

Empowerment and Embodiment for Collaborative Mixed Reality System

Abstract

We present several mixed reality based remote collaboration settings by using consumer head-mounted displays, including an AR system linked with an AR system, a VR system with virtual body, a VR system without virtual body and a desktop computer. We investigated how two people are able to work together in the these settings. We found that the person in the AR system will be regarded as the leader (greater contribution) in AR-to-VR and AR-to-Desktop settings, whereas two participants' performance in AR-to-AR and AR-to-VRBody settings are very similar to each other for the 3D interaction. However, no special pattern of leadership emerged for the 2D interaction. Results about participants' experience of leadership, collaboration, embodiment, presence, and co-presence shed further light on these findings.

Keywords: Augmented Reality and Virtual Reality, Telecollaboration

1 Introduction

Collaboration at a distance has long been an important research goal of networked or multi-user augmented reality (AR) and virtual reality (VR) systems. With the launch of low-cost head-mounted displays, networked mixed environments have rapidly increased in prevalence and popularity as a form of remote collaboration.

We present several collaborative mixed reality settings, allowing multiple users to visualize and edit a planet in a mixed reality environment. Table 1 gives an overview of the settings and technologies used and detailed in Section 3. For AR-to-AR setting, we provided each partic-

Table 1: Scenarios, labels, and technology used

Label	Site A	Site B
AR-to-AR	AR	AR
AR-to-VRBody	AR	VR with virtual body
AR-to-VR	AR	VR
AR-to-Desktop	AR	Desktop

ipant an AR system based on HTC Vive headset coupled with Ovrvision Pro stereo camera. The HTC Vive was chosen because Vive base stations provide space tracking, so we can realize markerless mixed reality easily. Also, the Vive controllers allow for high-quality, user-friendly interaction experiences. These two AR systems were then networked, enabling two users to interact with a shared virtual scene and each other in a face to face arrangement. This setting allowed the establishment of common ground for our study. For AR-to-VRBody setting, the VR system is a Vive headset. The participants were physically in two separate rooms while working together. Each users body could be represented by a jointed self-avatar that was dynamically controlled by head and hand controllers. For the AR-to-VR setting, each user was represented only by models of controllers. This representation is common in consumer virtual reality applications at the moment. For AR-to-Desktop, an AR system was linked with a Desktop computer.

To evaluate the effectiveness of our settings, we conducted a user study to investigate how people interact with each other in mixed reality environments, especially for spatially complex 3D environments. We expected that the more immersed participant was singled out as the leader (greater contribution to the task). The

AR-to-Desktop will have the highest leadership effect, next comes the AR-to-VR, then the AR-to-VRBody, and finally this advantage will be lost in the AR-to-AR setting. We further expected that this leadership effect only emerged in 3D interaction, but not in the 2D interaction.

The remainder of the paper is structured as follows. Section 2 provides a broad overview of related work on mixed reality systems, collaboration, and self-avatars. Section 3 and 4 presents the methodological basis of the experiment and details the analysis of the experimental results. Then, Section 5 discusses the main findings and Section 6 provides the concluding remarks.

2 Related work

2.1 Mixed reality system

Milgram & Kishino's virtuality continuum is the seminal taxonomy of the field [1]. It classifies systems that mixed real and virtual visual content from pure real environments (e.g., video) at one end to a purely synthetic virtual environment at the other. MR occupies the range of the continuum between these extremes, merging both real and virtual objects together. Mixed reality systems use a range of technologies including projection displays, situated displays and head-mounted augmented reality displays.

Inspired by these recently developed systems, we developed four mixed reality based telecommunication settings with different level of immersion and examine how these cutting-edge systems can be used in collaborative interactions.

2.2 Collaboration and leadership

A previous study of a puzzle-solving task with three participants found that leadership varies between a virtual setting in which the more immersed participant is singled out as the leader as against the same task performed in the real setting where no one is singled out as the leader [2]. There are a number of other studies of these issues (e.g. [3]), which cannot be elaborated here for reasons of space.

Most of these previous work have focused on different types of VR systems. Because AR systems provide with different levels of immersion,

there is a need for closer examination of the leadership/contribution to the task and different types of MR systems. Also, it is not so clear about task dependence, thus we include both 2D interaction and 3D interaction for our task design.

