

What is Weibayes analysis?

... there is a story in even the simplest data sets ...



Workbook

Presented by

Dr Chris Jackson

Reliability engineer



Copyright © 2024 Christopher Jackson

All rights reserved. No part of this publication may be reproduced, distributed or transmitted in any form or by any means, including photocopying or other electronic or mechanical methods without the prior written permission of the author (Christopher Jackson), except as permitted under Section 107 or 108 of the 1976 United States Copyright Act and certain other non-commercial uses permitted by copyright law.

For permission requests, write to Christopher Jackson using the details on the following pages.

Limit of Liability/Disclaimer of Warranty: While the author has used his best efforts in preparing this document, he makes no representations or warranties with respect to the accuracy or completeness of the contents of this document and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. The author shall not be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

Who is your teacher? ... Dr Chris Jackson

Dr Jackson holds a Ph.D. and MSc in Reliability Engineering from the University of Maryland's Center of Risk and Reliability. He also holds a BE in Mechanical Engineering, a Masters of Military Science from the Australian National University (ANU) and has substantial experience in the field of leadership, management, risk, reliability and maintainability. He is a Certified Reliability Engineer (CRE) through the American Society of Quality (ASQ) and a Chartered Professional Engineer (CPEng) through Engineers Australia.

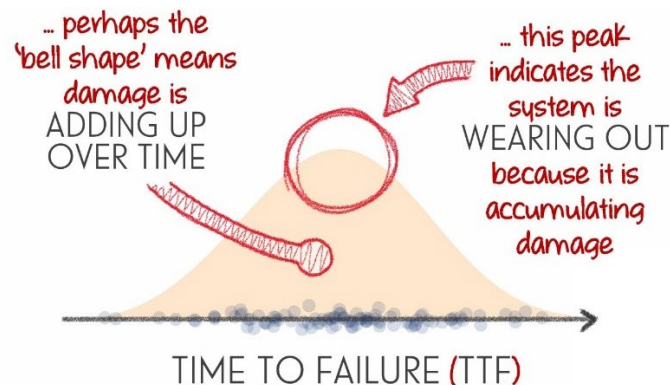
Dr Jackson is the cofounder of is4 online learning, was the inaugural director of the University of California, Los Angeles (UCLA's) Center of Reliability and Resilience Engineering and the founder of its Center for the Safety and Reliability of Autonomous Systems (SARAS). He has had an extensive career as a Senior Reliability Engineer in the Australian Defence Force, and is the Director of Acuitas Reliability Pty Ltd.

He has supported many complex and material systems develop their reliability performance and assisted in providing reliability management frameworks that support business outcomes (that is, make more money). He has been a reliability management consultant to many companies across industries ranging from medical devices to small satellites. He has also led initiatives in complementary fields such as systems engineering and performance-based management frameworks (PBMFs). He is the author of two reliability engineering books, co-author of another, and has authored several journal articles and conference papers.

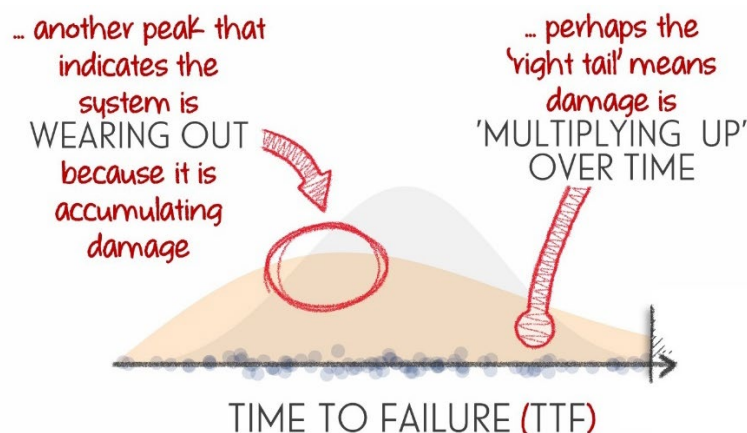


The Weibull distribution

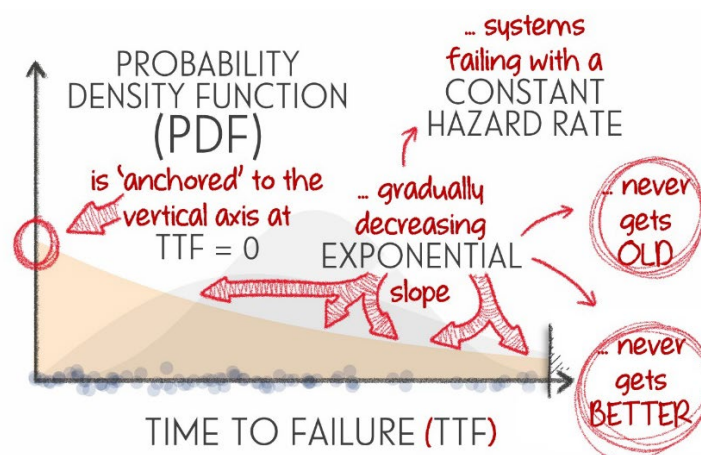
The **Weibull distribution** is talked about a *lot* in **reliability engineering**. And there is a really good reason why. But first, some revision. We know that the **probability density function (PDF)** defines a curve that defines the relative likelihood of a **random variable** having a certain value. That means, random variable values are more likely where the **PDF curve** is the highest (as in the illustration below). And the shape of that curve can tell us a lot about how failure is occurring ... if the random variable in question is **time to failure (TTF)**.



