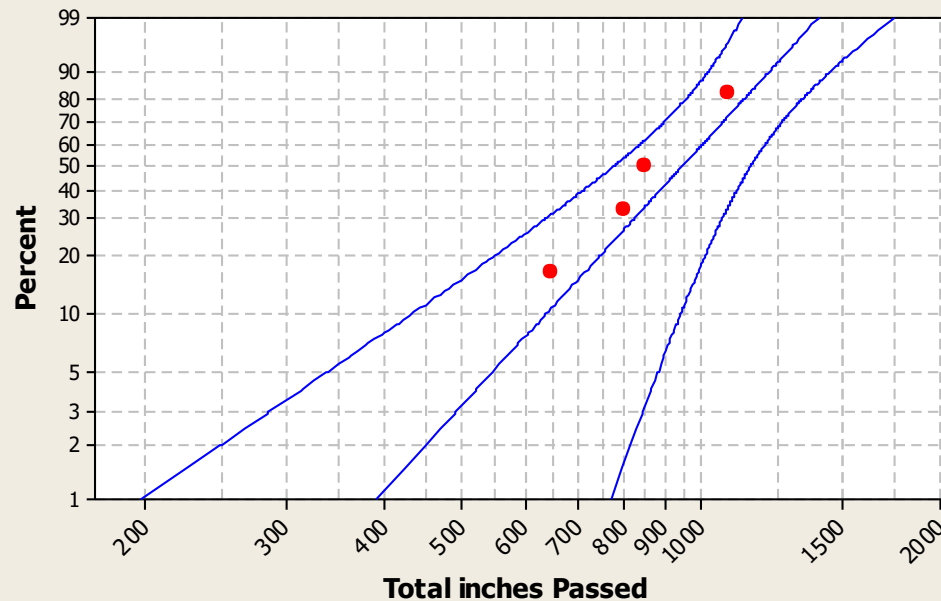
Arbitrary vs Right censoring

How Is It Different?

Probability Plot for Total inches Passed

Weibull - 95% CI

Arbitrary Censoring - ML Estimates

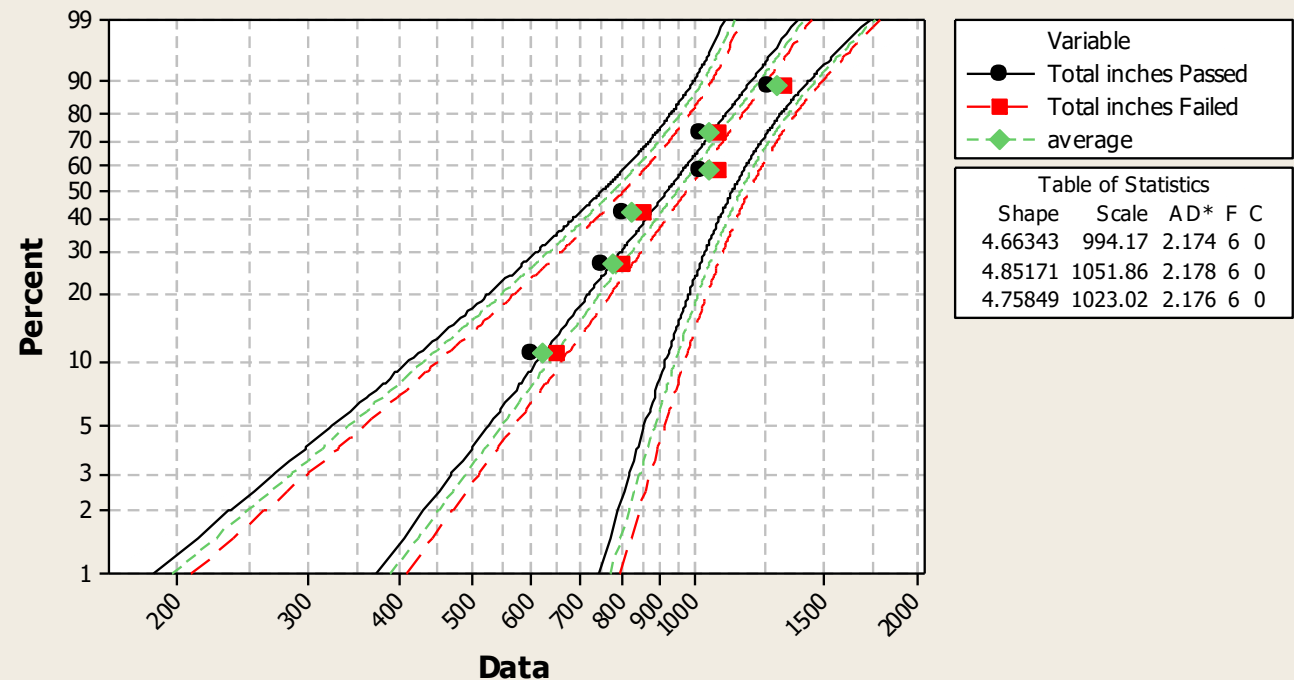


In this case, fairly close to the average.
But with wider intervals it becomes very different.

Probability Plot for Total inches Passed, Total inches Failed, average

Weibull - 95% CI

Complete Data - ML Estimates



Acceleration Factors

Acceleration Factors

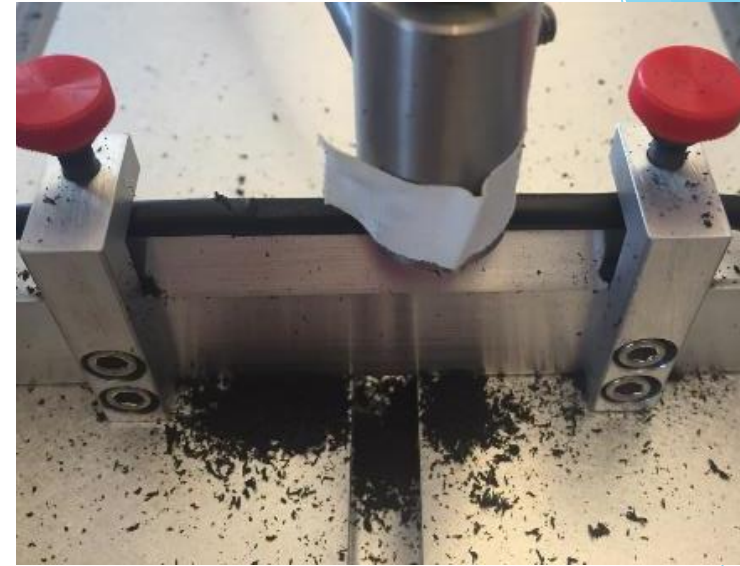
- ▶ Reliability engineers attempt to speed up testing by “accelerating” the test
 - ▶ Environmental:
 - ▶ Increased temperature
 - ▶ Increased temperature swing
 - ▶ Adding environmental factors together
 - ▶ Sequentially testing
 - ▶ Mechanical
 - ▶ Increased speed
 - ▶ Increased weight
 - ▶ Others?
 - ▶ Multiple factors

Back to:

