On the possible mathematical connections between several Ramanujan equations concerning the "Lost Notebook" and the Modular j-invariant, some equations concerning the $SO(2^{13})$ group in Bosonic String Theory, various parameters regarding Particle Physics, ϕ and $\zeta(2)$. IX

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Abstract

In this paper (part IX), we describe and analyze further new mathematical connections between some Ramanujan formulas concerning the "Lost Notebook" and the Modular j-invariant, several equations concerning the $SO(2^{13})$ group, in Bosonic String Theory, various parameters regarding Particle Physics, ϕ and $\zeta(2)$.

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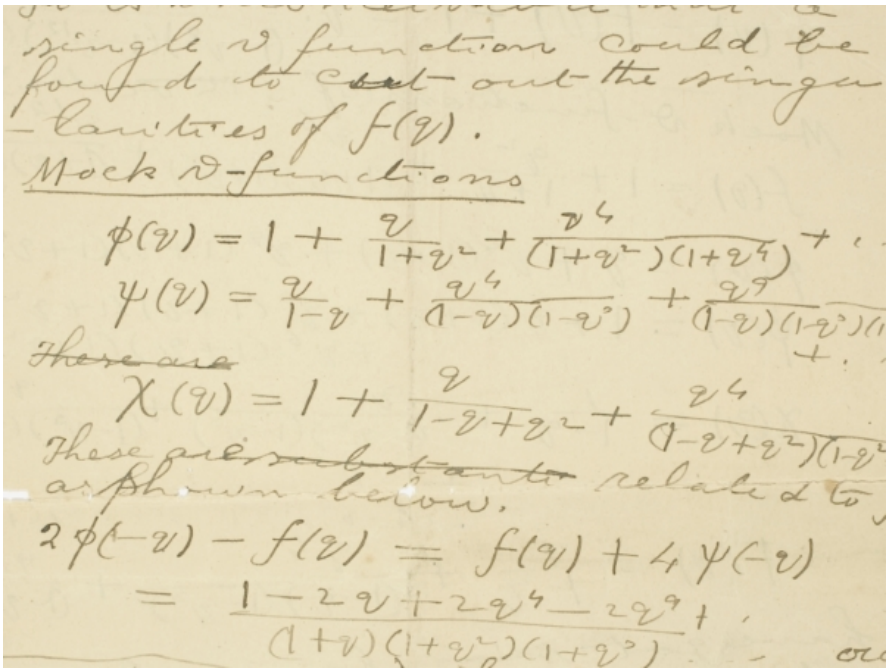

single ϑ function could be found to cut out the singularities of $f(q)$.

Mock ϑ-functions

$$\phi(q) = 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \cdots$$

$$\psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1\cdots} + \cdots$$

~~There are~~

$$\chi(q) = 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2\cdots}$$

These are ~~substantia~~ related to as shown below.

$$2\phi(-q) - f(q) = f(q) + 4\psi(-q)$$

$$= \frac{1-2q+2q^4-2q^9+\cdots}{(1+q)(1+q^2)(1+q^3)\cdots}$$

https://royalsociety.org/science-events-and-lectures/2018/10/srinivasa-ramanujan/



https://www.sapaviva.com/srinivasa-ramanujan-2/

From:

ANOMALY CANCELLATIONS IN SUPERSYMMETRIC D = 10 GAUGE THEORY AND SUPERSTRING THEORY

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and *John H. SCHWARZ* - California Institute of Technology, Pasadena, CA 91125, USA - Received 10 September 1984

We have:

matrices in the adjoint representation. The Lorentz group Chern–Simons form is given in identical fashion by

$$\omega_{3L}^{0} = \mathrm{tr}(\omega R - \tfrac{1}{3}\omega^3) \qquad (7)$$

and satisfies

$$\mathrm{d}\omega_{3L}^{0} = \mathrm{tr}\, R^2 . \qquad (8)$$

absent. We choose to study local Lorentz anomalies, rather than general-coordinate ones, since they bear the greater similarity to the Yang–Mills gauge ones. As described above for the Yang–Mills case, Lorentz anomaly structures arising from hexagon (and higher) diagrams with external gravitons only can be associated with the formal twelve-form [3]

$$-[\tfrac{1}{32} + (n - 496)/13824]\ (\mathrm{tr}\, R^2)^3$$

$$-[\tfrac{1}{8} + (n - 496)/5760]\ \mathrm{tr}\, R^2\ \mathrm{tr}\, R^4$$

$$-[(n - 496)/7560]\ \mathrm{tr}\, R^6. \qquad (20)$$

In this expression we have included the contributions of one left-handed spin 3/2 gravitino and one right-handed spin 1/2 field from the supergravity sector and n left-handed spin 1/2 fields from the matter sector, which only depends on the dimension of the gauge group.

From (20), for n = 16384 = 2^2 * 64^2 , R = 2 , we obtain:

-(1/32+(16384-496)/13824) * (2^2)^3 – (1/8+(16384-496)/5760) * 2^2*2^4 – ((16384-496)/7560) * 2^6

Input:

$$-\left(\frac{1}{32}+\frac{16\,384-496}{13\,824}\right)(2^2)^3-\left(\frac{1}{8}+\frac{16\,384-496}{5760}\right)\times 2^2\times 2^4-\frac{16\,384-496}{7560}\times 2^6$$

Exact result:

$$-\frac{41\,432}{105}$$

Decimal approximation:

–394.590476190476190476190476190476190476190476190476190476...

-394.590476…

We observe that:

-1/3 * (((-(1/32+(16384-496)/13824) * (2^2)^3 – (1/8+(16384-496)/5760) * 2^2*2^4 – ((16384-496)/7560) * 2^6)))+8

Input:

$$-\frac{1}{3}\left(-\left(\frac{1}{32}+\frac{16\,384-496}{13\,824}\right)(2^2)^3-\left(\frac{1}{8}+\frac{16\,384-496}{5760}\right)\times 2^2\times 2^4-\frac{16\,384-496}{7560}\times 2^6\right)+8$$

Exact result:

$$\frac{43\,952}{315}$$

Decimal approximation:

139.530158730158730158730

-1/3 * (((-(1/32+(16384-496)/13824) * (2^2)^3 – (1/8+(16384-496)/5760) * 2^2*2^4 – ((16384-496)/7560) * 2^6)))+8-11-3

Input:

$$-\frac{1}{3}\left(-\left(\frac{1}{32}+\frac{16\,384-496}{13\,824}\right)(2^2)^3-\left(\frac{1}{8}+\frac{16\,384-496}{5760}\right)\times 2^2\times 2^4-\frac{16\,384-496}{7560}\times 2^6\right)+8-11-3$$

Exact result:

$$\frac{39\,542}{315}$$

Decimal approximation:

125.530158730158730158730

Alternate forms:

$$c^2\left(c^2\left(c^4\left(\frac{x^2}{84}-\frac{c^2\,x^2}{504}\right)+\frac{x^2}{36}\right)-\frac{x^2}{6}\right)-\frac{5\,a\,x^2}{4}$$

$$x^2\left(c^2\left(c^2\left(\left(\frac{1}{84}-\frac{c^2}{504}\right)c^4+\frac{1}{36}\right)-\frac{1}{6}\right)-\frac{5\,a}{4}\right)$$

$$-\frac{1}{504}\,x^2\left(630\,a+\left(c^8-6\,c^6-14\,c^2+84\right)c^2\right)$$

Expanded form:

$$-\frac{5\,a\,x^2}{4}-\frac{1}{504}\,c^{10}\,x^2+\frac{c^8\,x^2}{84}+\frac{c^4\,x^2}{36}-\frac{c^2\,x^2}{6}$$

From

$$-\frac{1}{504}\,x^2\left(630\,a+\left(c^8-6\,c^6-14\,c^2+84\right)c^2\right)$$

For c = 4, we have

-1/504 (630 a + 4^2 (84 - 14 *4^2 - 6 *4^6 + 4^8))

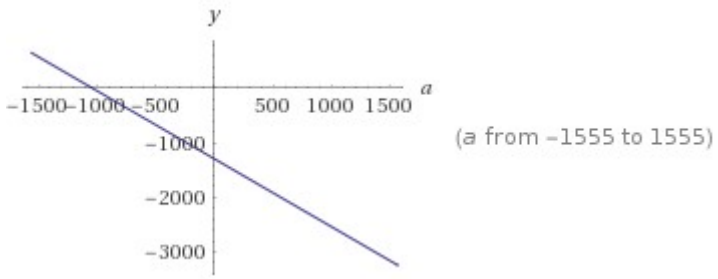
Input:

$$-\frac{1}{504}\left(630\,a+4^2\left(84-14\times4^2-6\times4^6+4^8\right)\right)$$

Result:

$$\frac{1}{504}\,(-630\,a-653\,120)$$

Plot:



Geometric figure:

line

Alternate forms:

$$-\frac{1}{252}\times 5\,(63\,a+65\,312)$$

$$-\frac{5\,a}{4}-\frac{81\,640}{63}$$

Root:

$$a\approx -1036.7$$

-1036.7

Derivative:

$$\frac{d}{da}\left(-\frac{1}{504}\left(630\,a+4^2\left(84-14\times 4^2-6\times 4^6+4^8\right)\right)\right)=-\frac{5}{4}$$

Indefinite integral:

$$\int \frac{1}{504}\,(-653\,120-630\,a)\,da=-\frac{5\,a^2}{8}-\frac{81\,640\,a}{63}+\text{constant}$$

From which:

-1/504 (630 (-1036.7) + 4^2 (84 - 14 *4^2 - 6 *4^6 + 4^8))

Input interpretation:

$$-\frac{1}{504}\left(630\times(-1036.7)+4^2\left(84-14\times 4^2-6\times 4^6+4^8\right)\right)$$

Result:

0.00198412698412698412698

and again:

16((1/ 0.0019841269841269)+8)

Input interpretation:

$$16\left(\frac{1}{0.0019841269841269}+8\right)$$

Result:

8192.000000000341913600000014497136640000614678593536026062...

$8192 = 2*64^2$

2((1/ 0.0019841269841269)+8)+9^3-24

Input interpretation:

$$2\left(\frac{1}{0.0019841269841269}+8\right)+9^3-24$$

Result:

1729.000000000042739200000001812142080000076834824192003257...

1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((2((1/ 0.0019841269841269)+8)+9^3-24)))^1/15

Input interpretation:

$$\sqrt[15]{2\left(\frac{1}{0.0019841269841269}+8\right)+9^3-24}$$

Result:

1.64381522874873...

$1.6438152\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Now, from:

The twelve-forms in (14), (20) and (22) are given with the correct relative normalization. The mixed anomalies can be cancelled by adding the local terms

$$S_3 = c\int \left(\tfrac{1}{8}B\,\mathrm{tr}\,R^2\,\mathrm{tr}\,F^2 + \tfrac{1}{48}\omega_{3\mathrm{L}}^0\omega_{3\mathrm{Y}}^0\,\mathrm{tr}\,R^2 - \tfrac{1}{24}\omega_{3\mathrm{Y}}^0\omega_{3\mathrm{L}}^0\,\mathrm{tr}\,F^2 - \tfrac{2}{3}\omega_{3\mathrm{L}}^0\omega_{7\mathrm{Y}}^0 + \tfrac{1}{12}\omega_{3\mathrm{Y}}^0\omega_{7\mathrm{L}}^0\right). \quad (23)$$

$$\omega_{3\mathrm{Y}}^0 = \mathrm{tr}(AF - \tfrac{1}{3}A^3)$$

$$\omega_{3\mathrm{L}}^0 = \mathrm{tr}(\omega\bar{R} - \tfrac{1}{3}\omega^3)$$

For:

(2c-1/3*c^3)*(2c-1/7*c^7) , and the previous data, we obtain:

integrate ((1/8*2^2*2^2 + 1/48*(2c-1/3*c^3)*(2c-1/3*c^3) (2^2)-1/24(2c-1/3*c^3)*(2c-1/3*c^3)*2^2-2/3(2c-1/3*c^3)*(2c-1/7*c^7)+1/12(2c-1/3*c^3)*(2c-1/7*c^7)))x

for c = 4:

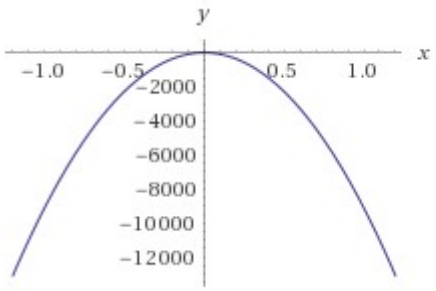
integrate [(1/8*16+1/48(8-1/3*64)(2*4-1/3*64)*4-1/24(2*4-1/3*64)(2*4-1/3*64)*4-2/3(2*4-1/3*64)(2*4-1/7*16384)+1/12(2*4-1/3*64)(2*4-1/7*16384))]x

Indefinite integral:

$$\int\left(\frac{16}{8}+\frac{1}{48}\left(8-\frac{64}{3}\right)\left(\left(2\times4-\frac{64}{3}\right)4\right)-\frac{1}{24}\left(2\times4-\frac{64}{3}\right)\left(\left(2\times4-\frac{64}{3}\right)4\right)-\frac{2}{3}\left(2\times4-\frac{64}{3}\right)\left(2\times4-\frac{16\,384}{7}\right)+\frac{1}{12}\left(2\times4-\frac{64}{3}\right)\left(2\times4-\frac{16\,384}{7}\right)\right)x\,dx = -\frac{245\,093\,x^2}{27}+\text{constant}$$

-9077.51851851851850 x^2

Plot of the integral:



(x from −1.2 to 1.2)

Input:

$$-\frac{245\,093}{27}$$

Exact result:

$$-\frac{245\,093}{27}$$ (irreducible)

Decimal approximation:

−9077.518518518518518518518518518518518518518518518518…

-9077.518518…

From which:

(-1/144(-245093/27) + 1)^2-5

Input:

$$\left(-\frac{1}{144}\left(-\frac{245\,093}{27}\right)+1\right)^2-5$$

Exact result:

$$\frac{61\,915\,955\,641}{15\,116\,544}$$

Decimal approximation:

4095.906818450037257193178546630764280512794458839269081610...

