On the new possible mathematical connections between some Ramanujan partition formulas and various parameters concerning the number of transverse light-cone directions in the bosonic string, ϕ and $\zeta(2)$.

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Abstract

In this paper we describe and analyze new possible mathematical connections between some Ramanujan partition formulas and various parameters concerning the number of transverse light-cone directions in the bosonic string, ϕ and $\zeta(2)$.

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4. Ramanujan -Hardy Asymptotic Formula:

Ramanujan's one of the major works was in the partition of numbers. In a joint paper with Hardy, Ramanujan gave an asymptotic formulas for $p(n)$. In fact, a careful analysis of the generating function for $p(n)$ leads to the Hardy – Ramanujan asymptotic formula given by,

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}} \quad , n \to \infty$$

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https://www.slideshare.net/KrishnaPriyaKB/contributions-of-sreenivasa-ramanujan

Partition Function

- Generating function by Indian genius **Srinivasa Ramanujan.**

$$p(n) \approx \frac{e^{\pi\sqrt{\frac{2n}{3}}}}{4n\sqrt{3}}$$



https://www.slideshare.net/klawdet/number-theory-59969950

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

For more information on the data entered for the development of the various equations, see the "Observations" section.

From:

Chapter 16
String thermodynamics and black holes -
http://fma.if.usp.br/~amsilva/Livros/Zwiebach/chapter16.pdf

We know that *p(N)* denotes the number of partitions of the integer N.

$$\ln p(N) \simeq 2\pi\sqrt{\frac{N}{6}}\,. \tag{16.2.29}$$

This was our goal, an estimate of $\ln p(N)$ for large N. Indeed, we must require large N since

$$N = \frac{E}{\hbar\omega_0} = \frac{\pi^2}{6}\left(\frac{kT}{\hbar\omega_0}\right)^2 >> 1\,, \tag{16.2.30}$$

because of our high temperature assumption (16.2.17).

The result (16.2.29) is only the leading term of the celebrated Hardy-Ramanujan asymptotic expansion of $p(N)$:

$$\boxed{p(N) \simeq \frac{1}{4N\sqrt{3}}\exp\left(2\pi\sqrt{\frac{N}{6}}\right).} \tag{16.2.31}$$

N	$p(N)$	$p(N)_{\text{est}}$	$p(N)/p_{\text{est}}(N)$
5	7	8.94	0.7829
10	42	48.10	0.8731
100	190569292	199281893.25	0.9563
1000	2.406×10^{31}	2.440×10^{31}	0.9860
10000	3.617×10^{106}	3.633×10^{106}	0.9956

Table 16.2: Comparing the exact values of $p(N)$ with the estimate $p(N)_{\text{est}}$ provided by the Hardy-Ramanujan formula.

Let us call $p_d(N)$ the number of partitions of N when we have a d-fold degeneracy. This means, for example, that the partition $\{3,2,1\}$ of 6 now gives rise to many partitions written like $\{3_{p_1}, 2_{p_2}, 1_{p_3}\}$, where we include subscripts p_i that can take all possible values from one to d. A partition with different subscripts is considered a different partition. We now see that, for a given energy, with associated number N, the number of states is $p_d(N)$. Therefore $S_d = k \ln p_d(N)$, and comparing with (16.2.39) we conclude that for large N

$$\ln p_d(N) \simeq 2\pi\sqrt{\frac{Nd}{6}}\,. \tag{16.2.40}$$

The more accurate version of this result can be shown to be

$$p_d(N) \simeq \frac{1}{\sqrt{2}}\left(\frac{d}{24}\right)^{(d+1)/4} N^{-(d+3)/4} \exp\left(2\pi\sqrt{\frac{Nd}{6}}\right). \tag{16.2.41}$$

You can see that for $d = 1$ this reduces to $p(N)$, as given in (16.2.31). For $d = 24$, the number of transverse light-cone directions in the bosonic string, the expression simplifies a little:

$$\boxed{p_{24}(N) \simeq \frac{1}{\sqrt{2}} N^{-27/4} \exp\left(4\pi\sqrt{N}\right).} \tag{16.2.42}$$

From the eq.(16.2.42), for n = 100, we obtain:

1/(sqrt2) * 100^(-27/4) * exp(4Pi*sqrt100)

Input:

$$\frac{1}{\sqrt{2}} \times 100^{-27/4} \exp\left(4\,\pi\,\sqrt{100}\right)$$

Exact result:

$$\frac{e^{40\,\pi}}{20\,000\,000\,000\,000\,\sqrt{5}}$$

Decimal approximation:

$8.405027831268191473727580191243496410225572639313728 9\ldots \times 10^{40}$

$8.405027831268 * 10^{40}$

Property:

$\frac{e^{40\pi}}{20\,000\,000\,000\,000\sqrt{5}}$ is a transcendental number

Series representations:

$$\frac{100^{-27/4}\exp(4\pi\sqrt{100})}{\sqrt{2}} = \frac{\exp\left(4\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(100-z_0)^k z_0^{-k}}{k!}\right)}{10\,000\,000\,000\,000\sqrt{10}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{100^{-27/4}\exp(4\pi\sqrt{100})}{\sqrt{2}} = \frac{\exp\left(4\pi\exp\left(i\pi\left\lfloor\frac{\arg(100-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(100-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{10\,000\,000\,000\,000\sqrt{10}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{100^{-27/4}\exp(4\pi\sqrt{100})}{\sqrt{2}} = \left(\exp\left(4\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(100-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(100-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(100-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\right)\Big/\left(10\,000\,000\,000\,000\sqrt{10}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

We observe that:

((1/(sqrt2) * 100^(-27/4) * exp(4Pi*sqrt100))) * 2.2673249372363586× 10^-33

Input interpretation:

$$\left(\frac{1}{\sqrt{2}}\times 100^{-27/4}\exp\left(4\pi\sqrt{100}\right)\right)\times 2.2673249372363586\times 10^{-33}$$

Result:

$1.9056929199999999... \times 10^8$

190569291.9999 ≈ 190569292 that is equal to the value of p(100)

For n = 200:

((1/(sqrt2) * 200^(-27/4) * exp(4Pi*sqrt200)))

Input:

$$\frac{1}{\sqrt{2}} \times 200^{-27/4} \exp\left(4\pi\sqrt{200}\right)$$

Exact result:

$$\frac{e^{40\sqrt{2}\,\pi}}{1\,280\,000\,000\,000\,000 \times 2^{3/4}\sqrt{5}}$$

Decimal approximation:

$3.150027047216587574212672161433182439214690646257201 8... \times 10^{61}$

$3.15002704721658757 * 10^{61}$

Property:

$$\frac{e^{40\sqrt{2}\,\pi}}{1\,280\,000\,000\,000\,000 \times 2^{3/4}\sqrt{5}}$$ is a transcendental number

Series representations:

$$\frac{200^{-27/4}\exp\left(4\pi\sqrt{200}\right)}{\sqrt{2}} = \frac{\exp\left(4\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (200-z_0)^k z_0^{-k}}{k!}\right)}{1\,280\,000\,000\,000\,000\sqrt[4]{2}\sqrt{5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \le 0))$

$$\frac{200^{-27/4}\exp(4\pi\sqrt{200})}{\sqrt{2}} = \frac{\exp\left(4\pi\exp\left(i\pi\left\lfloor\frac{\arg(200-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(200-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{1\,280\,000\,000\,000\,000\sqrt[4]{2}\ \sqrt{5}\ \exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{200^{-27/4}\exp(4\pi\sqrt{200})}{\sqrt{2}} = \left(\exp\left(4\pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(200-z_0)/(2\pi)]} z_0^{1/2\,(1+[\arg(200-z_0)/(2\pi)])}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(200-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\,[\arg(2-z_0)/(2\pi)]} z_0^{-1/2-1/2\,[\arg(2-z_0)/(2\pi)]}\right)\Big/\left(1\,280\,000\,000\,000\,000\sqrt[4]{2}\ \sqrt{5}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

We note that:

((1/(sqrt2) * 200^(-27/4) * exp(4Pi*sqrt200))) * 1.26125870344465872× 10^-49

Input interpretation:

$$\left(\frac{1}{\sqrt{2}}\times 200^{-27/4}\exp\left(4\pi\sqrt{200}\right)\right)\times 1.26125870344465872\times 10^{-49}$$

Result:

$3.97299902938800000\ldots \times 10^{12}$

3972999029388 that is equal to the value of p(200)

For n = 10000:

1/(sqrt2) * 10000^(-27/4) * exp(4Pi*sqrt10000)

Input:

$$\frac{1}{\sqrt{2}}\times 10\,000^{-27/4}\exp\left(4\pi\sqrt{10\,000}\right)$$

Exact result:

$$\frac{e^{400\,\pi}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\sqrt{2}}$$

Decimal approximation:

$3.98131498864032342891379719055599433649767822574041 40\ldots \times 10^{518}$

$3.98131498864*10^{518}$

Property:

$\frac{e^{400\,\pi}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\sqrt{2}}$ is a transcendental number

Series representations:

$$\frac{10\,000^{-27/4}\,\exp(4\,\pi\,\sqrt{10\,000})}{\sqrt{2}} = \frac{\exp\left(4\,\pi\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(10\,000-z_0)^k\,z_0^{-k}}{k!}\right)}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \le 0))$

$$\frac{10\,000^{-27/4}\,\exp(4\,\pi\,\sqrt{10\,000})}{\sqrt{2}} = \exp\left(4\,\pi\,\exp\left(i\,\pi\left\lfloor\frac{\arg(10\,000-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(10\,000-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/\left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R}\ \text{and}\ x < 0)$

$$\frac{10\,000^{-27/4}\,\exp\left(4\pi\sqrt{10\,000}\right)}{\sqrt{2}} =$$
$$\left(\exp\left(4\pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(10\,000-z_0)/(2\pi)]}\,z_0^{1/2\,(1+[\arg(10\,000-z_0)/(2\pi)])}\right.\right.$$
$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(10\,000-z_0)^k\,z_0^{-k}}{k!}\right)\right.$$
$$\left.\left(\frac{1}{z_0}\right)^{-1/2\,[\arg(2-z_0)/(2\pi)]}\,z_0^{-1/2-1/2\,[\arg(2-z_0)/(2\pi)]}\right)\Big/$$
$$\left(1\,000\,000\,000\,000\,000\,000\,000\,000\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}\right)$$

We note that

(((1/(sqrt2) * 10000^(-27/4) * exp(4Pi*sqrt10000))))*9.08493804263206×10^-413

Input interpretation:

$$\left(\frac{1}{\sqrt{2}}\times 10\,000^{-27/4}\,\exp\left(4\pi\sqrt{10\,000}\right)\right)\times\frac{9.08493804263206}{10^{413}}$$

Result:

$3.61699999999997\ldots\times 10^{106}$

$3.617*10^{106}$ that is equal to the value of p(10000)

Series representations:

$$\frac{9.084938042632060000\,(10\,000^{-27/4}\,\exp(4\pi\sqrt{10\,000}))}{10^{413}\sqrt{2}} =$$
$$\frac{9.084938042632060000\times 10^{-440}\,\exp\left(4\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(10000-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{9.084938042632060000\,(10\,000^{-27/4}\exp(4\pi\sqrt{10\,000}))}{10^{413}\sqrt{2}} =$$

$$\left(9.084938042632060000\times 10^{-440}\right.$$

$$\left.\exp\left(4\pi\exp\left(i\pi\left\lfloor\frac{\arg(10\,000-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(10\,000-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$

$$\left(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{9.084938042632060000\,(10\,000^{-27/4}\exp(4\pi\sqrt{10\,000}))}{10^{413}\sqrt{2}} =$$

$$\left(9.084938042632060000\times 10^{-440}\exp\left(4\pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(10\,000-z_0)/(2\pi)]}\right.\right.$$

$$\left.z_0^{1/2\,(1+[\arg(10000-z_0)/(2\pi)])}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(10\,000-z_0)^k\,z_0^{-k}}{k!}\right)$$

$$\left.\left(\frac{1}{z_0}\right)^{-1/2\,[\arg(2-z_0)/(2\pi)]}z_0^{-1/2-1/2\,[\arg(2-z_0)/(2\pi)]}\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}\right)$$

We have also the following calculation, from which we obtain:

sin((7 π)/(29+11))* [1/((2.2673249372363586× 10^-33) / (1.26125870344465872× 10^-49))]^2

Input interpretation:

$$\sin\left(\frac{7\pi}{29+11}\right)\left(\frac{1}{\frac{2.2673249372363586\times 10^{-33}}{1.26125870344465872\times 10^{-49}}}\right)^2$$

