On the new possible relationships between several Ramanujan formulas, equations concerning some sectors of String Theory (String Cosmology), various parameters regarding Particle Physics, ϕ, $\zeta(2)$, 8 and his multiples. II

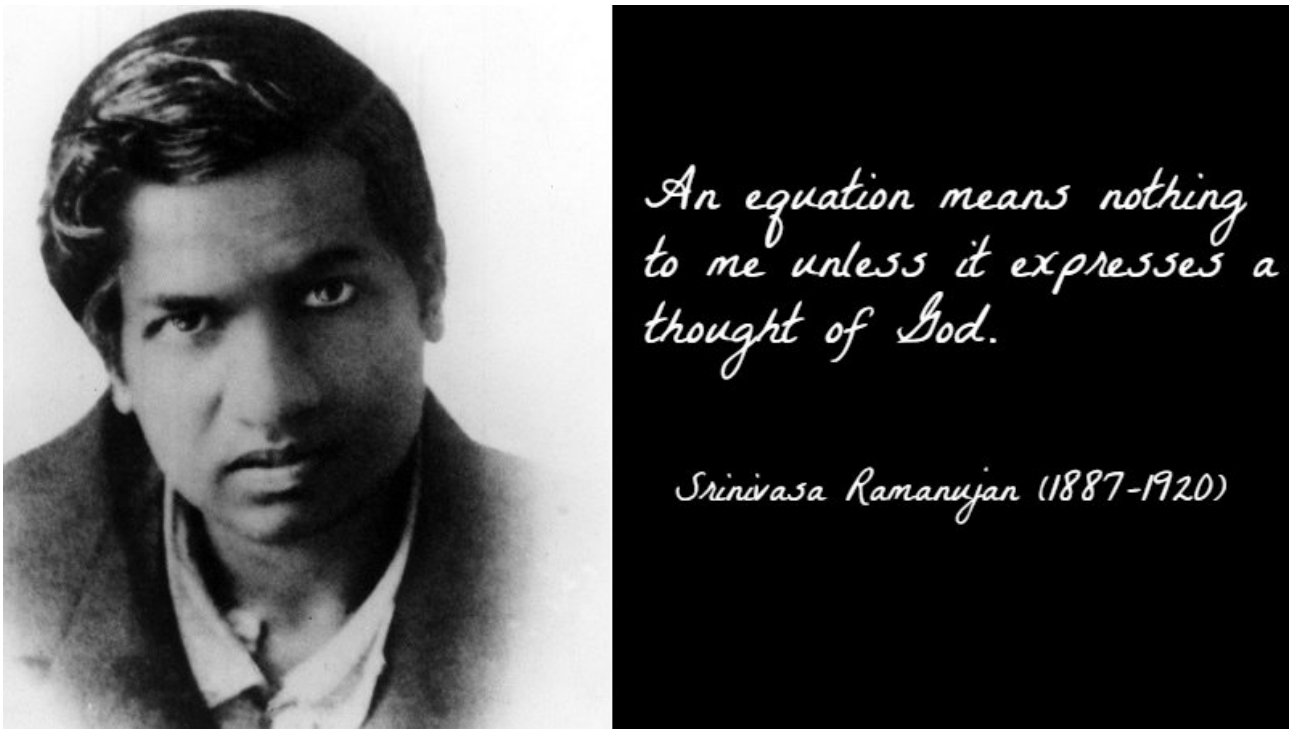
Michele Nardelli[1], **Antonio Nardelli**[2]

Abstract

In this paper (part II), we describe and analyze new possible relationships between some Ramanujan formulas, equations concerning some sectors of String Theory (String Cosmology), various parameters regarding Particle Physics, ϕ, $\zeta(2)$, 8 and his multiples.

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**



https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

From

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have:

$$\frac{32}{\pi} = (5\sqrt{5} - 1) + \frac{47\sqrt{5}+29}{64}\left(\frac{1}{2}\right)^3\left(\frac{\sqrt{5}-1}{2}\right)^8$$

$$+\frac{89\sqrt{5}+59}{64^2}\left(\frac{1\cdot 3}{2\cdot 4}\right)^3\left(\frac{\sqrt{5}-1}{2}\right)^{16} + \cdots,$$

(5sqrt5-1)+1/64(47sqrt5+29)*(1/2)^3*((sqrt5-1)/2)^8+1/64^2*(89sqrt5+59)(3/8)^3*((sqrt5-1)/2)^16

Input:

$$\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(47\sqrt{5}+29\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8+ \frac{1}{64^2}\left(89\sqrt{5}+59\right)\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)$$