Wire Scrape Test

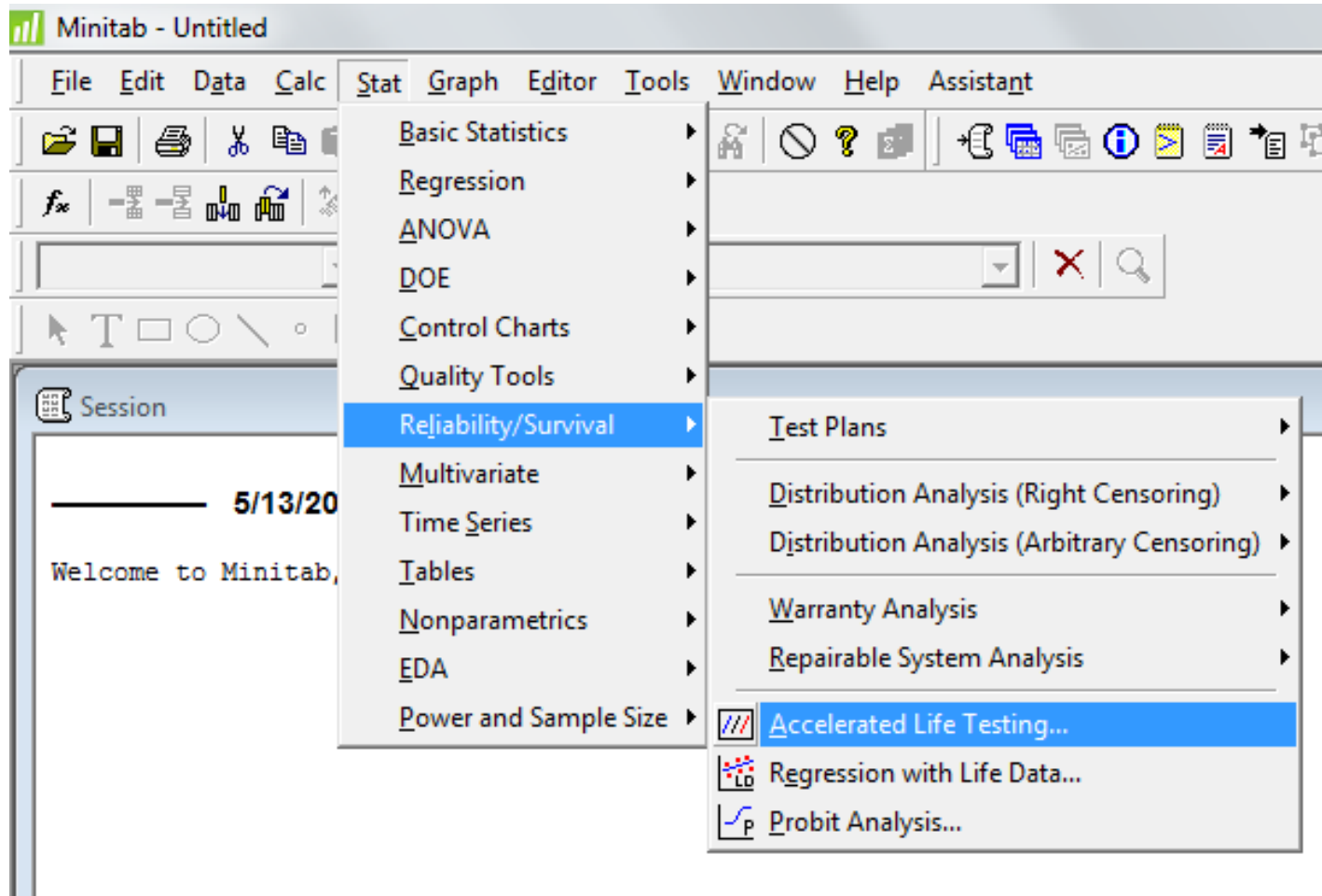
- ▶ This test can use different weights, what effect does that have?
- ▶ In this case, we used 4 weights: 0.85 kg, 1 kg, 1.1 kg, 1.25 kg

Acceleration
Factors



Acceleration Factors

Wire Scrape



cycles	weight
3055	0.85
3047	0.85
1728	0.85
3155	0.85
2498	0.85
2341	0.85
4140	0.85
1583	0.85
2908	0.85
4581	0.85
2286	0.85
1630	0.85
2521	0.85
2849	0.85
2283	0.85
6018	0.85
1394	0.85
2345	0.85
2984	0.85
1046	1
1199	1
1249	1
874	1
842	1
957	1
1181	1
1134	1
933	1
1145	1
802	1
988	1
949	1
874	1
728	1
651	1
818	1
414	1
596	1
610	1
968	1
684	1
845	1.1
591	1.1
852	1.1
655	1.1
382	1.1
397	1.1
315	1.1
299	1.1
471	1.1
806	1.1
699	1.1
440	1.1
547	1.1
317	1.1
270	1.1
1304	1.1
691	1.1
543	1.1
360	1.1
415	1.1
226	1.25
134	1.25
270	1.25
200	1.25
155	1.25
139	1.25
446	1.25
474	1.25
199	1.25
164	1.25
146	1.25
247	1.25
223	1.25
205	1.25
412	1.25
231	1.25
225	1.25
310	1.25
173	1.25
132	1.25

Acceleration Factors

Wire Scrape

Accelerated Life Testing

☒ Responses are uncens/right censored data
☐ Responses are uncens/arbitrarily censored data

Variables/Start variables: cycles

End variables:

Freq. columns: (optional)

Accelerating var: weight Relationship: Linear

☐ Second Variable

☒ Accelerating: Relationship: Linear

☐ Factor:

☐ Include interaction term between variables

Assumed distribution: Weibull

Select

Help

OK

Cancel

Censor...

Estimate...

Graphs...

Results...

Options...

Storage...

Accelerated Life Testing - Graphs

Plots based on fitted model

Design value to include on plots:

☒ Relation plot

Plot percentiles for percents: 50

☒ Display confidence intervals for middle percentile
☐ Display confidence intervals for all percentiles
☐ Display no confidence intervals

☒ Display failure times on plot

☒ Probability plot for each accelerating level based on fitted model

☒ Display confidence intervals for design value
☐ Display confidence intervals for all levels
☐ Display no confidence intervals

Diagnostic Plots

☐ Probability plot for each accelerating level based on individual fits
☐ Probability plot for standardized residuals
☐ Exponential probability plot for Cox-Snell residuals
☐ Display confidence intervals on diagnostic plots

