On the mathematical connections between some Ramanujan expressions, Product Formulas, D5-branes and various parameters of Particle Physics, ϕ and $\zeta(2)$.

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Abstract

In this paper we describe and analyze the mathematical connections between some Ramanujan expressions, Product Formulas, D5-branes and various parameters of Particle Physics, ϕ and $\zeta(2)$.

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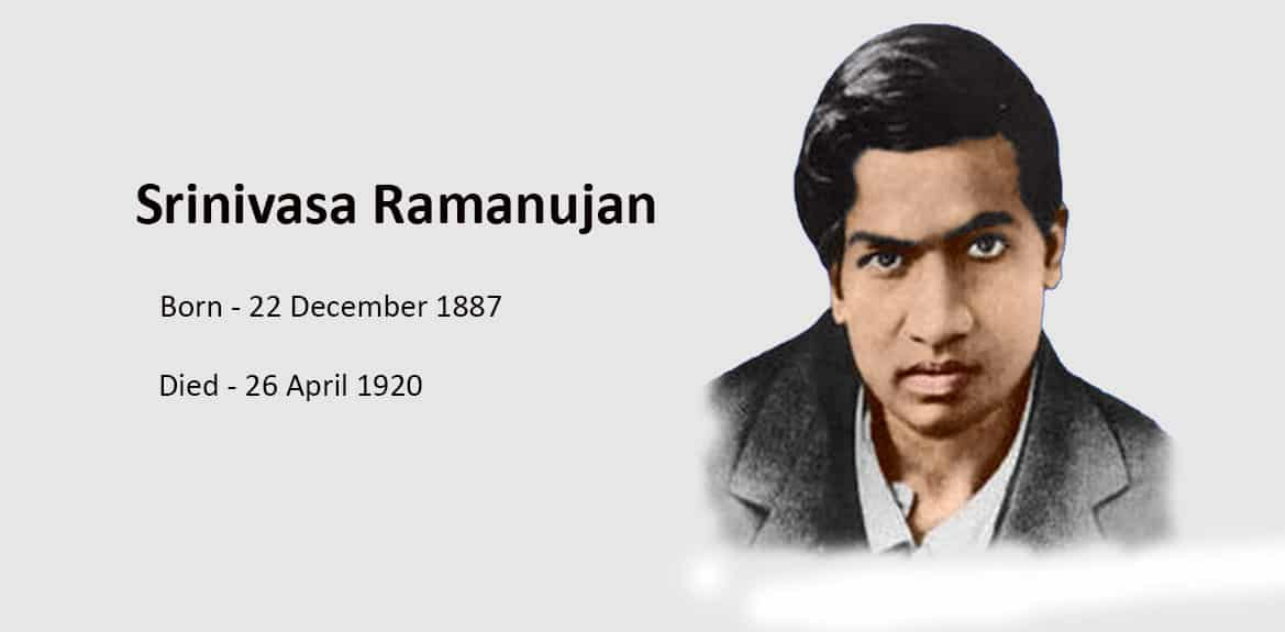


https://vedicmathschool.org/srinivasa-ramanujan/

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

For more information on the data entered for the development of the various equations, see the "Observations" section.

From:

George E. Andrews • *Bruce C. Berndt* - **Ramanujan's Lost Notebook**
Part IV

We have that

$$\prod_{n=1}^{\infty}\left\{1-\left(\frac{(\alpha-\beta)+i(\alpha+\beta)\sqrt{3}}{2n}\right)^2\right\}\left\{1-\left(\frac{(\alpha-\beta)-i(\alpha+\beta)\sqrt{3}}{2n}\right)^2\right\}$$
$$=\frac{\cosh\pi(\alpha+\beta)\sqrt{3}-\cos\pi(\alpha-\beta)}{2\pi^2(\alpha^2+\alpha\beta+\beta^2)}. \qquad (16.2.3)$$

And from

$$\prod_{n=1}^{n=\infty}\left[1-\left\{\frac{(\alpha-\beta)+i(\alpha+\beta)\sqrt{3}}{2n}\right\}^2\right]\left[1-\left\{\frac{(\alpha-\beta)-i(\alpha+\beta)\sqrt{3}}{2n}\right\}^2\right]$$
$$=\frac{\cosh\pi(\alpha+\beta)\sqrt{3}-\cos\pi(\alpha-\beta)}{2\pi^2(\alpha^2+\alpha\beta+\beta^2)}. \qquad (4)$$

$\alpha-\beta$ be any integer

If we put α = 8; β = 5, we obtain:

(((cosh Pi(8+5)*sqrt3 - cos Pi(8-5)))) / (((2Pi^2(8^2+8*5+5^2))))

Input:

$$\frac{\cosh(\pi)\,(8+5)\,\sqrt{3}\;-\cos(\pi)\,(8-5)}{2\,\pi^2\left(8^2+8\times5+5^2\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{3+13\sqrt{3}\,\cosh(\pi)}{258\,\pi^2}$$

Decimal approximation:

0.10368223286264106498305313924363066780234315200190549743 2…

0.10368223286…

Alternate forms:

$$\frac{1}{86\,\pi^2}+\frac{13\cosh(\pi)}{86\sqrt{3}\,\pi^2}$$

$$\frac{6+13\sqrt{3}\,e^{-\pi}+13\sqrt{3}\,e^{\pi}}{516\,\pi^2}$$

$$\frac{1}{86\,\pi^2}+\frac{13\,e^{-\pi}}{172\sqrt{3}\,\pi^2}+\frac{13\,e^{\pi}}{172\sqrt{3}\,\pi^2}$$

Alternative representations:

$$\frac{\cosh(\pi)(8+5)\sqrt{3}-\cos(\pi)(8-5)}{2\,\pi^2\left(8^2+8\times5+5^2\right)}=\frac{-3\cosh(-i\,\pi)+13\cos(-i\,\pi)\sqrt{3}}{2\left(40+5^2+8^2\right)\pi^2}$$

$$\frac{\cosh(\pi)(8+5)\sqrt{3}-\cos(\pi)(8-5)}{2\,\pi^2\left(8^2+8\times5+5^2\right)}=\frac{-3\cosh(i\,\pi)+13\cos(-i\,\pi)\sqrt{3}}{2\left(40+5^2+8^2\right)\pi^2}$$

$$\frac{\cosh(\pi)(8+5)\sqrt{3}-\cos(\pi)(8-5)}{2\,\pi^2\left(8^2+8\times5+5^2\right)}=\frac{-3\cosh(-i\,\pi)+13\cos(i\,\pi)\sqrt{3}}{2\left(40+5^2+8^2\right)\pi^2}$$

Series representations:

$$\frac{\cosh(\pi)(8+5)\sqrt{3}-\cos(\pi)(8-5)}{2\,\pi^2\left(8^2+8\times5+5^2\right)}=\frac{1}{86\,\pi^2}+\frac{13\sum_{k=0}^{\infty}\frac{\pi^{2k}}{(2k)!}}{86\sqrt{3}\,\pi^2}$$

$$\frac{\cosh(\pi)(8+5)\sqrt{3}-\cos(\pi)(8-5)}{2\,\pi^2\left(8^2+8\times5+5^2\right)}=\frac{1}{86\,\pi^2}+\frac{13\,i\sum_{k=0}^{\infty}\frac{\left(\left(1-\frac{i}{2}\right)\pi\right)^{1+2k}}{(1+2k)!}}{86\sqrt{3}\,\pi^2}$$

$$\frac{\cosh(\pi)\,(8+5)\,\sqrt{3}\;-\cos(\pi)\,(8-5)}{2\,\pi^2\,(8^2+8\times5+5^2)}=\frac{1}{86\,\pi^2}+\frac{13\,\sum_{k=0}^{\infty}\frac{(-i)^k\cos\left(\frac{k\,\pi}{2}-i\,z_0\right)(\pi-z_0)^k}{k!}}{86\,\sqrt{3}\;\pi^2}$$

Integral representations:

$$\frac{\cosh(\pi)\,(8+5)\,\sqrt{3}\;-\cos(\pi)\,(8-5)}{2\,\pi^2\,(8^2+8\times5+5^2)}=\frac{1}{86\,\pi^2}+\frac{13}{86\,\sqrt{3}\;\pi^2}\int_{\frac{i\,\pi}{2}}^{\pi}\sinh(t)\,dt$$

$$\frac{\cosh(\pi)\,(8+5)\,\sqrt{3}\;-\cos(\pi)\,(8-5)}{2\,\pi^2\,(8^2+8\times5+5^2)}=\frac{1}{86\,\pi^2}+\frac{13}{86\,\sqrt{3}\;\pi^2}+\frac{13}{86\,\sqrt{3}\;\pi}\int_{0}^{1}\sinh(\pi\,t)\,dt$$

$$\frac{\cosh(\pi)\,(8+5)\,\sqrt{3}\;-\cos(\pi)\,(8-5)}{2\,\pi^2\,(8^2+8\times5+5^2)}=$$
$$\frac{1}{86\,\pi^2}-\frac{13\,i}{172\,\sqrt{3}\;\pi^{5/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{\pi^2/(4\,s)+s}}{\sqrt{s}}\,ds\quad\text{for}\;\gamma>0$$

From:

The S Matrix of 6D Super Yang-Mills and Maximal Supergravity from Rational Maps - *Freddy Cachazo, Alfredo Guevara, Matthew Heydeman, Sebastian Mizera, John H. Schwarz and Congkao Wen* - arXiv:1805.11111v2 [hep-th] 2 Oct 2018

We have that:

6 Various Theories in D$\leq$ 6 and $\mathcal{N}=4$ SYM on the Coulomb Branch

6.5 Reduction to Four Dimensions: Special Sectors

We have checked numerically up to $n = 7$ that this expression coincides with $V_n^{-1} R^d(\rho) R^{\tilde{d}}(\tilde{\rho})$ for $\tilde{d} = d - 1$ on the support of the 4D equations (2.11). Hence for this sector ($d = \tilde{d}$ for even n or $d = \tilde{d} \pm 1$ for odd n) the integrand can be recast into the non-chiral form of the Witten–RSV formula, and the amplitude is given by [44]:

$$\mathcal{A}_{n,d}^{\mathcal{N}=4 \text{ SYM}} = \int \mu_{n,d}^{4\text{D}} R^d(\rho) R^{\tilde{d}}(\tilde{\rho}) \int \prod_{k=0}^{d} d^2\chi_k^I \prod_{k=0}^{\tilde{d}} d^2\tilde{\chi}_k^{\hat{I}} \times \prod_{i=1}^{n} \delta^4\left(q_i^{\alpha\hat{I}} - \frac{\rho^\alpha(\sigma_i)\tilde{\chi}^{\hat{I}}(\sigma_i)}{\prod_{j\neq i}\sigma_{ij}}\right) \delta^4\left(\tilde{q}_i^{\dot{\alpha}I} - \frac{\tilde{\rho}^{\dot{\alpha}}(\sigma_i)\chi^I(\sigma_i)}{\prod_{j\neq i}\sigma_{ij}}\right). \tag{6.70}$$