4095.9068…≈ 4096 = 64^2

And:

1/5(-(-9077.51851851851850)) -76 – 11

Input interpretation:

$\frac{1}{5}(-(-9077.51851851851850)) - 76 - 11$

Result:

1728.5037037037037

1728.5037…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

[1/5(-(-9077.51851851851850)) -76 - 11]^1/15

Input interpretation:

$\sqrt[15]{\frac{1}{5}(-(-9077.51851851851850)) - 76 - 11}$

Result:

1.643783768223893147...

1.6437837….≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

(-(-9077.51851851851850))^1/19

Input interpretation:

$\sqrt[19]{-(-9077.51851851851850)}$

Result:

1.6155264029246011 89...

1.6155264…. result that is a good approximation to the value of the golden ratio 1.618033988749...

We have that:

$$S_1 = c \int (B \operatorname{tr} F^4 + \tfrac{2}{3}\omega^0_{3Y}\omega^0_{7Y})$$

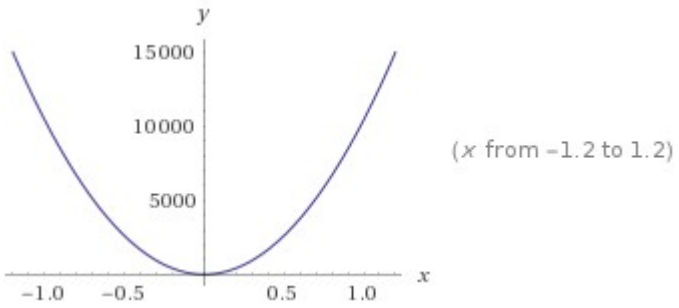
For F = 2, B = 3, and (2c-1/3*c^3)*(2c-1/7*c^7) , with c = 4, we obtain:

integrate ((3*2^4) + 2/3*(8-1/3*64)*(8-1/7*16384)))x

Indefinite integral:

$$\int \left(3 \times 2^4 + \frac{2}{3}\left(8 - \frac{64}{3}\right)\left(8 - \frac{16\,384}{7}\right)\right) x\,dx = \frac{654\,632\,x^2}{63} + \text{constant}$$

Plot of the integral:



(x from −1.2 to 1.2)

From

(654632 x^2)/63 , we obtain:

Input:

$$\frac{654\,632}{63}$$

Exact result:

$$\frac{654\,632}{63}$$ (irreducible)

Decimal approximation:

10390.984126984126984126984126984126984126984126984126984126...

10390.984126…

From which:

[(654632)/63]^1/18

Input:

$$\sqrt[18]{\frac{654\,632}{63}}$$

Result:

$$\frac{\sqrt[6]{2}\ \sqrt[18]{\frac{81829}{7}}}{\sqrt[9]{3}}$$

Decimal approximation:

1.671658624934192100594507806409285638435745225912025583965...

1.6716586249…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Alternate form:

root of $63\,x^{18} - 654\,632$ near $x = 1.67166$

From the three results, we obtain:

(-10797239/8064) * (654632)/63 * (-245093/27)

Input:

$$-\frac{10\,797\,239}{8064} \times \frac{654\,632}{63}\left(-\frac{245\,093}{27}\right)$$

Exact result:

$$\frac{216\,546\,349\,218\,217\,183}{1\,714\,608}$$

Decimal approximation:

$1.26294960258098167627819303304312122654274329759338 57\ldots \times 10^{11}$

$1.2629496\ldots*10^{11}$

From which:

[(-10797239/8064) * (654632)/63 * (-245093/27)]/(4096^2*2)-2*1024+13

Input:

$$\frac{-\frac{10\,797\,239}{8064} \times \frac{654\,632}{63}\left(-\frac{245\,093}{27}\right)}{4096^2 \times 2} - 2 \times 1024 + 13$$

Exact result:

$$\frac{99\,467\,309\,718\,912\,223}{57\,532\,697\,542\,656}$$

Decimal approximation:

$1728.88312155300878369269678903556235594374924180920653 5837\ldots$

$1728.883121\ldots$

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

CERN-TH/2002-025 ROM2F-2002/08 LPTENS 02/14 CPHT RR 020.0202
hep-th/0204089 - **Open Strings** - *C Angelantonj and A Sagnotti*

We have that:

Orbifold compactification 61

An equivalent description of this spectrum exhibits its orthogonal decomposition in different sectors [135]. It is rather effective and explicit for the oscillator excitations at the origin of the lattice, for which one can define the four combinations

$$\phi_{|\pm} = \tfrac{1}{2}\left(\frac{1}{\eta} + \sqrt{\frac{2\eta}{\vartheta_2}}\right), \qquad \phi_{\pm} = \tfrac{1}{2}\left(\sqrt{\frac{\eta}{\vartheta_4}} + \sqrt{\frac{\eta}{\vartheta_3}}\right), \tag{250}$$

and in terms of these expressions the torus partition function becomes

$$\mathcal{T} = |\phi_{++}|^2 + |\phi_{+-}|^2 + 2|\phi_{-+}|^2 + 2|\phi_{--}|^2 + \tfrac{1}{2}\sum{}'. \tag{251}$$

$$v = R/\sqrt{\alpha'}.$$

both D25 and D24 branes would be needed in this case if one insisted on cancelling all tadpoles, as can be seen from the transverse-channel open-string amplitudes

$$\tilde{\mathcal{A}} = \frac{2^{-13}}{4}\left[N^2\, v\, W_n + \frac{D^2}{v} P_n + 2\,N\,D\sqrt{\frac{2\eta}{\vartheta_2}} \right.$$
$$\left. + 2\left(R_N^2 + R_D^2\right)\sqrt{\frac{\eta}{\vartheta_4}} + 2\sqrt{2}\,R_N\,R_D\sqrt{\frac{\eta}{\vartheta_3}}\right] \tag{258}$$

and

$$\tilde{\mathcal{M}} = \tfrac{2}{4}\epsilon\left[N\,v\,W_{2n} + \frac{D}{v}P_{2m} + (N+D)\sqrt{\frac{2\hat{\eta}}{\hat{\vartheta}_2}}\right], \tag{259}$$

that in terms of the ϕ's would become

$$\begin{aligned}\tilde{\mathcal{A}} = \frac{2^{-13}}{4}\Bigg\{ & \left(N\sqrt{v} + \frac{D}{\sqrt{v}}\right)^2 \phi_{++} \\ & + \left(N\sqrt{v} - \frac{D}{\sqrt{v}}\right)^2 \phi_{+-} \\ & + 2\left[\left(\frac{R_N}{\sqrt{2}} + R_D\right)^2 + \left(\frac{R_N}{\sqrt{2}}\right)^2\right] \phi_{-+} \\ & + 2\left[\left(\frac{R_N}{\sqrt{2}} - R_D\right)^2 + \left(\frac{R_N}{\sqrt{2}}\right)^2\right] \phi_{--}\Bigg\} + \dots \end{aligned} \tag{260}$$

and

$$\begin{aligned}\tilde{\mathcal{M}} = \tfrac{2}{4}\epsilon\Bigg[& \left(\sqrt{v} + \frac{1}{\sqrt{v}}\right)\left(N\sqrt{v} + \frac{D}{\sqrt{v}}\right) \hat{\phi}_{++} \\ & + \left(\sqrt{v} - \frac{1}{\sqrt{v}}\right)\left(N\sqrt{v} - \frac{D}{\sqrt{v}}\right) \hat{\phi}_{+-}\Bigg] + \dots . \end{aligned} \tag{261}$$

These expressions match precisely the Klein-bottle amplitude (255), while the tadpole conditions would lead to

$$\begin{aligned} N &= -8192\,\epsilon\,, & R_N &= 0\,, \\ D &= -8192\,\epsilon\,, & R_D &= 0\,, \end{aligned} \tag{262}$$

thus also requiring that $\epsilon = -1$.

$$\begin{aligned}\tilde{\mathcal{M}} = \tfrac{2}{4}\epsilon\Bigg[& \left(\sqrt{v} + \frac{1}{\sqrt{v}}\right)\left(N\sqrt{v} + \frac{D}{\sqrt{v}}\right) \hat{\phi}_{++} \\ & + \left(\sqrt{v} - \frac{1}{\sqrt{v}}\right)\left(N\sqrt{v} - \frac{D}{\sqrt{v}}\right) \hat{\phi}_{+-}\Bigg] + \dots . \end{aligned}$$

$$\phi_{+\pm} = \tfrac{1}{2}\begin{pmatrix} 1 \\ \eta \end{pmatrix} \pm \sqrt{\begin{matrix} 2\eta \\ \vartheta_2 \end{matrix}}$$

2/4(-1) [((1+1)(8192+8192) * 1/2(1/a+sqrt((2a)/b))]

-1/2(2*(16384)*1/2(1/a+sqrt((2a)/b))

-1/2(2*(16384)*1/2(1/eta+sqrt((2*eta)/theta^2)))

Input:

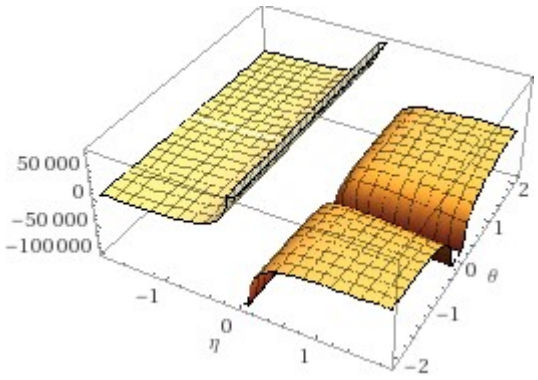
$$-\frac{1}{2}\left(2\times 16\,384\times\frac{1}{2}\left(\frac{1}{\eta}+\sqrt{\frac{2\,\eta}{\theta^2}}\right)\right)$$

Exact result:

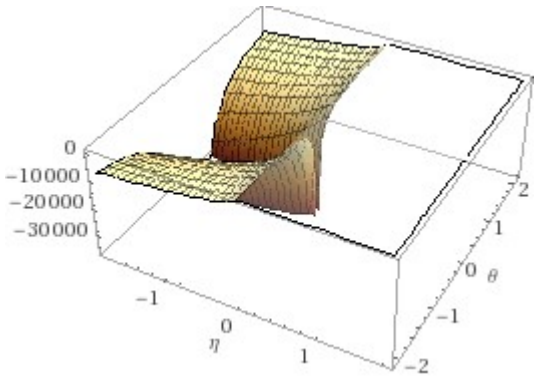
$$-8192\left(\sqrt{2}\,\sqrt{\frac{\eta}{\theta^2}}+\frac{1}{\eta}\right)$$

3D plots:

Real part

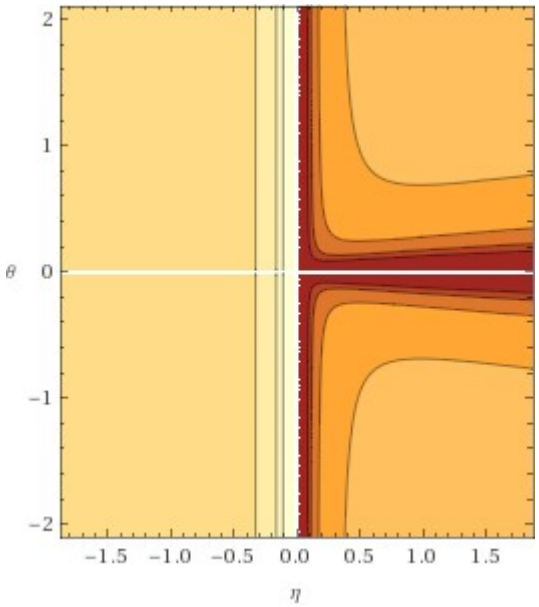


Imaginary part

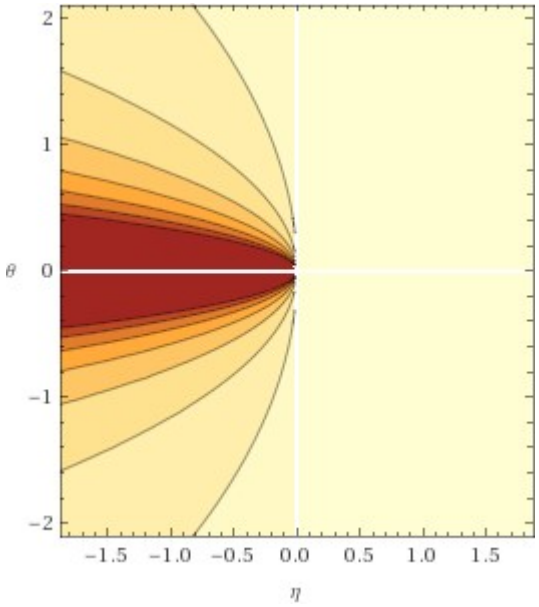


Contour plots:

Real part



Imaginary part



Alternate form assuming η and θ are real:

$$-8192\left(\frac{\sqrt{2}\ \sqrt{\eta}}{|\theta|}+\frac{1}{\eta}\right)$$

$|z|$ is the absolute value of z

Alternate form:

$$-\frac{8192\left(\sqrt{2}\ \eta\sqrt{\frac{\eta}{\theta^2}}+1\right)}{\eta}$$

Expanded form:

$$-8192\sqrt{2}\ \sqrt{\frac{\eta}{\theta^2}}-\frac{8192}{\eta}$$

Alternate forms assuming η and θ are positive:

$$-\frac{8192\left(\sqrt{2}\ \eta^{3/2}+\theta\right)}{\eta\,\theta}$$

$$-\frac{8192\sqrt{2}\ \sqrt{\eta}}{\theta}-\frac{8192}{\eta}$$

$$-8192\left(\frac{\sqrt{2}\ \sqrt{\eta}}{\theta}+\frac{1}{\eta}\right)$$

Roots for the variable θ:

$$\theta=-\sqrt{2}\ \eta^{3/2}$$

$$\theta=\sqrt{2}\ \eta^{3/2}$$

Series expansion at η = 0:

$$-\frac{8192}{\eta}-8192\sqrt{\eta}\left(\sqrt{2}\ \sqrt{\frac{1}{\theta^2}}\right)+O\left(\eta^{11/2}\right)$$

(Puiseux series)

Series expansion at η = ∞:

$$-8192\left(\sqrt{2}\,\sqrt{\frac{\eta}{\theta^2}}\right)-\frac{8192}{\eta}+O\left(\left(\frac{1}{\eta}\right)^6\right)$$

(Puiseux series)

Derivative:

$$\frac{\partial}{\partial\eta}\left(-\frac{2\times 16\,384\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)}{2\times 2}\right)=-\frac{4096\left(\sqrt{2}\,\eta\sqrt{\frac{\eta}{\theta^2}}-2\right)}{\eta^2}$$

Indefinite integral:

$$\int -8192\left(\frac{1}{\eta}+\sqrt{2}\,\sqrt{\frac{\eta}{\theta^2}}\right)d\eta=-8192\left(\frac{2}{3}\sqrt{2}\,\eta\sqrt{\frac{\eta}{\theta^2}}+\log(\eta)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Limit:

$$\lim_{\theta\to\pm\infty} -8192\left(\frac{1}{\eta}+\sqrt{2}\,\sqrt{\frac{\eta}{\theta^2}}\right)=-\frac{8192}{\eta}$$

Series representations:

$$\frac{\left(2\times 16\,384\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)\right)(-1)}{2\times 2}=-\frac{8192}{\eta}-8192\sqrt{-1+\frac{2\eta}{\theta^2}}\sum_{k=0}^{\infty}\left(-1+\frac{2\eta}{\theta^2}\right)^{-k}\binom{\frac{1}{2}}{k}$$

for $\left|1-\frac{2\eta}{\theta^2}\right|>1$

$$\frac{\left(2\times 16\,384\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)\right)(-1)}{2\times 2}=-\frac{8192}{\eta}-8192\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{2\eta}{\theta^2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

for $\left|1-\frac{2\eta}{\theta^2}\right|<1$

$$\frac{\left(2\times 16\,384\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)\right)(-1)}{2\times 2}=-\frac{8192}{\eta}-8192\sqrt{-1+\frac{2\eta}{\theta^2}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{2\eta}{\theta^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $\left|1-\frac{2\eta}{\theta^2}\right|>1$

Now, we take

$$-8192\left(\frac{\sqrt{2}\ \sqrt{\eta}}{|\theta|}+\frac{1}{\eta}\right)$$

For $\eta = \theta = 0.5$

-8192 ((sqrt(2) sqrt(0.5))/(0.5) + 1/(0.5))

Input:

$$-8192\left(\frac{\sqrt{2}\ \sqrt{0.5}}{0.5}+\frac{1}{0.5}\right)$$

Result:

$-32\,768$

-32768

For

$$\begin{aligned}\tilde{A}=\frac{2^{-13}}{4}\Bigg\{&\left(N\sqrt{v}+\frac{D}{\sqrt{v}}\right)^2\phi_{++}\\&+\left(N\sqrt{v}-\frac{D}{\sqrt{v}}\right)^2\phi_{+-}\\&+2\left[\left(\frac{R_N}{\sqrt{2}}+R_D\right)^2+\left(\frac{R_N}{\sqrt{2}}\right)^2\right]\phi_{-+}\\&+2\left[\left(\frac{R_N}{\sqrt{2}}-R_D\right)^2+\left(\frac{R_N}{\sqrt{2}}\right)^2\right]\phi_{--}\Bigg\}+\ldots\end{aligned}$$

we have that:

(2^(-13))/4 ((8192+8192)^2 * 1/2(1/eta+sqrt((2*eta)/theta^2)))

Input:

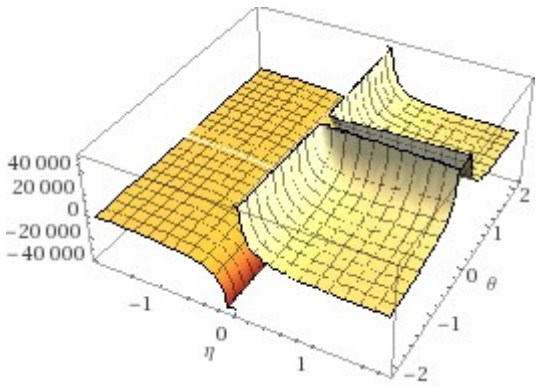
$$\frac{1}{2^{13}\times 4}\left((8192+8192)^2\times\frac{1}{2}\left(\frac{1}{\eta}+\sqrt{\frac{2\,\eta}{\theta^2}}\right)\right)$$

Exact result:

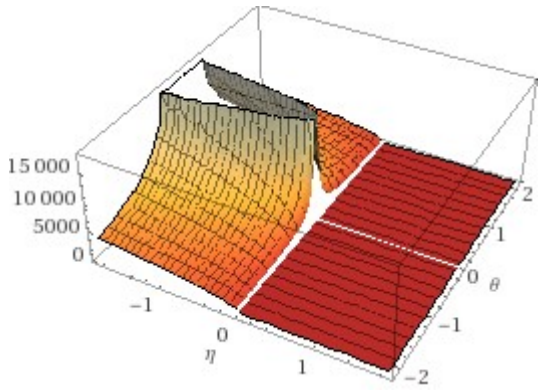
$$4096\left(\sqrt{2}\,\sqrt{\frac{\eta}{\theta^2}}+\frac{1}{\eta}\right)$$

3D plots:

Real part

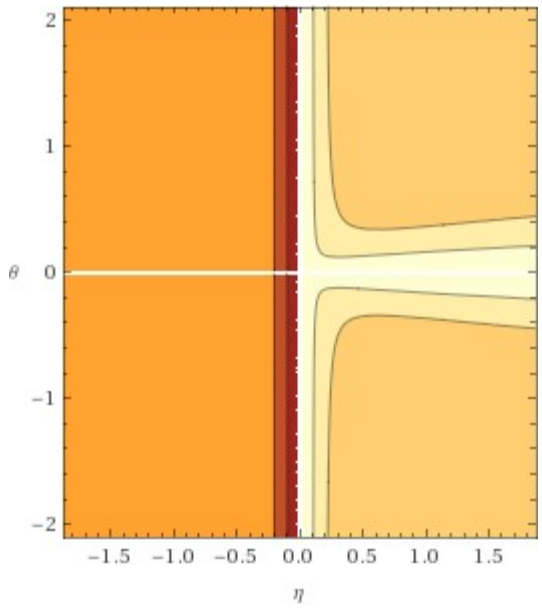


Imaginary part

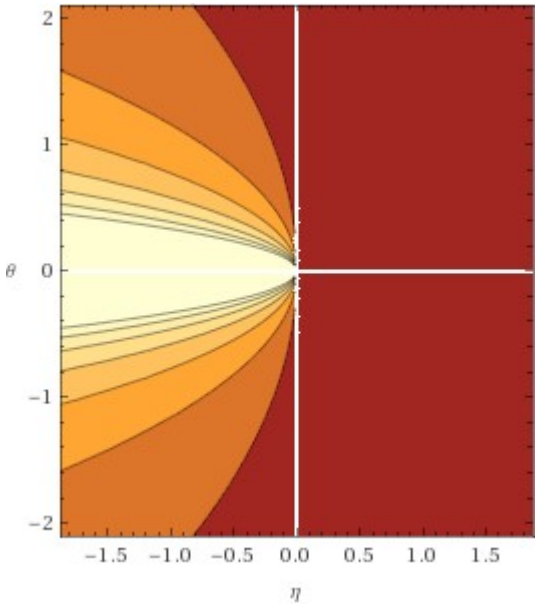


Contour plots:

Real part



Imaginary part



Alternate form assuming η and θ are real:

$$4096\left(\frac{\sqrt{2}\ \sqrt{\eta}}{|\theta|}+\frac{1}{\eta}\right)$$

$|z|$ is the absolute value of z

Alternate form:

$$\frac{4096\left(\sqrt{2}\ \eta\sqrt{\frac{\eta}{\theta^2}}+1\right)}{\eta}$$

Expanded form:

$$4096\sqrt{2}\ \sqrt{\frac{\eta}{\theta^2}}+\frac{4096}{\eta}$$

Alternate forms assuming η and θ are positive:

$$\frac{4096\left(\sqrt{2}\ \eta^{3/2}+\theta\right)}{\eta\,\theta}$$

$$\frac{4096\sqrt{2}\ \sqrt{\eta}}{\theta}+\frac{4096}{\eta}$$

$$4096\left(\frac{\sqrt{2}\ \sqrt{\eta}}{\theta}+\frac{1}{\eta}\right)$$

Roots for the variable θ:

$$\theta=-\sqrt{2}\ \eta^{3/2}$$

$$\theta=\sqrt{2}\ \eta^{3/2}$$

Series expansion at η = 0:

$$\frac{4096}{\eta}+4096\sqrt{2}\ \sqrt{\eta}\ \sqrt{\frac{1}{\theta^2}}+O\left(\eta^{11/2}\right)$$

(Puiseux series)

Series expansion at η = ∞:

$$4096\sqrt{2}\ \sqrt{\frac{\eta}{\theta^2}}+\frac{4096}{\eta}+O\left(\left(\frac{1}{\eta}\right)^6\right)$$

(Puiseux series)

Derivative:

$$\frac{\partial}{\partial\eta}\left(\frac{(8192+8192)^2\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)}{(2^{13}\times4)\times2}\right)=\frac{2048\left(\sqrt{2}\ \eta\sqrt{\frac{\eta}{\theta^2}}-2\right)}{\eta^2}$$

Indefinite integral:

$$\int 4096\left(\frac{1}{\eta}+\sqrt{2}\ \sqrt{\frac{\eta}{\theta^2}}\right)d\eta=4096\left(\frac{2}{3}\sqrt{2}\ \eta\sqrt{\frac{\eta}{\theta^2}}+\log(\eta)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Limit:

$$\lim_{\theta\to\pm\infty} 4096\left(\frac{1}{\eta}+\sqrt{2}\,\sqrt{\frac{\eta}{\theta^2}}\right)=\frac{4096}{\eta}$$

Series representations:

$$\frac{(8192+8192)^2\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)}{2\times 2^{13}\times 4}=\frac{4096}{\eta}+4096\sqrt{-1+\frac{2\eta}{\theta^2}}\sum_{k=0}^{\infty}\left(-1+\frac{2\eta}{\theta^2}\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$\text{for}\left|1-\frac{2\eta}{\theta^2}\right|>1$$

$$\frac{(8192+8192)^2\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)}{2\times 2^{13}\times 4}=\frac{4096}{\eta}+4096\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{2\eta}{\theta^2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\quad\text{for}\left|1-\frac{2\eta}{\theta^2}\right|<1$$

$$\frac{(8192+8192)^2\left(\frac{1}{\eta}+\sqrt{\frac{2\eta}{\theta^2}}\right)}{2\times 2^{13}\times 4}=\frac{4096}{\eta}+4096\sqrt{-1+\frac{2\eta}{\theta^2}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{2\eta}{\theta^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$\text{for}\left|1-\frac{2\eta}{\theta^2}\right|>1$$