Help

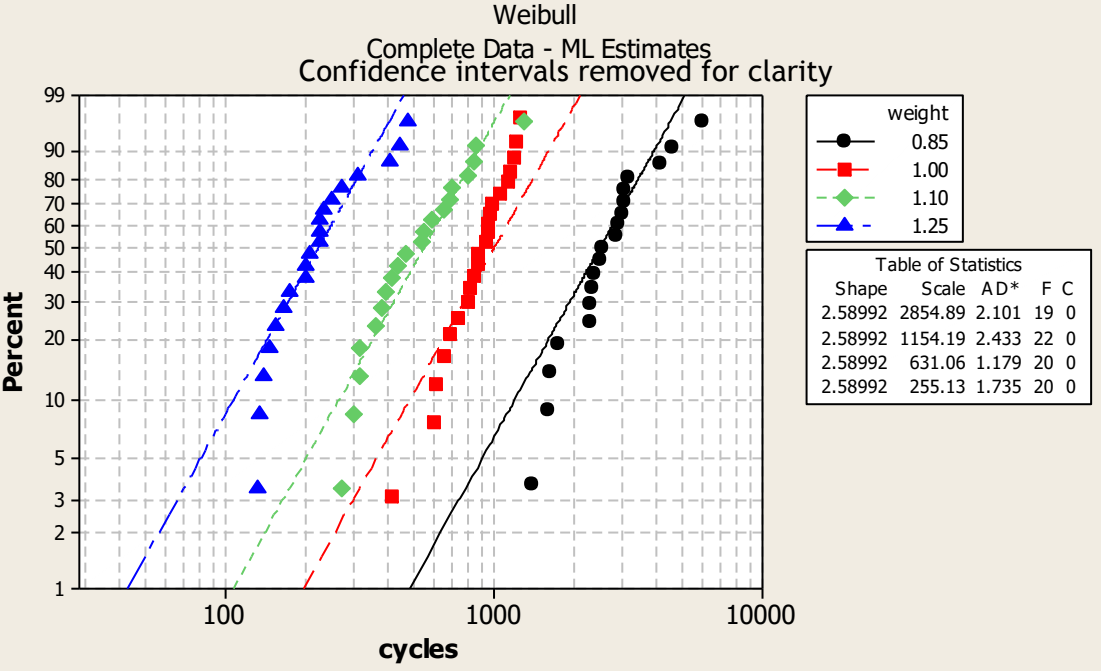
OK

Cancel

Acceleration Factors

Wire Scrape

Probability Plot (Fitted Linear) for cycles



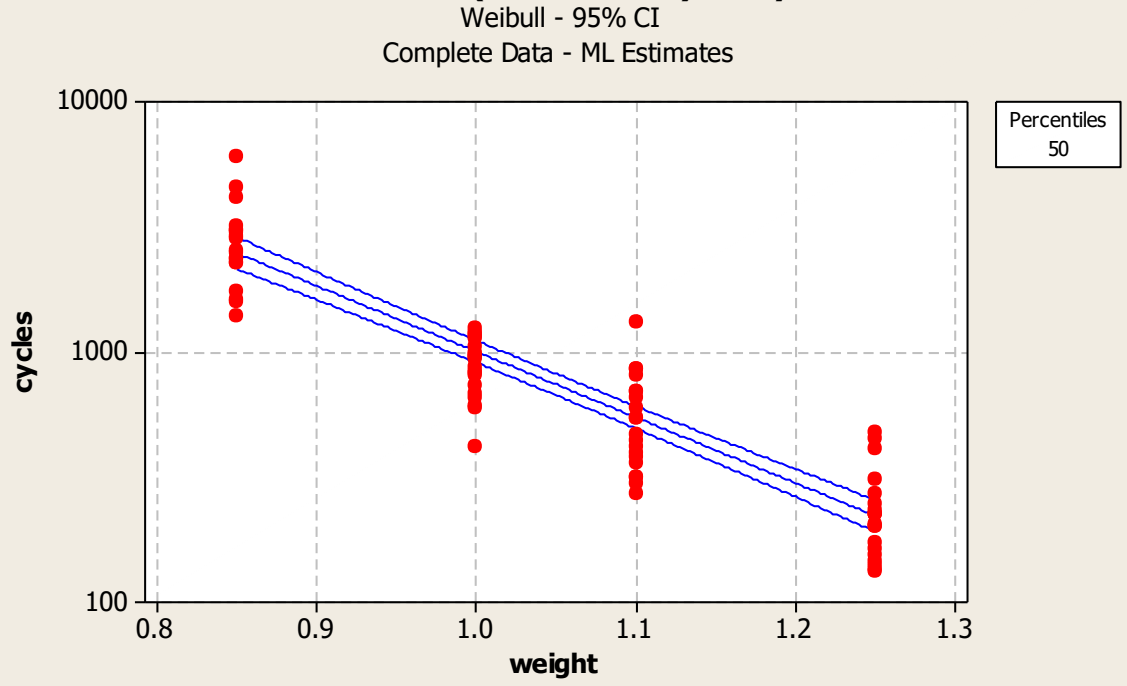
Relationship with accelerating variable(s): Linear

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI	
Intercept	13.0887	0.292537	44.74	0.000	12.5154	13.6621
weight	-6.03758	0.275137	-21.94	0.000	-6.57684	-5.49832
Shape	2.58992	0.202125			2.22257	3.01798

Log-Likelihood = -574.192

Relation Plot (Fitted Linear) for cycles



Acceleration Factors

Wire Scrape

Relationship with accelerating variable(s): Linear

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI	
					Lower	Upper
Intercept	13.0887	0.292537	44.74	0.000	12.5154	13.6621
weight	-6.03758	0.275137	-21.94	0.000	-6.57684	-5.49832
Shape	2.58992	0.202125			2.22257	3.01798

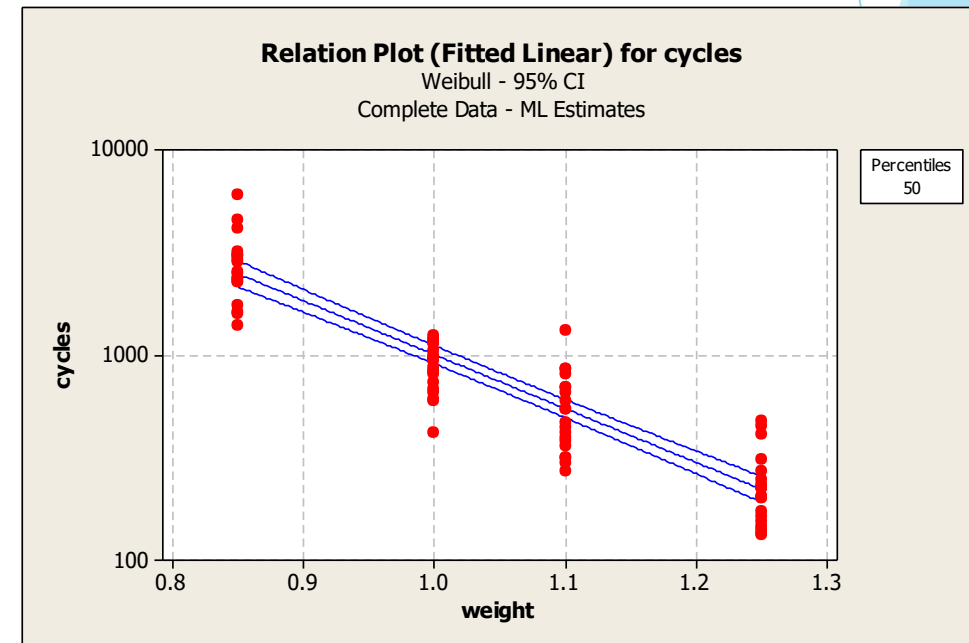
Log-Likelihood = -574.192

Prediction = $\exp [13.0887 - 6.03758 * (\text{weight}) + (1.0/2.58992)*\text{error term}]$

(The error term depends on the percentile of interest. For the 50 percentile, -0.3665 is the error term)

Weight (kg)	Prediction (cycles)
0.50	20504.5
0.75	4532.4
1.00	1001.9
1.25	221.5
1.50	49.0
1.75	10.8

point estimates



Wire Scrape

Acceleration
Factors

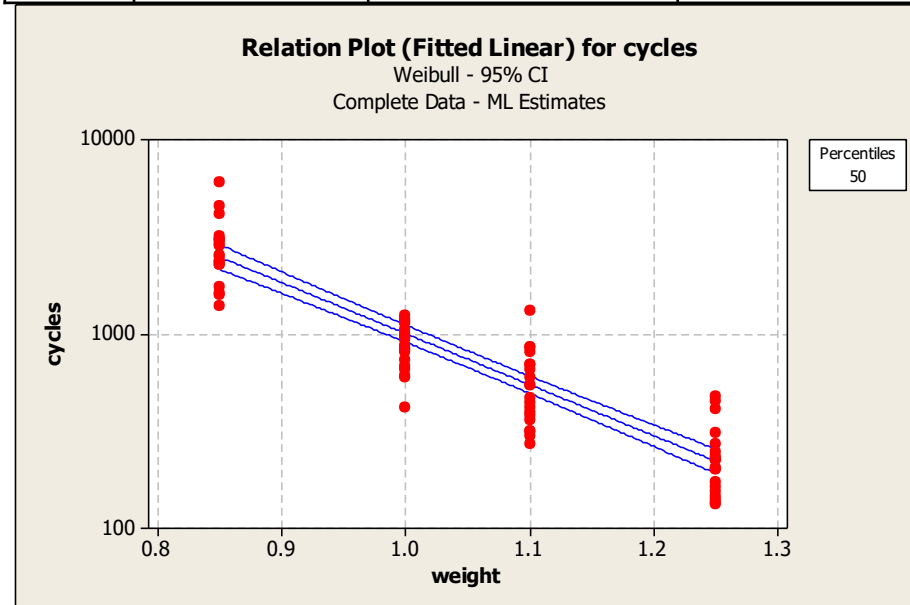
Acceleration factor in a traditional sense

Prediction = $\exp [13.0887 - 6.03758 * (\text{weight}) + (1.0/2.58992)*(-0.3665)]$

Weight (kg)	Prediction (cycles)	Acceleration factor per 1/4 Kg	Acceleration factor per 1Kg
0.50	20504.5		
0.75	4532.4		
1.00	1001.9		
1.25	221.5		
1.50	49.0		
1.75	10.8		

point
estimates

$$\frac{20504.5}{4532.4} = 4.52$$



Wire Scrape

Acceleration
Factors

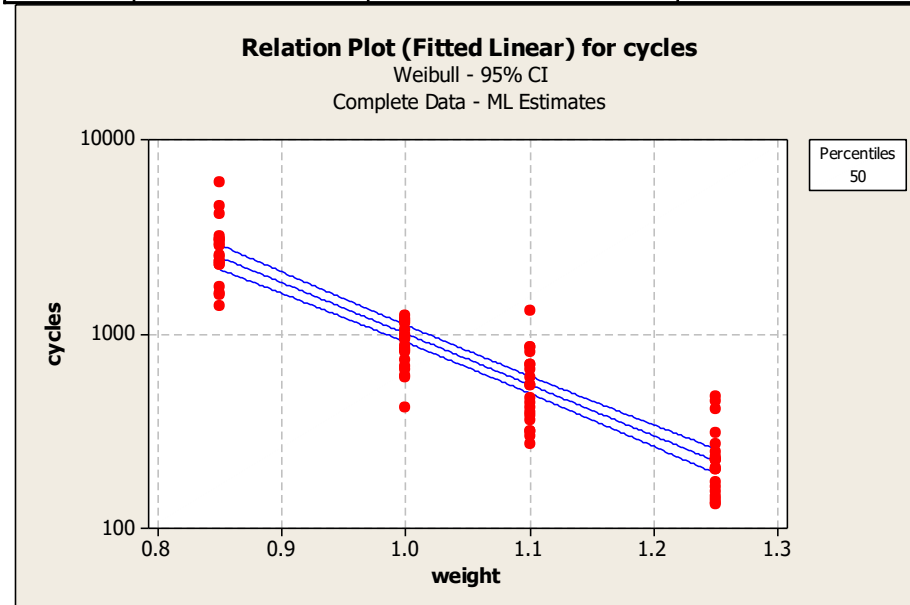
Acceleration factor in a traditional sense