And again:

As discussed in Section 3.1, the 6D $\mathcal{N} = (1,1)$ non-abelian SYM amplitudes for even n can be obtained from those of the D5-brane theory by replacing $(\text{Pf}'A_n)^2$ with the Parke–Taylor factor $\text{PT}(\alpha)$. It is natural to ask what happens if we apply the same replacement to the M5-brane formula [21], at least for an even number n of particles. This procedure leads to a formula with a non-abelian structure and $\mathcal{N} = (2,0)$ supersymmetry,

$$\mathcal{A}_n^{\mathcal{N}=(2,0)}(\alpha) = \int \frac{\prod_{i=1}^n d\sigma_i \prod_{k=0}^m d^8\rho_k\, d^4\chi_k}{\text{vol}\,(\text{SL}(2,\mathbb{C})_\sigma \times \text{SL}(2,\mathbb{C})_\rho)} \prod_{i=1}^{n} \delta^6\left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\,\rho^B(\sigma_i)\rangle}{\prod_{j\neq i}\sigma_{ij}}\right) \times \delta^8\left(q_i^{AI} - \frac{\langle \rho^A(\sigma_i)\,\chi^I(\sigma_i)\rangle}{\prod_{j\neq i}\sigma_{ij}}\right) \frac{\text{Pf}'A_n}{V_n}\,\text{PT}(\alpha)\,. \tag{7.1}$$

With regard the eq, (7.1), we observe that is possible to obtain a mathematical connection with the above Ramanujan product formula. Indeed, we have:

$$\prod_{i=1}^{n} \delta^6\left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\,\rho^B(\sigma_i)\rangle}{\prod_{j\neq i}\sigma_{ij}}\right) \times \delta^8\left(q_i^{AI} - \frac{\langle \rho^A(\sigma_i)\,\chi^I(\sigma_i)\rangle}{\prod_{j\neq i}\sigma_{ij}}\right) \frac{\text{Pf}'A_n}{V_n}\,\text{PT}(\alpha)$$

confronted with:

$$\prod_{n=1}^{n=\infty}\left[1 - \left\{\frac{(\alpha-\beta) + i(\alpha+\beta)\sqrt{3}}{2n}\right\}^2\right]\left[1 - \left\{\frac{(\alpha-\beta) - i(\alpha+\beta)\sqrt{3}}{2n}\right\}^2\right]$$

We know that (from Wikipedia):

The topology of the group SL(n, **C**) is the product of the topology of SU(n) and the topology of the group of hermitian matrices of unit determinant with positive eigenvalues. A hermitian matrix of unit determinant and having positive eigenvalues can be uniquely expressed as the exponential of a traceless hermitian matrix, and therefore the topology of this is that of (n^2-1)-dimensional Euclidean space. Since SU(n) is simply connected, we conclude that SL(n, **C**) is also simply connected, for all n.

In mathematics, an ***n*-sphere** is a topological space that is homeomorphic to a *standard n-sphere*, which is the set of points in (n+1)-dimensional Euclidean space, that are situated at a constant distance r of a fixed point, called the *center*. the $n-1$ dimensional boundary of a (n-dimensional) n-ball is a $n-1$-sphere.

From the following volume formula:

$$V_n(r) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} r^n,$$

for r = 2 and n = 5, we obtain:

((Pi^(5/2)*2^5)) / ((gamma (5/2+1)))

Input:

$$\frac{\pi^{5/2} \times 2^5}{\Gamma\left(\frac{5}{2}+1\right)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{256\,\pi^2}{15}$$

Decimal approximation:

168.441248445258387094775313064552979376020469883576160024 1…

168.4412484…

Property:

$\frac{256\,\pi^2}{15}$ is a transcendental number

Alternative representations:

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{2^5\,\pi^{5/2}}{\frac{5}{2}!}$$

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{2^5\,\pi^{5/2}}{e^{-\log G(7/2)+\log G(9/2)}}$$

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{2^5\,\pi^{5/2}}{(1)_{\frac{5}{2}}}$$

Series representations:

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{512}{5}\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = -\frac{1024}{5}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{2048}{15}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^2}$$

Integral representations:

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{4096}{15}\left(\int_0^1\sqrt{1-t^2}\;dt\right)^2$$

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{1024}{15}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{\pi^{5/2}\,2^5}{\Gamma\left(\frac{5}{2}+1\right)} = \frac{1024}{15}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

From (7.1), we consider:

$$\left(\mathrm{vol}\,(\mathrm{SL}(2,\mathbb{C})_\sigma \times \mathrm{SL}(2,\mathbb{C})_\rho)\right) = \left(\quad V_n(r) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} r^n, \right)$$

Thence, from:

$$\int \frac{\prod_{i=1}^{n} d\sigma_i \prod_{k=0}^{m} d^8\rho_k\, d^4\chi_k}{\mathrm{vol}\,(\mathrm{SL}(2,\mathbb{C})_\sigma \times \mathrm{SL}(2,\mathbb{C})_\rho)} \prod_{i=1}^{n} \delta^6\left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\,\rho^B(\sigma_i)\rangle}{\prod_{j\neq i}\sigma_{ij}}\right)$$
$$\times\ \delta^8\left(q_i^{AI} - \frac{\langle \rho^A(\sigma_i)\,\chi^I(\sigma_i)\rangle}{\prod_{j\neq i}\sigma_{ij}}\right) \frac{\mathrm{Pf}'A_n}{V_n}\,\mathrm{PT}(\alpha)\,.$$

for n = 4, we obtain the following n-sphere volume:

(((Pi^(4/2)*2^4)) / ((gamma (4/2+1))))

Input:

$$\frac{\pi^{4/2}\times 2^4}{\Gamma\left(\frac{4}{2}+1\right)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$8\,\pi^2$

Decimal approximation:

78.956835208714868950675927999009209082509595257926325011 30...

78.9568352…

Property:

$8\pi^2$ is a transcendental number

Alternative representations:

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = \frac{2^4\,\pi^2}{2\times\frac{1}{1}}$$

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = \frac{2^4\,\pi^2}{2!}$$

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = \frac{2^4\,\pi^2}{e^{\log(2)}}$$

Series representations:

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = 48\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = -96\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = 64\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^2}$$

Integral representations:

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = 128\left(\int_0^1\sqrt{1-t^2}\;dt\right)^2$$

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = 32\left(\int_0^{\infty}\frac{1}{1+t^2}\;dt\right)^2$$

$$\frac{\pi^{4/2}\,2^4}{\Gamma\left(\frac{4}{2}+1\right)} = 32\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\;dt\right)^2$$

Thence, performing the integral, we obtain:

integrate [(1/ (8Pi^2)) * (((cosh Pi(8+5)*sqrt3 - cos Pi(8-5)))) /
(((2Pi^2(8^2+8*5+5^2))))]x

Indefinite integral:

$$\int \frac{(\cosh(\pi)\,(8+5)\sqrt{3} - \cos(\pi)\,(8-5))\,x}{(8\pi^2)\,(2\pi^2\,(8^2+8\times 5+5^2))}\,dx = \frac{x^2}{1376\,\pi^4} + \frac{13\,x^2\cosh(\pi)}{1376\sqrt{3}\,\pi^4} + \text{constant}$$

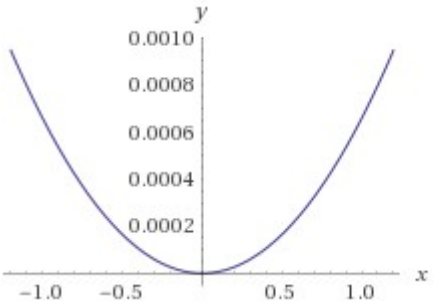
$\cosh(x)$ is the hyperbolic cosine function

Indefinite integral:

$$\int \frac{(\cosh(\pi)\,(8+5)\sqrt{3} - \cos(\pi)\,(8-5))\,x}{(8\pi^2)\,(2\pi^2\,(8^2+8\times 5+5^2))}\,dx \approx \text{constant} + 0.000656575\,x^2$$

0.000656575

Plot of the integral:



(x from −1.2 to 1.2)

Alternate forms of the integral:

$$\frac{x^2\,(3+13\sqrt{3}\,\cosh(\pi))}{4128\,\pi^4} + \text{constant}$$

$$\left(\frac{1}{1376\,\pi^4} + \frac{13\,e^{-\pi}}{2752\sqrt{3}\,\pi^4} + \frac{13\,e^{\pi}}{2752\sqrt{3}\,\pi^4}\right)x^2 + \text{constant}$$

$$\frac{13\,e^{\pi}\,x^2}{2752\sqrt{3}\,\pi^4} + \frac{13\,e^{-\pi}\,x^2}{2752\sqrt{3}\,\pi^4} + \frac{x^2}{1376\,\pi^4} + \text{constant}$$

For x = 31.6227766 , we obtain:

(((0.000656575 * 31.6227766^2)))

Input interpretation:

$0.000656575 \times 31.6227766^2$

Result:

0.656574999930079726207

0.6565749999…

and:

1+(((0.000656575 * 31.6227766^2)))

Input interpretation:

$1 + 0.000656575 \times 31.6227766^2$

Result:

1.656574999930079726207

1.6565749999….

