On the possible mathematical connections between some Ramanujan-Cardy-Rademacher formulas, various parameters of Open String, Particle Physics, ϕ and $\zeta(2)$

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Abstract

In this paper we describe and analyze new possible mathematical connections between some Ramanujan-Cardy-Rademacher formulas, various parameters of Open String, Particle Physics, ϕ and $\zeta(2$

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https://opc.mfo.de/detail?photo_id=10196&would_like_to_publish=1





https://blog.vedicmathsindia.org/srinivasa-ramanujan/

From:

OPEN STRING ORBIFOLDS
Gianfranco PRADISI and Augusto SAGNOTTI - Received 11 August 1988

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One can then argue as in ref. [9]. There are three distinct types of Klein bottle terms. They correspond to three distinct types of annulus terms, namely (NN,+), (ND,+) and (DD,+). Equality on the double leads again to the Chan–Paton multiplicities $\mathrm{Tr}(\mathbb{1}_N)$ and $\mathrm{Tr}(\mathbb{1}_D)$ of eq. (17). Then, in the closed channel, the amplitudes add up nicely to allow the principal part prescriptions

$$Z^{(a)}_{(NN,+)} + Z^{(k)}_{(++)} + Z^{(m)}_{(++)} = \frac{2^{13}}{4} \frac{V}{(4\pi^2\alpha')^{n/2}} \int_{-1}^{1} \frac{dq}{2\pi q} \frac{1}{\eta(q)^{24}}, \tag{20}$$

$$Z^{(a)}_{(DD,+)} + Z^{(k)}_{(+-)} + Z^{(m)}_{(+-)} = \frac{2^{13-n/2}}{4} \int_{-1}^{1} \frac{dq}{2\pi q} \left(\frac{-\log|q|}{2\pi} \right)^{n/2} \frac{1}{\eta(q)^{24}}, \tag{21}$$

where V denotes the (diverging) volume of the torus covering the orbifold. The term in eq. (21) deserves some comments, because of the presence of the logarithm. This is precisely what the proper choice of critical dimension is meant to avoid in the usual NN terms. In the DD terms, however, it signals the flow of momentum in the transverse channel, not forbidden by the Dirichlet conditions. Such a term appeared rather discouraging to the authors of refs. [14,15]. Here we see the corresponding cut as a sequence of poles that coalesce in the limit of infinite radius.

These results concern Z_2 orbifold compactifications of the SO(8192) model. We now sketch how they are

For n = 6

$$Z^{(a)}_{(NN,+)} + Z^{(k)}_{(++)} + Z^{(m)}_{(++)} = \frac{2^{13}}{4} \frac{V}{(4\pi^2\alpha')^{n/2}} \int_{-1}^{1} \frac{dq}{2\pi q} \frac{1}{\eta(q)^{24}},$$

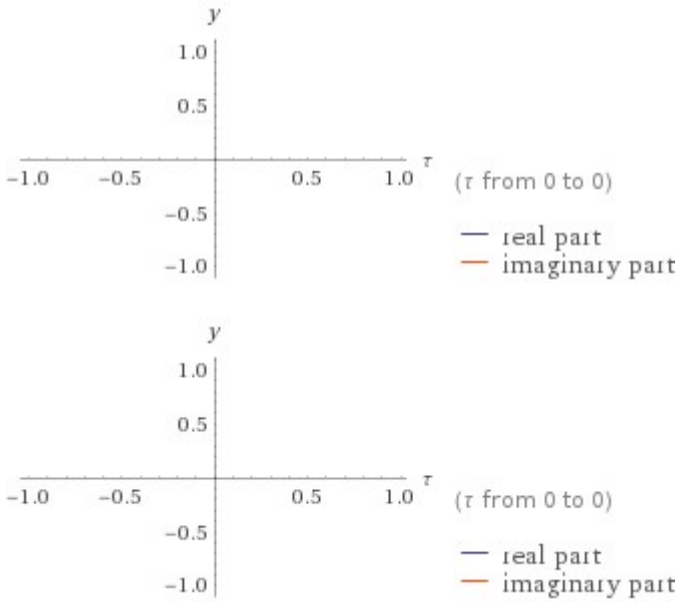
Dedekind eta function^24

Input:

$\eta(\tau)^{24}$

$\eta(\tau)$ is the Dedekind eta function

Plots:



Numerical roots:

$\tau \approx -0.711379 + 32.6616\,i\ldots$

$\tau \approx -0.499999 + 32.6718\,i\ldots$

$\tau \approx 1.51946 \times 10^{-6} + 32.8425\,i\ldots$

$\tau \approx 0.500002 + 32.6718\,i\ldots$

$\tau \approx 0.711381 + 32.6616\,i\ldots$

Input interpretation:

$0.711381 + 32.6616\,i$

i is the imaginary unit

Result:

0.711381... +
32.6616... i

Polar coordinates:

$r = 32.6693$ (radius), $\theta = 88.7523°$ (angle)
32.6693

Series expansion at $\tau = 0$:

$$\frac{e^{-(2\,i\,\pi)/\tau}}{\tau^{12}}$$

Alternative representations:

$$\eta(\tau)^{24} = \frac{g_2^3 - 27\,g_3^2}{\pi^{12}} \quad \text{for } (\{g_2,\, g_3\} = \text{WeierstrassInvariants}[\{1,\, \tau\}] \text{ and } \text{Im}(\tau) > 0)$$

$$\eta(\tau)^{24} = \frac{g_2^3 - 27\,g_3^2}{(2\,\pi)^{12}} \quad \text{for } \left(\{g_2,\, g_3\} = \text{WeierstrassInvariants}\left[\left\{\frac{1}{2},\, \frac{\tau}{2}\right\}\right] \text{ and } \text{Im}(\tau) > 0\right)$$

$$\eta(\tau)^{24} = \left(e^{1/12\,\pi\,(i\,\tau)}\,\vartheta_3\left(\frac{1}{2}\,(\tau+1)\,\pi,\, e^{3\,\pi\,i\,\tau}\right)\right)^{24} \quad \text{for } \text{Im}(\tau) > 0$$

Series representations:

$$\eta(\tau)^{24} = \exp\left(2\,i\,\pi\,\tau - 24\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\frac{e^{2\,i\,k\,n\,\pi\,\tau}}{k}\right)$$

$$\eta(\tau)^{24} = e^{2\,i\,\pi\,\tau}\left(\sum_{k=-\infty}^{\infty}(-1)^k\,e^{i\,k\,(-1+3\,k)\,\pi\,\tau}\right)^{24}$$

$$\eta(\tau)^{24} = \frac{1}{\left(\sum_{k=0}^{\infty}e^{2\,i\,(-1/24+k)\,\pi\,\tau}\,p(k)\right)^{24}}$$

Integral representation:

$$\eta(\tau)^{24} = \frac{\exp\left(\frac{24\,i}{\pi}\int_i^{\tau}\text{WeierstrassZeta}[1,\, \text{WeierstrassInvariants}[\{1,\, t\}]]\,dt\right)\Gamma\left(\frac{1}{4}\right)^{24}}{16\,777\,216\,\pi^{18}}$$

$\zeta(u;\, g_2,\, g_3)$ is the Weierstrass zeta function

(2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(32.6693))))x

Where $V = 2\pi^2 r R^2$ for r = 4 and R = 8; and α' = 0.988

Input interpretation:

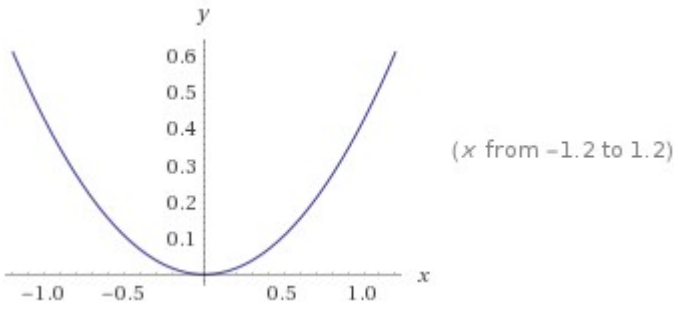
$$\frac{2^{13}}{4} \times \frac{2\pi^2 \times 4 \times 64}{(4\pi^2 \times 0.988)^3} \int \left(\frac{1}{2\pi} \times \frac{1}{32.6693}\right) x \, dx$$

Result:

$0.424815\, x^2$

0.424815

Plot:



Alternate form assuming x is real:

$0.424815\, x^2 + 0$

Indefinite integral assuming all variables are real:

$0.141605\, x^3$ + constant

(2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(32.6693))))x, [0, 1]

Definite integral:

$$174.401 \int_0^1 0.0048717\, x \, dx = 0.424815$$

1+1/sqrt[1/(((((2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3
integrate(((1/(2Pi)*1/(32.6693))))x, [0, 1])))]

Input interpretation:

$$1+\frac{1}{\sqrt{\frac{1}{\frac{2^{13}}{4}\times\frac{2\pi^2\times4\times64}{(4\pi^2\times0.988)^3}\int_0^1\left(\frac{1}{2\pi}\times\frac{1}{32.6693}\right)x\,dx}}}$$

Result:

1.65178

1.65178 result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Computation result:

$$1+\frac{1}{\sqrt{\frac{1}{\frac{2^{13}\left(2\pi^2\,4\times64\right)\int_0^1\frac{x}{(2\pi)\,32.6693}\,dx}{4\left(4\pi^2\,0.988\right)^3}}}} = 1.65178$$

or:

From

We can now display the contributions corresponding to the four sections for the Möbius strip. They are

$$Z^{(m)}_{(++)} = -\tfrac{1}{8}\,\mathrm{Tr}(\mathbb{1}_N)\int_0^\infty \frac{d\tau}{\tau^{14-n/2}}\,\frac{\sum_\Lambda \exp(-\pi\tau\alpha' p^2)}{\eta(\frac{1}{2}i\tau+\frac{1}{2})^{24}}. \tag{12}$$

$$Z^{(m)}_{(+-)} = -\tfrac{1}{8}\,\mathrm{Tr}(\mathbb{1}_D)\int_0^\infty \frac{d\tau}{\tau^{14-n/2}}\,\frac{2^n\sum_\Lambda \exp(-\pi\tau w^2/\alpha') + 2^n(2^n-1)\sum_{\Lambda'}\exp(-\pi\tau w'^2/\alpha')}{\eta(\frac{1}{2}i\tau+\frac{1}{2})^{24}}. \tag{13}$$

$$Z^{(m)}_{(-+)} = -\tfrac{1}{8}\cdot 2^2\,\mathrm{Tr}(\mathbb{1}_D)\int_0^\infty \frac{d\tau}{\tau^{14-n/2}}\,\frac{[\theta[^0_0](0|\frac{1}{2}i\tau+\frac{1}{2})\theta[^0_1](0|\frac{1}{2}i\tau+\frac{1}{2})]^{n/2}}{\eta(\frac{1}{2}i\tau+\frac{1}{2})^{24}}, \tag{14}$$

$$Z^{(m)}_{(--)} = -\tfrac{1}{8}\,\mathrm{Tr}(\mathbb{1}_N)\int_0^\infty \frac{d\tau}{\tau^{14-n/2}}\,\frac{[\theta[^0_0](0|\frac{1}{2}i\tau+\frac{1}{2})\theta[^0_1](0|\frac{1}{2}i\tau+\frac{1}{2})]^{n/2}}{\eta(\frac{1}{2}i\tau+\frac{1}{2})^{24}}. \tag{15}$$

considering τ = 1 in the following part of the above formulas:

$$\eta\left(\tfrac{1}{2}i\tau+\tfrac{1}{2}\right)^{24}$$

we obtain:

(2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(eta (1/2*i+1/2)^24))))x, [0, 1]

Input:

$$\frac{2^{13}}{4}\times\frac{2\pi^2\times4\times64}{(4\pi^2\times0.988)^3}\int_0^1\left(\frac{1}{2\pi}\times\frac{1}{\eta\left(\frac{1}{2}i+\frac{1}{2}\right)^{24}}\right)x\,dx$$

$\eta(\tau)$ is the Dedekind eta function

i is the imaginary unit

Result:

$$-121.46-3.28004\times10^{-14}\,i$$

Computation result:

$$\frac{2^{13}\,(2\pi^2\,4\times64)}{4\,(4\pi^2\,0.988)^3}\int_0^1\frac{x}{(2\pi)\,\eta\left(\frac{i}{2}+\frac{1}{2}\right)^{24}}\,dx=-121.46-3.28004\times10^{-14}\,i$$

Alternate form:

$$-121.46$$

-121.46

From which, we obtain:

(((((2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(eta (1/2*i+1/2)^24))))x, [0, 1])))))-((5+sqrt5)/2)

Input:

$$\frac{2^{13}}{4}\times\frac{2\pi^2\times4\times64}{(4\pi^2\times0.988)^3}\int_0^1\left(\frac{1}{2\pi}\times\frac{1}{\eta\left(\frac{1}{2}i+\frac{1}{2}\right)^{24}}\right)x\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)$$

$\eta(\tau)$ is the Dedekind eta function

i is the imaginary unit

Result:

$-125.078 - 3.28004\times10^{-14}\, i$

Computation result:

$$\frac{2^{13}\,(2\,\pi^2\,4\times64)}{4\,(4\,\pi^2\,0.988)^3}\int_0^1 \frac{x}{(2\,\pi)\,\eta\left(\frac{i}{2}+\frac{1}{2}\right)^{24}}\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)=-125.078-3.28004\times10^{-14}\,i$$

Alternate form:

-125.078

-125.078 result very near to the Higgs boson mass 125.18 GeV with negative sign

and:

(((((2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(eta (1/2*i+1/2)^24))))x, [0, 1])))))-((5+sqrt5)/2)-11-3

Input:

$$\frac{2^{13}}{4}\times\frac{2\,\pi^2\times4\times64}{(4\,\pi^2\times0.988)^3}\int_0^1\left(\frac{1}{2\,\pi}\times\frac{1}{\eta\left(\frac{1}{2}\,i+\frac{1}{2}\right)^{24}}\right)x\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)-11-3$$

$\eta(\tau)$ is the Dedekind eta function

i is the imaginary unit

Result:

$-139.078 - 3.28004\times10^{-14}\, i$

Computation result:

$$\frac{2^{13}\,(2\,\pi^2\,4\times64)}{4\,(4\,\pi^2\,0.988)^3}\int_0^1 \frac{x}{(2\,\pi)\,\eta\left(\frac{i}{2}+\frac{1}{2}\right)^{24}}\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)-11-3=$$
$$-139.078-3.28004\times10^{-14}\,i$$

Alternate form:

-139.078

-139.078 result practically equal to the rest mass of Pion meson 139.57 MeV with negative sign

27*1/2((((((((((2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(eta (1/2*i+1/2)^24))))x, [0, 1])))-((5+sqrt5)/2)-3)))))

Input:

$$27\times\frac{1}{2}\left(\frac{2^{13}}{4}\times\frac{2\pi^2\times4\times64}{(4\pi^2\times0.988)^3}\int_0^1\left(\frac{1}{2\pi}\times\frac{1}{\eta\left(\frac{1}{2}i+\frac{1}{2}\right)^{24}}\right)x\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)-3\right)$$

$\eta(\tau)$ is the Dedekind eta function

i is the imaginary unit

Result:

$$-1729.05-4.42806\times10^{-13}\,i$$

Computation result:

$$\frac{27}{2}\left(\frac{2^{13}\,(2\pi^2\,4\times64)}{4\,(4\pi^2\,0.988)^3}\int_0^1\frac{x}{(2\pi)\,\eta\left(\frac{i}{2}+\frac{1}{2}\right)^{24}}\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)-3\right)=$$

$$-1729.05-4.42806\times10^{-13}\,i$$

Alternate form:

$$-1729.05$$

-1729.05

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

[27*1/2((((((((((2^13)/4 (2Pi^2*4*64) / (4Pi^2*0.988)^3 integrate(((1/(2Pi)*1/(eta (1/2*i+1/2)^24))))x, [0, 1])))))-((5+sqrt5)/2)-3))))))]^(1/15)

