MATHEMATICAL NOTES

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NEW PROOF OF A MINIMUM PROPERTY OF THE REGULAR n-GON

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J. Kurschák gives in his paper *Über dem Kreis ein- und umgeschriebene Vielecke** among others a complete and entirely elementary geometrical proof of the well known fact according to which the regular n-gon $\overline{P}_n$ has a minimal area among all n-gons P circumscribed about a circle c. In this proof P_n is carried, after a dismemberment and a suitable reassembly, in $n-1$ steps into $\overline{P}_n$ so that the area increases at every step.

In this note we give an extremely simple proof,† which appears to be new, showing immediately that if P_n is not regular, then $P_n > \overline{P}_n$, where the area is denoted by the same symbol as the domain.

Consider the circle C circumscribed about $\overline{P}_n$. We show that already for the part $P_n \cdot C$ of P_n lying in C we have

$$P_n \cdot C > \overline{P}_n.$$

We have $P_n \cdot C = C - ns + (s_1 s_2 + s_2 s_3 + \cdots + s_n s_1)$, where we denote by $s_1, s_2, \cdots, s_n$ the circular sections of C cut off by the consecutive sides of P_n, and by s the circular section of C cut off by a tangent to c. Hence

$$P_n \cdot C \geqq C - ns.$$

Then

$$P_n \geqq P_n \cdot C \geqq C - ns = \overline{P}_n.$$

Equality holds in $P_n \geqq P_n C$ resp. in $P_n C \geqq \overline{P}_n$ only if no vertex of P_n lies in the outside resp. in the inside of C; this completes the proof.

BINOMIAL COEFFICIENTS MODULO A PRIME

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The following theorem, although given by Lucas in his *Theorie des Nombres* (pp. 417–420), does not appear to be as widely known as it deserves to be:

THEOREM 1. *Let p be a prime, and let*

$$M = M_0 + M_1 p + M_2 p^2 + \cdots + M_k p^k \qquad (0 \leqq M_r < p),$$
$$N = N_0 + N_1 p + N_2 p^2 + \cdots + N_k p^k \qquad (0 \leqq N_r < p).$$

* Mathematische Annalen 30 (1887), pp. 578–581.

† As P. Szász remarked [Bemerkung zu einer Arbeit von K. Kürschák, Matematikai es Fizikai Lapok XLIV (1937), p. 167, note 3] Kürschák's proof is independent of the axiom of parallels. This advantage is preserved in the present proof.