From:

$$4096\left(\frac{\sqrt{2}\,\sqrt{\eta}}{|\theta|}+\frac{1}{\eta}\right)$$

For $\eta = \theta = 0.5$

4096 (1/0.5 + (sqrt(2) sqrt(0.5))/(0.5))

Input:

$$4096\left(\frac{1}{0.5}+\frac{\sqrt{2}\,\sqrt{0.5}}{0.5}\right)$$

Result:

16 384

16384

From the algebraic sum of the two expression, we obtain:

$$-\left\{\tilde{\mathcal{M}} = \frac{2}{4}\epsilon\left[\left(\sqrt{v}+\frac{1}{\sqrt{v}}\right)\left(N\sqrt{v}+\frac{D}{\sqrt{v}}\right)\hat{\phi}_{++} + \left(\sqrt{v}-\frac{1}{\sqrt{v}}\right)\left(N\sqrt{v}-\frac{D}{\sqrt{v}}\right)\hat{\phi}_{+-}\right]+\dots.\right\}+$$

$$-\left\{\tilde{\mathcal{A}} = \frac{2^{-13}}{4}\left\{\left(N\sqrt{v}+\frac{D}{\sqrt{v}}\right)^2\phi_{++} + \left(N\sqrt{v}-\frac{D}{\sqrt{v}}\right)^2\phi_{+-} + 2\left[\left(\frac{R_N}{\sqrt{2}}+R_D\right)^2+\left(\frac{R_N}{\sqrt{2}}\right)^2\right]\phi_{-+} + 2\left[\left(\frac{R_N}{\sqrt{2}}-R_D\right)^2+\left(\frac{R_N}{\sqrt{2}}\right)^2\right]\phi_{--}\right\}+\dots\right\}=$$

-(-32768) - 16384 = 16384 = 8192*2 = 64^2 *2^2

We have also:

(-(-32768) * 16384)/8192*1/4096

Input:

$$-\frac{-32\,768\times 16\,384}{8192}\times\frac{1}{4096}$$

Exact result:

16

16

And:

-(-32768) / 16384

Input:

$$-\frac{-32\,768}{16\,384}$$

Result:

2

2

-2/4096 (-32768-16384)

Input:

$$-\frac{2}{4096}\,(-32\,768-16\,384)$$

Exact result:

24

24 value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

[-(-32768 * 16384)]^1/42

Input:

$$\sqrt[42]{-(-32\,768 \times 16\,384)}$$

Result:

$$2^{29/42}$$

Decimal approximation:

1.613816102964170993448924709883603332317823958925850514956...

1.6138161029…. result that is a good approximation to the value of the golden ratio 1.618033988749...

1/13*[-(-32768 * 16384)]^1/2-47-7

Input:

$$\frac{1}{13}\sqrt{-(-32\,768 \times 16\,384)} - 47 - 7$$

Result:

$$\frac{16\,384\sqrt{2}}{13} - 54$$

Decimal approximation:

1728.344231224676098428220619803976409175808000475070556225...

1728.3442312…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$\frac{2}{13}\left(8192\sqrt{2} - 351\right)$$

$$\frac{1}{13}\left(16\,384\sqrt{2}-702\right)$$

Minimal polynomial:

$$169x^2+18\,252x-536\,378\,108$$

(((1/13*[-(-32768 * 16384)]^1/2-47-7)))^1/15

Input:

$$\sqrt[15]{\frac{1}{13}\sqrt{-(-32\,768\times 16\,384)}-47-7}$$

Result:

$$\sqrt[15]{\frac{16\,384\sqrt{2}}{13}-54}$$

Decimal approximation:

1.643773657376422748600366112533294701826014418244381126822...

$1.643773657\ldots \approx \zeta(2)=\frac{\pi^2}{6}=1.644934\ldots$

Alternate forms:

$$\frac{1}{13}\sqrt[15]{2\left(8192\sqrt{2}-351\right)}\;13^{14/15}$$

$$\sqrt[15]{\frac{2}{13}\left(8192\sqrt{2}-351\right)}$$

Minimal polynomial:

$$169x^{30}+18\,252x^{15}-536\,378\,108$$

Now, we have that:

These can be associated to the standard bosonic O25 planes, whose contribution scales proportionally to the length of the circle, consistently with the fact that they invade the whole internal space, and to new bosonic O24 planes, whose contribution scales inversely with it, consistently with the fact that they are localized at the two fixed points. Thus, both D25 and D24 branes would be needed in this case if one insisted on cancelling all tadpoles, as can be seen from the transverse-channel open-string amplitudes

$$\tilde{\mathcal{A}} = \frac{2^{-13}}{4}\left[N^2\, v\, W_n + \frac{D^2}{v}\, P_n + 2\, N\, D\sqrt{\frac{2\eta}{\vartheta_2}} + 2\left(R_N^2 + R_D^2\right)\sqrt{\frac{\eta}{\vartheta_4}} + 2\sqrt{2}\, R_N\, R_D\sqrt{\frac{\eta}{\vartheta_3}}\right] \tag{258}$$

From the above data, we obtain:

(2^(-13))/4 [((8192^2* a + 8192^2 * b + 2*8192*8192 *sqrt(((2*0.5)/0.5))]

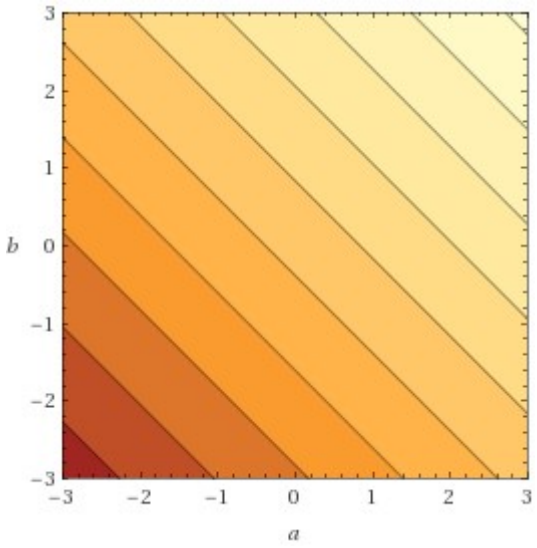
Input:

$$\frac{1}{2^{13}\times 4}\left(8192^2\, a + 8192^2\, b + 2\times 8192\times 8192\sqrt{\frac{2\times 0.5}{0.5}}\right)$$

Result:

$$\frac{67\,108\,864\, a + 67\,108\,864\, b + 1.89813\times 10^8}{32\,768}$$

Contour plot:



Geometric figure:

line

Alternate forms:

$4.08894\times10^{-8}\,(5.00863\times10^{10}\,a+5.00863\times10^{10}\,b+1.41665\times10^{11})$

$2048\,a+2048\,b+5792.62$

$2048\,(a+b+2.82843)$

Expanded form:

$2048\,a+2048\,b+5792.62$

Real root:

$b\approx -a-2.82843$

Root:

$b\approx -a-2.82843$

Root for the variable b:

$b\approx 0.000488281\,(-2048\,a-5792.62)$

Derivative:

$$\frac{\partial}{\partial a}\left(\frac{8192^2\,a+8192^2\,b+2\times8192\times8192\sqrt{\frac{2\times0.5}{0.5}}}{2^{13}\times4}\right)=2048$$

Indefinite integral:

$$\int \frac{1.89813\times10^8 + 67\,108\,864\,a + 67\,108\,864\,b}{32\,768}\,da = 2048\,(0.5\,a^2 + a\,b + 2.82843\,a) + \text{constant}$$

Definite integral over a disk of radius R:

$$\iint_{a^2+b^2<R^2} \frac{67\,108\,864\,a + 67\,108\,864\,b + 1.89813\times10^8}{32\,768}\,da\,db = 18\,198.\,R^2 + 0$$

Definite integral over a square of edge length 2 L:

$$\int_{-L}^{L}\int_{-L}^{L} \frac{1.89813\times10^8 + 67\,108\,864\,a + 67\,108\,864\,b}{32\,768}\,db\,da = 23\,170.5\,L^2 + 0$$

From

$$\frac{67\,108\,864\,a + 67\,108\,864\,b + 1.89813\times10^8}{32\,768}$$

For a = b = 1.5, we obtain:

(67108864 * 1.5 + 67108864 * 1.5 + 1.89813×10^8)/32768

Input interpretation:

$$\frac{67\,108\,864\times1.5 + 67\,108\,864\times1.5 + 1.89813\times10^8}{32\,768}$$

Result:

11936.633056640625

11936.633056640625

1/16*[(67108864 * 1.5 + 67108864 * 1.5 + 1.89813×10^8)/32768]+29

Input interpretation:

$$\frac{1}{16} \times \frac{67\,108\,864 \times 1.5 + 67\,108\,864 \times 1.5 + 1.89813 \times 10^8}{32\,768} + 29$$

Result:

775.0395660400390625

775.0395660400390625 result practically equal to the rest mass of Charged rho meson 775.11

1/7*[(67108864 * 1.5 + 67108864 * 1.5 + 1.89813×10^8)/32768]+24

Input interpretation:

$$\frac{1}{7} \times \frac{67\,108\,864 \times 1.5 + 67\,108\,864 \times 1.5 + 1.89813 \times 10^8}{32\,768} + 24$$

Result:

1729.233293805803571428571428571428571428571428571428571428...

1729.2332938…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

1/3*[(67108864 * 1.5 + 67108864 * 1.5 + 1.89813×10^8)/32768]+128-11

Input interpretation:

$$\frac{1}{3} \times \frac{67\,108\,864 \times 1.5 + 67\,108\,864 \times 1.5 + 1.89813 \times 10^8}{32\,768} + 128 - 11$$

Result:

4095.877685546875

4095.877685546875 ≈ 4096

Now, we have that:

To conclude our description of the $S^1/\mathbb{Z}_2$ orbifold, we now turn to the open spectrum, rewriting also $\mathcal{A}$ and $\mathcal{M}$ in terms of the ϕ's. The result is

$$\begin{aligned}\mathcal{A}_0 \sim \tfrac{1}{4}\Big\{ & \left[(N^2+R_N^2)+(D^2+R_D^2)\right]\,\phi_{++} \\ & +\left[(N^2-R_N^2)+(D^2-R_D^2)\right]\,\phi_{+-} \\ & +2\,(N\,D+R_N\,R_D)\,\phi_{-+}+2\,(N\,D+R_N\,R_D)\,\phi_{--}\Big\}\end{aligned} \tag{263}$$

for the annulus amplitude, and

$$\mathcal{M}_0 \sim \tfrac{1}{2}\,\epsilon\,(N+D)\,\phi_{++} \tag{264}$$

for the Möbius amplitude. A consistent particle interpretation of the direct-channel

$$\phi_{+\pm} = \tfrac{1}{2}\left(\frac{1}{\eta} \pm \sqrt{\frac{2\eta}{\vartheta_2}}\right) \qquad \phi_{-\pm} = \tfrac{1}{2}\left(\sqrt{\frac{\eta}{\vartheta_4}} \pm \sqrt{\frac{\eta}{\vartheta_3}}\right)$$

1/4 ((((((8192^2+8192^2)*1/2(1/0.5+sqrt((2*0.5)/0.5))+(8192^2+8192^2)*1/2(1/0.5-sqrt((2*0.5)/0.5))+2*8192^2))))

Input:

$$\frac{1}{4}\left((8192^2+8192^2)\times\frac{1}{2}\left(\frac{1}{0.5}+\sqrt{\frac{2\times0.5}{0.5}}\right)+(8192^2+8192^2)\times\frac{1}{2}\left(\frac{1}{0.5}-\sqrt{\frac{2\times0.5}{0.5}}\right)+2\times8192^2\right)$$

Result:

100 663 296

100663296

1/(8192*16)
((((((8192^2+8192^2)*1/2(1/0.5+sqrt((2*0.5)/0.5))+(8192^2+8192^2)*1/2(1/0.5-sqrt((2*0.5)/0.5))+2*8192^2))))

Input:

$$\frac{1}{8192\times 16}\left((8192^2+8192^2)\times\frac{1}{2}\left(\frac{1}{0.5}+\sqrt{\frac{2\times 0.5}{0.5}}\right)+(8192^2+8192^2)\times\frac{1}{2}\left(\frac{1}{0.5}-\sqrt{\frac{2\times 0.5}{0.5}}\right)+2\times 8192^2\right)$$

Result:

3072

3072 = 24 * 128

From

Topics in Analytic Number Theory - The Circle Method – Chapter 14 – Analytic Theory of Partitions – *Hans Rademacher*

We have:

Conjecture.