Result:

$1.6168358298664251\ldots\times 10^{-33}$

1.61683582…*10^{-33} result practically equal to the value of Planck length in cm

1.6162552×10^{-33}

From:

The Theory of Partitions - *George E. Andrews* - Cambridge University Press, 28 lug 1998

We have that:

of all rational points, we restrict our attention to the discrete set of those rational points $\exp(2\pi ih/k)$ with $0 < k \leqslant N$, a fixed positive integer. Thus

We assume $0 < \varepsilon < N^{-2}$ and we shall mostly be interested in what happens as $\varepsilon \to 0$. For brevity, we rewrite (5.2.21) as

The formula that we shall prove is in the final form obtained by Rademacher:

THEOREM 5.1

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n) k^{\frac{1}{2}} \left[\frac{d}{dx} \frac{\sinh((\pi/k)(\frac{2}{3}(x - 1/24))^{\frac{1}{2}})}{(x - 1/24)^{\frac{1}{2}}} \right]_{x=n}, \quad (5.1.1)$$

where

$$A_k(n) = \sum_{\substack{h \bmod k \\ (h,k)=1}} \omega_{h,k} e^{-2\pi inh/k}$$

4*. Apart from the modular functions, there exists a class of functions called mock-theta functions (originally studied by S. Ramanujan and G. N. Watson) for which reasonable asymptotic series for the coefficients may be derived. For example, let

$$f(q) = 1 + \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1+q)^2(1+q^2)^2\cdots(1+q^n)^2}$$

$$= \sum_{n=0}^{\infty} a_n q^n.$$

Then it can be shown that

$$a_n = \tfrac{1}{2} \sum_{k=1}^{\sqrt{n}} (-1)^{[(k+1)/2]} A_{2k}[n - \tfrac{1}{4}(1 + (-1)^k)k][k(n - 1/24)]^{-\frac{1}{4}}$$

$$\times \exp\left[\frac{\pi}{k\sqrt{6}}(n - 1/24)^{\frac{1}{2}}\right] + O(n^{\varepsilon})$$

where $A_k(n)$ is the same as the exponential sum appearing after (5.1.1).

Dragonette has pointed out the possibility that the error inyolved in the expression for a_n may generally be less than $\frac{1}{2}$ in absolute value. In particular, the series at $n = 100$ is -18520.206 while $a_{100} = -18520$, at $n = 200$ the series yields -2660007.847 while $a_{200} = -2660008$.

For n = 100 and ε = 0.015, from

$$a_n = \tfrac{1}{2} \sum_{k=1}^{\sqrt{n}} (-1)^{[(k+1)/2]} A_{2k}[n - \tfrac{1}{4}(1 + (-1)^k)k][k(n - 1/24)]^{-\frac{1}{4}}$$

$$\times \exp\left[\frac{\pi}{k\sqrt{6}}(n - 1/24)^{\frac{1}{2}}\right] + O(n^{\varepsilon})$$

we obtain:

(((((1/2 sum ((((-1)^((k+1)/2)*(0.009994722786) [100-1/4(1+(-1)^k)k][k(100-1/24)]^(-1/2) * exp((Pi/(ksqrt6))(100-1/24)^(1/2))))), k = 1..10))))) + 100^0.015

where $A_{2k} = 0.009994722786$

Input interpretation:

$$\frac{1}{2}\sum_{k=1}^{10}(-1)^{(k+1)/2}\times 0.009994722786$$
$$\left(\left(100-\frac{1}{4}\left(1+(-1)^k\right)k\right)\left(k\left(100-\frac{1}{24}\right)\right)^{-1/2}\exp\left(\frac{\pi}{k\sqrt{6}}\sqrt{100-\frac{1}{24}}\right)\right)+100^{0.015}$$

Result:

$-18520.2 - 20.834\,i$

$-18520.2... -$
$20.834...\,i$

Polar coordinates:

$r = 18520.2$ (radius), $\theta = -179.936°$ (angle)

-18520.2

For n = 200 and ε = 0.015, we obtain

(((((1/2 sum ((((-1)^((k+1)/2)*(0.0049997) [200-1/4(1+(-1)^k)k][k(200-1/24)]^(-1/2) * exp((Pi/(ksqrt6))(200-1/24)^(1/2))))), k = 1..10))))) + 200^0.015

where $A_{2k} = 0.0049997$

Input interpretation:

$$\frac{1}{2}\sum_{k=1}^{10}(-1)^{(k+1)/2}\times 0.0049997$$
$$\left(\left(200-\frac{1}{4}\left(1+(-1)^k\right)k\right)\left(k\left(200-\frac{1}{24}\right)\right)^{-1/2}\exp\left(\frac{\pi}{k\sqrt{6}}\sqrt{200-\frac{1}{24}}\right)\right)+200^{0.015}$$

Result:

$-2.66001\times10^{6} - 214.379\,i$

$-2.66001...\times10^{6} - 214.379...\,i$

Polar coordinates:

$r = 2.66001\times10^{6}$ (radius), $\theta = -179.995°$ (angle)

-2660010

For n = 10000 and ε = 0.015, we obtain:

(((((1/2 sum ((((-1)^((k+1)/2)*(0.0001 - 9.98352×10^-33 i) [10000-1/4(1+(-1)^k)k][k(10000-1/24)]^(-1/2) * exp((Pi/(ksqrt6))(10000-1/24)^(1/2))))), k = 1..10))))) + 10000^0.015

where A_{2k} = 0.0001 - 9.98352×10^-33 i

Input interpretation:

$$\frac{1}{2}\sum_{k=1}^{10}(-1)^{(k+1)/2}\times(0.0001 - 9.98352\times10^{-33}\,i)\left(\left(10\,000 - \frac{1}{4}\left(1+(-1)^{k}\right)k\right)\left(k\left(10\,000-\frac{1}{24}\right)\right)^{-1/2}\exp\left(\frac{\pi}{k\sqrt{6}}\sqrt{10\,000-\frac{1}{24}}\right)\right) + 10\,000^{0.015}$$

i is the imaginary unit

Result:

$-2.50776\times10^{53} - 1.14913\times10^{19}\,i$

Alternate form:

-2.50776×10^{53}

-2.50776*10^53

The Ramanujan's formula (see OEIS-A000025) is:

-(-1)^n * exp(Pi*sqrt(n/6)) / (2*sqrt(n))

For n = 100:

-(-1)^100 * exp(Pi*sqrt(100/6)) / (2*sqrt(100))

Input:

$$-(-1)^{100} \times \frac{\exp\left(\pi \sqrt{\frac{100}{6}}\right)}{2\sqrt{100}}$$

Exact result:

$$-\frac{1}{20} e^{5\sqrt{2/3}\,\pi}$$

Decimal approximation:

−18578.606843879997192741450493640978804180533721649473213 4...

-18578.60684387…

Property:

$-\frac{1}{20} e^{5\sqrt{2/3}\,\pi}$ is a transcendental number

Series representations:

$$-\frac{(-1)^{100} \exp\left(\pi \sqrt{\frac{100}{6}}\right)}{2\sqrt{100}} = -\frac{\exp\left(\pi \sqrt{\frac{47}{3}} \sum_{k=0}^{\infty} \left(\frac{47}{3}\right)^{-k} \binom{\frac{1}{2}}{k}\right)}{2\sqrt{99} \sum_{k=0}^{\infty} 99^{-k} \binom{\frac{1}{2}}{k}}$$

$$-\frac{(-1)^{100} \exp\left(\pi \sqrt{\frac{100}{6}}\right)}{2\sqrt{100}} = -\frac{\exp\left(\pi \sqrt{\frac{47}{3}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{47}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}{2\sqrt{99} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{99}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

$$-\frac{(-1)^{100} \exp\left(\pi \sqrt{\frac{100}{6}}\right)}{2\sqrt{100}} = -\frac{\exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{50}{3} - z_0\right)^k z_0^{-k}}{k!}\right)}{2\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (100 - z_0)^k z_0^{-k}}{k!}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

And for n = 200:

-(-1)^200 * exp(Pi*sqrt(200/6)) / (2*sqrt(200))

Input:

$$-(-1)^{200} \times \frac{\exp\left(\pi \sqrt{\frac{200}{6}}\right)}{2\sqrt{200}}$$

Exact result:

$$-\frac{e^{(10\pi)/\sqrt{3}}}{20\sqrt{2}}$$

Decimal approximation:

$-2.664928480157840091163823914130856951077674916362603 9\ldots \times 10^6$

-2664928.4801578…..

Property:

$-\frac{e^{(10\pi)/\sqrt{3}}}{20\sqrt{2}}$ is a transcendental number

Series representations:

$$-\frac{(-1)^{200}\exp\left(\pi\sqrt{\frac{200}{6}}\right)}{2\sqrt{200}} = -\frac{\exp\left(\pi\sqrt{\frac{97}{3}}\sum_{k=0}^{\infty}\left(\frac{97}{3}\right)^{-k}\binom{\frac{1}{2}}{k}\right)}{2\sqrt{199}\sum_{k=0}^{\infty}199^{-k}\binom{\frac{1}{2}}{k}}$$

$$-\frac{(-1)^{200}\exp\left(\pi\sqrt{\frac{200}{6}}\right)}{2\sqrt{200}} = -\frac{\exp\left(\pi\sqrt{\frac{97}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{97}\right)^{k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{2\sqrt{199}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{199}\right)^{k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$-\frac{(-1)^{200}\exp\left(\pi\sqrt{\frac{200}{6}}\right)}{2\sqrt{200}} = -\frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{100}{3}-z_0\right)^k z_0^{-k}}{k!}\right)}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(200-z_0)^k z_0^{-k}}{k!}}$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \leq 0))$

For n = 10000:

-(-1)^10000 * exp(Pi*sqrt(10000/6)) / (2*sqrt(10000))

Input:

$$-(-1)^{10000}\times\frac{\exp\left(\pi\sqrt{\frac{10000}{6}}\right)}{2\sqrt{10\,000}}$$

Exact result:

$$-\frac{1}{200}e^{50\sqrt{2/3}\,\pi}$$

Decimal approximation:

$-2.508426642528190415670693529697551569276936961922065\ldots\times 10^{53}$

-2.50842664…*10^{53}

Property:

$-\frac{1}{200}e^{50\sqrt{2/3}\,\pi}$ is a transcendental number

Series representations:

$$-\frac{(-1)^{10000}\exp\left(\pi\sqrt{\frac{10000}{6}}\right)}{2\sqrt{10\,000}} = -\frac{\exp\left(\pi\sqrt{\frac{4997}{3}}\sum_{k=0}^{\infty}\left(\frac{4997}{3}\right)^{-k}\binom{\frac{1}{2}}{k}\right)}{2\sqrt{9999}\sum_{k=0}^{\infty}9999^{-k}\binom{\frac{1}{2}}{k}}$$

$$-\frac{(-1)^{10000}\exp\left(\pi\sqrt{\frac{10000}{6}}\right)}{2\sqrt{10\,000}}=-\frac{\exp\left(\pi\sqrt{\frac{4997}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4997}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{2\sqrt{9999}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{9999}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$-\frac{(-1)^{10000}\exp\left(\pi\sqrt{\frac{10000}{6}}\right)}{2\sqrt{10\,000}}=-\frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5000}{3}-z_0\right)^k z_0^{-k}}{k!}\right)}{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(10000-z_0)^k z_0^{-k}}{k!}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

Results that, as we can see, are good approximations to the real values previously obtained

Now, we note that:

For n = 100, we have that:

$$\begin{bmatrix} -18520.2 \\ 8.4050278831268\times 10^{40} \\ 190569292 \end{bmatrix}$$

From the following calculation, we obtain:

1/2+ [-(8.405027831268e+40 / 190569292 / -18520.2)]^1/12 + 24

Input interpretation:

$$\frac{1}{2}+12\sqrt{-\left(-\frac{\frac{8.405027831268\times 10^{40}}{190\,569\,292}}{18\,520.2}\right)}+24$$

Result:

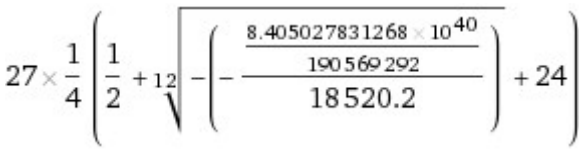
256.0990...

256.0990… ≈ 256

and:

27*1/4(((1/2+ [-(8.405027831268e+40 / 190569292 / -18520.2)]^1/12 + 24)))

Input interpretation:



Result:

1728.668...