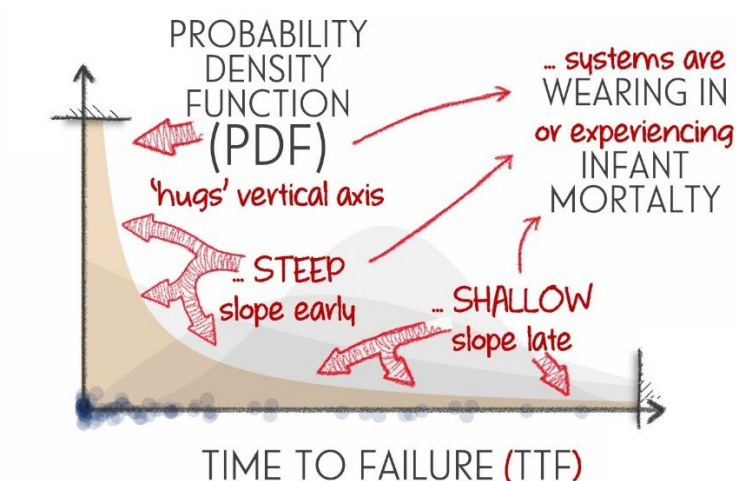
If we become really proficient, at this we can not only tell if our system is wearing out, but HOW it is wearing out. Another PDF curve with a 'wear-out' peak is illustrated below.



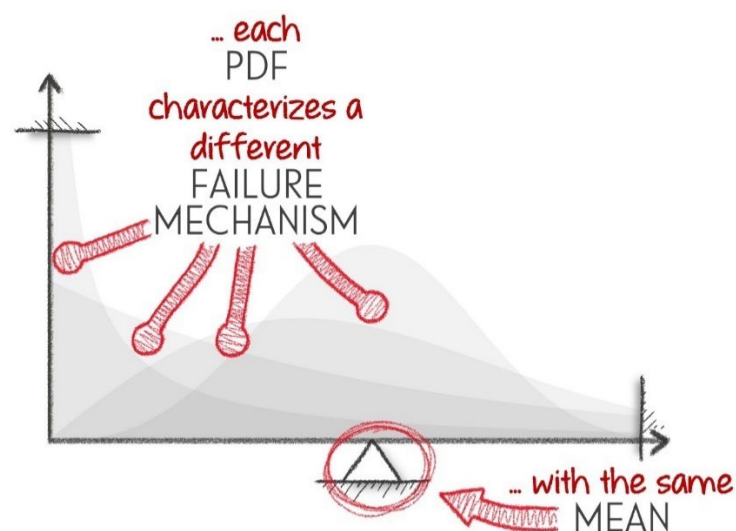
Then there are **PDF** curves with this shape ...



And **PDF curves** with this shape ...



All the **PDF curves** in the previous illustrations (each one modelling a different **failure mechanism**) were all specific examples of the **Weibull distribution**. And ... they all had the same **mean** or **mean time between failure (MTBF)**. This further illustrates that just understanding the mean of the **random process** that is **failure** reveals very, very little about the **reliability characteristics**.



The ability to model lots of different, well known failure mechanisms is primarily why the **Weibull distribution** is so popular. If you don't know what your data 'says' about the nature of **failure**, conducting what we call a '**Weibull analysis**' can not only find (perhaps) a really good **TTF** model for your product, system or service, but it can tell you where you need to look in order to improve **reliability**.

... Weibull distribution PARAMETERS

The parameters of the **Weibull distribution** are talked about ... a lot. Beyond helping us define the shape of the **PDF** and **reliability curves**, they tell us a lot about the nature of **failure**.

shape parameter – β

The **shape parameter** is represented by the lowercase Greek letter beta (β). The **shape parameter** tells us if our product, system or service is wearing in, wearing out, or has a constant **hazard rate**

if the shape parameter (β) is ... then the product, system or service is/has ...

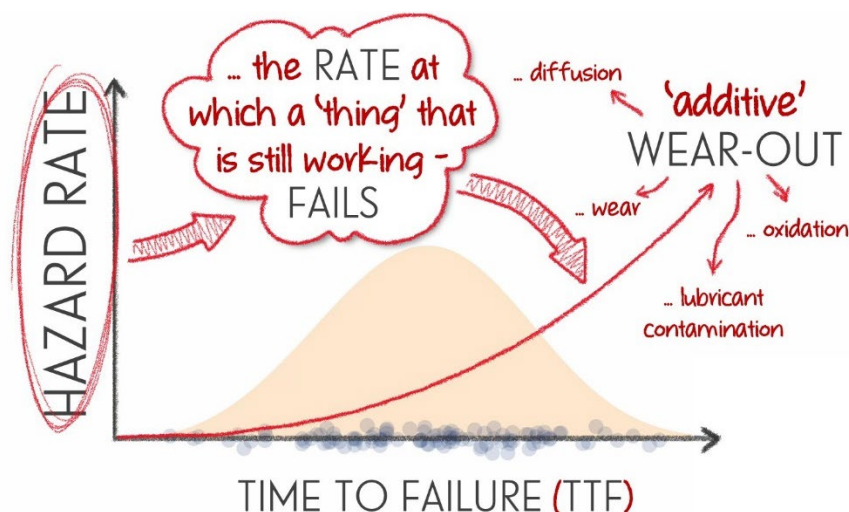
| | |
|----------------|------------------------|
| less than 1 | wearing in |
| equal to 1 | a constant hazard rate |
| greater than 1 | wearing out |