Prediction = $\exp [13.0887 - 6.03758 * (\text{weight}) + (1.0/2.58992)*(-0.3665)]$

Weight (kg)	Prediction (cycles)	Acceleration factor per 1/4 Kg	Acceleration factor per 1Kg
0.50	20504.5	4.52	
0.75	4532.4	4.52	
1.00	1001.9	4.52	
1.25	221.5	4.52	
1.50	49.0	4.52	
1.75	10.8	...	

point
estimates

4.52



Wire Scrape

Acceleration
Factors

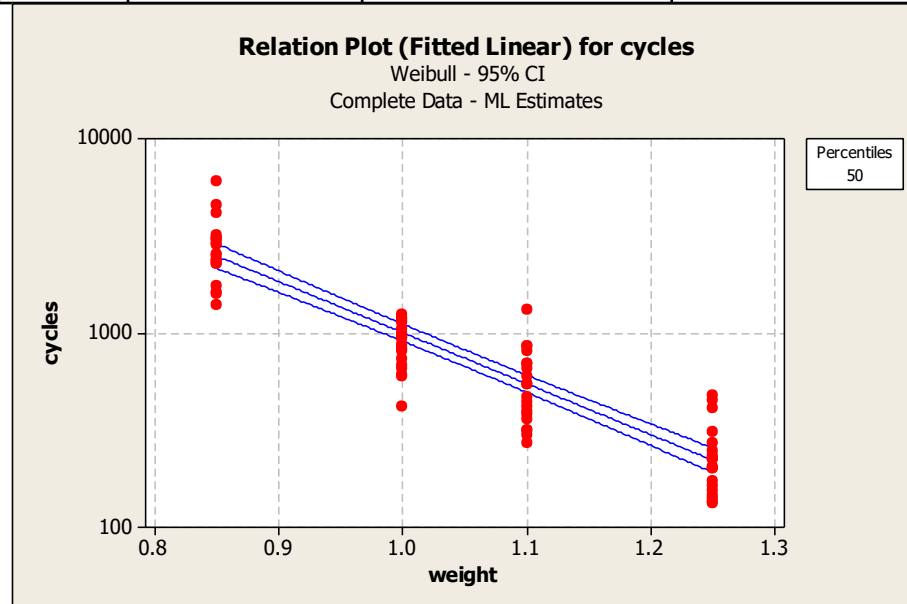
Acceleration factor in a traditional sense

Prediction = $\exp [13.0887 - 6.03758 * (\text{weight}) + (1.0/2.58992)*(-0.3665)]$

Weight (kg)	Prediction (cycles)	Acceleration factor per 1/4 Kg	Acceleration factor per 1Kg
0.50	20504.5	4.52	
0.75	4532.4	4.52	
1.00	1001.9	4.52	
1.25	221.5	4.52	
1.50	49.0	4.52	
1.75	10.8	...	

point
estimates

$$\frac{20504.5}{49.0} = 418.9$$



Wire Scrape

Acceleration
Factors

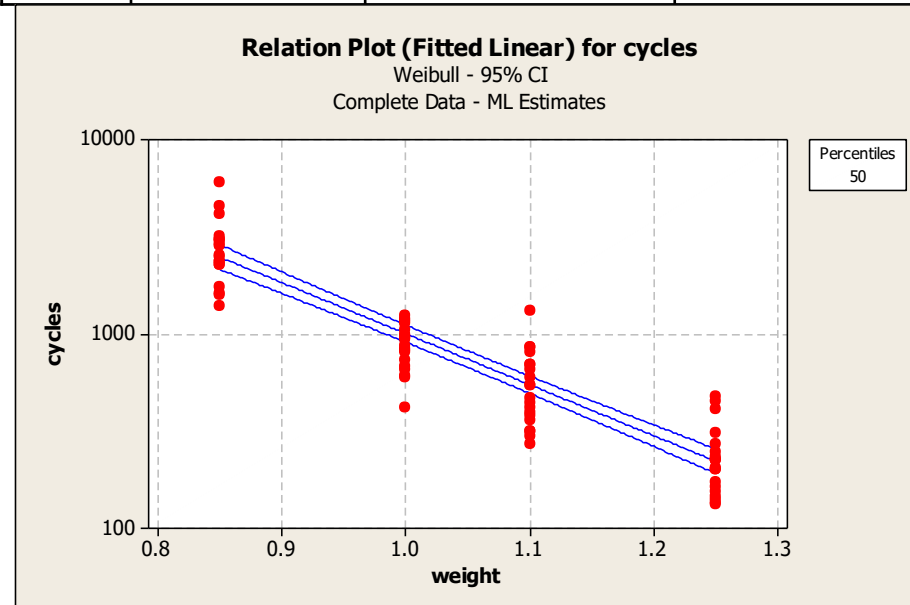
Acceleration factor in a traditional sense

Prediction = $\exp [13.0887 - 6.03758 * (\text{weight}) + (1.0/2.58992)*(-0.3665)]$

Weight (kg)	Prediction (cycles)	Acceleration factor per 1/4 Kg	Acceleration factor per 1Kg
0.50	20504.5	4.52	418.9
0.75	4532.4	4.52	418.9
1.00	1001.9	4.52	...
1.25	221.5	4.52	...
1.50	49.0	4.52	...
1.75	10.8	...	...

point
estimates

418.9



Acceleration Factors

Multiple factors

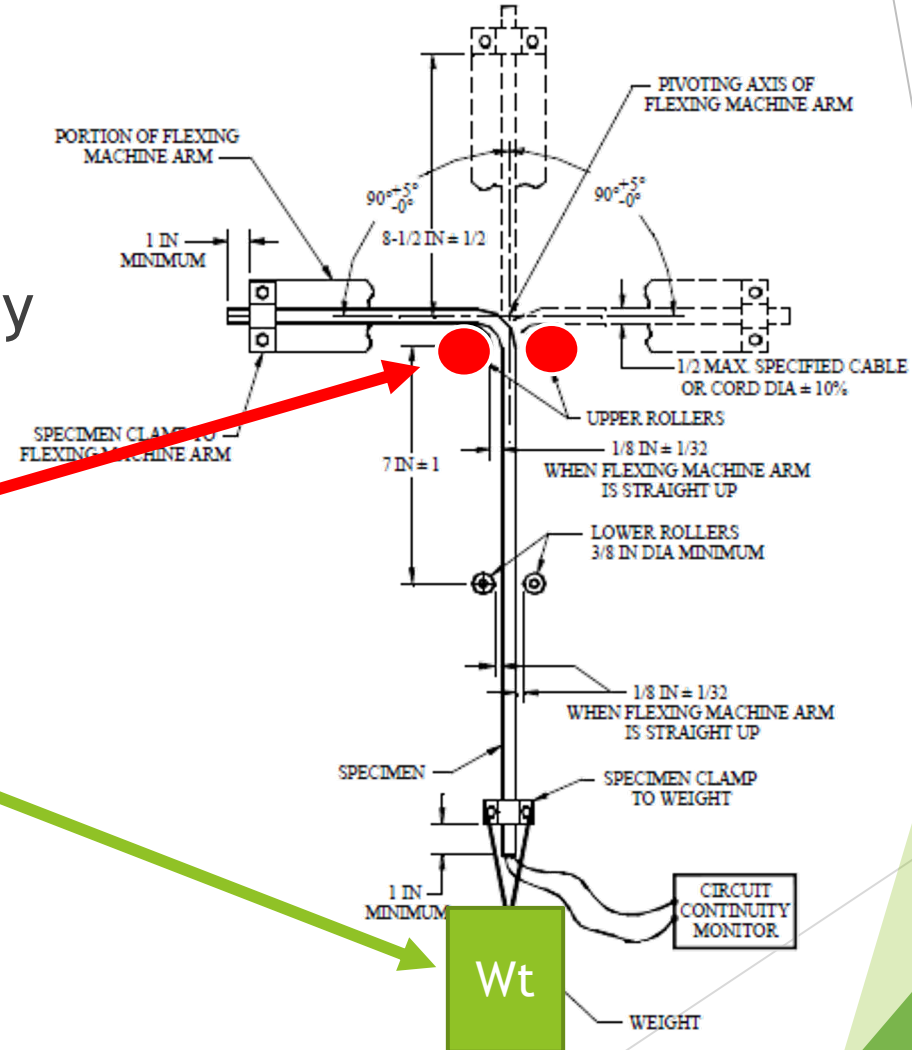
Acceleration Factors

- ▶ Reliability engineers attempt to speed up testing by “accelerating” the test
 - ▶ Environmental:
 - ▶ Increased temperature
 - ▶ Increased temperature swing
 - ▶ Adding environmental factors together
 - ▶ Sequentially testing
 - ▶ Mechanical
 - ▶ Increased speed
 - ▶ Increased weight
 - ▶ Others?
 - ▶ Multiple factors