2.3 Avatars

The impact of a self-avatar has been investigated in many ways, including the visual embodiment of the user, means of interaction with the world, means of sensing various attributes of the world etc [4]. The self-avatar in a collaborative MR has crucial functions in addition to those of single-user MR environments, as the avatar is used for communication, including determining position, identification, visualization of focus of attention and recognition of gesture and actions [5, 6, 7].

Various papers demonstrated avatars exhibiting higher levels of visual quality or tracking quality (e.g. eye tracking, facial expression, finger tracking etc.) can potentially communicate more subtleties of human nonverbal communication, enhancing the perceived authenticity of the interaction [8, 9, 10]. However, there are problems in providing a self-avatar, due to uncanny valley and the different discrepancies [11, 12, 13].

The general thrust of the work indicates that self-avatars are important and that animation of the avatar can improve the effect of the self-avatar for most tasks. In this study, we aim to grow the existing knowledge on how the self-avatar alters users' behavior in collaborative MR.

3 System Design

In this section we discuss the system design and implementation of the experiment application. The experiment were conducted at two very similar cubicles with a size of 2.5m × 2.5m on the same floor of a building. These two sites were networked, so that the users were physically separated while working together in the MR environment (see Figure 1).

Each participant was supported by an application on a computer in the laboratory they used.

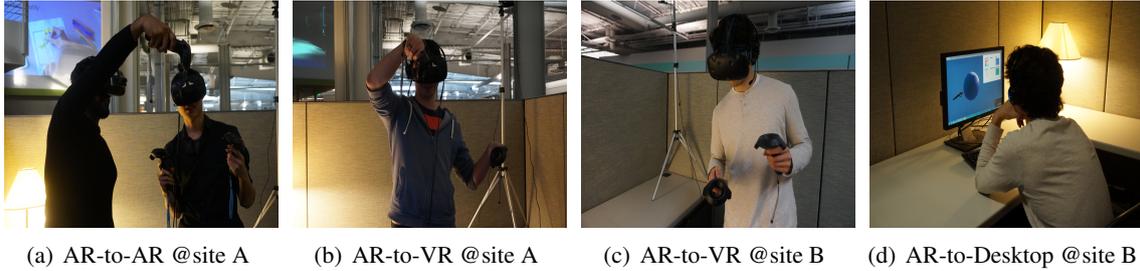


Figure 1: Photos taken from 3rd person views for different conditions

Each application ran on a Windows 7 computer with an Intel Xeon processor, 16GB ram and a GeForce TitanX graphics card. We provided each user with the HTC Vive headset (combining with Ovrvision Pro stereo camera for AR system) and controllers to view the virtual world and control the self-avatar. The MR environment was created using Unity 5.6.2f and written in C#. All scenes were rendered at 90Hz. Audio extension cables were used, and we also ensured HMD cables were long enough to not obstruct participant movements.

The scene consisted of three elements: a background scene, a planet and self-avatars. The background scene was a model of the cubicle.

3.1 Planet

The planet’s appearance is of a textured sphere, procedurally generated using Unity shader code. The structure of the planet landmass is determined by a set of nodes (points on the surface with associated radii, representing continental landmasses) and links (terrain ‘bridges’ between nodes). This graph-like structure is used to create a distance field cubemap texture representing the shortest distance to the nearest node or link. A few noise functions based upon simplex noise [14], fractal brownian motion [15] and ridged multifractals [16] are used to perturb the distance field and simulate more realistic terrain boundaries; terrain is then colorized according to the perturbed distances, and some basic lighting effects are added to create a more pleasing visual appearance. The terrain generation is performant enough that discrete edits to the terrain can be smoothly interpolated and animated in real time on consumer-grade desktop computers - for instance, a newly created node will appear to ‘grow’ outwards from the centre

until it reaches the appropriate radius.

3.2 Avatars

Some participants had a self-avatar. We provided both male and female avatars in generic clothing, taken from the Rocketbox Complete Characters HD set. We used each participant’s height information to scale the height of the avatar. The participant held the two Vive controllers and wore the Vive HMD with tracking. This gave three points of tracking to animate the self-avatar. We linked these tracking points to the avatar’s hands, and head respectively. We then used the VR IK solver from the Final IK plugin to map participant’s movements in real world space to the self-avatar’s movements.