$$\lim_{N\to\infty} C_{hkl}(N)$$

exists and is equal to

$$C_{hkl}(\infty) = -2\pi\left(\frac{\pi}{12}\right)^{3/2}\frac{\omega_{hk}e^{\frac{2\pi i h l}{k}}}{k^{5/2}}\Delta_\alpha^{l-1}L_{3/2}\left(-\frac{\pi^2}{6k^2}(\alpha+1)\right),$$

$$\alpha = \frac{1}{24}. \qquad (130.6)$$

And:

$$C_{012}(\infty) = \frac{24}{25\cdot 49}\left(6+\frac{109\sqrt{3}}{35\pi}\right) = 0.15119\ldots,$$

We perform the following calculation:

-2Pi*(Pi/12)^1.5 * a * b * c (-(Pi^2)/6 (1/24+1)) = 24/(25*49) (6+(109sqrt3)/35pi)

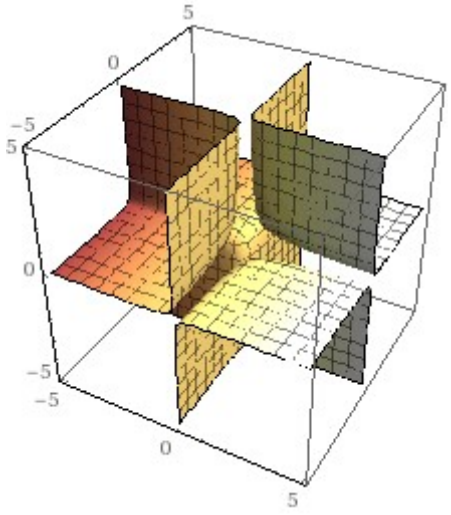
Input:

$$-2\pi\left(\frac{\pi}{12}\right)^{1.5} a\,b\,c\left(-\frac{\pi^2}{6}\left(\frac{1}{24}+1\right)\right) = \frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\pi}\right)$$

Result:

$$1.44215\,a\,b\,c = \frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}$$

Plot:



Alternate forms:

$a\,b\,c = 0.104837$

$$1.44215\,a\,b\,c-\frac{2616\sqrt{3}}{42875\,\pi}-\frac{144}{1225}=0$$

$$1.44215\,a\,b\,c=\frac{24\left(109\sqrt{3}+210\,\pi\right)}{42875\,\pi}$$

Expanded form:

$$1.44215\,a\,b\,c=\frac{144}{1225}+\frac{2616\sqrt{3}}{42875\,\pi}$$

Solution:

$$a\,b\neq 0\,,\quad c\approx\frac{0.104837}{a\,b}$$

Solution for the variable c:

$$c\approx\frac{0.104837}{a\,b}$$

We put:

$\frac{\omega_{hk}\,e^{\frac{2\pi i h l}{k}}}{k^{5/2}} = 2$; $\Delta_{\alpha}^{l-1} = 3$; and $L_{3/2} = (0.104837/6) = 0.01747283$

we obtain:

-2Pi*(Pi/12)^1.5 * 2 * 3 * (0.104837/6) (-(Pi^2)/6 (1/24+1))

Input interpretation:

$$-2\,\pi\left(\frac{\pi}{12}\right)^{1.5}\times 2\times 3\times\frac{0.104837}{6}\left(-\frac{\pi^2}{6}\left(\frac{1}{24}+1\right)\right)$$

Result:

0.151190552829316008659820977586899608355137957504466344740...

0.15119055…

Alternative representations:

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=$$
$$226.448^\circ\left(\frac{1}{6}\right)^2\left(1+\frac{1}{24}\right)(180^\circ)^2\left(\frac{180^\circ}{12}\right)^{1.5}$$

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=$$
$$1.25804\cos^{-1}(-1)\left(\frac{1}{6}\right)^2\left(1+\frac{1}{24}\right)\cos^{-1}(-1)^2\left(\frac{1}{12}\cos^{-1}(-1)\right)^{1.5}$$

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=$$
$$-1.25804\,i\log(-1)\left(\frac{1}{6}\right)^2\left(1+\frac{1}{24}\right)(-i\log(-1))^2\left(-\frac{1}{12}\,i\log(-1)\right)^{1.5}$$

Series representations:

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=0.448353\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{4.5}$$

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=0.0198146\left(-1+\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}\right)^{4.5}$$

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=0.00087569\left(\sum_{k=0}^{\infty}\frac{2^{-k}\,(-6+50k)}{\binom{3k}{k}}\right)^{4.5}$$

Integral representations:

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=0.0198146\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{4.5}$$

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=0.448353\left(\int_0^1\sqrt{1-t^2}\;dt\right)^{4.5}$$

$$-\frac{(2\times 0.104837\,\pi)\left(\frac{\pi}{12}\right)^{1.5}\left(2\times 3\left(-\pi^2\left(\frac{1}{24}+1\right)\right)\right)}{6\times 6}=0.0198146\left(\int_0^\infty\frac{\sin(t)}{t}\,dt\right)^{4.5}$$

And:

24/(25*49) (6+(109sqrt3)/(35pi))

Input:

$$\frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)$$

Result:

$$\frac{24\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)}{1225}$$

Decimal approximation:

0.151190119893257329481554055267328934313599137787501619771...

0.15119011…

Property:

$\frac{24\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)}{1225}$ is a transcendental number

Alternate forms:

$$\frac{24\left(109\sqrt{3}+210\,\pi\right)}{42\,875\,\pi}$$

$$\frac{144}{1225}+\frac{2616\sqrt{3}}{42\,875\,\pi}$$

Series representations:

$$\frac{\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)24}{25\times 49}=\frac{144}{1225}+\frac{2616\sqrt{2}\ \sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}{42\,875\,\pi}$$

$$\frac{\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)24}{25\times 49}=\frac{144}{1225}+\frac{2616\sqrt{2}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}{42\,875\,\pi}$$

$$\frac{\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)24}{25\times 49}=\frac{144}{1225}+\frac{1308\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{42\,875\,\pi\sqrt{\pi}}$$

From which:

1+(((24/(25*49) (6+(109sqrt3)/(35pi)))))^1/4

Input:

$$1+\sqrt[4]{\frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)}$$

Exact result:

$$1+\frac{2^{3/4}\sqrt[4]{3\left(6+\frac{109\sqrt{3}}{35\,\pi}\right)}}{\sqrt{35}}$$

Decimal approximation:

1.623563739525094483582949536738483636708229612401517569770…

1.62356373…

Property:

$$1+\frac{2^{3/4}\sqrt[4]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}}{\sqrt{35}}$$ is a transcendental number

Alternate forms:

$$1+\left(\frac{2}{35}\right)^{3/4}\sqrt[4]{630+\frac{327\sqrt{3}}{\pi}}$$

$$1+\left(\frac{2}{35}\right)^{3/4}\sqrt[4]{\frac{3\left(109\sqrt{3}+210\pi\right)}{\pi}}$$

$$\frac{1}{35}\left(35+2^{3/4}\sqrt[4]{\frac{105\left(109\sqrt{3}+210\pi\right)}{\pi}}\right)$$

Series representations:

$$1+\sqrt[4]{\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}}=1+\left(\frac{2}{35}\right)^{3/4}\sqrt[4]{630+\frac{327\sqrt{2}\ \sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}{\pi}}$$

$$1+\sqrt[4]{\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}}=1+\left(\frac{2}{35}\right)^{3/4}\sqrt[4]{630+\frac{327\sqrt{2}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}{\pi}}$$

$$1+\sqrt[4]{\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}}=1+\frac{\sqrt{2}\ \sqrt[4]{1260+\frac{327\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\pi\sqrt{\pi}}}}{35^{3/4}}$$

Furthermore, we note that performing the 128th root, we obtain:

(((24/(25*49) (6+(109sqrt3)/(35pi)))))^1/128

Input:

$$\sqrt[128]{\frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\pi}\right)}$$

Exact result:

$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}}{\sqrt[64]{35}}$$

Decimal approximation:

0.985348878571314690339042945130110938499428673407196612491...

0.985348878…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d}=255-390 \mid 0.988-1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243 = \phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or 'glueballs').

A_1^{**}	0.943(39) [2.5]	0.988(38)	0.152(53)
A_4	1.03(10) [2.5]	0.999(32)	0.035(21)

(**Glueball Regge trajectories -** *Harvey Byron Meyer*, Lincoln College -Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford Trinity Term, 2004)

Property:

$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}}{\sqrt[64]{35}} \text{ is a transcendental number}$$

Alternate forms:

$$\left(\frac{2}{35}\right)^{3/128}\sqrt[128]{630+\frac{327\sqrt{3}}{\pi}}$$

$$\left(\frac{2}{35}\right)^{3/128}\sqrt[128]{\frac{3\left(109\sqrt{3}+210\pi\right)}{\pi}}$$

All 128th roots of (24 (6 + (109 sqrt(3))/(35 π)))/1225:

$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}\,e^{0}}{\sqrt[64]{35}} \approx 0.985349 \text{ (real, principal root)}$$

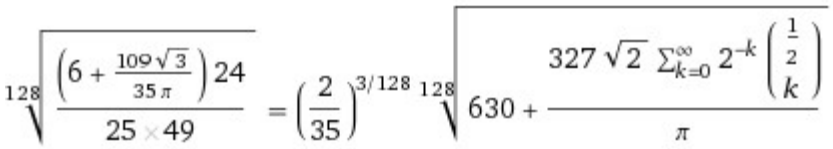
$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}\,e^{(i\pi)/64}}{\sqrt[64]{35}} \approx 0.984162 + 0.048349\,i$$

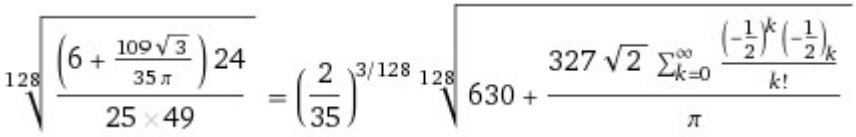
$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}\,e^{(i\pi)/32}}{\sqrt[64]{35}} \approx 0.980604 + 0.09658\,i$$

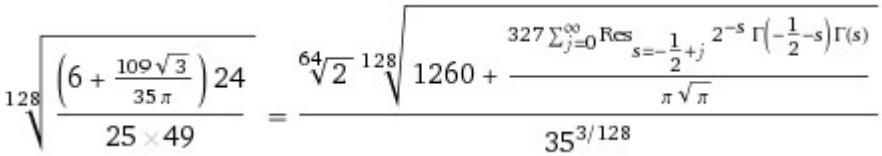
$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}\,e^{(3i\pi)/64}}{\sqrt[64]{35}} \approx 0.974684 + 0.14458\,i$$

$$\frac{2^{3/128}\sqrt[128]{3\left(6+\frac{109\sqrt{3}}{35\pi}\right)}\,e^{(i\pi)/16}}{\sqrt[64]{35}} \approx 0.96642 + 0.19223\,i$$

Series representations:







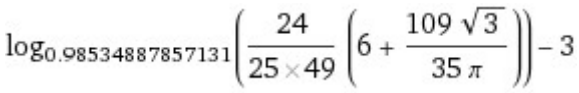
Integral representation:

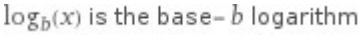


From which:

log base 0.98534887857131(((24/(25*49) (6+(109sqrt3)/(35pi))))) – 3

Input interpretation:





Result:

125.000000000...

125 result very near to the Higgs boson mass 125.18 GeV

Alternative representation:

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)-3=-3+\frac{\log\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}\right)}{\log(0.985348878571310000)}$$

Series representations:

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)-3=$$

$$-3+\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{2}\ \sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}{35\pi}\right)}{1225}\right)$$

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)-3=-3-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}\right)^k}{k}}{\log(0.985348878571310000)}$$

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)-3=$$

$$-3+\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{2}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{35\pi}\right)}{1225}\right)$$

log base 0.98534887857131(((24/(25*49) (6+(109sqrt3)/(35pi))))) +11

Input interpretation:

$$\log_{0.98534887857131}\left(\frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\pi}\right)\right)+11$$

$\log_b(x)$ is the base-b logarithm

Result:

139.000000000...

139 result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)+11=11+\frac{\log\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}\right)}{\log(0.985348878571310000)}$$

Series representations:

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)+11=$$

$$11+\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}{35\pi}\right)}{1225}\right)$$

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)+11=11-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}\right)^k}{k}}{\log(0.985348878571310000)}$$

$$\log_{0.985348878571310000}\left(\frac{\left(6+\frac{109\sqrt{3}}{35\pi}\right)24}{25\times 49}\right)+11=$$

$$11+\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{35\pi}\right)}{1225}\right)$$

And again:

27*1/2*(((log base 0.98534887857131(((24/(25*49) (6+(109sqrt3)/(35pi)))))))+1

Input interpretation:

$$27\times\frac{1}{2}\log_{0.98534887857131}\left(\frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\pi}\right)\right)+1$$

$\log_b(x)$ is the base-b logarithm

Result:

1729.00000000...