Multiple Acceleration Factors

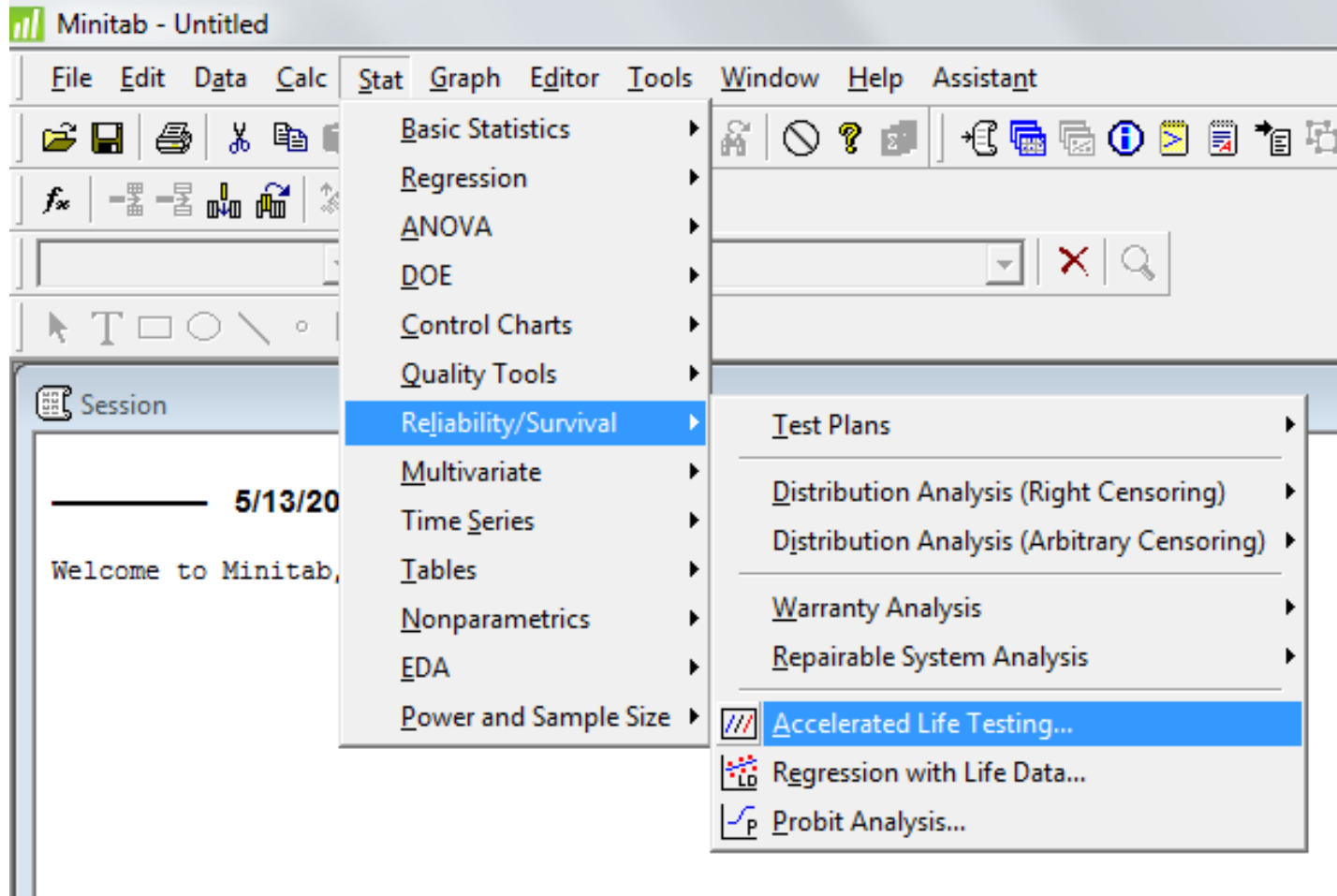
Back to: **Cable Flex**

- ▶ Checking for continuity
- ▶ Failure mode - open on any conductor
- ▶ Varying Mandrel
- ▶ Varying Weight



Multiple Acceleration Factors

Cable Flex



cycles	susp	weight (kg)	mandrel (mm)	qty
942	n	0.5	4	1
735	n	0.5	4	1
1289	n	0.5	4	1
1487	n	0.5	4	1
1222	n	0.5	4	1
2750	n	0.25	4	1
3074	n	0.25	4	1
2547	n	0.25	4	1
2992	n	0.25	4	1
4338	n	0.25	4	1
1954	n	0.25	4	1
1845	n	0.4	4	1
867	n	0.4	4	1
1004	n	0.4	4	1
1182	n	0.4	4	1
3451	n	0.4	4	1
2715	n	0.4	4	1
1071	n	0.5	4	1
831	n	0.5	4	1
1114	n	0.5	4	1
1159	n	0.5	4	1
934	n	0.5	4	1
1130	n	0.5	4	1
1150	n	0.5	4	1
1115	n	0.5	4	1
1560	n	0.5	4	1
1465	n	0.5	4	1
939	n	0.5	4	1
1343	n	0.5	4	1
1331	n	0.5	4	1
1329	n	0.5	4	1
2264	n	0.5	4	1
17701	n	1	25.4	1
12741	n	1	25.4	1
12805	n	1	25.4	1
14467	n	1	25.4	1
17301	n	1	25.4	1
18815	n	1	25.4	1
21800	n	0.5	25.4	1
19710	n	0.5	25.4	1
29224	n	0.5	25.4	1
35248	n	0.5	25.4	1
24332	n	0.5	25.4	1
24865	n	0.5	25.4	1
33948	n	0.5	25.4	1
33944	n	0.25	25.4	1
29157	n	0.25	25.4	1
32291	n	0.25	25.4	1
34150	n	0.25	25.4	1
31960	n	0.25	25.4	1
29278	n	0.25	25.4	1
70000	s	0.25	25.4	1

Multiple Acceleration Factors

Cable Flex

Accelerated Life Testing

☒ Responses are uncens/right censored data
☐ Responses are uncens/arbitrarily censored data

Variables/Start variables: cycles

End variables:

Freq. columns: (optional) qty

Accelerating var: weight Relationship: Linear

☒ Second Variable
☒ Accelerating: mandrelmm Relationship: Linear

☐ Factor:

☐ Include interaction term between variables

Assumed distribution: Weibull

Select

Help

OK

Cancel

Accelerated Life Testing - Graphs

Plots based on fitted model

Design value to include on plots:

☒ Relation plot
Plot percentiles for percents: 50

☒ Display confidence intervals for middle percentile
☐ Display confidence intervals for all percentiles
☐ Display no confidence intervals
☒ Display failure times on plot

☒ Probability plot for each accelerating level based on fitted model
☒ Display confidence intervals for design value
☐ Display confidence intervals for all levels
☐ Display no confidence intervals

Diagnostic Plots

☐ Probability plot for each accelerating level based on individual fits
☐ Probability plot for standardized residuals
☐ Exponential probability plot for Cox-Snell residuals
☐ Display confidence intervals on diagnostic plots