3.3 Interaction techniques

There are four main editing operations used by participants to edit the planet. Participants in AR and VR modes use the Vive controller to complete the operations, and Desktop participants use mouse and keyboard. When in VR or AR mode, the virtual Vive controller appears almost identical to the real controller, except that a laser-like beam is emitted from the front of the controller to show the user which objects are being pointed at by the controller, and a colour picker dial is superimposed over the Vive controller’s touchpad. The operations are as follows:

Change Colour In AR and VR modes, a vertical board is shown near to both users; the board contains a series of coloured rectangles, labelled with the planet’s terrain types. To change the colour of the planet’s terrain, the user aims the

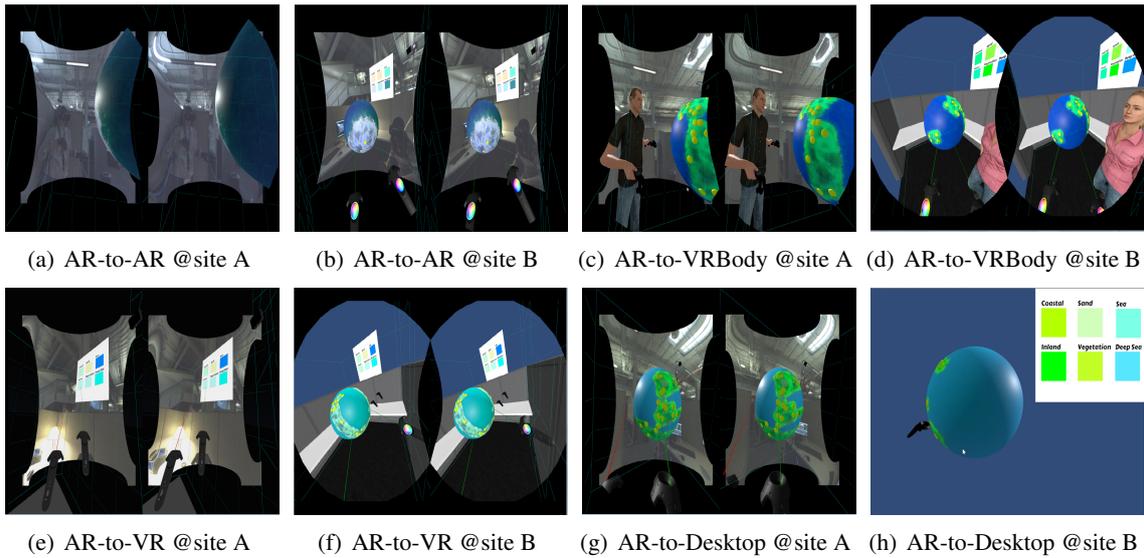


Figure 2: Screenshots for different conditions. Each pair of screenshots was simultaneously captured from the first-person view of each participant within the dyad.

Vive controller at the coloured rectangle and manipulates the touchpad; the colour of the rectangle, and that of all the corresponding terrain on the planet, is changed to the colour which matches the colour picker overlay on the virtual Vive controller.

Desktop users have an inset with the same board as that shown to the AR and VR users. Holding down the left mouse button with the pointer over one of the coloured rectangles turns the coloured rectangle into a colour picker overlay. Moving the mouse pointer over a colour on the picker changes the rectangle's colour, and that of the corresponding terrain, to the corresponding colour, and releasing the mouse button removes the picker, with the terrain colour changed to the appropriate colour.

Create Node Nodes are terrain points where landmasses are centred; these are signified by a yellow node marker. There is a terrain radius associated with these nodes. Temporary nodes - those which are created in an ongoing edit operation - are signified by a cyan marker until the operation is either completed (in which case it turns yellow), or aborted (in which case the marker disappears).

To create a node in AR/VR mode, the user aims the controller beam at the planet and holds down the trigger; the radius of the terrain ex-

pands, with a real time animation, until the user releases the trigger.

