A Not So Famous Goldbach Conjecture

Goldbach to Euler
Petersburg, November 18th, 1752

Sir,

you kindly wrote me some time ago that the books which Mr. Spener addressed to me were already dispatched from Berlin in July; however, up to this moment I do not know to which merchant in Petersburg they were addressed or if they were even left behind at Lübeck. As it is not known to me by what shortcuts you obtained the difference of the series $\frac{1}{3} + \frac{1}{7} + \frac{1}{11} + \frac{1}{19} + \dots$ and $\frac{1}{5} + \frac{1}{13} + \frac{1}{17} + \frac{1}{29} + \dots$,if you have already published the method, please tell me where it is to be found. I take it to be certain that the number of all primes of the form a^2+1 is infinitely large, even if I cannot immediately prove it, and I do not think your reason for doubting it - that the number of primes of the form $a^2 + 1$ is infinitely smaller than the number of all primes - is at all relevant, since no infinite number can be taken so small that it is not infinitely greater than some other infinite. Mersenne assuredly did not say that only ten perfect numbers are possible, as he himself indicates eleven, among which your eighth one, Sir, is however not comprised; he does not state either that the number of perfect numbers is finite, but only that no range of numbers can be indicated which is so large that it could not be devoid of perfect numbers. You will be able to see that for yourself - in case tome II of the Commentarii has still not arrived - from the general preface to Mersenne's Cogitata Physico Mathematica, §19, as cited by Mr.Winsheim. As regards Leuneschloß's Paradoxa mathematica, they were printed at Heidelberg in 1658, in octavo; however, I never possessed this book myself, but borrowed the copy which I read in 1716 from the Old City Library at Königsberg, and returned it there before I departed for the last time in 1718, so it is impossible that you could have seen the book at my place in Petersburg. On the other hand, I remember - though not with utter certainty - that some years ago you wrote me from Berlin you had got the book, along with Bongo's 1591 list, from the Royal Library. I am not aware of anything further about this Leuneschloß than that I read somewhere he had been a professor at Heidelberg; if I am not mistaken, he is also referred to by some as 'Luneschlos', so his name should have to be looked up in the encyclopedias in both spellings. It seems that he found his paradox on perfect numbers in Mersenne and afterwards, when writing it down from memory, deviated from Mersenne's true meaning. I had already observed in one of my earlier letters that no algebraic formula can yield only prime numbers; indeed, taking the formula to be, for example, $x^3 + bx^2 + cx + e$, it is obvious that whenever x is a multiple of the absolute term e, then (and consequently infinitely often) the formula will yield a non-prime number; but if e were to equal 1, I merely substitute $x+p$ for x, as then the formula is changed into

$$\begin{aligned} x^3 + 3px^2 + 3p^2x + p^3 \\ +bx^2 + 2bpx + bp^2 \\ +cx + cp \\ +1 \end{aligned} \tag{1}$$

and whenever x is a multiple of the number $p^3 + bp^2 + cp + 1$, the formula yields a non-prime number. Now since this case where the highest power of x equals 3 can be extended to all other whims of nature, whatever the power of x is, it is impossible to indicate an algebraic series in which there should not occur infinitely many terms consisting of non-prime numbers. I have yet another small theorem to add, which is quite new and which I take to be true until the contrary is proved: Any odd number equals the sum of twice a square and a prime, or: $2n - 1 = 2a^2 + p$, where a is to signify a whole number, including 0, and p a prime number; for example, $17 = 2 \cdot 0^2 + 17; 21 = 2 \cdot 1^2 + 19;\ 27 = 2 \cdot 2^2 + 19$, and so on. With a dutiful recommendation to your dearest family, I remain, Sir, your most devoted servant

Goldbach.