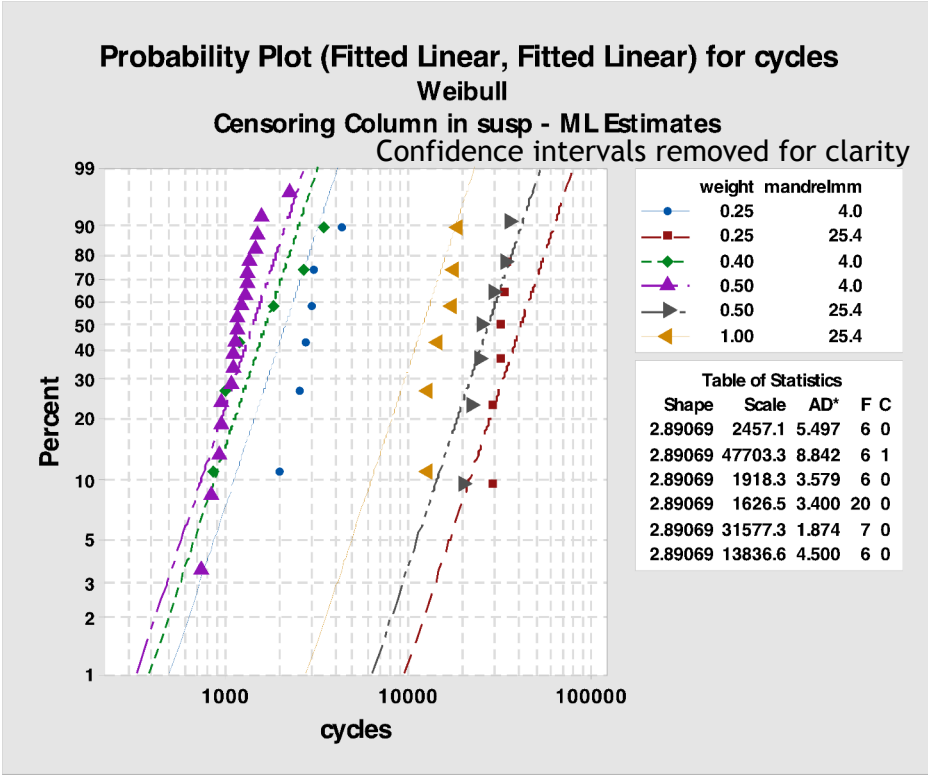
Help

OK

Cancel

Multiple Acceleration Factors

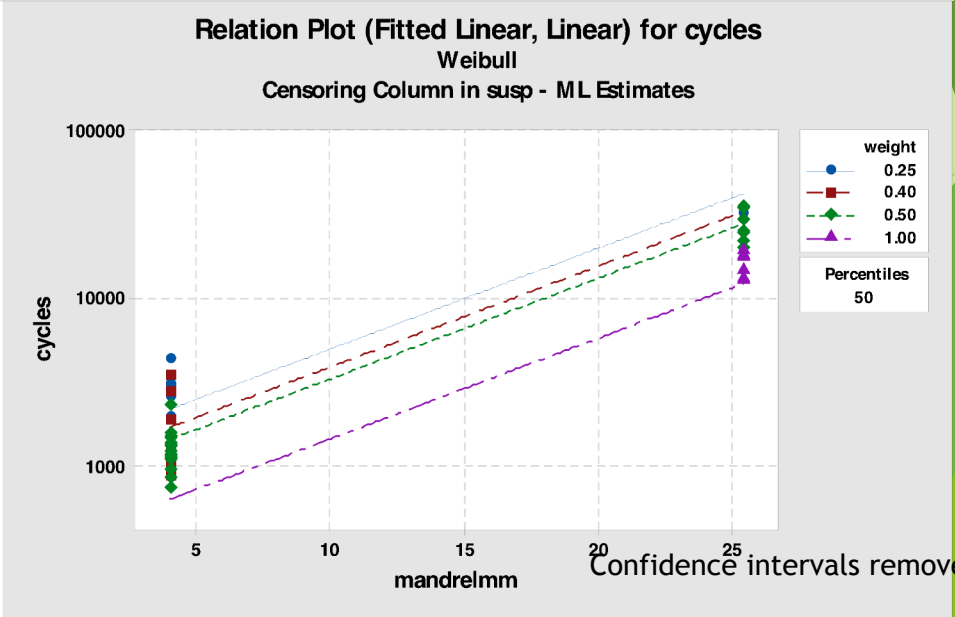
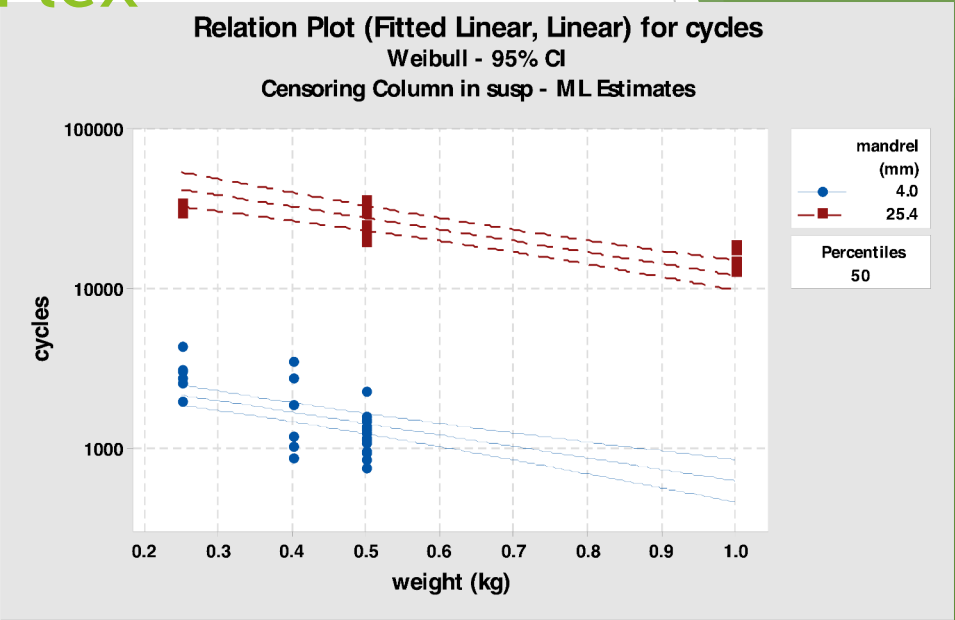
Cable Flex



Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI	
Intercept	7.66490	0.106594	71.91	0.000	7.45598	7.87382
weight	-1.65024	0.225415	-7.32	0.000	-2.09205	-1.20843
Mandrel-mm	0.138599	0.0056508	24.53	0.000	0.127524	0.149675
Shape	2.89069	0.296798			2.36377	3.53506