Repeating decimal:

1.656574999930079726207

But, more exactly, for x = 32 = 2^5, we obtain:

(((0.000656575 * 32^2)))

Input interpretation:

0.000656575×32^2

Result:

0.6723328

0.6723328 (Amplitude)

From which:

1+(((0.000656575 * 32^2)))

Input interpretation:

$1 + 0.000656575 \times 32^2$

Result:

1.6723328

1.6723328

and, multiplying by $1/10^{27}$, we obtain:

1/10^27*((1+ (0.000656575 * 32^2)))

Input interpretation:

$\frac{1}{10^{27}} \left(1 + 0.000656575 \times 32^2\right)$

Result:

$1.6723328 \times 10^{-27}$

$1.6723328*10^{-27}$ result practically equal to the proton mass in kg

Thence, we have the following connection and consequent result:

$$\left[\int \frac{\prod_{i=1}^{n} d\sigma_i \prod_{k=0}^{m} d^8\rho_k \, d^4\chi_k}{\mathrm{vol}\,(\mathrm{SL}(2,\mathbb{C})_\sigma \times \mathrm{SL}(2,\mathbb{C})_\rho)} \prod_{i=1}^{n} \delta^6 \left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\, \rho^B(\sigma_i) \rangle}{\prod_{j \neq i} \sigma_{ij}} \right) \times \delta^8 \left(q_i^{AI} - \frac{\langle \rho^A(\sigma_i)\, \chi^I(\sigma_i) \rangle}{\prod_{j \neq i} \sigma_{ij}} \right) \frac{\mathrm{Pf}' A_n}{V_n} \mathrm{PT}(\alpha) \, . \right] =$$

$$= \left[\int \frac{(\cosh(\pi)(8+5)\sqrt{3} - \cos(\pi)(8-5))\,x}{(8\pi^2)(2\pi^2(8^2+8\times 5+5^2))}\,dx \approx \text{constant} + 0.000656575\,x^2 \right] =$$

$$= 0.000656575 \times 32^2 \quad = 0.6723328$$

Note that, from the first result, we obtain also:

(0.6723328*24)/10

Input interpretation:

$$\frac{0.6723328 \times 24}{10}$$

Result:

1.61359872

1.61359872 result that is a good approximation to the value of the golden ratio 1.618033988749...

Now, we have:

$$\prod_{n=1}^{n=\infty} \left\{ \frac{\left(1+\frac{\alpha+2\beta}{n}\right)\left(1+\frac{\beta+2\alpha}{n}\right)}{\left(1+\frac{\alpha}{n}\right)^3\left(1+\frac{\beta}{n}\right)^3} \right\} = \frac{\{\Gamma(1+\alpha)\Gamma(1+\beta)\}^3}{\Gamma(1+\alpha+2\beta)\Gamma(1+\beta+2\alpha)}; \qquad (3)$$

and:

$$\left(\prod_{i=1}^{n}\sigma_{n+1,i}\right)^2\int \tilde{t}d\tilde{t}\,\delta\left(\tilde{t}-\frac{[\tilde{\lambda}_{n+1}\,q]}{[\tilde{\rho}(\sigma_{n+1})\,q]}\right)\delta\left([\tilde{\lambda}_{n+1}\,\tilde{\rho}(\sigma_{n+1})]\right)\delta^2(\rho(\sigma_{n+1}))$$

$$=\left(\prod_{i=1}^{n}\sigma_{n+1,i}\right)\frac{1}{t}\,\delta\left([\tilde{\lambda}_{n+1}\,\tilde{\rho}(\sigma_{n+1})]\right)\delta^2(\rho(\sigma_{n+1})), \qquad \text{(B.5)}$$

where now

$$t=\frac{1}{\prod_{i=1}^{n}\sigma_{n+1,i}}\frac{[\tilde{\rho}(\sigma_{n+1})\,q]}{[\tilde{\lambda}_{n+1}\,q]}. \qquad \text{(B.6)}$$

From (3), if we put α = 8; β = 5, we obtain:

[(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))]

Input:

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{1}{2\,887\,883\,194\,276\,080}$$

Decimal approximation:

$3.462743929470717248113091232643856638092158259000387 0\ldots \times 10^{-16}$

$3.462743929\ldots*10^{-16}$

Alternative representations:

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}=\frac{(5!\times 8!)^3}{18!\times 21!}$$

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}=\frac{(\Gamma(6,0)\,\Gamma(9,0))^3}{\Gamma(19,0)\,\Gamma(22,0)}$$

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}=\frac{((1)_5\,(1)_8)^3}{(1)_{18}\,(1)_{21}}$$

Series representations:

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)} = \frac{\left(\sum_{k=0}^{\infty} \frac{(6-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^3 \left(\sum_{k=0}^{\infty} \frac{(9-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^3}{\left(\sum_{k=0}^{\infty} \frac{(19-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right) \sum_{k=0}^{\infty} \frac{(22-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)} =$$

$$\left(\pi^4 \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} (19-z_0)^{k_1}\,(22-z_0)^{k_2} \sum_{j_1=0}^{k_1}\sum_{j_2=0}^{k_2} \left((-1)^{j_1+j_2}\,\pi^{-j_1-j_2+k_1+k_2} \sin\left(\frac{1}{2}\pi\,(-j_1+k_1+2\,z_0)\right)\sin\left(\frac{1}{2}\pi\,(-j_2+k_2+2\,z_0)\right) \Gamma^{(j_1)}(1-z_0)\,\Gamma^{(j_2)}(1-z_0)\right) \Big/ (j_1!\,j_2!\,(-j_1+k_1)!\,(-j_2+k_2)!)\right) \Big/$$

$$\left(\left(\sum_{k=0}^{\infty}(6-z_0)^k \sum_{j=0}^{k} \frac{(-1)^j\,\pi^{-j+k} \sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^3 \left(\sum_{k=0}^{\infty}(9-z_0)^k \sum_{j=0}^{k} \frac{(-1)^j\,\pi^{-j+k} \sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^3\right)$$

Integral representations:

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)} = \exp\left(\int_0^1 \frac{x\left(-4+3\,x^5+3\,x^8-x^{18}-x^{21}\right)}{(-1+x)\log(x)}\,dx\right)$$

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)} = \exp\left(-4\,\gamma + \int_0^1 \frac{4-3\,x^6-3\,x^9+x^{19}+x^{22}+3\log(x^6)+3\log(x^9)-\log(x^{19})-\log(x^{22})}{\log(x)-x\log(x)}\,dx\right)$$

$$\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)} = \frac{\left(\int_0^1 \log^5\left(\frac{1}{t}\right)dt\right)^3 \left(\int_0^1 \log^8\left(\frac{1}{t}\right)dt\right)^3}{\left(\int_0^1 \log^{18}\left(\frac{1}{t}\right)dt\right) \int_0^1 \log^{21}\left(\frac{1}{t}\right)dt}$$

From which, we obtain:

48*-ln((([(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))]))))+21-1/sqrt3

Input:

$$48\times(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

Exact result:

$$21-\frac{1}{\sqrt{3}}+48\log(2\,887\,883\,194\,276\,080)$$

Decimal approximation:

1729.189057887357875085793918804480190989925071720143810992...

1729.189057…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Property:

$$21-\frac{1}{\sqrt{3}}+48\log(2\,887\,883\,194\,276\,080)$$ is a transcendental number

Alternate forms:

$$\frac{1}{3}\left(63-\sqrt{3}+144\log(2\,887\,883\,194\,276\,080)\right)$$

$$\frac{1}{3}\left(63-\sqrt{3}\right)+48\log(2\,887\,883\,194\,276\,080)$$

$$21-\frac{1}{\sqrt{3}}+192\log(2)+384\log(3)+48\log(95)+96\log(17\,017)$$

Alternative representations:

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=21-48\log_e\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right)-\frac{1}{\sqrt{3}}$$

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=$$
$$21-48\log(a)\log_a\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right)-\frac{1}{\sqrt{3}}$$

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=21-48\log\left(\frac{((1)_5\,(1)_8)^3}{(1)_{18}\,(1)_{21}}\right)-\frac{1}{\sqrt{3}}$$

Series representations:

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=$$
$$21-\frac{1}{\sqrt{3}}+48\log(2\,887\,883\,194\,276\,079)-48\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\,887\,883\,194\,276\,079}\right)^k}{k}$$

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=$$
$$21-\frac{1}{\sqrt{3}}+96\,i\,\pi\left\lfloor\frac{\arg(2\,887\,883\,194\,276\,080-x)}{2\,\pi}\right\rfloor+48\log(x)-$$
$$48\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=$$
$$21-\frac{1}{\sqrt{3}}+96\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+$$
$$48\log(z_0)-48\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=$$
$$21-\frac{1}{\sqrt{3}}+48\int_1^{2\,887\,883\,194\,276\,080}\frac{1}{t}\,dt$$

$$48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}=$$
$$21-\frac{1}{\sqrt{3}}-\frac{24\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2\,887\,883\,194\,276\,079^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

Note that:

(1729.189057 / 12) – 5 = 139.09908808…

(1729.189057 / 12) – 18 – 1/golden ratio = 125.4810541…

Results very near to the Higgs boson and Pion meson masses, i.e. 125.18 GeV and 139.57 MeV

Performing the 15th root:

((((48*-ln(((([(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))])))+21-1/sqrt3))))^1/15

Input:

$$\sqrt[15]{48\times(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}}$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

Exact result:

$$\sqrt[15]{21-\frac{1}{\sqrt{3}}+48\log(2\,887\,883\,194\,276\,080)}$$

Decimal approximation:

1.643827211026624917166074301339365264928756871682025554273...

$1.643827211\ldots \approx \zeta(2)=\frac{\pi^2}{6}=1.644934\ldots$

Property:

$\sqrt[15]{21 - \frac{1}{\sqrt{3}} + 48 \log(2\,887\,883\,194\,276\,080)}$ is a transcendental number

Alternate forms:

$\sqrt[15]{\frac{1}{3}\left(63 - \sqrt{3} + 144 \log(2\,887\,883\,194\,276\,080)\right)}$

$\sqrt[15]{\frac{1}{3}\left(63 - \sqrt{3}\right) + 48 \log(2\,887\,883\,194\,276\,080)}$

$\frac{\sqrt[15]{-1 + 21\sqrt{3} + 192\sqrt{3}\log(2) + 384\sqrt{3}\log(3) + 48\sqrt{3}\log(95) + 96\sqrt{3}\log(17\,017)}}{\sqrt[30]{3}}$

All 15th roots of 21 - 1/sqrt(3) + 48 log(2887883194276080):

$e^{0}\sqrt[15]{21 - \frac{1}{\sqrt{3}} + 48 \log(2\,887\,883\,194\,276\,080)} \approx 1.64383$ (real, principal root)

$e^{(2i\pi)/15}\sqrt[15]{21 - \frac{1}{\sqrt{3}} + 48 \log(2\,887\,883\,194\,276\,080)} \approx 1.50171 + 0.6686\,i$

$e^{(4i\pi)/15}\sqrt[15]{21 - \frac{1}{\sqrt{3}} + 48 \log(2\,887\,883\,194\,276\,080)} \approx 1.0999 + 1.2216\,i$

$e^{(2i\pi)/5}\sqrt[15]{21 - \frac{1}{\sqrt{3}} + 48 \log(2\,887\,883\,194\,276\,080)} \approx 0.5080 + 1.5634\,i$