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{2^{13}}{4}\times\frac{2\pi^2\times4\times64}{(4\pi^2\times0.988)^3}\int_0^1\left(\frac{1}{2\pi}\times\frac{1}{\eta\left(\frac{1}{2}i+\frac{1}{2}\right)^{24}}\right)x\,dx-\frac{1}{2}\left(5+\sqrt{5}\right)-3\right)}$$

$\eta(\tau)$ is the Dedekind eta function

i is the imaginary unit

Result:

$1.6079 - 0.341769\,i$

Computation result:

$$\sqrt[15]{\frac{27}{2}\left(\frac{2^{13}\left(2\pi^2\,4\times64\right)\int_0^1\frac{x}{(2\pi)\eta\left(\frac{i}{2}+\frac{1}{2}\right)^{24}}\,dx}{4\left(4\pi^2\,0.988\right)^3}-\frac{1}{2}\left(5+\sqrt{5}\right)-3\right)}=1.6079-0.341769\,i$$

1.6079 - 0.341769 i

Input interpretation:

$1.6079+i\times(-0.341769)$

i is the imaginary unit

Result:

1.6079... −
0.341769... i

Polar coordinates:

$r = 1.64382$ (radius), $\theta = -12.°$ (angle)

$1.64382 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Now, from (21)

$$Z^{(a)}_{(DD,+)} + Z^{(k)}_{(+-)} + Z^{(m)}_{(+-)} = \frac{2^{13-n/2}}{4} \int_{-1}^{1} \frac{dq}{2\pi q} \left(\frac{-\log|q|}{2\pi} \right)^{-n/2} \frac{1}{\eta(q)^{24}},$$

and for

$\eta(\frac{1}{2}i\tau)^{24}$ that is a part of the following formula

$$Z^{(a)}_{(DD,+)} = \frac{1}{8} \mathrm{Tr}(1_D)^2 \int_0^{\infty} \frac{d\tau}{\tau^{14-n/2}} \frac{2^n \sum_A \exp(-\pi\tau w^2/\alpha') + 2^n(2^n-1) \sum_{A'} \exp(-\pi\tau w'^2/\alpha')}{\eta(\frac{1}{2}i\tau)^{24}},$$

we obtain:

(2^10)/4 integrate(((1/(2Pi*(eta(1/2*i)))*(((-ln(eta(1/2*i))/(2Pi))))^-3 * 1/(eta (1/2*i)^24))))x, [0, 1]

Input:

$$\frac{2^{10}}{4} \int_0^1 \frac{\frac{1}{2\pi\eta\left(\frac{1}{2}i\right)} \times \frac{1}{\eta\left(\frac{1}{2}i\right)^{24}}}{\left(-\frac{\log\left(\eta\left(\frac{1}{2}i\right)\right)}{2\pi}\right)^3} x\,dx$$

$\eta(\tau)$ is the Dedekind eta function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$-\frac{512\pi^2}{\eta\left(\frac{i}{2}\right)^{25} \log^3\left(\eta\left(\frac{i}{2}\right)\right)} \approx 7.61208 \times 10^7$$

$7.61208*10^7$

Computation result:

$$\frac{2^{10}}{4}\int_0^1 \frac{x}{\left(2\pi\eta\left(\frac{i}{2}\right)\right)\left(-\frac{\log\left(\eta\left(\frac{i}{2}\right)\right)}{2\pi}\right)^3 \eta\left(\frac{i}{2}\right)^{24}}\,dx = -\frac{512\pi^2}{\eta\left(\frac{i}{2}\right)^{25}\log^3\left(\eta\left(\frac{i}{2}\right)\right)}$$

From which:

((((((2^10)/4 integrate(((1/(2Pi*(eta(1/2*i)))*(((-ln(eta(1/2*i))/(2Pi))))^-3 * 1/(eta (1/2*i)^24))))x, [0, 1])))))^1/38

Input:

$$\sqrt[38]{\frac{2^{10}}{4}\int_0^1 \frac{\frac{1}{2\pi\eta\left(\frac{1}{2}i\right)}\times\frac{1}{\eta\left(\frac{1}{2}i\right)^{24}}}{\left(-\frac{\log\left(\eta\left(\frac{1}{2}i\right)\right)}{2\pi}\right)^3}\,x\,dx}$$

$\eta(\tau)$ is the Dedekind eta function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$\frac{2^{9/38}\sqrt[19]{\pi}\ \sqrt[38]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{\eta\left(\frac{i}{2}\right)^{25/38}} \approx 1.61216$$

1.61216 result that is a good approximation to the value of the golden ratio 1.618033988749...

Computation result:

$$\sqrt[38]{\frac{2^{10}}{4}\int_0^1 \frac{x}{\left(2\pi\eta\left(\frac{i}{2}\right)\right)\left(-\frac{\log\left(\eta\left(\frac{i}{2}\right)\right)}{2\pi}\right)^3 \eta\left(\frac{i}{2}\right)^{24}}\,dx} = \frac{2^{9/38}\sqrt[19]{\pi}\ \sqrt[38]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{\eta\left(\frac{i}{2}\right)^{25/38}}$$

Alternate form:

$$\frac{2^{9/38}\, e^{(3\,i\,\pi)/38}\, \sqrt[19]{\pi}}{\eta\left(\frac{i}{2}\right)^{25/38} \log^{3/38}\left(\eta\left(\frac{i}{2}\right)\right)}$$

Alternate forms:

$$\frac{2^{9/38}\, \sqrt[19]{\pi}}{\eta\left(\frac{i}{2}\right)^{25/38} \left(-\log\left(\eta\left(\frac{i}{2}\right)\right)\right)^{3/38}}$$

$$\frac{2^{9/38}\, e^{(3\,i\,\pi)/38}\, \sqrt[19]{\pi}}{\eta\left(\frac{i}{2}\right)^{25/38} \log^{3/38}\left(\eta\left(\frac{i}{2}\right)\right)}$$

and also:

1/3((((((2^10)/4 integrate(((1/(2Pi*(eta(1/2*i)))*(((-ln(eta(1/2*i))/(2Pi))))^-3 * 1/(eta (1/2*i)^24))))x, [0, 1])))))^1/3 – 2

Input:

$$\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1 \frac{\frac{1}{2\,\pi\,\eta\left(\frac{1}{2}\,i\right)} \times \frac{1}{\eta\left(\frac{1}{2}\,i\right)^{24}}}{\left(-\frac{\log\left(\eta\left(\frac{1}{2}\,i\right)\right)}{2\,\pi}\right)^3}\, x\, dx} - 2$$

$\eta(\tau)$ is the Dedekind eta function
$\log(x)$ is the natural logarithm
i is the imaginary unit

Result:

$$-2 + \frac{8\,\pi^{2/3}\, \sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}} \approx 139.269$$

139.269 result practically equal to the rest mass of Pion meson 139.57 MeV

Computation result:

$$\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1 \frac{x}{\left(2\pi\,\eta\left(\frac{i}{2}\right)\right)\left(-\frac{\log\left(\eta\left(\frac{i}{2}\right)\right)}{2\pi}\right)^3 \eta\left(\frac{i}{2}\right)^{24}}\,dx} - 2 = -2 + \frac{8\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}$$

Alternate forms:

$$-2 - \frac{8\sqrt[3]{-1}\;e^{-(i\pi)/3}\,\pi^{2/3}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}\log\left(\eta\left(\frac{i}{2}\right)\right)}$$

$$\frac{2\left(4\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}} - 3\,\eta\left(\frac{i}{2}\right)^{25/3}\right)}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}$$

$$-2 - \frac{8\,\pi^{2/3}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}\log\left(\eta\left(\frac{i}{2}\right)\right)}$$

1/3((((((2^10)/4 integrate(((1/(2Pi*(eta(1/2*i)))*(((-ln(eta(1/2*i))/(2Pi))))^-3 * 1/(eta(1/2*i)^24))))x, [0, 1])))))^1/3 - 13-3

Input:

$$\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1 \frac{\frac{1}{2\pi\,\eta\left(\frac{1}{2}i\right)}\times\frac{1}{\eta\left(\frac{1}{2}i\right)^{24}}}{\left(-\frac{\log\left(\eta\left(\frac{1}{2}i\right)\right)}{2\pi}\right)^3}\,x\,dx} - 13 - 3$$

$\eta(\tau)$ is the Dedekind eta function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$-16+\frac{8\,\pi^{2/3}\,\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}\approx 125.269$$

125.269 result very near to the Higgs boson mass 125.18 GeV

Computation result:

$$\frac{1}{3}\sqrt[3]{\frac{1}{4}\times 2^{10}\int_0^1\frac{x}{\left(2\,\pi\,\eta\left(\frac{i}{2}\right)\right)\left(-\frac{\log\left(\eta\left(\frac{i}{2}\right)\right)}{2\,\pi}\right)^3\eta\left(\frac{i}{2}\right)^{24}}\,dx}-13-3=-16+\frac{8\,\pi^{2/3}\,\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}$$

Alternate forms:

$$-16-\frac{8\,\pi^{2/3}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}\log\left(\eta\left(\frac{i}{2}\right)\right)}$$

$$-16-\frac{8\sqrt[3]{-1}\;e^{-(i\,\pi)/3}\,\pi^{2/3}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}\log\left(\eta\left(\frac{i}{2}\right)\right)}$$

$$\frac{8\left(\pi^{2/3}\,\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}-6\,\eta\left(\frac{i}{2}\right)^{25/3}\right)}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}$$

27*1/2((((((1/3((((((2^10)/4 integrate(((1/(2Pi*(eta(1/2*i)))*(((-ln(eta(1/2*i))/(2Pi))))^-3 * 1/(eta (1/2*i)^24))))x, [0, 1])))))^1/3 - 13))))))-3

Input:

$$27\times\frac{1}{2}\left(\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1 \frac{\frac{1}{2\pi\eta\left(\frac{1}{2}i\right)}\times\frac{1}{\eta\left(\frac{1}{2}i\right)^{24}}}{\left(-\frac{\log\left(\eta\left(\frac{1}{2}i\right)\right)}{2\pi}\right)^3}\,x\,dx}-13\right)-3$$

$\eta(\tau)$ is the Dedekind eta function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$-3+\frac{27}{2}\left(-13+\frac{8\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}\right)\approx 1728.63$$

1728.63

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Computation result:

$$\frac{27}{2}\left(\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1 \frac{x}{\left(2\pi\eta\left(\frac{i}{2}\right)\right)\left(-\frac{\log\left(\eta\left(\frac{i}{2}\right)\right)}{2\pi}\right)^3\eta\left(\frac{i}{2}\right)^{24}}\,dx}-13\right)-3=$$

$$-3+\frac{27}{2}\left(-13+\frac{8\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}\right)$$

Alternate forms:

$$-\frac{357}{2}+\frac{36\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{\eta\left(\frac{i}{2}\right)^{25/3}}$$

$$-3+\frac{27}{2}\left(-13-\frac{8\sqrt[3]{-1}\;e^{-(i\pi)/3}\,\pi^{2/3}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}\log\left(\eta\left(\frac{i}{2}\right)\right)}\right)$$

$$\frac{3\left(24\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}-119\,\eta\left(\frac{i}{2}\right)^{25/3}\right)}{2\,\eta\left(\frac{i}{2}\right)^{25/3}}$$

[27*1/2((((((1/3((((((2^10)/4 integrate(((1/(2Pi*(eta(1/2*i)))*(((-ln(eta(1/2*i))/(2Pi))))^-3 * 1/(eta (1/2*i)^24))))x, [0, 1])))))^1/3 - 13)))))-3]^1/15

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1\frac{\frac{1}{2\,\pi\,\eta\left(\frac{1}{2}\,i\right)}\times\frac{1}{\eta\left(\frac{1}{2}\,i\right)^{24}}}{\left(-\frac{\log\left(\eta\left(\frac{1}{2}\,i\right)\right)}{2\,\pi}\right)^3}\,x\,dx}-13\right)}-3$$

$\eta(\tau)$ is the Dedekind eta function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$\sqrt[15]{-3+\frac{27}{2}\left(-13+\frac{8\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}\right)}\approx 1.64379$$

$1.64379\;\approx\zeta(2)=\frac{\pi^2}{6}=1.644934\ldots$

Computation result:

$$\sqrt[15]{\frac{27}{2}\left(\frac{1}{3}\sqrt[3]{\frac{2^{10}}{4}\int_0^1 \frac{x}{\left(2\pi\,\eta\left(\frac{i}{2}\right)\right)\left(-\frac{\log\left(\eta\left(\frac{i}{2}\right)\right)}{2\pi}\right)^3 \eta\left(\frac{i}{2}\right)^{24}}\,dx} - 13\right) - 3} = \sqrt[15]{-3+\frac{27}{2}\left(-13+\frac{8\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{3\,\eta\left(\frac{i}{2}\right)^{25/3}}\right)}$$

Alternate forms:

$$\sqrt[15]{-\frac{357}{2}-\frac{36\,\pi^{2/3}}{\eta\left(\frac{i}{2}\right)^{25/3}\log\left(\eta\left(\frac{i}{2}\right)\right)}}$$

$$\sqrt[15]{-\frac{357}{2}+\frac{36\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}}{\eta\left(\frac{i}{2}\right)^{25/3}}}$$

$$\frac{\sqrt[15]{\frac{3}{2}\left(24\,\pi^{2/3}\sqrt[3]{-\frac{1}{\log^3\left(\eta\left(\frac{i}{2}\right)\right)}}-119\,\eta\left(\frac{i}{2}\right)^{25/3}\right)}}{\eta\left(\frac{i}{2}\right)^{5/9}}$$

From:

Theory and Phenomenology of Type I strings and M-theory
Emilian Dudas - arXiv:hep-ph/0006190v2 6 Oct 2000

We have that:

The breaking of supersymmetry gives rise to a vacuum energy localized on the $D\bar{5}$ branes, and thus to a tree-level potential for the NS moduli, that can be extracted from the corresponding uncancelled NS tadpoles. A simple inspection shows that the only non-vanishing ones correspond to the NS characters V_4O_4 and O_4V_4 associated to the 6D dilaton ϕ_6 and to the internal volume v:

$$\frac{2^{-5}}{4}\left\{\left((N-32)\sqrt{v}+\frac{D+32}{\sqrt{v}}\right)^2 V_4O_4+\left((N-32)\sqrt{v}-\frac{D+32}{\sqrt{v}}\right)^2 O_4V_4\right\}. \qquad (140)$$

Using factorization and the values $N = D = 32$ needed to cancel the RR tadpoles, the potential (in the string frame) is:

$$V_{\rm eff}=c\frac{e^{-\phi_6}}{\sqrt{v}}=ce^{-\phi_{10}}=\frac{c}{g^2_{\rm YM}}, \qquad (141)$$

where ϕ_{10} is the 10D dilaton, that determines the Yang-Mills coupling $g_{\rm YM}$ on the $D\bar{5}$ branes, and c is some *positive* numerical constant. The potential (141) is clearly localized on the $D\bar{5}$ branes, and is positive. This can be understood by noticing that the $O9_+$ plane contribution to vacuum energy is negative and exactly cancels for $N = 32$. This fixes the $D\bar{5}$ brane contribution to the vacuum energy, that is thus positive, consistently with the interpretation of this mechanism as global supersymmetry breaking. The potential (141) has the usual runaway behavior in the dilaton field, as expected by general arguments.

