

Reliability, Availability and Maintainability (RAM) Practitioners Course

If reliability engineering isn't making you money, then you are doing it wrong



Workbook

Presented by

Dr Chris Jackson

Reliability engineer



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Who is your teacher? ... Dr Chris Jackson

Dr Jackson holds a Ph.D. and MSc in Reliability Engineering from the University of Maryland's Center of Risk and Reliability. He also holds a BE in Mechanical Engineering, a Masters of Military Science from the Australian National University (ANU) and has substantial experience in the field of leadership, management, risk, reliability and maintainability. He is a Certified Reliability Engineer (CRE) through the American Society of Quality (ASQ) and a Chartered Professional Engineer (CPEng) through Engineers Australia.

Dr Jackson is the cofounder of www.is4.com online learning, was the inaugural director of the University of California, Los Angeles (UCLA's) Center of Reliability and Resilience Engineering and the founder of its Center for the Safety and Reliability of Autonomous Systems (SARAS). He has had an extensive career as a Senior Reliability Engineer in the Australian Defence Force, and is the Director of Acuitas Reliability Pty Ltd.

He has supported many complex and material systems develop their reliability performance and assisted in providing reliability management frameworks that support business outcomes (that is, make more money). He has been a reliability management consultant to many companies across industries ranging from medical devices to small satellites. He has also led initiatives in complementary fields such as systems engineering and performance-based management frameworks (PBMFs). He is the author of two reliability engineering books, co-author of another, and has authored several journal articles and conference papers.

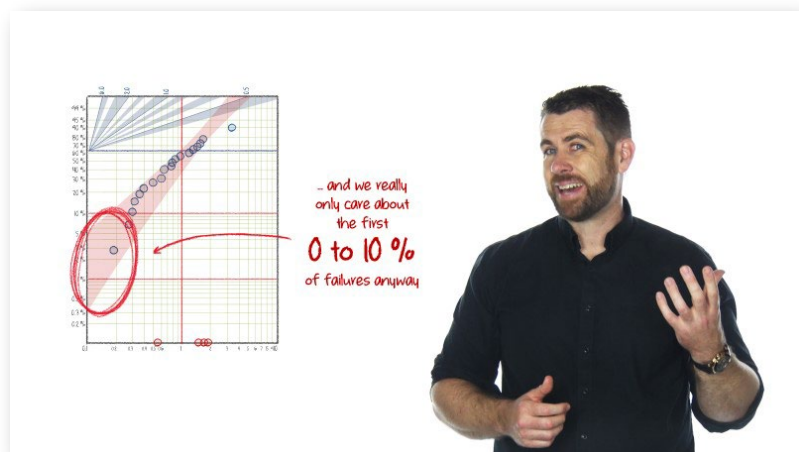
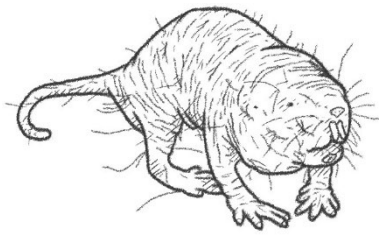


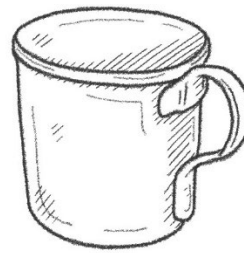
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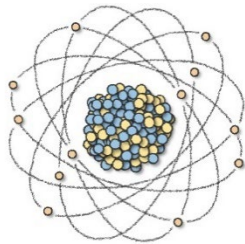
We are going to start with four 'different' things that will (hopefully) help us understand the nature of the exponential distribution ...



NAKED MOLE RAT



COFFEE MUG



RADIOACTIVE DECAY



COMPOUND INTEREST

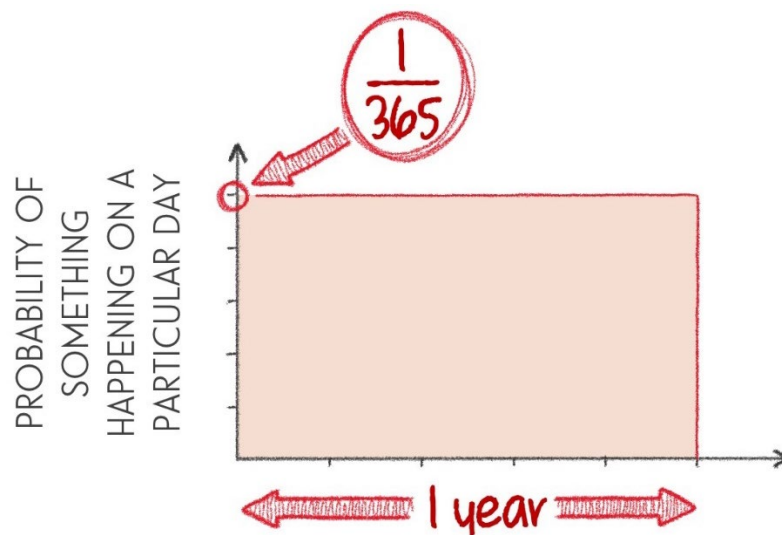
What is 'RANDOM'?



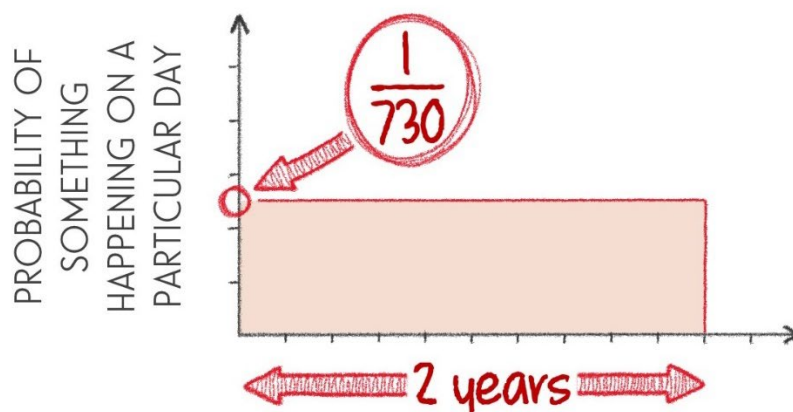
... something that has the same likelihood of occurring on any given day is 'random' ...

... is this really true?

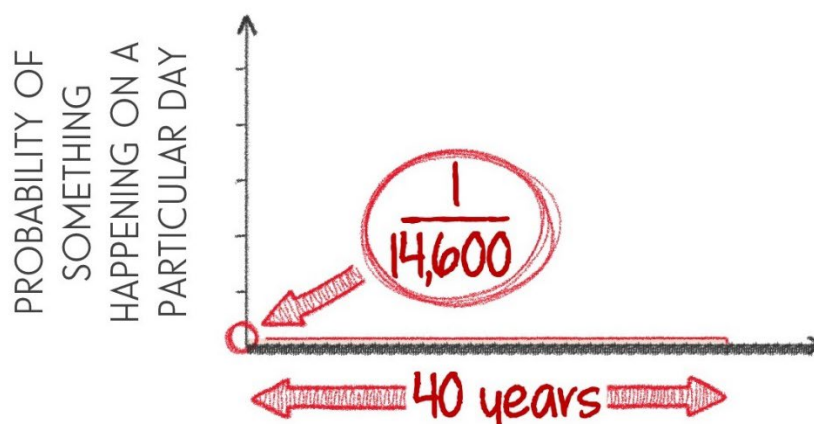
If we focus (for example) on something happening with equal probability over one year ...