$e^{(8i\pi)/15}\sqrt[15]{21 - \frac{1}{\sqrt{3}} + 48 \log(2\,887\,883\,194\,276\,080)} \approx -0.17183 + 1.63482\,i$

Alternative representations:

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}} = \sqrt[15]{21-48\log_e\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right)-\frac{1}{\sqrt{3}}}$$

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}} = \sqrt[15]{21-48\log(a)\log_a\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right)-\frac{1}{\sqrt{3}}}$$

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}} = \sqrt[15]{21-48\log\left(\frac{((1)_5\,(1)_8)^3}{(1)_{18}\,(1)_{21}}\right)-\frac{1}{\sqrt{3}}}$$

Series representations:

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}} = \sqrt[15]{21-\frac{1}{\sqrt{3}}+48\left(\log(2\,887\,883\,194\,276\,079)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\,887\,883\,194\,276\,079}\right)^k}{k}\right)}$$

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}} = \left(21-\frac{1}{\sqrt{3}}+48\left(2\,i\,\pi\left\lfloor\frac{\arg(2\,887\,883\,194\,276\,080-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-x)^k\,x^{-k}}{k}\right)\right)\text{^}\,(1/15) \quad \text{for } x<0$$

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}}\;=$$
$$\left(21-\frac{1}{\sqrt{3}}+48\left(\log(z_0)+\left\lfloor\frac{\arg(2\,887\,883\,194\,276\,080-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-z_0)^k\,z_0^{-k}}{k}\right)\right)\text{^}\,(1/15)$$

Integral representations:

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}}\;=$$
$$\sqrt[15]{21-\frac{1}{\sqrt{3}}+48\int_1^{2\,887\,883\,194\,276\,080}\frac{1}{t}\,dt}$$

$$\sqrt[15]{48\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)+21-\frac{1}{\sqrt{3}}}\;=$$
$$\sqrt[15]{21-\frac{1}{\sqrt{3}}-\frac{24\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2\,887\,883\,194\,276\,079^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad\text{for}\quad -1<\gamma<0$$

Furthermore, we obtain also:

(24*3^3*2)((([(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))]))))^3

Input:

$$(24\times 3^3\times 2)\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{1}{18\,583\,772\,006\,455\,971\,342\,386\,498\,846\,753\,868\,293\,472\,000}$$

Decimal approximation:

$5.381038895938895886989541339769760331772184239263556 1\ldots \times 10^{-44}$

5.381038895…*10^{-44} result very near to the value of Planck time 5.39124760×10^{-44} s

Alternative representations:

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = 1296\left(\frac{(5!\times 8!)^3}{18!\times 21!}\right)^3$$

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = 1296\left(\frac{(\Gamma(6,0)\,\Gamma(9,0))^3}{\Gamma(19,0)\,\Gamma(22,0)}\right)^3$$

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = 1296\left(\frac{((1)_5\,(1)_8)^3}{(1)_{18}\,(1)_{21}}\right)^3$$

Series representations:

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = \frac{1296\left(\sum_{k=0}^{\infty}\frac{(6-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^9\left(\sum_{k=0}^{\infty}\frac{(9-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^9}{\left(\sum_{k=0}^{\infty}\frac{(19-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^3\left(\sum_{k=0}^{\infty}\frac{(22-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^3} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = \left(1296\,\pi^{12}\left(\sum_{k=0}^{\infty}(19-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^3\left(\sum_{k=0}^{\infty}(22-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^3\right)\Bigg/\left(\left(\sum_{k=0}^{\infty}(6-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^9\left(\sum_{k=0}^{\infty}(9-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^9\right)$$

Integral representations:

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = 1296\exp\left(\int_0^1 \frac{3\,x\,(-4+3\,x^5+3\,x^8-x^{18}-x^{21})}{(-1+x)\log(x)}\,dx\right)$$

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = 1296\exp\left(-12\,\gamma+\int_0^1 -\frac{3\,(4-3\,x^6-3\,x^9+x^{19}+x^{22}+3\log(x^6)+3\log(x^9)-\log(x^{19})-\log(x^{22}))}{(-1+x)\log(x)}\,dx\right)$$

$$\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 24\,(3^3\times 2) = \frac{1296\left(\int_0^1 \log^5\left(\frac{1}{t}\right)dt\right)^9\left(\int_0^1 \log^8\left(\frac{1}{t}\right)dt\right)^9}{\left(\int_0^1 \log^{18}\left(\frac{1}{t}\right)dt\right)^3\left(\int_0^1 \log^{21}\left(\frac{1}{t}\right)dt\right)^3}$$

or:

1.668510441^14((([(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))]))^3

where $1.668510441^{14} = 1296$

Input interpretation:

$$1.668510441^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3$$

$\Gamma(x)$ is the gamma function

Result:

$5.381038894727714432643378337925533273108376297187887 7\ldots \times 10^{-44}$

$5.38103889472\ldots*10^{-44}$ as above

Alternative representations:

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = 1.66851^{14}\left(\frac{(5!\times 8!)^3}{18!\times 21!}\right)^3$$

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = 1.66851^{14}\left(\frac{(\Gamma(6,\,0)\,\Gamma(9,\,0))^3}{\Gamma(19,\,0)\,\Gamma(22,\,0)}\right)^3$$

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = 1.66851^{14}\left(\frac{((1)_5\,(1)_8)^3}{(1)_{18}\,(1)_{21}}\right)^3$$

Series representations:

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = \frac{1296.\left(\sum_{k=0}^{\infty}\frac{(6-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^9\left(\sum_{k=0}^{\infty}\frac{(9-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^9}{\left(\sum_{k=0}^{\infty}\frac{(19-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^3\left(\sum_{k=0}^{\infty}\frac{(22-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^3} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = \left(1296.\,\pi^{12}\left(\sum_{k=0}^{\infty}(19-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\,\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^3\left(\sum_{k=0}^{\infty}(22-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\,\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^3\right)\Bigg/\left(\left(\sum_{k=0}^{\infty}(6-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\,\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^9\left(\sum_{k=0}^{\infty}(9-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\,\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^9\right)$$

Integral representations:

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = 1296.\exp\left(\int_0^1\frac{3\,x\left(-4+3\,x^5+3\,x^8-x^{18}-x^{21}\right)}{(-1+x)\log(x)}\,dx\right)$$

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = 1296.\exp\left(-12\,\gamma + \int_0^1 -\frac{3\,(4-3\,x^6-3\,x^9+x^{19}+x^{22}+3\log(x^6)+3\log(x^9)-\log(x^{19})-\log(x^{22}))}{(-1+x)\log(x)}\,dx\right)$$

$$1.66851^{14}\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)^3 = \frac{1296.\left(\int_0^1 \log^5\left(\frac{1}{t}\right)dt\right)^9\left(\int_0^1 \log^8\left(\frac{1}{t}\right)dt\right)^9}{\left(\int_0^1 \log^{18}\left(\frac{1}{t}\right)dt\right)^3\left(\int_0^1 \log^{21}\left(\frac{1}{t}\right)dt\right)^3}$$

Thence, we have the following mathematical connection:

$$\left(\prod_{n=1}^{n=\infty}\left\{\frac{\left(1+\frac{\alpha+2\beta}{n}\right)\left(1+\frac{\beta+2\alpha}{n}\right)}{\left(1+\frac{\alpha}{n}\right)^3\left(1+\frac{\beta}{n}\right)^3}\right\} = \frac{\{\Gamma(1+\alpha)\Gamma(1+\beta)\}^3}{\Gamma(1+\alpha+2\beta)\Gamma(1+\beta+2\alpha)};\right) =$$

$= 3.462743929\ldots * 10^{-16} \Rightarrow$

$$\Rightarrow \left[\left(\prod_{i=1}^{n}\sigma_{n+1,i}\right)^2\int \tilde{t}\,d\tilde{t}\,\delta\left(\tilde{t}-\frac{[\tilde{\lambda}_{n+1}\,q]}{[\tilde{\rho}(\sigma_{n+1})\,q]}\right)\delta\left([\tilde{\lambda}_{n+1}\,\tilde{\rho}(\sigma_{n+1})]\right)\delta^2(\rho(\sigma_{n+1})) = \left(\prod_{i=1}^{n}\sigma_{n+1,i}\right)\frac{1}{t}\,\delta\left([\tilde{\lambda}_{n-1}\,\tilde{\rho}(\sigma_{n+1})]\right)\delta^2(\rho(\sigma_{n+1})).\right]$$

We obtain from the previous formula also the following results:

4*colog(((([(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))])))-3

Input:

$$4\left(-\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)\right)-3$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

Exact result:

$$4\log(2\,887\,883\,194\,276\,080)-3$$

Decimal approximation:

139.397200679712291737525255632081845703797722789284494822 3...

139.3972006… result practically equal to the rest mass of Pion meson 139.57 MeV

Property:

$-3+4\log(2\,887\,883\,194\,276\,080)$ is a transcendental number

Alternate forms:

$$-3+16\log(2)+32\log(3)+4\log(95)+8\log(17\,017)$$

$$-3+16\log(2)+32\log(3)+4\log(5)+8\log(7)+8\log(11)+8\log(13)+8\log(17)+4\log(19)$$

$$4\,(4\log(2)+8\log(3)+\log(5)+2\log(7)+2\log(11)+2\log(13)+2\log(17)+\log(19))-3$$

Alternative representations:

$$4\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=-3-4\log\left(\frac{(5!\times 8!)^3}{18!\times 21!}\right)$$

$$4\,(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=-3-4\log_e\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right)$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=-3-4\log(a)\log_a\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right)$$

Series representations:

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=$$
$$-3+4\log(2\,887\,883\,194\,276\,079)-4\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\,887\,883\,194\,276\,079}\right)^k}{k}$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=-3+8\,i\,\pi\left\lfloor\frac{\arg(2\,887\,883\,194\,276\,080-x)}{2\,\pi}\right\rfloor+$$
$$4\log(x)-4\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=-3+8\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+$$
$$4\log(z_0)-4\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=-3+4\int_1^{2\,887\,883\,194\,276\,080}\frac{1}{t}\,dt$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-3=$$
$$-3-\frac{2\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2\,887\,883\,194\,276\,079^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

4*colog((((([(((gamma (1+8) gamma (1+5))))^3] / [((gamma (1+8+10) gamma (1+5+16)))])))))-18+1/golden ratio

Input:

$$4\left(-\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)\right)-18+\frac{1}{\phi}$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact result:

$$\frac{1}{\phi} - 18 + 4\log(2\,887\,883\,194\,276\,080)$$

Decimal approximation:

125.015234668462186585729842466447483821518031969090257684 5...

