

What is the 3 Parameter Weibull Distribution?

Use it if you know when it is cool!



Workbook

Presented by

Dr Chris Jackson

Reliability engineer



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Who is your teacher? ... Dr Chris Jackson

Dr Jackson holds a Ph.D. and MSc in Reliability Engineering from the University of Maryland's Center of Risk and Reliability. He also holds a BE in Mechanical Engineering, a Masters of Military Science from the Australian National University (ANU) and has substantial experience in the field of leadership, management, risk, reliability and maintainability. He is a Certified Reliability Engineer (CRE) through the American Society of Quality (ASQ) and a Chartered Professional Engineer (CPEng) through Engineers Australia.

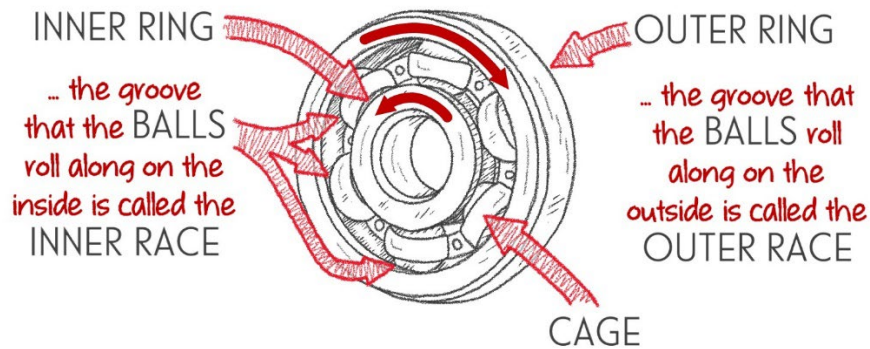
Dr Jackson is the cofounder of www.is4.com online learning, was the inaugural director of the University of California, Los Angeles (UCLA's) Center of Reliability and Resilience Engineering and the founder of its Center for the Safety and Reliability of Autonomous Systems (SARAS). He has had an extensive career as a Senior Reliability Engineer in the Australian Defence Force, and is the Director of Acuitas Reliability Pty Ltd.

He has supported many complex and material systems develop their reliability performance and assisted in providing reliability management frameworks that support business outcomes (that is, make more money). He has been a reliability management consultant to many companies across industries ranging from medical devices to small satellites. He has also led initiatives in complementary fields such as systems engineering and performance-based management frameworks (PBMFs). He is the author of two reliability engineering books, co-author of another, and has authored several journal articles and conference papers.

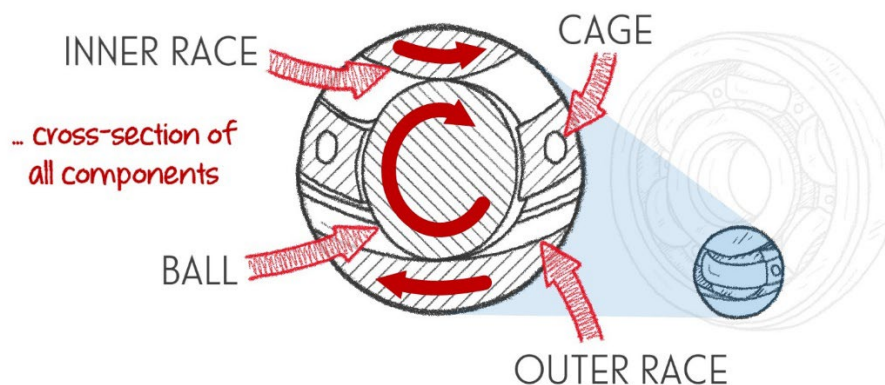


... let's start with an EXAMPLE

Ball bearings are one of the most common and important components of machines across the world. Each ball bearing is made up of a number of different parts.

