On the mathematical connections between some Ramanujan expressions, Modular j- Invariant, various parameters of Particle Physics, ϕ and $\zeta(2)$.

Michele Nardelli[1], **Antonio Nardelli**[2]

Abstract

In this paper we describe and analyze the mathematical connections between some Ramanujan expressions, Modular j-Invariant, various parameters of Particle Physics, ϕ and $\zeta(2)$.

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**



https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

For more information on the data entered for the development of the various equations, see the "Observations" section.

From

RAMANUJAN AND THE MODULAR *j*-INVARIANT
BRUCE C. BERNDT AND HENG HUAT CHAN

We have:

$$t_{35} = \left(2\sqrt{64\cdot 5\left(\frac{\sqrt{5}+1}{2}\right)^8 - 24\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 + 9} - \left(16\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 - 3\right)\right)^{1/6}$$
$$= \left(2\sqrt{7349+3276\sqrt{5}} - 117 - 56\sqrt{5}\right)^{1/6}.$$

From which, we obtain:

(2(7349+3276sqrt5)^0.5-117-56sqrt5)^(1/6)

Input:

$$\sqrt[6]{2\sqrt{7349+3276\sqrt{5}} - 117 - 56\sqrt{5}}$$

Decimal approximation:

0.618033988749894848204586834365638117720309179805762862135...

0.6180339887…. result that is equal to the value of the conjugate of golden ratio 0.618033988749...

Alternate forms:

$$\frac{1}{2}\left(\sqrt{5}-1\right)$$

$$\sqrt[6]{9-4\sqrt{5}}$$

$$\sqrt[6]{-117-56\sqrt{5}+2\left(63+26\sqrt{5}\right)}$$

Minimal polynomial:

$$x^2+x-1$$

All 6th roots of -117 - 56 sqrt(5) + 2 sqrt(7349 + 3276 sqrt(5)):

$$\sqrt[6]{-117-56\sqrt{5}+2\sqrt{7349+3276\sqrt{5}}}\quad e^{0}\approx 0.6 \text{ (real, principal root)}$$

$$\sqrt[6]{-117-56\sqrt{5}+2\sqrt{7349+3276\sqrt{5}}}\quad e^{(i\pi)/3}\approx 0.31+0.5\,i$$

$$\sqrt[6]{-117-56\sqrt{5}+2\sqrt{7349+3276\sqrt{5}}}\quad e^{(2i\pi)/3}\approx -0.31+0.5\,i$$

$$\sqrt[6]{-117-56\sqrt{5}+2\sqrt{7349+3276\sqrt{5}}}\quad e^{i\pi}\approx -0.6 \text{ (real root)}$$

$$\sqrt[6]{-117-56\sqrt{5}+2\sqrt{7349+3276\sqrt{5}}}\quad e^{-(2i\pi)/3}\approx -0.31-0.5\,i$$

and:

1/(((2(7349+3276sqrt5)^0.5-117-56sqrt5)^(1/6)))

Input:

$$\frac{1}{\sqrt[6]{2\sqrt{7349+3276\sqrt{5}}-117-56\sqrt{5}}}$$

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621 35...

1.6180339887…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{1}{\sqrt[6]{9-4\sqrt{5}}}$$

$$\frac{1}{\sqrt[6]{-117-56\sqrt{5}+2\left(63+26\sqrt{5}\right)}}$$

Minimal polynomial:

$x^2 - x - 1$

We obtain also:

(2(7349+(2*x-55-13)sqrt5)^0.5-117-56sqrt5)^(1/6) = 1/golden ratio

Input:

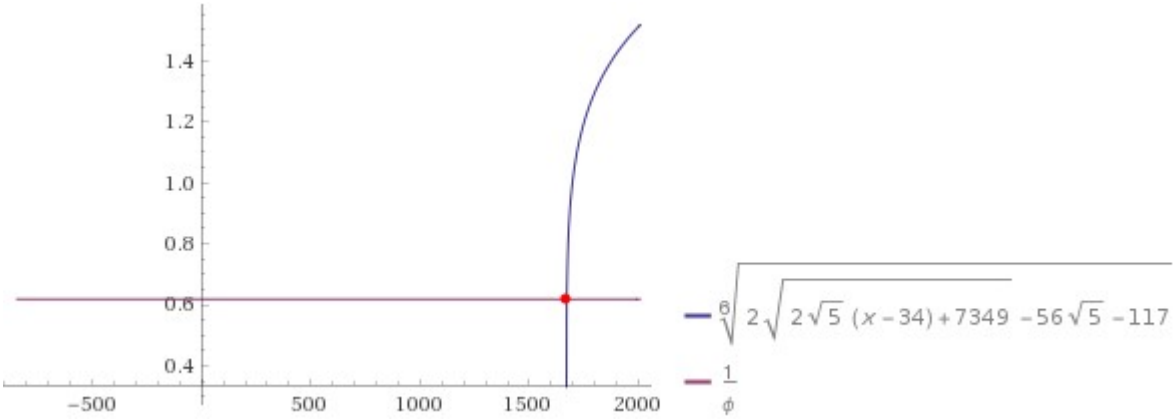
$$\sqrt[6]{2\sqrt{7349+(2x-55-13)\sqrt{5}}-117-56\sqrt{5}}=\frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\sqrt[6]{2\sqrt{\sqrt{5}\,(2x-68)+7349}-56\sqrt{5}-117}=\frac{1}{\phi}$$

Plot:



Alternate forms:

$$\sqrt[6]{2\sqrt{2\sqrt{5}\,(x-34)+7349}-56\sqrt{5}-117}=\frac{1}{\phi}$$

$$\sqrt[6]{2\sqrt{2\sqrt{5}\,x-68\sqrt{5}+7349}-56\sqrt{5}-117}=\frac{1}{\phi}$$

$$\sqrt[6]{2\sqrt{\sqrt{5}\,(2x-68)+7349}-56\sqrt{5}-117}=\frac{1}{2}\left(\sqrt{5}-1\right)$$

Solution:

$x = 1672$

1672 result practically equal to the rest mass of Omega baryon 1672.45

and:

(2(7349+(2*x-144-34-2)sqrt5)^0.5-117-56sqrt5)^(1/6) = 1/golden ratio

Input:

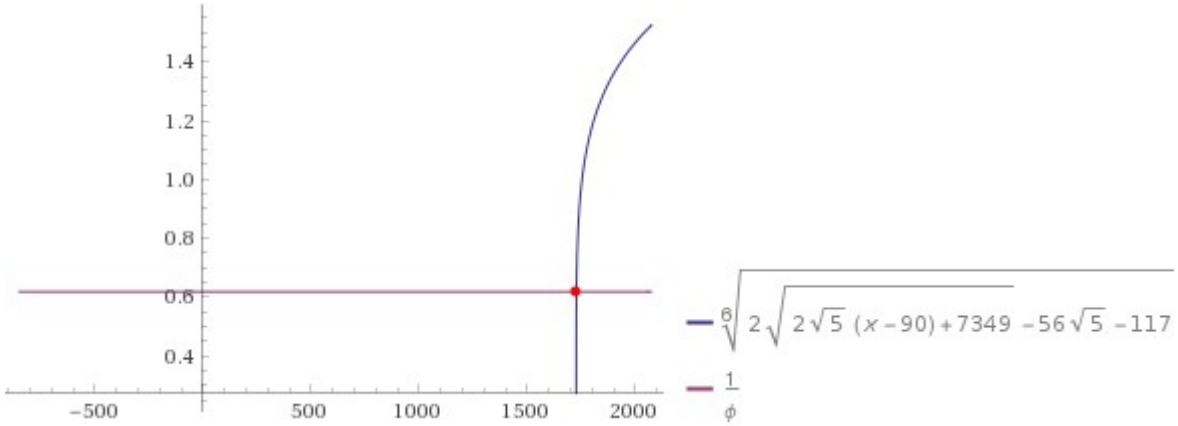
$$\sqrt[6]{2\sqrt{7349+(2x-144-34-2)\sqrt{5}}-117-56\sqrt{5}} = \frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\sqrt[6]{2\sqrt{\sqrt{5}\,(2x-180)+7349}-56\sqrt{5}-117} = \frac{1}{\phi}$$

Plot:



Alternate forms:

$$\sqrt[6]{2\sqrt{2\sqrt{5}\,(x-90)+7349}-56\sqrt{5}-117} = \frac{1}{\phi}$$

$$\sqrt[6]{2\sqrt{2\sqrt{5}\,x-180\sqrt{5}+7349}-56\sqrt{5}-117} = \frac{1}{\phi}$$

$$\sqrt[6]{2\sqrt{\sqrt{5}\,(2x-180)+7349}-56\sqrt{5}-117} = \frac{1}{2}\left(\sqrt{5}-1\right)$$

Solution:

$x = 1728$

1728

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From which:

(2(7349+(2*x^15-144-34-2)sqrt5)^0.5-117-56sqrt5)^(1/6) = 1/golden ratio

Input:

$$\sqrt[6]{2\sqrt{7349+(2x^{15}-144-34-2)\sqrt{5}}-117-56\sqrt{5}} = \frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\sqrt[6]{2\sqrt{\sqrt{5}\,(2x^{15}-180)+7349}-56\sqrt{5}-117} = \frac{1}{\phi}$$

Alternate forms:

$$\sqrt[6]{2\sqrt{2\sqrt{5}\,(x^{15}-90)+7349}-56\sqrt{5}-117} = \frac{1}{\phi}$$

$$\sqrt[6]{2\sqrt{2\sqrt{5}\,x^{15}-180\sqrt{5}+7349}-56\sqrt{5}-117} = \frac{1}{\phi}$$

$$\sqrt[6]{2\sqrt{\sqrt{5}\,(2x^{15}-180)+7349}-56\sqrt{5}-117} = \frac{1}{2}\left(\sqrt{5}-1\right)$$

Solutions:

$x \approx -1.32982 - 0.966173\,i$

$x \approx 0.507947 + 1.5633\,i$

$x \approx 1.64375 = 1.64375$

$x \approx 0.507947 - 1.5633\,i$

$1.64375 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

And again:

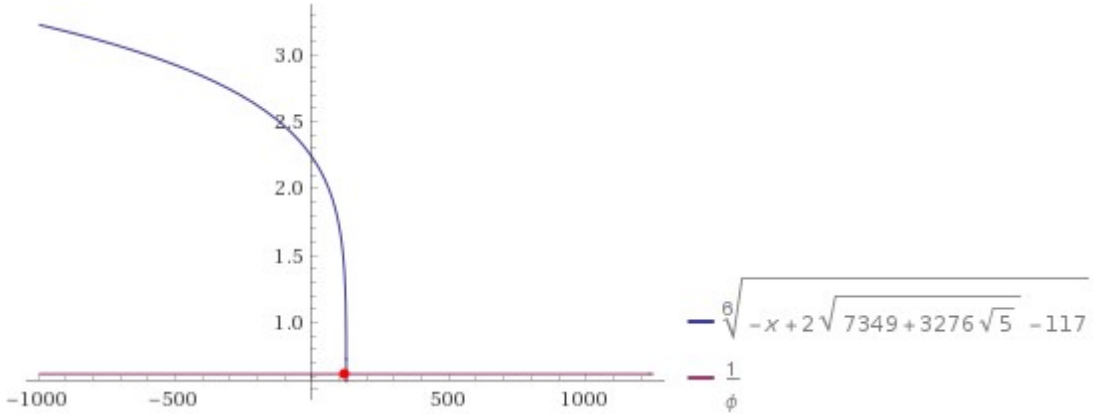
(2(7349+3276sqrt5)^0.5-117-x)^(1/6) = 1/golden ratio

Input:

$$\sqrt[6]{2\sqrt{7349+3276\sqrt{5}}-117-x}=\frac{1}{\phi}$$

ϕ is the golden ratio

Plot:



Alternate forms:

$$\sqrt[6]{-x+52\sqrt{5}+9}=\frac{1}{2}\left(\sqrt{5}-1\right)$$

$$\sqrt[6]{-x+2\left(63+26\sqrt{5}\right)-117}=\frac{2}{1+\sqrt{5}}$$

Alternate form assuming x is positive:

$$2\sqrt[6]{-x+52\sqrt{5}+9}+1=\sqrt{5}$$

Solution:

$$x=56\sqrt{5}$$

Solution:

$$x\approx 125.22$$

125.22 result very near to the Higgs boson mass 125.18 GeV

(2(7349+3276sqrt5)^0.5-117-x+13+1)^(1/6) = 1/golden ratio

Input:

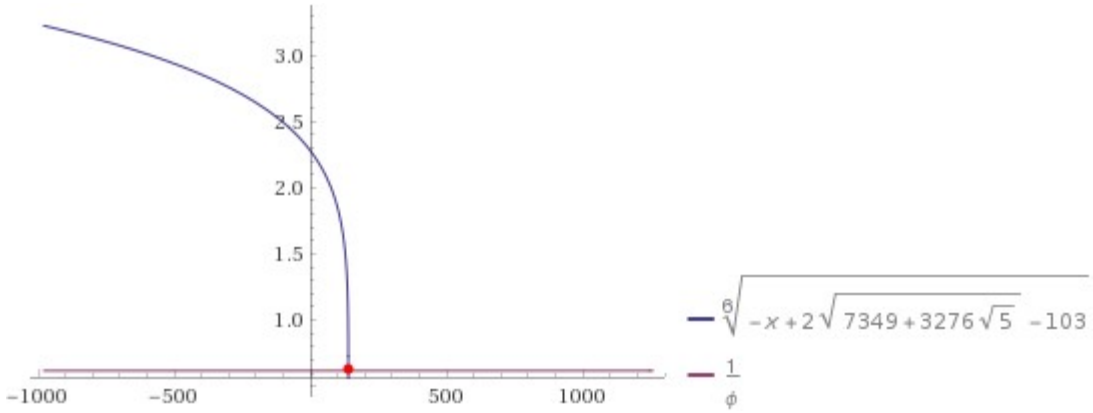
$$\sqrt[6]{2\sqrt{7349+3276\sqrt{5}}-117-x+13+1}=\frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\sqrt[6]{-x+2\sqrt{7349+3276\sqrt{5}}-103}=\frac{1}{\phi}$$

Plot:



Alternate forms:

$$\sqrt[6]{-x+52\sqrt{5}+23}=\frac{1}{2}\left(\sqrt{5}-1\right)$$

$$\sqrt[6]{-x+2\left(63+26\sqrt{5}\right)-103}=\frac{2}{1+\sqrt{5}}$$

Alternate form assuming x is positive:

$$2\sqrt[6]{-x+52\sqrt{5}+23}+1=\sqrt{5}$$

Solution:

$$x=14+56\sqrt{5}$$

$$x\approx 139.22$$

139.22 result practically equal to the rest mass of Pion meson 139.57 MeV

From

EISENSTEIN SERIES AND RAMANUJAN-TYPE SERIES FOR 1/π
NAYANDEEP DEKA BARUAH and BRUCE C. BERNDT
2000 Mathematical Reviews Classification Numbers: 33C05, 33E05, 11F11, 11R29.

We have that:

$$P(e^{-2\pi\sqrt{13}}) = \frac{3}{\pi\sqrt{13}} + (7\sqrt{13}-25)\frac{7+\sqrt{13}}{4\sqrt{13}} \cdot \sqrt{\frac{17+5\sqrt{13}}{2}}\, z_{13}^2.$$

$$P(e^{-2\pi\sqrt{13}}) = \frac{3}{\pi\sqrt{13}} + (7\sqrt{13}-25)\frac{7+\sqrt{13}}{4\sqrt{13}} \cdot \sqrt{\frac{17+5\sqrt{13}}{2}} \sum_{k=0}^{\infty} A_k X_{13}^k, \qquad (12.18)$$

where $X_{13} = 4x_{13}(1-x_{13}) = \{(\sqrt{13}-3)/2\}^6$.

From which, we obtain:

3/(Pi*sqrt13)+(7sqrt13-25)*((7+sqrt13)/(4sqrt13))*(1/2(17+5sqrt13))^0.5

Input:

$$\frac{3}{\pi\sqrt{13}} + \left(7\sqrt{13}-25\right) \times \frac{7+\sqrt{13}}{4\sqrt{13}} \sqrt{\frac{1}{2}\left(17+5\sqrt{13}\right)}$$

Exact result:

$$\frac{1}{4}\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(7\sqrt{13}-25\right)+\frac{3}{\sqrt{13}\,\pi}$$

Decimal approximation:

0.999929183846144757446892351882842151206387828760302947150...

0.999929183846…. result very near to the 16th root of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

= 0.9991104684; $\sqrt[16]{0.9991104684} = 0.99994438108$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d} = 255-390 \mid 0.988-1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243 = \phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or 'glueballs').

A_1^{**}	0.943(39) [2.5]	0.988(38)	0.152(53)
A_4	1.03(10) [2.5]	0.999(32)	0.035(21)

(**Glueball Regge trajectories -** *Harvey Byron Meyer*, Lincoln College -Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford Trinity Term, 2004)

Property:

$$\frac{1}{4}\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(-25+7\sqrt{13}\right)+\frac{3}{\sqrt{13}\,\pi}$$

is a transcendental number

Alternate forms:

$$\sqrt{\frac{261}{2\sqrt{13}}-\frac{927}{26}}+\frac{3}{\sqrt{13}\,\pi}$$

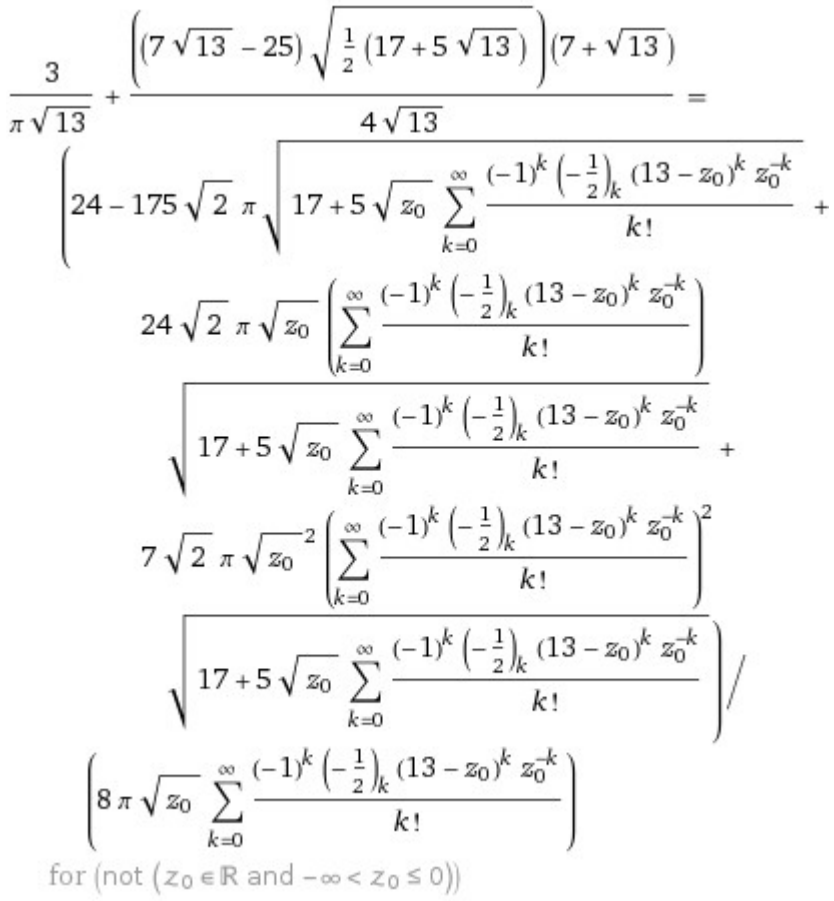
$$\frac{3\left(2+\sqrt{58\sqrt{13}-206}\,\pi\right)}{2\sqrt{13}\,\pi}$$

$$\frac{6+3\left(2\sqrt{13}-7\right)\sqrt{34+10\sqrt{13}}\,\pi}{2\sqrt{13}\,\pi}$$

Series representations:

$$\frac{3}{\pi\sqrt{13}}+\frac{\left((7\sqrt{13}-25)\sqrt{\frac{1}{2}(17+5\sqrt{13})}\right)(7+\sqrt{13})}{4\sqrt{13}}=$$
$$\left(24-175\sqrt{2}\,\pi\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}+\right.$$
$$24\sqrt{2}\,\pi\sqrt{12}\left(\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}+7\sqrt{2}\,\pi\sqrt{12}^{2}$$
$$\left.\left(\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^{2}\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}\right)\Big/\left(8\pi\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)$$

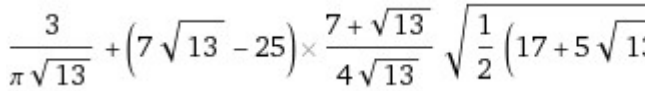
$$\frac{3}{\pi\sqrt{13}}+\frac{\left((7\sqrt{13}-25)\sqrt{\frac{1}{2}(17+5\sqrt{13})}\right)(7+\sqrt{13})}{4\sqrt{13}}=$$
$$\left(24-175\sqrt{2}\,\pi\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}+\right.$$
$$24\sqrt{2}\,\pi\sqrt{12}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}+$$
$$7\sqrt{2}\,\pi\sqrt{12}^{2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}\left.\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}\right)\Big/$$
$$\left(8\pi\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$



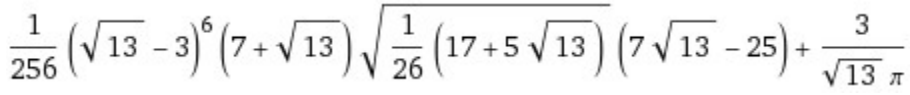
and:

3/(Pi*sqrt13)+(7sqrt13-25)*((7+sqrt13)/(4sqrt13))*(1/2(17+5sqrt13))^0.5 * (((sqrt13-3)/2))^6

Input:



Exact result:



Decimal approximation:

0.265416151736136976413223210859563175861431652676639105391...

0.26541615…

Property:

$$\frac{1}{256}\left(-3+\sqrt{13}\right)^6\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(-25+7\sqrt{13}\right)+\frac{3}{\sqrt{13}\,\pi}$$

is a transcendental number

Alternate forms:

$$27\sqrt{\frac{2}{13\left(174\,849\,583+48\,494\,549\sqrt{13}\right)}}+\frac{3}{\sqrt{13}\,\pi}$$

$$\frac{\pi\ \text{root of}\ x^4+6\,294\,584\,988\,x^2-104\,976\ \text{near}\ x=0.00408377+6}{2\sqrt{13}\,\pi}$$

$$\frac{6+3\sqrt{34+10\sqrt{13}}\,\left(2558\sqrt{13}-9223\right)\pi}{2\sqrt{13}\,\pi}$$

Series representations:

$$\frac{3}{\pi\sqrt{13}}+\frac{\left(7\sqrt{13}-25\right)\left(7+\sqrt{13}\right)\sqrt{\frac{1}{2}\left(17+5\sqrt{13}\right)}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^6}{4\sqrt{13}}=$$

$$\frac{3}{\pi\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}+$$

$$\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^6\left(7+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}\left(-25+7\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(256\sqrt{2}\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}=$$

$$\frac{3}{\pi\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6\left(7+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$

$$\left.\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\left(-25+7\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$

$$\left(256\sqrt{2}\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}=$$

$$\frac{3}{\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}}+\left(\left(-3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)^6\right.$$

$$\left(7+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)$$

$$\sqrt{17+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}}$$

$$\left.\left(-25+7\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left(256\sqrt{2}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

2Pi*(((3/(Pi*sqrt13)+(7sqrt13-25)*((7+sqrt13)/(4sqrt13))*(1/2(17+5sqrt13))^0.5 * (((sqrt13-3)/2))^6)))

Input:

$$2\pi\left(\frac{3}{\pi\sqrt{13}}+\left(7\sqrt{13}-25\right)\times\frac{7+\sqrt{13}}{4\sqrt{13}}\sqrt{\frac{1}{2}\left(17+5\sqrt{13}\right)}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^6\right)$$

Exact result:

$$2\left(\frac{1}{256}\left(\sqrt{13}-3\right)^6\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(7\sqrt{13}-25\right)+\frac{3}{\sqrt{13}\,\pi}\right)\pi$$

Decimal approximation:

1.667658864876643542748329813263980691334415564444893996898...