125.015234… result very near to the Higgs boson mass 125.18 GeV

Property:

$-18 + \frac{1}{\phi} + 4\log(2\,887\,883\,194\,276\,080)$ is a transcendental number

Alternate forms:

$$\frac{1}{2}\left(\sqrt{5} - 37\right) + 4\log(2\,887\,883\,194\,276\,080)$$

$$-18 + \frac{2}{1+\sqrt{5}} + 4\log(2\,887\,883\,194\,276\,080)$$

$$\frac{1}{\phi} - 18 + 16\log(2) + 32\log(3) + 4\log(95) + 8\log(17\,017)$$

Alternative representations:

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right) - 18 + \frac{1}{\phi} = -18 - 4\log\left(\frac{(5!\times 8!)^3}{18!\times 21!}\right) + \frac{1}{\phi}$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right) - 18 + \frac{1}{\phi} = -18 - 4\log_e\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right) + \frac{1}{\phi}$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right) - 18 + \frac{1}{\phi} = -18 - 4\log(a)\log_a\left(\frac{(\Gamma(6)\,\Gamma(9))^3}{\Gamma(19)\,\Gamma(22)}\right) + \frac{1}{\phi}$$

Series representations:

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-18+\frac{1}{\phi}=$$
$$-18+\frac{1}{\phi}+4\log(2\,887\,883\,194\,276\,079)-4\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\,887\,883\,194\,276\,079}\right)^k}{k}$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-18+\frac{1}{\phi}=$$
$$-18+\frac{1}{\phi}+8\,i\,\pi\left\lfloor\frac{\arg(2\,887\,883\,194\,276\,080-x)}{2\,\pi}\right\rfloor+4\log(x)-$$
$$4\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-18+\frac{1}{\phi}=$$
$$-18+\frac{1}{\phi}+8\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+$$
$$4\log(z_0)-4\sum_{k=1}^{\infty}\frac{(-1)^k\,(2\,887\,883\,194\,276\,080-z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-18+\frac{1}{\phi}=-18+\frac{1}{\phi}+4\int_1^{2\,887\,883\,194\,276\,080}\frac{1}{t}\,dt$$

$$4(-1)\log\left(\frac{(\Gamma(1+8)\,\Gamma(1+5))^3}{\Gamma(1+8+10)\,\Gamma(1+5+16)}\right)-18+\frac{1}{\phi}=$$
$$-18+\frac{1}{\phi}-\frac{2\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2\,887\,883\,194\,276\,079^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

Now, we have that:

$$\int d\mu_{n\ \text{even}}^{6\text{D}} = \int \frac{J_\rho\, J_\sigma}{V_n^2} \left(\prod_{i=4}^{n} d\sigma_i \right) d\rho_0^{2,-} d^2\rho_0^3 d^2\rho_0^4 \left(\prod_{k=1}^{m} d^8\rho_k \right) \prod_{i=1}^{n} \delta^6 \left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\, \rho^B(\sigma_i) \rangle}{\prod_{j\neq i} \sigma_{ij}} \right),$$

where the Jacobians are[2]

$$J_\sigma = \sigma_{12}\sigma_{23}\sigma_{31}, \qquad J_\rho = \rho_0^{1,+} \langle \rho_0^1\, \rho_0^2 \rangle. \tag{2.37}$$

It is convenient to use a short-hand notation for the bosonic delta functions:

$$\Delta_B = \prod_{i=1}^{n} \delta^6 \left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\, \rho^B(\sigma_i) \rangle}{\prod_{j\neq i} \sigma_{ij}} \right) = \delta^6 (\textstyle\sum_{i=1}^{n} p_i^{AB}) \left(\prod_{i=1}^{n} \delta(p_i^2) \right) \hat{\Delta}_B, \tag{2.38}$$

where $\hat{\Delta}_B$ is

$$\hat{\Delta}_B = \delta^4 \left(p_n^{AB} - \frac{\langle \rho^A(\sigma_n)\, \rho^B(\sigma_n) \rangle}{\prod_{i\neq n} \sigma_{ni}} \right) \prod_{i=1}^{n-2} \delta^5 \left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\, \rho^B(\sigma_i) \rangle}{\prod_{j\neq i} \sigma_{ij}} \right) \prod_{i=1}^{n} p_i^{12} \left(\frac{p_{n-1}^{24}}{p_{n-1}^{12}} - \frac{p_n^{24}}{p_n^{12}} \right). \tag{2.39}$$

From

Similarly supposing that $\beta = \alpha + 1$ in (16.2.4) we obtain

$$\prod_{n=1}^{\infty} \left\{ 1 + \left(\frac{1+2\alpha}{n+\alpha} \right)^3 \right\} = \frac{\{\Gamma(1+\alpha)\}^3}{\Gamma(2+3\alpha)} \cdot \frac{\cosh \pi(\tfrac{1}{2}+\alpha)\sqrt{3}}{\pi}. \tag{16.3.2}$$

we obtain, for α = 8 :

((gamma (1+8)))^3 / (gamma (2+3*8)) * ((cosh (Pi(1/2+8))*sqrt3)) / Pi

Input:

$$\frac{\Gamma(1+8)^3}{\Gamma(2+3\times 8)} \times \frac{\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}}{\pi}$$

$\Gamma(x)$ is the gamma function

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{\cosh\left(\frac{17\pi}{2}\right)}{78\,879\,264\,750\,\sqrt{3}\,\pi}$$

Decimal approximation:

0.460782161990980595480582175911071320157229128241438499580...

0.46078216199…

Alternate forms:

$$\frac{e^{-(17\pi)/2}}{157\,758\,529\,500\,\sqrt{3}\,\pi}+\frac{e^{(17\pi)/2}}{157\,758\,529\,500\,\sqrt{3}\,\pi}$$

$$\frac{1}{78\,879\,264\,750\,\sqrt{3}\,\pi}\cosh\left(\frac{\pi}{2}\right)(1-2\cosh(\pi)+2\cosh(2\pi)-2\cosh(3\pi)+2\cosh(4\pi)-2\cosh(5\pi)+2\cosh(6\pi)-2\cosh(7\pi)+2\cosh(8\pi))$$

$$\frac{\cosh^{17}\left(\frac{\pi}{2}\right)}{78\,879\,264\,750\,\sqrt{3}\,\pi}+\frac{4\sinh^{2}\left(\frac{\pi}{2}\right)\cosh^{15}\left(\frac{\pi}{2}\right)}{2\,319\,978\,375\,\sqrt{3}\,\pi}+\frac{14\sinh^{4}\left(\frac{\pi}{2}\right)\cosh^{13}\left(\frac{\pi}{2}\right)}{463\,995\,675\,\sqrt{3}\,\pi}+\frac{28\sinh^{6}\left(\frac{\pi}{2}\right)\cosh^{11}\left(\frac{\pi}{2}\right)}{178\,459\,875\,\sqrt{3}\,\pi}+\frac{\sinh^{8}\left(\frac{\pi}{2}\right)\cosh^{9}\left(\frac{\pi}{2}\right)}{3\,244\,725\,\sqrt{3}\,\pi}+\frac{4\sinh^{10}\left(\frac{\pi}{2}\right)\cosh^{7}\left(\frac{\pi}{2}\right)}{16\,223\,625\,\sqrt{3}\,\pi}+\frac{14\sinh^{12}\left(\frac{\pi}{2}\right)\cosh^{5}\left(\frac{\pi}{2}\right)}{178\,459\,875\,\sqrt{3}\,\pi}+\frac{4\sinh^{14}\left(\frac{\pi}{2}\right)\cosh^{3}\left(\frac{\pi}{2}\right)}{463\,995\,675\,\sqrt{3}\,\pi}+\frac{\sinh^{16}\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi}{2}\right)}{4\,639\,956\,750\,\sqrt{3}\,\pi}$$

$\sinh(x)$ is the hyperbolic sine function

Alternative representations:

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)}=\frac{\cos\left(-\frac{17\,i\,\pi}{2}\right)((1)_8)^3\,\sqrt{3}}{\pi\,(1)_{25}}$$

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)}=\frac{\left(e^{-(17\pi)/2}+e^{(17\pi)/2}\right)((1)_8)^3\,\sqrt{3}}{2\,\pi\,(1)_{25}}$$

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} = \left(\cos\left(\frac{17\,i\,\pi}{2}\right)\left(\frac{5\,056\,584\,744\,960\,000}{125\,411\,328\,000}\right)^3\sqrt{3}\right)\Big/(\pi$$
$$18\,208\,281\,810\,188\,658\,276\,327\,614\,252\,643\,415\,193\,394\,793\,745\,060\,606\,\ddots$$
$$890\,429\,618\,992\,193\,283\,129\,810\,291\,626\,991\,262\,221\,974\,029\,109\,363\,\ddots$$
$$817\,787\,141\,461\,966\,252\,773\,116\,546\,051\,694\,612\,770\,469\,960\,217\,933\,\ddots$$
$$012\,913\,302\,274\,713\,118\,105\,435\,774\,192\,291\,006\,757\,570\,987\,304\,784\,\ddots$$
$$651\,878\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\ddots$$
$$000\,000\,000\,000)/$$
$$1\,173\,878\,875\,943\,484\,045\,145\,854\,663\,582\,065\,069\,715\,341\,161\,356\,797\,835\,\ddots$$
$$752\,331\,993\,963\,556\,125\,154\,689\,322\,181\,575\,091\,217\,882\,971\,654\,578\,156\,\ddots$$
$$844\,286\,587\,738\,831\,291\,224\,576\,726\,252\,820\,647\,026\,675\,651\,753\,423\,450\,\ddots$$
$$654\,917\,092\,461\,536\,374\,019\,784\,808\,857\,600\,000\,000\,000\,000\,000\,000\,000\,\ddots$$
$$000\,000\,000\,000\,000\,000\,000\,000\,000$$

Series representations:

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} = \frac{\sum_{k=0}^{\infty}\frac{\left(\frac{17\,\pi}{2}\right)^{2k}}{(2k)!}}{78\,879\,264\,750\,\sqrt{3}\,\pi}$$

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} = -\frac{\sum_{k=0}^{\infty} I_{2k}\left(\frac{17}{2}\right) T_{2k}(\pi)\,(-2+\delta_k)}{78\,879\,264\,750\,\sqrt{3}\,\pi}$$

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} = \frac{i\sum_{k=0}^{\infty}\frac{\left(\left(\frac{17}{2}-\frac{i}{2}\right)\pi\right)^{1+2k}}{(1+2k)!}}{78\,879\,264\,750\,\sqrt{3}\,\pi}$$