scale parameter – η

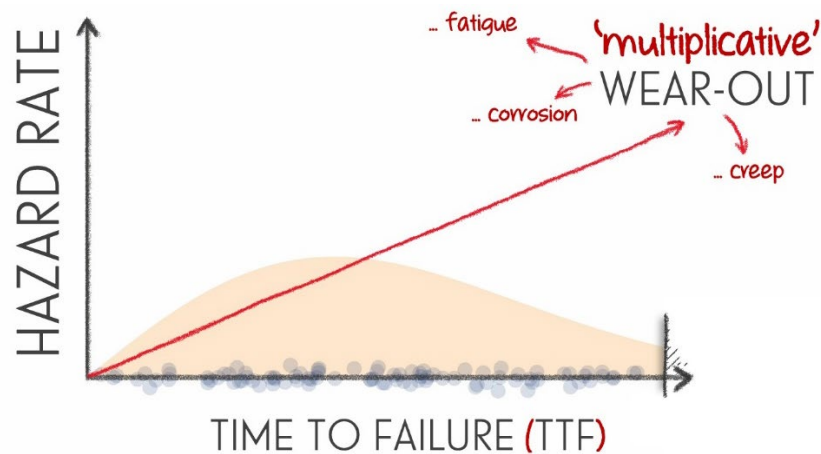
The **scale parameter** is represented by the lowercase Greek letter eta (η). The **scale parameter** tells represents an indicative size of the **Weibull random variable** in question. The **scale parameter** is the 63rd percentile (which is just a value that makes it easy for us to express the **probability distribution functions**).

... some examples

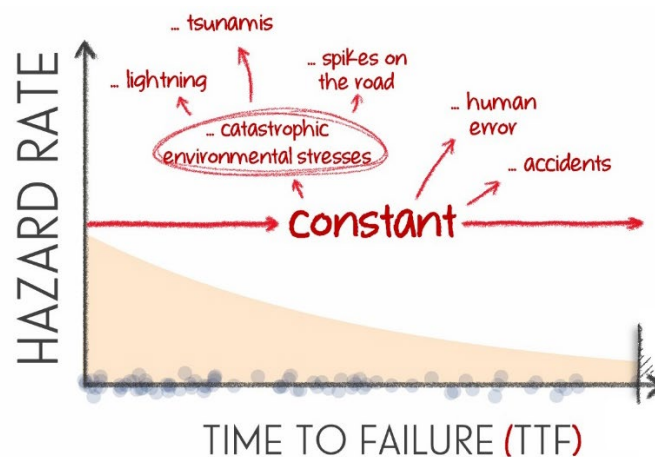
Because the **Weibull distribution** can model lots of different **failure mechanisms**, it (very understandably) it can look very similar to lots of other **probability distribution**. For example, the **PDF** and **hazard rate curves** for a **Weibull distribution** with a **shape parameter of 3.5** are illustrated below. This closely mimics the **normal distribution**.



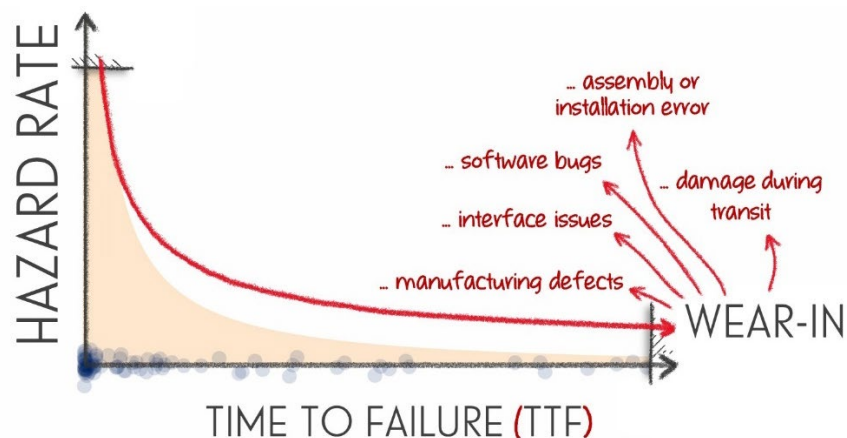
The **PDF** and **hazard rate curves** for a **Weibull distribution** with a **shape parameter of 2** are illustrated below. This closely mimics some **lognormal distributions**.



The **PDF** and **hazard rate curves** for a **Weibull distribution** with a **shape parameter of 1** are illustrated below. This is the **exponential distribution**.

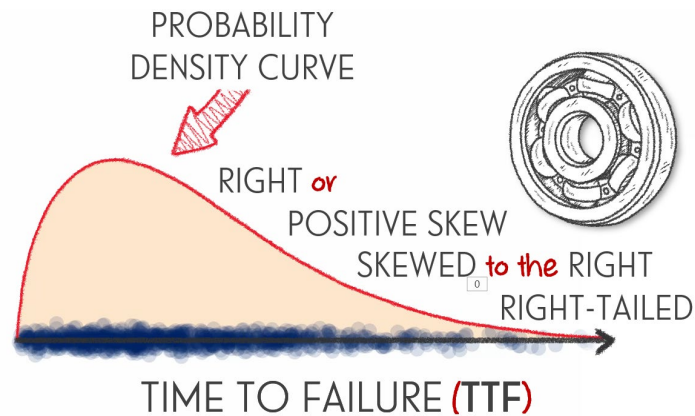


And finally, the **PDF** and **hazard rate curves** for a **Weibull distribution** with a **shape parameter of 0.4** are illustrated below. This does not mimic a specific distribution, but has been found to accurately model wear-in failure mechanisms.