1.6676588648…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Property:

$$2\left(\frac{1}{256}\left(-3+\sqrt{13}\right)^6\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(-25+7\sqrt{13}\right)+\frac{3}{\sqrt{13}\,\pi}\right)\pi$$

is a transcendental number

Alternate forms:

$$\frac{\pi\ \text{root of}\ x^4+6\,294\,584\,988\,x^2-104\,976\ \text{near}\ x=0.00408377\ +6}{\sqrt{13}}$$

$$\frac{6+3\sqrt{34+10\sqrt{13}}\ (2558\sqrt{13}-9223)\,\pi}{\sqrt{13}}$$

$$2\left(27\sqrt{\frac{2}{13\,(174\,849\,583+48\,494\,549\sqrt{13})}}+\frac{3}{\sqrt{13}\,\pi}\right)\pi$$

Series representations:

$$2\pi\left(\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}\right)=$$

$$2\pi\left(\frac{3}{\pi\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}+\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^6\left(7+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right.\right.$$
$$\left.\left.\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}\left(-25+7\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\right.$$
$$\left.\left(256\sqrt{2}\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$2\pi\left(\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}\right)=$$

$$2\pi\left(\frac{3}{\pi\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\right.$$
$$\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6\left(7+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$
$$\left.\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\left(-25+7\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$
$$\left.\left(256\sqrt{2}\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$2\pi\left(\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}\right)=$$

$$2\pi\left(\frac{3}{\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}}+\left(\left(-3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)^6\left(7+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)\sqrt{17+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}}\left(-25+7\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)\right)\Bigg/\left(256\sqrt{2}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(13-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \le 0))$

and:

1+(((3*(((3/(Pi*sqrt13)+(7sqrt13-25)*((7+sqrt13)/(4sqrt13))*(1/2(17+5sqrt13))^0.5 * (((sqrt13-3)/2))^6)))))^2)

Input:

$$1+\left(3\left(\frac{3}{\pi\sqrt{13}}+\left(7\sqrt{13}-25\right)\times\frac{7+\sqrt{13}}{4\sqrt{13}}\sqrt{\frac{1}{2}\left(17+5\sqrt{13}\right)}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^6\right)\right)^2$$

Exact result:

$$1+9\left(\frac{1}{256}\left(\sqrt{13}-3\right)^6\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(7\sqrt{13}-25\right)+\frac{3}{\sqrt{13}\,\pi}\right)^2$$

Decimal approximation:

1.6340116024217807858678376547289409871424177180644551261 59...

1.63401160242…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

Property:

$$1+9\left(\frac{1}{256}\left(-3+\sqrt{13}\right)^6\left(7+\sqrt{13}\right)\sqrt{\frac{1}{26}\left(17+5\sqrt{13}\right)}\left(-25+7\sqrt{13}\right)+\frac{3}{\sqrt{13}\,\pi}\right)^2$$

is a transcendental number

Alternate forms:

$$\frac{81\left(\pi\ \text{root of}\ x^4+699\,398\,332x^2-1296\ \text{near}\ x=0.00136126+2\right)^2}{52\pi^2}+1$$

$$1+\frac{81\left(2+\sqrt{34+10\sqrt{13}}\ (2558\sqrt{13}-9223)\pi\right)^2}{52\pi^2}$$

$$1+9\left(27\sqrt{\frac{2}{13\,(174\,849\,583+48\,494\,549\sqrt{13})}}+\frac{3}{\sqrt{13}\,\pi}\right)^2$$

Series representations:

$$1+\left(3\left(\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}\right)\right)^2=$$

$$1+9\left(\frac{3}{\pi\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}+\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^6\left(7+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right.\right.$$

$$\left.\left.\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}\left(-25+7\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\right.$$

$$\left.\left(256\sqrt{2}\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)\right)^2$$

$$1+\left(3\left(\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}\right)\right)^2=$$

$$1+9\left(\frac{3}{\pi\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6\right.\right.$$

$$\left(7+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{17+5\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\left.\left(-25+7\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$

$$\left(256\sqrt{2}\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

$$1+\left(3\left(\frac{3}{\pi\sqrt{13}}+\frac{(7\sqrt{13}-25)(7+\sqrt{13})\sqrt{\frac{1}{2}(17+5\sqrt{13})}\left(\frac{1}{2}(\sqrt{13}-3)\right)^6}{4\sqrt{13}}\right)\right)^2=$$

$$1+9\left(\frac{6\sqrt{\pi}}{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}+\right.$$

$$\left(\sqrt{\pi}\left(-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^6\right.$$

$$\left(7+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

$$\sqrt{17+\frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$$\left.\left.\left(-25+\frac{7\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)\right)\Big/\right.$$

$$\left(128\sqrt{2}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2$$

Now, we have that:

Theorem 15.1. *If B_k, $k \geq 0$, is defined by (3.2), then*

$$\frac{16\sqrt{38+10\sqrt{17}}}{\pi} = \sum_{k=0}^{\infty}(-1)^k\{(60\sqrt{17}+340)k+33+3\sqrt{17}\} \times B_k\left(\frac{25\sqrt{17}-103}{128}\right)^k. \tag{15.1}$$

The representation (15.1) is new. We note that

$$f_{17}(e^{-\pi/\sqrt{17}}) = 17P(e^{-2\pi\sqrt{17}}) - P(e^{-2\pi/\sqrt{17}}) = \frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2x_{17})\,z_{17}^2,$$

where

$$x_{17} = \frac{1}{2}\left(\sqrt{\frac{7+\sqrt{17}}{4}} - \sqrt{\frac{3+\sqrt{17}}{4}}\right)^4 \left(\sqrt{\frac{3+\sqrt{4+\sqrt{17}}}{4}} - \sqrt{\frac{\sqrt{4+\sqrt{17}}-1}{4}}\right)^8.$$

From:

$$x_{17} = \frac{1}{2}\left(\sqrt{\frac{7+\sqrt{17}}{4}} - \sqrt{\frac{3+\sqrt{17}}{4}}\right)^4 \left(\sqrt{\frac{3+\sqrt{4+\sqrt{17}}}{4}} - \sqrt{\frac{\sqrt{4+\sqrt{17}}-1}{4}}\right)^8.$$

we obtain:

1/2((1/4(7+sqrt17))^0.5-(1/4(3+sqrt17))^0.5)^4 (((((1/4(3+(((4+sqrt17))^0.5)))^0.5-((((1/4((4+sqrt17)^0.5)-1)^0.5)))))))^8