From

$$\frac{2^{-5}}{4}\left\{\left((N-32)\sqrt{v}+\frac{D+32}{\sqrt{v}}\right)^2 V_4O_4+\left((N-32)\sqrt{v}-\frac{D+32}{\sqrt{v}}\right)^2 O_4V_4\right\}.$$

we consider: N = D = 32 , v = 2 and $V_4O_4 = O_4V_4 = 1$; we obtain

2^(-5)/(4) * [((32+32)/(sqrt2))^2+(-((32+32)/sqrt2))^2]

Input:

$$\frac{1}{2^5\times 4}\left(\left(\frac{32+32}{\sqrt{2}}\right)^2+\left(-\frac{32+32}{\sqrt{2}}\right)^2\right)$$

Result:

32

32

$$\frac{1}{\alpha_a(\mu)} = \frac{1}{\alpha_a(\mu_0)} + \frac{1}{2\pi}\sum_r \mathrm{Str}\int_{1/\mu^2}^{1/\mu_0^2}\frac{dt}{t}Q_{a,r}^2\left(\frac{1}{12}-\chi_r^2\right)\left(\sum_n e^{-tm_{n,r}^2(R_{||})}+e^{-tm_r^2}\right), \qquad (153)$$

where we separated the mass operator into a part containing fields with KK modes and a part containing fields without KK modes. Indeed, consider again for concreteness gauge couplings of a D9 brane and consider δ large compact dimensions $R_{||}M_i >> 1$ parallel to D9 and orthogonal to D5. Then the 99 states will have associated KK states, but 95 states will not. Evaluating (153) with $\mu_0 = M_Z$, one finds

$$\begin{aligned}\frac{1}{\alpha_a(\mu)} &= \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} - \frac{\tilde{b}_a}{2\pi}\int_{1/\mu^2}^{1/M_Z^2}\frac{dt}{t}\theta_3^\delta\left(\frac{it}{\pi R_{||}^2}\right)\\ &\simeq \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} + \frac{\tilde{b}_a}{2\pi}\ln(\mu R_{||}) - \frac{\tilde{b}_a}{2\pi}[(\mu R_{||})^\delta - 1]\,. \qquad (154)\end{aligned}$$

Note that $\mu \to 0$ smoothly.

From (154)

$$\begin{aligned}\frac{1}{\alpha_a(\mu)} &= \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} - \frac{\tilde{b}_a}{2\pi}\int_{1/\mu^2}^{1/M_Z^2}\frac{dt}{t}\theta_3^\delta\left(\frac{it}{\pi R_{||}^2}\right)\\ &\simeq \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} + \frac{\tilde{b}_a}{2\pi}\ln(\mu R_{||}) - \frac{\tilde{b}_a}{2\pi}[(\mu R_{||})^\delta - 1]\,.\end{aligned}$$

we consider:

60 = 1/a-b/(2Pi) * ln(1/12*1/c)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)

and obtain:

Input:

$$60 = \frac{1}{a} - \frac{b}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{c}\right) - \frac{6}{2\pi}\log(25) + \frac{6}{2\pi}(25-1)$$

$\log(x)$ is the natural logarithm

Exact result:

$$60 = \frac{1}{a} - \frac{b\log\left(\frac{1}{12c}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi}$$

Alternate form assuming a, b, and c are real:

$$b\log\left(\frac{1}{12c}\right) + 12(10\pi + \log(5)) = \frac{2\pi}{a} + 144$$

Alternate forms:

$$60 = \frac{1}{a} - \frac{b\log\left(\frac{1}{12c}\right)}{2\pi} - \frac{6(\log(5)-12)}{\pi}$$

$$ab\left(\log\left(\frac{1}{3c}\right) - 2\log(2)\right) = a(144 - 120\pi - 12\log(5)) + 2\pi$$

$$60 = \frac{a(-b)\log\left(\frac{1}{12c}\right) + 144a - 6a\log(25) + 2\pi}{2\pi a}$$

Alternate forms assuming a, b, and c are positive:

$$60 = \frac{ab\log(c) + a(b\log(12) + 144 - 12\log(5)) + 2\pi}{2\pi a}$$

$$60 = \frac{1}{a} - \frac{b(-\log(c) - \log(3) - 2\log(2))}{2\pi} + \frac{72}{\pi} - \frac{6\log(5)}{\pi}$$

$$6a(-24 + 20\pi + \log(25)) = ab\log(12c) + 2\pi$$

Real solutions:

$$a = \frac{\pi}{6(-12 + 10\pi + \log(5))}, \quad b = 0, \quad c > 0$$

$$a \neq 0, \quad b \neq 0, \quad c = \frac{1}{12}\times 5^{12/b}\, e^{-(2\pi)/(ab)+(120\pi)/b-144/b}$$

Solution for the variable c:

$$c = \frac{1}{12}\, e^{\frac{2\,(60\,\pi\,a - 72\,a + 3\,a\log(25) - \pi)}{a\,b}}$$

Implicit derivatives:

$$\frac{\partial a(b,c)}{\partial c} = \frac{a^2\,b}{2\,c\,\pi}$$

$$\frac{\partial a(b,c)}{\partial b} = -\frac{a^2\log\left(\frac{1}{12\,c}\right)}{2\,\pi}$$

$$\frac{\partial b(a,c)}{\partial c} = \frac{b}{c\log\left(\frac{1}{12\,c}\right)}$$

$$\frac{\partial b(a,c)}{\partial a} = -\frac{2\,\pi}{a^2\log\left(\frac{1}{12\,c}\right)}$$

$$\frac{\partial c(a,b)}{\partial b} = \frac{c\log\left(\frac{1}{12\,c}\right)}{b}$$

$$\frac{\partial c(a,b)}{\partial a} = \frac{2\,c\,\pi}{a^2\,b}$$

From

$$60 = \frac{1}{a} - \frac{b\log\left(\frac{1}{12\,c}\right)}{2\,\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi}$$

$$a = \frac{\pi}{6\,(-12 + 10\,\pi + \log(5))}, \quad b = 0, \quad c > 0$$

we obtain, inserting the value of *a*

60 = 1/(π/(6 (-12 + 10 π + log(5)))) - (b log(1/(12 c)))/(2 π) + 72/π - (3 log(25))/π

Input:

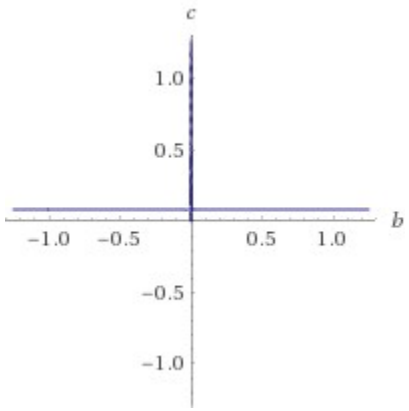
$$60 = \frac{1}{\frac{\pi}{6\,(-12+10\,\pi+\log(5))}} - \frac{b\log\left(\frac{1}{12\,c}\right)}{2\,\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi}$$

$\log(x)$ is the natural logarithm

Exact result:

$$60 = -\frac{b \log\left(\frac{1}{12\,c}\right)}{2\,\pi} + \frac{72}{\pi} - \frac{3 \log(25)}{\pi} + \frac{6\,(-12 + 10\,\pi + \log(5))}{\pi}$$

Implicit plot:



Alternate form assuming b and c are real:

$$b \log\left(\frac{1}{c}\right) = b \log(12)$$

Alternate forms:

$$60 = \frac{b\left(-\log\left(\frac{1}{c}\right)\right) + b \log(12) + 120\,\pi}{2\,\pi}$$

$$60 = \frac{-b \log\left(\frac{1}{12\,c}\right) + 120\,\pi - 6 \log(25) + 12 \log(5)}{2\,\pi}$$

$$60 = \frac{3\,(20\,\pi + 2 \log(5) - \log(25))}{\pi} - \frac{b \log\left(\frac{1}{12\,c}\right)}{2\,\pi}$$

Expanded form:

$$60 = -\frac{b \log\left(\frac{1}{12\,c}\right)}{2\,\pi} + 60 - \frac{3 \log(25)}{\pi} + \frac{6 \log(5)}{\pi}$$

Alternate forms assuming b and c are positive:

$$60 = \frac{b\log(c) + b\log(12) + 120\pi}{2\pi}$$

$$60 = -\frac{b(-\log(c) - \log(3) - 2\log(2))}{2\pi} + \frac{72}{\pi} + \frac{6(-12 + 10\pi + \log(5))}{\pi} - \frac{6\log(5)}{\pi}$$

$$\log(12c) = 0$$

Solutions:

$$c \neq 0, \quad b = 0$$

$$c = \frac{1}{12}$$

1/12

Implicit derivatives:

$$\frac{\partial b(c)}{\partial c} = \frac{b}{c\log\left(\frac{1}{12c}\right)}$$

$$\frac{\partial c(b)}{\partial b} = \frac{c\log\left(\frac{1}{12c}\right)}{b}$$

For

$$c = \frac{1}{12}$$

1/(π/(6 (-12 + 10 π + log(5)))) - (b log(1/(12/12)))/(2 π) + 72/π - (3 log(25))/π

Input:

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{1}{\frac{12}{12}}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi}$$

$\log(x)$ is the natural logarithm

Result:

60

60

Alternative representations:

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} =$$
$$\frac{72}{\pi} - \frac{3\log_e(25)}{\pi} - \frac{b\log_e\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{1}{\frac{\pi}{6(-12+10\pi+\log_e(5))}}$$

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} =$$
$$\frac{72}{\pi} - \frac{3\log(a)\log_a(25)}{\pi} - \frac{b\log(a)\log_a\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{1}{\frac{\pi}{6(-12+10\pi+\log(a)\log_a(5))}}$$

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} =$$
$$\frac{72}{\pi} + \frac{3\,\mathrm{Li}_1(-24)}{\pi} + \frac{b\,\mathrm{Li}_1\left(1-\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{1}{\frac{\pi}{6(-12+10\pi-\mathrm{Li}_1(-4))}}$$

Series representations:

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} =$$
$$60 - \frac{\log\left(\frac{27}{8}\right)}{\pi} + \sum_{k=1}^{\infty} \frac{3\left(-\frac{1}{8}\right)^k\left(-2^{1+k}+3^{-k}\right)}{k\pi}$$

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{\frac{1}{12}}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} =$$
$$60 + 6i\left\lfloor\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi}\right\rfloor + \frac{3\log(z_0)}{\pi} + \sum_{k=1}^{\infty} -\frac{3(-1)^k\left(2(5-z_0)^k - (25-z_0)^k\right)z_0^{-k}}{k\pi}$$

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{1}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} = 60 + 12i\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor - 6i\left\lfloor\frac{\arg(25-x)}{2\pi}\right\rfloor + \frac{3\log(x)}{\pi} + \sum_{k=1}^{\infty} -\frac{3(-1)^k\left(2(5-x)^k - (25-x)^k\right)x^{-k}}{k\pi} \quad \text{for } x < 0$$

Integral representations:

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{1}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} = 60 + \int_1^5 \frac{30-18t}{5\pi t - 6\pi t^2}\,dt$$

$$\frac{1}{\frac{\pi}{6(-12+10\pi+\log(5))}} - \frac{b\log\left(\frac{1}{12}\right)}{2\pi} + \frac{72}{\pi} - \frac{3\log(25)}{\pi} = 60 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\,2^{-1-3s}\times 3^{1-s}\left(-1+2^{1+s}\times 3^s\right)\Gamma(-s)^2\,\Gamma(1+s)}{\pi^2\,\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

$(\pi/(6\,(-12 + 10\,\pi + \log(5)))) = a$

Input:

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

0.024903196179309823497244279261254587428523168158418288455...

0.024903196…

Alternative representations:

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\,(-12+10\,\pi+\log_e(5))}$$

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\,(-12+10\,\pi+\log(a)\,\log_a(5))}$$

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\,(-12+10\,\pi-\mathrm{Li}_1(-4))}$$

Series representations:

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\left(-12+10\,\pi+\log(4)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k}{k}\right)}$$

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\left(-12+10\,\pi+2\,i\,\pi\left\lfloor\frac{\arg(5-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(5-x)^k\,x^{-k}}{k}\right)} \quad \text{for } x<0$$

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\left(-12+10\,\pi+\log(z_0)+\left\lfloor\frac{\arg(5-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(5-z_0)^k\,z_0^{-k}}{k}\right)}$$

Integral representations:

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi}{6\left(-12+10\,\pi+\int_1^5\frac{1}{t}\,dt\right)}$$

$$\frac{\pi}{6\,(-12+10\,\pi+\log(5))} = \frac{\pi^2}{12\,\pi\,(-6+5\,\pi)-3\,i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{4^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

60 = 1/(0.024903196)-b/(2Pi) * ln(1/12*1/12)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)

Input interpretation:

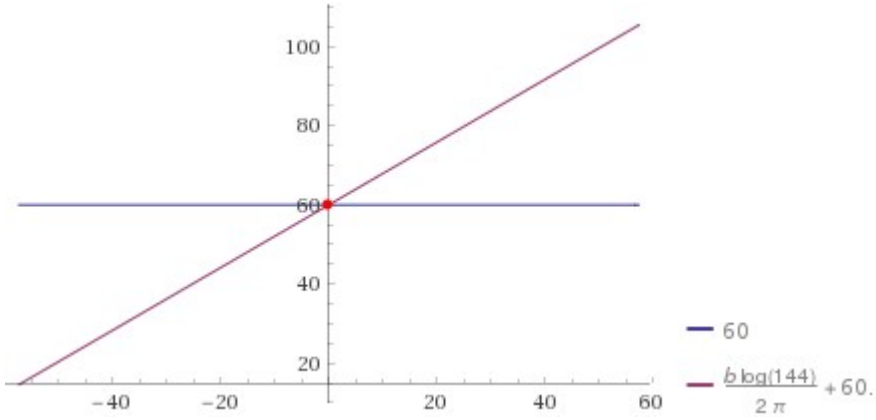
$$60 = \frac{1}{0.024903196}-\frac{b}{2\,\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right)-\frac{6}{2\,\pi}\log(25)+\frac{6}{2\,\pi}\,(25-1)$$

$\log(x)$ is the natural logarithm

Result:

$$60 = \frac{b \log(144)}{2\pi} + 60.$$

Plot:



Alternate forms:

$$60 = 0.79097\, b + 60.$$

$$60 = 19.0986\,(0.0414151\, b + 3.14159)$$

$$-\frac{b \log(144)}{2\pi} - 2.89131 \times 10^{-7} = 0$$

Alternate form assuming b>0:

$$60 = \frac{b\,(4 \log(2) + 2 \log(3))}{2\pi} + 60.$$

Solution:

$$b \approx -3.65539 \times 10^{-7}$$

-3.65539*10^-7

In conclusion, we obtain:

1/(0.024903196)-(-3.65539e-7)/(2Pi) * ln(1/12*1/12)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)

Input interpretation:

$$\frac{1}{0.024903196} - -\frac{3.65539\times10^{-7}}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right) - \frac{6}{2\pi}\log(25) + \frac{6}{2\pi}(25-1)$$

$\log(x)$ is the natural logarithm

Result:

60.000000...

