

Confidence in Reliability

Is good is our best guess?



WORKBOOK

Presented by

Dr Chris Jackson

Reliability engineer



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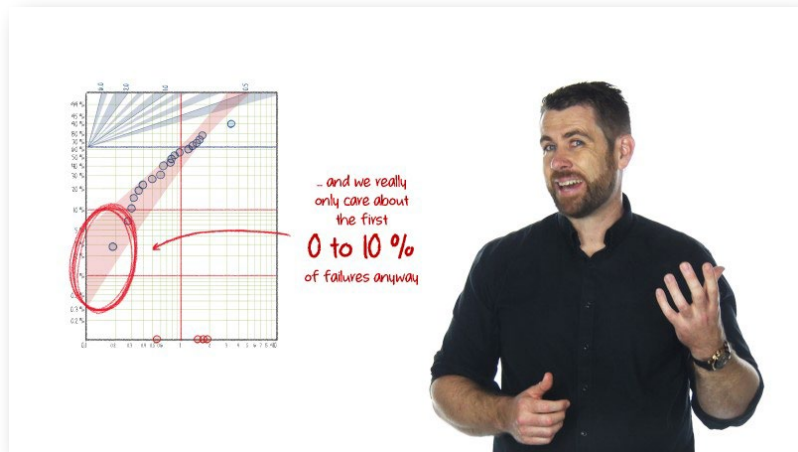
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Who is your teacher? ... Dr Chris Jackson

Dr Jackson holds a Ph.D. and MSc in Reliability Engineering from the University of Maryland's Center of Risk and Reliability. He also holds a BE in Mechanical Engineering, a Masters of Military Science from the Australian National University (ANU) and has substantial experience in the field of leadership, management, risk, reliability and maintainability. He is a Certified Reliability Engineer (CRE) through the American Society of Quality (ASQ) and a Chartered Professional Engineer (CPEng) through Engineers Australia.

Dr Jackson is the cofounder of IS4 online learning, was the inaugural director of the University of California, Los Angeles (UCLA's) Center of Reliability and Resilience Engineering and the founder of its Center for the Safety and Reliability of Autonomous Systems (SARAS). He has had an extensive career as a Senior Reliability Engineer in the Australian Defence Force, and is the Director of Acuitas Reliability Pty Ltd.



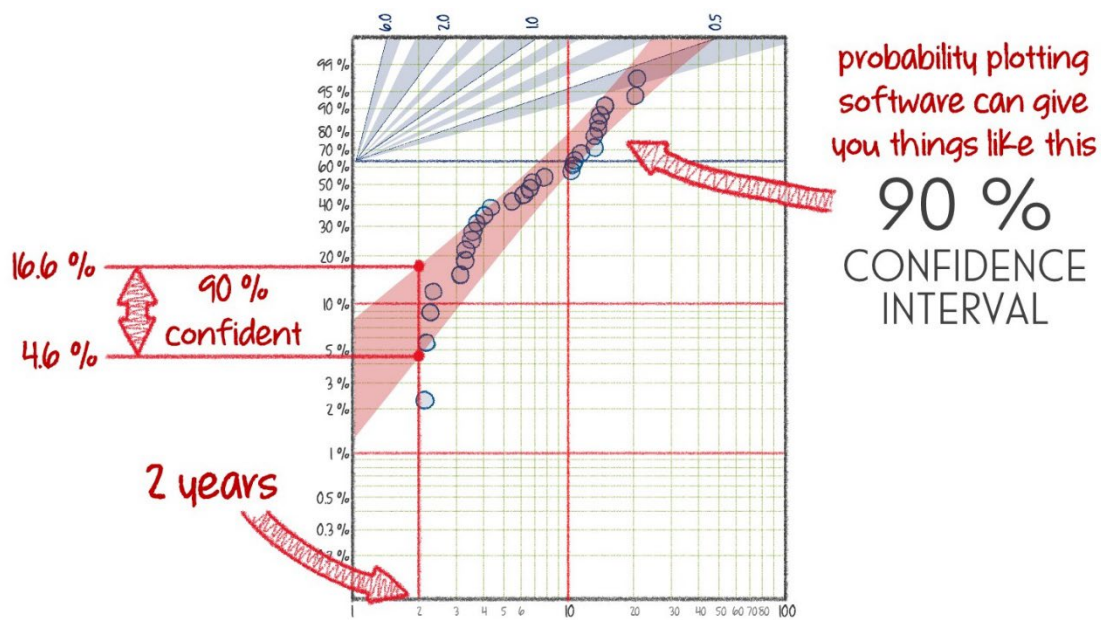
He has supported many complex and material systems develop their reliability performance and assisted in providing reliability management frameworks that support business outcomes (that is, make more money). He has been a reliability management consultant to many companies across industries ranging from medical devices to small satellites. He has also led initiatives in complementary fields such as systems engineering and performance-based management frameworks (PBMFs). He is the author of two reliability engineering books, co-author of another, and has authored several journal articles and conference papers.