In Desktop mode, the operation takes a similar form. Holding down the left mouse button with the mouse button over the planet creates a node with terrain which expands until the user releases the mouse button.

Create Link Links are strips of terrain along the geodesic lines between two nodes, at least one of which is newly created. To create a link, the user first creates a node by either using the Vive controller trigger or the left mouse button, as above. Then, while holding down the button or trigger, the user drags the controller pointer, or mouse pointer, to another point on the planet, and releases the trigger or mouse button. If the release point is not an already existing node, two nodes will be created, one at the position initially pointed at by the user, and the other at the position when the user released the trigger or mouse button, and there will be line of terrain between them. Both nodes would have the same terrain radius, which is determined by the length of time that the trigger or mouse button was held down.

If the user drags the pointer over another node while creating a link, then a geodesic link is created between the newly created node, and the node dragged over. Only the newly cre-

ated node's radius will be determined by the length of time that the trigger or mouse pointer is held down; the node terrain radius of the node dragged over remains constant. The width of the geodesic terrain line is linearly interpolated so that it matches the node terrain radiuses at either end, and there are no sharp edges or discontinuities in the resulting landmass.

Delete Node Nodes can be deleted in AR/VR modes by aiming the controller at an already created node marker and pulling the trigger. Desktop users delete nodes by left-clicking on a node marker. In both cases, there is an animation showing the terrain radius decreasing, and any geodesic terrain links receding, until the terrain vanishes and the node marker is removed.

3.4 Networking

To ensure all participants were receiving the same state for the virtual environment, we implemented a client-server system using Unity's built-in multiplayer networking system. We first tracked each participant's physical movement and behaviour, obtaining 3D coordinate frames for all the tracked objects to animate the self-avatar at the local each client. Then, these 3D coordinates were submitted to the server, and propagated to all the remote clients. At the remote client, the corresponding avatar would be animated based on these 3D coordinate frames. Aural communication was supported using Skype. We identify spatialized 3D audio as an area of future work.

4 Experiment

The goal of the study was to investigate leadership and collaboration for several MR settings. We manipulated the levels of immersion to examine users' performance.

4.1 Method

4.1.1 Participants

16 participants, recruited from ANONYMOUS, worked in pairs to complete the build your own earth task in the four conditions. The average age of the participants was 25.94 years, with a

range between 21 and 33 years old. 50% were male. All participants reported some familiarity with AR or VR. They were naive to the purposes of the study.

4.1.2 Material

This task was chosen because it demonstrates our MR settings supporting multiple users to visualize and edit a planet in real time. Also, it requires collaboration between the users since it is difficult for one participant to remember various characteristics of a planet. The task can be divided by each participant creating a different part of the planet; or one working on continent and the other working on colour.

4.1.3 Design

A repeated measures design was used. There were two independent variables: sites (site A or site B), and settings (AR-to-AR, AR-to-VRBody, AR-to-VR, or AR-to-Desktop). Each group of two participants took part in all four conditions. To minimize any practice or carry-over effects, the order of the settings was counterbalanced using a latin square.

4.1.4 Procedure

Before beginning the experiment, participants at both sites were asked to fill out a brief demographic survey as well as a consent form. The experimenters in both sites gave the participants an overview of the build your own earth task that the participants would engage in. The experimenters calibrated matched size self-avatars for them for some VR and AR conditions. They then guided the participant on how to create continents and change colors using controllers in the VR and AR conditions, or using mouse in the desktop conditions.

For each trial, participants were asked to complete the build your own earth task. Then, the experimenters at both sites terminated the connection, participants were taken to a nearby computer, where in private they completed a questionnaire featuring questions relating to the experience.

Finally, when participants completed all trials, an experimenter conducted an interview

with participants individually, in order to collect general comments on their experience during the experiment. Participants received chocolates as compensation. The experiment took about 40 minutes.

4.1.5 Post-questionnaire

Participants were presented post-experimental questionnaire consisted of 14 statements (see Table 2) with randomising the order of the statements. The greatest part of the questionnaire is based on previous work, since it has been shown to be a reliable indicator for leadership, collaboration, embodiment, presence and co-presence. Participants responded to a set of statements each with an associated 1–7 Likert scale, where an answer of 1 indicated complete disagreement, and 7 indicated complete agreement.