And because there are 365 days in one year, we know that for there to be equal probability of the event occurring on any given day, then that probability is $\frac{1}{365}$. But if we now focus on a period of two years ...



And now 40 years ...



For anything to have an 'equal' probability of something occurring at any particular time, you must also specify the interval over which this equal probability occurs. And if this is (for example) one year, it means that there is absolutely no possibility of that event occurring after one year has elapsed. And that probability changes as the interval you specify changes.

... what else (constant HAZARD or FAILURE RATE)?

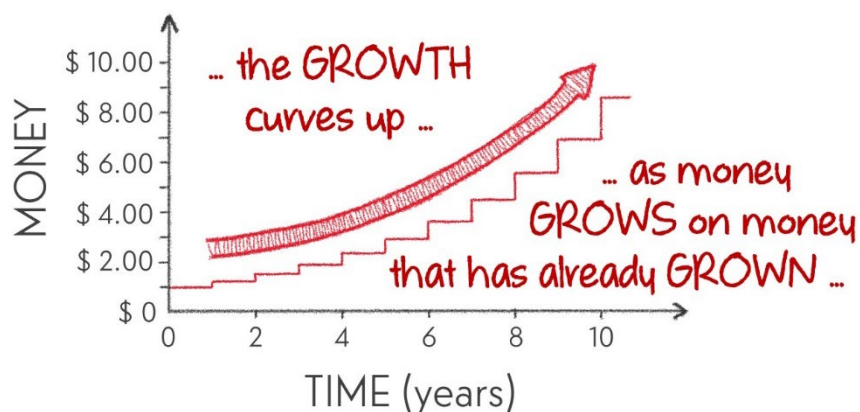


So let's start with looking at compound interest, and what it is.

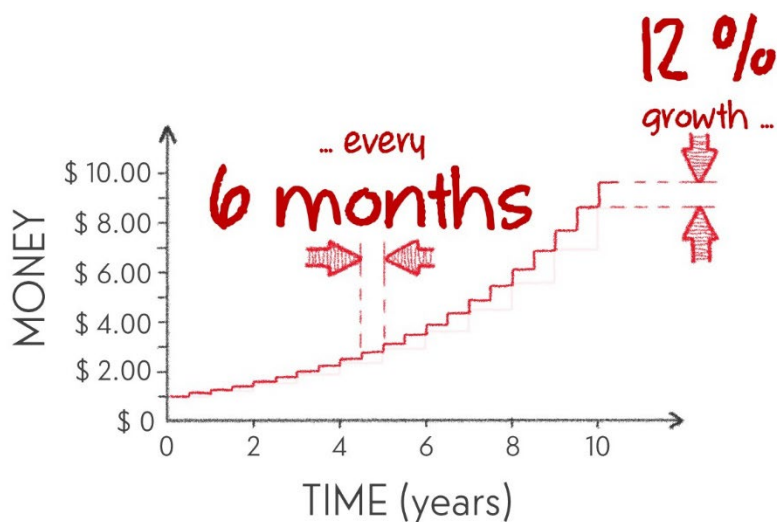


COMPOUND INTEREST

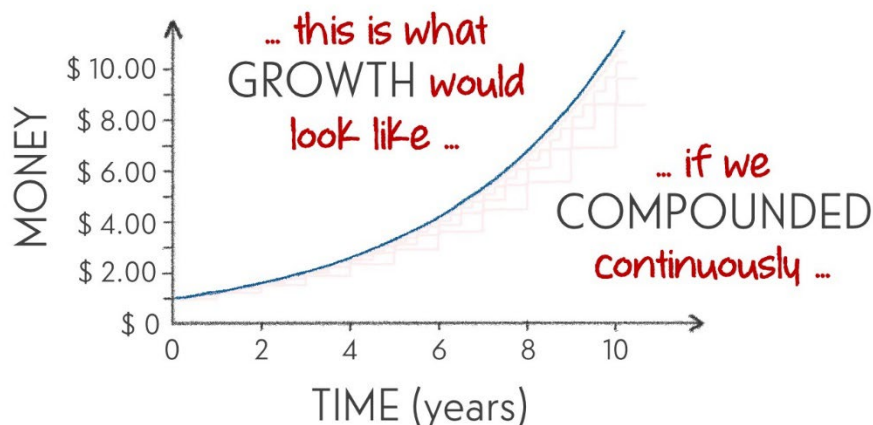
Let's say you have one dollar, and you invest it at a bank that offers an annual interest rate of 24 %. After one year, \$ 0.24 (or 24 % of \$ 1.00) is added to your bank account. And after the next year, \$ 0.30 (or 24 % of \$ 1.24) is added to your bank account. So the amount of interest you own starts to grow.



This is what happens when you compound interest annually. That is, interest is added to your account every year. But we can compound over different periods. For example, you could invest your money into another bank that offers the same interest rate (24 % per year), but the interest is added every six months or half a year. But if we are adding interest over half a year, we also need to halve the interest rate we use to calculate that amount. So instead of adding 24 % per year, we are now adding 12 % (half of 24 %) every six months. And our money grows in a different, faster way.



We could compound interest every 3 months (6 % every 3 months), every 2 months (4 % every 2 months), 1 month (2 % per month), or any other period. The smaller this period, the more money grows. Eventually, the curve stops being stepped in is completely smooth.



There is a special equation that describes this line of continuous growth.

$$\text{\$} = 2.71828 \dots^{0.24 \times t}$$

You can see the interest rate of 24 % is in the equation above represented by 0.24. But the number 2.71828 is something 'new.' No one knows why this number helps us model continuous growth like that described above. We know it just does. And this number appears in lots of interesting mathematical (and reliability engineering) applications as well.

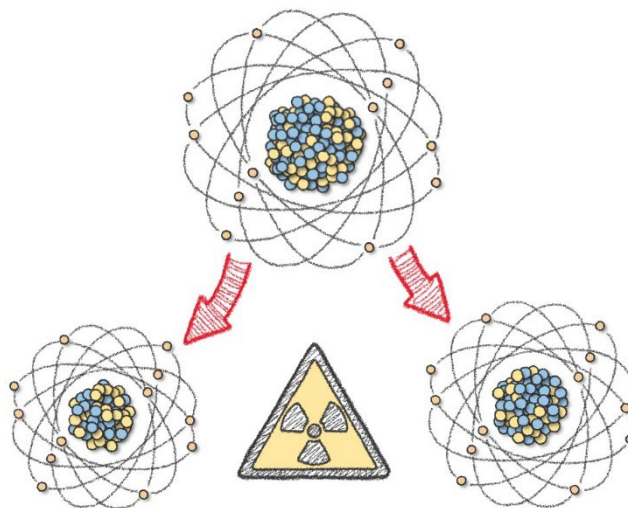
The number is named after this man ...



Euler was a Swiss mathematician and scientist. He was not the first person to find this 'special' number, but he was the first to represent it with the letter '*e*.' And for that reason, $e = 2.71828 \dots$ is usually known as 'Euler's number.'