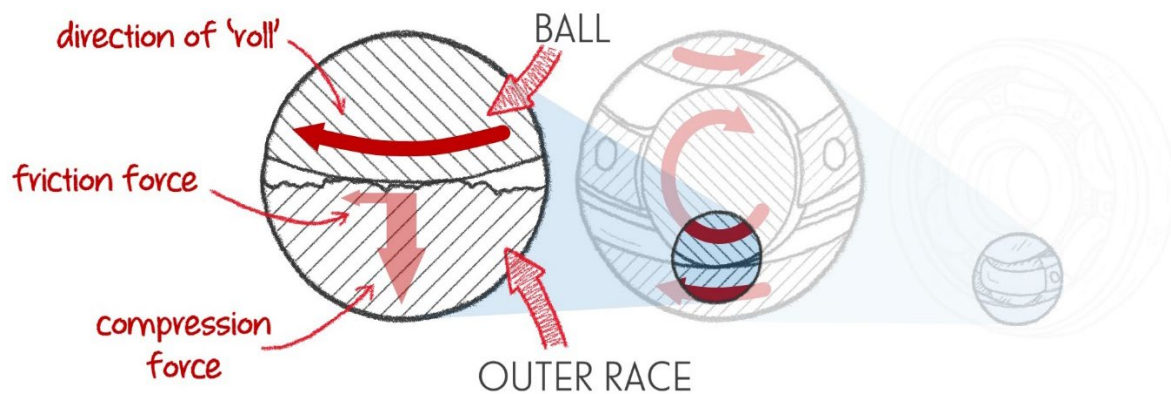


Ball bearings are designed to minimize the friction a rotating shaft experiences when it is supported within a machine. Because freely supported balls are the only physical connections between the inner and outer rings, it is very difficult for 'torque' to be transferred. Every time torque is generated, the balls simply roll. This greatly minimizes any frictional forces.

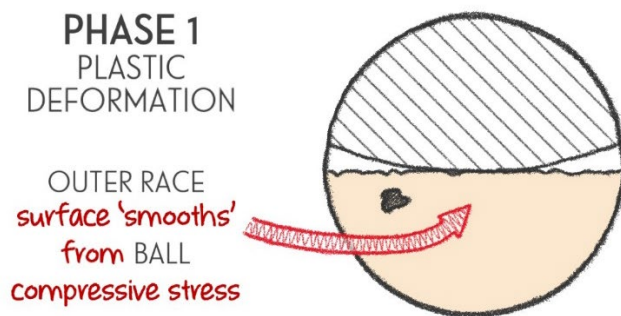


This also makes ball bearings relatively reliable, being able to withstand millions of revolutions before failing.

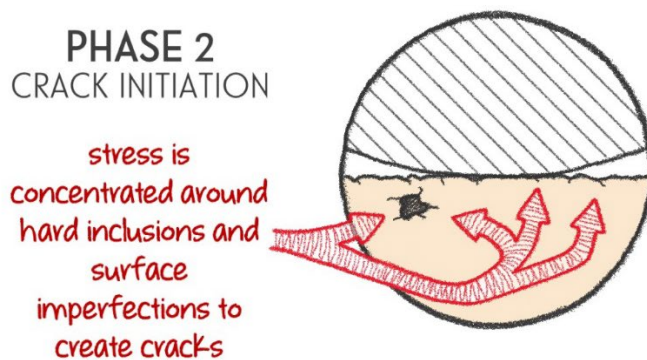
One of the more common ways ball bearings fail is from 'spalling.' This is essentially a manifestation of different types of fatigue. There are two forces that races experience during normal operation: compression and the (smaller) friction force.



The first step in spalling is actually plastic deformation or smoothing of the race surface. This absorbs energy and otherwise protects the race material

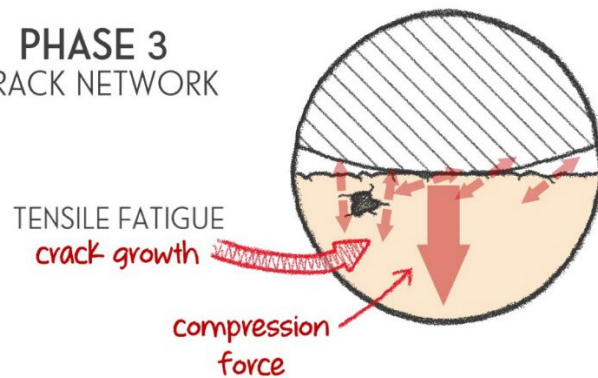


The predominantly compressive forces then start to add tension to the very small crack tips and inclusions (i.e. the sides of each crack tip are pulled away from each other).