Log-Likelihood = -449.260



Multiple Acceleration Factors

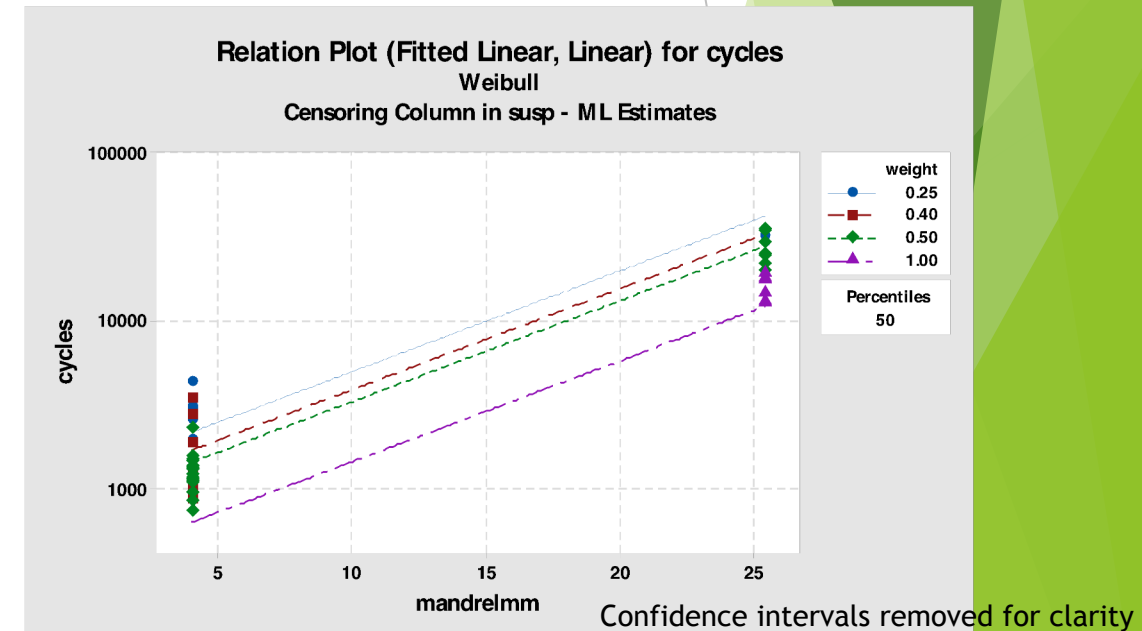
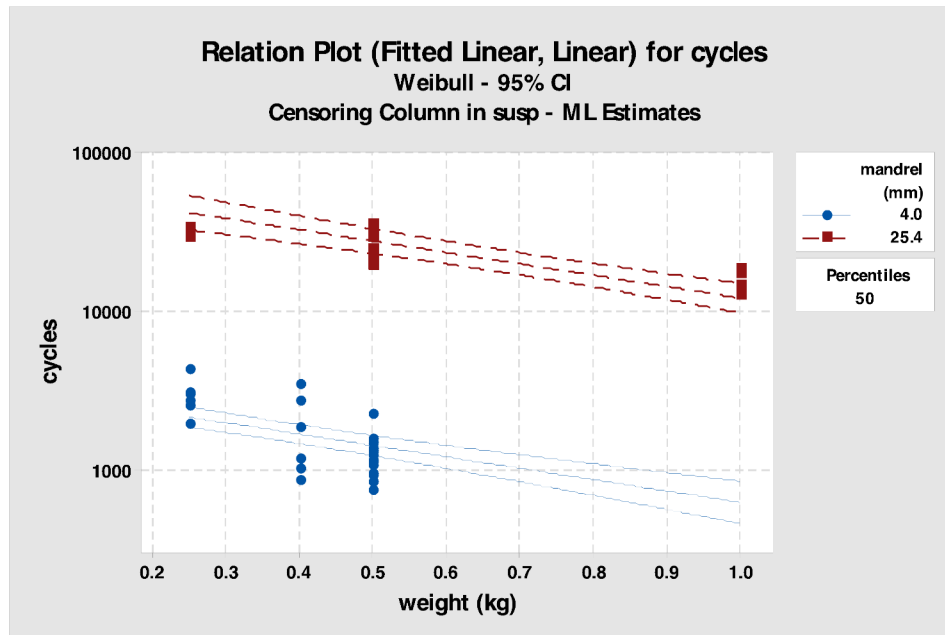
Cable Flex

Regression Table

Predictor	Coef	Standard Error	Z	P	95.0% Normal CI	
Intercept	7.66490	0.106594	71.91	0.000	7.45598	7.87382
weight	-1.65024	0.225415	-7.32	0.000	-2.09205	-1.20843
Mandrel-mm	0.138599	0.0056508	24.53	0.000	0.127524	0.149675
Shape	2.89069	0.296798			2.36377	3.53506

Log-Likelihood = -449.260

Prediction = $\exp [7.66490 - 1.65024 * (\text{weight}) + 0.138599 (\text{Mandrel-mm}) + (1.0/2.89069) * \text{error term}]$
 *Error term = -0.3665 for 50%



Multiple Acceleration Factors

Cable Flex

Acceleration factor in a traditional sense

Prediction = $\exp [7.66490 - 1.65024 * (\text{weight}) + 0.138599 (\text{Mandrel-mm}) + (1.0/2.89069) * (-0.3665)]$

Weight (kg)	Mandrel (mm)	Prediction (cycle)	Acceleration factor per 1/2 Kg	Acceleration factor per 5mm mandrel increase
0.5	5.0	1649.0		
0.5	10.0	3304.0		
0.5	15.0	6620.0		
1.0	5.0	722.6		
1.0	10.0	1447.8		
1.0	15.0	2900.7		

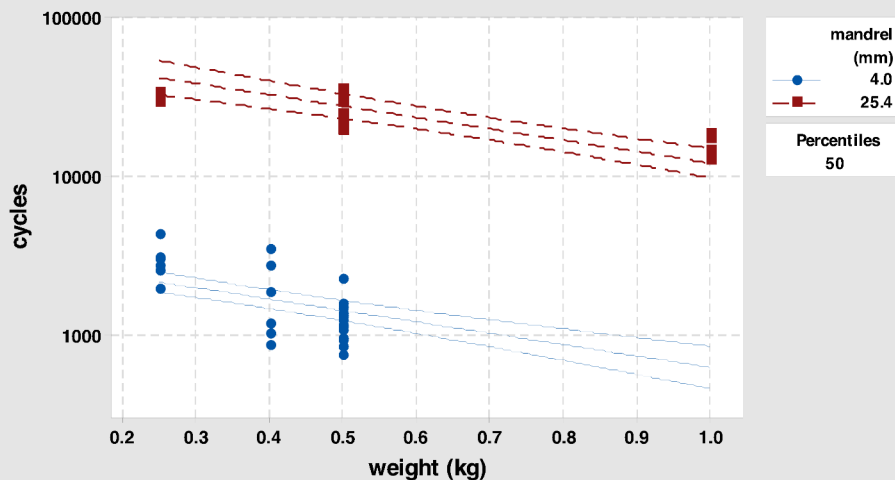
point estimates

$$\frac{1621.8}{710.6} = 2.28$$

Relation Plot (Fitted Linear, Linear) for cycles

Weibull - 95% CI

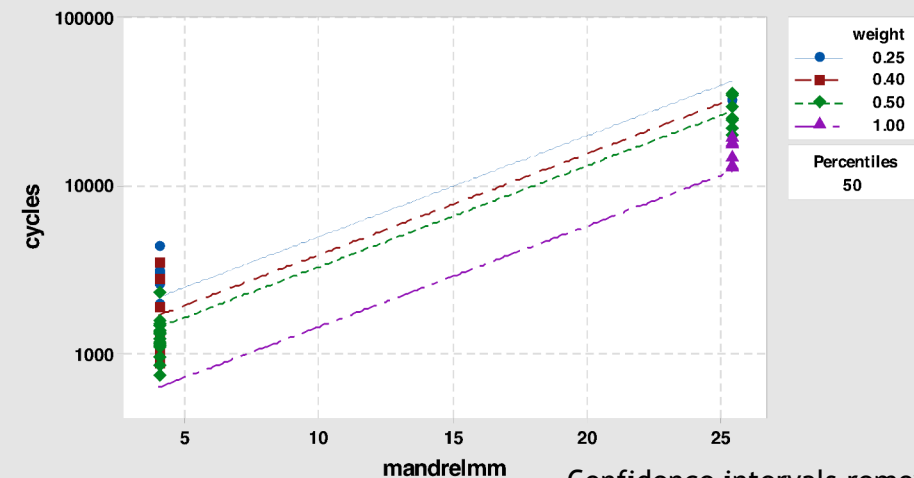
Censoring Column in susp - ML Estimates



Relation Plot (Fitted Linear, Linear) for cycles

Weibull

Censoring Column in susp - ML Estimates



Confidence intervals removed for clarity

Multiple Acceleration Factors

Cable Flex

Acceleration factor in a traditional sense

Prediction = $\exp [7.66490 - 1.65024 * (\text{weight}) + 0.138599 (\text{Mandrel-mm}) + (1.0/2.89069) * (-0.3665)]$

Weight (kg)	Mandrel (mm)	Prediction (cycle)	Acceleration factor per 1/2 Kg	Acceleration factor per 5mm mandrel increase
0.5	5.0	1649.0	2.28	
0.5	10.0	3304.0	2.28	
0.5	15.0	6620.0	2.28	
1.0	5.0	722.6	...	
1.0	10.0	1447.8	...	
1.0	15.0	2900.7	...	