Input:

$$\frac{1}{2}\left(\sqrt{\frac{1}{4}\left(7+\sqrt{17}\right)} - \sqrt{\frac{1}{4}\left(3+\sqrt{17}\right)}\right)^4 \left(\sqrt{\frac{1}{4}\left(3+\sqrt{4+\sqrt{17}}\right)} - \sqrt{\frac{1}{4}\sqrt{4+\sqrt{17}}-1}\right)^8$$

Exact result:

$$\frac{1}{2}\left(\frac{\sqrt{7+\sqrt{17}}}{2}-\frac{1}{2}\sqrt{3+\sqrt{17}}\right)^4\left(\frac{1}{2}\sqrt{3+\sqrt{4+\sqrt{17}}}-i\sqrt{1-\frac{\sqrt{4+\sqrt{17}}}{4}}\right)^8$$

Decimal approximation:

$-0.056623037009761753153011034718910948969579378030660171 2\ldots +$
$0.011294809538440885623971789639966359572713103149990813 2\ldots i$

Polar coordinates:

$r \approx 0.0577386$ (radius), $\theta \approx 168.719°$ (angle)

0.0577386

Alternate forms:

$$\frac{\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\left(\sqrt{4-\sqrt{4+\sqrt{17}}}+i\sqrt{3+\sqrt{4+\sqrt{17}}}\right)^8}{8192}$$

$$\frac{1}{1024}\left(-30360-6710\sqrt{17}-24320\sqrt{4+\sqrt{17}}-\right.$$
$$5440\sqrt{17\left(4+\sqrt{17}\right)}+1023\sqrt{34\left(19+5\sqrt{17}\right)}+2299\sqrt{38+10\sqrt{17}}+$$
$$\left.800\sqrt{34\left(161+39\sqrt{17}\right)}+1952\sqrt{322+78\sqrt{17}}\right)+$$
$$\frac{1}{128}i\left(6040\sqrt{8-\sqrt{17}+\sqrt{4+\sqrt{17}}}+1450\sqrt{17\left(8-\sqrt{17}+\sqrt{4+\sqrt{17}}\right)}-\right.$$
$$1840\sqrt{15+4\sqrt{17}+4\sqrt{4+\sqrt{17}}+\sqrt{17\left(4+\sqrt{17}\right)}}-$$
$$420\sqrt{17\left(15+4\sqrt{17}+4\sqrt{4+\sqrt{17}}+\sqrt{17\left(4+\sqrt{17}\right)}\right)}-$$
$$653\sqrt{2\left(67+21\sqrt{17}+19\sqrt{4+\sqrt{17}}+5\sqrt{17\left(4+\sqrt{17}\right)}\right)}-$$
$$169\sqrt{34\left(67+21\sqrt{17}+19\sqrt{4+\sqrt{17}}+5\sqrt{17\left(4+\sqrt{17}\right)}\right)}+$$
$$58\sqrt{34\left(625+151\sqrt{17}+161\sqrt{4+\sqrt{17}}+39\sqrt{17\left(4+\sqrt{17}\right)}\right)}+$$
$$\left.162\sqrt{1250+302\sqrt{17}+322\sqrt{4+\sqrt{17}}+78\sqrt{17\left(4+\sqrt{17}\right)}}\right)$$

From which:

25*4*1/[1/2((1/4(7+sqrt17))^0.5-(1/4(3+sqrt17))^0.5)^4
(((((1/4(3+(((4+sqrt17))^0.5)))^0.5-((((1/4((4+sqrt17)^0.5)-1)^0.5)))))))^8]+3

where 25 can be related to the D25-brane, while 4 and 3 are Lucas numbers

Input:

$$25\times 4\times 1\Big/\left(\frac{1}{2}\left(\sqrt{\frac{1}{4}\left(7+\sqrt{17}\right)}-\sqrt{\frac{1}{4}\left(3+\sqrt{17}\right)}\right)^4\left(\sqrt{\frac{1}{4}\left(3+\sqrt{4+\sqrt{17}}\right)}-\sqrt{\frac{1}{4}\sqrt{4+\sqrt{17}}-1}\right)^8\right)+3$$

Exact result:

$$3+\frac{200}{\left(\frac{\sqrt{7+\sqrt{17}}}{2}-\frac{1}{2}\sqrt{3+\sqrt{17}}\right)^4\left(\frac{1}{2}\sqrt{3+\sqrt{4+\sqrt{17}}}-i\sqrt{1-\frac{\sqrt{4+\sqrt{17}}}{4}}\right)^8}$$

Decimal approximation:

$$-1695.48336401043338475375933326790914938707509520595337 47\ldots - 338.80284622318820643635749751151251598211886815915618846\ldots\, i$$

Polar coordinates:

$r \approx 1729.$ (radius), $\theta \approx -168.7°$ (angle)

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$3+\frac{819\,200}{\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\left(\sqrt{4-\sqrt{4+\sqrt{17}}}+i\sqrt{3+\sqrt{4+\sqrt{17}}}\right)^8}$$

$$-\left(\left(8\left(-767244920-136899230\sqrt{17}-49152000\sqrt{4+\sqrt{17}}+26214400\sqrt{17\left(4+\sqrt{17}\right)}+17294403\sqrt{34\left(19+5\sqrt{17}\right)}+86472015\sqrt{38+10\sqrt{17}}\right)\right)/\left(5764801\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\right)\right)+\left(6553600\,i\sqrt{8-\sqrt{17}+\sqrt{4+\sqrt{17}}}\left(-49-24\sqrt{17}-22\sqrt{4+\sqrt{17}}+16\sqrt{17\left(4+\sqrt{17}\right)}\right)\right)/\left(5764801\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\right)$$

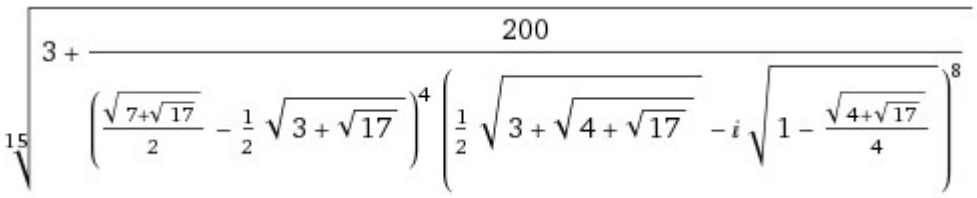
And, performing the 15th root:

((((25*4*1/[1/2((1/4(7+sqrt17))^0.5-(1/4(3+sqrt17))^0.5)^4
(((((1/4(3+(((4+sqrt17))^0.5)))^0.5-((((1/4((4+sqrt17)^0.5)-
1)^0.5)))))))^8]+3))))^1/15

Input:

$$\left(25\times4\times1\Big/\left(\frac{1}{2}\left(\sqrt{\frac{1}{4}\left(7+\sqrt{17}\right)}-\sqrt{\frac{1}{4}\left(3+\sqrt{17}\right)}\right)^4\left(\sqrt{\frac{1}{4}\left(3+\sqrt{4+\sqrt{17}}\right)}-\sqrt{\frac{1}{4}\sqrt{4+\sqrt{17}}-1}\right)^8\right)+3\right)\wedge(1/15)$$

Exact result:



Decimal approximation:

1.612248761452440285976221176208535661707308493208378430 6... –
0.320597960820806952216850041831060240703885222794747644 17... i

Polar coordinates:

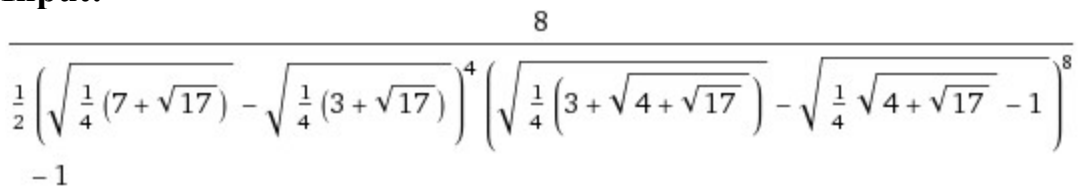
$r \approx 1.64382$ (radius), $\theta \approx -11.2466°$ (angle)

$1.64382 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

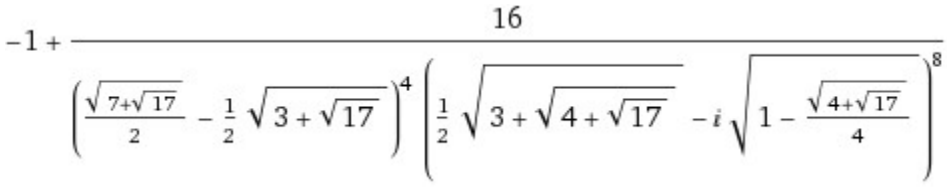
And again:

8/[1/2((1/4(7+sqrt17))^0.5-(1/4(3+sqrt17))^0.5)^4
(((((1/4(3+(((4+sqrt17))^0.5)))^0.5-((((1/4((4+sqrt17)^0.5)-1)^0.5)))))))^8]-1

Input:



Exact result:



Decimal approximation:

– 136.878669120834670780300746661432731950966007616476269 97... –
27.104227697855056514908599800921001278569509452732495077... i

Polar coordinates:

$r \approx 139.536$ (radius), $\theta \approx -168.799°$ (angle)

139.536 result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$-1+\frac{65536}{\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\left(\sqrt{4-\sqrt{4+\sqrt{17}}}+i\sqrt{3+\sqrt{4+\sqrt{17}}}\right)^8}$$

$$-\Bigg(\Bigg(8\Big(224554536+60531594\sqrt{17}-3932160\sqrt{4+\sqrt{17}}+2097152\sqrt{17\left(4+\sqrt{17}\right)}-5764801\sqrt{34\left(19+5\sqrt{17}\right)}-28824005\sqrt{38+10\sqrt{17}}\Big)\Bigg)\Big/\left(5764801\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\right)\Bigg)+\left(524288\,i\sqrt{8-\sqrt{17}+\sqrt{4+\sqrt{17}}}\left(-49-24\sqrt{17}-22\sqrt{4+\sqrt{17}}+16\sqrt{17\left(4+\sqrt{17}\right)}\right)\right)\Big/\left(5764801\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\right)$$

8/[1/2((1/4(7+sqrt17))^0.5-(1/4(3+sqrt17))^0.5)^4
(((((1/4(3+(((4+sqrt17))^0.5)))^0.5-((((1/4((4+sqrt17)^0.5)-1)^0.5)))))))^8]+13

Input:

$$\frac{8}{\frac{1}{2}\left(\sqrt{\frac{1}{4}\left(7+\sqrt{17}\right)}-\sqrt{\frac{1}{4}\left(3+\sqrt{17}\right)}\right)^4\left(\sqrt{\frac{1}{4}\left(3+\sqrt{4+\sqrt{17}}\right)}-\sqrt{\frac{1}{4}\sqrt{4+\sqrt{17}}-1}\right)^8}+13$$

Exact result:

$$13+\frac{16}{\left(\frac{\sqrt{7+\sqrt{17}}}{2}-\frac{1}{2}\sqrt{3+\sqrt{17}}\right)^4\left(\frac{1}{2}\sqrt{3+\sqrt{4+\sqrt{17}}}-i\sqrt{1-\frac{\sqrt{4+\sqrt{17}}}{4}}\right)^8}$$

Decimal approximation:

$-122.878669120834670780300746661432731950966007616476269 97\ldots -$
$27.104227697855056514908599800921001278569509452732495077\ldots\, i$

Polar coordinates:

$r \approx 125.832$ (radius), $\theta \approx -167.561°$ (angle)

125.832 result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$13+\frac{65536}{\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\left(\sqrt{4-\sqrt{4+\sqrt{17}}}+i\sqrt{3+\sqrt{4+\sqrt{17}}}\right)^8}$$

$$-\left(\left(8\left(-3003734024-746540546\sqrt{17}-\right.\right.\right.$$

$$3932160\sqrt{4+\sqrt{17}}+2097152\sqrt{17\left(4+\sqrt{17}\right)}+$$

$$\left.\left.\left.74942413\sqrt{34\left(19+5\sqrt{17}\right)}+374712065\sqrt{38+10\sqrt{17}}\right)\right)\Big/$$

$$\left.\left(5764801\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\right)\right)+$$

$$\left(524288\,i\sqrt{8-\sqrt{17}+\sqrt{4+\sqrt{17}}}\right.$$

$$\left.\left(-49-24\sqrt{17}-22\sqrt{4+\sqrt{17}}+16\sqrt{17\left(4+\sqrt{17}\right)}\right)\right)\Big/$$

$$\left(5764801\left(\sqrt{3+\sqrt{17}}-\sqrt{7+\sqrt{17}}\right)^4\right)$$

From:

$$\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2x_{17})\,z_{17}^2$$

we obtain:

(sqrt17)/2 * (sqrt17+3(4+sqrt17)^0.5) (1-2*(-0.05662303700976 + 0.01129480953844i))

Input interpretation:

$$\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\times(-0.05662303700976+0.01129480953844\,i))$$

i is the imaginary unit

Result:

29.085706320821... −
0.59019747897879... i

Polar coordinates:

$r = 29.091693750025$ (radius), $\theta = -1.162467369926°$ (angle)

29.091693750025 ≈ 29 that is a Lucas number

Dividing 47, that is a Lucas number, by the whole expression:

47/[(sqrt17)/2 * (sqrt17+3(4+sqrt17)^0.5) (1-2*(-0.05662303700976 + 0.01129480953844i))]

Input interpretation:

$$\frac{47}{\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\times(-0.05662303700976+0.01129480953844\,i))}$$

i is the imaginary unit

Result:

1.6152489169031... +
0.032776093802369... i

Polar coordinates:

$r = 1.6155814234762$ (radius), $\theta = 1.162467369926°$ (angle)

1.6155814234762 result that is a good approximation to the value of the golden ratio 1.618033988749...

and:

5 * ((((sqrt17)/2 * (sqrt17+3(4+sqrt17)^0.5) (1-2*(-0.05662303700976 + 0.01129480953844i)))))-2Pi

Input interpretation:

$$5\left(\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\times(-0.05662303700976+0.01129480953844\,i))\right)-2\pi$$

i is the imaginary unit

Result:

139.14534629693... −
2.9509873948940... i

Polar coordinates:

$r = 139.17663497404$ (radius), $\theta = -1.214943835031°$ (angle)

139.17663497404 result practically equal to the rest mass of Pion meson 139.57 MeV

Series representations:

$$\frac{5}{2}\sqrt{17}\left(\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\,(-0.056623037009760000+0.011294809538440000\,i))\right)-2\pi =$$

$$-2.0000000000000\left(1.0000000000000\,\pi-1.3915575925244\sqrt{16}^{2}\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{2}\atop k\right)\right)^{2}+0.028237023846100\,i\sqrt{16}^{2}\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{2}\atop k\right)\right)^{2}-4.1746727775732\sqrt{16}\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{2}\atop k\right)\right)\sqrt{4+\sqrt{16}\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{2}\atop k\right)}+0.084711071538300\,i\sqrt{16}\left(\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{2}\atop k\right)\right)\sqrt{4+\sqrt{16}\sum_{k=0}^{\infty}16^{-k}\left(\frac{1}{2}\atop k\right)}\right)$$

$$\frac{5}{2}\sqrt{17}\left(\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)\right.$$
$$\left.(1-2\,(-0.056623037009760000+0.011294809538440000\,i))\right)-2\,\pi=$$
$$-2.0000000000000\left(1.0000000000000\,\pi-1.3915575925244\,\sqrt{16}^{\,2}\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2+0.028237023846100\,i\,\sqrt{16}^{\,2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2-$$
$$4.1746727775732\,\sqrt{16}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{4+\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\left.0.084711071538300\,i\,\sqrt{16}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt{4+\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\frac{5}{2}\sqrt{17}\left(\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)\right.$$
$$\left.(1-2\,(-0.056623037009760000+0.011294809538440000\,i))\right)-2\,\pi=$$
$$-2.0000000000000\left(1.0000000000000\,\pi-\right.$$
$$1.3915575925244\,\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}\right)^2+$$
$$0.028237023846100\,i\,\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}\right)^2-$$
$$4.1746727775732\,\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}\right)$$
$$\sqrt{4+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}}+0.084711071538300\,i\,\sqrt{z_0}$$
$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}\right)\sqrt{4+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

5 * ((((sqrt17)/2 * (sqrt17+3(4+sqrt17)^0.5) (1-2*(-0.05662303700976 + 0.01129480953844i)))))-18-2

Input interpretation:

$$5\left(\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\times(-0.05662303700976+0.01129480953844\,i))\right)-18-2$$

i is the imaginary unit

Result:

125.42853160411... −
2.9509873948940... i

Polar coordinates:

$r = 125.46324109861$ (radius), $\theta = -1.347763020640°$ (angle)

125.46324109861…. result very near to the Higgs boson mass 125.18 GeV

Multiplying by 27*1/2, and adding 8, that is a Fibonacci number, we obtain:

27*1/2(((5 * ((((sqrt17)/2 * (sqrt17+3(4+sqrt17)^0.5) (1-2*(-0.0566230 + 0.0112948i)))))-18)))+8

Input interpretation:

$$27\times\frac{1}{2}\left(5\left(\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\times(-0.0566230+0.0112948\,i))\right)-18\right)+8$$

i is the imaginary unit

Result:

1728.285... −
39.83830... i

Polar coordinates:

$r = 1728.74$ (radius), $\theta = -1.32048°$ (angle)

1728.74

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\boldsymbol{Z}/3\boldsymbol{Z}$, and its outer automorphism group is the cyclic group $\boldsymbol{Z}/2\boldsymbol{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".

In conclusion, performing the 15th root:

((((27*1/2(((5 * ((((sqrt17)/2 * (sqrt17+3(4+sqrt17)^0.5) (1-2*(-0.0566230 + 0.0112948i)))))-18)))+8))))^1/15

Input interpretation:

$$\left(27\times\frac{1}{2}\left(5\left(\frac{\sqrt{17}}{2}\left(\sqrt{17}+3\sqrt{4+\sqrt{17}}\right)(1-2\times(-0.0566230+0.0112948\,i))\right)-18\right)+8\right)\text{^}(1/15)$$

i is the imaginary unit

Result:

1.6437971... −
0.0025256070... i

Polar coordinates:

$r = 1.6438$ (radius), $\theta = -0.0880319°$ (angle)

$1.6438 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

RAMANUJAN AND THE MODULAR *j*-INVARIANT
BRUCE C. BERNDT AND HENG HUAT CHAN

EISENSTEIN SERIES AND RAMANUJAN-TYPE SERIES FOR 1/π
NAYANDEEP DEKA BARUAH and BRUCE C. BERNDT
2000 Mathematical Reviews Classification Numbers: 33C05, 33E05, 11F11, 11R29.