1728.668… ≈ 1729

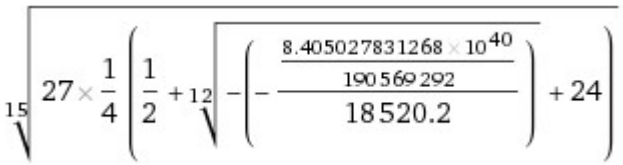
This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

((((27*1/4(((1/2+ [-(8.405027831268e+40 / 190569292 / -18520.2)]^1/12 + 24)))))))^1/15

Input interpretation:



Result:

1.6437942...

1.6437942…. ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

For n = 200, we have that:

$$\begin{bmatrix} -2660010 \\ 3.150027047216587 57 \times 10^{61} \\ 3972999029388 \end{bmatrix}$$

1/2+ [-(3.150027047216587 57e+61 /3972999029388 / -2660010)]^1/12 + 24*5

Input interpretation:

$$\frac{1}{2} + \sqrt[12]{-\left(-\frac{3.150027047216587 57 \times 10^{61}}{\frac{3972999029388}{2660010}}\right)} + 24 \times 5$$

Result:

3584.086563672024456...

3584.0865636… ≈ 3584 = 256*14

1/14 (((1/2+ [-(3.150027047216587 57e+61 /3972999029388 / -2660010)]^1/12 + 24*5)))

Input interpretation:

$$\frac{1}{14}\left(\frac{1}{2} + \sqrt[12]{-\left(-\frac{3.150027047216587 57 \times 10^{61}}{\frac{3972999029388}{2660010}}\right)} + 24 \times 5\right)$$

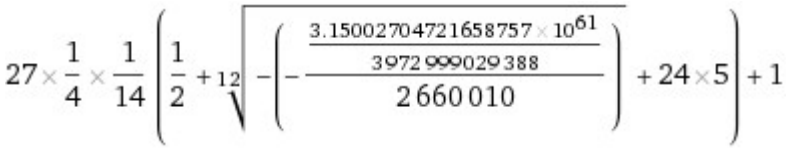
Result:

256.0061831194303183...

256.006183… ≈ 256

27*1/4*1/14 (((1/2+ [-(3.15002704721658757e+61 /3972999029388 / -2660010)]^1/12 + 24*5)))+1

Input interpretation:



Result:

1729.041736056154648...

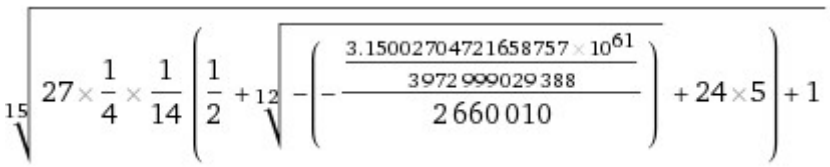
1729.041736…

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From which:

((((27*1/4*1/14 (((1/2+ [-(3.15002704721658757e+61 /3972999029388 / -2660010)]^1/12 + 24*5)))+1))))^1/15

Input interpretation:



Result:

1.6438178740385631213...

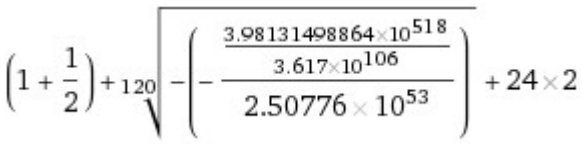
1.643817874…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

For n = 10000, we have that:

$$\begin{bmatrix} -2.50776 \times 10^{53} \\ 3.617 \times 10^{106} \\ 3.98131498864 \times 10^{518} \end{bmatrix}$$

(1+1/2)+ [-(3.98131498864e+518 / 3.617e+106 / -2.50776e+53)]^1/120 +24*2

Input interpretation:



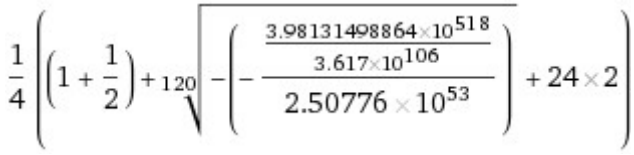
Result:

1023.786...

1023.786… ≈ 1024

1/4*((((1+1/2)+ [-(3.98131498864e+518 / 3.617e+106 / -2.50776e+53)]^1/120 +24*2)))

Input interpretation:



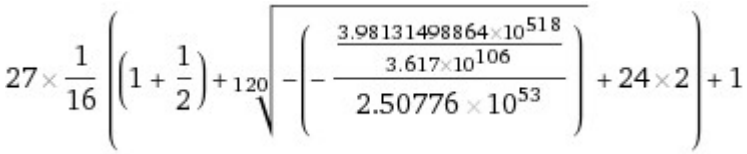
Result:

255.9466...

255.9466… ≈ 256

27*1/16*((((1+1/2)+ [-(3.98131498864e+518 / 3.617e+106 / -2.50776e+53)]^1/120 +24*2)))+1

Input interpretation:



Result:

1728.639...

1728.639… ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic

curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From which:

(((27*1/16*((((1+1/2)+ [-(3.98131498864e+518 / 3.617e+106 / -2.50776e+53)]^1/120 +24*2)))+1)))^1/15

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{16}\left(\left(1+\frac{1}{2}\right)+\sqrt[120]{-\left(-\frac{3.98131498864\times10^{518}}{3.617\times10^{106}}\right)}+24\times2\right)+1}$$

Result:

1.64379237...

$1.64379237\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Now, we have the following formula:

$$\frac{1}{i} L_k = \frac{1}{i} \int_{-\infty}^{(0+)} \omega^{\frac{1}{2}} \exp\left[\frac{\pi}{12k^2\omega} + 2\pi\left(n - \frac{1}{24}\right)\omega\right] d\omega$$

$$= \frac{1}{i} \int_{-\infty}^{(0+)} \omega^{\frac{1}{2}} \exp\left[2\pi\left(n - \frac{1}{24}\right)\omega\right] \sum_{s=0}^{\infty} \frac{(\pi/12k^2\omega)^s}{s!} d\omega$$

$$= \frac{1}{i} \sum_{s=0}^{\infty} \frac{(\pi/12k^2)^s}{s!} \int_{-\infty}^{(0+)} \omega^{\frac{1}{2}-s} \exp\left[2\pi\left(n - \frac{1}{24}\right)\omega\right] d\omega$$

$$= 2\pi \sum_{s=0}^{\infty} \frac{(\pi/12k^2)^s}{s!} \left[2\pi\left(n - \frac{1}{24}\right)\right]^{s-3/2} \frac{1}{2\pi i} \int_{-\infty}^{(0+)} e^z z^{-s+\frac{1}{2}} dz$$

$$= (2\pi)^{-\frac{1}{2}} \left(n - \frac{1}{24}\right)^{-3/2} \sum_{s=0}^{\infty} \frac{[\pi^2(n - 1/24)/6k^2]^s}{s!\Gamma(s - 1/2)}$$

where the final equation follows from Hankel's loop integral formula for the reciprocal of the gamma function:

$$\frac{1}{\Gamma(s - \frac{1}{2})} = \frac{1}{2\pi i} \int_{-\infty}^{(0+)} e^z z^{-s+\frac{1}{2}} dz.$$

From

$$2\pi \sum_{s=0}^{\infty} \frac{(\pi/12k^2)^s}{s!} \left[2\pi\left(n - \frac{1}{24}\right)\right]^{s-3/2} \frac{1}{2\pi i} \int_{-\infty}^{(0+)} e^z z^{-s+\frac{1}{2}} dz$$

$$= (2\pi)^{-\frac{1}{2}} \left(n - \frac{1}{24}\right)^{-3/2} \sum_{s=0}^{\infty} \frac{[\pi^2(n - 1/24)/6k^2]^s}{s!\Gamma(s - 1/2)}$$

For n = 72, we obtain:

(2Pi)^(-1/2) (72-1/24)^(-1.5) * sum (((((Pi^2(72-1/24)/(6*3^2)))^s /(((s! gamma (s-1/2))))))), s = 0..infinity

Input interpretation:

$$\frac{(2\,\pi)^{-1/2}\sum_{s=0}^{\infty}\frac{\left(\pi^2\times\frac{72-\frac{1}{24}}{6\times 3^2}\right)^s}{s!\,\Gamma\left(s-\frac{1}{2}\right)}}{\left(72-\frac{1}{24}\right)^{1.5}}$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

Result:

0.814196

0.814196

From which:

2*((((2Pi)^(-1/2) (72-1/24)^(-1.5) * sum (((((Pi^2(72-1/24)/(6*3^2)))^s /(((s! gamma (s-1/2))))))), s = 0..infinity)))

Input interpretation:

$$2\times\frac{(2\,\pi)^{-1/2}\sum_{s=0}^{\infty}\frac{\left(\pi^2\times\frac{72-\frac{1}{24}}{6\times 3^2}\right)^s}{s!\,\Gamma\left(s-\frac{1}{2}\right)}}{\left(72-\frac{1}{24}\right)^{1.5}}$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

Result:

1.62839

1.62839

For n = 65:

(2Pi)^(-1/2) (65-1/24)^(-1.5) * sum (((((Pi^2(65-1/24)/(6*3^2)))^s /(((s! gamma (s-1/2)))))))), s = 0..infinity

Input interpretation:

$$\frac{(2\pi)^{-1/2}\sum_{s=0}^{\infty}\frac{\left(\pi^2\times\frac{65-\frac{1}{24}}{6\times 3^2}\right)^s}{s!\,\Gamma\left(s-\frac{1}{2}\right)}}{\left(65-\frac{1}{24}\right)^{1.5}}$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

Result:

0.622847

0.622847

For n = $\left[\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\times 15^{3/4}}{\pi^{11/4}}\right)\right]$

we obtain the following value:

Input:

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\times 15^{3/4}}{\pi^{11/4}}\right)$$

Exact result:

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\;15^{3/4}}{\pi^{11/4}}\right)$$

Decimal approximation:

91.058378785040254657031655380810531017905868738037342340 79...

91.0583787…

Property:

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right) \text{is a transcendental number}$$

Alternate forms:

$$91+\frac{3\times 3^{3/4}}{2\times 2^{3/4}\sqrt[4]{5}\ \pi^{11/4}}$$

$$\frac{3\sqrt[4]{2}\ 15^{3/4}+1820\,\pi^{11/4}}{20\,\pi^{11/4}}$$

Alternative representations:

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{(180^\circ)^{11/4}}\right)$$

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{(-i\log(-1))^{11/4}}\right)$$

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\cos^{-1}(-1)^{11/4}}\right)$$

Series representations:

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=91+\frac{3\times 3^{3/4}}{128\sqrt[4]{10}\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{11/4}}$$

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=$$

$$91+\frac{3\times 3^{3/4}}{2\times 2^{3/4}\sqrt[4]{5}\left(\sum_{k=0}^{\infty}-\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{11/4}}$$

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=91+\frac{3\times 3^{3/4}}{2\times 2^{3/4}\sqrt[4]{5}\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{11/4}}$$

Integral representations:

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\ 15^{3/4}}{\pi^{11/4}}\right)=91+\frac{3\times 3^{3/4}}{128\sqrt[4]{10}\left(\int_0^1\sqrt{1-t^2}\ dt\right)^{11/4}}$$

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\;15^{3/4}}{\pi^{11/4}}\right)=91+\frac{3\times 3^{3/4}}{16\sqrt{2}\;\sqrt[4]{5}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^{11/4}}$$

$$\frac{1}{20}\left(1820+\frac{3\sqrt[4]{2}\;15^{3/4}}{\pi^{11/4}}\right)=91+\frac{3\times 3^{3/4}}{16\sqrt{2}\;\sqrt[4]{5}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{11/4}}$$

Thence, with n = 91.0583787 we obtain:

(2Pi)^(-1/2) (91.0583787-1/24)^(-1.5) * sum (((((Pi^2(91.0583787-1/24)/(6*3^2)))^s /(((s! gamma (s-1/2)))))), s = 0..infinity

Input interpretation:

$$\frac{(2\,\pi)^{-1/2}\sum_{s=0}^{\infty}\frac{\left(\pi^2\times\frac{91.0583787-\frac{1}{24}}{6\times 3^2}\right)^s}{s!\,\Gamma\left(s-\frac{1}{2}\right)}}{\left(91.0583787-\frac{1}{24}\right)^{1.5}}$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

Result:

1.61803

1.61803 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Now, we have that:

The formula that we shall prove is in the final form obtained by Rademacher:

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n) k^{\frac{1}{2}} \left[\frac{d}{dx} \frac{\sinh((\pi/k)(\frac{2}{3}(x - 1/24))^{\frac{1}{2}})}{(x - 1/24)^{\frac{1}{2}}} \right]_{x=n}, \quad (5.1.1)$$

where

$$A_k(n) = \sum_{\substack{h \bmod k \\ (h,k)=1}} \omega_{h,k} e^{-2\pi i n h/k}$$

For n = 100:

1/(Pi*sqrt2) sum (((((0.00038320*[e^((200Pi)/k)]^(1/24)))sinh(Pi/k) (2/3(100-1/24))^1/2)) / (((100-1/24)^1/2))), k = 1..100

Input interpretation:

$$\frac{1}{\pi\sqrt{2}} \sum_{k=1}^{100} \frac{\left(0.0003832 \sqrt[24]{e^{(200\pi)/k}}\right) \sinh\left(\frac{\pi}{k}\right) \sqrt{\frac{2}{3}\left(100 - \frac{1}{24}\right)}}{\sqrt{100 - \frac{1}{24}}}$$

$\sinh(x)$ is the hyperbolic sine function

Result:

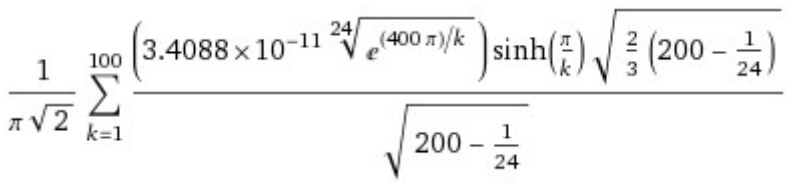
1.90569×10^8

$1.90569*10^8$ (p(100) = 190569292)

For n = 200:

1/(Pi*sqrt2) sum (((((0.000000000034088*[e^((400Pi)/k)]^(1/24)))sinh(Pi/k) (2/3(200-1/24))^1/2)) / (((200-1/24)^1/2))), k = 1..100

Input interpretation:



$\sinh(x)$ is the hyperbolic sine function

Result:

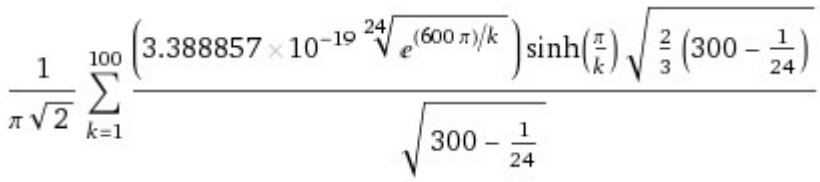
3.9722×10^{12}

$3.9722*10^{12}$ (p(200) = 3972999029388)

For n = 300:

1/(Pi*sqrt2) sum (((((3.388857 × 10^-19*[e^((600Pi)/k)]^(1/24)))sinh(Pi/k) (2/3(300-1/24))^1/2)) / (((300-1/24)^1/2))), k = 1..100

Input interpretation:



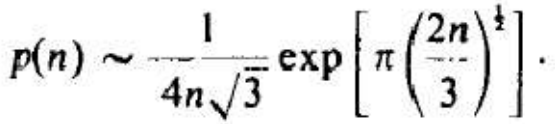
$\sinh(x)$ is the hyperbolic sine function

Result:

9.25308×10^{15}

$9.25308*10^{15}$ (p(300) = 9253082936723602)

And we have also, from the following approximated Ramanujan partition function:



For n = 100:

1/(4*100*sqrt3)*exp(Pi*(200/3)^0.5)

Input:

$$\frac{1}{4\times 100\sqrt{3}}\exp\left(\pi\sqrt{\frac{200}{3}}\right)$$

Exact result:

$$\frac{e^{10\sqrt{2/3}\,\pi}}{400\sqrt{3}}$$

Decimal approximation:

$1.992808933497401125156583028630957468582794501694286 64\ldots \times 10^8$

$1.992808933497*10^8 \approx 199281893.25$ (see above Table 16.2)

Property:

$\frac{e^{10\sqrt{2/3}\,\pi}}{400\sqrt{3}}$ is a transcendental number

Series representations:

$$\frac{\exp\left(\pi\sqrt{\frac{200}{3}}\right)}{4\times 100\sqrt{3}} = \frac{\exp\left(10\sqrt{\frac{2}{3}}\,\pi\right)}{400\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{200}{3}}\right)}{4\times 100\sqrt{3}} = \frac{\exp\left(10\sqrt{\frac{2}{3}}\,\pi\right)}{400\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{200}{3}}\right)}{4\times 100\sqrt{3}} = \frac{\exp\left(10\sqrt{\frac{2}{3}}\,\pi\right)\sqrt{\pi}}{200\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

For n = 1000:

1/(4*1000*sqrt3)*exp(Pi*(2000/3)^0.5)

Input:

$$\frac{1}{4\times 1000\sqrt{3}}\exp\left(\pi\sqrt{\frac{2000}{3}}\right)$$

Exact result:

$$\frac{e^{20\sqrt{5/3}\,\pi}}{4000\sqrt{3}}$$

Decimal approximation:

$2.440199631680247628826341494306241407249504150667722 6\ldots \times 10^{31}$

2.440×10^{31}

Property:

$\frac{e^{20\sqrt{5/3}\,\pi}}{4000\sqrt{3}}$ is a transcendental number

Series representations:

$$\frac{\exp\left(\pi\sqrt{\frac{2000}{3}}\right)}{4\times 1000\sqrt{3}} = \frac{\exp\left(20\sqrt{\frac{5}{3}}\,\pi\right)}{4000\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{2000}{3}}\right)}{4\times 1000\sqrt{3}} = \frac{\exp\left(20\sqrt{\frac{5}{3}}\,\pi\right)}{4000\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{2000}{3}}\right)}{4\times 1000\sqrt{3}} = \frac{\exp\left(20\sqrt{\frac{5}{3}}\,\pi\right)\sqrt{\pi}}{2000\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

For n = 10000:

1/(4*10000*sqrt3)*exp(Pi*(20000/3)^0.5)

Input:

$$\frac{1}{4\times 10\,000\sqrt{3}}\exp\left(\pi\sqrt{\frac{20\,000}{3}}\right)$$

Exact result:

$$\frac{e^{100\sqrt{2/3}\,\pi}}{40\,000\sqrt{3}}$$

Decimal approximation:

$3.6328058007588395493685612737048337177501237961537430\ldots \times 10^{106}$

3.633×10^{106}

Property:

$\frac{e^{100\sqrt{2/3}\,\pi}}{40\,000\sqrt{3}}$ is a transcendental number

Series representations:

$$\frac{\exp\left(\pi\sqrt{\frac{20000}{3}}\right)}{4\times 10\,000\sqrt{3}} = \frac{\exp\left(100\sqrt{\frac{2}{3}}\,\pi\right)}{40\,000\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{20000}{3}}\right)}{4\times 10\,000\sqrt{3}} = \frac{\exp\left(100\sqrt{\frac{2}{3}}\,\pi\right)}{40\,000\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{20000}{3}}\right)}{4\times 10\,000\sqrt{3}} = \frac{\exp\left(100\sqrt{\frac{2}{3}}\,\pi\right)\sqrt{\pi}}{20\,000\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

From:

$$p_{24}(N) \simeq \frac{1}{\sqrt{2}}\,N^{-27/4}\exp\left(4\pi\sqrt{N}\right).$$

that is the expression concerning the number of transverse light-cone directions in the bosonic string, we obtain:

For n = 100:

1/(sqrt2) * 100^(-27/4) * exp(4Pi*sqrt100)

Input:

$$\frac{1}{\sqrt{2}}\times 100^{-27/4}\exp\left(4\,\pi\,\sqrt{100}\right)$$

Exact result:

$$\frac{e^{40\,\pi}}{20\,000\,000\,000\,000\sqrt{5}}$$

Decimal approximation:

$8.405027831268191473727580191243496410225572639313728 9\ldots \times 10^{40}$

$8.405027831268 * 10^{40}$

For n = 1000:

1/(sqrt2) * 1000^(-27/4) * exp(4Pi*sqrt1000)

Input:

$$\frac{1}{\sqrt{2}} \times 1000^{-27/4} \exp\left(4\pi\sqrt{1000}\right)$$

Exact result:

$$\frac{e^{40\sqrt{10}\,\pi}}{100\,000\,000\,000\,000\,000\,000 \times 2^{3/4}\sqrt[4]{5}}$$

Decimal approximation:

$1.516909093424422589280167958890658548734595159534734 9\ldots \times 10^{152}$

$1.516909093424\ldots*10^{152}$

For n = 10000:

1/(sqrt2) * 10000^(-27/4) * exp(4Pi*sqrt10000)

Input:

$$\frac{1}{\sqrt{2}} \times 10\,000^{-27/4} \exp\left(4\pi\sqrt{10\,000}\right)$$

Exact result:

$$\frac{e^{400\,\pi}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\sqrt{2}}$$

Decimal approximation:

$3.981314988640323428913797190555994336497678225740414 0\ldots \times 10^{518}$

$3.98131498864*10^{518}$

Ramanujan partition numbers (100, 1000, 10000)

$p(100) = 1.992808933497 * 10^8$

$p(1000) = 2.440 \times 10^{31}$

$p(10000) = 3.633 \times 10^{106}$

Number of transverse light-cone directions in the bosonic string (100, 1000, 10000)

$p_{24}(100) = 8.405027831268 * 10^{40}$

$p_{24}(1000) = 1.516909093424 * 10^{152}$

$p_{24}(10000) = 3.98131498864 * 10^{518}$

Dividing the results between them, we obtain:

(1.992808933497 * 10^8)^5 / (8.405027831268 * 10^40)

Input interpretation:

$$\frac{(1.992808933497 \times 10^8)^5}{8.405027831268 \times 10^{40}}$$

Result:

3.739290027112342949131167627455290174557890505626776098305...

3.739290027…

and:

(2.440 × 10^31)^5 / (1.516909093424 * 10^152)

Input interpretation:

$$\frac{(2.44 \times 10^{31})^5}{1.516909093424 \times 10^{152}}$$

Result:

57015.058843889213241199137294466442879412700718202507930 69...

57015.05884388…

(3.633 × 10^106)^5 / (3.98131498864 * 10^518)

Input interpretation:

$$\frac{(3.633\times10^{106})^5}{3.98131498864\times10^{518}}$$

Result:

$1.589646105537369861190215198526327270075107794297417 9... \times 10^{14}$

$1.5896461055…*10^{14}$

(((3.633 × 10^106)^5 / (3.98131498864 * 10^518))) *1/ (((2.440 × 10^31)^5 / (1.516909093424 * 10^152))) *1/ (((1.992808933497 * 10^8)^5 / (8.405027831268 * 10^40)))

Input interpretation:

$$\frac{(3.633\times10^{106})^5}{3.98131498864\times10^{518}}\times\frac{1}{\frac{(2.44\times10^{31})^5}{1.516909093424\times10^{152}}}\times\frac{1}{\frac{(1.992808933497\times10^{8})^5}{8.405027831268\times10^{40}}}$$

Result:

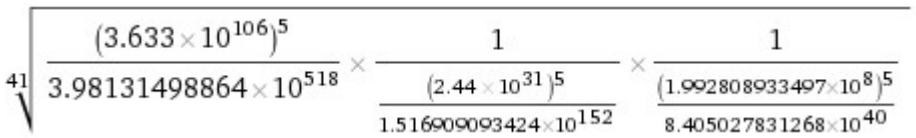
$7.456271654276685625779855030300000509141134672727328 79... \times 10^{8}$

$7.45627165…*10^{8}$

[(((3.633 × 10^106)^5 / (3.98131498864 * 10^518))) *1/ (((2.440 × 10^31)^5 / (1.516909093424 * 10^152))) *1/ (((1.992808933497 * 10^8)^5 / (8.405027831268 * 10^40)))]^1/41

where 41 is an Eisenstein prime number

Input interpretation:



Result:

1.645898...