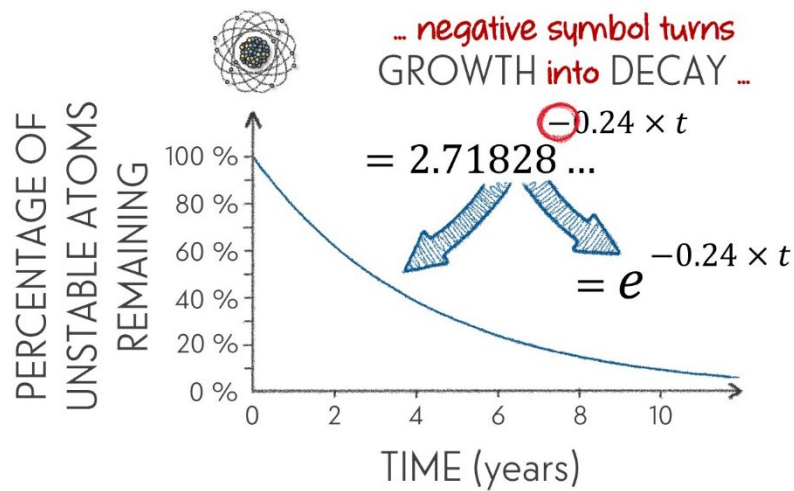
... what about things that DECAY and not GROW?

Radioactive decay involves atoms that split, in various different ways, into two or more smaller atoms.



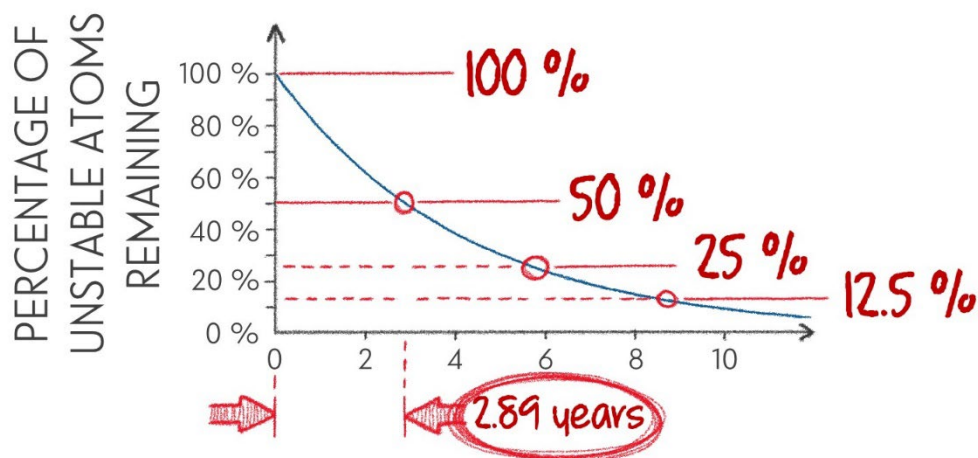
This split often also involves energy, radiation and particles being emitted as well. Let's say that the atom above decays at a rate of 24 % per year (remember that when we were talking about compound interest, our money was growing at a rate of 24 % per year).

We can use Euler's number (e) and the equation we used to describe continuously compounding interest growth, but with a twist.



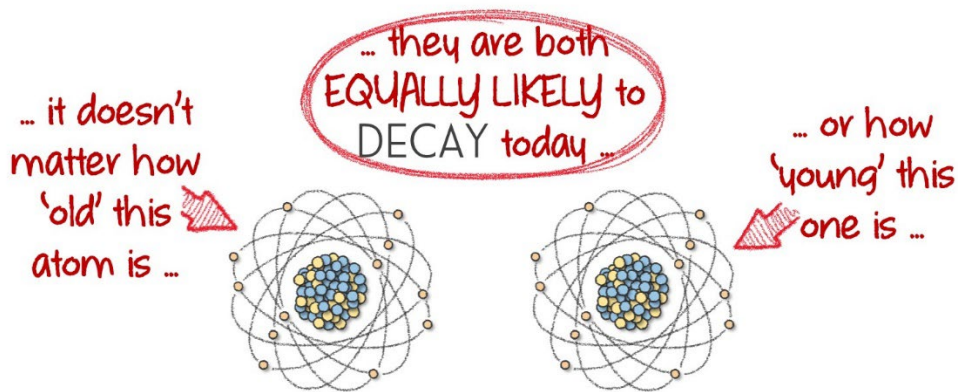
And while the continuously compounding interest curve 'speeds up' as our money grows on money that has 'already grown,' the same thing happens to our decaying atoms, but in a different way. The decay curve above 'slows' down as there are fewer atoms left to decay.

This decay curve has a particular characteristic. And that involves the concept of a 'half-life.' A half-life of a radioactive atom is the time by which we expect half of the total number of atoms to decay.



Polonium²⁰⁸ is a radioactive element that has a half-life of 2.89 years. And that means that after every 2.89 years, half of the atoms will have decayed. But if you wait another 2.89 years, than half of the half of the remaining atoms (now 25 % of the original number of atoms) will decay. So after 2.89 years, we have 50 % of the atoms remaining. After another 2.89 years, we have 25 % of the original atoms remaining. And then after another 2.89 years, we have 12.5 % remaining.

And there is a reason why this is the case ...

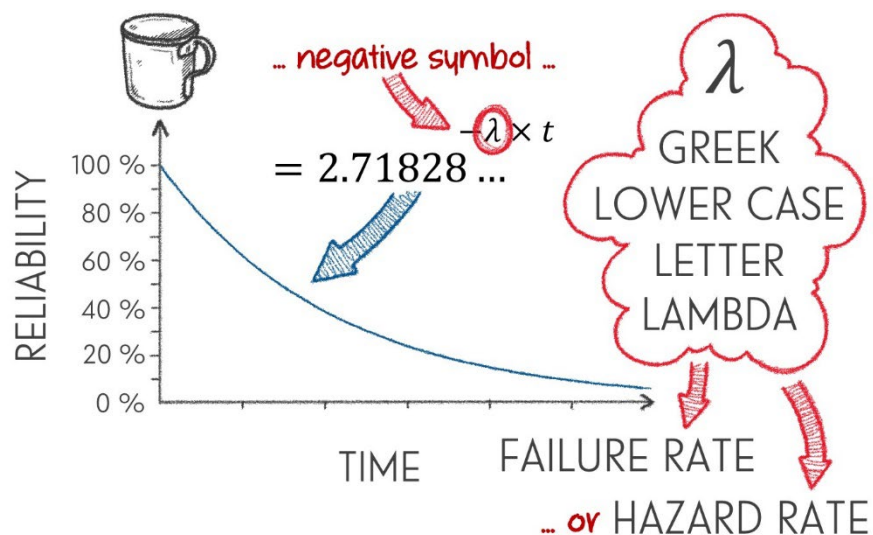


... so what's this got to do with RELIABILITY?

This is where the coffee mug comes into the discussion.



So it doesn't matter how old or young the coffee mug is, if it only fails when someone drops it, then every coffee mug is equally likely to fail (be dropped) today. So this means that the failure of the coffee mug can be modelled in the same way as the radioactive decay of polonium²⁰⁸.