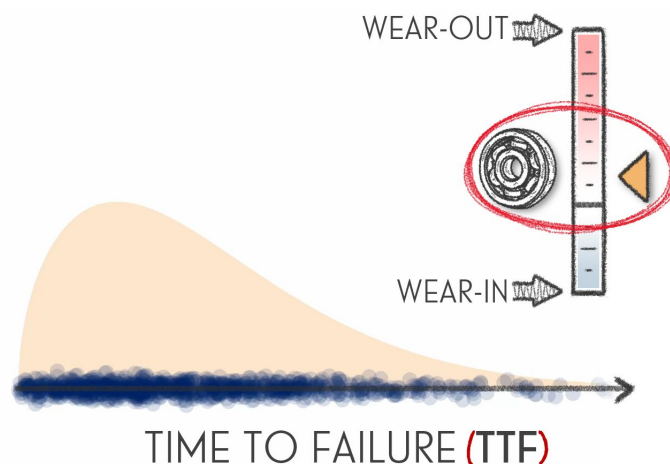


... let's look at a BALL BEARING

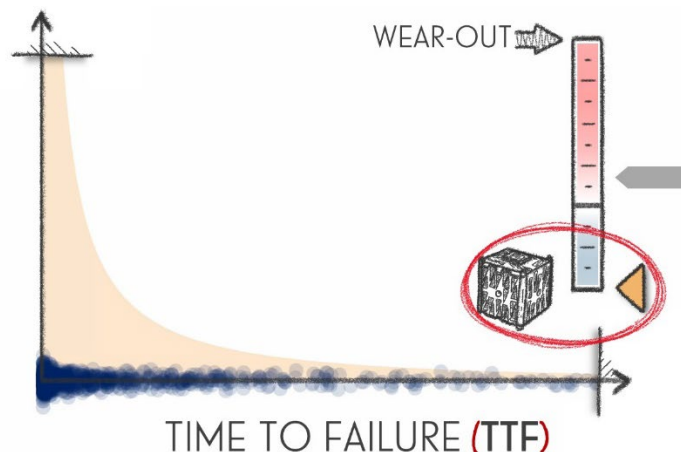
The probability density function of a typical ball bearing looks something like this ...



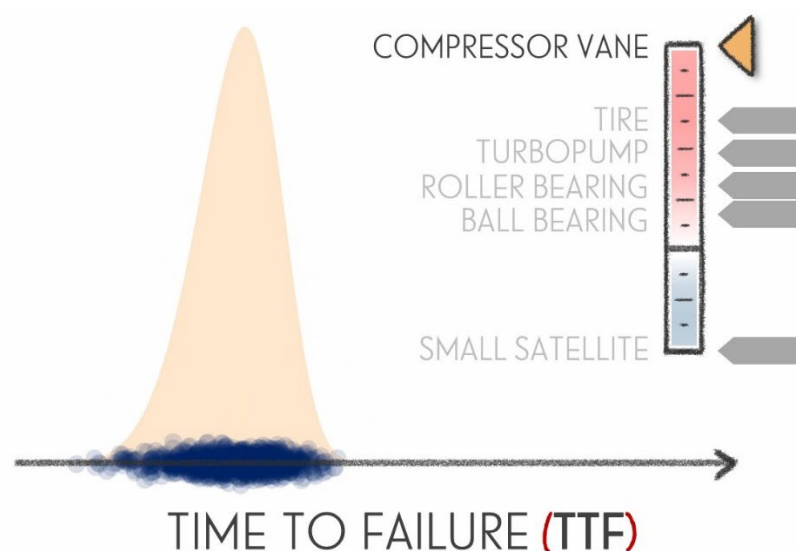
All ball bearings wear out. This means that if they can be modelled by a Weibull distribution, then the shape parameter will be greater than one. We can visualize the extent to which something (like a ball bearing) wears out (or in) by introducing a visual gauge, with the yellow marker indicating where on this gauge the extent to which ball bearing wear out lies.



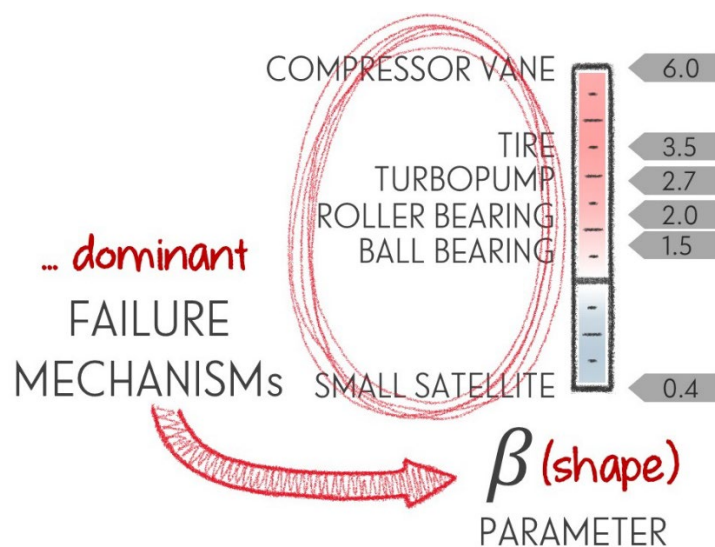
Small satellites on the other hand, wear-in and have a very different probability density curve shape.



In fact, different products with different probability density curves for times to failure (like the compressor vane below) each have their own 'place' on the gauge that represents the extent to which something wears-in or wears-out.