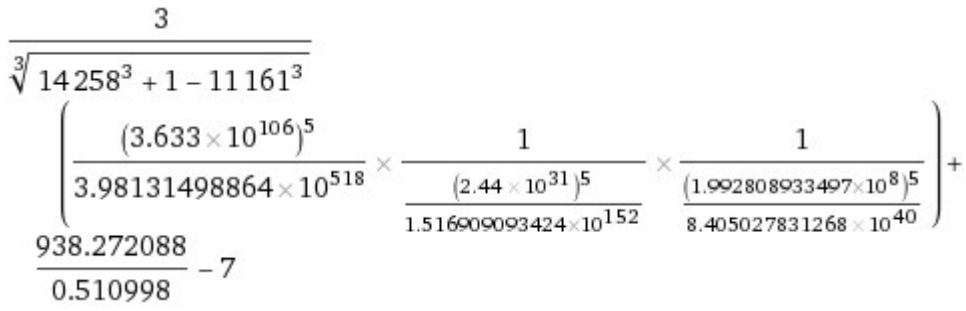
$1.645898\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

And, in conclusion, we obtain:

3/((14258^3+1-11161^3)^1/3)[(((3.633×10^106)^5/(3.98131498864*10^518)))*1/(((2.440×10^31)^5/(1.516909093424*10^152)))*1/(((1.992808933497 * 10^8)^5/(8.405027831268*10^40)))]+(938.272088/0.510998)-7

where $(14258^3 + 1 - 11161^3)^{1/3} = 11468$ (see Ramanujan taxicab numbers) and (938.272088/0.510998) i.e. the ratio between Proton and Electron mass, that is equal to 1836.1560867…

Input interpretation:



Result:

196883.350042332704116615813616357983251913488624531489641 6...

196883.3500423…

196884 is a fundamental number of the following *j*-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's j-invariant or j function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of j have to do with its q expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that j has a simple pole at the cusp, so its q-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

From which, performing the ln:

ln[3/((14258^3+1-11161^3)^1/3)[(((3.633×10^106)^5/(3.98131498864*10^518)))
1/(((2.440×10^31)^5/(1.516909093424*10^152)))1/(((1.992808933497*10^8)^5/(8.4
05027831268*10^40)))]+(938.272088/0.510998)-7]

we obtain:

Input interpretation:

$$\log\left(\frac{3}{\sqrt[3]{14258^3 + 1 - 11161^3}}\left(\frac{(3.633\times10^{106})^5}{3.98131498864\times10^{518}} \times \frac{1}{\frac{(2.44\times10^{31})^5}{1.516909093424\times10^{152}}} \times \frac{1}{\frac{(1.992808933497\times10^{8})^5}{8.405027831268\times10^{40}}}\right) + \frac{938.272088}{0.510998} - 7\right)$$

log(x) is the natural logarithm

Result:

12.1904...

12.1904… result equal to the black hole entropy 12.1904

And again:

144ln[3/((14258^3+1-11161^3)^1/3)[(((3.633×10^106)^5/(3.98131498*10^518)))1/(((2.440×10^31)^5/(1.516909093*10^152)))1/(((1.99280893*10^8)^5/(8.405027831*10^40)))]+(938.272088/0.510998)-7]-26

where 144 is a Fibonacci number and 26 is the dimensions number of Bosonic String Theory

Input interpretation:

$$144 \log\left(\frac{3}{\sqrt[3]{14258^3+1-11161^3}}\left(\frac{(3.633\times10^{106})^5}{3.98131498\times10^{518}}\times\frac{1}{\frac{(2.44\times10^{31})^5}{1.516909093\times10^{152}}}\times\frac{1}{\frac{(1.99280893\times10^{8})^5}{8.405027831\times10^{40}}}\right)+\frac{938.272088}{0.510998}-7\right)-26$$

log(x) is the natural logarithm

Result:

1729.41...

1729.41…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, from:

A CONVERGENT SERIES FOR THE PARTITION FUNCTION P(n)
BY HANS RADEMACHER - DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA - Communicated January 9, 1937

We have the following formulas:

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k \leqq \alpha\sqrt{n}} A_k(n)\, \sqrt{k}\, \frac{d}{dn} \left(\frac{e^{\frac{C\sqrt{n - 1/24}}{k}}}{\sqrt{n - 1/24}} \right) + O(n^{-1/4}) \quad (2)$$

with α as an arbitrary constant, and

$$A_k(n) = \sum_{\substack{h \bmod k \\ (h,k) = 1}} \omega_{h,k}\, e^{-\frac{2\pi i h n}{k}}, \quad C = \pi\sqrt{2/3}. \quad (3)$$

The following revision and modification of the Hardy-Ramanujan method permits us to replace the asymptotic formula (2) by the equality

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n) \sqrt{k}\, \frac{d}{dn} \left(\frac{\sinh \frac{C\sqrt{n - 1/24}}{k}}{\sqrt{n - 1/24}} \right), \quad (4)$$

in which the series is absolutely convergent. The asymptotic formula (2) is then an immediate consequence of (4).

From (4) it is possible to derive a definite expression for the O-term in (2). We obtain

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k=1}^{N} A_k(n)\, \sqrt{k}\, \frac{d}{dn} \left(\frac{e^{\frac{C\sqrt{n - 1/24}}{k}}}{\sqrt{n - 1/24}} \right) \quad (12)$$

$$+ \theta \cdot \left\{ \frac{44\pi^2}{225\sqrt{3}} N^{-1/2} + \frac{\pi\sqrt{2}}{75} \left(\frac{N}{n-1} \right)^{1/2} \sinh \frac{C\sqrt{n}}{N} \right.$$

$$\left. + \frac{1}{2\sqrt{3}} \frac{(N+1)^{3/2}}{n-1} e^{-\frac{C\sqrt{n-1}}{N}} \left(\frac{1}{3} + \frac{\sqrt{3}}{5\pi\sqrt{2}} \frac{N+1}{(n-1)^{1/2}} \right) \right\}, \quad |\theta| < 1,$$

We know that:

THEOREM. *Suppose that*

$$\phi_q(n) = \frac{\sqrt{q}}{2\pi\sqrt{2}} \frac{d}{dn}\left(\frac{e^{C\lambda_n/q}}{\lambda_n}\right), \tag{1.71}$$

where C and λ_n are defined by the equations (1.53), for all positive integral values of q; that p is a positive integer less than and prime to q; that $\omega_{p,q}$ is a $24q$-th root of unity, defined when p is odd by the formula

$$\omega_{p,q} = \left(\frac{-q}{p}\right) \exp\left[-\left\{\tfrac{1}{4}(2-pq-p) + \tfrac{1}{12}\left(q-\tfrac{1}{q}\right)(2p-p'+p^2p')\right\}\pi i\right], \tag{1.721}$$

and when q is odd by the formula

$$\omega_{p,q} = \left(\frac{-p}{q}\right) \exp\left[-\left\{\tfrac{1}{4}(q-1) + \tfrac{1}{12}\left(q-\tfrac{1}{q}\right)(2p-p'+p^2p')\right\}\pi i\right], \tag{1.722}$$

where $\left(\frac{a}{b}\right)$ is the symbol of Legendre and Jacobi[†], *and p' is any positive integer such that $1+pp'$ is divisible by q; that*

$$A_q(n) = \textstyle\sum_{(p)} \omega_{p,q} e^{-2np\pi i/q}; \tag{1.73}$$

and that α is any positive constant, and ν the integral part of $\alpha\sqrt{n}$.

Then

$$p(n) = \textstyle\sum_1^\nu A_q\phi_q + O(n^{-\frac{1}{4}}), \tag{1.74}$$

so that $p(n)$ is, for all sufficiently large values of n, the integer nearest to

$$\textstyle\sum_1^\nu A_q\phi_q. \tag{1.75}$$

For p = 3, p' = 5, q = 12

From

$$\omega_{p,q} = \left(\frac{-q}{p}\right) \exp\left[-\left\{\tfrac{1}{4}(2-pq-p) + \tfrac{1}{12}\left(q-\tfrac{1}{q}\right)(2p-p'+p^2p')\right\}\pi i\right],$$

we obtain:

(-12/3) exp [-(((1/4(2-36-3)+1/12(12-1/12)(6-5+45))))Pi*i]

Input:

$$-\frac{12}{3}\exp\left(-\left(\frac{1}{4}(2-36-3)+\frac{1}{12}\left(12-\frac{1}{12}\right)(6-5+45)\right)(\pi\, i)\right)$$

i is the imaginary unit

Exact result:

$$-4\,e^{-(31\,i\,\pi)/72}$$

Decimal approximation:

$$-0.865758455752411519038214678471762914693548774200248167 1\ldots + 3.905184028479733463883545958421710308661341956514036252\ldots\,i$$

-0.86575845575+3.905184028479i

Polar coordinates:

$r = 4$ (radius), $\theta \approx 102.5°$ (angle)

4

Alternate forms:

$$4\,(-1)^{41/72}$$

$$-4\sin\left(\frac{5\,\pi}{72}\right) + 4\,i\cos\left(\frac{5\,\pi}{72}\right)$$

$$2\,(-1)^{7/12}\,2^{23/24}\,\sqrt[24]{1 - i\,\sqrt{3}}$$

Series representations:

$$\frac{1}{3}\exp\left(-\left(\frac{1}{4}\,(2-36-3) + \frac{1}{12}\left(12 - \frac{1}{12}\right)(6-5+45)\right)(\pi\,i)\right)(-12) = -4\,e^{-31/18\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2\,k)}$$

$$\frac{1}{3}\exp\left(-\left(\frac{1}{4}\,(2-36-3) + \frac{1}{12}\left(12 - \frac{1}{12}\right)(6-5+45)\right)(\pi\,i)\right)(-12) = -4\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-(31\,i\,\pi)/72}$$

$$\frac{1}{3}\exp\left(-\left(\frac{1}{4}\,(2-36-3) + \frac{1}{12}\left(12 - \frac{1}{12}\right)(6-5+45)\right)(\pi\,i)\right)(-12) = -4\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-(31\,i\,\pi)/72}$$

Integral representations:

$$\frac{1}{3}\exp\left(-\left(\frac{1}{4}(2-36-3)+\frac{1}{12}\left(12-\frac{1}{12}\right)(6-5+45)\right)(\pi i)\right)(-12)=-4\,e^{-(31\,i)/18\int_0^1\sqrt{1-t^2}\,dt}$$

$$\frac{1}{3}\exp\left(-\left(\frac{1}{4}(2-36-3)+\frac{1}{12}\left(12-\frac{1}{12}\right)(6-5+45)\right)(\pi i)\right)(-12)=$$
$$-4\,e^{-(31\,i)/36\int_0^1 1/\sqrt{1-t^2}\,dt}$$

$$\frac{1}{3}\exp\left(-\left(\frac{1}{4}(2-36-3)+\frac{1}{12}\left(12-\frac{1}{12}\right)(6-5+45)\right)(\pi i)\right)(-12)=-4\,e^{-(31\,i)/36\int_0^\infty 1/(1+t^2)\,dt}$$

Thence for (3), we have:

$$A_k(n)=\sum_{\substack{h \bmod k\\(h,k)=1}}\omega_{h,k}\,e^{-\frac{2\pi i h n}{k}},\quad C=\pi\sqrt{2/3}. \qquad (3)$$

For h = 1

(-0.86575845575+3.905184028479i) * e^((-2Pi*200*i)/12)

Input interpretation:

$$(-0.86575845575+3.905184028479\,i)\,e^{1/12\,(-2\,\pi\times 200\,i)}$$

i is the imaginary unit

Result:

$$-2.94910934724\ldots-2.70236083046\ldots\,i$$

(using the principal branch of the logarithm for complex exponentiation)

-2.94910934724-2.70236083046 i = $A_k(n)$

Polar coordinates:

$r = 4.00000000000$ (radius), $\theta = -137.500000000°$ (angle)

4

Alternative representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\,e^{-(72\,000\,°\,i)/12}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\,e^{400/12\,i^2\log(-1)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\exp^{-\frac{2}{12}\,(\pi\,200\,i)}(z) \text{ for } z = 1$$

Series representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = 3.90518402848$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-400/3\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = 3.90518402848$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{-400/3\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = 3.905184028479$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-100/3\,i\,\sum_{k=1}^{\infty}4^{-k}\left(-1+3^k\right)\zeta(1+k)}$$

Integral representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = e^{-(200\,i)/3\int_0^{\infty}1/\left(1+t^2\right)dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = e^{-100\,i\int_0^\infty \sin^4(t)/t^4\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,200\,i)} = e^{-(400\,i)/3\int_0^1 \sqrt{1-t^2}\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

For: $A_k(n) = -2.94910934724-2.70236083046\,i$, $n = 200$, $N = 1729$,

$C = Pi*(2/3)^0.5$, we obtain:

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k=1}^{N} A_k(n)\,\sqrt{k}\,\frac{d}{dn}\left(\frac{e^{\frac{C\sqrt{n-1/24}}{k}}}{\sqrt{n-1/24}}\right) \qquad (12)$$

$$+\ \theta\cdot\left\{\frac{44\pi^2}{225\sqrt{3}}\,N^{-1/2} + \frac{\pi\sqrt{2}}{75}\left(\frac{N}{n-1}\right)^{1/2}\sinh\frac{C\sqrt{n}}{N}\right.$$

$$\left. +\ \frac{1}{2\sqrt{3}}\,\frac{(N+1)^{3/2}}{n-1}\,e^{-\frac{C\sqrt{n-1}}{N}}\left(\frac{1}{3} + \frac{\sqrt{3}}{5\pi\sqrt{2}}\,\frac{N+1}{(n-1)^{1/2}}\right)\right\}, \quad |\theta| < 1,$$

For

$$\frac{d}{dn}\left(\frac{e^{\frac{C\sqrt{n-1/24}}{k}}}{\sqrt{n-1/24}}\right)$$

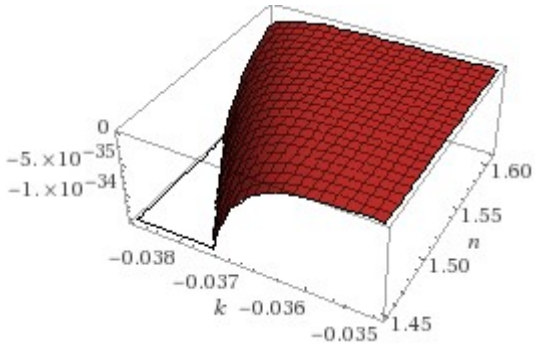
we obtain :

partial derivative [exp((((Pi*(2/3)^0.5)*sqrt(n-1/24)/k)))/(sqrt(n-1/24))]

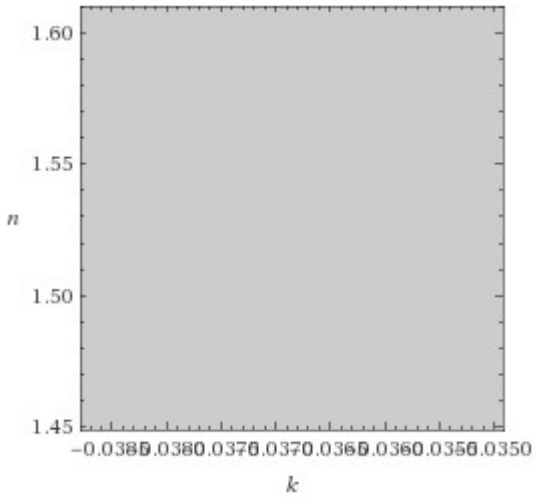
Derivative:

$$\frac{\partial}{\partial n}\left(\frac{\exp\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{n-\frac{1}{24}}}{k}\right)}{\sqrt{n-\frac{1}{24}}}\right)=\frac{\pi\, e^{\left(\sqrt{\frac{2}{3}}\,\pi\sqrt{n-\frac{1}{24}}\right)/k}}{\sqrt{6}\;k\left(n-\frac{1}{24}\right)}-\frac{e^{\left(\sqrt{\frac{2}{3}}\,\pi\sqrt{n-\frac{1}{24}}\right)/k}}{2\left(n-\frac{1}{24}\right)^{3/2}}$$

3D plot:



Contour plot:



Alternate forms:

$$e^{\left(\pi\sqrt{24n-1}\right)/(6k)}\left(\frac{4\sqrt{6}\ \pi}{k\,(24n-1)}-\frac{24\sqrt{6}}{(24n-1)^{3/2}}\right)$$

$$\frac{4\sqrt{6}\ e^{\left(\pi\sqrt{24n-1}\right)/(6k)}\left(-144kn+6k+\pi(24n-1)^{3/2}\right)}{k\,(24n-1)^{5/2}}$$

$$\frac{4\sqrt{6}\ \pi e^{\left(\pi\sqrt{24n-1}\right)/(6k)}}{k\,(24n-1)}-\frac{24\sqrt{6}\ e^{\left(\pi\sqrt{24n-1}\right)/(6k)}}{(24n-1)^{3/2}}$$

Series expansion at n = 0:

$$\begin{cases} -\frac{4\left(\sqrt{6}\ e^{-(i\pi)/(6k)}(\pi-6ik)\right)}{k}+\frac{8i\sqrt{6}\ e^{-(i\pi)/(6k)}\left(108k^2+18i\pi k-\pi^2\right)n}{k^2}+ \\ \frac{8\sqrt{6}\ e^{-(i\pi)/(6k)}\left(3240ik^3-540\pi k^2-36i\pi^2k+\pi^3\right)n^2}{k^3}+ \\ \frac{16\sqrt{\frac{2}{3}}\ e^{-(i\pi)/(6k)}\left(136080ik^4-22680\pi k^3-1620i\pi^2k^2+60\pi^3k+i\pi^4\right)n^3}{k^4}-\frac{1}{k^5} \\ 8\left(\sqrt{\frac{2}{3}}\ e^{-(i\pi)/(6k)}\left(-7348320ik^5+1224720\pi k^4+90720i\pi^2k^3-\right.\right. \\ \left.\left.3780\pi^3k^2-90i\pi^4k+\pi^5\right)\right)n^4+\frac{1}{5k^6}16\sqrt{\frac{2}{3}}\ e^{-(i\pi)/(6k)} \\ \left(484989120ik^6-80831520\pi k^5-6123600i\pi^2k^4+\right. \\ \left.272160\pi^3k^3+7560i\pi^4k^2-126\pi^5k-i\pi^6\right)n^5+O\left(n^6\right) & \text{Im}(n)<0 \\ -\frac{4\left(\sqrt{6}\ e^{(i\pi)/(6k)}(\pi+6ik)\right)}{k}+\frac{8i\sqrt{6}\ e^{(i\pi)/(6k)}\left(-108k^2+18i\pi k+\pi^2\right)n}{k^2}+ \\ \frac{8\sqrt{6}\ e^{(i\pi)/(6k)}\left(-3240ik^3-540\pi k^2+36i\pi^2k+\pi^3\right)n^2}{k^3}+ \\ \frac{16\sqrt{\frac{2}{3}}\ e^{(i\pi)/(6k)}\left(-136080ik^4-22680\pi k^3+1620i\pi^2k^2+60\pi^3k-i\pi^4\right)n^3}{k^4}-\frac{1}{k^5} \\ 8\left(\sqrt{\frac{2}{3}}\ e^{(i\pi)/(6k)}\left(7348320ik^5+1224720\pi k^4-90720i\pi^2k^3-\right.\right. \\ \left.\left.3780\pi^3k^2+90i\pi^4k+\pi^5\right)\right)n^4+\frac{1}{5k^6}16i\sqrt{\frac{2}{3}}\ e^{(i\pi)/(6k)} \\ \left(-484989120k^6+80831520i\pi k^5+6123600\pi^2k^4-\right. \\ \left.272160i\pi^3k^3-7560\pi^4k^2+126i\pi^5k+\pi^6\right)n^5+O\left(n^6\right) & \text{(otherwise)} \end{cases}$$

Series expansion at n = ∞:

$$e^{\left(\sqrt{\frac{2}{3}}\,\pi\sqrt{n}\right)/k}\left(\frac{\pi}{\sqrt{6}\,k\,n}-\frac{1}{2}\left(\frac{1}{n}\right)^{3/2}+\frac{\pi\left(\sqrt{6}\,k-\sqrt{n}\,\pi\right)}{144\,k^2\,n^2}+\frac{1}{288}\left(\frac{\sqrt{6}\,\sqrt{n}\,\pi}{k}-9\right)\left(\frac{1}{n}\right)^{5/2}+\frac{\pi\left(12\sqrt{6}\,k^2-15\sqrt{n}\,\pi k+\sqrt{6}\,n\pi^2\right)}{41\,472\,k^3\,n^3}-\frac{\left(45\,k^2-7\sqrt{6}\,\sqrt{n}\,\pi k+2\,n\pi^2\right)\left(\frac{1}{n}\right)^{7/2}}{27\,648\,k^2}+\frac{\pi\left(72\sqrt{6}\,k^3-99\sqrt{n}\,\pi k^2+9\sqrt{6}\,n\pi^2 k-2\,n^{3/2}\pi^3\right)}{5\,971\,968\,k^4\,n^4}+\frac{\left(-945\,k^3+171\sqrt{6}\,\sqrt{n}\,\pi k^2-72\,n\pi^2 k+2\sqrt{6}\,n^{3/2}\pi^3\right)\left(\frac{1}{n}\right)^{9/2}}{11\,943\,936\,k^3}+\frac{\pi\left(1728\sqrt{6}\,k^4-2511\sqrt{n}\,\pi k^3+261\sqrt{6}\,n\pi^2 k^2-84\,n^{3/2}\pi^3 k+2\sqrt{6}\,n^2\pi^4\right)}{3\,439\,853\,568\,k^5\,n^5}-\frac{\left(8505\,k^4-1683\sqrt{6}\,\sqrt{n}\,\pi k^3+846\,n\pi^2 k^2-36\sqrt{6}\,n^{3/2}\pi^3 k+4\,n^2\pi^4\right)\left(\frac{1}{n}\right)^{11/2}}{2\,293\,235\,712\,k^4}+O\left(\left(\frac{1}{n}\right)^6\right)\right)$$

Indefinite integral:

$$\int\frac{4\sqrt{6}\;e^{\left(\sqrt{-1+24\,n}\,\pi\right)/(6k)}\left(-6\,k+\sqrt{-1+24\,n}\,\pi\right)}{k\,(-1+24\,n)^{3/2}}\,dn=\frac{2\,e^{\left(\pi\sqrt{24\,n-1}\right)/(6k)}}{\sqrt{4\,n-\frac{1}{6}}}+\text{constant}$$

From the following alternate form, for n = 200, we obtain:

$$e^{\left(\pi\sqrt{24\,n-1}\right)/(6k)}\left(\frac{4\sqrt{6}\,\pi}{k\,(24\,n-1)}-\frac{24\sqrt{6}}{(24\,n-1)^{3/2}}\right)$$

1/(2Pi*sqrt2) * sum(((((-2.94910934724-2.70236083046i) * sqrt(k) (e^((π sqrt(24*200 - 1))/(6 k)) ((4 sqrt(6) π)/(k (24*200- 1)) - (24 sqrt(6))/(24*200 - 1)^(3/2)))))), k = 1..1729

Input interpretation:

$$\frac{1}{2\pi\sqrt{2}}\sum_{k=1}^{1729}(-2.94910934724 + i\times(-2.70236083046))\sqrt{k}\left(e^{\left(\pi\sqrt{24\times200-1}\right)/(6k)}\left(\frac{4\sqrt{6}\,\pi}{k\,(24\times200-1)}-\frac{24\sqrt{6}}{(24\times200-1)^{3/2}}\right)\right)$$

i is the imaginary unit

Result:

$$-1.17168085746\times10^{13} - 1.07364769569\times10^{13}\,i$$

Input interpretation:

$$-1.17168085746\times10^{13} - 1.07364769569\times10^{13}\,i$$

i is the imaginary unit

Result:

$$-1.1716808575...\times10^{13} - 1.0736476957...\times10^{13}\,i$$

Polar coordinates:

$r = 1.5891996118\times10^{13}$ (radius), $\theta = -137.500000000°$ (angle)

-1.1716808575 × 10^13 -1.0736476957 × 10^13 i

From

$$\frac{44\pi^2}{225\sqrt{3}}\,N^{-1/2} + \frac{\pi\sqrt{2}}{75}\left(\frac{N}{n-1}\right)^{1/2}\sinh\frac{C\sqrt{n}}{N}$$

we obtain:

((44Pi^2) / (225sqrt3)) * 1729^(-1/2) + (Pi*sqrt2 / 75) (1729/(200-1))^0.5 sinh[(((Pi*(2/3)^0.5)*sqrt200))/1729]

Input:

$$\frac{44\,\pi^2}{225\,\sqrt{3}} \times 1729^{-1/2} + \left(\pi \times \frac{\sqrt{2}}{75}\right)\sqrt{\frac{1729}{200-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{44\,\pi^2}{225\,\sqrt{5187}} + \frac{1}{75}\sqrt{\frac{3458}{199}}\ \pi \sinh\left(\frac{20\,\pi}{1729\,\sqrt{3}}\right)$$

Decimal approximation:

0.030462370708745781092864447108830746965860771838127892100...