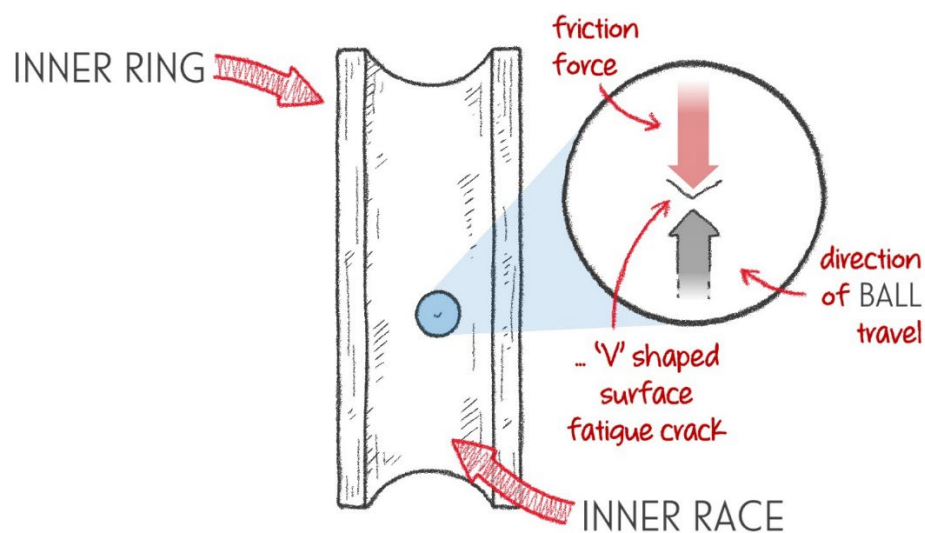


Because the balls repeated roll over the surface, the tensile stresses cycle that allows fatigue cracking to occur.

PHASE 3 CRACK NETWORK

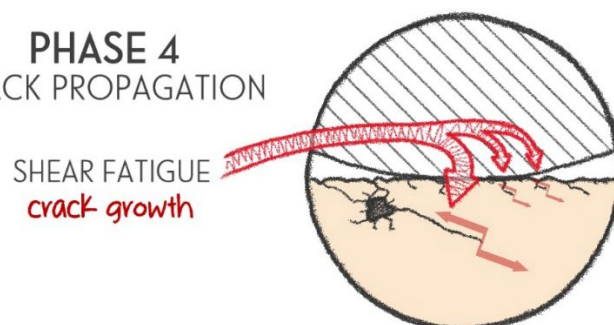


Eventually, this crack network will reach the surface, or a surface crack will grow to be relatively 'large.' From above, this will create a 'v' shape surface crack.

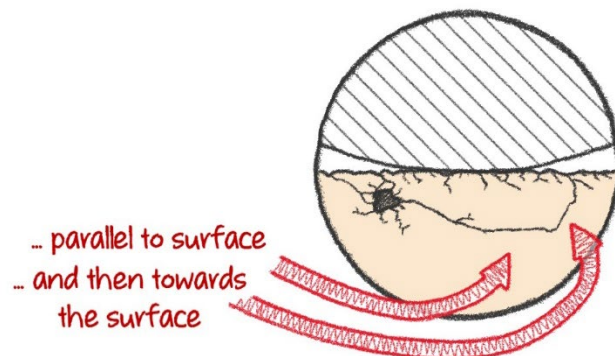


This then leads to another type of fatigue crack that grows not due to tension, but to shear. That is where one side of the crack tip is being 'pulled back' while the other side is being 'pushed forward.'