60

Now, the previous expression:

$$\frac{2^{-5}}{4}\left\{\left((N-32)\sqrt{v}+\frac{D+32}{\sqrt{v}}\right)^2 V_4O_4 + \left((N-32)\sqrt{v}-\frac{D+32}{\sqrt{v}}\right)^2 O_4V_4\right\}$$

$$\frac{1}{2^5\times4}\left(\left(\frac{32+32}{\sqrt{2}}\right)^2+\left(-\frac{32+32}{\sqrt{2}}\right)^2\right)$$

32

multiplied by 60, that is the result of the last expression:

$$\frac{1}{0.024903196} - -\frac{3.65539\times10^{-7}}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right) - \frac{6}{2\pi}\log(25) + \frac{6}{2\pi}(25-1)$$

60.000000...

give us:

2^(-5)/(4) * [((32+32)/(sqrt2))^2+(-((32+32)/sqrt2))^2]
[1/(0.024903196)+(3.65539e-7)/(2Pi) * ln(1/12*1/12)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)]

Input interpretation:

$$\frac{1}{2^5\times4}\left(\left(\frac{32+32}{\sqrt{2}}\right)^2+\left(-\frac{32+32}{\sqrt{2}}\right)^2\right)$$
$$\left(\frac{1}{0.024903196}+\frac{3.65539\times10^{-7}}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right) - \frac{6}{2\pi}\log(25) + \frac{6}{2\pi}(25-1)\right)$$

$\log(x)$ is the natural logarithm

Result:

1920.0000...

1920

From which:

27/30 * (((2^(-5)/(4) * [((32+32)/(sqrt2))^2+(-((32+32)/sqrt2))^2] [1/(0.024903196)+(3.65539e-7)/(2Pi) * ln(1/12*1/12)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)]))+1

Input interpretation:

$$\frac{27}{30}\left(\frac{1}{2^5\times 4}\left(\left(\frac{32+32}{\sqrt{2}}\right)^2+\left(-\frac{32+32}{\sqrt{2}}\right)^2\right)\left(\frac{1}{0.024903196}+\frac{3.65539\times 10^{-7}}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right)-\frac{6}{2\pi}\log(25)+\frac{6}{2\pi}(25-1)\right)\right)+1$$

$\log(x)$ is the natural logarithm

Result:

1729.0000...

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and:

2/30 * (((2^(-5)/(4) * [((32+32)/(sqrt2))^2+(-((32+32)/sqrt2))^2] [1/(0.024903196)+(3.65539e-7)/(2Pi) * ln(1/12*1/12)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)]))-3

Input interpretation:

$$\frac{2}{30}\left(\frac{1}{2^5\times 4}\left(\left(\frac{32+32}{\sqrt{2}}\right)^2+\left(-\frac{32+32}{\sqrt{2}}\right)^2\right)\left(\frac{1}{0.024903196}+\frac{3.65539\times 10^{-7}}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right)-\frac{6}{2\pi}\log(25)+\frac{6}{2\pi}(25-1)\right)\right)-3$$

$\log(x)$ is the natural logarithm

Result:

125.00000...

125 result very near to the Higgs boson mass 125.18 GeV

2/30 * (((2^(-5)/(4) * [((32+32)/(sqrt2))^2+(-((32+32)/sqrt2))^2] [1/(0.024903196)+(3.65539e-7)/(2Pi) * ln(1/12*1/12)-6/(2Pi) * ln(25)+6/(2Pi)*(25-1)]))+11

Input interpretation:

$$\frac{2}{30}\left(\frac{1}{2^5\times 4}\left(\left(\frac{32+32}{\sqrt{2}}\right)^2+\left(-\frac{32+32}{\sqrt{2}}\right)^2\right)\left(\frac{1}{0.024903196}+\frac{3.65539\times 10^{-7}}{2\pi}\log\left(\frac{1}{12}\times\frac{1}{12}\right)-\frac{6}{2\pi}\log(25)+\frac{6}{2\pi}(25-1)\right)\right)+11$$

$\log(x)$ is the natural logarithm

Result:

139.00000...

139 result practically equal to the rest mass of Pion meson 139.57 MeV

Now, we have that:

$$V_{eff}\sim e^{-\phi_6}\left[(N_+ + N_- - 32)\sqrt{v}+\frac{(D_+ + D_-)}{\sqrt{v}}\right] \qquad (176)$$

in string units, where Φ_6 is the 6d dilaton. The potential has a minimum and stabilizes the internal space at the value $v_0 = (D_+ + D_-)/(N_+ + N_- - 32)$. We see that, in order to have a very large (very small) compact space, we need a very large number of D9+$\bar{D9}$ (D5+$\bar{D5}$) branes, compatible with (175). This is in principle possible, but of course asks for a dynamical explanation of the very large number of branes and antibranes in the model[20].

From (176), for D = 32 and v = 2 , we obtain:

exp(-x)*(64/(sqrt2))

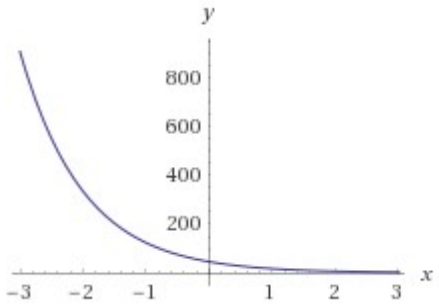
Input:

$$\exp(-x) \times \frac{64}{\sqrt{2}}$$

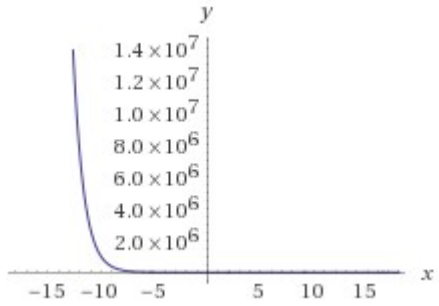
Exact result:

$$32\sqrt{2}\, e^{-x}$$

Plots:



(x from −3 to 3)



(x from −18 to 18)

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{y \in \mathbb{R} : y > 0\}$ (all positive real numbers)

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Periodicity:

periodic in x with period $2\,i\,\pi$

Series expansion at x = 0:

$$32\sqrt{2} - 32\sqrt{2}\,x + 16\sqrt{2}\,x^2 - \frac{16\sqrt{2}\,x^3}{3} + \frac{4\sqrt{2}\,x^4}{3} + O(x^5)$$

(Taylor series)

Derivative:

$$\frac{d}{dx}\left(\frac{\exp(-x)\times 64}{\sqrt{2}}\right) = -32\sqrt{2}\,e^{-x}$$

Indefinite integral:

$$\int 32\sqrt{2}\,e^{-x}\,dx = -32\sqrt{2}\,e^{-x} + \text{constant}$$

Limit:

$$\lim_{x\to\infty} 32\sqrt{2}\,e^{-x} = 0$$

Series representations:

$$\frac{\exp(-x)\,64}{\sqrt{2}} = 32\sqrt{2}\sum_{k=0}^{\infty}\frac{(-x)^k}{k!}$$

$$\frac{\exp(-x)\,64}{\sqrt{2}} = 32\sqrt{2}\sum_{k=-\infty}^{\infty} I_k(-x)$$

$$\frac{\exp(-x)\,64}{\sqrt{2}} = \frac{32\sqrt{2}\,\sum_{k=0}^{\infty}\frac{(-x)^{2k}(-2k+x)}{(2k)!}}{x}$$

Definite integral:

$$\int_0^{\infty} 32\sqrt{2}\,e^{-x}\,dx = 32\sqrt{2} \approx 45.2548$$

Definite integral over a half-period:

$$\int_0^{-i\pi} 32\sqrt{2}\,e^{-x}\,dx = 64\sqrt{2} \approx 90.5097$$

For ϕ_6 = [1/5 (e^π + π - 2 log(π) - 2 tan^(-1)(π))] =

$$\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)$$

(result in radians)

4.293514200662487705803246176262107230830158135059185495489...

(result in radians)

ϕ_6 = 4.2935142… , we obtain:

exp(-(1/5 (e^π + π - 2 log(π) - 2 tan^(-1)(π))))*(64/(sqrt2))

Input:

$$\exp\left(-\left(\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)\right)\times\frac{64}{\sqrt{2}}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$32\sqrt{2}\ \exp\left(1/5\left(-e^{\pi}-\pi+2\log(\pi)+2\tan^{-1}(\pi)\right)\right)$$

(result in radians)

Decimal approximation:

0.618038387608134989824723666741477891726341143235122477813...

(result in radians)

0.6180383876….. result that is a very good approximation to the value of the golden ratio conjugate 0.618033988749...

Property:

$32\sqrt{2}\ e^{1/5\left(-e^{\pi}-\pi+2\tan^{-1}(\pi)+2\log(\pi)\right)}$ is a transcendental number

Alternate forms:

$$32\sqrt{2}\ \exp\left(\frac{1}{5}\left(-e^{\pi}-\pi+2\left(\log(\pi)+\tan^{-1}(\pi)\right)\right)\right)$$

$$32\sqrt{2}\ \pi^{2/5}\ e^{-e^{\pi}/5-\pi/5+2/5\tan^{-1}(\pi)}$$

$$32\sqrt{2}\ \exp\left(\frac{1}{5}\left(-e^{\pi}-\pi+i\left(\log(1-i\,\pi)-\log(1+i\,\pi)\right)+2\log(\pi)\right)\right)$$

Alternative representations:

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=\frac{\exp\left(-\frac{1}{5}\left(z^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(1,\,\pi)\right)\right)64}{\sqrt{2}}\quad\text{for } z=e$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=\frac{64\exp\left(-\frac{1}{5}\left(\pi-2\tan^{-1}(1,\,\pi)-2\log_e(\pi)+e^{\pi}\right)\right)}{\sqrt{2}}$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=\frac{64\exp\left(-\frac{1}{5}\left(\pi-2\tan^{-1}(1,\,\pi)-2\log(a)\log_a(\pi)+e^{\pi}\right)\right)}{\sqrt{2}}$$

Series representations:

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=32\sqrt{2}\ \exp\left(-\frac{e^{\pi}}{5}-\frac{\pi}{5}+\frac{2}{5}\tan^{-1}(\pi)-\frac{2}{5}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-\pi}\right)^k}{k}\right)(-1+\pi)^{2/5}$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=32\sqrt{2}\ \exp\left(-\frac{e^{\pi}}{5}-\frac{\pi}{5}+\frac{2}{5}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\left(\frac{2\,(-1)^{-1+k}\,(-1+\pi)^{-k}}{5\,k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(\pi-z_0)^k}{5\,k}\right)\right)(-1+\pi)^{2/5}$$

for $(i\,z_0\notin\mathbb{R}$ or $((\text{not } 1\le i\,z_0<\infty)$ and $(\text{not } -\infty<i\,z_0\le -1)))$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=32\sqrt{2}$$

$$\exp\left(-\frac{e^{\pi}}{5}-\frac{\pi}{5}+\frac{2}{5}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\frac{-2\left(\frac{1}{1-\pi}\right)^k+i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(\pi-z_0)^k}{5k}\right)$$

$(-1+\pi)^{2/5}$ for $(i\,z_0\notin\mathbb{R}$ or $((\text{not } 1\le i\,z_0<\infty)$ and $(\text{not } -\infty<i\,z_0\le -1)))$

Integral representations:

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=$$

$$32\sqrt{2}\,\exp\left(-\frac{e^{\pi}}{5}-\frac{\pi}{5}+\frac{2\pi}{5}\int_0^1\frac{1}{1+\pi^2\,t^2}\,dt\right)\pi^{2/5}$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=$$

$$32\sqrt{2}\,\exp\left(-\frac{e^{\pi}}{5}-\frac{\pi}{5}+\int_1^{\pi}\frac{2}{5}\left(\frac{1}{t}+\frac{(-1+\pi)\pi}{1-2\pi+\pi^2\left(2-2t+t^2\right)}\right)dt\right)$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=$$

$$32\sqrt{2}\,\exp\left(-\frac{e^{\pi}}{5}-\frac{\pi}{5}-\frac{i}{10\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\pi^2\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\right)\pi^{2/5}\ \text{for}$$

$0<\gamma<\frac{1}{2}$

Continued fraction representations:

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=$$

$$32\sqrt{2}\,\exp\left(\frac{1}{5}\left(-e^{\pi}+2\log(\pi)+\pi\left(-1+\frac{2}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2\pi^2}{1+2k}}\right)\right)\right)=$$

$$32\sqrt{2}\,\exp\left(\frac{1}{5}\left(-e^{\pi}+2\log(\pi)+\pi\left(-1+\cfrac{2}{1+\cfrac{\pi^2}{3+\cfrac{4\pi^2}{5+\cfrac{9\pi^2}{7+\cfrac{16\pi^2}{9+\ldots}}}}}\right)\right)\right)$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=$$

$$32\sqrt{2}\,\exp\left(\frac{1}{5}\left(-e^{\pi}-\pi+2\log(\pi)+\frac{2\pi}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2\pi^2}{1+2k}}\right)\right)=$$

$$32\sqrt{2}\,\exp\left(\frac{1}{5}\left(-e^{\pi}-\pi+2\log(\pi)+\cfrac{2\pi}{1+\cfrac{\pi^2}{3+\cfrac{4\pi^2}{5+\cfrac{9\pi^2}{7+\cfrac{16\pi^2}{9+\ldots}}}}}\right)\right)$$

$$\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}=$$

$$32\sqrt{2}\,\exp\left(\frac{1}{5}\left(-e^{\pi}+\pi-\frac{2\pi^3}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left(1+(-1)^{1+k}+k\right)^2\pi^2}{3+2k}}\right)\right)\pi^{2/5}=$$

$$32\sqrt{2}\,\exp\left(\frac{1}{5}\left(-e^{\pi}+\pi-\frac{2\pi^3}{3+\frac{9\pi^2}{5+\frac{4\pi^2}{7+\frac{25\pi^2}{9+\frac{16\pi^2}{11+\ldots}}}}}\right)\right)\pi^{2/5}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}\, a_k/b_k$ is a continued fraction

1/(((exp(-(1/5 (e^π + π - 2 log(π) - 2 tan^(-1)(π))))*(64/(sqrt2)))))

Input:

$$\frac{1}{\exp\left(-\left(\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)\times\frac{64}{\sqrt{2}}\right)}$$

Exact Result:

$$\frac{e^{1/5\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)}}{32\sqrt{2}}$$

(result in radians)

Decimal approximation:

1.618022472471477601224442616699208723081781684516080315977...