0.030462370708745781

Alternate forms:

$$\frac{\pi\left(88\,\pi + 10\,374\sqrt{\frac{6}{199}}\ \sinh\left(\frac{20\,\pi}{1729\sqrt{3}}\right)\right)}{450\,\sqrt{5187}}$$

$$\frac{8756\,\sqrt{5187}\ \pi^2 + 15\,561\,\sqrt{688\,142}\ \pi \sinh\left(\frac{20\,\pi}{1729\sqrt{3}}\right)}{232\,247\,925}$$

$$\frac{\pi\left(8756\,\sqrt{3}\ \pi + 15\,561\,\sqrt{398}\ \sinh\left(\frac{20\,\pi}{1729\sqrt{3}}\right)\right)}{134\,325\,\sqrt{1729}}$$

Alternative representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}} + \frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\sqrt{2} =$$

$$\frac{44\,\pi^2}{\sqrt{1729}\ (225\,\sqrt{3}\,)} + \frac{1}{75}\,i\,\pi\cos\left(\frac{\pi}{2} + \frac{i\,\pi\sqrt{\frac{2}{3}}\ \sqrt{200}}{1729}\right)\sqrt{\frac{1729}{199}}\ \sqrt{2}$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{\sqrt{1729}\,(225\,\sqrt{3}\,)}+\frac{1}{75}\,i\,\pi\cosh\left(\frac{i\,\pi}{2}-\frac{\pi\,\sqrt{\frac{2}{3}}\,\sqrt{200}}{1729}\right)\sqrt{\frac{1729}{199}}\,\sqrt{2}$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{\sqrt{1729}\,(225\,\sqrt{3}\,)}-\frac{1}{75}\,i\left(\pi\cosh\left(\frac{i\,\pi}{2}+\frac{\pi\,\sqrt{\frac{2}{3}}\,\sqrt{200}}{1729}\right)\sqrt{\frac{1729}{199}}\,\sqrt{2}\right)$$

Series representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{2}{75}\sqrt{\frac{3458}{199}}\,\pi\sum_{k=0}^{\infty}I_{1+2\,k}\left(\frac{20\,\pi}{1729\,\sqrt{3}}\right)$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{1}{75}\sqrt{\frac{3458}{199}}\,\pi\sum_{k=0}^{\infty}\frac{3^{-1/2-k}\times 1729^{-1-2\,k}\,(20\,\pi)^{1+2\,k}}{(1+2\,k)!}$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{1}{75}\sqrt{\frac{3458}{199}}\;\pi\sum_{k=0}^{\infty}\frac{3^{1/2\,(-1-2\,k)}\left(\frac{1729}{20\,\pi}\right)^{-1-2\,k}}{(1+2\,k)!}$$

Integral representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{4}{15}\sqrt{\frac{2}{1\,032\,213}}\;\pi^2\int_0^1\cosh\left(\frac{20\,\pi\,t}{1729\,\sqrt{3}}\right)dt$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{200-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{200}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}-\frac{1}{15}\,i\sqrt{\frac{2}{1\,032\,213}}\;\pi^{3/2}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{(100\,\pi^2)/(8\,968\,323\,s)+s}}{s^{3/2}}\,ds\quad\text{for}\;\gamma>0$$

And, from

$$+\;\theta\cdot\left\{\frac{44\pi^2}{225\sqrt{3}}\;N^{-1/2}+\frac{\pi\sqrt{2}}{75}\left(\frac{N}{n-1}\right)^{1/2}\sinh\frac{C\sqrt{n}}{N}\right.$$

$$\left.+\;\frac{1}{2\sqrt{3}}\;\frac{(N+1)^{3/2}}{n-1}\;e^{-\frac{C\sqrt{n-1}}{N}}\left(\frac{1}{3}+\frac{\sqrt{3}}{5\pi\sqrt{2}}\;\frac{N+1}{(n-1)^{1/2}}\right)\right\},$$

inserting the value above obtained, we have:

0.5*[0.030462370708745781 + 1/(2sqrt3) * ((1729+1)^1.5) / (200-1) * exp-(((((((Pi*(2/3)^0.5)*sqrt(200-1))/1729)))))*(((1/3+sqrt3/(5Pi*sqrt2)*(1729+1)/(200-1)^0.5)))]

Input interpretation:

$$0.5\left(0.030462370708745781 + \frac{1}{2\sqrt{3}} \times \frac{(1729+1)^{1.5}}{200-1} \exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{200-1}}{1729}\right)\left(\frac{1}{3} + \frac{\sqrt{3}}{5\pi\sqrt{2}} \times \frac{1729+1}{\sqrt{200-1}}\right)\right)$$

Result:

505.762...

505.762

Series representations:

$$0.5\left(0.0304623707087457810000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{200-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,(1729+1)}{\left(5\pi\sqrt{2}\right)^{\sqrt{200-1}}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(200-1)}\right)=$$

$$\left(30.1325\left(\pi\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(199-z_0)^k z_0^{-k}}{k!}}{1729}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}+73.5819\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(199-z_0)^k z_0^{-k}}{k!}}{1729}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}+0.000505474\,\pi\sqrt{z_0}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}(3-z_0)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$0.5\left(0.0304623707087457810000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{200-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,(1729+1)}{\left(5\pi\sqrt{2}\right)\sqrt{200-1}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(200-1)}\right)=$$

$$\left(30.1325\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\right.\right.$$

$$\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\exp\left(i\pi\left\lfloor\frac{\arg(199-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(199-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1729}\right)$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+73.5819\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)$$

$$\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\exp\left(i\pi\left\lfloor\frac{\arg(199-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(199-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1729}\right)$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$

$$0.000505474\,\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,(2-x)^{k_1}\,(3-x)^{k_2}\,x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$0.5\left(0.0304623707087457810000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{200-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\ (1729+1)}{\left(5\pi\sqrt{2}\right)^{\sqrt{200-1}}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(200-1)}\right)=$$

$$\left(30.1325\left(\frac{1}{z_0}\right)^{-1/2\,[\arg(2-z_0)/(2\pi)]-1/2\,[\arg(3-z_0)/(2\pi)]}z_0^{-1-1/2\,[\arg(2-z_0)/(2\pi)]-1/2\,[\arg(3-z_0)/(2\pi)]}\right.$$

$$\left(\pi\exp\left(-\frac{1}{1729}\sqrt{\frac{2}{3}}\ \pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(199-z_0)/(2\pi)]}z_0^{1/2\,(1+[\arg(199-z_0)/(2\pi)])}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(199-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\,[\arg(2-z_0)/(2\pi)]}$$

$$z_0^{1/2+1/2\,[\arg(2-z_0)/(2\pi)]}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}+$$

$$73.5819\exp\left(-\frac{1}{1729}\sqrt{\frac{2}{3}}\ \pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(199-z_0)/(2\pi)]}z_0^{1/2\,(1+[\arg(199-z_0)/(2\pi)])}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(199-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$z_0^{1/2+1/2\,[\arg(3-z_0)/(2\pi)]}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}+$$

$$0.000505474\,\pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(2-z_0)/(2\pi)]+1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$z_0^{1+1/2\,[\arg(2-z_0)/(2\pi)]+1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\,(3-z_0)^{k_2}\,z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)$$

Thence:

(-1.1716808575 × 10^13 -1.0736476957 × 10^13 i) + 505.762

Input interpretation:

$(-1.1716808575 \times 10^{13} - 1.0736476957 \times 10^{13}\, i) + 505.762$

i is the imaginary unit

Result:

$-1.171680857... \times 10^{13} - 1.073647696... \times 10^{13}\, i$

Polar coordinates:

$r = 1.5892 \times 10^{13}$ (radius), $\theta = -137.5°$ (angle)

$1.5892*10^{13}$

From

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k \leqq \alpha\sqrt{n}} A_k(n)\, \sqrt{k}\, \frac{d}{dn} \left(\frac{e^{\frac{C\sqrt{n - 1/24}}{k}}}{\sqrt{n - 1/24}} \right) + O(n^{-1/4}) \quad (2)$$

we obtain, in conclusion:

1/4((((-1.1716808575 × 10^13 -1.0736476957 × 10^13 i) + (((505.762*(200)^(-0.25))))

Input interpretation:

$\frac{1}{4}\left((-1.1716808575 \times 10^{13} - 1.0736476957 \times 10^{13}\, i) + \frac{505.762}{200^{0.25}}\right)$

i is the imaginary unit

Result:

$-2.929202144... \times 10^{12} - 2.684119239... \times 10^{12}\, i$

Polar coordinates:

$r = 3.973 \times 10^{12}$ (radius), $\theta = -137.5°$ (angle)

$3.973*10^{12} \cong 3.972999029388 \times 10^{12} = p(200)$

As further prove, for n = 100, we obtain, performing the same calculations as previously:

((44Pi^2) / (225sqrt3)) * 1729^(-1/2) + (Pi*sqrt2 / 75) (1729/(100-1))^0.5 sinh[(((Pi*(2/3)^0.5)*sqrt100))/1729]

Input:

$$\frac{44\,\pi^2}{225\,\sqrt{3}} \times 1729^{-1/2} + \left(\pi \times \frac{\sqrt{2}}{75}\right)\sqrt{\frac{1729}{100-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{44\,\pi^2}{225\,\sqrt{5187}} + \frac{1}{225}\sqrt{\frac{3458}{11}}\ \pi \sinh\left(\frac{10\sqrt{\frac{2}{3}}\ \pi}{1729}\right)$$

Decimal approximation:

0.030471476320358143604963836923736421925612828070861000275...

0.0304714763…

Alternate forms:

$$\frac{\pi\left(88\,\pi + 3458\sqrt{\frac{6}{11}}\ \sinh\left(\frac{10\sqrt{\frac{2}{3}}\ \pi}{1729}\right)\right)}{450\,\sqrt{5187}}$$

$$\frac{484\,\sqrt{5187}\ \pi^2 + 5187\,\sqrt{38\,038}\ \pi \sinh\left(\frac{10\sqrt{\frac{2}{3}}\ \pi}{1729}\right)}{12\,837\,825}$$

$$\frac{\pi\left(484\sqrt{3}\ \pi+5187\sqrt{22}\ \sinh\left(\frac{10\sqrt{\frac{2}{3}}\ \pi}{1729}\right)\right)}{7425\sqrt{1729}}$$

Alternative representations:

$$\frac{1729^{-1/2}\left(44\pi^2\right)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\sqrt{2}=$$

$$\frac{44\pi^2}{\sqrt{1729}\ (225\sqrt{3}\,)}+\frac{1}{75}\,i\pi\cos\left(\frac{\pi}{2}+\frac{i\pi\sqrt{\frac{2}{3}}\ \sqrt{100}}{1729}\right)\sqrt{\frac{1729}{99}}\ \sqrt{2}$$

$$\frac{1729^{-1/2}\left(44\pi^2\right)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\sqrt{2}=$$

$$\frac{44\pi^2}{\sqrt{1729}\ (225\sqrt{3}\,)}+\frac{1}{75}\,i\pi\cosh\left(\frac{i\pi}{2}-\frac{\pi\sqrt{\frac{2}{3}}\ \sqrt{100}}{1729}\right)\sqrt{\frac{1729}{99}}\ \sqrt{2}$$

$$\frac{1729^{-1/2}\left(44\pi^2\right)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\sqrt{2}=$$

$$\frac{44\pi^2}{\sqrt{1729}\ (225\sqrt{3}\,)}-\frac{1}{75}\,i\left(\pi\cosh\left(\frac{i\pi}{2}+\frac{\pi\sqrt{\frac{2}{3}}\ \sqrt{100}}{1729}\right)\sqrt{\frac{1729}{99}}\ \sqrt{2}\right)$$

Series representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{2}{225}\,\sqrt{\frac{3458}{11}}\;\pi\sum_{k=0}^{\infty}I_{1+2\,k}\left(\frac{10\,\sqrt{\frac{2}{3}}\;\pi}{1729}\right)$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{1}{225}\,\sqrt{\frac{3458}{11}}\;\pi\sum_{k=0}^{\infty}\frac{2^{3/2+3\,k}\times 3^{-1/2-k}\times 1729^{-1-2\,k}\,(5\,\pi)^{1+2\,k}}{(1+2\,k)!}$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{1}{225}\,\sqrt{\frac{3458}{11}}\;\pi\sum_{k=0}^{\infty}\frac{2^{1+2\,k+1/2\,(1+2\,k)}\times 3^{1/2\,(-1-2\,k)}\left(\frac{1729}{5\,\pi}\right)^{-1-2\,k}}{(1+2\,k)!}$$

Integral representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\,\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{4\,\pi^2}{45\,\sqrt{57\,057}}\int_0^1\cosh\left(\frac{10\,\sqrt{\frac{2}{3}}\;\pi\,t}{1729}\right)dt$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}} + \frac{1}{75}\left(\sqrt{\frac{1729}{100-1}}\,\sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100}}{1729}\right)\right)\pi\sqrt{2} =$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}} - \frac{i\,\pi^{3/2}}{45\,\sqrt{57057}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{(50\,\pi^2)/(8\,968\,323\,s)+s}}{s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

0.5*[0.030471476320358143604963 83 + 1/(2sqrt3) * ((1729+1)^1.5) / (100-1) * exp-(((((((Pi*(2/3)^0.5)*sqrt(100-1))/1729)))))*(((1/3+sqrt3/(5Pi*sqrt2)*(1729+1)/(100-1)^0.5)))]

Input interpretation:

$$0.5\left(0.030471476320358143604963 83 + \frac{1}{2\sqrt{3}} \times \frac{(1729+1)^{1.5}}{100-1}\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100-1}}{1729}\right)\left(\frac{1}{3} + \frac{\sqrt{3}}{5\,\pi\sqrt{2}} \times \frac{1729+1}{\sqrt{100-1}}\right)\right)$$

Result:

1435.86...