Confidence in our conclusions



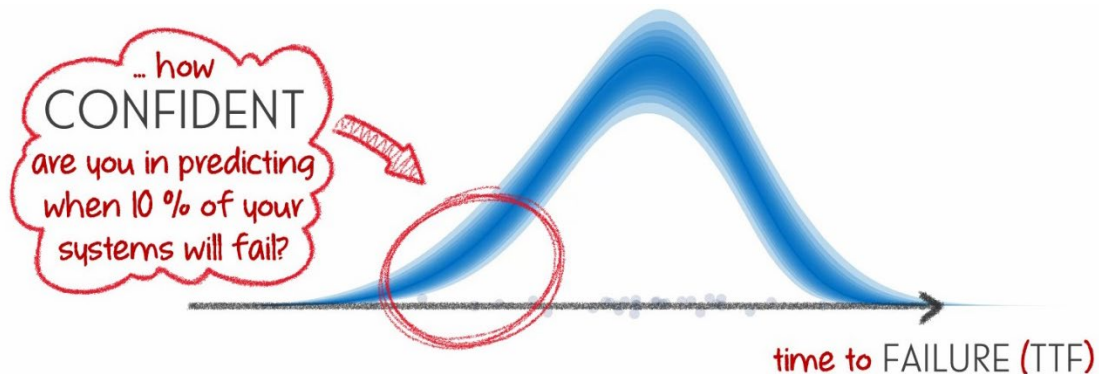
You might remember from our **Weibull Probability Plotting webinar** or any other statistical analysis course that based on the way some **random variables** behave (**probability distribution**), we can create a straight line on specially scaled charts (like the one below). In these charts, the vertical axis represents the **probability of failure** or the **Cumulative Distribution Function (CDF)**. The horizontal axis represents usage. The probability plot below is a **Weibull probability plot**. That means that if plotted data creates a straight line then the **Weibull distribution** may be a good model for our process. The example below contains time to failure data of an amazing new wireless modem router that we might want to launch soon ... perhaps with a competitive warranty. But even though we might have a 'line of best fit' ... how certain are we that it is 'right'?

If we use some commercially available **Weibull probability plotting** software, we might get the following:

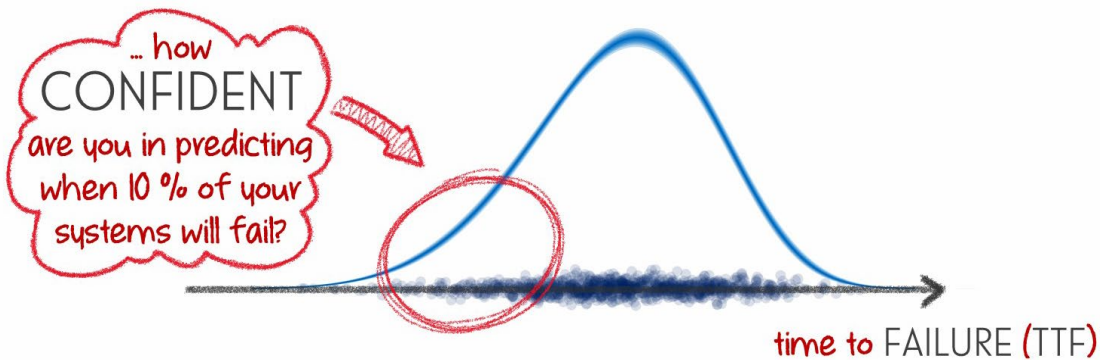


How confident are you in your conclusions now?

Remembering that data contains information, one thing we can do to reduce this uncertainty is to get more data. For example, here is the inferred **PDF curve** with **confidence intervals** based on 30 data points. The outer-most 'contours' represent a region of 90 % confidence, followed by 80 % and so on. This scenario already takes into account 'a lot' of information because we have assumed it is modelled with a **Weibull distribution**.



Compare that to the same scenario ... but with a total of 1 000 data points.



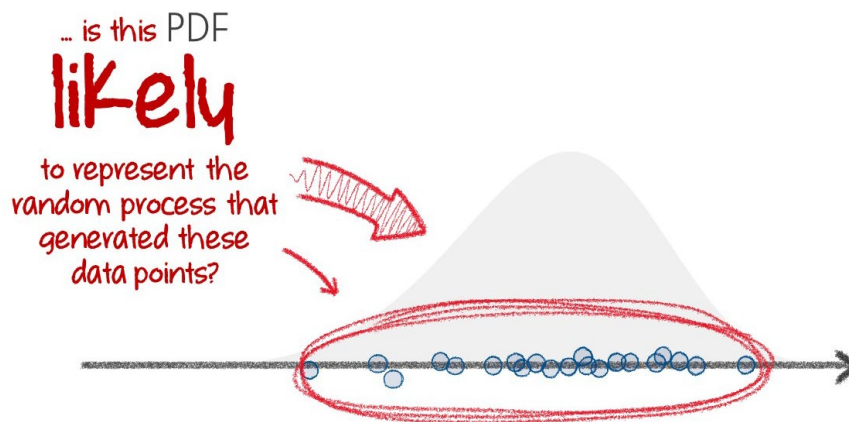
This makes sense ... the more data we have (the more information we have) and the less uncertainty we have. But how do we come up with these **confidence intervals**? ... what do they mean? ... and how do we use them?

... so where does the CONFIDENCE stuff come from? (LIKELIHOOD)

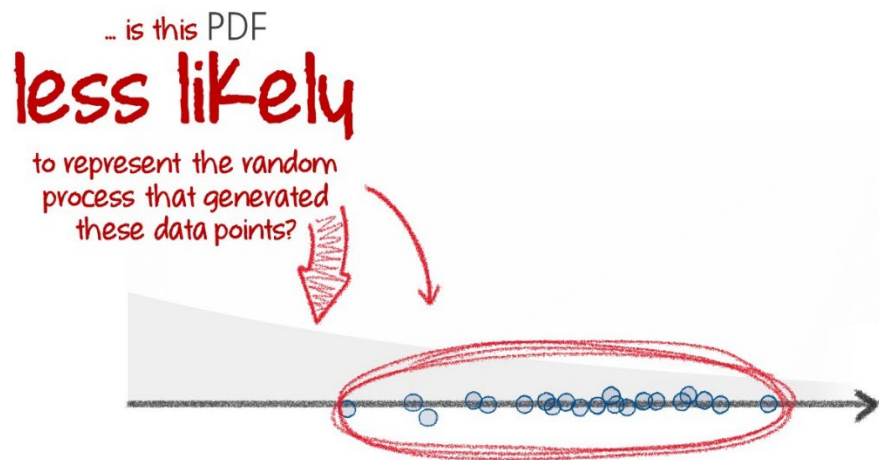
*we use the term **PROBABILITY** to describe the chances of certain things occurring – like failure*

*we use the term **LIKELIHOOD** to describe the chances of certain things **WE THINK** are true*

Let's look at a scenario where we conduct a reliability test and observe 21 **TTFs**. These values look like they are 'clustering' around a central value. This automatically suggests that the **probability distribution curve** that describes this random (failure) process has a 'bell-like' shape.