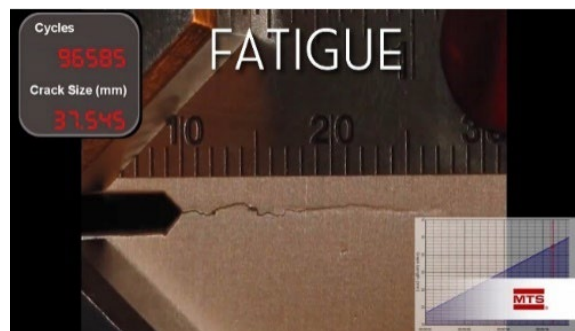
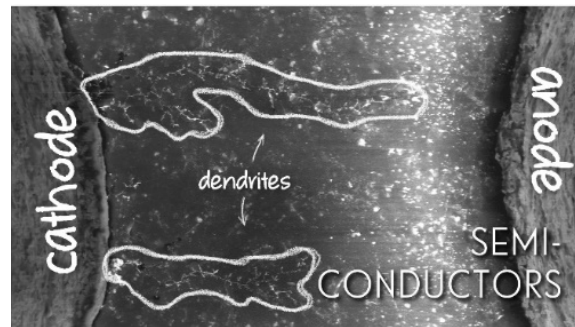
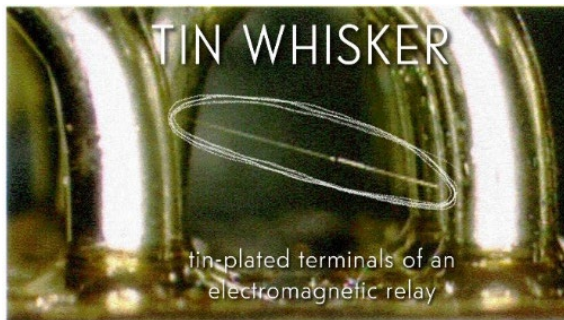


If any of these components have a dominant failure mechanism, then there is a really good chance that their time to failure can be modelled by a Weibull distribution. This also means that virtually every place on the gauge the visualizes wear-in and wear-out has a corresponding shape parameter value.



FAILURE MECHANISMS

Failure mechanisms are the ONLY things we can model when it comes to **reliability**. A failure mechanism is the physical, chemical, electrical, thermal or other process which results in failure. As they are naturally occurring processes, they are based on environmental factors and material properties. Some example **failure mechanisms** are illustrated below.

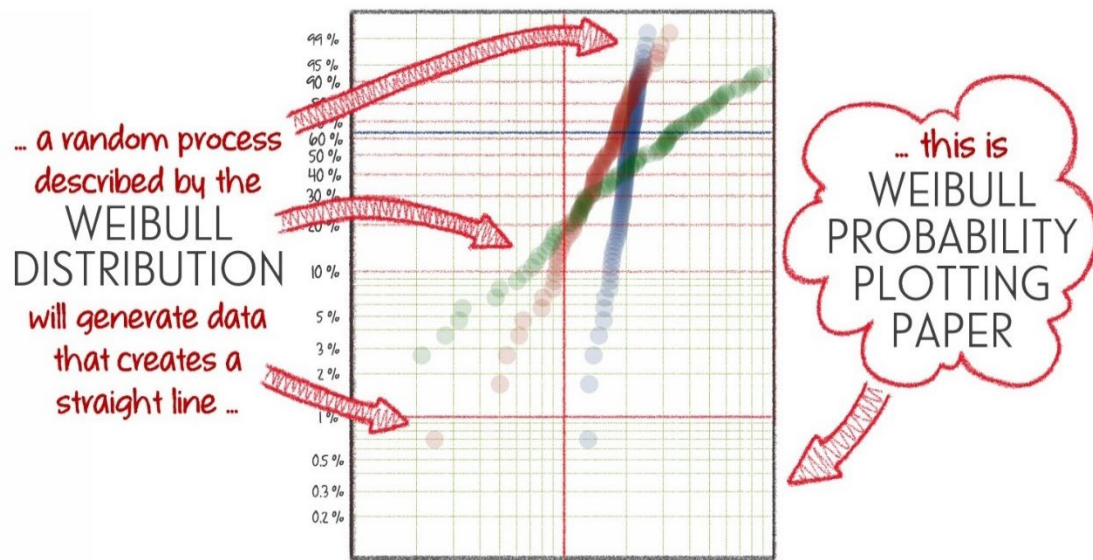


There are thousands of different **failure mechanisms**, and only a some of them will apply to your **product**. Of these some, only the **VITAL FEW failure mechanisms** will dominate how your product fails.