Integral representations:

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} = \frac{1}{78\,879\,264\,750\,\sqrt{3}\,\pi}\int_{\frac{i\,\pi}{2}}^{\frac{17\,\pi}{2}}\sinh(t)\,dt$$

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} =$$
$$\frac{1}{78\,879\,264\,750\,\sqrt{3}\,\pi} + \frac{1}{9\,279\,913\,500\,\sqrt{3}}\int_0^1 \sinh\left(\frac{17\,\pi\,t}{2}\right)dt$$

$$\frac{\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)\Gamma(1+8)^3}{\pi\,\Gamma(2+3\times 8)} =$$
$$-\frac{i}{157\,758\,529\,500\,\sqrt{3}\,\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{(289\,\pi^2)/(16\,s)+s}}{\sqrt{s}}\,ds \quad \text{for } \gamma > 0$$

Thence, we obtain:

$$\int d\mu_{n\ \text{even}}^{6\text{D}} = \int \frac{J_\rho\, J_\sigma}{V_n^2} \left(\prod_{i=4}^{n} d\sigma_i \right) d\rho_0^{2,-} d^2\rho_0^3 d^2\rho_0^4 \left(\prod_{k=1}^{m} d^8\rho_k \right) \prod_{i=1}^{n} \delta^6 \left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\, \rho^B(\sigma_i) \rangle}{\prod_{j\neq i} \sigma_{ij}} \right)$$

$$\left[\Delta_B = \prod_{i=1}^{n} \delta^6 \left(p_i^{AB} - \frac{\langle \rho^A(\sigma_i)\, \rho^B(\sigma_i) \rangle}{\prod_{j\neq i} \sigma_{ij}} \right) \right] =$$

$$= \left[\frac{\Gamma(1+8)^3}{\Gamma(2+3\times 8)} \times \frac{\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}}{\pi} \right] = 0.46078216199\ldots$$

From which:

sqrt13[((gamma (1+8)))^3 / (gamma (2+3*8)) * ((cosh (Pi(1/2+8))*sqrt3)) / Pi]

Input:

$$\sqrt{13} \left(\frac{\Gamma(1+8)^3}{\Gamma(2+3\times 8)} \times \frac{\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}}{\pi} \right)$$

Exact result:

$$\frac{\cosh\left(\frac{17\pi}{2}\right)}{6\,067\,635\,750\,\sqrt{39}\,\pi}$$

Decimal approximation:

1.661373711877634614159413894352551892525484414755739777512...

1.66137371187…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Alternate forms:

$$\frac{e^{-(17\pi)/2}}{12\,135\,271\,500\sqrt{39}\,\pi}+\frac{e^{(17\pi)/2}}{12\,135\,271\,500\sqrt{39}\,\pi}$$

$$\frac{1}{6\,067\,635\,750\sqrt{39}\,\pi}\cosh\left(\frac{\pi}{2}\right)(1-2\cosh(\pi)+2\cosh(2\pi)-2\cosh(3\pi)+2\cosh(4\pi)-2\cosh(5\pi)+2\cosh(6\pi)-2\cosh(7\pi)+2\cosh(8\pi))$$

$$\frac{\cosh^{17}\left(\frac{\pi}{2}\right)}{6\,067\,635\,750\sqrt{39}\,\pi}+\frac{4\sinh^2\left(\frac{\pi}{2}\right)\cosh^{15}\left(\frac{\pi}{2}\right)}{178\,459\,875\sqrt{39}\,\pi}+\frac{14\sinh^4\left(\frac{\pi}{2}\right)\cosh^{13}\left(\frac{\pi}{2}\right)}{35\,691\,975\sqrt{39}\,\pi}+\frac{28\sqrt{\frac{13}{3}}\sinh^6\left(\frac{\pi}{2}\right)\cosh^{11}\left(\frac{\pi}{2}\right)}{178\,459\,875\,\pi}+\frac{\sqrt{\frac{13}{3}}\sinh^8\left(\frac{\pi}{2}\right)\cosh^9\left(\frac{\pi}{2}\right)}{3\,244\,725\,\pi}+\frac{4\sqrt{\frac{13}{3}}\sinh^{10}\left(\frac{\pi}{2}\right)\cosh^7\left(\frac{\pi}{2}\right)}{16\,223\,625\,\pi}+\frac{14\sqrt{\frac{13}{3}}\sinh^{12}\left(\frac{\pi}{2}\right)\cosh^5\left(\frac{\pi}{2}\right)}{178\,459\,875\,\pi}+\frac{4\sinh^{14}\left(\frac{\pi}{2}\right)\cosh^3\left(\frac{\pi}{2}\right)}{35\,691\,975\sqrt{39}\,\pi}+\frac{\sinh^{16}\left(\frac{\pi}{2}\right)\cosh\left(\frac{\pi}{2}\right)}{356\,919\,750\sqrt{39}\,\pi}$$

$\sinh(x)$ is the hyperbolic sine function

Alternative representations:

$$\frac{\sqrt{13}\;\Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi}=\frac{\cos\left(-\frac{17i\pi}{2}\right)((1)_8)^3\sqrt{3}\;\sqrt{13}}{\pi\,(1)_{25}}$$

$$\frac{\sqrt{13}\;\Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi}=\frac{\left(e^{-(17\pi)/2}+e^{(17\pi)/2}\right)((1)_8)^3\sqrt{3}\;\sqrt{13}}{2\pi\,(1)_{25}}$$

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} =$$
$$\left(\cos\left(\frac{17\,i\,\pi}{2}\right)\left(\frac{5\,056\,584\,744\,960\,000}{125\,411\,328\,000}\right)^3\sqrt{3}\ \sqrt{13}\right)\Big/(\pi$$
$$18\,208\,281\,810\,188\,658\,276\,327\,614\,252\,643\,415\,193\,394\,793\,745\,060\,606\,\ddots$$
$$890\,429\,618\,992\,193\,283\,129\,810\,291\,626\,991\,262\,221\,974\,029\,109\,363\,\ddots$$
$$817\,787\,141\,461\,966\,252\,773\,116\,546\,051\,694\,612\,770\,469\,960\,217\,933\,\ddots$$
$$012\,913\,302\,274\,713\,118\,105\,435\,774\,192\,291\,006\,757\,570\,987\,304\,784\,\ddots$$
$$651\,878\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\ddots$$
$$000\,000\,000\,000)/$$
$$1\,173\,878\,875\,943\,484\,045\,145\,854\,663\,582\,065\,069\,715\,341\,161\,356\,797\,835\,\ddots$$
$$752\,331\,993\,963\,556\,125\,154\,689\,322\,181\,575\,091\,217\,882\,971\,654\,578\,156\,\ddots$$
$$844\,286\,587\,738\,831\,291\,224\,576\,726\,252\,820\,647\,026\,675\,651\,753\,423\,450\,\ddots$$
$$654\,917\,092\,461\,536\,374\,019\,784\,808\,857\,600\,000\,000\,000\,000\,000\,000\,000\,\ddots$$
$$000\,000\,000\,000\,000\,000\,000\,000\,000$$

Series representations:

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} = \frac{\sum_{k=0}^{\infty}\frac{\left(\frac{17\,\pi}{2}\right)^{2k}}{(2k)!}}{6\,067\,635\,750\,\sqrt{39}\ \pi}$$

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} = -\frac{\sum_{k=0}^{\infty} I_{2k}\left(\frac{17}{2}\right) T_{2k}(\pi)\,(-2+\delta_k)}{6\,067\,635\,750\,\sqrt{39}\ \pi}$$

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} = \frac{i\sum_{k=0}^{\infty}\frac{\left(\left(\frac{17}{2}-\frac{i}{2}\right)\pi\right)^{1+2k}}{(1+2k)!}}{6\,067\,635\,750\,\sqrt{39}\ \pi}$$

Integral representations:

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} = \frac{1}{6\,067\,635\,750\,\sqrt{39}\ \pi}\int_{\frac{i\,\pi}{2}}^{\frac{17\,\pi}{2}}\sinh(t)\,dt$$

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} =$$
$$\frac{1}{6\,067\,635\,750\,\sqrt{39}\ \pi} + \frac{1}{713\,839\,500\,\sqrt{39}}\int_0^1 \sinh\left(\frac{17\,\pi\,t}{2}\right)dt$$

$$\frac{\sqrt{13}\ \Gamma(1+8)^3\left(\cosh\left(\pi\left(\frac{1}{2}+8\right)\right)\sqrt{3}\right)}{\Gamma(2+3\times 8)\,\pi} =$$

$$-\frac{i}{12\,135\,271\,500\,\sqrt{39}\ \pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{(289\pi^2)/(16\,s)+s}}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

Now, we have that:

W_i. This will allow us to show that these constraints apply to the elements of a *symplectic* Grassmannian. Let us begin with the even-n case and recast the bosonic measure:

$$\int d\mu_{n\ \text{even}}^{6\text{D}} = \int \frac{\left(\prod_{i=1}^{n} d\sigma_i\, d^4W_i\right)\prod_{k=0}^{m} d^8\rho_k}{\text{vol}\,(\text{SL}(2,\mathbb{C})_\sigma \times \text{SL}(2,\mathbb{C})_\rho)}\prod_{i-1}^{n}\delta^8\left(\lambda_{ia}^A - (W_i)_a^b\rho_b^A(\sigma_i)\right)\delta\left(|W_i| - \frac{1}{\prod_{j\neq i}\sigma_{ij}}\right)$$

$$= \int \frac{\prod_{i=1}^{n} d\sigma_i\, d^4W_i}{\text{vol}\,(\text{SL}(2,\mathbb{C})_\sigma \times \text{SL}(2,\mathbb{C})_W)}\prod_{k=0}^{m}\delta^8\left(\sum_{i=1}^{n}(W_i)_a^b\sigma_i^k\lambda_{ib}^A\right)\prod_{i=1}^{n}\delta\left(|W_i| - \frac{1}{\prod_{j\neq i}\sigma_{ij}}\right). \quad (5.27)$$