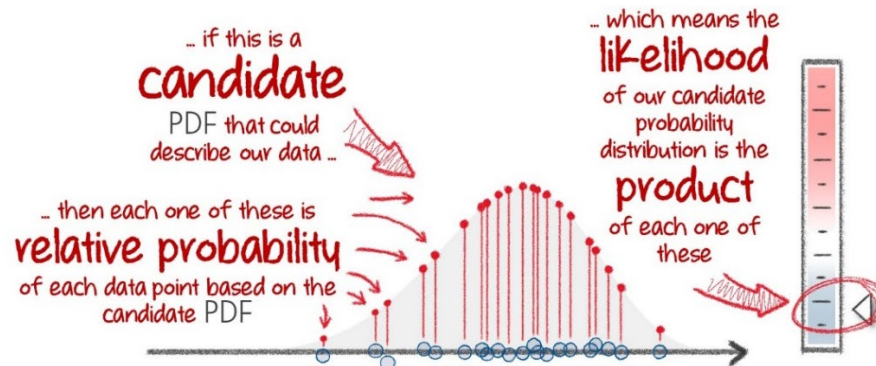


What about the **exponential distribution**?

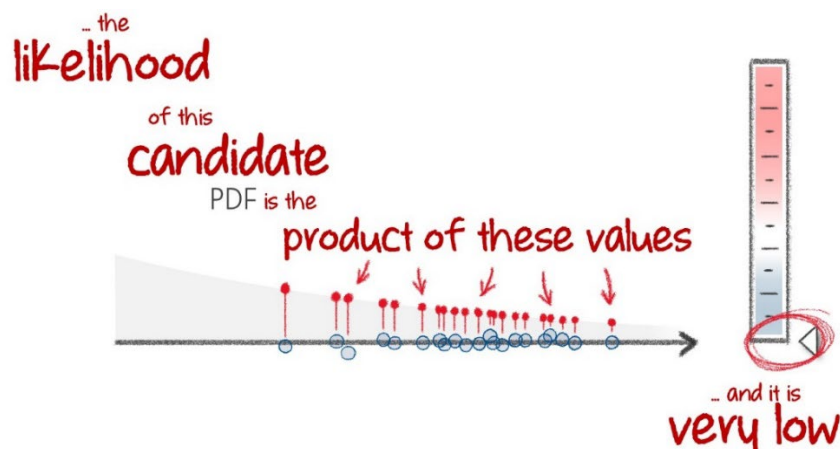


Hopefully you agree that the top **PDF curve** represents a **probability distribution** that is MUCH more likely to describe the underlying **random (failure) process**. But how do we quantify this **likelihood**?

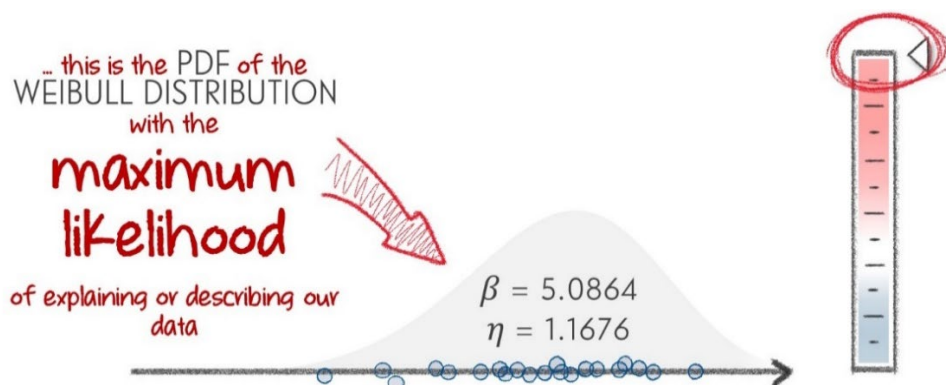
Well ... to do that, all we need to do is multiply the **PDF values** of any hypothetical **probability distribution** at the data point values. The **PDF** provides us the relative probability of a **random variable** having a particular value ... so the product of all these **PDF values** represents the **intersection** of all the random variable value events. Which is why we multiply them.



The product of these **PDF values (likelihood)** for the hypothetical **probability distribution** represented by the **PDF curve** above is represented on the scale on the right-hand side of the illustration. Now let's compare this likelihood with the likelihood of our hypothetical **exponential distribution**.