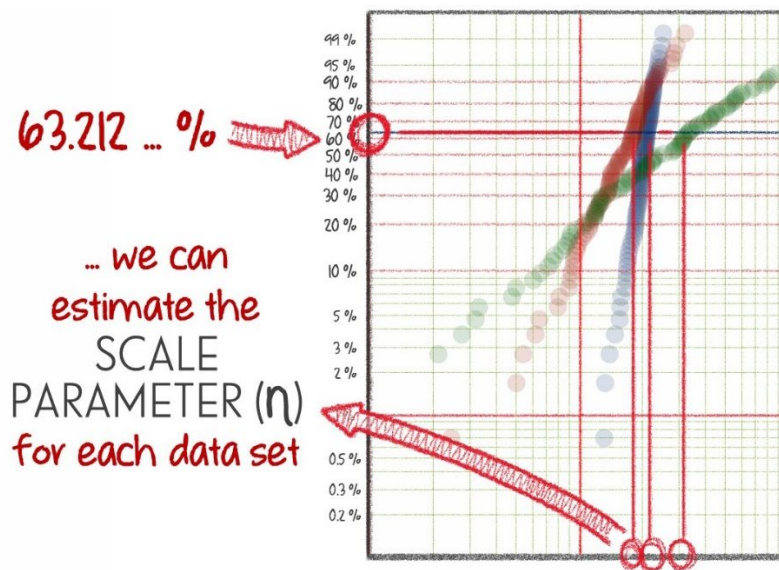
... WEIBULL PROBABILITY PLOTTING

The key to a lot of data analysis activities is to make a 'straight line.' We can manipulate the **Weibull CDF** a little bit to create a 'special' chart that is often referred to as **Weibull probability plotting paper**. The reason we do this is that any data that is described by a **Weibull distribution** will create a straight line on **Weibull probability plotting paper**.

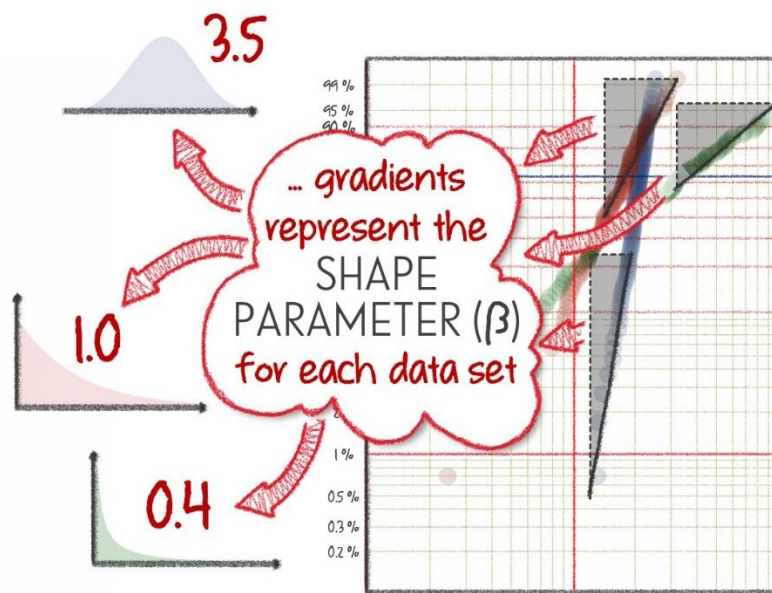
The data from three **failure processes** are illustrated on **Weibull plotting paper** below. Note that the vertical axis represents the **CDF** or **failure probability**.



And that straight line can be used to estimate our **Weibull distribution parameters** directly. We know that because the **CDF** of any **Weibull** will be 63.212 % at the **characteristic life**. So we can find the **scale parameter** from the **Weibull plot** where the **CDF** is 63.212 %.



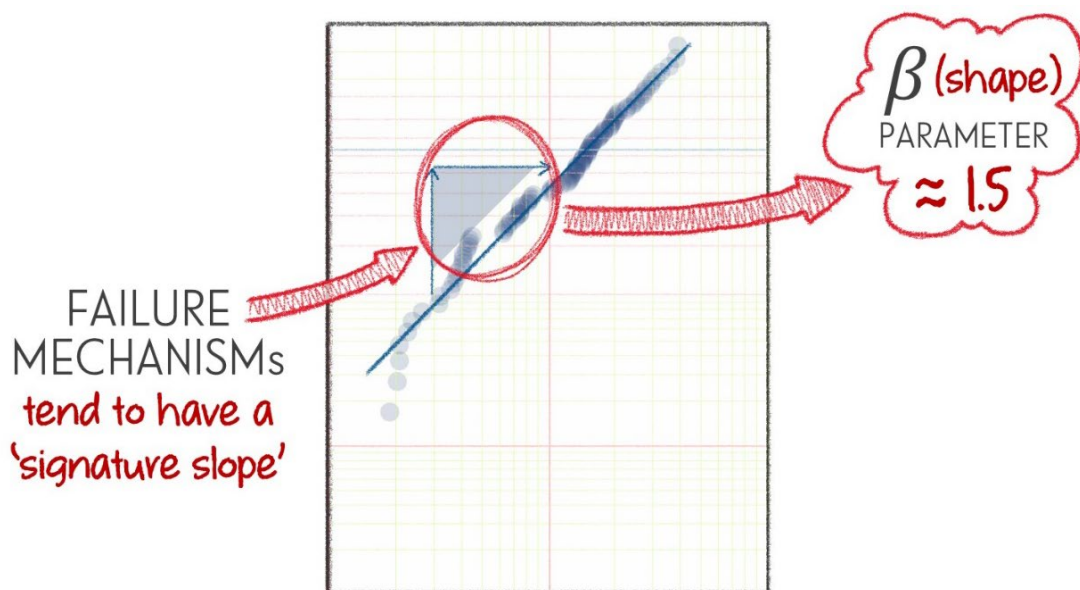
We can use the slopes of the lines of best fit to estimate the **shape parameter** of each process.