So instead of dealing with interest or decay rates, we now deal with hazard or failure rates (λ). And if we were able to characterize the rate at which we are likely to drop a coffee mug, we can then create a reliability function for our coffee mug.

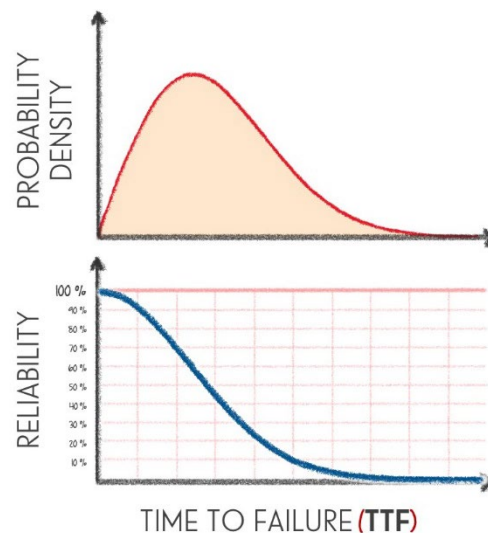


RELIABILITY

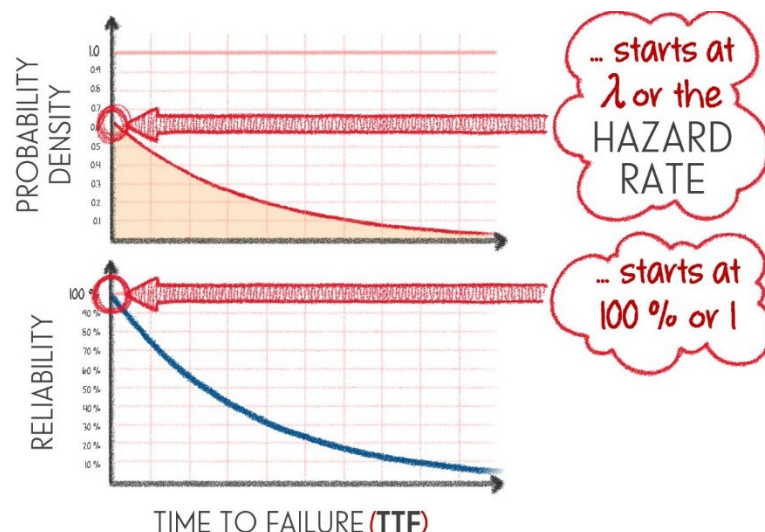
$$R(t) = e^{-\lambda t}$$

... and now the PROBABILITY DENSITY FUNCTION (PDF)

We typically like to visualize or conceptualize the nature of randomness with a probability density curve which has a 'peak' that represents how there is a 'central value' that is more likely than all others, and as we move away from this central value the likelihood of that number being relevant is less likely. We see this below, with its corresponding reliability curve.



But the exponential distribution probability density function (PDF) actually has the same shape as the reliability curve, with one key difference ...



And what this means is that we now have two equations to describe an exponential random variable.

$$R(t) = e^{-\lambda t} \dots \text{and} \dots f(t) = \lambda e^{-\lambda t}$$

So the PDF (in this case, and this case only), is simply the reliability function multiplied by the hazard rate. And this is because all the remaining systems, products or items are continuing to fail at the same rate, regardless of how old it is.

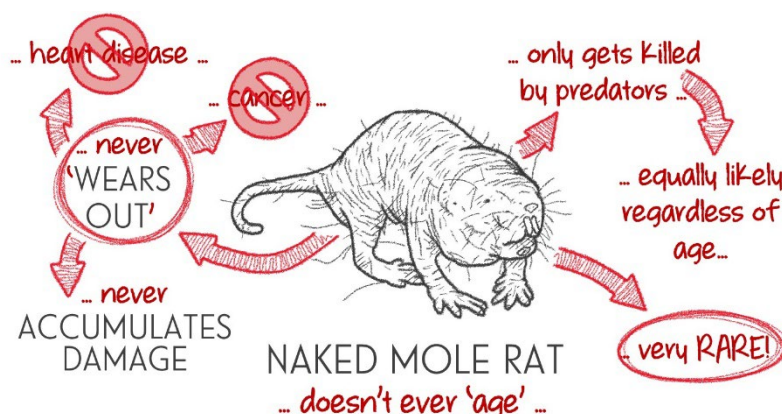
... so why is this called the EXPONENTIAL DISTRIBUTION

The following function is called the exponential function ...

$$f(x) = e^x$$

An exponential function is simply Euler's number raised to the power of some number. The reliability function and PDF above are similar in structure to the exponential function above, which means that this probability distribution is called the exponential distribution.

... what about the NAKED MOLE RAT?

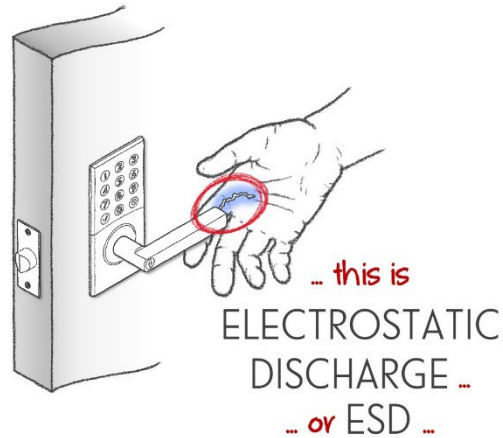


The reason this is here is to highlight the rarity of a real-world scenario where an animal, product, or anything else 'never ages.' There are naked mole rats and coffee mugs, but these examples are incredibly rare in terms of how they fail.

Any product, item or system that is more complex than a coffee mug, or isn't made out of the extremely durable ceramic, will have some failure mechanisms that do not have constant or hazard rates. And so the exponential distribution only applies to a tiny number of real-world scenarios, OR applies to a specific failure mechanism.

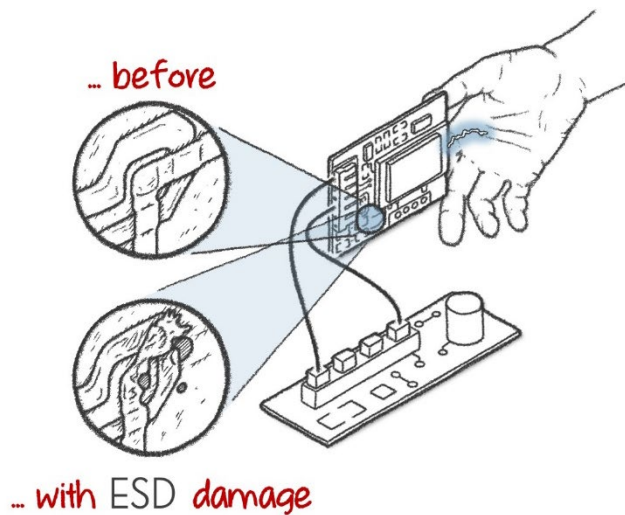
... but most ITEMS have at least one CONSTANT HAZARD RATE FAILURE MECHANISM

Smart locks and other products are susceptible to electrostatic discharge.