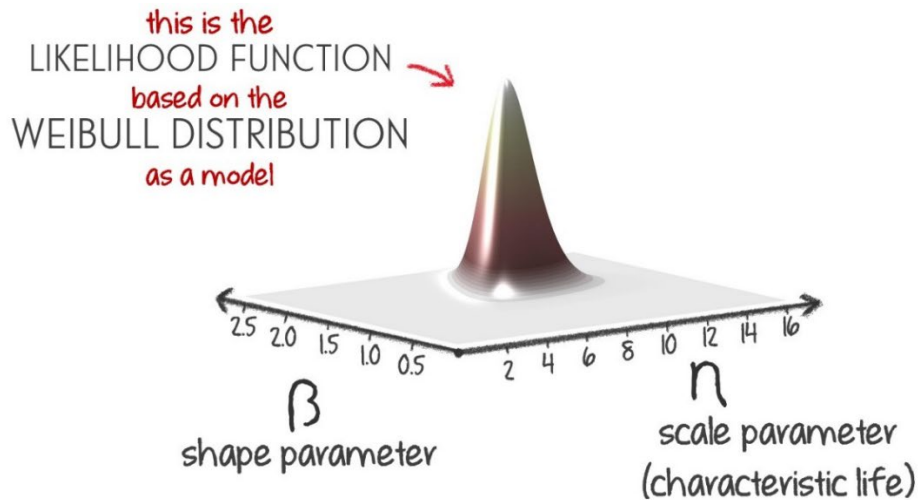


And what is the 'most likely' hypothetical **Weibull distribution**? The one below. And look how high its **likelihood** is.

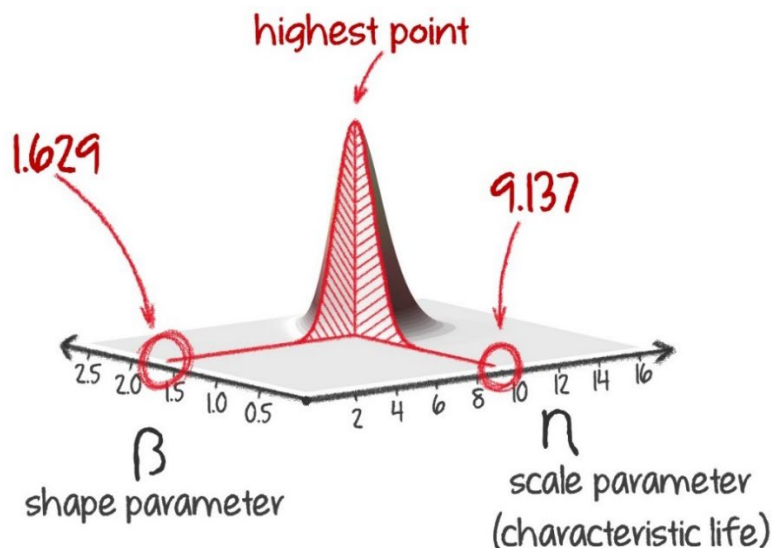


... it is hard to use these LIKELIHOODs to understand CONFIDENCE

Remembering that the **exponential distribution** is a particular form of the **Weibull distribution** ($\beta = 1$), the three hypothetical distributions on the previous page were all different versions of the **Weibull distribution**. And each different Weibull distribution is described by two parameters (**scale** - η and **shape** - β). So for each parameter (η, β) pair, we can work out the **likelihood** that they are 'the correct' parameter values. As is illustrated below.

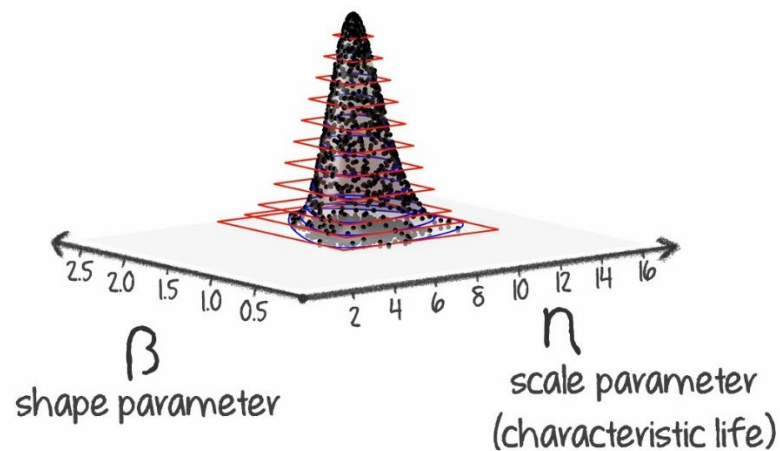


You can see that there is a nice 'smooth' surface that clearly peaks around a particular point.

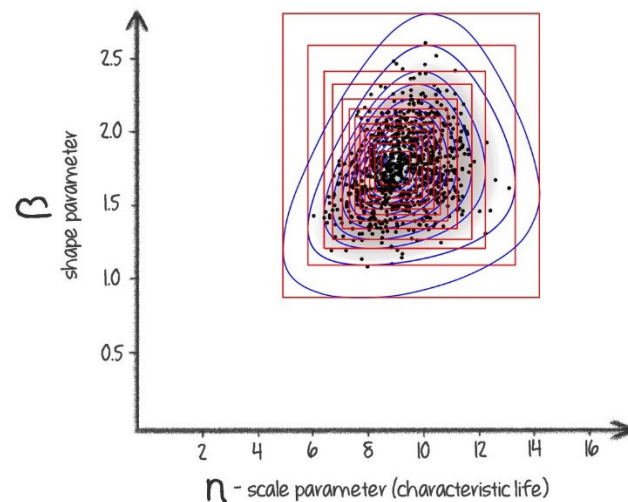


The 'highest' point occurs when the **scale parameter** (η) is equal to 9.137 and the **shape parameter** (β) is equal to 1.629. This pair of values are what we call the **Maximum Likelihood Estimate (MLE)**. You might remember in our [Probability plotting](#) lesson that the parameters we got from our 'line of best fit' were $\eta = 9.2$ and $\beta = 1.6$. Not bad!