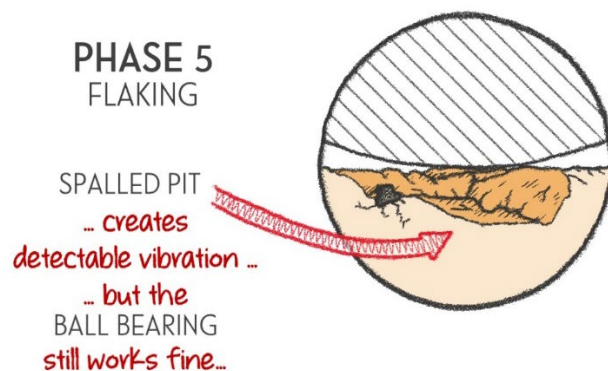
PHASE 4 CRACK PROPAGATION



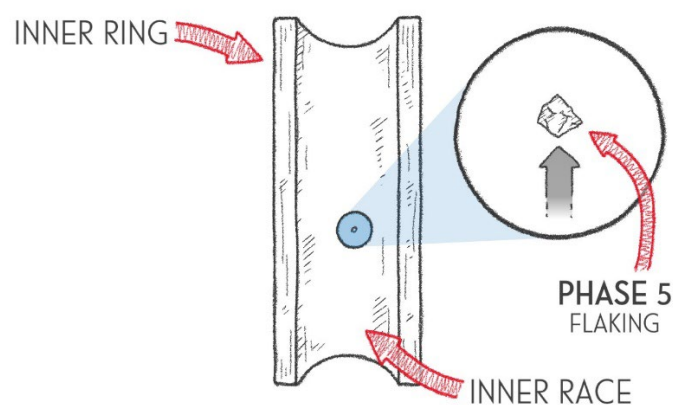
This crack initially grows at a shallow angle (approximately 35 °) to the surface. Once the crack is deep enough, it then turns and travels parallel to the race surface. Eventually, the shearing forces will cause the crack to go back to the surface.



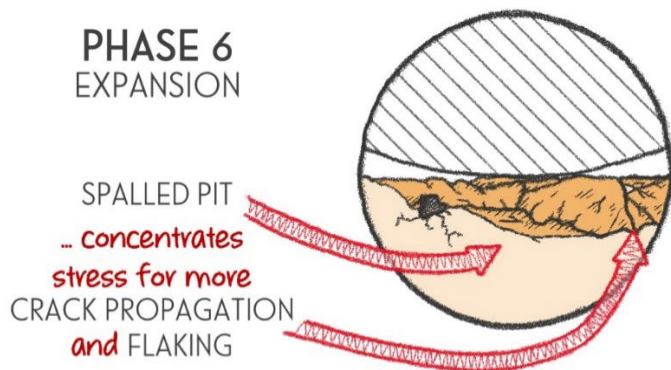
Because the crack has reached the surface, chunks of the race material can now break free.



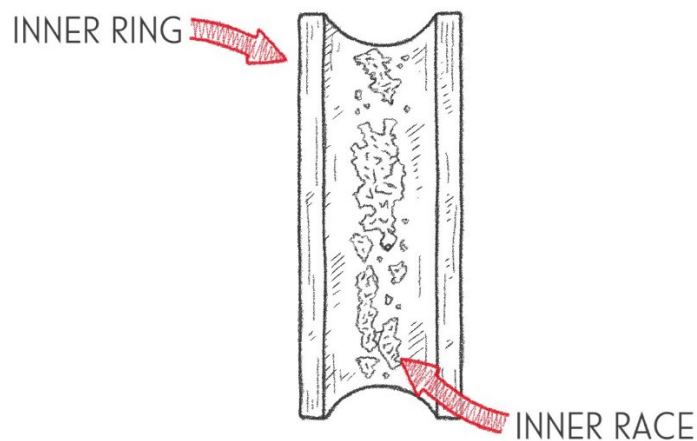
These pits will be visually detectable (for the first time) if the ball bearing can be accessed.



Once a pit is formed, it becomes much easier for material to flake away.



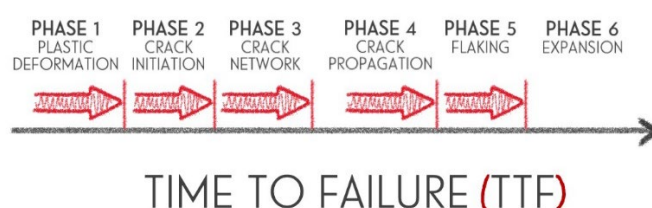
When enough material has flaked away, the ball bearing will vibrate excessively, be noisy, and generate a lot of heat that will either directly or indirectly lead to catastrophic failure.



... how do we deal with a FAILURE FREE PERIOD?