Electrostatic discharge (ESD) is an instantaneous flow of electric current between two differently-charged objects, like a smart lock and the hand of someone opening it after that person has gathered enough static electricity, which can often happen when people walk on carpet in a dry environment.

ESD can create thousands of volts in electric potential. And this can cause damage to small electronics.

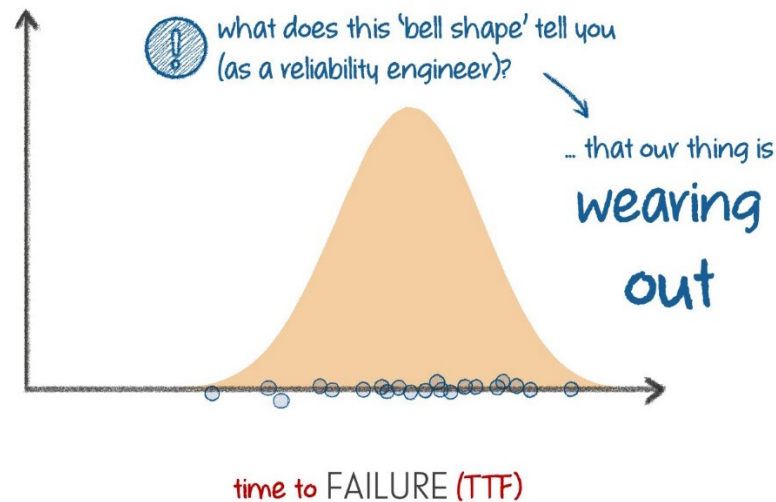


This is an example of a failure mechanism that will cause failure no matter how old or how young the smart lock is. So this failure mechanism will manifest with a constant hazard rate.

NORMAL DISTRIBUTINO

The **normal distribution PDF curve** is a 'perfectly symmetrical' bell shape. This shape isn't there because it looks 'pretty.' This shape is actually based on an underlying process that appears in lots of probability and statistics scenarios, random processes and the natural world.

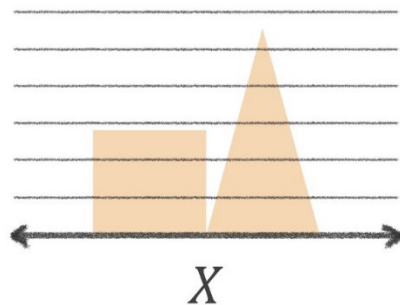
But first, remember what it means when a **TTF random variable** is described by a **PDF curve** with a 'bell shape?'



So ... where does this nice shape come from?

... the CENTRAL LIMIT THEOREM

Let's make up a **random variable**, which has a clearly 'absurd' **PDF curve**. Let's make this absurd **PDF** look like it has a square on the left and a triangle on the right.

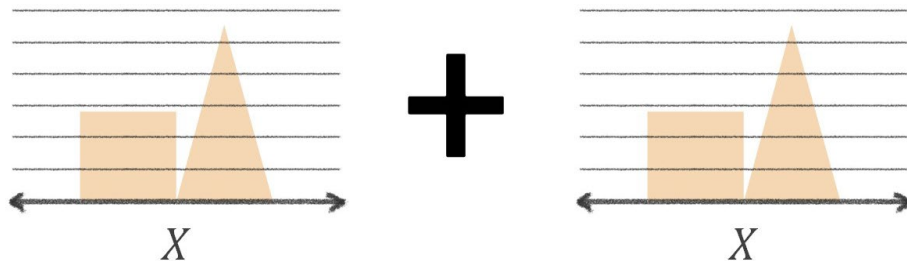


Hopefully you can agree that this **random variable** is not likely to exist! But now let's add this **random variable** to itself. That is ...

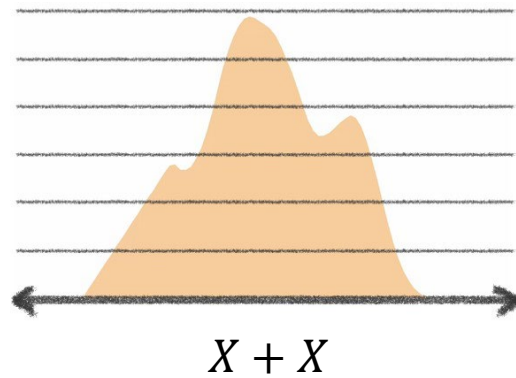
$$X + X$$

What does the **PDF** of the sum of these two **random variables** look like?

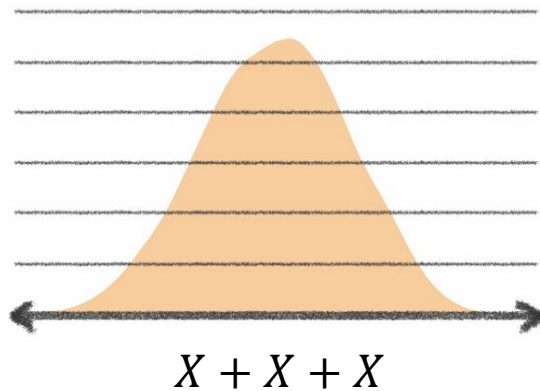
So we are trying to ...



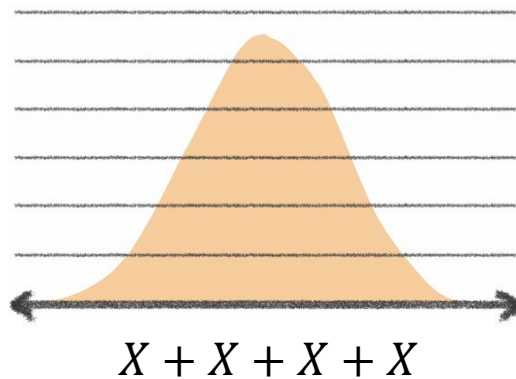
It turns out the PDF of this new **random variable** ($X + X$) looks like ...



You can already see that the shape of this new **PDF curve** is substantially different. It is smoother, and perhaps more like a bell shape! But let's add another X and see what happens ...

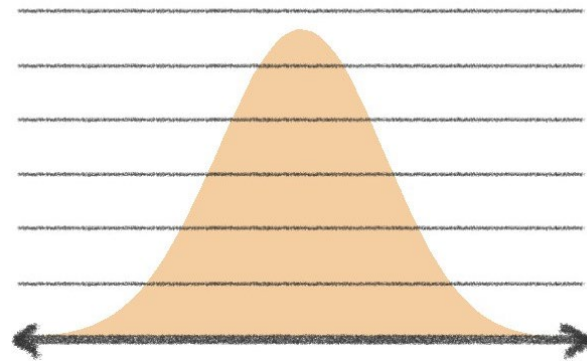


And let's add another X ...



It should be really obvious that the **PDF** of this **random variable** that is the sum of lots of 'X's' has a **PDF curve** that looks like a bell shape. And this is where the **normal distribution** comes from. If there a **random process** is based on the sum of lots of random processes, the output random variable will probably be described by a **normal distribution** with its bell-shaped **PDF curve**.

In other words, **random variable 'Y'** is a **normally distributed random variable** if ...



$$Y = X_1 + X_2 + X_3 + \dots$$

INDEPENDENT random variables

This phenomenon is known as the **Central Limit Theorem**.