So what do we do with this surface? Well ... this is where we need some 'heavy duty' computer power. We use this approach called **Markov Chain Monte Carlo (MCMC) simulation** that takes this surface and outputs a large number of parameter 'pairs.'

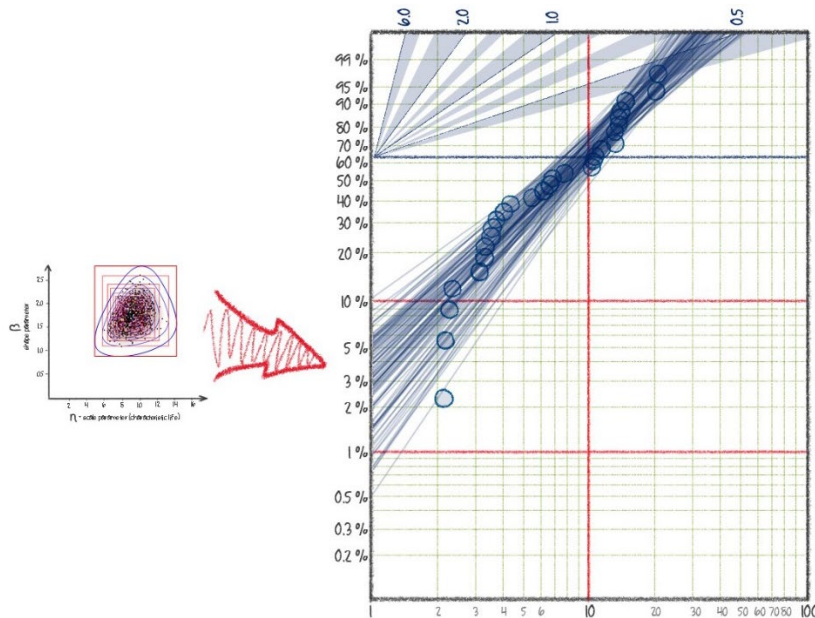


The idea is that the density of these parameter 'pairs' is proportional to the height of this **likelihood surface**. So when we look at the **likelihood surface** from above, we can see that the 'sampled pair' density is highest where the **likelihood surface** is highest.

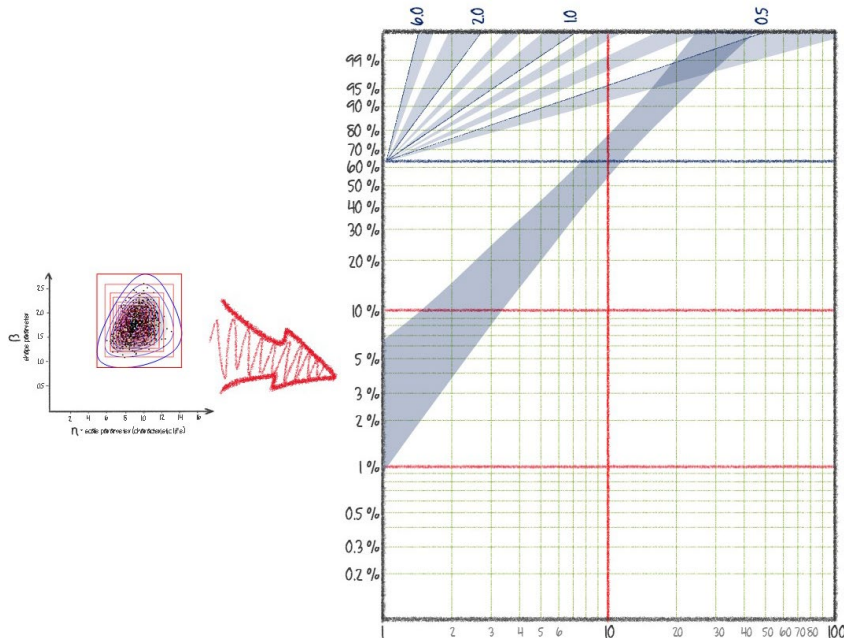


This is only a relatively small number of samples. **MCMC** can spit out tens of thousands, hundreds of thousands, or millions of these sampled 'parameter pairs.' So what do we do with these now?

Well, every one of these parameter pairs defines a specific **Weibull distribution**. In fact, we can plot a line for each and every one of these on our Weibull probability plot.

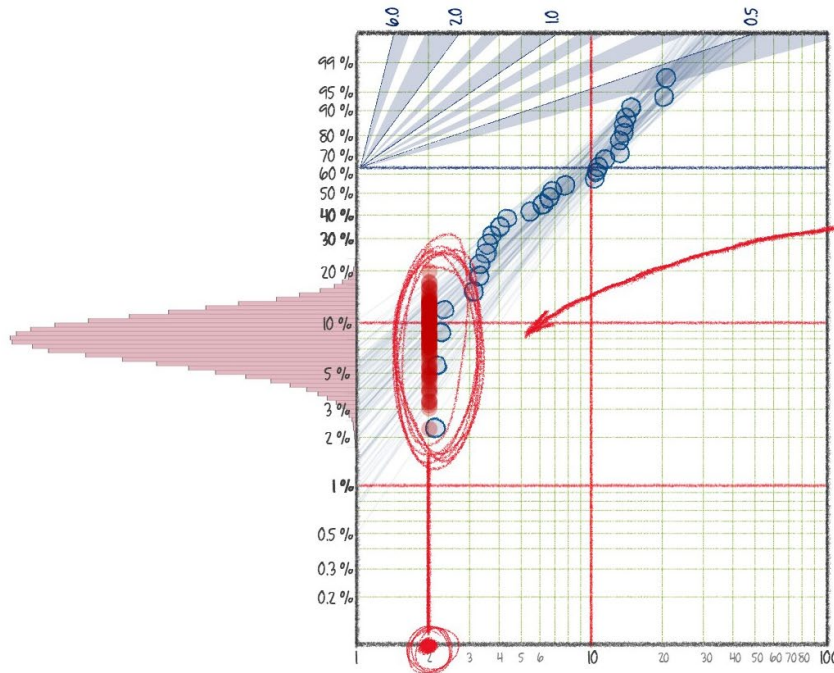


And if you have tens of thousands, hundreds of thousands, or millions of sampled 'parameter pairs' we can get a confidence interval ... like the 90 % **confidence interval** below.