(result in radians)

1.6180224724714…… result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$$\frac{e^{1/5\left(e^{\pi}+\pi-2\tan^{-1}(\pi)-2\log(\pi)\right)}}{32\sqrt{2}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{e^{1/5\left(e^{\pi}+\pi-2\left(\log(\pi)+\tan^{-1}(\pi)\right)\right)}}{32\sqrt{2}}$$

$$\frac{e^{e^{\pi}/5+\pi/5-2/5\tan^{-1}(\pi)}}{32\sqrt{2}\ \pi^{2/5}}$$

$$\frac{\exp\left(\frac{1}{5}\left(e^{\pi}+\pi-i\left(\log(1-i\pi)-\log(1+i\pi)\right)-2\log(\pi)\right)\right)}{32\sqrt{2}}$$

Alternative representations:

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{1}{\frac{64\exp\left(-\frac{1}{5}\left(\pi-2\tan^{-1}(1,\pi)-2\log_e(\pi)+e^{\pi}\right)\right)}{\sqrt{2}}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{1}{\frac{64\exp\left(-\frac{1}{5}\left(\pi-2\tan^{-1}(1,\pi)-2\log(a)\log_a(\pi)+e^{\pi}\right)\right)}{\sqrt{2}}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{1}{\frac{\exp\left(-\frac{1}{5}\left(z^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(1,\pi)\right)\right)64}{\sqrt{2}}} \quad \text{for } z = e$$

Series representations:

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{\exp\left(\frac{e^{\pi}}{5}+\frac{\pi}{5}-\frac{2}{5}\tan^{-1}(\pi)+\frac{2}{5}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{1-\pi}\right)^k}{k}\right)}{32\sqrt{2}\ (-1+\pi)^{2/5}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{\exp\left(\frac{e^{\pi}}{5}+\frac{\pi}{5}-\frac{2}{5}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\frac{2\left(\frac{1}{1-\pi}\right)^k+i\left((-i-z_0)^{-k}-(i-z_0)^{-k}\right)(\pi-z_0)^k}{5k}\right)}{32\sqrt{2}\,(-1+\pi)^{2/5}}$$

for $(i\,z_0 \notin \mathbb{R}$ or $((\text{not } 1 \le i\,z_0 < \infty)$ and $(\text{not } -\infty < i\,z_0 \le -1)))$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{\exp\left(\frac{e^{\pi}}{5}+\frac{\pi}{5}-\frac{2}{5}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\left(-\frac{2(-1)^{-1+k}(-1+\pi)^{-k}}{5k}-\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(\pi-z_0)^k}{5k}\right)\right)}{32\sqrt{2}\,(-1+\pi)^{2/5}}$$

for $(i\,z_0 \notin \mathbb{R}$ or $((\text{not } 1 \le i\,z_0 < \infty)$ and $(\text{not } -\infty < i\,z_0 \le -1)))$

Integral representations:

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{e^{\frac{e^{\pi}}{5}+\frac{\pi}{5}-\frac{2\pi}{5}\int_0^1\frac{1}{1+\pi^2t^2}\,dt}}{32\sqrt{2}\,\pi^{2/5}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{\exp\left(\frac{e^{\pi}}{5}+\frac{\pi}{5}+\int_1^{\pi}\left(-\frac{2}{5t}-\frac{2(-1+\pi)\pi}{5\left(1-2\pi+\pi^2\left(2-2t+t^2\right)\right)}\right)dt\right)}{32\sqrt{2}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}} = \frac{\exp\left(\frac{e^{\pi}}{5}+\frac{\pi}{5}+\frac{i}{10\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\pi^2\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\right)}{32\sqrt{2}\,\pi^{2/5}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}}=\frac{\exp\left(\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-\frac{2\pi}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k^2\pi^2}{1+2k}}\right)\right)}{32\sqrt{2}}=$$

$$\frac{\exp\left(\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-\cfrac{2\pi}{1+\cfrac{\pi^2}{3+\cfrac{4\pi^2}{5+\cfrac{9\pi^2}{7+\cfrac{16\pi^2}{9+\ddots}}}}}\right)\right)}{32\sqrt{2}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}}=$$

$$\frac{\exp\left(\frac{1}{5}\left(e^{\pi}-\pi+\frac{2\pi^3}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left(1+(-1)^{1+k}+k\right)^2\pi^2}{3+2k}}\right)\right)}{32\sqrt{2}\,\pi^{2/5}}=\frac{\exp\left(\frac{1}{5}\left(e^{\pi}-\pi+\cfrac{2\pi^3}{3+\cfrac{9\pi^2}{5+\cfrac{4\pi^2}{7+\cfrac{25\pi^2}{9+\cfrac{16\pi^2}{11+\ddots}}}}}\right)\right)}{32\sqrt{2}\,\pi^{2/5}}$$

$$\frac{1}{\frac{\exp\left(-\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\tan^{-1}(\pi)\right)\right)64}{\sqrt{2}}}=$$

$$\frac{\exp\left(\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\left(\pi-\frac{\pi^3}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left(1+(-1)^{1+k}+k\right)^2\pi^2}{3+2k}}\right)\right)\right)}{32\sqrt{2}}=$$

$$\frac{\exp\left(\frac{1}{5}\left(e^{\pi}+\pi-2\log(\pi)-2\left(\pi-\cfrac{\pi^3}{3+\cfrac{9\pi^2}{5+\cfrac{4\pi^2}{7+\cfrac{25\pi^2}{9+\cfrac{16\pi^2}{11+\ddots}}}}}\right)\right)\right)}{32\sqrt{2}}$$

$\mathop{\mathrm{K}}_{k=k_1}^{k_2} a_k / b_k$ is a continued fraction

From

[21]Actually, our field theory approach breaks down for KK masses of the order of the fundamental string scale M_s. If we cut our summation at $k_{max} = RM_s$, the physical neutrino is not exactly massless anymore, but aquires a small mass $m_\nu \sim m^2/M_s$. For phenomenologically interesting values $m \sim R^{-1} \simeq 10^{-2}eV$ and $M_s \sim$ TeV, this mass is however negligibly small $m_\nu \sim 10^{-15}eV$.

in the limit $mR \ll 1$, we can solve (186) iteratively by power-expanding the cotangent function. To order $\mathcal{O}(m^5R^5)$, this gives the solutions

$$\lambda_\pm = \pm m\left(1 - \frac{\pi^2}{6}m^2R^2 + ...\right), \qquad \lambda_{\pm k} = \pm\frac{k}{R}\left(1 + \frac{m^2R^2}{k^2} - \frac{m^4R^4}{k^4} + ...\right), \quad (187)$$

For m = 10^{-2} ; mR = 1/24 ; we obtain from

$$\lambda_\pm = \pm m\left(1 - \frac{\pi^2}{6}m^2R^2 + ...\right)$$

10^(-2)*(1-((Pi^2)/6))*(1/24)^2

Input:

$$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}$$

Result:

$$\frac{1-\frac{\pi^2}{6}}{57600}$$

Decimal approximation:

−0.000011196771993892820077646096643160159535051213562618 02...

-0.0000111967719……

Property:

$\frac{1-\frac{\pi^2}{6}}{57\,600}$ is a transcendental number

Alternate forms:

$-\frac{\pi^2-6}{345\,600}$

$\frac{1}{57\,600}-\frac{\pi^2}{345\,600}$

$\frac{6-\pi^2}{345\,600}$

Alternative representations:

$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}=\frac{\left(1-\frac{1}{6}\,(180\,^\circ)^2\right)\left(\frac{1}{24}\right)^2}{10^2}$

$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}=\frac{\left(\frac{1}{24}\right)^2(1-\zeta(2))}{10^2}$

$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}=\frac{\left(1-\frac{1}{6}\,(-i\log(-1))^2\right)\left(\frac{1}{24}\right)^2}{10^2}$

Series representations:

$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}=\frac{1}{57\,600}-\frac{\sum_{k=1}^{\infty}\frac{1}{k^2}}{57\,600}$

$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}=\frac{1}{57\,600}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}}{28\,800}$

$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2}=\frac{1}{57\,600}-\frac{\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}}{43\,200}$

Integral representations:

$$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2} = \frac{1}{57\,600} - \frac{\left(\int_0^1 \sqrt{1-t^2}\ dt\right)^2}{21\,600}$$

$$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2} = \frac{1}{57\,600} - \frac{\left(\int_0^\infty \frac{1}{1+t^2}\ dt\right)^2}{86\,400}$$

$$\frac{\left(1-\frac{\pi^2}{6}\right)\left(\frac{1}{24}\right)^2}{10^2} = \frac{1}{57\,600} - \frac{\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\ dt\right)^2}{86\,400}$$

From

$$\lambda R = -\pi(mR)^2 \tan(\pi\lambda R)$$

-Pi(1/24)^2* tan(Pi*-0.0000111967719*10^-2)

Input interpretation:

$$-\pi\left(\left(\frac{1}{24}\right)^2 \tan(\pi(-0.0000111967719\times 10^{-2}))\right)$$

Result:

$1.91853662\ldots \times 10^{-9}$

1.91853662…*10^{-9}

From which:

1+[-Pi*((((1/24)^2)))* (((tan(Pi*-0.0000111967719*10^-2))))]^1/48

Input interpretation:

$$1+\sqrt[48]{-\pi\left(\left(\frac{1}{24}\right)^2 \tan(\pi(-0.0000111967719\times 10^{-2}))\right)}$$

Result:

1.6582565818...

1.6582565818…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

From:

$$M_H = \left(\frac{\alpha_{GUT}}{8}\right)^{1/2} M_P \ , \ \lambda_H = 2(\alpha_{GUT} V)^{1/2} M_H^3 \ , \qquad (4)$$

where $M_P = G_N^{-1/2}$ is the Planck mass. Then $M_H \sim 5 \times 10^{17}$ GeV, and therefore there is some (slight) discrepancy between the GUT scale M_{GUT} and the string scale M_H. Indeed, from (1) and (4) we find $M_{GUT}/M_H = (4\alpha_{GUT}/\lambda_H^2)^{1/6}$ which asks, in order to find $M_{GUT} \sim 2 - 3 \times 10^{16}$ GeV, for a very large string coupling λ_H. The problem might be alleviated by considering an anisotropic Calabi-Yau space and a lot of effort in this direction was made over the years [7].

The above picture evolved considerably in the last few years. First of all, it was a puzzle that the heterotic $SO(32)$ and the Type I ten dimensional strings share the same low-energy theory. Indeed, the two low-energy actions coincide if the following identifications are made

$$\lambda_I = \frac{1}{\lambda_H} \ , \quad M_I = \frac{M_H}{\sqrt{\lambda_H}} \ , \qquad (5)$$

2.435e+18/((((4 sqrt(2) (5e+17)^3 sqrt(4.2217e-105)))))^0.5

Input interpretation:

$$\frac{2.435 \times 10^{18}}{\sqrt{4\sqrt{2}\,(5 \times 10^{17})^3 \sqrt{\frac{4.2217}{10^{105}}}}}$$

Result:

$3.59240... \times 10^{17}$

$3.59240 * 10^{17} = M_I$

Now, we have that:

where A_4^{FT} is the field-theory amplitude [101]. Eq. (227) obviously reduces to the field theory result in the limit of low energy ($s,t,u << M_I^2$) and small graviton mass ($w^2 << M_I^2$). This string amplitude has poles for $s,t,u = (2n-2)M_I^2$, with n a positive integer, corresponding to tree-level open string state exchanges in the s, t and u channels. Moreover,

$$s = -2p_1p_2 = -2p_3p_4 + w^2 \ , \ t = -2p_2p_3 = -2p_1p_4 + w^2 \ ,$$

$$u = -2p_1p_3 = -2p_2p_4 + w^2 \ , \ s+t+u = w^2 \ .$$

$$A_4 = \frac{1}{\sqrt{\pi}}\, 2^{-\frac{w^2}{M_I^2}}\, \frac{\Gamma\left(-w^2/2M_I^2+1/2\right)\Gamma\left(-s/2M_I^2+1\right)\Gamma\left(-t/2M_I^2+1\right)\Gamma\left(-u/2M_I^2+1\right)}{\Gamma\left((s-w^2)/2M_I^2+1\right)\Gamma\left((t-w^2)/2M_I^2+1\right)\Gamma\left((u-w^2)/2M_I^2+1\right)}\, A_4^{FT} \ ,$$

s + t + u = -16/(1/4) = -16/0.25 = -64; s = -16; t = -16; u = -32

or:

s + t + u = 16/(1/4) = 16/0.25 = 64; s = 16; t = 16; u = 32

$s + t + u = 8^2 = w^2$

gamma (((64/(2*(3.59240e+17)^2))+1/2)) gamma (((-16/(2*(3.59240e+17)^2))+1)) gamma (((-16/(2*(3.59240e+17)^2))+1)) gamma (((-32/(2*(3.59240e+17)^2))+1))

Input interpretation:

$$\Gamma\left(\frac{64}{2\,(3.59240\times 10^{17})^2}+\frac{1}{2}\right)\Gamma\left(-\frac{16}{2\,(3.59240\times 10^{17})^2}+1\right)$$
$$\Gamma\left(-\frac{16}{2\,(3.59240\times 10^{17})^2}+1\right)\Gamma\left(-\frac{32}{2\,(3.59240\times 10^{17})^2}+1\right)$$

$\Gamma(x)$ is the gamma function

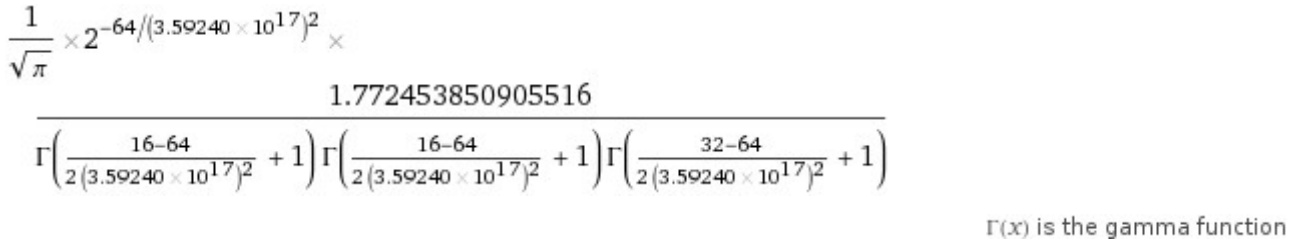
Result:

1.772453850905516027298167483341144573 5...

1.7724538509…

1/(sqrtPi) * 2^(-64/(3.59240e+17)^2) *1.772453850905516 / [gamma ((((16-64)/(2*(3.59240e+17)^2))+1)) gamma ((((16-64)/(2*(3.59240e+17)^2))+1)) gamma ((((32-64)/(2*(3.59240e+17)^2))+1))]

Input interpretation:



$\Gamma(x)$ is the gamma function

Result:

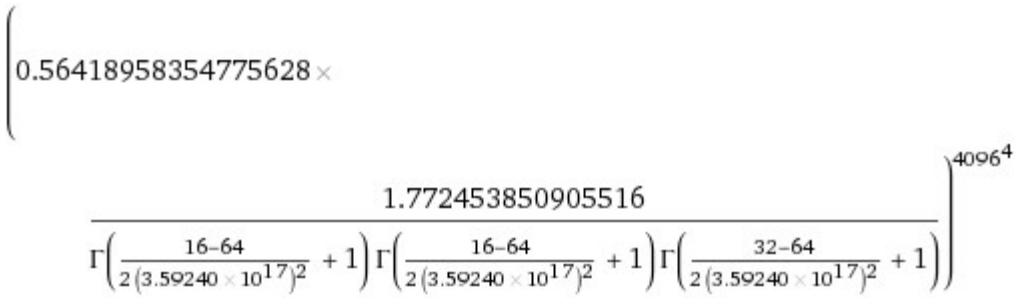
0.99999999999999998459865825595685636141639185485656279348 1...

0.9999…

From which:

(((((0.56418958354775628 *1.772453850905516 / [gamma ((((16-64)/(2*(3.59240e+17)^2))+1)) gamma ((((16-64)/(2*(3.59240e+17)^2))+1)) gamma ((((32-64)/(2*(3.59240e+17)^2))+1))]))))^(4096^4)

Input interpretation:



$\Gamma(x)$ is the gamma function

Result:

0.99222885379028229134816045196610954284386418577415822318 4...

0.99222885…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d}=255-390 \mid 0.988-1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243 = \phi}$

From which:

1/ log base 0.99222885379(((0.56418958354775628 *1.772453850905516 / [gamma ((((16-64)/(2*(3.59240e+17)^2))+1)) gamma ((((16-64)/(2*(3.59240e+17)^2))+1)) gamma ((((32-64)/(2*(3.59240e+17)^2))+1))]))

Input interpretation:

$$1 \Big/ \log_{0.99222885379}\left(0.56418958354775628 \times \frac{1.772453850905516}{\Gamma\left(\frac{16-64}{2(3.59240\times 10^{17})^2}+1\right)\Gamma\left(\frac{16-64}{2(3.59240\times 10^{17})^2}+1\right)\Gamma\left(\frac{32-64}{2(3.59240\times 10^{17})^2}+1\right)}\right)$$

$\Gamma(x)$ is the gamma function

$\log_b(x)$ is the base-b logarithm

Result:

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation:

$2.81474976720920728219266348802856458373922204533761 48\ldots \times 10^{14}$

$2.8147497672\ldots*10^{14}$

2(2.81474976720920728219 × 10^14)^1/8-3

Input interpretation:

$$2\sqrt[8]{2.81474976720920728219\times 10^{14}}-3$$

Result:

125.00000000058348224 56...