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representation:

$$\frac{27}{2}\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{25\times 49}\right)+1=1+\frac{27\log\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}\right)}{2\log(0.985348878571310000)}$$

Series representations:

$$\frac{27}{2}\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{25\times 49}\right)+1=$$

$$1+\frac{27}{2}\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{2}\ \sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}{35\pi}\right)}{1225}\right)$$

$$\frac{27}{2}\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{25\times 49}\right)+1=1-\frac{27\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225}\right)^k}{k}}{2\log(0.985348878571310000)}$$

$$\frac{27}{2}\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{25\times 49}\right)+1=$$

$$1+\frac{27}{2}\log_{0.985348878571310000}\left(\frac{24\left(6+\frac{109\sqrt{2}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{35\pi}\right)}{1225}\right)$$

[27*1/2*(((log base 0.98534887857131(((24/(25*49)
(6+(109sqrt3)/(35pi)))))))+1]^1/15

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\log_{0.98534887857131}\left(\frac{24}{25\times 49}\left(6+\frac{109\sqrt{3}}{35\pi}\right)\right)+1}$$

$\log_b(x)$ is the base-b logarithm

Result:

1.643815228749...

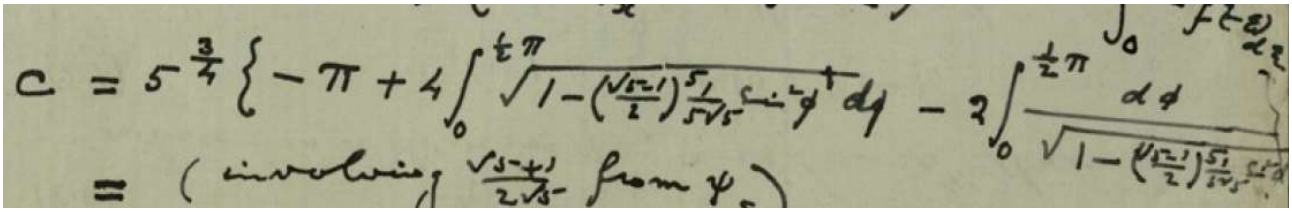
$1.6438152\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From

https://archives.trin.cam.ac.uk/index.php/papers-of-srinivasa-ramanujan

https://mss-cat.trin.cam.ac.uk/manuscripts/uv/view.php?n=Add.Ms.a.94.15

https://mss-cat.trin.cam.ac.uk/manuscripts/uv/view.php?n=Add.Ms.a.94.15#?c=0&m=0&s=0&cv=75&xywh=-1524%2C578%2C7291%2C4547&r=180



5^(3/4) [-Pi+4 * integrate (sqrt((((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(x))))x, 0 to Pi/2 – 2 * integrate (Pi / sqrt((((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(Pi))))x, 0 to Pi/2]

5^(3/4) [-Pi+4 * integrate sqrt(((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(x))))x, 0 to Pi/2]

Input:

$$5^{3/4}\left(-\pi+4\int_0^{\frac{\pi}{2}}\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(x)}\;x\,dx\right)$$

ϕ is the golden ratio

Decimal approximation:

5.949126845157503555501996346393103445650355365967416841234...

5.949126845….

5^(3/4) (((– 2 * integrate [Pi / sqrt(((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(Pi))))x, 0 to Pi/2])))

Input:

$$5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi}{\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(\pi)}}\,x\,dx\right)$$

ϕ is the golden ratio

Result:

$$-\frac{1}{4}\times 5^{3/4}\,\pi^3 \approx -25.9189$$

-25.9189

Computation result:

$$5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi\,x}{\sqrt{1-\frac{\left(\frac{1}{\phi}\right)^5\sin^2(\pi)}{5\sqrt{5}}}}\,dx\right)=-\frac{1}{4}\times 5^{3/4}\,\pi^3$$

Thence:

Input interpretation:

$$5.9491268451575035-5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi}{\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(\pi)}}\,x\,dx\right)$$

ϕ is the golden ratio

Result:

31.8680604993662835

31.8680604….

From which:

4*((((5.949126845-(5^(3/4) (((– 2 * integrate [Pi / sqrt(((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(Pi))))x, 0 to Pi/2])))))))-2

Input interpretation:

$$4\left(5.949126845-5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi}{\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(\pi)}}\,x\,dx\right)\right)-2$$

ϕ is the golden ratio

Result:

125.472

125.472 result very near to the Higgs boson mass 125.18 GeV

4*((((5.949126845-(5^(3/4) (((– 2 * integrate [Pi / sqrt(((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(Pi))))x, 0 to Pi/2])))))))+11+1

Input interpretation:

$$4\left(5.949126845-5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi}{\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(\pi)}}\,x\,dx\right)\right)+11+1$$

ϕ is the golden ratio

Result:

139.472

139.472 result practically equal to the rest mass of Pion meson 139.57 MeV

55*((((5.949126845-(5^(3/4) (((– 2 * integrate [Pi / sqrt(((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(Pi))))x, 0 to Pi/2])))))))-24

Input interpretation:

$$55\left(5.949126845-5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi}{\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(\pi)}}\,x\,dx\right)\right)-24$$

ϕ is the golden ratio

Result:

1728.74

1728.74

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

[[[55*((((5.949126845-(5^(3/4) (((– 2 * integrate [Pi / sqrt(((1-(1/golden ratio)^5*1/(5sqrt5) sin^2(Pi))))x, 0 to Pi/2])))))))-24]]]^1/15

Input interpretation:

$$\sqrt[15]{55\left(5.949126845-5^{3/4}\left(-2\int_0^{\frac{\pi}{2}}\frac{\pi}{\sqrt{1-\left(\left(\frac{1}{\phi}\right)^5\times\frac{1}{5\sqrt{5}}\right)\sin^2(\pi)}}\,x\,dx\right)\right)-24}$$

ϕ is the golden ratio

Result:

1.6438

$1.6438 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From

Canad. Math. Bull. Vol. **42** (4), 1999 pp. 427–440
Ramanujan and the Modular *j*-Invariant - *Bruce C. Berndt and Heng Huat Chan*

Page 436, we have that:

$$t_{35} = \left(2\sqrt{64\cdot 5\left(\frac{\sqrt{5}+1}{2}\right)^8 - 24\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 + 9} - \left(16\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 - 3\right)\right)^{1/6}$$

$$= \left(2\sqrt{7349 + 3276\sqrt{5}} - 117 - 56\sqrt{5}\right)^{1/6}.$$

We take:

$$t_{35} = \left(2\sqrt{64\cdot 5\left(\frac{\sqrt{5}+1}{2}\right)^8 \quad 24\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 \; 9} \quad \left(16\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 \quad 3\right)\right)^{1/6}$$

and obtain:

((((2*(((64*5((sqrt5+1)/2)^8-24sqrt5((sqrt5+1)/2)^4+9)))^0.5 –
(((16sqrt5((sqrt5+1)/2)^4-3)))))))^(1/6)

Input:

$$\sqrt[6]{2\sqrt{64\times 5\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^8 - 24\sqrt{5}\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^4 + 9} - \left(16\sqrt{5}\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^4 - 3\right)}$$

Exact result:

$$\sqrt[6]{3-\sqrt{5}\left(1+\sqrt{5}\right)^4 + 2\sqrt{9-\frac{3}{2}\sqrt{5}\left(1+\sqrt{5}\right)^4 + \frac{5}{4}\left(1+\sqrt{5}\right)^8}}$$

Decimal approximation:

0.618033988749894848204586834365638117720309179805762862135...

0.61803398…. result that is the value of the golden ratio conjugate

Alternate forms:

$$\frac{1}{2}\left(\sqrt{5}-1\right)$$

$$\sqrt[6]{-117-56\sqrt{5}+2\sqrt{7349+3276\sqrt{5}}}$$

$$\sqrt[6]{3-\sqrt{5}\left(1+\sqrt{5}\right)^4+2\sqrt{7529-36\sqrt{5}\left(\sqrt{5}-91\right)}}$$

Now, we have that:

Now, since

$$7349^2 - 5 \cdot 3276^2 = 589^2,$$

we find that

$$\sqrt{7349 + 3276\sqrt{5}} = \sqrt{\frac{7349 + 589}{2}} + \sqrt{\frac{7349 - 589}{2}} = \sqrt{3969} + \sqrt{3380} = 63 + 26\sqrt{5},$$

by (4.1). Hence,

$$t_{35} = \left(2\left(63 + 26\sqrt{5}\right) - 117 - 56\sqrt{5}\right)^{1/6} = (9 - 4\sqrt{5})^{1/6} = \frac{\sqrt{5} - 1}{2}.$$

From the above expression, we obtain:

(2((7349+3276sqrt5)^0.5)-117-56sqrt5)^(1/6)

Input:

$$\sqrt[6]{2\sqrt{7349+3276\sqrt{5}}-117-56\sqrt{5}}$$

Decimal approximation:

0.618033988749894848204586834365638117720309179805762862135...

0.61803398…. as above

Alternate forms:

$$\frac{1}{2}\left(\sqrt{5}-1\right)$$

$$\sqrt[6]{9-4\sqrt{5}}$$

$$\sqrt[6]{-117-56\sqrt{5}+2\left(63+26\sqrt{5}\right)}$$

Minimal polynomial:

$$x^2+x-1$$

1/[(2((7349+3276sqrt5)^0.5)-117-56sqrt5)^(1/6)]

Input:

$$\frac{1}{\sqrt[6]{2\sqrt{7349+3276\sqrt{5}}-117-56\sqrt{5}}}$$

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621 35...

1.61803398…. result that is the value of the golden ratio

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{1}{\sqrt[6]{9-4\sqrt{5}}}$$

$$\frac{1}{\sqrt[6]{-117-56\sqrt{5}+2\left(63+26\sqrt{5}\right)}}$$

Minimal polynomial:

$$x^2-x-1$$

From page 8, we have:

$$\alpha_7 = \frac{1}{2} - \frac{13}{36}\sqrt{3} + \frac{1}{36}\sqrt{21}.$$

And

$$q = e^{-2\pi\sqrt{7/3}}$$

1/2 – 13/36(sqrt3)+1/36(sqrt21)

Input:

$$\frac{1}{2} - \frac{13}{36}\sqrt{3} + \frac{1}{36}\sqrt{21}$$

Result:

$$\frac{1}{2} + \frac{\sqrt{\frac{7}{3}}}{12} - \frac{13}{12\sqrt{3}}$$

Decimal approximation:

0.001830977682234310853645687615324103297999674367583992718...

0.001830977…

Alternate forms:

$$\frac{1}{36}\left(18 - 13\sqrt{3} + \sqrt{21}\right)$$

$$\frac{-13 + 6\sqrt{3} + \sqrt{7}}{12\sqrt{3}}$$

$$-\frac{13\sqrt{3}}{36} + \frac{\sqrt{21}}{36} + \frac{1}{2}$$

Minimal polynomial:

$$1728x^4 - 3456x^3 + 1184x^2 + 544x - 1$$

e^(-2Pi*sqrt(7/3))

Input:

$$e^{-2\pi\sqrt{7/3}}$$

Decimal approximation:

0.000067883056415811498227867669709186646997148775901098748...

0.0000678830564…

Property:

$e^{-2\sqrt{7/3}\,\pi}$ is a transcendental number

Series representations:

$$e^{-2\pi\sqrt{7/3}} = e^{-2\pi\sqrt{\frac{4}{3}}\,\sum_{k=0}^{\infty}\left(\frac{4}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}$$

$$e^{-2\pi\sqrt{7/3}} = \exp\left(-2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-2\pi\sqrt{7/3}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\left(\frac{3}{4}\right)^s\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

Dividing the two results, we obtain:

(((1/2 – 13/36(sqrt3)+1/36(sqrt21)))) / ((((e^(-2Pi*sqrt(7/3)))))

Input:

$$\frac{\frac{1}{2}-\frac{13}{36}\sqrt{3}+\frac{1}{36}\sqrt{21}}{e^{-2\pi\sqrt{7/3}}}$$

Exact result:

$$\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\,\pi}$$

Decimal approximation:

26.972528623620352561382377803657728931922043447514522520 95...