1435.86…

Series representations:

$$0.5\left(0.030471476320358143604963830000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{100-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,(1729+1)}{\left(5\pi\sqrt{2}\right)^{\sqrt{100-1}}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(100-1)}\right)=\left(60.5693\right.$$

$$\left(\pi\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\infty-z_0)^k\,z_0^{-k}}{k!}}{1729}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}+\right.$$

$$104.323\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\infty-z_0)^k\,z_0^{-k}}{k!}}{1729}\right)$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}+0.000251542\,\pi\sqrt{z_0}\right.\right.$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\,(3-z_0)^{k_2}\,z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$0.5\left(0.030471476320358143604963830000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{100-1}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,(1729+1)}{\left(5\pi\sqrt{2}\right)\sqrt{100-1}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(100-1)}\right)=$$

$$\left(60.5693\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\exp\left(i\pi\left\lfloor\frac{\arg(99-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(99-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1729}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+104.323\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\exp\left(i\pi\left\lfloor\frac{\arg(99-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(99-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1729}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+0.000251542\,\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,(2-x)^{k_1}\,(3-x)^{k_2}\,x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\Bigg/$$

$$\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$0.5\left(0.030471476320358143604963830000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{100-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\ (1729+1)}{\left(5\pi\sqrt{2}\right)^{\sqrt{100-1}}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(100-1)}\right)=$$

$$\left(60.5693\left(\frac{1}{z_0}\right)^{-1/2\,[\arg(2-z_0)/(2\pi)]-1/2\,[\arg(3-z_0)/(2\pi)]}z_0^{-1-1/2\,[\arg(2-z_0)/(2\pi)]-1/2\,[\arg(3-z_0)/(2\pi)]}\right.$$

$$\left(\pi\exp\left(-\frac{1}{1729}\sqrt{\frac{2}{3}}\ \pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(99-z_0)/(2\pi)]}z_0^{1/2\,(1+[\arg(99-z_0)/(2\pi)])}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(99-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\,[\arg(2-z_0)/(2\pi)]}$$

$$z_0^{1/2+1/2\,[\arg(2-z_0)/(2\pi)]}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}+$$

$$104.323\exp\left(-\frac{1}{1729}\sqrt{\frac{2}{3}}\ \pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(99-z_0)/(2\pi)]}z_0^{1/2\,(1+[\arg(99-z_0)/(2\pi)])}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(99-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$z_0^{1/2+1/2\,[\arg(3-z_0)/(2\pi)]}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}+$$

$$0.000251542\,\pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(2-z_0)/(2\pi)]+1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$z_0^{1+1/2\,[\arg(2-z_0)/(2\pi)]+1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\,(3-z_0)^{k_2}\,z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)$$

We have:

(-0.86575845575+3.905184028479i) * e^((-2Pi*100*i)/12)

Input interpretation:

$$(-0.86575845575 + 3.905184028479\,i)\,e^{1/12\,(-2\,\pi\times 100\,i)}$$

i is the imaginary unit

Result:

3.81486780299... −
1.20282319802... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 4.00000000000$ (radius), $\theta = -17.500000000°$ (angle)

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Alternative representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\,e^{-(36000°\,i)/12}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\,e^{200/12\,i^2\log(-1)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\exp^{-\frac{2}{12}\,(\pi\,100\,i)}(z) \quad \text{for } z = 1$$

Series representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = 3.905184028479$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-200/3\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = 3.905184028479$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{-200/3\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = 3.905184028479$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-50/3\,i\,\sum_{k=1}^{\infty}4^{-k}\left(-1+3^{k}\right)\zeta(1+k)}$$

Integral representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = e^{-(100\,i)/3\int_0^{\infty}1/(1+t^2)\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = e^{-50\,i\int_0^{\infty}\sin^4(t)/t^4\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,100\,i)} = e^{-(200\,i)/3\int_0^{1}\sqrt{1-t^2}\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

Thence:

1/(2Pi*sqrt2) * sum(((((3.81486780299 - 1.20282319802i) * sqrt(k) (e^((π sqrt(24*100 - 1))/(6 k)) ((4 sqrt(6) π)/(k (24*100- 1)) - (24 sqrt(6))/(24*100 - 1)^(3/2)))))), k = 1..1729

Input interpretation:

$$\frac{1}{2\,\pi\,\sqrt{2}}\sum_{k=1}^{1729}(3.81486780299 + i\times(-1.20282319802))\sqrt{k}\left(e^{\left(\pi\,\sqrt{24\times100-1}\right)/(6k)}\left(\frac{4\,\sqrt{6}\,\pi}{k\,(24\times100-1)}-\frac{24\,\sqrt{6}}{(24\times100-1)^{3/2}}\right)\right)$$

i is the imaginary unit

Result:

$7.26996668883\times10^{8} - 2.29221169219\times10^{8}\,i$

7.26996668883×10^8 - 2.29221169219×10^8 i

From which:

1/4((((7.26996668883×10^8 - 2.29221169219×10^8 i) + (((1435.86*(100)^(-0.25))))

Input interpretation:

$$\frac{1}{4}\left((7.26996668883\times10^8 - 2.29221169219\times10^8\, i) + \frac{1435.86}{100^{0.25}}\right)$$

i is the imaginary unit

Result:

$1.81749280735... \times 10^8 - 5.73052923048... \times 10^7\, i$

Polar coordinates:

$r = 1.90569\times10^8$ (radius), $\theta = -17.5°$ (angle)

$1.90569*10^8 \cong 190569292 = 1.90569292 \times 10^8 = p(100)$

From the expression concerning the number of transverse light-cone directions in the bosonic string, we obtain:

For n = 100:

1/(sqrt2) * 100^(-27/4) * exp(4Pi*sqrt100)

Input:

$$\frac{1}{\sqrt{2}}\times100^{-27/4}\exp\left(4\pi\sqrt{100}\right)$$

Exact result:

$$\frac{e^{40\pi}}{20\,000\,000\,000\,000\sqrt{5}}$$

Decimal approximation:

$8.40502783126819147372758019124349641022557263931372 89... \times 10^{40}$

$8.405027831268 * 10^{40}$ for $p_{24}(100)$

For n = 200:

1/(sqrt2) * 200^(-27/4) * exp(4Pi*sqrt200)

Input:

$$\frac{1}{\sqrt{2}} \times 200^{-27/4} \exp\left(4\pi\sqrt{200}\right)$$

Exact result:

$$\frac{e^{40\sqrt{2}\,\pi}}{1\,280\,000\,000\,000\,000 \times 2^{3/4}\sqrt{5}}$$

Decimal approximation:

$3.150027047216587574212672161433182439214690646257201 8\ldots \times 10^{61}$

3.15002704721658…*10^{61} for $p_{24}(200)$

Property:

$$\frac{e^{40\sqrt{2}\,\pi}}{1\,280\,000\,000\,000\,000 \times 2^{3/4}\sqrt{5}}$$ is a transcendental number

Series representations:

$$\frac{200^{-27/4}\exp(4\pi\sqrt{200})}{\sqrt{2}} = \frac{\exp\left(4\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (200-z_0)^k z_0^{-k}}{k!}\right)}{1\,280\,000\,000\,000\,000\sqrt[4]{2}\sqrt{5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \le 0))$

$$\frac{200^{-27/4}\exp\left(4\pi\sqrt{200}\right)}{\sqrt{2}} = \frac{\exp\left(4\pi\exp\left(i\pi\left\lfloor\frac{\arg(200-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(200-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{1\,280\,000\,000\,000\,000\sqrt[4]{2}\sqrt{5}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{200^{-27/4}\exp\left(4\pi\sqrt{200}\right)}{\sqrt{2}} = \left(\exp\left(4\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(200-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(200-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(200-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\right)\Big/\left(1\,280\,000\,000\,000\,000\sqrt[4]{2}\sqrt{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

Thence, from the partition number formula, we obtain:

$p(100) = 190569292 = 1.90569292 \times 10^8$

$p(200) = 3972999029388 = 3.972999029388 \times 10^{12}$

while, from the formula concerning the number of transverse light-cone directions in the bosonic string, we obtain:

$p_{24}(100) = 8.405027831268 * 10^{40}$

$p_{24}(200) = 3.15002704721658 * 10^{61}$

Now, we perform the following multiplication:

p(100) * p(200) = 7.571316121471583532 96 × 10^20

Input interpretation:

$1.90569292 \times 10^8 \times 3.972999029388 \times 10^{12}$

Result:

757 131 612 147 158 353 296

Scientific notation:

$7.57131612147158353296 \times 10^{20}$

$7.57131612147158353296*10^{20}$

and the following division:

$p_{24}(200) / p_{24}(100) = 3.74778895496097406795968 \times 10^{20}$

Input interpretation:

$$\frac{3.15002704721658 \times 10^{61}}{8.405027831268 \times 10^{40}}$$

Result:

$3.74778895496097406795968751501767677442152929195399 23... \times 10^{20}$

$3.747788954960974067959 \times 10^{20}$

And again, dividing the two results, we obtain:

(7.57131612147158353296 × 10^20 / 3.747788954960974067959 × 10^20)

Input interpretation:

$$\frac{7.57131612147158353296 \times 10^{20}}{3.747788954960974067959 \times 10^{20}}$$

Result:

2.02020877174777416547321585117771745245291848913966588220 2...

2.020208771747…

From which:

((26 / sqrt(7.57131612147158353296 × 10^20 / 3.747788954960974067959 × 10^20)))^(25/151)

where 26 is the dimensions number of the Bosonic String Theory (D25-brane) and 151 is a Prime number

Input interpretation:

$$\left(\frac{26}{\sqrt{\frac{7.571316121471583532 96\times 10^{20}}{3.747788954960974067959\times 10^{20}}}}\right)^{25/151}$$

Result:

1.6180277431105619791 10...

1.6180277431…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Input interpretation:

1.618027

Possible closed forms:

$\phi \approx 1.618033988$

$2-\frac{6}{5\pi} \approx 1.6180281365$

$\frac{25\,e}{42} \approx 1.6180248978$

From this last closed form, we obtain:

(5^2* e)/(21*2)

where 2, 5 and 21 are Fibonacci numbers

Input:

$$\frac{5^2\,e}{21\times 2}$$

Result:

$$\frac{25\,e}{42}$$

Decimal approximation:

1.6180248978922888305715996853289657724745518414880711755 75...

1.618024897…. as above

Property:

$\frac{25\,e}{42}$ is a transcendental number

Alternative representation:

$$\frac{5^2\,e}{21\times 2} = \frac{5^2\,\exp(z)}{21\times 2} \quad \text{for } z = 1$$

Series representations:

$$\frac{5^2\,e}{21\times 2} = \frac{25}{42}\sum_{k=0}^{\infty}\frac{1}{k!}$$

$$\frac{5^2\,e}{21\times 2} = \frac{25}{84}\sum_{k=0}^{\infty}\frac{1+k}{k!}$$

$$\frac{5^2\,e}{21\times 2} = \frac{25\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}{42\,z}$$

Observations

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden

ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

References

Chapter 16
String thermodynamics and black holes -
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A CONVERGENT SERIES FOR THE PARTITION FUNCTION P(n)
BY HANS RADEMACHER - DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA - Communicated January 9, 1937