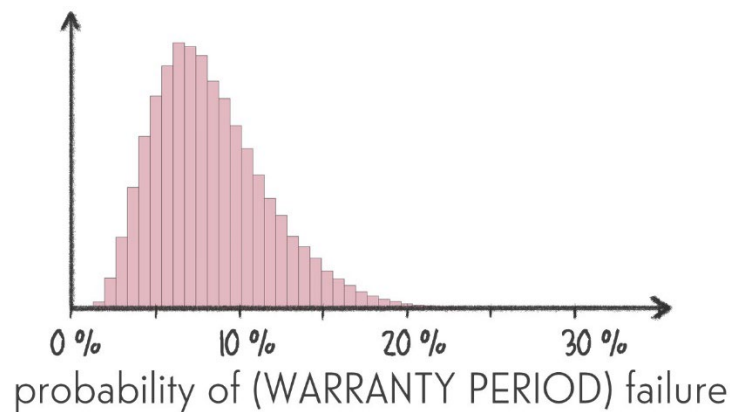
This might seem like progress (and it is) ... but in reality ... those sampled 'parameter pairs' can be used in a much better way.

Let's go back to all those lines for each sampled 'parameter pair.' Remember we want to understand the 2-year warranty reliability of our wireless modem router? Well, we can find the **CDF** value of each sampled 'parameter pair' or line at 2 years. Which we can (for example) turn into a **histogram**.

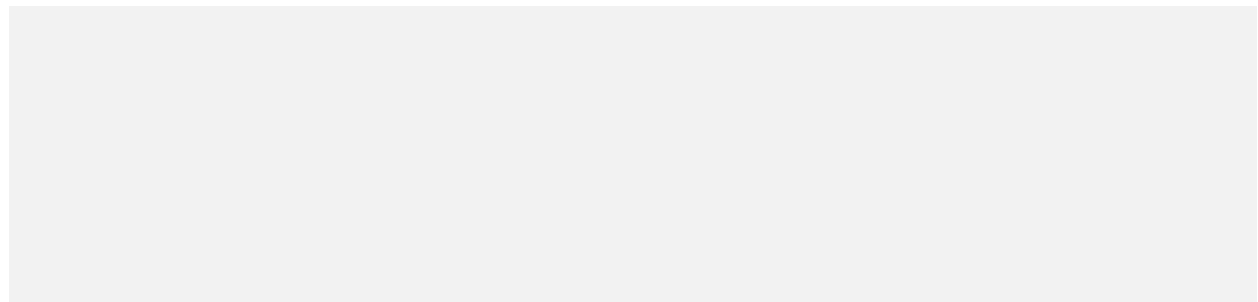


... which
means a
sample
of CDF
estimates at
2 years

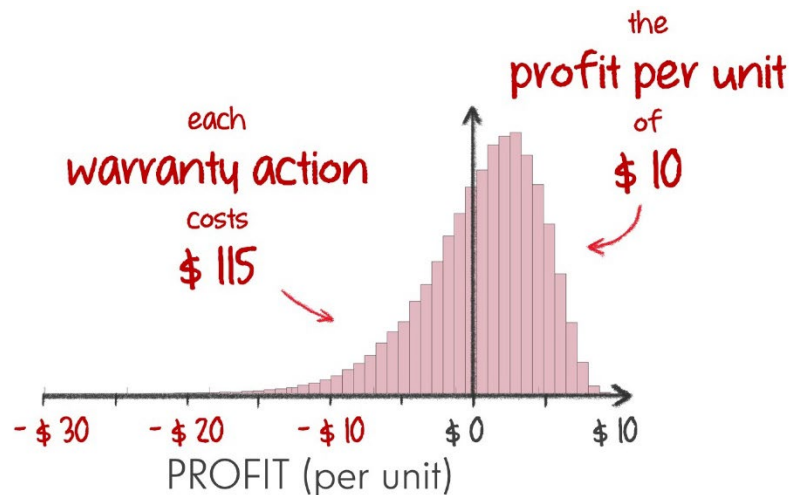
Now we are getting somewhere! Because this **histogram** can go on its own chart:



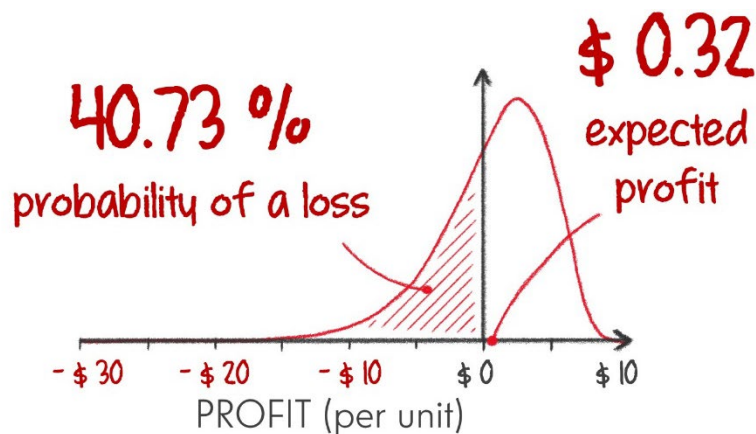
But there is still lots more we can do.