125 result very near to the Higgs boson mass 125.18 GeV

2(2.81474976720920728219 × 10^14)^1/8+11

Input interpretation:

$$2\sqrt[8]{2.81474976720920728219\times 10^{14}}+11$$

Result:

139.00000000058348224 56...

139 result practically equal to the rest mass of Pion meson 139.57 MeV

27*(2.81474976720920728219 × 10^14)^1/8+1

Input interpretation:

$$27\sqrt[8]{2.81474976720920728219\times 10^{14}}+1$$

Result:

1729.000000078770103 16...

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((27*(2.81474976720920728219 × 10^14)^1/8+1)))^1/15

Input interpretation:

$$\sqrt[15]{27\sqrt[8]{2.81474976720920728219\times 10^{14}}+1}$$

Result:

1.643815228749227392176 9...

$1.64381522\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From:

Beyond Logarithmic Corrections to Cardy Formula
Farhang Lorana, M. M. Sheikh-Jabbari and Massimiliano Vincon -
arXiv:1010.3561v3 [hep-th] 31 Jan 2011

$$\begin{aligned}
\rho(\Delta,\bar{\Delta}) = (2\pi)^2 & \sum_{\Delta_n<\frac{c_L}{24},\bar{\Delta}_n<\frac{c_R}{24}} \rho(\Delta_n,\bar{\Delta}_n)\frac{|\frac{c_L}{24}-\Delta_n|}{u_n} I_1(4\pi u_n)\cdot\frac{|\frac{c_R}{24}-\bar{\Delta}_n|}{v_n} I_1(4\pi v_n) \\
- (2\pi)^2 & \sum_{\Delta_n>\frac{c_L}{24},\bar{\Delta}_n<\frac{c_R}{24}} \rho(\Delta_n,\bar{\Delta}_n)\frac{|\frac{c_L}{24}-\Delta_n|}{u_n} J_1(4\pi u_n)\cdot\frac{|\frac{c_R}{24}-\bar{\Delta}_n|}{v_n} I_1(4\pi v_n) \\
- (2\pi)^2 & \sum_{\Delta_n<\frac{c_L}{24},\bar{\Delta}_n>\frac{c_R}{24}} \rho(\Delta_n,\bar{\Delta}_n)\frac{|\frac{c_L}{24}-\Delta_n|}{u_n} I_1(4\pi u_n)\cdot\frac{|\frac{c_R}{24}-\bar{\Delta}_n|}{v_n} J_1(4\pi v_n) \\
+ (2\pi)^2 & \sum_{\Delta_n>\frac{c_L}{24},\bar{\Delta}_n>\frac{c_R}{24}} \rho(\Delta_n,\bar{\Delta}_n)\frac{|\frac{c_L}{24}-\Delta_n|}{u_n} J_1(4\pi u_n)\cdot\frac{|\frac{c_R}{24}-\bar{\Delta}_n|}{v_n} J_1(4\pi v_n)\,,
\end{aligned}$$

$$\rho(\Delta,\bar{\Delta})_{\text{Cardy}} \sim e^{2\pi\sqrt{\frac{c_L\Delta}{6}}}\cdot e^{2\pi\sqrt{\frac{c_R\bar{\Delta}}{6}}}\,.$$

$$I_1(z) \sim \frac{1}{\sqrt{2\pi z}} e^z$$

A first solution, can be:

(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*1.0844375*4Pi * (1/24-1/48)*1.0844375*4Pi

-(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)* -0.1699162315*4Pi * (1/24-1/48)*1.0844375*4Pi

-(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*1.0844375*4Pi * (1/24-1/48)*-0.1699162315*4Pi

(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*-0.1699162315*4Pi * (1/24-1/48)*-0.1699162315*4Pi

$c_L = c_R = 1$

For z = 1; e/(sqrt(2Pi))

Input:

$$\frac{e}{\sqrt{2\pi}}$$

Decimal approximation:

1.084437551419227546611577313422947985839596931964726268225...

$I_1 = 1.0844375\ldots.$

$$J_1(z) \sim \sqrt{\frac{2}{\pi z}} \sin\left(\frac{\pi}{4} - z\right)$$

sqrt(2/Pi) sin(Pi/4-1)

Input:

$$\sqrt{\frac{2}{\pi}} \sin\left(\frac{\pi}{4} - 1\right)$$

Exact result:

$$-\sqrt{\frac{2}{\pi}}\,\sin\left(1-\frac{\pi}{4}\right)$$

Decimal approximation:

−0.16991623154864930237035917496465197940349898485584809571...

-0.1699162315….

$\Delta_n = 1/48$; $\bar{\Delta}_n = 1/48$

(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*1.0844375 * (4Pi) * (1/24-1/48)*1.0844375 * (4Pi)

Input interpretation:

$$(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)$$

Result:

6.67256226203866480077565886519934526062241667735471087957 3...

6.672562262…

Series representations:

$$\left((2\pi)^2\,4\pi\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)=$$

$$0.0326668\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left((2\pi)^2\, 4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)=0.0326668\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$\left((2\pi)^2\, 4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)=0.0326668\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty < z_0 \le 0))$

-(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)* -0.1699162315* (4Pi) * (1/24-1/48)*1.0844375* (4Pi)

Input interpretation:

$$-(2\pi)^2\left(\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{48}\right)\times(-0.1699162315)\,(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)\right)$$

Result:

1.0454974...

1.0454974…

Series representations:

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(-0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)\right)=$$
$$0.00511843\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(-0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)\right)=$$
$$0.00511843\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(-0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)\right)=$$
$$0.00511843\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

-(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*1.0844375* (4Pi) * (1/24-1/48)*-0.1699162315 * (4Pi)

Input interpretation:

$$-(2\pi)^2\left(\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)\times(-0.1699162315)\,(4\pi)\right)$$

Result:

1.0454974…

1.0454974…

Series representations:

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)(-0.169916)(4\pi)\right)\right)=$$
$$0.00511843\,\pi^4 \exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)(-0.169916)(4\pi)\right)\right)=$$
$$0.00511843\,\pi^4 \exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)(-0.169916)(4\pi)\right)\right)=$$
$$0.00511843\,\pi^4 \exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*-0.1699162315*(4Pi) * (1/24-1/48)*-0.1699162315* (4Pi)

Input interpretation:

$$(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{48}\right)\times$$
$$(-0.1699162315)\,(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)\times(-0.1699162315)\,(4\pi)$$

Result:

0.1638148678...

0.1638148678…

Series representations:

$$\left((2\pi)^2\,(-1)\,0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times6}}\right)\right)$$
$$\exp\left(2\pi\sqrt{\frac{1}{48\times6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)(-0.169916)\,(4\pi)\right)=$$
$$0.000801987\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left((2\pi)^2\,(-1)\,0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times6}}\right)\right)$$
$$\exp\left(2\pi\sqrt{\frac{1}{48\times6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)(-0.169916)\,(4\pi)\right)=$$
$$0.000801987\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$\left((2\pi)^2\,(-1)\,0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times6}}\right)\right)$$
$$\exp\left(2\pi\sqrt{\frac{1}{48\times6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)(-0.169916)\,(4\pi)\right)=$$
$$0.000801987\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$
$$\text{for}\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le0\right)\right)$$

Thence, we obtain:

(6.672562262 - 1.0454974-1.0454974+0.1638148678)

Input interpretation:

$$6.672562262 - 1.0454974 - 1.0454974 + 0.1638148678$$

Result:

4.7453823298

4.7453823298

From which:

5(6.672562262 - 1.0454974-1.0454974+0.1638148678)

Input interpretation:

$$5\,(6.672562262 - 1.0454974 - 1.0454974 + 0.1638148678)$$

Result:

23.726911649

23.726911649

And also, performing the following calculations:

(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/12)*-0.1699162315*(4Pi) * (1/24-1/12)*-0.1699162315* (4Pi)

Input interpretation:

$$(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{12}\right)\times (-0.1699162315)\,(4\pi)\left(\frac{1}{24}-\frac{1}{12}\right)\times(-0.1699162315)\,(4\pi)$$

Result:

0.6552594711...

0.6552594711…

Series representations:

$$\left((2\pi)^2(-1)\,0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{12}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{12}\right)(-0.169916)(4\pi)\right)=0.00320795\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left((2\pi)^2(-1)\,0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{12}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{12}\right)(-0.169916)(4\pi)\right)=0.00320795\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$\left((2\pi)^2(-1)\,0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{12}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{12}\right)(-0.169916)(4\pi)\right)=0.00320795\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

-(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*1.0844375* (4Pi) * (1/24-1/12)*-0.1699162315* (4Pi)

Input interpretation:

$$-(2\pi)^2\left(\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)\left(\frac{1}{24}-\frac{1}{12}\right)\times(-0.1699162315)(4\pi)\right)$$

Result:

−2.0909949…

-2.0909949…

Series representations:

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(4\pi\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{12}\right)(-0.169916)(4\pi)\right)\right)=$$
$$-0.0102369\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(4\pi\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{12}\right)(-0.169916)(4\pi)\right)\right)=$$
$$-0.0102369\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(4\pi\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{12}\right)(-0.169916)(4\pi)\right)\right)=$$
$$-0.0102369\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

-(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/12)* -0.1699162315* (4Pi) * (1/24-1/48)*1.0844375* (4Pi)

Input interpretation:

$$-(2\pi)^2 \left(\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{12}\right)\times(-0.1699162315)\,(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)\right)$$

Result:

$-2.0909949\ldots$

-2.0909949…

Series representations:

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(-0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{12}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)\right)=$$
$$-0.0102369\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(-0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{12}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)\right)=$$
$$-0.0102369\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$-(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(-0.169916\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{12}\right)(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)\right) = -0.0102369\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

(2Pi)^2 * exp(2Pi*sqrt(1/48*1/6)) * exp(2Pi*sqrt(1/48*1/6)) * (1/24-1/48)*1.0844375* (4Pi) * (1/24-1/48)*1.0844375* (4Pi)

Input interpretation:

$$(2\pi)^2 \exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\exp\left(2\pi\sqrt{\frac{1}{48}\times\frac{1}{6}}\right)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)\left(\frac{1}{24}-\frac{1}{48}\right)\times 1.0844375\,(4\pi)$$

Result:

6.6725623...

6.6725623…

Series representations:

$$\left((2\pi)^2\,4\pi\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right) = 0.0326668\,\pi^4\exp^2\left(2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{287}{288}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left((2\pi)^2\, 4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)=0.0326668\,\pi^4\exp^2\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{287}{288}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

$$\left((2\pi)^2\, 4\pi \exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\right)\exp\left(2\pi\sqrt{\frac{1}{48\times 6}}\right)\left(\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\left(\frac{1}{24}-\frac{1}{48}\right)1.08444\,(4\pi)\right)=0.0326668\,\pi^4\exp^2\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{288}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From the eq.(2.8), we have, from the above results:

(6.6725623+2.0909949+2.0909949+0.6552594711)

Input interpretation:

6.6725623 + 2.0909949 + 2.0909949 + 0.6552594711

Result:

11.5098115711

11.5098115711 = density of states

From

In the previous two sections, starting from the canonical partition function we derived the expression for the microcanonical density of states through a Laplace transform. Then, employing modular invariance of the partition function we fixed the form of the microcanonical density of states, up to exponentially suppressed contributions in the saddle point approximation. By means of the usual thermodynamical equations, one can then read the *microcanonical entropy* $S_{\text{m.c.}}$ upon taking the logarithm of the density of states. Let us first consider the pure imaginary τ case, corresponding to the $L_0 = \bar{L}_0 = E$ sector, for which the microcanonical entropy is

$$S_{\text{m.c.}} = \ln\left(\rho(E)\right) = \ln\left(\frac{\pi^2}{3}c_{\text{tot}}\right) + \ln\left(\frac{I_1(S_{\text{Cardy}})}{S_{\text{Cardy}}}\right) \tag{4.1a}$$

$$= \ln\left(\frac{\pi^{3/2}}{3\sqrt{2}}c_{\text{tot}}\right) + S_{\text{Cardy}} - \frac{3}{2}\ln S_{\text{Cardy}} + \mathcal{O}\left(\frac{1}{S_{\text{Cardy}}}\right), \tag{4.1b}$$

for the previous equation result, performing the ln, we obtain:

ln(6.6725623+2.0909949+2.0909949+0.6552594711)

Input interpretation:

$\log(6.6725623 + 2.0909949 + 2.0909949 + 0.6552594711)$

$\log(x)$ is the natural logarithm

Result:

2.4431999...

2.4431999…

Alternative representations:

$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = \log_e(11.5098)$

$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = \log(a)\log_a(11.5098)$

$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = -\text{Li}_1(-10.5098)$

Series representations:

$$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = \log(10.5098) - \sum_{k=1}^{\infty} \frac{(-1)^k\, e^{-2.35231k}}{k}$$

$$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = 2\,i\,\pi\left\lfloor \frac{\arg(11.5098 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(11.5098 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = \left\lfloor \frac{\arg(11.5098 - z_0)}{2\,\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(11.5098 - z_0)}{2\,\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(11.5098 - z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = \int_1^{11.5098} \frac{1}{t}\,dt$$

$$\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = \frac{1}{2\,i\,\pi} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{-2.35231\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

5ln(6.6725623+2.0909949+2.0909949+0.6552594711)

Input interpretation:

$$5\log(6.6725623 + 2.0909949 + 2.0909949 + 0.6552594711)$$

$\log(x)$ is the natural logarithm

Result:

12.215999...

12.215999… result very near to the black hole entropy 12.1904, that is equal to ln(196884)

Alternative representations:

$$5\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = 5\log_e(11.5098)$$

$$5\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = 5\log(a)\log_a(11.5098)$$

$$5\log(6.67256 + 2.09099 + 2.09099 + 0.655259) = -5\,\mathrm{Li}_1(-10.5098)$$

Series representations:

$$5\log(6.67256+2.09099+2.09099+0.655259)=5\log(10.5098)-5\sum_{k=1}^{\infty}\frac{(-1)^k\,e^{-2.35231k}}{k}$$

$$5\log(6.67256+2.09099+2.09099+0.655259)=10\,i\,\pi\left\lfloor\frac{\arg(11.5098-x)}{2\,\pi}\right\rfloor+5\log(x)-5\sum_{k=1}^{\infty}\frac{(-1)^k\,(11.5098-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$5\log(6.67256+2.09099+2.09099+0.655259)=5\left\lfloor\frac{\arg(11.5098-z_0)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+5\log(z_0)+5\left\lfloor\frac{\arg(11.5098-z_0)}{2\,\pi}\right\rfloor\log(z_0)-5\sum_{k=1}^{\infty}\frac{(-1)^k\,(11.5098-z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$5\log(6.67256+2.09099+2.09099+0.655259)=5\int_1^{11.5098}\frac{1}{t}\,dt$$

$$5\log(6.67256+2.09099+2.09099+0.655259)=\frac{5}{2\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-2.35231\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

exp((5ln(6.6725623+2.0909949+2.0909949+0.6552594711)))-4096-1024+8

where $4096 = 64^2$ and $1024 = 64*16$

Input interpretation:

$$\exp(5\log(6.6725623+2.0909949+2.0909949+0.6552594711))-4096-1024+8$$

$\log(x)$ is the natural logarithm

Result:

196883.209058117456051034933773913545895389626797718795 51

Repeating decimal:

196 883.209058117456051034933773913545895389626797718795 51

196883.209058…

196884 is a fundamental number of the following j-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's *j*-invariant or *j* function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of *j* have to do with its *q* expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that *j* has a simple pole at the cusp, so its *q*-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

Alternative representations:

$$\exp(5\log(6.67256 + 2.09099 + 2.09099 + 0.655259)) - 4096 - 1024 + 8 = -5112 + \exp(5\log_e(11.5098))$$

$$\exp(5\log(6.67256 + 2.09099 + 2.09099 + 0.655259)) - 4096 - 1024 + 8 = -5112 + \exp(5\log(a)\log_a(11.5098))$$