26.9725286.... ≈ 27

Property:

$$\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\,\pi}$$ is a transcendental number

Alternate forms:

$$\frac{1}{36}\left(18-13\sqrt{3}+\sqrt{21}\right)e^{2\sqrt{7/3}\,\pi}$$

$$\frac{\left(13\sqrt{7}-34\right)e^{2\sqrt{7/3}\,\pi}}{36\left(3+\sqrt{\frac{1}{6}\left(88-13\sqrt{7}\right)}\right)}$$

$$\frac{1}{2}e^{2\sqrt{7/3}\,\pi}+\frac{1}{12}\sqrt{\frac{7}{3}}\,e^{2\sqrt{7/3}\,\pi}-\frac{13\,e^{2\sqrt{7/3}\,\pi}}{12\sqrt{3}}$$

Series representations:

$$\frac{\frac{1}{2}-\frac{\sqrt{3}\ 13}{36}+\frac{\sqrt{21}}{36}}{e^{-2\pi\sqrt{7/3}}} = \frac{1}{36}\,e^{2\pi\sqrt{4/3}\ \sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}}\left(18+\sum_{k=0}^{\infty}-20^{-k}\binom{\frac{1}{2}}{k}\right)\left(13\times 10^{k}\sqrt{2}\ -\sqrt{20}\right)$$

$$\frac{\frac{1}{2}-\frac{\sqrt{3}\ 13}{36}+\frac{\sqrt{21}}{36}}{e^{-2\pi\sqrt{7/3}}} = \frac{1}{36}\exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(18+\sum_{k=0}^{\infty}-\frac{20^{-k}\left(-\frac{1}{2}\right)_k\left(13\,(-10)^{k}\sqrt{2}\ +(-1)^{1+k}\sqrt{20}\right)}{k!}\right)$$

$$\frac{\frac{1}{2}-\frac{\sqrt{3}\ 13}{36}+\frac{\sqrt{21}}{36}}{e^{-2\pi\sqrt{7/3}}} = -\frac{1}{36}\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_k\left(\frac{7}{3}-z_0\right)^{k}z_0^{-k}}{k!}\right)\left(-18+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_k\left(13\,(3-z_0)^{k}-(21-z_0)^{k}\right)z_0^{-k}}{k!}\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0\right)\right)$

5* (((1/2 – 13/36(sqrt3)+1/36(sqrt21)))) / (((e^(-2Pi*sqrt(7/3))))) + 5

Input:

$$5\times\frac{\frac{1}{2}-\frac{13}{36}\sqrt{3}\ +\frac{1}{36}\sqrt{21}}{e^{-2\pi\sqrt{7/3}}}+5$$

Exact result:

$$5+5\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\ \pi}$$

Decimal approximation:

139.862643118101762806911889018288644659610217237572612604 7...

139.862643… result practically equal to the rest mass of Pion meson 139.57 MeV

Property:

$5+5\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\,\pi}$ is a transcendental number

Alternate forms:

$$\frac{5\left(12\sqrt{3}+\left(-13+6\sqrt{3}+\sqrt{7}\right)e^{2\sqrt{7/3}\,\pi}\right)}{12\sqrt{3}}$$

$$\frac{5}{36}\left(36+18\,e^{2\sqrt{7/3}\,\pi}-13\sqrt{3}\,e^{2\sqrt{7/3}\,\pi}+\sqrt{21}\,e^{2\sqrt{7/3}\,\pi}\right)$$

$$5+\frac{5}{2}e^{2\sqrt{7/3}\,\pi}+\frac{5}{12}\sqrt{\frac{7}{3}}\,e^{2\sqrt{7/3}\,\pi}-\frac{65\,e^{2\sqrt{7/3}\,\pi}}{12\sqrt{3}}$$

Series representations:

$$\frac{5\left(\frac{1}{2}-\frac{13\sqrt{3}}{36}+\frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}}+5=-\frac{5}{36}\left(-36-18\,e^{2\pi\sqrt{4/3}\,\sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}}+e^{2\pi\sqrt{4/3}\,\sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}20^{-k}\binom{\frac{1}{2}}{k}\left(13\times10^{k}\sqrt{2}-\sqrt{20}\right)\right)$$

$$\frac{5\left(\frac{1}{2}-\frac{13\sqrt{3}}{36}+\frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}}+5=-\frac{5}{36}\left(-36-18\exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)+\exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\sum_{k=0}^{\infty}\frac{20^{-k}\left(-\frac{1}{2}\right)_{k}\left(13\,(-10)^{k}\sqrt{2}+(-1)^{1+k}\sqrt{20}\right)}{k!}\right)$$

$$\frac{5\left(\frac{1}{2}-\frac{13\sqrt{3}}{36}+\frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}}+5=\frac{5}{2}\left(2+\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7}{3}-z_0\right)^k z_0^{-k}}{k!}\right)+2\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}-\frac{(-1)^k\left(-\frac{1}{2}\right)_k\sqrt{z_0}\left(13(3-z_0)^k-(21-z_0)^k\right)z_0^{-k}}{36\,k!}\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

4* (((1/2 – 13/36(sqrt3)+1/36(sqrt21)))) / (((e^(-2Pi*sqrt(7/3))))) + 21 – Pi

Input:

$$4\times\frac{\frac{1}{2}-\frac{13}{36}\sqrt{3}+\frac{1}{36}\sqrt{21}}{e^{-2\pi\sqrt{7/3}}}+21-\pi$$

Exact result:

$$21+4\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\,\pi}-\pi$$

Decimal approximation:

125.748521840891617007066867831351412843491004390682984262 8...

125.74852184… result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$\frac{\left(-13+6\sqrt{3}+\sqrt{7}\right)e^{2\sqrt{7/3}\,\pi}-3\sqrt{3}\,(\pi-21)}{3\sqrt{3}}$$

$$\frac{1}{9}\left(189+18\,e^{2\sqrt{7/3}\,\pi}-13\sqrt{3}\,e^{2\sqrt{7/3}\,\pi}+\sqrt{21}\,e^{2\sqrt{7/3}\,\pi}-9\pi\right)$$

$$21 + 2\,e^{2\sqrt{7/3}\,\pi} + \frac{1}{3}\sqrt{\frac{7}{3}}\;e^{2\sqrt{7/3}\,\pi} - \frac{13\,e^{2\sqrt{7/3}\,\pi}}{3\sqrt{3}} - \pi$$

Series representations:

$$\frac{4\left(\frac{1}{2} - \frac{13\sqrt{3}}{36} + \frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}} + 21 - \pi = \frac{1}{9}\left(189 + 18\,e^{2\pi\sqrt{4/3}\;\sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}} - 9\pi + e^{2\pi\sqrt{4/3}\;\sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}-20^{-k}\binom{\frac{1}{2}}{k}\left(13\times 10^{k}\sqrt{2} - \sqrt{20}\right)\right)$$

$$\frac{4\left(\frac{1}{2} - \frac{13\sqrt{3}}{36} + \frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}} + 21 - \pi = \frac{1}{9}\left(189 + 18\exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) - 9\pi + \exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\sum_{k=0}^{\infty}-\frac{20^{-k}\left(-\frac{1}{2}\right)_{k}\left(13\,(-10)^{k}\sqrt{2} + (-1)^{1+k}\sqrt{20}\right)}{k!}\right)$$

$$\frac{4\left(\frac{1}{2} - \frac{13\sqrt{3}}{36} + \frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}} + 21 - \pi = 21 + 2\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{7}{3} - z_0\right)^{k}z_0^{-k}}{k!}\right) - \pi + 4\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{7}{3} - z_0\right)^{k}z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}-\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\sqrt{z_0}\left(13\,(3 - z_0)^{k} - (21 - z_0)^{k}\right)z_0^{-k}}{36\,k!}$$

for $\left(\text{not}\left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right)$

64* (((1/2 – 13/36(sqrt3)+1/36(sqrt21)))) / (((e^(-2Pi*sqrt(7/3))))) +3

Input:

$$64\times\frac{\frac{1}{2} - \frac{13}{36}\sqrt{3} + \frac{1}{36}\sqrt{21}}{e^{-2\pi\sqrt{7/3}}} + 3$$

Exact result:

$$3+64\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\,\pi}$$

Decimal approximation:

1729.241831911702563928472179434094651643010780640929441341...

1729.2418319…

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Property:

$3+64\left(\frac{1}{2}+\frac{\sqrt{\frac{7}{3}}}{12}-\frac{13}{12\sqrt{3}}\right)e^{2\sqrt{7/3}\,\pi}$ is a transcendental number

Alternate forms:

$$\frac{9\sqrt{3}+16\left(-13+6\sqrt{3}+\sqrt{7}\right)e^{2\sqrt{7/3}\,\pi}}{3\sqrt{3}}$$

$$\frac{1}{9}\left(27+288\,e^{2\sqrt{7/3}\,\pi}-208\sqrt{3}\,e^{2\sqrt{7/3}\,\pi}+16\sqrt{21}\,e^{2\sqrt{7/3}\,\pi}\right)$$

$$3+32\,e^{2\sqrt{7/3}\,\pi}+\frac{16}{3}\sqrt{\frac{7}{3}}\,e^{2\sqrt{7/3}\,\pi}-\frac{208\,e^{2\sqrt{7/3}\,\pi}}{3\sqrt{3}}$$

Series representations:

$$\frac{64\left(\frac{1}{2}-\frac{13\sqrt{3}}{36}+\frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}}+3=\frac{1}{9}\left(27+288\,e^{2\pi\sqrt{4/3}\sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}}-16\,e^{2\pi\sqrt{4/3}\sum_{k=0}^{\infty}(4/3)^{-k}\binom{1/2}{k}}\sum_{k=0}^{\infty}20^{-k}\binom{\frac{1}{2}}{k}\left(13\times10^{k}\sqrt{2}-\sqrt{20}\right)\right)$$

$$\frac{64\left(\frac{1}{2}-\frac{13\sqrt{3}}{36}+\frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}}+3=\frac{1}{9}\left(27+288\exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)-16\exp\left(2\pi\sqrt{\frac{4}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\sum_{k=0}^{\infty}\frac{20^{-k}\left(-\frac{1}{2}\right)_{k}\left(13\,(-10)^{k}\sqrt{2}+(-1)^{1+k}\sqrt{20}\right)}{k!}\right)$$

$$\frac{64\left(\frac{1}{2}-\frac{13\sqrt{3}}{36}+\frac{\sqrt{21}}{36}\right)}{e^{-2\pi\sqrt{7/3}}}+3=3+32\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{7}{3}-z_0\right)^{k}z_0^{-k}}{k!}\right)+64\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{7}{3}-z_0\right)^{k}z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}-\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\sqrt{z_0}\left(13\,(3-z_0)^{k}-(21-z_0)^{k}\right)z_0^{-k}}{36\,k!}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

From

Topics in Analytic Number Theory - Authors: *Rademacher Hans* - 1973

Now, we have

The restriction $\sigma \geqq (1 - c)/2$ may be stronger than is needed for (34.1). However, some restriction stronger than $\sigma \geqq 0$ is necessary as the examples

$$\left| \frac{\Gamma\left(\frac{1}{3} + \frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\left(\frac{i}{\sqrt{3}}\right)^{1/3}} \right| = 1.0017\ldots, \quad \left| \frac{\Gamma\left(\frac{1}{4} + \frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\left(\frac{i}{\sqrt{3}}\right)^{1/4}} \right| = 1.0142\ldots$$

Indeed, we have

gamma(1/3+i/sqrt3) / (((gamma(i/sqrt3)(i/sqrt3)^(1/3))))

Input:

$$\frac{\Gamma\left(\frac{1}{3} + \frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Exact result:

$$-\frac{(-1)^{5/6}\sqrt[6]{3}\,\Gamma\left(\frac{1}{3} + \frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

Decimal approximation:

0.976728045889611921530056804148335234746951737035209211 63... +
0.222360686398725970762637367534532433784851353739218994 03... i

Input interpretation:

0.976728045889611921530056804148335234746951737035209211 63 +
0.222360686398725970762637367534532433784851353739218994 03 i

i is the imaginary unit

Result:

0.976728045889611921530056804148335234746951737035209211 63... +
0.222360686398725970762637367534532433784851353739218994 03... i

Polar coordinates:

$r = 1.001719496906720094298401276110550033831744487354903054 74$ (radius),
$\theta = 12.825295484451908013353770447113949345756064550762285621°$ (angle)

1.001719496906…… result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}}-\varphi} = 1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\frac{}{1+\ldots}}}}} \approx 1.0018674362$$

= 1.0018674362

Alternate forms:

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right) \text{ root of } x^6+3 \text{ near } x = 1.04004 - 0.600468\,i}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$-\frac{\sqrt[6]{-3}\,(-1)^{2/3}\,\Gamma\left(\frac{1}{3}\left(1+i\sqrt{3}\right)\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$-\frac{(-1)^{5/6}\,3^{2/3}\left(\frac{1}{3}\left(1+i\sqrt{3}\right)\right)!}{\left(\sqrt{3}+-i\right)\frac{i}{\sqrt{3}}!}$$

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}} = \frac{\left(-\frac{2}{3}+\frac{i}{\sqrt{3}}\right)!}{\left(-1+\frac{i}{\sqrt{3}}\right)!\sqrt[3]{\frac{i}{\sqrt{3}}}}$$

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\frac{\exp\left(-\log G\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)+\log G\left(\frac{4}{3}+\frac{i}{\sqrt{3}}\right)\right)}{e^{-\log G\left(i/\sqrt{3}\right)+\log G\left(1+i/\sqrt{3}\right)}\sqrt[3]{\frac{i}{\sqrt{3}}}}$$