4.1.6 Data analysis

Because our experiment involved pairs of participants rather than individuals, we were unable to assume independence in measurements from participants in two sites. Therefore, we employed dyadic analysis methods in order to compare data across experiment conditions, while taking the potential interdependencies in data from members of dyads into consideration.

4.2 Result

4.2.1 2D interaction

A two-way repeated measures ANOVA was run to determine the effect of different settings at two sites on the number of 2D interactions. There were no outliers and the data was normally distributed for each conditions, as assessed by boxplot and Shapiro-Wilk test ($p > .05$), respectively. Mauchly's test of sphericity indicated that the assumption of sphericity was met for the main effect of conditions, $\chi^2(5) = 4.695, p = .458$; but not for the two-way interaction $\chi^2(5) = 12.192, p = .034$. Results revealed that here were no statistically significant differences the for two-way interaction, $F(3, 24) = 3.356, p = .036$, the main effect of conditions, $F(1.775, 14.2) = .495, p = .689$, and the main effect of sites, $F(1, 8) = .007, p = .936$.

4.2.2 3D interaction

A two-way repeated measures ANOVA was run to determine the effect of different conditions at two sites on the number of 3D interactions. There were no outliers and the data was normally distributed for each conditions, as assessed by boxplot and Shapiro-Wilk test ($p > .05$), respectively. Mauchly's test of sphericity indicated that the assumption of sphericity was met for the two-way interaction, $\chi^2(5) = 1.738, p = .885$. There was a statistically significant two-way interaction between site and condition, $F(3, 24) = 3.356, p = .036$. Therefore, simple main effects were run. The number of 3D interaction between two sites was not statistically significantly different in the condition AR-to-AR $t(8) = -1.101, p = .303$ and condition the AR-to-VRBody $t(8) = 1.151, p = .283$. However, there was a statistically significant mean difference in the condition AR-to-VR $t(8) = 2.239, p = .044$ and condition AR-to-Desktop $t(8) = 3.594, p = .007$.

4.2.3 Post-questionnaire

Leadership Three pair of questions were asked to allow the participants to evaluate their own and their partners contribution to the task. The Q1 & Q2 concerned contribution to the task in general, the Q3 & Q4 the contribution in editing the terrain, and the Q5 & Q6 the amount of verbal communication.

We first looked at the estimation of contribution regarded themselves and their partners for participants at the site B (see Figure 4, blue box in Q1 & Q3, and red box in Q2 & Q4). We can see a clear downward trend from AR-to-AR, AR-to-VRBody, AR-to-VR to AR-to-Desktop. A Friedman test was run to determine if there were differences in these four conditions. Pairwise comparisons were performed with a Bonferroni correction for multiple comparisons. Adjusted p-value are presented, and only significant results are shown. Only participant in the Desktop condition was evaluated their contribution as being statistically significant less that participant in the AR condition ($p = .028$), and this in respect both to their contribution in solving the task and to editing the terrain ($p = .016$). The verbal contribu-

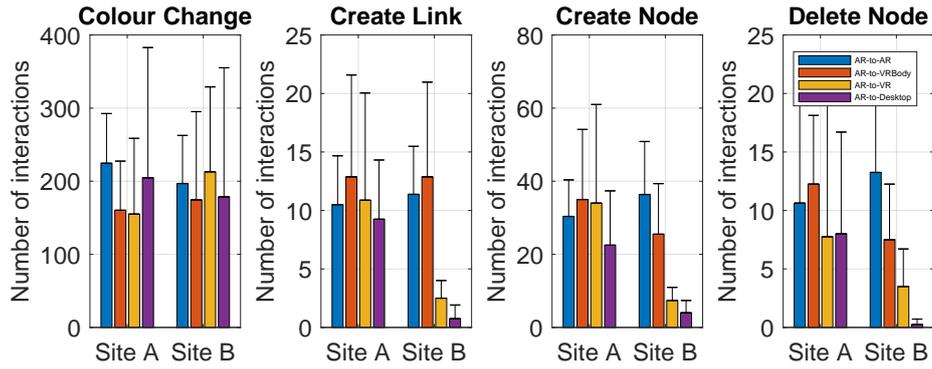


Figure 3: Bars showing the number of interactions for each condition and site.