Series representations:

$$\exp(5\log(6.67256 + 2.09099 + 2.09099 + 0.655259)) - 4096 - 1024 + 8 = -5112 + \exp\left(5\left(\log(10.5098) - \sum_{k=1}^{\infty}\frac{(-1)^k\, e^{-2.35231k}}{k}\right)\right)$$

$$\exp(5\log(6.67256 + 2.09099 + 2.09099 + 0.655259)) - 4096 - 1024 + 8 = -5112 + \exp\left(5\left(2i\pi\left\lfloor\frac{\arg(11.5098 - x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k\,(11.5098 - x)^k\, x^{-k}}{k}\right)\right) \quad \text{for } x < 0$$

$$\exp(5\log(6.67256+2.09099+2.09099+0.655259))-4096-1024+8=$$
$$-5112+\exp\left(5\left(\log(z_0)+\left\lfloor\frac{\arg(11.5098-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(11.5098-z_0)^k\,z_0^{-k}}{k}\right)\right)$$

Integral representations:

$$\exp(5\log(6.67256+2.09099+2.09099+0.655259))-4096-1024+8=$$
$$-5112+\exp\left(5\int_1^{11.5098}\frac{1}{t}\,dt\right)$$

$$\exp(5\log(6.67256+2.09099+2.09099+0.655259))-4096-1024+8=$$
$$-5112+\exp\left(\frac{5}{2i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-2.35231s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\quad\text{for}\quad -1<\gamma<0$$

We have from:

A CONVERGENT SERIES FOR THE PARTITION FUNCTION P(n)
BY HANS RADEMACHER - DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA - Communicated January 9, 1937

We have the following formulas:

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k \leqq \alpha\sqrt{n}} A_k(n)\, \sqrt{k}\, \frac{d}{dn} \left(\frac{e^{\frac{C\sqrt{n - 1/24}}{k}}}{\sqrt{n - 1/24}} \right) + O(n^{-1/4}) \quad (2)$$

with α as an arbitrary constant, and

$$A_k(n) = \sum_{\substack{h \bmod k \\ (h,k) = 1}} \omega_{h,k}\, e^{-\frac{2\pi i h n}{k}}, \quad C = \pi\sqrt{2/3}. \quad (3)$$

The following revision and modification of the Hardy-Ramanujan method permits us to replace the asymptotic formula (2) by the equality

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n) \sqrt{k}\, \frac{d}{dn} \left(\frac{\sinh \frac{C\sqrt{n - 1/24}}{k}}{\sqrt{n - 1/24}} \right), \quad (4)$$

in which the series is absolutely convergent. The asymptotic formula (2) is then an immediate consequence of (4).

From (4) it is possible to derive a definite expression for the O-term in (2). We obtain

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k=1}^{N} A_k(n)\, \sqrt{k}\, \frac{d}{dn} \left(\frac{e^{\frac{C\sqrt{n-1/24}}{k}}}{\sqrt{n-1/24}} \right) \quad (12)$$

$$+ \theta \cdot \left\{ \frac{44\pi^2}{225\sqrt{3}} N^{-1/2} + \frac{\pi\sqrt{2}}{75} \left(\frac{N}{n-1} \right)^{1/2} \sinh \frac{C\sqrt{n}}{N} \right.$$

$$\left. + \frac{1}{2\sqrt{3}} \frac{(N+1)^{3/2}}{n-1}\, e^{-\frac{C\sqrt{n-1}}{N}} \left(\frac{1}{3} + \frac{\sqrt{3}}{5\pi\sqrt{2}} \frac{N+1}{(n-1)^{1/2}} \right) \right\}, \quad |\theta| < 1,$$

From the following alternate form, for n = 100, we obtain:

$$e^{\left(\pi\sqrt{24n-1}\right)/(6k)} \left(\frac{4\sqrt{6}\,\pi}{k\,(24n-1)} - \frac{24\sqrt{6}}{(24n-1)^{3/2}} \right)$$

For n = 100

((44Pi^2) / (225sqrt3)) * 1729^(-1/2) + (Pi*sqrt2 / 75) (1729/(100-1))^0.5 sinh[(((Pi*(2/3)^0.5)*sqrt100))/1729]

= n

0.5*[n + 1/(2sqrt3) * ((1729+1)^1.5) / (100-1) * exp-(((((((Pi*(2/3)^0.5)*sqrt(100-1))/1729)))))*(((1/3+sqrt3/(5Pi*sqrt2)*(1729+1)/(100-1)^0.5)))]

= m

(-0.86575845575+3.905184028479i) * e^((-2Pi*100*i)/12)

= p

1/(2Pi*sqrt2) * sum(((((p) * sqrt(k) (e^((π sqrt(24*100 - 1))/(6 k)) ((4 sqrt(6) π)/(k (24*100- 1)) - (24 sqrt(6))/(24*100 - 1)^(3/2))))))), k = 1..1729

= q

1/4((((q) + (((m*(100)^(-0.25))))

R = 1.90569*10^8 ≅ 190569292 = 1.90569292 × 10^8 = p(100)

For p(1000) = 2.406146786403262247369214972799 1 * 10^{31} (OEIS A000041)

We obtain:

((44Pi^2) / (225sqrt3)) * 1729^(-1/2) + (Pi*sqrt2 / 75) (1729/(1000-1))^0.5 sinh[(((Pi*(2/3)^0.5)*sqrt1000))/1729]

Input:

$$\frac{44\,\pi^2}{225\,\sqrt{3}} \times 1729^{-1/2} + \left(\pi \times \frac{\sqrt{2}}{75}\right)\sqrt{\frac{1729}{1000-1}}\ \sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{1}{225}\sqrt{\frac{3458}{111}}\,\pi\sinh\left(\frac{20\sqrt{\frac{5}{3}}\,\pi}{1729}\right)$$

Decimal approximation:

0.030456101540054260279220873823403564613535943535044828637...

0.03045610154005426…..

Alternate forms:

$$\frac{\pi\left(1628\,\pi+1729\sqrt{74}\,\sinh\left(\frac{20\sqrt{\frac{5}{3}}\,\pi}{1729}\right)\right)}{8325\,\sqrt{5187}}$$

$$\frac{\pi\left(88\,\pi+3458\sqrt{\frac{2}{37}}\,\sinh\left(\frac{20\sqrt{\frac{5}{3}}\,\pi}{1729}\right)\right)}{450\,\sqrt{5187}}$$

$$\frac{1628\sqrt{5187}\,\pi^2+1729\sqrt{383\,838}\,\pi\sinh\left(\frac{20\sqrt{\frac{5}{3}}\,\pi}{1729}\right)}{43\,181\,775}$$

Alternative representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}=$$

$$\frac{44\,\pi^2}{\sqrt{1729}\,(225\sqrt{3})}+\frac{1}{75}\,i\,\pi\cos\left(\frac{\pi}{2}+\frac{i\,\pi\sqrt{\frac{2}{3}}\,\sqrt{1000}}{1729}\right)\sqrt{\frac{1729}{999}}\,\sqrt{2}$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{\sqrt{1729}\,(225\,\sqrt{3}\,)}+\frac{1}{75}\,i\,\pi\cosh\left(\frac{i\,\pi}{2}-\frac{\pi\,\sqrt{\frac{2}{3}}\,\sqrt{1000}}{1729}\right)\sqrt{\frac{1729}{999}}\,\sqrt{2}$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{\sqrt{1729}\,(225\,\sqrt{3}\,)}-\frac{1}{75}\,i\left(\pi\cosh\left(\frac{i\,\pi}{2}+\frac{\pi\,\sqrt{\frac{2}{3}}\,\sqrt{1000}}{1729}\right)\sqrt{\frac{1729}{999}}\,\sqrt{2}\right)$$

Series representations:

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{2}{225}\sqrt{\frac{3458}{111}}\,\pi\sum_{k=0}^{\infty}I_{1+2\,k}\left(\frac{20\,\sqrt{\frac{5}{3}}\,\pi}{1729}\right)$$

$$\frac{1729^{-1/2}\,(44\,\pi^2)}{225\,\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\,\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\,\sqrt{5187}}+\frac{1}{225}\sqrt{\frac{3458}{111}}\,\pi\sum_{k=0}^{\infty}\frac{3^{-1/2-k}\times 5^{3/2+3\,k}\times 1729^{-1-2\,k}\,(4\,\pi)^{1+2\,k}}{(1+2\,k)!}$$

$$\frac{1729^{-1/2}\left(44\,\pi^2\right)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\sqrt{5187}}+\frac{1}{225}\sqrt{\frac{3458}{111}}\;\pi\sum_{k=0}^{\infty}\frac{3^{1/2\left(-1-2\,k\right)}\times 5^{1+2\,k+1/2\left(1+2\,k\right)}\left(\frac{1729}{4\,\pi}\right)^{-1-2\,k}}{(1+2\,k)!}$$

Integral representations:

$$\frac{1729^{-1/2}\left(44\,\pi^2\right)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\sqrt{5187}}+\frac{4}{27}\sqrt{\frac{2}{319\,865}}\;\pi^2\int_0^1\cosh\left(\frac{20\sqrt{\frac{5}{3}}\,\pi\,t}{1729}\right)dt$$

$$\frac{1729^{-1/2}\left(44\,\pi^2\right)}{225\sqrt{3}}+\frac{1}{75}\left(\sqrt{\frac{1729}{1000-1}}\,\sinh\left(\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{1000}}{1729}\right)\right)\pi\sqrt{2}\;=$$

$$\frac{44\,\pi^2}{225\sqrt{5187}}-\frac{1}{27}\,i\sqrt{\frac{2}{319\,865}}\;\pi^{3/2}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{\left(500\,\pi^2\right)/\left(8\,968\,323\,s\right)+s}}{s^{3/2}}\,ds\quad\text{for}\;\gamma>0$$

0.030456101540054 26 = n

0.5*[0.030456101540054 26 + 1/(2sqrt3) * ((1729+1)^1.5) / (1000-1) * exp-(((((((Pi*(2/3)^0.5)*sqrt(1000-1))/1729)))))*(((1/3+sqrt3/(5Pi*sqrt2)*(1729+1)/(1000-1)^0.5)))]

Input interpretation:

$$0.5\left(0.030456101540054 26+\frac{1}{2\sqrt{3}}\times\frac{(1729+1)^{1.5}}{1000-1}\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)\sqrt{1000-1}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}}{5\pi\sqrt{2}}\times\frac{1729+1}{\sqrt{1000-1}}\right)\right)$$

Result:

45.6577...

45.6577…

Series representations:

$$0.5\left(0.030456101540054260000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{1000-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,(1729+1)}{\left(5\pi\sqrt{2}\right)^{\sqrt{1000-1}}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(1000-1)}\right)=$$

$$\left(6.00237\left(\pi\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(999-z_0)^k\,z_0^{-k}}{k!}}{1729}\right)\right.\right.$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}+$$

$$32.8409\exp\left(-\frac{\sqrt{\frac{2}{3}}\;\pi\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(999-z_0)^k\,z_0^{-k}}{k!}}{1729}\right)$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(3-z_0)^k\,z_0^{-k}}{k!}+0.00253701\,\pi\sqrt{z_0}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\,(2-z_0)^{k_1}\,(3-z_0)^{k_2}\,z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(3-z_0)^k\,z_0^{-k}}{k!}\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$0.5\left(0.030456101540054260000 + \frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{1000-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\ (1729+1)}{\left(5\pi\sqrt{2}\right)^{\sqrt{1000-1}}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(1000-1)}\right) =$$

$$\left(6.00237\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\right.\right.$$

$$\exp\left(-\frac{\sqrt{\frac{2}{3}}\ \pi\exp\left(i\pi\left\lfloor\frac{\arg(999-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,(999-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1729}\right)$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 32.8409\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)$$

$$\exp\left(-\frac{\sqrt{\frac{2}{3}}\ \pi\exp\left(i\pi\left\lfloor\frac{\arg(999-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,(999-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1729}\right)$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!} +$$

$$0.00253701\,\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,(2-x)^{k_1}\,(3-x)^{k_2}\,x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$0.5\left(0.030456101540054260000+\frac{\left(\exp\left(-\frac{\left(\pi\sqrt{\frac{2}{3}}\right)^{\sqrt{1000-1}}}{1729}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\ (1729+1)}{\left(5\pi\sqrt{2}\right)\sqrt{1000-1}}\right)\right)(1729+1)^{1.5}}{\left(2\sqrt{3}\right)(1000-1)}\right)=$$

$$\left(6.00237\left(\frac{1}{z_0}\right)^{-1/2\,[\arg(2-z_0)/(2\pi)]-1/2\,[\arg(3-z_0)/(2\pi)]}z_0^{-1-1/2\,[\arg(2-z_0)/(2\pi)]-1/2\,[\arg(3-z_0)/(2\pi)]}\right.$$

$$\left(\pi\exp\left(-\frac{1}{1729}\sqrt{\frac{2}{3}}\ \pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(999-z_0)/(2\pi)]}z_0^{1/2\,(1+[\arg(999-z_0)/(2\pi)])}\right.\right.$$

$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(999-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\,[\arg(2-z_0)/(2\pi)]}\right.$$

$$z_0^{1/2+1/2\,[\arg(2-z_0)/(2\pi)]}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}+$$

$$32.8409\exp\left(-\frac{1}{1729}\sqrt{\frac{2}{3}}\ \pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(999-z_0)/(2\pi)]}z_0^{1/2\,(1+[\arg(999-z_0)/(2\pi)])}\right.$$

$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(999-z_0)^k\,z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\,[\arg(3-z_0)/(2\pi)]}\right.$$

$$z_0^{1/2+1/2\,[\arg(3-z_0)/(2\pi)]}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}+$$

$$0.00253701\,\pi\left(\frac{1}{z_0}\right)^{1/2\,[\arg(2-z_0)/(2\pi)]+1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$z_0^{1+1/2\,[\arg(2-z_0)/(2\pi)]+1/2\,[\arg(3-z_0)/(2\pi)]}$$

$$\left.\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\,(3-z_0)^{k_2}\,z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\right/$$

$$\left(\pi\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k\,z_0^{-k}}{k!}\right)$$

45.6577 = m

(-0.86575845575+3.905184028479i) * e^((-2Pi*1000*i)/12)

Input interpretation:

$(-0.86575845575 + 3.905184028479\,i)\,e^{1/12\,(-2\,\pi\times 1000\,i)}$

i is the imaginary unit

Result:

3.81486780299... −
1.20282319802... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 4.00000000000$ (radius), $\theta = -17.500000000°$ (angle)

4 that is a Lucas number

Alternative representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\ 1000\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\,e^{-(360\,000\,°\,i)/12}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\ 1000\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\,e^{2000/12\,i^2\log(-1)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\ 1000\,i)} = (-0.865758455750000 + 3.9051840284790000\,i)\exp^{-\frac{2}{12}\,(\pi\ 1000\,i)}(z) \quad \text{for } z = 1$$

Series representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\ 1000\,i)} = 3.90518402848$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-2000/3\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2\,k)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,1000\,i)} = 3.90518402848$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{-2000/3\,i\,\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,1000\,i)} = 3.90518402848$$
$$(-0.22169466264236 + 1.00000000000000\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-500/3\,i\,\sum_{k=1}^{\infty}4^{-k}\left(-1+3^k\right)\zeta(1+k)}$$

Integral representations:

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,1000\,i)} = e^{-(1000\,i)/3\int_0^{\infty}1/(1+t^2)\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,1000\,i)} = e^{-500\,i\int_0^{\infty}\sin^4(t)/t^4\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

$$(-0.865758455750000 + 3.9051840284790000\,i)\,e^{-2/12\,(\pi\,1000\,i)} = e^{-(2000\,i)/3\int_0^{1}\sqrt{1-t^2}\,dt}\,(-0.865758455750000 + 3.9051840284790000\,i)$$