In this example, we might perhaps know that each wireless modem router sells for \$ 125. We also know that the cost of each wireless modem router is \$ 115. This leaves \$ 10 profit per unit. And when a warranty action is initiated, we simply provide our customer a new unit. Which of course costs us \$ 115. So we can turn the **histrogram** on the previous page that helps us understand the **warranty period failure probability** into a **histrogram** that represents **expected** profit per unit.



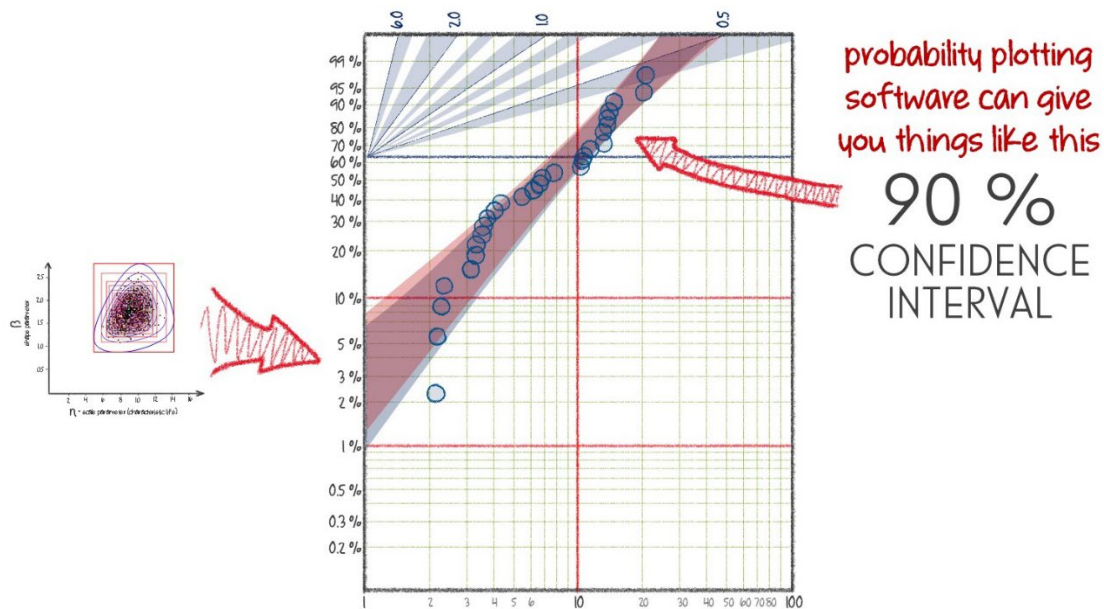
If we have enough sampled 'parameter pairs,' this **histrogram** will eventually 'smooth' out to a **likelihood curve**, like the one below. Which tells us a lot more than a simple **failure probability** estimate.



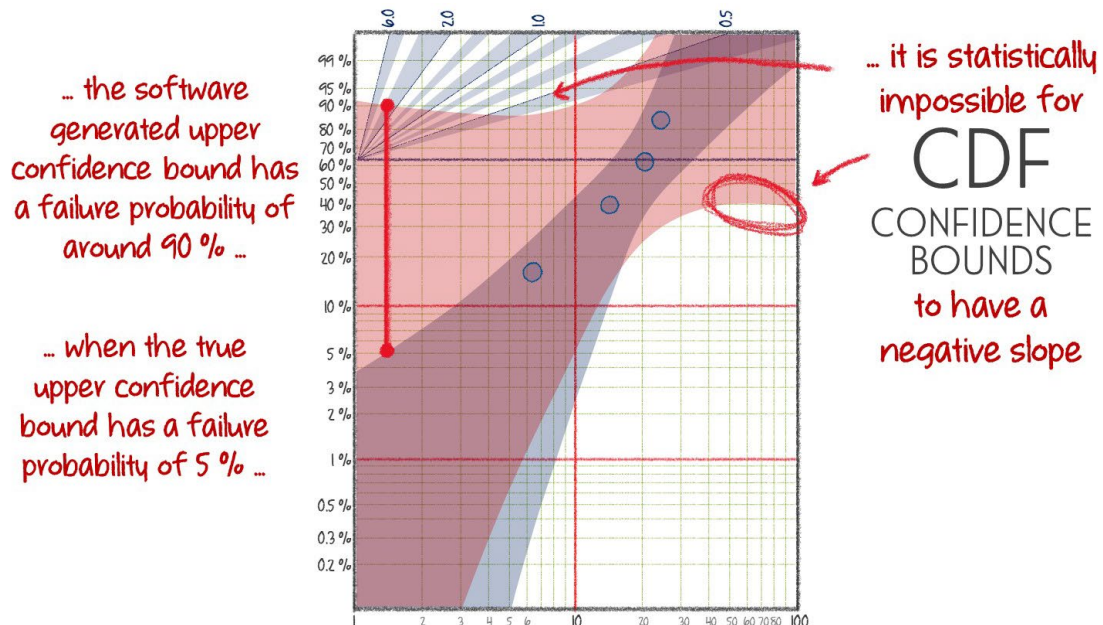
Not enough reliability engineers can do this level of analysis using **MCMC**. But they should. Because this is really useful information for a decision-maker. Do we launch? Do we invest more into wireless modem router reliability? Do we just have a 1-year warranty reliability (in which case we do the analysis again but at one year)?