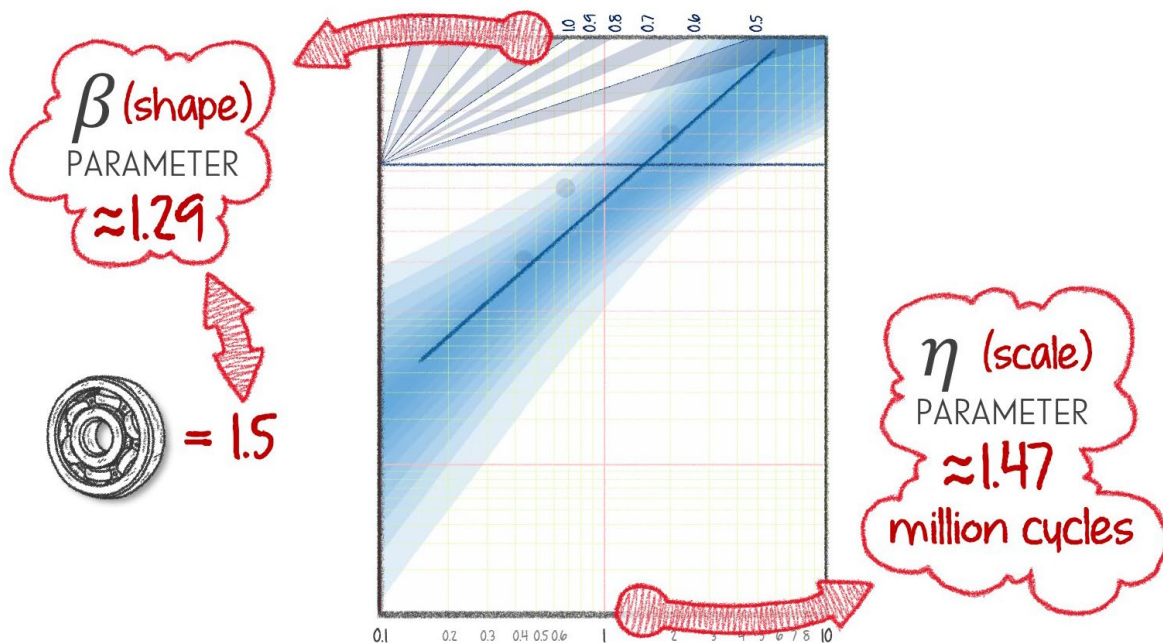
Weibull probability plotting is a lot easier than you might expect. You can do it on specially printed paper or using software to automatically generate **Weibull plots**. Neither approach requires you to use any equations ... but you do need to really understand what every **Weibull plot** represents.

... back to the BALL BEARING

Let's say that we have a 'typical' ball bearing that we have a lot of data for. You would expect to see a Weibull plot that creates a straight line with a slope of 1.5.



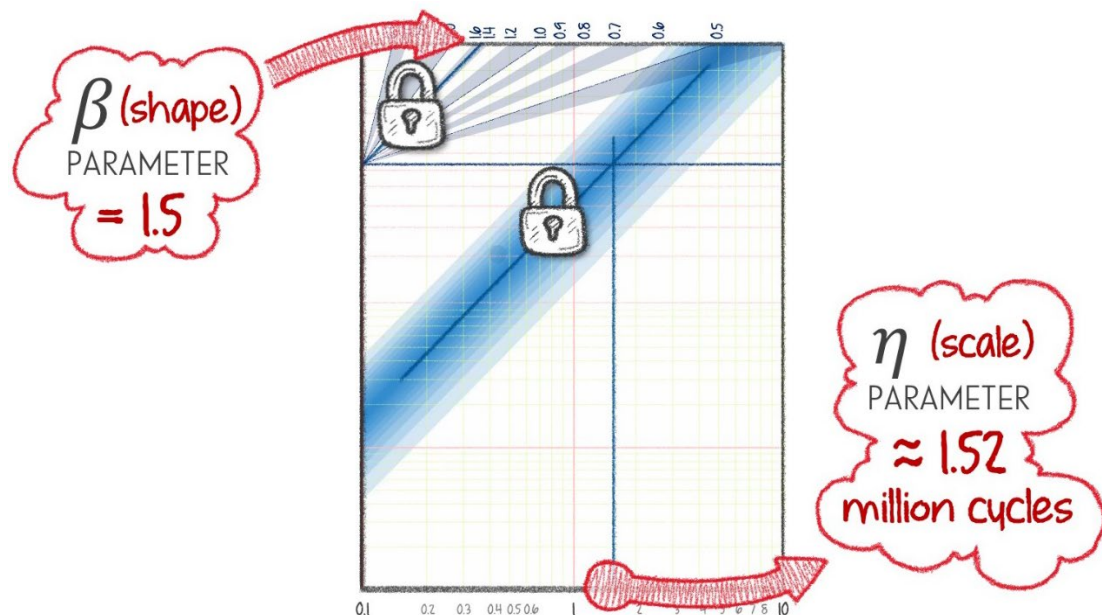
This all changes when you don't have as much data. For example, if we only have the times to failure three ball bearings, then the line of best fit on a Weibull plot might not have a slope of 1.5, even though we might know it is the 'right' shape parameter. This is because failure is a random process. We can also get computers to help us create what we call 'confidence regions.' This helps us visualize the uncertainty in any data analysis, like the blue contoured regions below that represent confidence regions ranging from 90 % (the outer most blue region) down to 10 % (the smallest, innermost blue region).