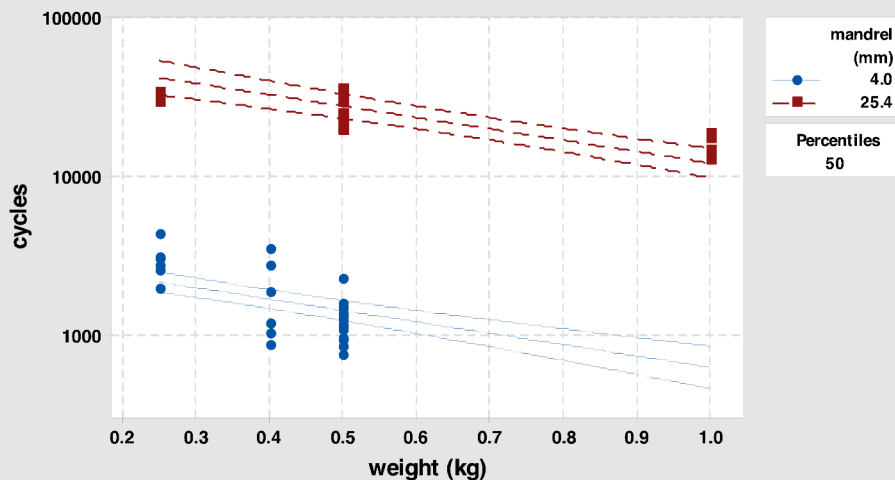
point estimates

$$\frac{3304.0}{1649.0} = 2.00$$

Relation Plot (Fitted Linear, Linear) for cycles

Weibull - 95% CI

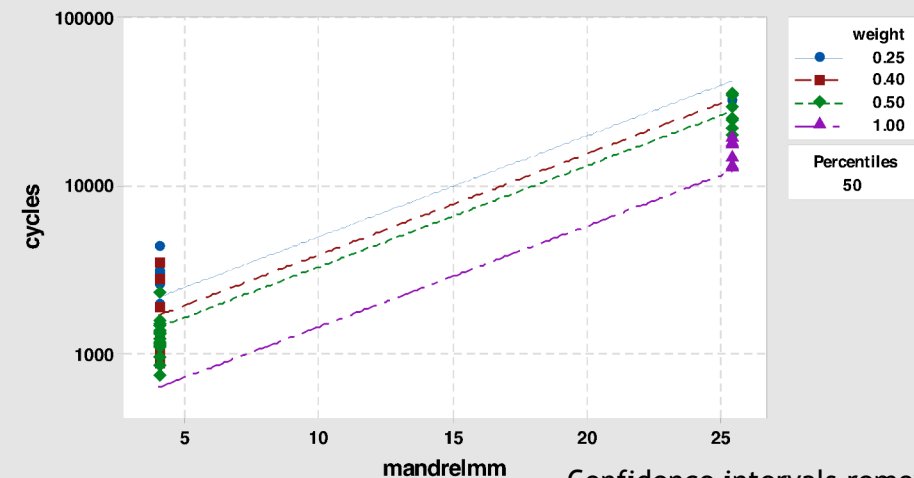
Censoring Column in susp - ML Estimates



Relation Plot (Fitted Linear, Linear) for cycles

Weibull

Censoring Column in susp - ML Estimates



Confidence intervals removed for clarity

Multiple Acceleration Factors

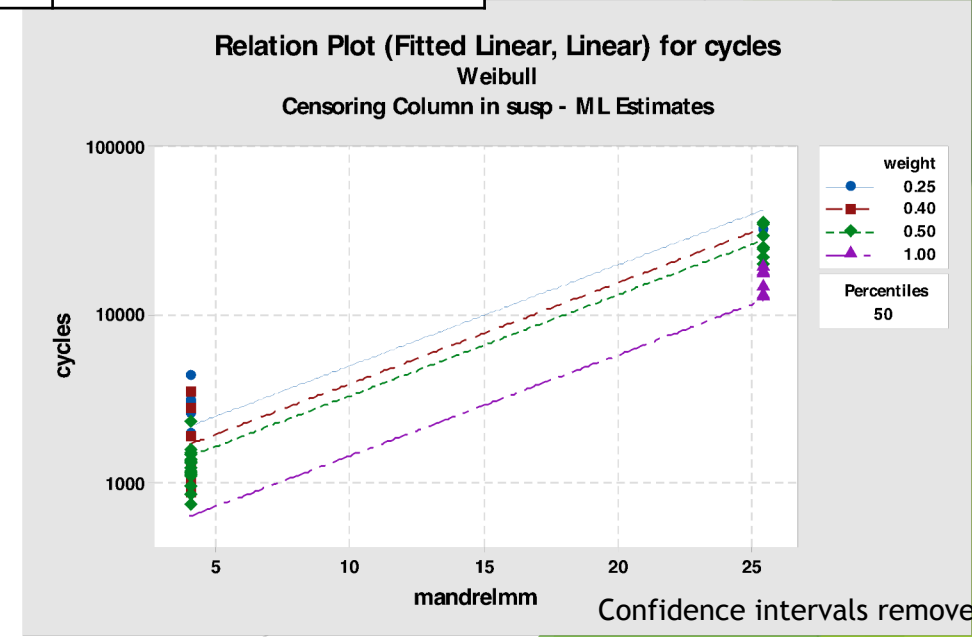
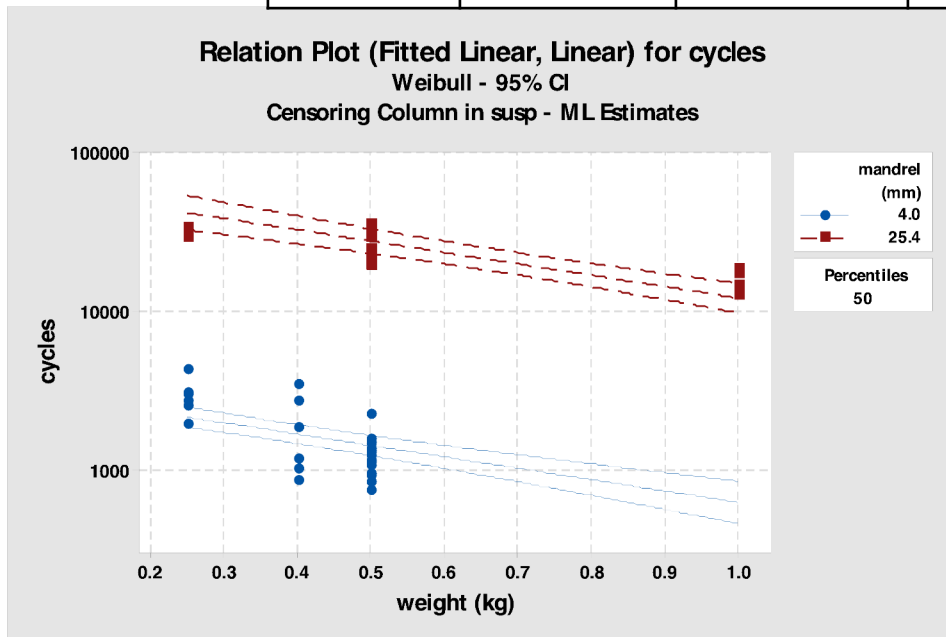
Cable Flex

Acceleration factor in a traditional sense

Prediction = $\exp [7.66490 - 1.65024 * (\text{weight}) + 0.138599 (\text{Mandrel-mm}) + (1.0/2.89069) * (-0.3665)]$

Weight (kg)	Mandrel (mm)	Prediction (cycle)	Acceleration factor per 1/2 Kg	Acceleration factor per 5mm mandrel increase
0.5	5.0	1649.0	2.28	2.00
0.5	10.0	3304.0	2.28	2.00
0.5	15.0	6620.0	2.28	...
1.0	5.0	722.6	...	2.00
1.0	10.0	1447.8	...	2.00
1.0	15.0	2900.7	...	...

point estimates



Summary

- ▶ Distribution selection can be assisted with statistics, but knowledge about your data is key
- ▶ Use Right Censoring if an exact count is available, Arbitrary Censoring is for periodic testing
- ▶ Accelerated testing can provide faster answers and testing at different levels can assist in understanding the relationship for the acceleration

Author

Rob Schubert - Robert.J.Schubert@gmail.com

Certified Reliability Engineer (ASQ), Ford Certified Six Sigma Black Belt

Bachelor's in Mechanical Engineering, Purdue

Master's in Acoustical Engineering, Penn State

Thesis: Use of Multiple-Input Single-Output Methods to Increase Repeatability of Measurements of Road Noise in Automobiles

Previous work experience:

Quality/Reliability Engineer, Shure Inc - 13 years

Quality/Reliability Engineer, 6 Sigma Black belt, Noise & Vibration Engineer, Ford Motor Company - 13 years

Previous presentations:

2 Parameter vs. 3 Parameter Weibull with a Cable Flex Test - ARS 2015

This presentation: Overview of Reliability Functions for Life Testing in Minitab - Minitab Insights 2016

Practical Use of Stress-Strength Models to Develop Specifications - ARDC 2018 - **Perhaps Accendo Reliability?**

Thank you for your attention.

What are your questions?