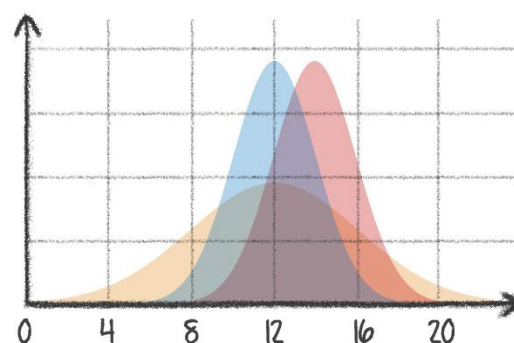
the CENTRAL LIMIT THEOREM states that any sum of (independently individually distributed) random variables will be approximately NORMALLY DISTRIBUTED.

So ... the **normal distribution** IS the 'bell curve.'

NORMAL DISTRIBUTION - PROBABILITY DENSITY FUNCTION (PDF)

The equations and formulae for the **PDF** and other functions and statistics based on the **normal distribution** (including the **mean**) are at the end of this chapter. The most important thing to understand when dealing with **PDFs** is knowing what they represent in terms of failure. Three example **lognormal distribution PDF curves** are illustrated below.

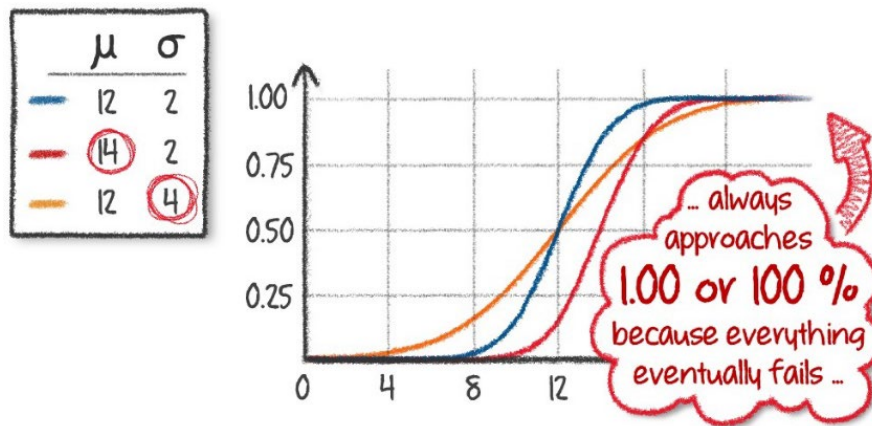
... a symmetric
bell-shaped
curve centered
on the MEAN



	μ	σ
Blue	12	2
Red	14	2
Orange	12	4

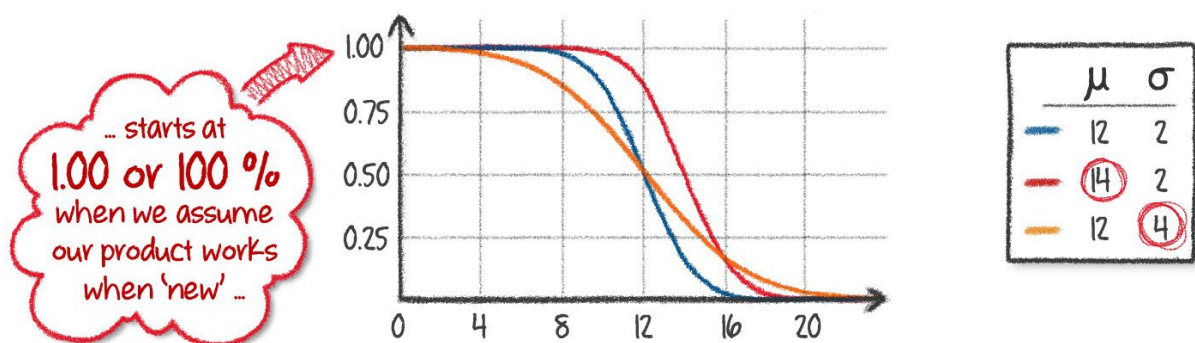
NORMAL DISTRIBUTION - CUMULATIVE DISTRIBUTION FUNCTION (CDF)

Three example **normal distribution CDF curves** are illustrated below. Each **CDF curve** corresponds to one of the **PDF curves** above. You can see here that the **Weibull distribution CDF** (like all other **CDFs**) starts at 0 and approaches 1 (or 100 %) as **TTF** increases.



NORMAL DISTRIBUTION - RELIABILITY FUNCTION

Three example **normal distribution reliability curves** are illustrated below. Each **reliability curve** corresponds to one of the **PDF and CDF curves** above. You can see here that the **normal distribution reliability curve** (like all other **reliability curves**) starts at 1 (or 100 %) and approaches 0 as **TTF** increases.



NORMAL DISTRIBUTION - HAZARD RATE

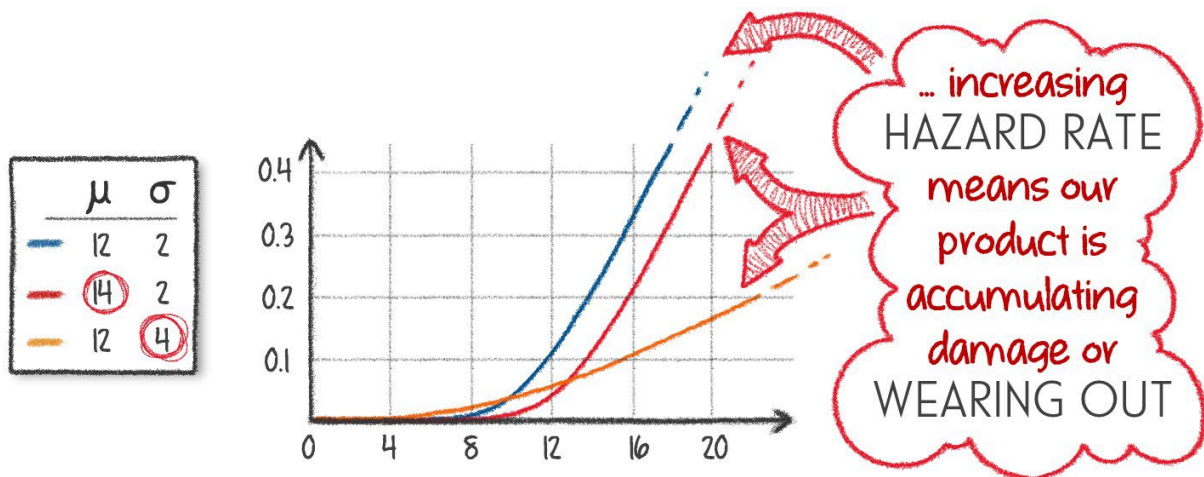
One way of looking at the **hazard rate** is by seeing it as the rate at which working (non-failed) systems are **failing**.

$$\frac{f(t)}{R(t)}$$

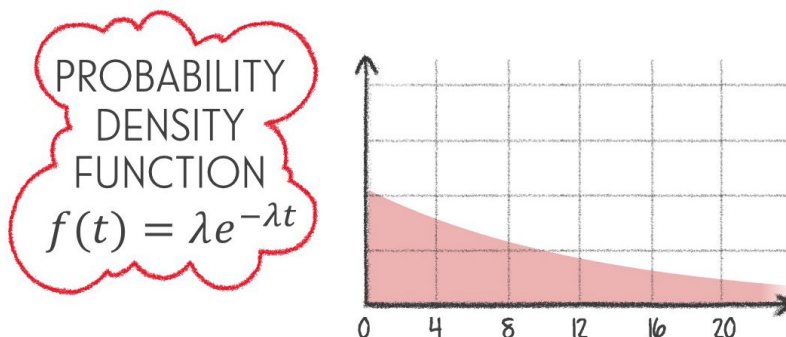
... the RATE at which a 'thing' that is still working - FAILS

PROBABILITY DENSITY FUNCTION (PDF)
RELIABILITY FUNCTION

Three example **normal distribution hazard rate curves** are illustrated below. Each **hazard rate curve** corresponds to one of the **PDF, CDF and reliability curves** above.