This result can be obtained by using the following identity for each of the eight components separately [2, 41, 48],

$$\prod_{k=0}^{m}\delta\left(\sum_{i=1}^{n}\sigma_i^k X_i\right) = V_n\int\left(\prod_{k=0}^{m}d\rho_k\right)\prod_{i=1}^{n}\delta\left(\rho(\sigma_i) - X_i\prod_{j\neq i}\sigma_{ij}\right), \quad (5.28)$$

where $\rho(z) = \sum_{k=0}^{m}\rho_k z^k$ denotes any component of the polynomial map. Starting with (5.4), one can obtain a similar result for the fermions. Specifically,

$$\int\left(\prod_{k=0}^{m}d^2\chi_k\right)\prod_{i=1}^{n}\delta^2\left(\eta_{i,a} - (W_i)_a^b\chi_b(\sigma_i)\right) = \prod_{k=0}^{m}\delta^2\left(\sum_{i=1}^{n}(W_i)_a^b\sigma_i^k\eta_{i,b}\right). \quad (5.29)$$

From

$$\prod_{n=1}^{\infty}\left(1+\left(\frac{\alpha}{n}\right)^3\right)\left\{1+3\left(\frac{\alpha}{2n+\alpha}\right)^2\right\} = \frac{\Gamma(\frac{1}{2}\alpha)}{\Gamma(\frac{1}{2}(1+\alpha))}\cdot\frac{\cosh\pi\alpha\sqrt{3}-\cos\pi\alpha}{2^{\alpha+2}\pi\alpha\sqrt{\pi}}. \quad (16.3.3)$$

we obtain, for α = 8 :

gamma (8/2) / gamma (1/2(1+8)) * ((cosh (8Pi*sqrt3) – cos (8Pi))) / ((2^10*8Pi*sqrtPi))

Input:

$$\frac{\Gamma\left(\frac{8}{2}\right)}{\Gamma\left(\frac{1}{2}(1+8)\right)} \times \frac{\cosh(8\pi\sqrt{3}) - \cos(8\pi)}{2^{10} \times 8\,(\pi\sqrt{\pi})}$$

Exact result:

$$\frac{\cosh(8\sqrt{3}\,\pi) - 1}{8960\,\pi^2}$$

Decimal approximation:

$4.546901784169447302210657756547755026623713613248106 5\ldots \times 10^{13}$

$4.54690178\ldots * 10^{13}$

Alternate forms:

$$\frac{\sinh^2(4\sqrt{3}\,\pi)}{4480\,\pi^2}$$

$$\frac{2\cosh(8\sqrt{3}\,\pi) - 2}{17920\,\pi^2}$$

$$\frac{\cosh(8\sqrt{3}\,\pi)}{8960\,\pi^2} - \frac{1}{8960\,\pi^2}$$

Alternative representations:

$$\frac{(\cosh(8\pi\sqrt{3}) - \cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10} \times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{e^{-\log(2)+\log(12)}\left(-\cosh(-8\,i\,\pi) + \frac{1}{2}\left(e^{-8\pi\sqrt{3}} + e^{8\pi\sqrt{3}}\right)\right)}{e^{-\log G(9/2)+\log G(11/2)}\,(8\,\pi\,2^{10}\sqrt{\pi})}$$

$$\frac{(\cosh(8\pi\sqrt{3}) - \cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10} \times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{\left(\cos(-8\,i\,\pi\sqrt{3}) + \frac{1}{2}\left(-e^{-8\,i\,\pi} - e^{8\,i\,\pi}\right)\right)e^{-\log(2)+\log(12)}}{e^{-\log G(9/2)+\log G(11/2)}\,(8\,\pi\,2^{10}\sqrt{\pi})}$$

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{e^{-\log(2)+\log(12)}\left(\frac{1}{2}\left(-e^{-8i\pi}-e^{8i\pi}\right)+\frac{1}{2}\left(e^{-8\pi\sqrt{3}}+e^{8\pi\sqrt{3}}\right)\right)}{e^{-\log G(9/2)+\log G(11/2)}\,(8\pi 2^{10}\sqrt{\pi})}$$

Series representations:

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = -\frac{1}{8960\,\pi^2}+\frac{\sum_{k=0}^{\infty}\frac{192^k\,\pi^{2k}}{(2k)!}}{8960\,\pi^2}$$

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = -\frac{1}{8960\,\pi^2}-\frac{\sum_{k=0}^{\infty} I_{2k}(8)\,T_{2k}(\sqrt{3}\ \pi)\,(-2+\delta_k)}{8960\,\pi^2}$$

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = -\frac{1}{8960\,\pi^2}+\frac{i\sum_{k=0}^{\infty}\frac{\left(\left(-\frac{i}{2}+8\sqrt{3}\right)\pi\right)^{1+2k}}{(1+2k)!}}{8960\,\pi^2}$$

Integral representations:

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{\sqrt{3}}{1120\,\pi}\int_0^1 \sinh\left(8\sqrt{3}\ \pi\,t\right)dt$$

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = -\frac{1}{8960\,\pi^2}+\frac{1}{8960\,\pi^2}\int_{\frac{i\pi}{2}}^{8\sqrt{3}\ \pi}\sinh(t)\,dt$$

$$\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = -\frac{1}{8960\,\pi^2}-\frac{i}{17920\,\pi^{5/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{(48\pi^2)/s+s}}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

Multiple-argument formulas:

$$\frac{(\cosh(8\pi\sqrt{3}) - \cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10} \times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{\sinh^2(4\sqrt{3}\,\pi)}{4480\,\pi^2}$$

$$\frac{(\cosh(8\pi\sqrt{3}) - \cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10} \times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{-2 + 2\cosh^2(4\sqrt{3}\,\pi)}{8960\,\pi^2}$$

$$\frac{(\cosh(8\pi\sqrt{3}) - \cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10} \times 8\,(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)} = \frac{-1 - 3\cosh\left(\frac{8\pi}{\sqrt{3}}\right) + 4\cosh^3\left(\frac{8\pi}{\sqrt{3}}\right)}{8960\,\pi^2}$$

Thence, we obtain:

$$\left[\int \left(\prod_{k=0}^{m} d^2\chi_k \right) \prod_{i=1}^{n} \delta^2 \left(\eta_{i,a} - (W_i)_a^b \chi_b(\sigma_i) \right) - \prod_{k=0}^{m} \delta^2 \left(\sum_{i=1}^{n} (W_i)_a^b \sigma_i^k \eta_{i,b} \right) \right] =$$

$$= \left[\prod_{n=1}^{\infty} \left(1 \mid \binom{\alpha}{n}^3 \right) \left\{ 1 \mid 3 \binom{\alpha}{2n \mid \alpha}^2 \right\} = \frac{\Gamma(\frac{1}{2}\alpha)}{\Gamma(\frac{1}{2}(1+\alpha))} \cdot \frac{\cosh \pi\alpha\sqrt{3} - \cos \pi\alpha}{2^{\alpha+2}\pi\alpha\sqrt{\pi}} \cdot \right] =$$

$= 4.54690178\ldots * 10^{13}$

From which:

((ln[gamma (8/2) / gamma (1/2(1+8)) * ((cosh (8Pi*sqrt3) – cos (8Pi))) / ((2^10*8Pi*sqrtPi))]))^1/((728-12)/10^2)

where $728 = 9^3 - 1$ and $12 = \sqrt[3]{1728}$ (728 = Ramanujan Taxicab number)

Input:

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{\Gamma\left(\frac{8}{2}\right)}{\Gamma\left(\frac{1}{2}(1+8)\right)}\times\frac{\cosh(8\pi\sqrt{3})-\cos(8\pi)}{2^{10}\times 8\left(\pi\sqrt{\pi}\right)}\right)}$$

$\Gamma(x)$ is the gamma function

$\cosh(x)$ is the hyperbolic cosine function

$\log(x)$ is the natural logarithm

Exact result:

$$\log^{25/179}\left(\frac{\cosh(8\sqrt{3}\,\pi)-1}{8960\,\pi^2}\right)$$

Decimal approximation:

1.6186805141513562869256722000488142931255924567549927711 08...

1.61868051415…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\log^{25/179}\left(\frac{\sinh^2(4\sqrt{3}\,\pi)}{4480\,\pi^2}\right)$$

$$\left(-\log(8960)-2\log(\pi)+\log\left(\cosh\left(8\sqrt{3}\,\pi\right)-1\right)\right)^{25/179}$$

$$\log^{25/179}\left(\frac{\frac{1}{2}\left(e^{-8\sqrt{3}\,\pi}+e^{8\sqrt{3}\,\pi}\right)-1}{8960\,\pi^2}\right)$$

All values of log^(25/179)((-1 + cosh(8 sqrt(3) π))/(8960 π^2)):

$$e^0\log^{25/179}\left(\frac{\cosh(8\sqrt{3}\,\pi)-1}{8960\,\pi^2}\right)\approx 1.61868 \text{ (real root)}$$

$$e^{(50\,i\,\pi)/179}\log^{25/179}\left(\frac{\cosh(8\sqrt{3}\ \pi)-1}{8960\,\pi^2}\right)\approx 1.0344+1.2450\,i$$

$$e^{(100\,i\,\pi)/179}\log^{25/179}\left(\frac{\cosh(8\sqrt{3}\ \pi)-1}{8960\,\pi^2}\right)\approx -0.2966+1.59127\,i$$

$$e^{(150\,i\,\pi)/179}\log^{25/179}\left(\frac{\cosh(8\sqrt{3}\ \pi)-1}{8960\,\pi^2}\right)\approx -1.4135+0.7888\,i$$

$$e^{-(158\,i\,\pi)/179}\log^{25/179}\left(\frac{\cosh(8\sqrt{3}\ \pi)-1}{8960\,\pi^2}\right)\approx -1.5100-0.5832\,i$$

Alternative representations:

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\,\pi\,\sqrt{3})-\cos(8\,\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\,\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)}=\sqrt[\frac{716}{10^2}]{\log_e\left(\frac{(\cosh(8\,\pi\,\sqrt{3})-\cos(8\,\pi))\,\Gamma(4)}{\Gamma\left(\frac{9}{2}\right)(8\,\pi\,2^{10}\,\sqrt{\pi})}\right)}$$

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\,\pi\,\sqrt{3})-\cos(8\,\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\,\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)}=$$
$$\sqrt[\frac{716}{10^2}]{\log(a)\log_a\left(\frac{(\cosh(8\,\pi\,\sqrt{3})-\cos(8\,\pi))\,\Gamma(4)}{\Gamma\left(\frac{9}{2}\right)(8\,\pi\,2^{10}\,\sqrt{\pi})}\right)}$$

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\,\pi\,\sqrt{3})-\cos(8\,\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8\,(\pi\,\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)}=$$
$$\sqrt[\frac{716}{10^2}]{\log\left(\frac{\left(\cos(-8\,i\,\pi\,\sqrt{3})+\frac{1}{2}\left(-e^{-8\,i\,\pi}-e^{8\,i\,\pi}\right)\right)e^{-\log(2)+\log(12)}}{e^{-\log G(9/2)+\log G(11/2)}\,(8\,\pi\,2^{10}\,\sqrt{\pi})}\right)}$$