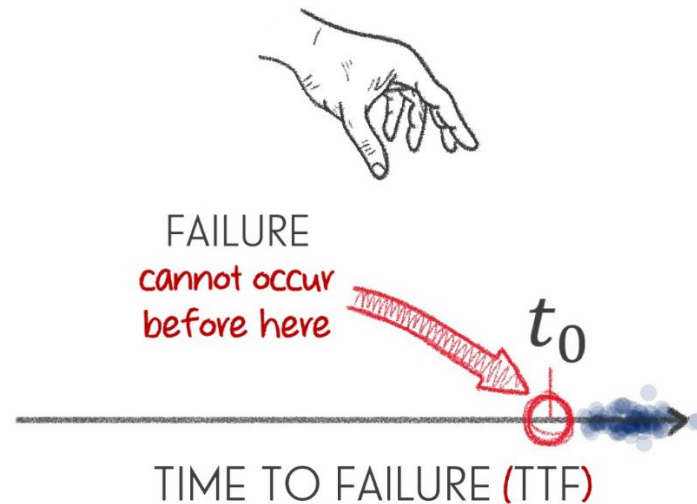
For the ball bearing that fails due to spalling, there will generally be an initial period where failure cannot occur. Spalling damage takes a long time to start, and then a long time to accumulate to the point the bearing fails.

the (random) hand of FAILURE



This might imply that there is some time ' t_0 ,' before which failure cannot occur.

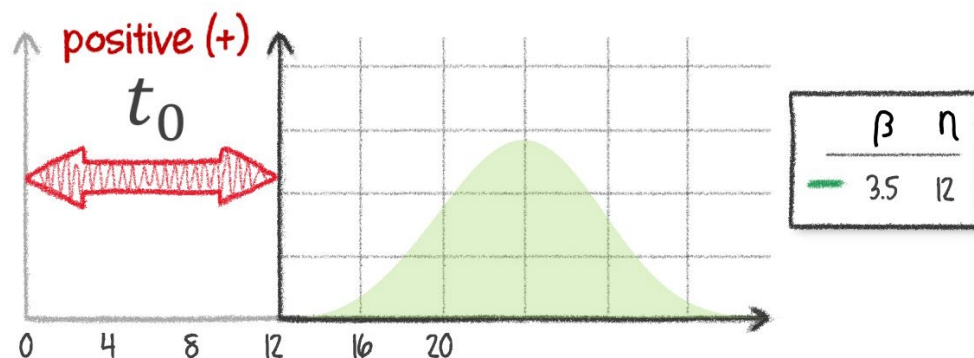
the (random) hand of FAILURE



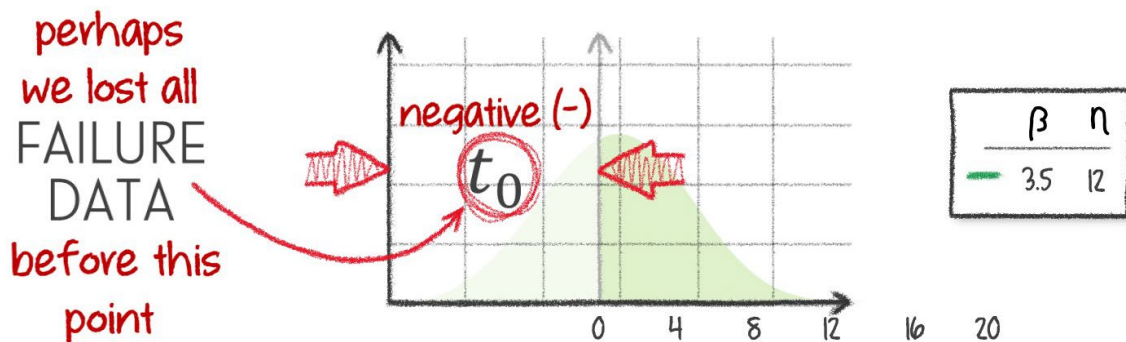
Generating models where 'failure free periods' are incorporated should only be developed with extreme caution. Of course, the bearing can fail due to other failure mechanisms beyond spalling. But there are some scenarios where a 'failure free period' can be used.