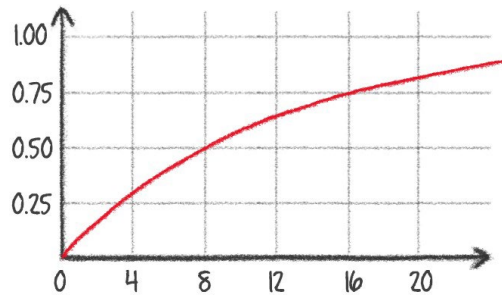


EXPONENTIAL DISTRIBUTION - PROBABILITY DENSITY FUNCTION (PDF)

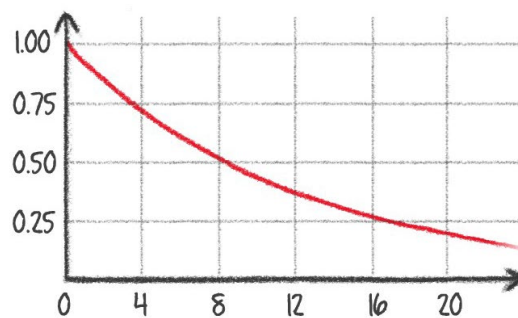


EXPONENTIAL DISTRIBUTION - CUMULATIVE DISTRIBUTION FUNCTION (CDF)

The area under the PDF curve creates the failure probability at a particular usage, which gives us the CDF.



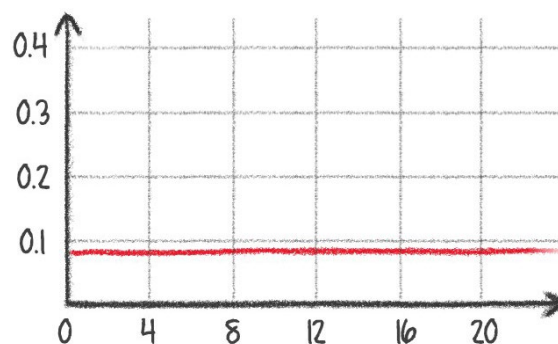
EXPONENTIAL DISTRIBUTION - RELIABILITY FUNCTION



RELIABILITY
 $R(t) = e^{-\lambda t}$

You can see here that the **exponential distribution reliability function** (like all other **reliability functions**) approaches 0 as **TTF** increases. This is another way of saying that everything eventually **fails**! We know that $R(t) = 1 - F(t)$ and $F(t) = 1 - R(t)$.

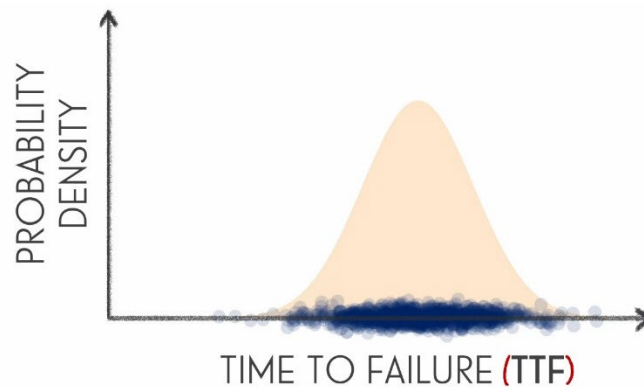
EXPONENTIAL DISTRIBUTION - HAZARD RATE



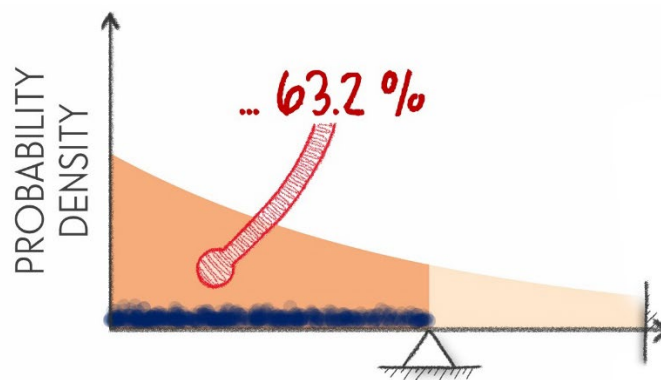
This **hazard rate** essentially means that the **exponential distribution** is simple. You only need one parameter to define it (the **hazard rate**). Which usually makes it too simple.

AND THE MEAN TIME TO FAILURE (MTTF)

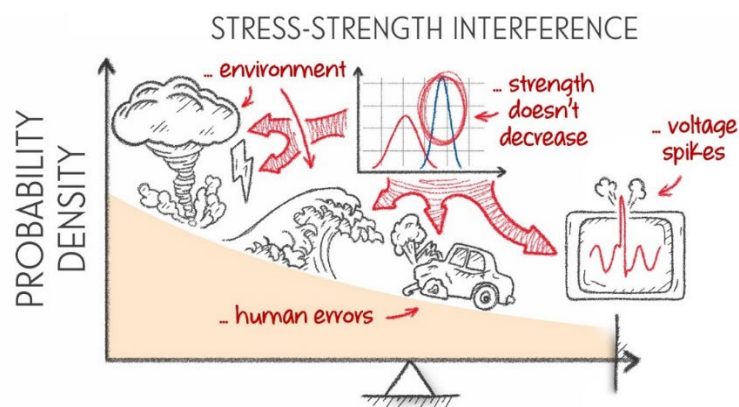
This is how the human brain is 'wired' to characterize uncertainty and randomness, with a PDF that indicates some central value is more likely than others, with that likelihood decreasing as we move away from that central value.



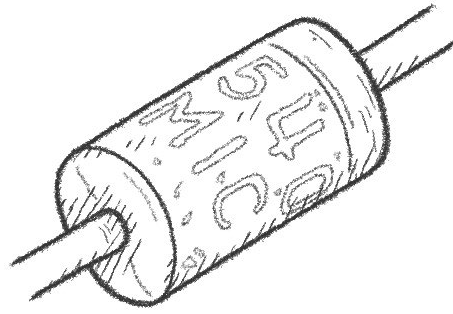
And we know this is not what it looks like for the exponential distribution. And we also know that the MTTF (balance point) is the inverse of constant hazard rate. And when we put the balance point under the PDF, we find the following ...



And this doesn't change when the MTTF changes. So we know that we should expect 63.2 % of items to fail by the MTTF, if it has a constant hazard rate. And this happens when we see the following causes of failure ...