... it is hard to use these LIKELIHOODs ... so we APPROXIMATE(!@#)

Let's go back to the 90 % **confidence interval** our software package generated and compare it to the 'real' 90 % **confidence interval** that we generate using the method above.



You can see that there is a difference. A small difference, but a difference nonetheless. But look what happens when we only have a small number of data points.



How confident are you in confidence now?

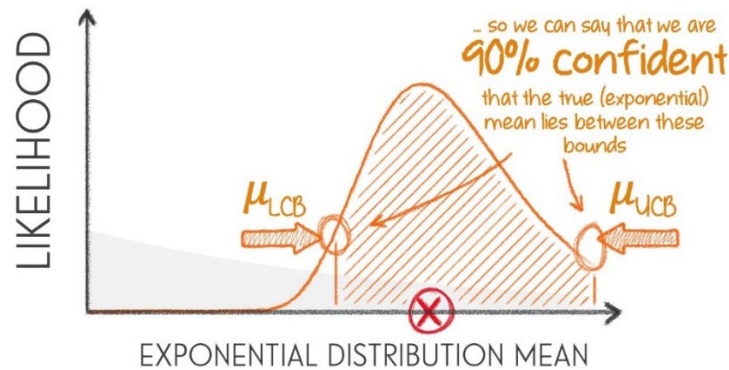
... but there are some SIMPLE SCENARIOS for confidence

... but they all tend to deal with MTBF only

Let's look at some **TTF** data below ...



We can calculate the **likelihood** for the **MTBF**, which is the only parameter an **exponential distribution** needs. And when we do, we can create a **likelihood line** that summarizes our confidence in hypothetical **MTBF** values.



And there are equations that can capture these **confidence bounds** exactly.

$$\mu_{LCB} = \frac{2T}{\chi^2_{\frac{1+CL}{2}; 2r+2}}$$

$$\mu_{UCB} = \frac{2T}{\chi^2_{\frac{1-CL}{2}; 2r+2}}$$

... where μ_{UCB} and μ_{LCB} are the **upper** and **lower confidence bounds** on our understanding of the mean respectively, T is the sum of all random data values (or the total time of a reliability test), CL is the level of **confidence** you are after for your interval, r is the total number of data points (failures), $\chi^2_{p; 2r+2}$ is the chi-squared random variable with a CDF of p and $2r + 2$ degrees of freedom.

But as a rule, we are after the one-sided **lower confidence bound** on the **mean** only. This is usually the MTBF metric we see in **reliability specifications**.

... the MTBF shall be equal to or greater than <XXX> demonstrated with a confidence of <XX>%...

The Microsoft Excel function you can use to find the **lower confidence bound** on the mean of a set of data points that are assumed to be described by the **exponential distribution** is ...

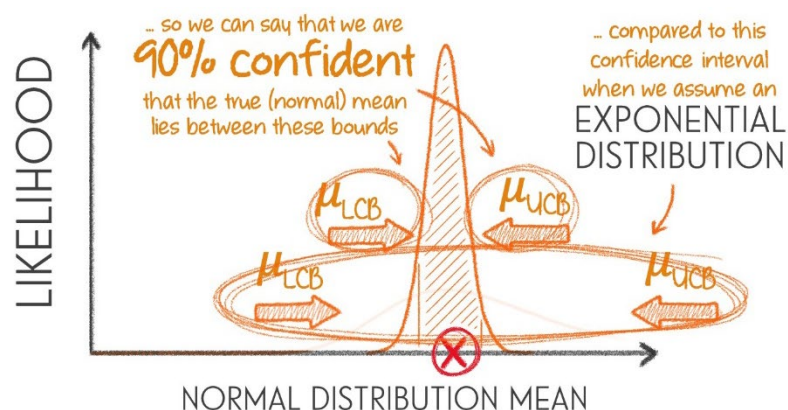


$$=2 * [T] / \text{CHISQ.INV}([CL], 2 * [r])$$

But ... we know that the **exponential distribution** is not a good fit (just by looking at the data).



If we now assume that **TTF** is modelled by the **normal distribution**, the change to our confidence is considerable.



You can see how important it is to get the model right – as the model represents information.

And there are equations that can capture these **confidence bounds** exactly.

$$\mu_{LCB} = \bar{x} + \left(t_{\frac{1-CL}{2}; r-1} \right) \frac{s}{\sqrt{r}}$$

$$\mu_{UCB} = \bar{x} + \left(t_{\frac{1+CL}{2}; r-1} \right) \frac{s}{\sqrt{r}}$$

... where μ_{UCB} and μ_{LCB} are the **upper** and **lower confidence bounds** on our understanding of the **mean** respectively,
 \bar{x} and s are the sample **mean** and **standard deviation** respectively,
 r is the total number of data points (failures),
and $t_{p; r-1}$ is the student's t random variable with a CDF of p and $r - 1$ degrees of freedom.

But again, as a rule, we are after the one-sided **lower confidence bound** on the mean only. The Microsoft Excel function you can use to find the **lower confidence bound** on the **mean** of a set of data points that are assumed to be described by the **normal distribution** is ...



= [X-BAR] + (T . INV (1 - [CL] ,
[r] - 1) * [s] / SQRT ([R]))

EXERCISE

Why do you think we are **MORE CONFIDENT** about the mean when we assume a **NORMAL DISTRIBUTION**?

... reliability measures your SYSTEM ... CONFIDENCE is about YOU



Regardless, some sort of data analysis is inevitable when it comes to **reliability engineering**. And a lot of the time we forget that our 'line of best fit' is simply our 'best guess.' And with guesses come uncertainty. Some 'best guesses' have lots of uncertainty. And if you can't understand that, then you risk making disastrous business or design decisions. Over and over again.