... the THREE (3) PARAMETER WEIBULL DISTRIBUTION

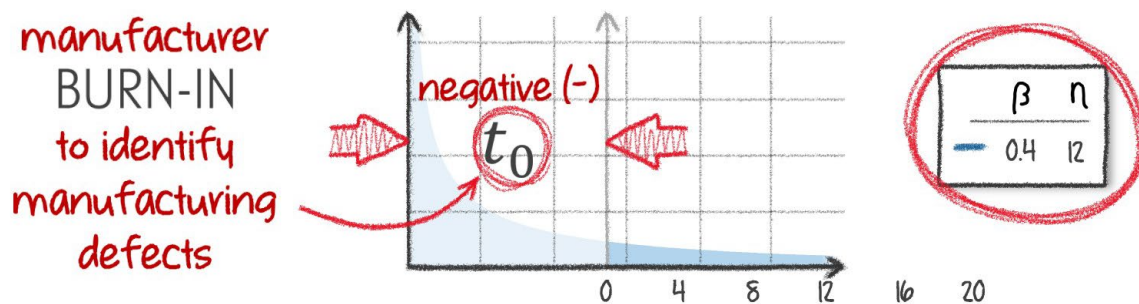
The PDF of the ball bearing's spalling-based TTF is illustrated below. The failure free period (t_0) is equivalent to 12×10^6 revolutions).



But t_0 does not always have to be positive. For example, if maintenance or failure data for the first period of a system's life was lost, then essentially the 'odometer' starts well after the systems start being used.



More commonly, negative t_0 apply to manufacturers where components that are subject to manufacturing defects are 'burnt-in' before being shipped. This means that they are tested in a way to identify manufacturing defects early, but this also means that a fixed amount of usage is applied to the components before the customers or users ever use them.



... the RULES for the THREE (3) PARAMETER WEIBULL DISTRIBUTION

Adding an additional parameter (especially one that implies a failure free period) should never be undertaken lightly. Adding any parameter to any statistical analysis process will *a/ways* make the model fit better ... even if there is no justification for that additional parameter. And we don't want to incorrectly include a 'failure-free period' just because it makes the model fit the data better ... because this risks us assuming there is a 'failure-free period' when there might not be one.

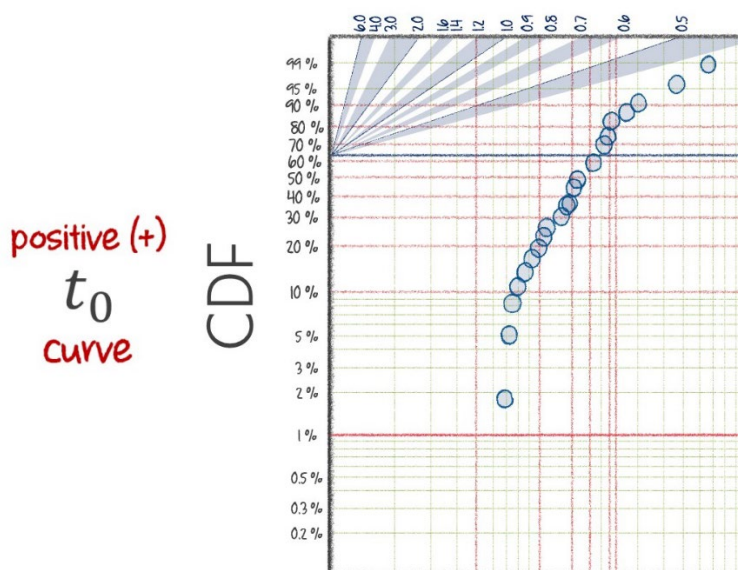
There are four rules that need to be satisfied when 3 parameter Weibull distribution is used.