(3.81486780299 -1.20282319802 i) = p

1/(2Pi*sqrt2) * sum(((((3.81486780299 -1.20282319802 i) * sqrt(k) (e^((π sqrt(24*1000 - 1))/(6 k)) ((4 sqrt(6) π)/(k (24*1000- 1)) - (24 sqrt(6))/(24*1000 - 1)^(3/2))))))), k = 1..1729

Input interpretation:

$$\frac{1}{2\,\pi\,\sqrt{2}}\sum_{k=1}^{1729}(3.81486780299 + i\times(-1.20282319802))\sqrt{k}\left(e^{\left(\pi\,\sqrt{24\times1000-1}\right)/(6k)}\left(\frac{4\,\sqrt{6}\,\pi}{k\,(24\times1000-1)} - \frac{24\,\sqrt{6}}{(24\times1000-1)^{3/2}}\right)\right)$$

i is the imaginary unit

Result:

$9.17913190472\times10^{31} - 2.89416917253\times10^{31}\,i$

(9.17913190472×10^31 - 2.89416917253×10^31 i) = q

1/4((((9.17913190472×10^31 - 2.89416917253×10^31 i) + (((45.6577*(1000)^(-0.25))))

Input interpretation:

$\frac{1}{4}\left((9.17913190472\times10^{31} - 2.89416917253\times10^{31}\,i) + \frac{45.6577}{1000^{0.25}}\right)$

i is the imaginary unit

Result:

$2.29478297618...\times10^{31} -$
$7.23542293132...\times10^{30}\,i$

Polar coordinates:

$r = 2.40615\times10^{31}$ (radius), $\theta = -17.5°$ (angle)

$2.40615*10^{31} \approx 2.406146786403262247369214972799 1 * 10^{31}$ = p(1000)

From:

String thermodynamics and black holes -
http://fma.if.usp.br/~amsilva/Livros/Zwiebach/chapter16.pdf

we have:

$$p_{24}(N) \simeq \frac{1}{\sqrt{2}} N^{-27/4} \exp\left(4\pi\sqrt{N}\right).$$

that is the expression concerning the number of transverse light-cone directions in the bosonic string, we obtain:

For n = 1000:

1/(sqrt2) * 1000^(-27/4) * exp(4Pi*sqrt1000)

Input:

$$\frac{1}{\sqrt{2}} \times 1000^{-27/4} \exp\left(4\pi\sqrt{1000}\right)$$

Exact result:

$$\frac{e^{40\sqrt{10}\,\pi}}{100\,000\,000\,000\,000\,000\,000 \times 2^{3/4}\sqrt[4]{5}}$$

Decimal approximation:

$1.516909093424422589280167958890658548734595159534734 9\ldots \times 10^{152}$

$1.516909093424\ldots * 10^{152} = p_{24}(1000)$

Interesting is the following Table

N	$p(N)$	$p(N)_{\text{est}}$	$p(N)/p_{\text{est}}(N)$
5	7	8.94	0.7829
10	42	48.10	0.8731
100	190569292	199281893.25	0.9563
1000	2.406×10^{31}	2.440×10^{31}	0.9860
10000	3.617×10^{106}	3.633×10^{106}	0.9956

Table 16.2: Comparing the exact values of $p(N)$ with the estimate $p(N)_{\text{est}}$ provided by the Hardy-Ramanujan formula.

We see that the result for p(N) for N = 1000 , is $2.406 * 10^{31}$, practically equal to the solution of the Rademacher's partition formula (2):

From:

A CONVERGENT SERIES FOR THE PARTITION FUNCTION P(n) - *BY HANS RADEMACHER* - DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA - Communicated January 9, 1937

$$p(n) = \frac{1}{2\pi\sqrt{2}} \sum_{k \leqq \alpha\sqrt{n}} A_k(n)\, \sqrt{k}\, \frac{d}{dn} \left(\frac{e^{\frac{C\sqrt{n - 1/24}}{k}}}{\sqrt{n - 1/24}} \right) + O(n^{-1/4}) \quad (2)$$

with α as an arbitrary constant, and

$$A_k(n) = \sum_{\substack{h \bmod k \\ (h,k) = 1}} \omega_{h,k}\, e^{-\frac{2\pi i h n}{k}}, \quad C = \pi\sqrt{2/3}. \quad (3)$$

The following revision and modification of the Hardy-Ramanujan method permits us to replace the asymptotic formula (2) by the equality

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} A_k(n) \sqrt{k}\, \frac{d}{dn} \left(\frac{\sinh \frac{C\sqrt{n - 1/24}}{k}}{\sqrt{n - 1/24}} \right), \quad (4)$$

in which the series is absolutely convergent. The asymptotic formula (2) is then an immediate consequence of (4).

Now, $p_{24}(1000) = 1.516909093424\ldots * 10^{152}$ represents the number of transverse light-cone directions in the bosonic string, while $p(1000) = 2.40615 * 10^{31}$ is the number of possible partitions of a non-negative integer, i.e. 1000.

We observe that:

(2.40615e+31)^5

Input interpretation:

$(2.40615 \times 10^{31})^5$

Result:

8.06517×10^{156}

Scientific notation:

$8.065169319781769590526343 75 \times 10^{156}$

$8.06516931978\ldots *10^{156}$

From which:

1/(16*3323) ((((2.40615e+31)^5)))

Input interpretation:

$$\frac{1}{16 \times 3323} (2.40615 \times 10^{31})^5$$

Result:

$1.516921704743787539596438412202828769184471862774601 2\ldots \times 10^{152}$

$1.5169217074\ldots.*10^{152}$ result very near to the value of $p_{24}(1000) =$ $1.516909093424\ldots*10^{152}$

Further, we have that:

1/3ln[(2.40615*10^31) * (1.516909093424*10^152)]-1

Input interpretation:

$$\frac{1}{3} \log(2.40615 \times 10^{31} (1.516909093424 \times 10^{152})) - 1$$

$\log(x)$ is the natural logarithm

Result:

139.88926...

139.88926…. result practically equal to the rest mass of Pion meson 139.57 MeV

1/3ln[(2.40615*10^31) * (1.516909093424*10^152)]-19+4

Input interpretation:

$$\frac{1}{3} \log(2.40615 \times 10^{31} (1.516909093424 \times 10^{152})) - 19 + 4$$

$\log(x)$ is the natural logarithm

Result:

125.88926...

125.88926… result very near to the Higgs boson mass 125.18 GeV

1/3ln[(2.40615*10^31) * (1.516909093424*10^152)]-13

27*1/2(((1/3ln[(2.40615*10^31) * (1.516909093424*10^152)]-13)))+golden ratio^2

Input interpretation:

$$27\times\frac{1}{2}\left(\frac{1}{3}\log(2.40615\times10^{31}\,(1.516909093424\times10^{152}))-13\right)+\phi^2$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

1729.1230...

1729.1230…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

[27*1/2(((1/3ln[(2.40615*10^31) * (1.516909093424*10^152)]-13)))+golden ratio^2]^1/15

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{3}\log(2.40615\times10^{31}\,(1.516909093424\times10^{152}))-13\right)+\phi^2}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

1.64382303...

$1.64382303\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From:

FOURIER EXPANSIONS OF MODULAR FORMS AND PROBLEMS OF PARTITION[1] - *HANS RADEMACHER* - An address delivered before the Williamsburg meeting of the Society, December 29, 1938, by invitation of the Program Committee.

For N = 1729 ; k = 2 ; m = 1 and z > 1 = 2

$$\Phi_k(z) = \sum_{m=0}^{\infty} \frac{d}{dm}\left(\frac{\sin\frac{\pi}{k}\left[\frac{2}{3}(m+1/24)\right]^{1/2}}{(m+1/24)^{1/2}} \right) z^{-m}$$

we obtain:

[((((sin(Pi/2)((2/3(1+1/24))^0.5))) / (((1+1/24)^0.5))))]*2^(-1)

Input:

$$\frac{1}{2} \times \frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}$$

Exact result:

$$\frac{1}{\sqrt{6}}$$

Decimal approximation:

0.40824829046386301636621401245098189866099124677611168807 2...

0.40824829…

Alternate form:

$$\frac{\sqrt{6}}{6}$$

Alternative representations:

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\cos(0)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\cosh(0)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}\,\sec(0)}$$

Series representations:

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}\pi^{1+2k}}{(1+2k)!}}{\sqrt{6}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}+2k\right)\left(\left(\frac{1}{2}\right)_k\right)^3}{(k!)^3}}{\sqrt{6}}$$

Integral representations:

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\pi}{2\sqrt{6}}\int_0^1 \cos\left(\frac{\pi t}{2}\right)dt$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\sqrt{\pi}}{8\sqrt{6}\,i}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(16s)+s}}{s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\sqrt{\pi}}{2\sqrt{6}\,i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^{-1+2s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0 < \gamma < 1$$

Half-argument formulas:

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{\sqrt{\frac{1}{2}\left(1-\cos(\pi)\right)}}{\sqrt{6}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{(-1)^{\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{\frac{1}{2}(1-\cos(\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)}{\sqrt{6}}$$

Multiple-argument formulas:

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)}{\sqrt{6}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \sqrt{\frac{2}{3}}\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{2\sqrt{1+\frac{1}{24}}} = \frac{U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)}{\sqrt{6}}$$

From which, multiplying by 4 (that is a Lucas number), we obtain:

4*[((((sin(Pi/2)((2/3(1+1/24))^0.5))) / (((1+1/24)^0.5))))]*2^(-1)

Input:

$$\frac{4}{2}\times\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}$$

Exact result:

$$2\sqrt{\frac{2}{3}}$$

Decimal approximation:

1.6329931618554520654648560498039275946439649871044467522 88...

1.6329931618…

Alternate form:

$$\frac{2\sqrt{6}}{3}$$

Alternative representations:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = \frac{2\cos(0)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = \frac{2\cosh(0)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = \frac{2\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}\,\sec(0)}$$

Series representations:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 2\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 4\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}\left(\frac{\pi}{2}\right)$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 2\sqrt{\frac{2}{3}}\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}+2k\right)\left(\left(\frac{1}{2}\right)_k\right)^3}{(k!)^3}$$

Integral representations:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = \sqrt{\frac{2}{3}}\,\pi\int_0^1\cos\left(\frac{\pi t}{2}\right)dt$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = \frac{\sqrt{\pi}}{2\sqrt{6}\,i}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(16s)+s}}{s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = \frac{\sqrt{\frac{2}{3}}\,\sqrt{\pi}}{i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{4^{-1+2s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0 < \gamma < 1$$

Half-argument formulas:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 2\sqrt{\frac{2}{3}}\ \sqrt{\frac{1}{2}\left(1-\cos(\pi)\right)}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} =$$

$$2\,(-1)^{\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{\frac{2}{3}}\ \sqrt{\frac{1}{2}\left(1-\cos(\pi)\right)}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)$$

Multiple-argument formulas:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 2\sqrt{\frac{2}{3}}\ \left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 4\sqrt{\frac{2}{3}}\ \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} = 2\sqrt{\frac{2}{3}}\ U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)$$

From which:

4*[((((sin(Pi/2)((2/3(1+1/24))^0.5))) / (((1+1/24)^0.5))))]*2^(-1)+11/10^3

Input:

$$\frac{4}{2}\times\frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}+\frac{11}{10^3}$$

Exact result:

$$\frac{11}{1000}+2\sqrt{\frac{2}{3}}$$

Decimal approximation:

1.643993161855452065464856049803927594643964987104446752288...

$1.64399316185\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms:

$$\frac{33+2000\sqrt{6}}{3000}$$

$$\frac{2\sqrt{6}}{3}+\frac{11}{1000}$$

$$\frac{11+2000\sqrt{\frac{2}{3}}}{1000}$$

Minimal polynomial:

$3\,000\,000\,x^2 - 66\,000\,x - 7\,999\,637$

and:

4*[((((sin(Pi/2)((2/3(1+1/24))^0.5))) / (((1+1/24)^0.5))))]*2^(-1)+(11-26)/10^3

Input:

$$\frac{4}{2} \times \frac{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}} + \frac{11-26}{10^3}$$

Exact result:

$$2\sqrt{\frac{2}{3}} - \frac{3}{200}$$

Decimal approximation:

1.617993161855452065464856049803927594643964987104446752288...

1.61799316185…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\frac{1}{600}\left(400\sqrt{6} - 9\right)$$

$$\frac{2\sqrt{6}}{3} - \frac{3}{200}$$

$$\frac{1}{200}\left(400\sqrt{\frac{2}{3}} - 3\right)$$

Minimal polynomial:

$$120\,000\,x^2 + 3600\,x - 319\,973$$

Alternative representations:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}} + \frac{11-26}{10^3} = -\frac{15}{10^3} + \frac{2\cos(0)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{15}{10^3}+\frac{2\cosh(0)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{15}{10^3}+\frac{2\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}}{\sqrt{1+\frac{1}{24}}\,\sec(0)}$$

Series representations:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+2\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+4\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}\left(\frac{\pi}{2}\right)$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+2\sqrt{\frac{2}{3}}\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}+2k\right)\left(\left(\frac{1}{2}\right)_k\right)^3}{(k!)^3}$$

Integral representations:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+\sqrt{\frac{2}{3}}\,\pi\int_0^1\cos\left(\frac{\pi t}{2}\right)dt$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+\frac{\sqrt{\pi}}{2\sqrt{6}\,i}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(16\,s)+s}}{s^{3/2}}\,ds \quad \text{for } \gamma>0$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+\frac{\sqrt{\frac{2}{3}}\,\sqrt{\pi}}{i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{4^{-1+2\,s}\,\pi^{1-2\,s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds$$

for $0<\gamma<1$

Half-argument formulas:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+2\sqrt{\frac{2}{3}}\,\sqrt{\frac{1}{2}\,(1-\cos(\pi))}$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+$$

$$2\,(-1)^{\lfloor \mathrm{Re}(\pi)/(2\,\pi)\rfloor}\sqrt{\frac{2}{3}}\,\sqrt{\frac{1}{2}\,(1-\cos(\pi))}\,\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\,\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\,\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)$$

Multiple-argument formulas:

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+2\sqrt{\frac{2}{3}}\,\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+4\sqrt{\frac{2}{3}}\,\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\frac{4\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{2}{3}\left(1+\frac{1}{24}\right)}\right)}{2\sqrt{1+\frac{1}{24}}}+\frac{11-26}{10^3}=-\frac{3}{200}+2\sqrt{\frac{2}{3}}\,U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24}+g_{22}^{-24})=e^{\pi\sqrt{22}}-24+4372e^{-\pi\sqrt{22}}+\cdots=64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of

the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

OPEN STRING ORBIFOLDS
Gianfranco PRADISI and Augusto SAGNOTTI - Received 11 August 1988

5 January 1989

Theory and Phenomenology of Type I strings and M-theory
Emilian Dudas - arXiv:hep-ph/0006190v2 6 Oct 2000

Beyond Logarithmic Corrections to Cardy Formula
Farhang Lorana, M. M. Sheikh-Jabbari and Massimiliano Vincon -
arXiv:1010.3561v3 [hep-th] 31 Jan 2011

A CONVERGENT SERIES FOR THE PARTITION FUNCTION P(n)
BY HANS RADEMACHER - DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA - Communicated January 9, 1937

String thermodynamics and black holes -
http://fma.if.usp.br/~amsilva/Livros/Zwiebach/chapter16.pdf

FOURIER EXPANSIONS OF MODULAR FORMS AND PROBLEMS OF PARTITION[1] - *HANS RADEMACHER* - An address delivered before the Williamsburg meeting of the Society, December 29, 1938, by invitation of the Program Committee.