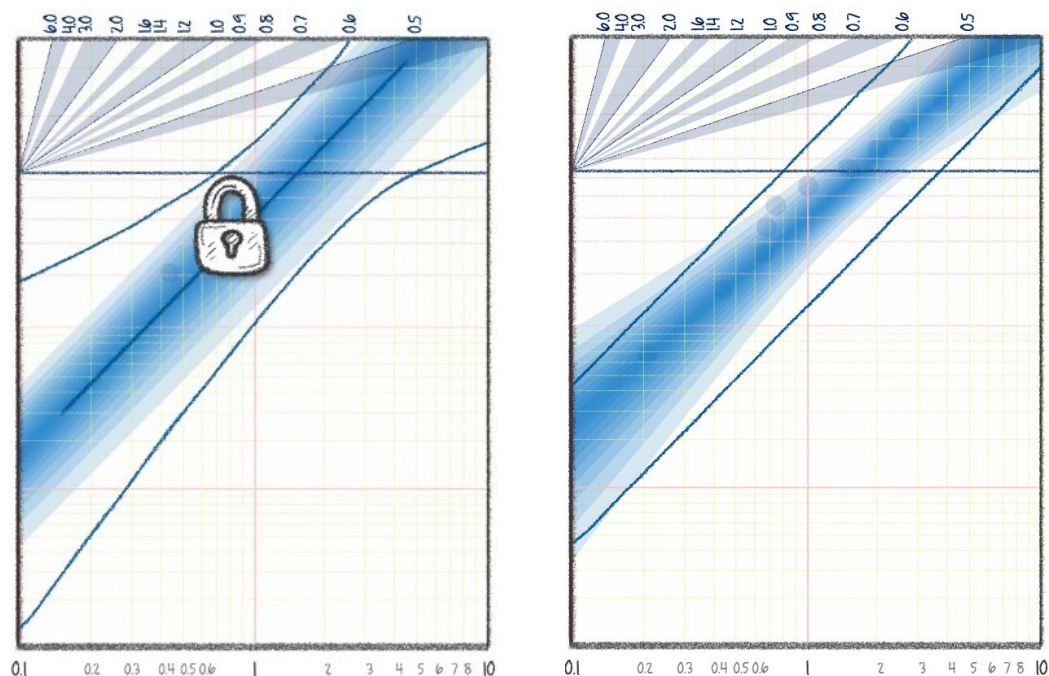
... so what is WEIBAYES ANALYSIS?

Weibayes analysis tries to take advantage of any knowledge we might have about how something fails. We already know that ball bearings fail in a way that allows us to assume a shape parameter with a value of around 1.5 is used to model time to failure with a Weibull distribution. This means that we can include information into our analysis, improving its outcomes.

For example, if we assume a shape parameter of 1.5 for the times to failure of 3 ball bearings, we essentially 'lock' the slope of the line of best fit and confidence regions. An example of what this looks like is illustrated below.



By locking in the slope of the line of best fit, we reduce uncertainty. The chart below on the left compares the confidence regions for the Weibayes approach with basic Weibull analysis. The chart below on the right compares the confidence region for the three-point Weibayes analysis along with the confidence regions for Weibull analysis with 10 data points.



... and the WEIBULL LIBRARY

A Weibull library is something that some organizations create to store different shape parameters for different components to help them with their analysis. You should never simply use values from the internet or textbooks, as different components from different suppliers used in the different operating conditions of your system will change the way it fails.

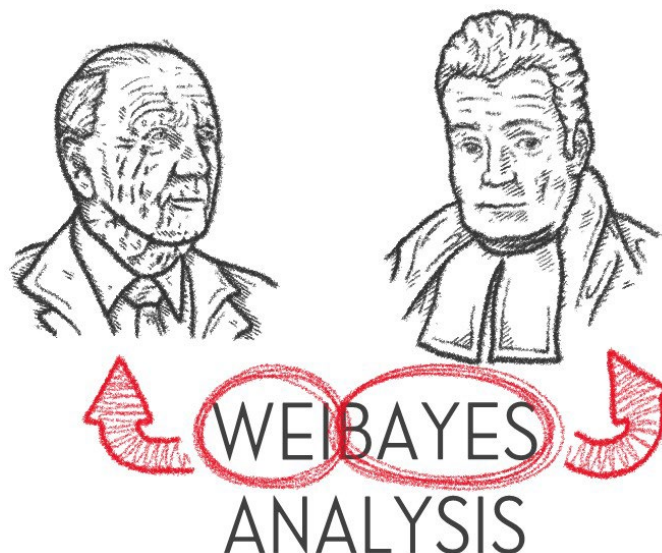
You can conduct Accelerated Life Testing (ALT) where you increase the stress a component experiences in order to accelerate failure (perhaps turning years of equivalent use into days of accelerated testing). If you don't change the dominant failure mechanism when you accelerate or increase the stresses, then you can reasonably use the shape parameter you identify in ALT for Weibayes analysis.

Another source is existing failure data for a component that is then modified or used in a different set of operating conditions. By knowing the likely shape parameter, you might only need to test one or two components to ensure that it is 'reliable enough.'

So this might save you a lot of time and money?

... so where does the term 'WEIBAYES' come from?

The term 'Weibayes' is an amalgam of the names of Waloddi Weibull and Thomas Bayes.



'Wei' represents the role the Weibull distribution plays in Weibayes analysis. And then there is 'Bayes.' He came up with a theorem that revolves around incorporating 'prior information' to improve analysis. A full explanation of what is known as 'Bayes' Theorem' is beyond the scope of this webinar, but it is used because of how we incorporate prior knowledge of the shape parameter into our analysis.

Many statisticians don't like Bayesian analysis as it is seen to include 'subjective data,' and not just the 'objective data' associated with test data. But if prior information is used well, it can incorporate a lot of useful information that reduces the need to test.

But technically ... Weibayes analysis is not true 'Bayesian analysis.' This is because Weibayes analysis involves point estimates of the shape parameter and not a distribution. But nonetheless ... the term 'Weibayes' has stuck.