1. you have a PHYSICAL EXPLANATION for t_0
2. the 2 parameter Weibull plot SHOWS A CURVE
3. you have a SAMPLE SIZE GREATER THAN 20 ... ideally MUCH greater than 20
4. a GOODNESS OF FIT that exceeds other distributions

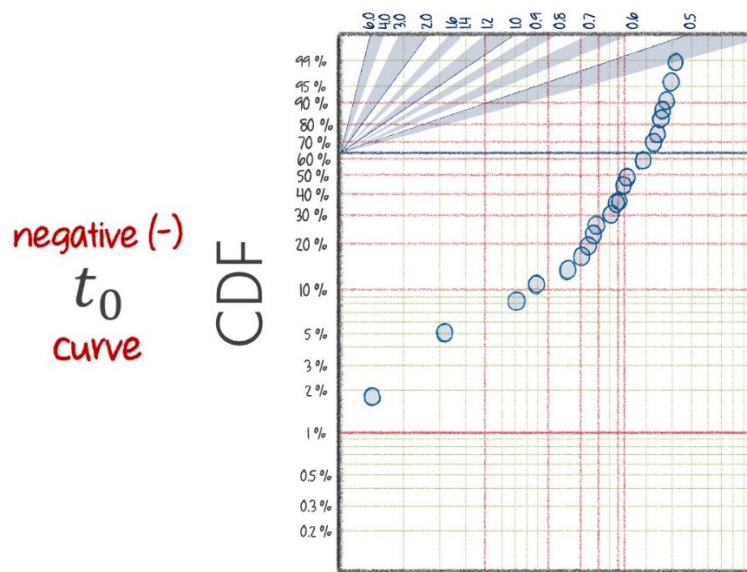
The first rule is the most important. If you are simply trying to make the model fit better without understanding what is causing failure ... you are asking for trouble. So you need to think about how your system fails. If your bearing is primarily failing due to spalling, and you study this failure mechanism like we have earlier in this chapter ... then you can justifiably claim to have a physical explanation for t_0 .

The second rule is also very important. If you can't see a curve, then you might be adding an additional parameter to make the model fit better without any evidence for the process being governed by an additional parameter. A straight line usually means a 2 parameter Weibull distribution is a more appropriate model.

The other benefit of examining a data set with a 2 parameter Weibull distribution is that it helps you identify if t_0 is positive or negative. If the data curves to the right, it implies a positive t_0 .



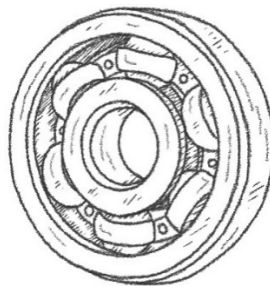
IF the data curves up, it implies a negative t_0 .



The other issue with 3 parameter Weibull distributions is that they are difficult to statistically distinguish from other distributions. The data with the apparent positive t_0 is often also well described by a lognormal distribution. So we need to explore other probability distributions to ensure that we do not incorrectly (and usually devastatingly) assume a failure free period. This also means that you should have at least 20 data points to make this distinction. Ideally, you would like to have much more than 20 data points.

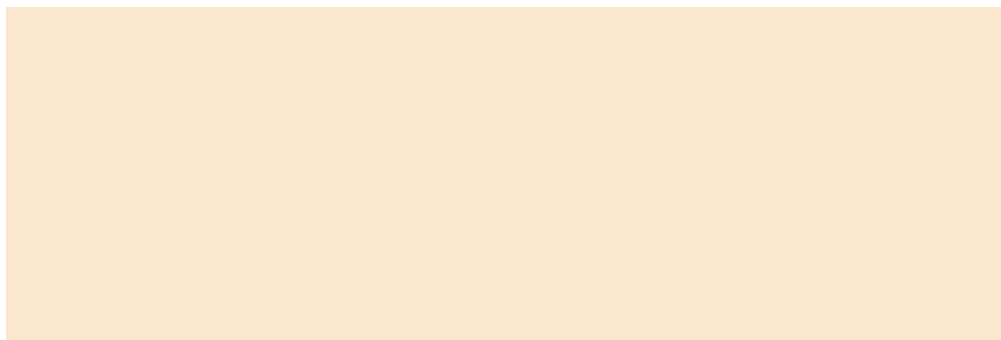
EXERCISE

An electricity generation plant has failure data on a number of bearing that they suspect are failing due to spalling.



Is their suspicion possible?

Does modelling the data with a 3 Parameter Weibull distribution help?



EXERCISE

A utility company's maintenance data management system failed!

All failure data for a number of valves that are known to wear out was lost for a number of months.

Using the failure data that was generated when the system became available (on 1st September) ... when do you think the original valves were installed?