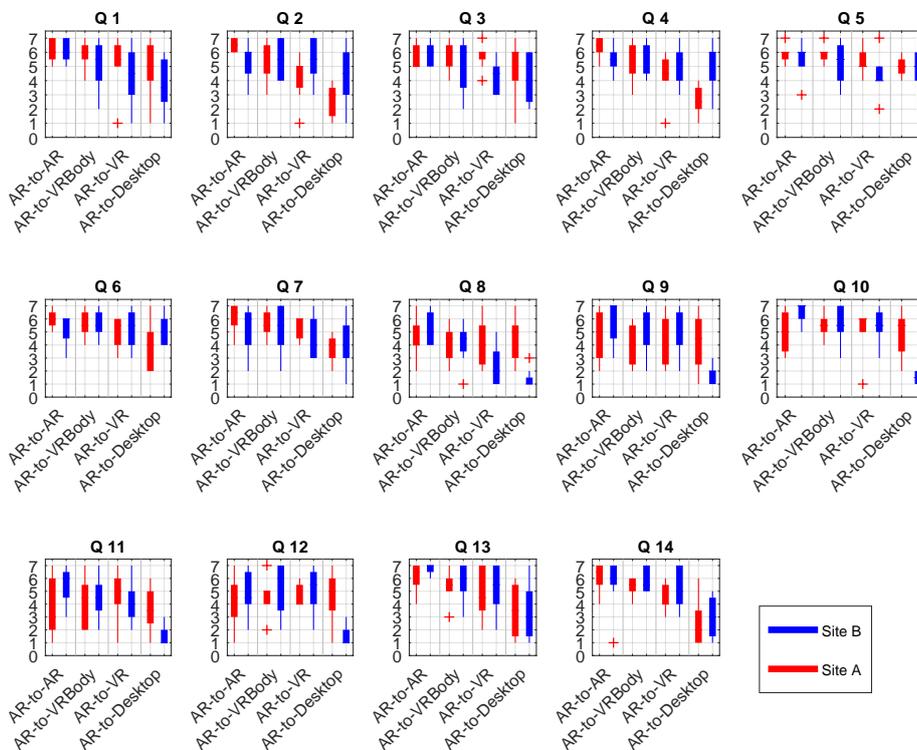


Figure 4: Box-plots for questionnaire items associated with Table 2. Medians, interquartile ranges, and full ranges are shown.

tion, however, was regarded as equal in all cases. These results were not surprising in as much as we would not expect there to be any difference in verbal contribution, but we would expect differences for the spatial part of the task.

Figure 4 also showed that participants at the site A were evaluated by both partners (red box in Q1 & Q3, and blue box in Q2 & Q4) as being more active in the task generally and con-

tributing more to editing the terrain. This point can spelled out in more detail for emphasis: both partners agreed about the difference in their contributions, and there was agreement that this difference applied in terms of contribution to overall contribution, editing the terrain, and verbal communication.

Collaboration We also asked the participants to evaluate collaboration (Q7). A Friedman test showed that there was a significant difference, $\chi^2(3) = 15, p = .002$, among multiple conditions at site A, but no such difference was found at site B. Post hoc analysis revealed participants from the Desktop condition reported a lower degree of collaboration than the VRBody condition ($p = .04$) and the AR condition ($p = .003$).

From the observations of all trials, it appears some groups maintained a conversation while collaborating, constantly updating each other on the choices of what colour might be, and strategies for editing the terrain. Some groups didn't feel a need to constantly update the partner verbally on progress, as a quick glance was sufficient for sharing the partner's work. One participant in the VRBody condition commented:

"We can see each other, we don't necessarily have to communicate verbally all the time."

Participants in the AR-to-VR setting gave detailed instructions. In contrast to deictic references such as "here" or "there", which were more frequently observed in the AR-to-VRBody setting and the AR-to-AR setting. In addition, they often asked for "confirmation" to ensure the other partner could clearly understand while pointing, for example,

"Can you see my controller at least?"

In addition, we also looked at verbal communication during while collaborating:

"You are ruining my drawing! " "I am sorry. I am using a desktop. I cannot see your drawing. I am going to rotate the earth and make it facing us again."

