A One-Line Proof of the Infinitude of Primes

Sam Northshield

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Proof. If the set of primes is finite, then

$$0 < \prod_p \sin\left(\frac{\pi}{p}\right) = \prod_p \sin\left(\frac{\pi \cdot (1 + 2\prod_{p'} p')}{p}\right) = 0$$

□