Diodes are used in virtually every electronic product we use.



OEMs usually advertise electronic components as having a certain number of 'FITs.' A 'FIT' stands for 'failure in time' and is equivalent to 1 failure per billion hours. The diode above is advertised as having a hazard rate of '100 FITs' This means a hazard rate of 100 failures per billion hours or 1 failure per 10 million hours.

This single hazard rate more or less 'forces' us to use the exponential distribution to model reliability, as the exponential distribution is the only probability distribution that only needs one parameter.

So let's calculate the probability of this diode surviving the first year of continuous operation (reliability), assuming TTF is modelled with the exponential distribution.

The first thing we need to do is get the 'units' right. We are trying to find the probability of the diode failing in the first year. The failure rate above (1 failure per 10 million hours) needs to be converted into years.

What is the failure rate in terms of failures per year?

Calculate the reliability of the diode at one year
(assuming an exponential distribution).

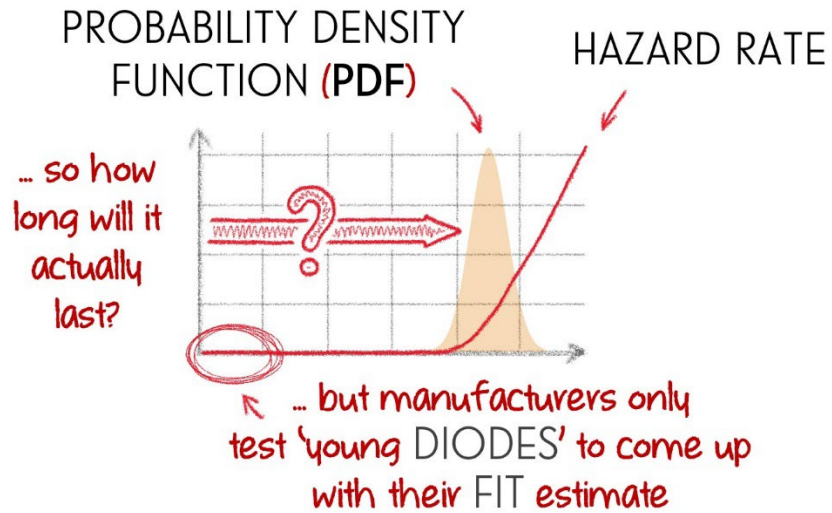
$$R(t) = e^{-\lambda t}$$

$$R(\text{ }) = e^{-\text{ } \times \text{ } = e^{-\text{ } = \text{ }}$$

What is the MTTF or MTBF of this diode?

EXERCISE

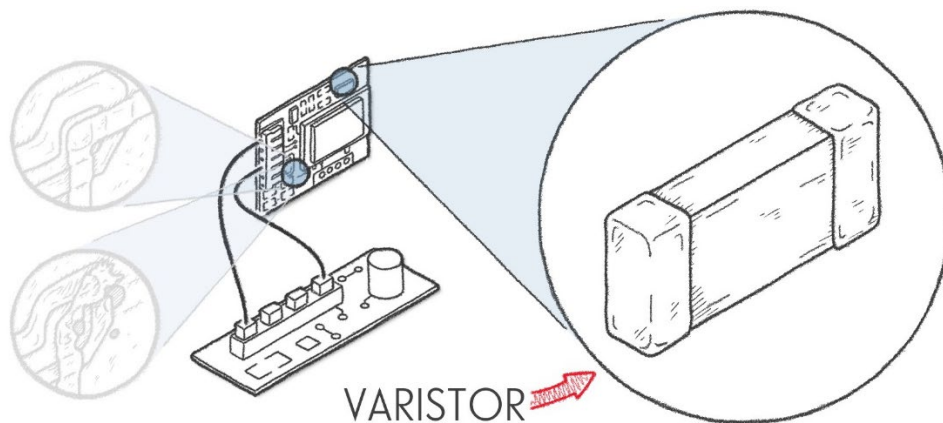
But the BIG problem with all of this is that we now know that diodes (and every other electronic component) wears out. Which means that hazard rates increase. So how do OEMs come up with outlandishly low constant hazard rates.



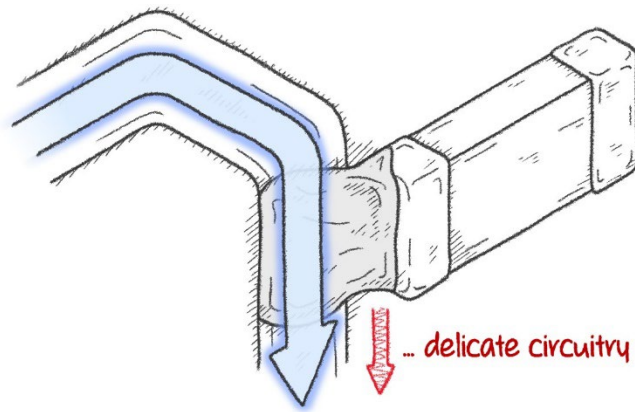
So make sure you check, double check, and otherwise interrogate anything that your OEMs say.

but we aren't here to ADMIRE PROBLEMS

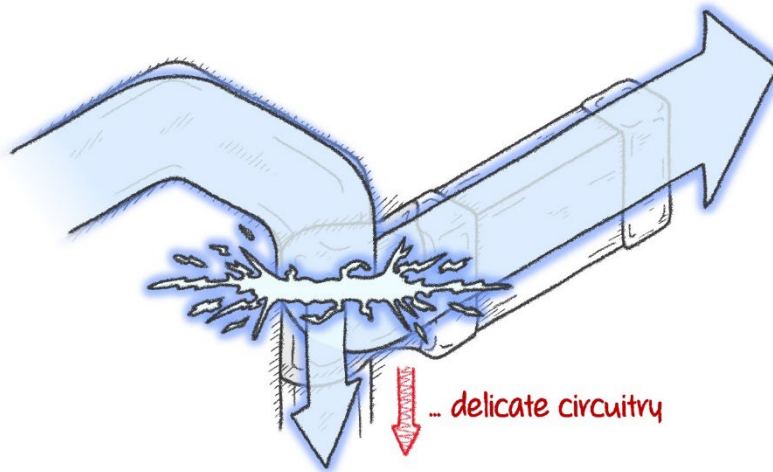
There is a way we can tackle ESD.



A varistor (variable resistor) has very high resistance when the voltage is low. And so no current flows through it when operating normally.



But when the high voltage from ESD 'comes into contact' with the varistor, its resistance dramatically drops. This allows the high energy electricity to then 'drain' through the varistor.



... and the FINAL WORD

The **exponential distribution** is usually too simple, and usually overused. It can model **failure** that has a constant **hazard rate**. That means the system is equally likely to **fail** when it is 100 years old as it is when it is brand new. There are some **failure mechanisms** where this is appropriate (like a tire puncture as described above), but in reality there are virtually no systems that have a system level constant **hazard rate**.

One of the reasons we see **exponential distribution** so much in **reliability engineering** is that it is simple. Textbooks were originally written before we had personal calculators (let alone computers). This means that textbooks needed to have really simple methods. But unfortunately, these simple methods have remained embedded in **reliability engineering**. So make sure you really understand how your product, system or service fails!