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\frac{G\left(\frac{4}{3}+\frac{i}{\sqrt{3}}\right)}{\frac{G\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)\left(G\left(1+\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}\right)}{G\left(\frac{i}{\sqrt{3}}\right)}}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{5/6}\sqrt[6]{3}\sum_{k=0}^{\infty}\frac{i^k\,3^{-k/2}\,\Gamma^{(k)}\left(\frac{1}{3}\right)}{k!}}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{5/6}\sqrt[6]{3}\sum_{k=1}^{\infty}i^k\,3^{-k/2}\,c_k}{\sum_{k=1}^{\infty}\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)^k c_k}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\frac{3^{2/3}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)^k\Gamma^{(k)}(1)}{k!}}{2\sum_{k=0}^{\infty}\frac{3^{-k/2}\,e^{(i\,k\,\pi)/2}\,\Gamma^{(k)}(1)}{k!}}$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

Integral representations:

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\frac{\sqrt[3]{-1}\,\sqrt[6]{3}\,\csc\left(\frac{1}{6}\left(1+i\sqrt{3}\right)\pi\right)\sinh\left(\frac{\pi}{2\sqrt{3}}\right)\int_0^{\infty}t^{-2/3+i/\sqrt{3}}\,\sin(t)\,dt}{\int_0^{\infty}t^{-1+i/\sqrt{3}}\,\sin(t)\,dt}$$

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{5/6}\sqrt[6]{3}\left(\int_1^\infty e^{-t}\,t^{-2/3+i/\sqrt{3}}\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{\left(\frac{1}{3}+\frac{i}{\sqrt{3}}+k\right)k!}\right)}{\int_1^\infty e^{-t}\,t^{-1+i/\sqrt{3}}\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{\left(\frac{i}{\sqrt{3}}+k\right)k!}}$$

$$\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{5/6}\sqrt[6]{3}}{\oint_L e^t\,t^{-1/3-i/\sqrt{3}}\,dt}\oint_L e^t\,t^{-i/\sqrt{3}}\,dt$$

And:

gamma(1/4+i/sqrt3) / (((gamma(i/sqrt3)(i/sqrt3)^(1/4))))

Input:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Exact result:

$$-\frac{(-1)^{7/8}\sqrt[8]{3}\,\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

Decimal approximation:

0.995945119658634082960884295743858913778417077177901177 50... +
0.191830479467430526748824525182230980348197976635954461 98... i

Input interpretation:

$$0.995945119658634082960884295743858913778417077177901177 50 + 0.191830479467430526748824525182230980348197976635954461 98\, i$$

i is the imaginary unit

Result:

$$0.995945119658634082960884295743858913778417077177901177 5\ldots + 0.191830479467430526748824525182230980348197976635954461 98\ldots\, i$$

Polar coordinates:

$r = 1.014251257935899849089932077990443309256511603984943595 72$ (radius),

$\theta = 10.902311999947990726173571943177771544819989546586149699°$ (angle)

1.014251257…

Alternate forms:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right) \text{ root of } x^8+3 \text{ near } x = 1.05988 - 0.439015\, i}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$-\frac{\sqrt[8]{-3}\,(-1)^{3/4}\,\Gamma\left(\frac{1}{12}\left(3+4\,i\,\sqrt{3}\right)\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$-\frac{4\,(-1)^{7/8}\,3^{5/8}\left(\frac{1}{12}\left(3+4\,i\,\sqrt{3}\right)\right)!}{\left(4\sqrt{3}+-3\,i\right)\frac{i}{\sqrt{3}}!}$$

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}} = \frac{\left(-1+\frac{1}{4}+\frac{i}{\sqrt{3}}\right)!}{\left(-1+\frac{i}{\sqrt{3}}\right)!\sqrt[4]{\frac{i}{\sqrt{3}}}}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}} = \frac{\exp\left(-\log G\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)+\log G\left(1+\frac{1}{4}+\frac{i}{\sqrt{3}}\right)\right)}{e^{-\log G\left(i/\sqrt{3}\right)+\log G\left(1+i/\sqrt{3}\right)}\sqrt[4]{\frac{i}{\sqrt{3}}}}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}=\frac{G\left(1+\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\frac{G\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)\left(G\left(1+\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}\right)}{G\left(\frac{i}{\sqrt{3}}\right)}}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{7/8}\sqrt[8]{3}\sum_{k=0}^{\infty}\frac{i^k\,3^{-k/2}\,\Gamma^{(k)}\left(\frac{1}{4}\right)}{k!}}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{7/8}\sqrt[8]{3}\sum_{k=1}^{\infty}i^k\,3^{-k/2}\,c_k}{\sum_{k=1}^{\infty}\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)^k c_k}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}=-\frac{4\,(-1)^{7/8}\,3^{5/8}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)^k\Gamma^{(k)}(1)}{k!}}{\left(-3\,i+4\sqrt{3}\right)\sum_{k=0}^{\infty}\frac{i^k\,3^{-k/2}\,\Gamma^{(k)}(1)}{k!}}$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}=\frac{(-1)^{3/8}\sqrt[8]{3}\,\csc\left(\frac{1}{2}\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)\pi\right)\sinh\left(\frac{\pi}{2\sqrt{3}}\right)\int_0^{\infty}t^{-3/4+i/\sqrt{3}}\,\sin(t)\,dt}{\int_0^{\infty}t^{-1+i/\sqrt{3}}\,\sin(t)\,dt}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}=-\frac{(-1)^{7/8}\sqrt[8]{3}\left(\int_1^{\infty}e^{-t}\,t^{-3/4+i/\sqrt{3}}\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{\left(\frac{1}{4}+\frac{i}{\sqrt{3}}+k\right)k!}\right)}{\int_1^{\infty}e^{-t}\,t^{-1+i/\sqrt{3}}\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{\left(\frac{i}{\sqrt{3}}+k\right)k!}}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}} = -\frac{(-1)^{7/8}\sqrt[8]{3}}{\oint_L e^t\, t^{-1/4-i/\sqrt{3}}\, dt}\oint_L e^t\, t^{-i/\sqrt{3}}\, dt$$

From which, subtracting the two expressions:

gamma(1/4+i/sqrt3) / (((gamma(i/sqrt3)(i/sqrt3)^(1/4)))) - gamma(1/3+i/sqrt3) / (((gamma(i/sqrt3)(i/sqrt3)^(1/3))))

Input:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}} - \frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Exact result:

$$\frac{(-1)^{5/6}\sqrt[6]{3}\,\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)} - \frac{(-1)^{7/8}\sqrt[8]{3}\,\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

Decimal approximation:

0.019217073769022161430827491595523679031465340142691965 86... –
0.030530206931295444013812842352301453436653377103264532 05... i

Alternate forms:

$$\frac{(-1)^{5/6}\sqrt[6]{3}\,\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right) - (-1)^{7/8}\sqrt[8]{3}\,\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$-\frac{(-1)^{5/6}\sqrt[8]{3}\left(\sqrt[24]{-1}\,\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right) - \sqrt[24]{3}\,\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$\frac{1}{\Gamma\left(\frac{i}{\sqrt{3}}\right)}\left(\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)\ \text{root of } x^8+3 \text{ near } x=1.05988-0.439015\,i+\right.$$
$$\left.\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)\ \text{root of } x^6+3 \text{ near } x=-1.04004+0.600468\,i\right)$$

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=-\frac{\left(-\frac{2}{3}+\frac{i}{\sqrt{3}}\right)!}{\left(-1+\frac{i}{\sqrt{3}}\right)!\sqrt[3]{\frac{i}{\sqrt{3}}}}+\frac{\left(-1+\frac{1}{4}+\frac{i}{\sqrt{3}}\right)!}{\left(-1+\frac{i}{\sqrt{3}}\right)!\sqrt[4]{\frac{i}{\sqrt{3}}}}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=$$
$$-\frac{\exp\left(-\log G\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)+\log G\left(\frac{4}{3}+\frac{i}{\sqrt{3}}\right)\right)}{e^{-\log G\left(i/\sqrt{3}\right)+\log G\left(1+i/\sqrt{3}\right)}\sqrt[3]{\frac{i}{\sqrt{3}}}}+\frac{\exp\left(-\log G\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)+\log G\left(1+\frac{1}{4}+\frac{i}{\sqrt{3}}\right)\right)}{e^{-\log G\left(i/\sqrt{3}\right)+\log G\left(1+i/\sqrt{3}\right)}\sqrt[4]{\frac{i}{\sqrt{3}}}}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=-\frac{(1)_{-\frac{2}{3}+\frac{i}{\sqrt{3}}}}{(1)_{-1+\frac{i}{\sqrt{3}}}\sqrt[3]{\frac{i}{\sqrt{3}}}}+\frac{(1)_{-1+\frac{1}{4}+\frac{i}{\sqrt{3}}}}{(1)_{-1+\frac{i}{\sqrt{3}}}\sqrt[4]{\frac{i}{\sqrt{3}}}}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\sum_{k=0}^{\infty}\frac{(-1)^{5/6}\,i^k\,3^{1/8-k/2}\left(-\sqrt[24]{-1}\,\Gamma^{(k)}\left(\frac{1}{4}\right)+\sqrt[24]{3}\,\Gamma^{(k)}\left(\frac{1}{3}\right)\right)}{k!\,\Gamma\left(\frac{i}{\sqrt{3}}\right)}$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=$$
$$\sum_{k=0}^{\infty}\left(-\frac{(-1)^{7/8}\,i^k\,3^{1/8-k/2}\,\Gamma^{(k)}\left(\frac{1}{4}\right)}{k!\,\Gamma\left(\frac{i}{\sqrt{3}}\right)}+\frac{(-1)^{5/6}\,i^k\,3^{1/6-k/2}\,\Gamma^{(k)}\left(\frac{1}{3}\right)}{k!\,\Gamma\left(\frac{i}{\sqrt{3}}\right)}\right)$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\frac{\sum_{k=0}^{\infty}\frac{\sqrt[3]{-1}\,\sqrt[8]{3}\left(-4\sqrt[24]{-1}\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)^{k}\left(3i+\sqrt{3}\right)+3\sqrt[24]{3}\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)^{k}\left(4i+\sqrt{3}\right)\right)\Gamma^{(k)}(1)}{k!}}{\left(-i+\sqrt{3}\right)\left(-3i+4\sqrt{3}\right)\sum_{k=0}^{\infty}\frac{i^{k}\,3^{-k/2}\,\Gamma^{(k)}(1)}{k!}}$$

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=\frac{1}{\int_{0}^{\infty}t^{-1+i/\sqrt{3}}\sin(t)\,dt}$$

$$\sqrt[3]{-1}\,\sqrt[8]{3}\left(\sqrt[24]{-1}\csc\left(\frac{1}{2}\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)\pi\right)\int_{0}^{\infty}t^{-3/4+i/\sqrt{3}}\sin(t)\,dt-\sqrt[24]{3}\csc\left(\frac{1}{2}\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)\pi\right)\int_{0}^{\infty}t^{-2/3+i/\sqrt{3}}\sin(t)\,dt\right)\sinh\left(\frac{\pi}{2\sqrt{3}}\right)$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=$$

$$-\left(\left((-1)^{5/6}\,\sqrt[8]{3}\left(\sqrt[24]{-1}\int_{1}^{\infty}e^{-t}\,t^{-3/4+i/\sqrt{3}}\,dt-\sqrt[24]{3}\int_{1}^{\infty}e^{-t}\,t^{-2/3+i/\sqrt{3}}\,dt+\sqrt[24]{-1}\sum_{k=0}^{\infty}\frac{(-1)^{k}}{\left(\frac{1}{4}+\frac{i}{\sqrt{3}}+k\right)k!}-\sqrt[24]{3}\sum_{k=0}^{\infty}\frac{(-1)^{k}}{\left(\frac{1}{3}+\frac{i}{\sqrt{3}}+k\right)k!}\right)\right)\Big/\left(\int_{1}^{\infty}e^{-t}\,t^{-1+i/\sqrt{3}}\,dt+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{\left(\frac{i}{\sqrt{3}}+k\right)k!}\right)\right)$$

$$\frac{\Gamma\left(\frac{1}{4}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[4]{\frac{i}{\sqrt{3}}}}-\frac{\Gamma\left(\frac{1}{3}+\frac{i}{\sqrt{3}}\right)}{\Gamma\left(\frac{i}{\sqrt{3}}\right)\sqrt[3]{\frac{i}{\sqrt{3}}}}=$$

$$\frac{(-1)^{5/6}\,\sqrt[6]{3}}{\oint_{L}e^{t}\,t^{-1/3-i/\sqrt{3}}\,dt}\oint_{L}e^{t}\,t^{-i/\sqrt{3}}\,dt-\frac{(-1)^{7/8}\,\sqrt[8]{3}}{\oint_{L}e^{t}\,t^{-1/4-i/\sqrt{3}}\,dt}\oint_{L}e^{t}\,t^{-i/\sqrt{3}}\,dt$$

and:

64/(0.0192170737690221 - 0.0305302069312954i)-8i

Input interpretation:

$$\frac{64}{0.0192170737690221 + i\times(-0.0305302069312954)} - 8\,i$$

i is the imaginary unit

Result:

945.061228388031... +
1493.42083088330... i

Polar coordinates:

$r = 1767.32750318622$ (radius), $\theta = 57.673699502488°$ (angle)

1767.32750318622 result in the range of the mass of candidate "glueball" $f_0(1710)$ ("glueball" =1760 ± 15 MeV).

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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