Series representations:

$$\frac{728-12}{10^2}\sqrt{\log\left(\frac{\left(\cosh\left(8\pi\sqrt{3}\right)-\cos(8\pi)\right)\Gamma\left(\frac{8}{2}\right)}{\left(2^{10}\times 8\left(\pi\sqrt{\pi}\right)\right)\Gamma\left(\frac{1+8}{2}\right)}\right)}=$$

$$\left(\log\left(-1+\frac{-1+\cosh\left(8\sqrt{3}\,\pi\right)}{8960\,\pi^2}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{-1+\cosh\left(8\sqrt{3}\,\pi\right)}{8960\,\pi^2}}\right)^k}{k}\right)^{25/179}$$

$$\frac{728-12}{10^2}\sqrt{\log\left(\frac{\left(\cosh\left(8\pi\sqrt{3}\right)-\cos(8\pi)\right)\Gamma\left(\frac{8}{2}\right)}{\left(2^{10}\times 8\left(\pi\sqrt{\pi}\right)\right)\Gamma\left(\frac{1+8}{2}\right)}\right)}=$$

$$\left(\log\left(-1+\frac{\sinh^2\left(4\sqrt{3}\,\pi\right)}{4480\,\pi^2}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{\sinh^2\left(4\sqrt{3}\,\pi\right)}{4480\,\pi^2}}\right)^k}{k}\right)^{25/179}$$

$$\frac{728-12}{10^2}\sqrt{\log\left(\frac{\left(\cosh\left(8\pi\sqrt{3}\right)-\cos(8\pi)\right)\Gamma\left(\frac{8}{2}\right)}{\left(2^{10}\times 8\left(\pi\sqrt{\pi}\right)\right)\Gamma\left(\frac{1+8}{2}\right)}\right)}=$$

$$\left(\log\left(-1+\frac{-1+\cosh\left(8\sqrt{3}\,\pi\right)}{8960\,\pi^2}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{\sinh^2\left(4\sqrt{3}\,\pi\right)}{4480\,\pi^2}}\right)^k}{k}\right)^{25/179}$$

Integral representations:

$$\frac{728-12}{10^2}\sqrt{\log\left(\frac{\left(\cosh\left(8\pi\sqrt{3}\right)-\cos(8\pi)\right)\Gamma\left(\frac{8}{2}\right)}{\left(2^{10}\times 8\left(\pi\sqrt{\pi}\right)\right)\Gamma\left(\frac{1+8}{2}\right)}\right)}=\left(\int_1^{\frac{\sinh^2\left(4\sqrt{3}\,\pi\right)}{4480\,\pi^2}}\frac{1}{t}\,dt\right)^{25/179}$$

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)} = \log^{25/179}\left(\frac{\sqrt{3}}{1120\,\pi}\int_0^1 \sinh\left(8\sqrt{3}\,\pi t\right)dt\right)$$

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)} = \log^{25/179}\left(\frac{-1+\int_{\frac{i\pi}{2}}^{8\sqrt{3}\,\pi}\sinh(t)\,dt}{8960\,\pi^2}\right)$$

Multiple-argument formulas:

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)} = \log^{25/179}\left(\frac{\sinh^2(4\sqrt{3}\,\pi)}{4480\,\pi^2}\right)$$

$$\sqrt[\frac{728-12}{10^2}]{\log\left(\frac{(\cosh(8\pi\sqrt{3})-\cos(8\pi))\,\Gamma\left(\frac{8}{2}\right)}{(2^{10}\times 8(\pi\sqrt{\pi}))\,\Gamma\left(\frac{1+8}{2}\right)}\right)} = \left(-\log(8960)+\log\left(\frac{2\sinh^2(4\sqrt{3}\,\pi)}{\pi^2}\right)\right)^{25/179}$$

From:

STRING THEORY
JOSEPH POLCHINSKI - Institute for Theoretical Physics, University of California at Santa Barbara - **VOLUME I** - An Introduction to the Bosonic String - © Cambridge University Press 2001, 2005

The slope in this inequality is known as the *Regge slope*. Meson resonances obey a linear relation of this form, with $\alpha' \sim (1\ \text{GeV})^{-2}$. For this and other reasons, string theory was originally proposed in 1970 as a theory of the strong interaction. Within a few years, however, the $SU(3)$ gauge theory QCD was discovered to be the correct theory, with string-like behavior only in certain limits.

$$\kappa = (8\pi G_N)^{1/2} = \frac{(8\pi)^{1/2}}{M_P} = (2.43 \times 10^{18}\ \text{GeV})^{-1}\ .$$

From:

TASI LECTURES ON D-BRANES
JOSEPH POLCHINSKI - arXiv:hep-th/9611050v2 23 Apr 1997

D-Brane Tension

$$T_p = \frac{\sqrt{\pi}}{16\kappa_0}(4\pi^2\alpha')^{(11-p)/2}\ . \qquad (54)$$

For p = 5, we obtain:

((sqrt(Pi)* (4Pi^2*1/(0.988)^2 GeV)^3)) / (((16*1/(2.43e+18) GeV)))

Input interpretation:

$$\frac{\sqrt{\pi}\left(4\times\pi^2\times\frac{1}{0.988^2}\ \text{GeV (gigaelectronvolts)}\right)^3}{16\times\frac{1}{2.43\times10^{18}}\ \text{GeV (gigaelectronvolts)}}$$

Result:

$1.7807\times10^{22}\ \text{GeV}^2$ (gigaelectronvolts squared)

$1.7807*10^{22}\ \text{GeV}^2$

Converting, we obtain:

1.7807×10^22 gigaelectronvolts squared = kg

Input interpretation:

convert $1.7807\times10^{22}\ \text{GeV}^2$ (gigaelectronvolts squared) to kilograms-force

Result:

2.346×10^{26} kgf (kilograms-force)

(using [energy] = (hc) [length]$^{-1}$)

Additional conversions:

2.301×10^{27} N (newtons)
2.301×10^27 newtons

Input interpretation:
2.301×10^{27} N (newtons)

Corresponding quantity:
Mass m of a body from $w = mg$:
2.346×10^{26} kg (kilograms)
5.173×10^{26} lb (pounds)

2.346×10^26 kilograms = joule

Input interpretation:
convert 2.346×10^{26} kg (kilograms) to joules

Result:
2.108×10^{43} J (joules)
(using $E = mc^2$)

Additional conversions:
1.316×10^{62} eV (electronvolts)

Comparisons as energy:
≈ (0.00012 ≈ 1/8480) × relativistic mass-energy equivalent of a solar mass ($1 M_\odot c^2$)
≈ (0.00021 ≈ 1/4740) × energy released from a gamma ray burst ($\approx 1 \times 10^{47}$ J)
≈ (0.21 ≈ 1/5) × energy released from a supernova ($\approx 1 \times 10^{44}$ J)

Input interpretation:
convert 1.316×10^{62} eV (electronvolts) to gigaelectronvolts

Result:
1.316×10^{53} GeV (gigaelectronvolts)

Dividing by c^2:

(1.316×10^53 gigaelectronvolts) / (9e+16)

Input interpretation:

$$\frac{1.316\times10^{53}\ \text{GeV (gigaelectronvolts)}}{9\times10^{16}}$$

Result:

$1.462222222222222354094062278679806906432317885012741 69\times10^{36}$ GeV (gigaelectronvolts)

1.4622222…*10^{36} GeV

Corresponding quantity:

Relativistic mass m from $E = mc^2$:

2.607×10^{9} kg (kilograms)

$2.607*10^{9}$ kg that is the result of:

D-Brane Tension

$$T_p = \frac{\sqrt{\pi}}{16\kappa_0}(4\pi^2\alpha')^{(11-p)/2} .$$

Performing the 43th root of the above value:

(2.607×10^9)^1/43

Input interpretation:

$\sqrt[43]{2.607\times10^{9}}$

Result:

1.65569380562...

1.65569380562…..

We note that, the result 1.6556938... is practically equal to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Indeed:

From:

TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY - Volume 349, Number 6, June 1997, Pages 2125-2173 S 0002-9947(97)01738-8 - **RAMANUJAN'S CLASS INVARIANTS, KRONECKER'S LIMIT FORMULA, AND MODULAR EQUATIONS -** *BRUCE C. BERNDT, HENG HUAT CHAN, AND LIANG-CHENG ZHANG*

We have the following class invariant:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4} \times \left((130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}} = \left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. □

Performing the 14th root of the result, i.e. the right-hand side of the last expression, we obtain:

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+\sqrt{505}}{8}}\right)^3} = 1.65578\ldots \Rightarrow 1.6556938\ldots$$

Thence, we observe that performing the following computation

[(((((((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8))))))^3))))^(1/14)]^43

We obtain:

Input:

$$\sqrt[14]{\left(\sqrt{\frac{1}{8}\left(113+5\sqrt{505}\right)}+\sqrt{\frac{1}{8}\left(105+5\sqrt{505}\right)}\right)^{3}}^{43}$$

Exact result:

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(105+5\sqrt{505}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(113+5\sqrt{505}\right)}\right)^{129/14}$$

Decimal approximation:

$2.613150966986558294392121412484628035075933801044378 78\ldots \times 10^{9}$

$2.6131509669\ldots*10^{9}$

result practically equal to the value obtained previously concerning the D-Brane tension for p = 5, that is equal to 2.607×10^{9} kg

Alternate forms:

$$\frac{\left(5\sqrt{5}+\sqrt{101}+\sqrt{105-40\,i}+\sqrt{105+40\,i}\right)^{129/14}}{262\,144\times 2^{3/7}\left(\sqrt{5\left(21+\sqrt{505}\right)}+\sqrt{113+5\sqrt{505}}\right)^{129/14}}{8192\times 2^{23/28}}$$

So, as you can see, by making the forty-third power of the value obtained from the 14th root of G_{505} class invariant, we obtain a value practically equal to the tension of a D-brane that has p = 5. It is evident, then, the connection between a expression of Ramanujan's mathematics and a physical parameter, such as the tension of a D-brane.

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY - Volume 349, Number 6, June 1997, Pages 2125-2173 S 0002-9947(97)01738-8 - **RAMANUJAN'S CLASS INVARIANTS, KRONECKER'S LIMIT FORMULA, AND MODULAR EQUATIONS -** *BRUCE C. BERNDT, HENG HUAT CHAN, AND LIANG-CHENG ZHANG*