Thus, this indicated the desktop condition introduces a possibility of interference and confusion, where one participant's actions potentially disturb the productivity of others.

Embodiment For embodiment Q8, we find a rank order: for participants at site B, the AR-to-AR has the highest reported embodiment, next comes the AR-to-VRBody, then AR-to-VR, and

finally AR-to-Desktop. A Friedman test showed that there was a significant difference, $\chi^2(3) = 20.186, p < .001$, among multiple conditions at site B. Post hoc analysis revealed participants from the AR-to-AR and AR-to-VRBody reported a higher degree of embodiment than the AR-to-Desktop condition, ($p = .04$) and ($p = .001$), respectively. Also, the difference between the AR-to-AR and the AR-to-VR was significant, ($p = .022$).

Presence Our findings are as expected; namely, at site B, that the only major difference is that AR-to-Desktop participants report a lower degree of presence. A Friedman test showed that there were a significant difference among multiple conditions for Q10, $\chi^2(3) = 21.286, p < .001$, Q11, $\chi^2(3) = 19.875, p < .001$, and Q12, $\chi^2(3) = 16.757, p = .001$, respectively. Post hoc analysis revealed participants from the AR-to-Desktop condition reported a lower degree of presence than the AR condition for Q10 ($p < .001$), Q11 ($p = .001$) and Q12 ($p = .004$); and the VRBody condition for Q10 ($p = .03$), Q11 ($p = .04$) and Q12 ($p = .016$). Also, the difference between the desktop and the VR was significant, Q14 ($p = .22$).

Co-presence By co-presence we mean the subjective sense of being together or being co-located with another person in a computer-generated environment.

At site A, Friedman test showed that there were a significant difference among multiple conditions for Q13, $\chi^2(3) = 12.785, p = .005$, and Q14, $\chi^2(3) = 19.708, p < .001$, respectively. Post hoc analysis revealed participants from the desktop condition reported a lower degree of co-presence than the AR condition for Q13 ($p = .008$), and Q14 ($p = .001$). Also, the difference between the desktop and the AR-to-VRBody condition was significant for Q14, ($p = .03$).

Interestingly, at site B, Friedman test also showed that there were a significant difference among multiple conditions for Q13, $\chi^2(3) = 14.304, p = .003$, and Q14, $\chi^2(3) = 12.422, p = .006$, respectively. Post hoc analysis revealed participants from the desktop condi-

Table 2: Post-questionnaire (7–Likert scale).

NO.	Questionnaire Item
Q1	How would you evaluate your and your partner's level of activity in solving the task. Please rate YOUR level of activity.
Q2	How would you evaluate your and your partner's level of activity in solving the task. Please rate YOUR PARTNER's level of activity.
Q3	To what extent did you and your partner contribute to editing the terrain. Please rate YOUR level of contribution.
Q4	To what extent did you and your partner contribute to editing the terrain. Please rate YOUR PARTNER's level of contribution.
Q5	Who talked the most, you or your partner. Please rate YOUR the amount of verbal contribution.
Q6	Who talked the most, you or your partner. Please rate PARTNER's the amount of verbal contribution.
Q7	To what extent did you experience that you and your partner collaborated while editing the terrain?
Q8	During the experience I felt that the body I saw when looking down towards myself was my own body (even though it didn't look like me).
Q9	During the experience I tried to avoid the virtual planet while performing the task.
Q10	There was a sense of being in the room which has the planet.
Q11	I think the virtual place is somewhere I visited, rather than just images I saw.
Q12	There were times during the experience when the real world of the laboratory in which the experience was really taking place was forgotten.
Q13	The experience was more like working with other people rather than interacting with a computer.
Q14	There was a sense of being with the other people.

tion reported a lower degree of co-presence than the AR condition for Q13 ($p = .012$), and Q14 ($p = .03$).

We can note, from the post interview, that participants at site B sometimes misperceived what type of system their partner was working on: that is, persons in the VRBody or VR condition tended to think that their partner was also using a VRBody or VR system like their own, and desktop persons thought their partners were also using a desktop system.

5 Discussion

6 Conclusion

Acknowledgements

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