Result:

$$-1+5\sqrt{5}+\frac{\left(\sqrt{5}-1\right)^8\left(29+47\sqrt{5}\right)}{131\,072}+\frac{27\left(\sqrt{5}-1\right)^{16}\left(59+89\sqrt{5}\right)}{137\,438\,953\,472}$$

Decimal approximation:

10.185916357452345299336727734394539078233439350749910096 94…

10.185916357…

Alternate forms:

$$\frac{15\left(1\,041\,875\sqrt{5}-905\,609\right)}{2\,097\,152}$$

$$\frac{15\,628\,125\sqrt{5}}{2\,097\,152}-\frac{13\,584\,135}{2\,097\,152}$$

$$\frac{15\,628\,125\sqrt{5}-13\,584\,135}{2\,097\,152}$$

Minimal polynomial:

$$1\,099\,511\,627\,776\,x^2 + 14\,243\,997\,941\,760\,x - 259\,165\,682\,844\,975$$

With regard the result:

10.185916357452345299336727734394539078233439350749910096 94…

We have:

Possible closed forms:

$$-\frac{220\,(91\,A+3)}{9\,(123\,A-445)} \approx 10.185916357452345\,30500$$

$$\pi \left[\text{root of } 739\,x^4 - 1106\,x^3 - 4076\,x^2 - 39\,x - 995 \text{ near } x = 3.24228\right] \approx 10.185916357452345\,299\,69998$$

$$\frac{5\,125\,730\,774\,\pi}{1\,580\,904\,219} \approx 10.185916357452345\,299\,03237$$

$$\left[\text{root of } 3573\,x^3 - 34\,627\,x^2 - 17\,714\,x - 2927 \text{ near } x = 10.1859\right] \approx 10.185916357452345\,30355$$

$$\left[\text{root of } 126\,x^4 - 1345\,x^3 + 910\,x^2 - 2781\,x - 1015 \text{ near } x = 10.1859\right] \approx 10.185916357452345\,30862$$

$$\pi \left[\text{root of } 7329\,x^3 - 35\,262\,x^2 + 43\,113\,x - 18\,899 \text{ near } x = 3.24228\right] \approx 10.185916357452345\,299\,03614$$

$$\frac{1}{\left[\text{root of } 2927\,x^3 + 17\,714\,x^2 + 34\,627\,x - 3573 \text{ near } x = 0.0981748\right]} \approx 10.185916357452345\,30355$$

$$\frac{1}{\left[\text{root of } 1015\,x^4 + 2781\,x^3 - 910\,x^2 + 1345\,x - 126 \text{ near } x = 0.0981748\right]} \approx 10.185916357452345\,30862$$

$$\sqrt[3]{\frac{1}{38}\,(1801 + 17\,802\,e - 3962\,\pi + 3483\,\log(2))} \approx 10.185916357452345\,299\,40575$$

$$\pi \left[\text{root of } 451x^5 - 1497x^4 + 466x^3 - 1142x^2 + 191x - 659 \text{ near } x = 3.24228\right] \approx 10.185916357452345299318617$$

$$\frac{27\left(\frac{3}{2}\right)^{52/301} e^{190/301} \log^{53/301}(2)}{4\log^{103/43}(3)} \approx 10.1859163574523452989670$$

$$\frac{374 - 59\pi - 837\pi^2}{355 - 648\pi + 90\pi^2} \approx 10.185916357452345299999372$$

$$\frac{166 - 143e - 367e^2}{179 - 479e + 113e^2} \approx 10.185916357452345\,31237$$

$$\frac{-\sqrt{2} + \sqrt{3} - 6e - \pi + 3\pi^2 + \log\left(\frac{243}{4}\right)}{4\sqrt{2} + 3\sqrt{3} + 2e + \pi - 2\pi^2 + \log\left(\frac{729}{128}\right)} \approx 10.185916357452345\,86672$$

and also:

10.18591635745…

Possible closed forms:

$$\frac{32}{\pi} \approx 10.185916357\,88130$$

$$\frac{31ee! + 91 + 26e - 33e^2}{10e} \approx 10.185916357\,45\,19863$$

$$\sqrt{\frac{1}{14}(1 + 404e + 86\pi + 120\log(2))} \approx 10.185916357\,49387$$

$$\pi \left[\text{root of } 16x^3 + 35x^2 - 385x + 335 \text{ near } x = 3.24228\right] \approx 10.185916357\,67734$$

We have also:

(5sqrt5-1)+1/64(47sqrt5+29)*(1/2)^3*((sqrt5-1)/2)^8+1/x*(89sqrt5+59)(3/8)^3*((sqrt5-1)/2)^16 = 32/Pi

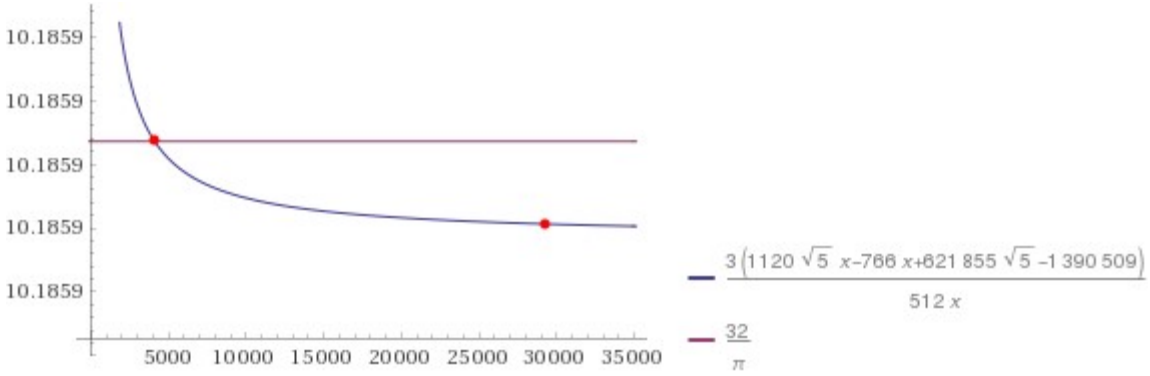
Input:

$$\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(47\sqrt{5}+29\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8+\frac{1}{x}\left(89\sqrt{5}+59\right)\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)=\frac{32}{\pi}$$

Exact result:

$$\frac{27\left(\sqrt{5}-1\right)^{16}\left(59+89\sqrt{5}\right)}{33554432\,x}+\frac{\left(\sqrt{5}-1\right)^8\left(29+47\sqrt{5}\right)}{131072}+5\sqrt{5}-1=\frac{32}{\pi}$$

Plot:



Alternate form assuming x is real:

$$\frac{3\pi\left(1120\sqrt{5}\,x-766\,x+621855\sqrt{5}-1390509\right)}{x}=16384$$

Alternate forms:

$$\frac{3\left(1120\sqrt{5}\,x-766\,x+621855\sqrt{5}-1390509\right)}{512\,x}=\frac{32}{\pi}$$

$$\frac{1865565\sqrt{5}-4171527}{512\,x}+\frac{1}{256}\left(1680\sqrt{5}-1149\right)=\frac{32}{\pi}$$

Alternate form assuming x is positive:

$$3\pi\left(1120\sqrt{5}\,x-766\,x+621855\sqrt{5}-1390509\right)=16384\,x \quad (\text{for } x\neq 0)$$

Expanded form:

$$\frac{1\,865\,565\sqrt{5}}{512\,x} - \frac{4\,171\,527}{512\,x} + \frac{105\sqrt{5}}{16} - \frac{1149}{256} = \frac{32}{\pi}$$

Solution:

$$x = \frac{27\,(154\,501 - 69\,095\sqrt{5}\,)\,\pi}{6\,(560\sqrt{5}\, - 383)\,\pi - 16\,384}$$

Solution:

$x \approx 4094.8$

$4094.8 \approx 4096 = 64^2$

From:

Vietnam Journal of Mathematics **36**:1(2008) 103–124 - **On Some New Explicit Evaluations of Class Invariants** - *M. S. Mahadeva Naika and K. Sushan Bairy*

We have that:

$$\alpha_{39} = \left(\frac{\sqrt{13}-3}{2}\right)^4 \left(\sqrt{\frac{5+\sqrt{13}}{8}} - \sqrt{\frac{-3+\sqrt{13}}{8}}\right)^{24} \left(\frac{11+3\sqrt{13}}{2}\right)^2 \left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2 \left(\sqrt{\frac{348+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}}{8}} - \sqrt{\frac{340+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}}{8}}\right)^2, \tag{7.9}$$

((Sqrt13-3)/2)^4 [((5+sqrt13)/8)^0.5-((-3+sqrt13)/8)^0.5]^24 * [((11+3sqrt13)/2)^2 (((2sqrt2-(-3+3sqrt13)^0.5)/(4sqrt2)))^2]

Input:

$$\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}\left(\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\right)$$

Exact result:

$$\frac{1}{2048}\left(\sqrt{13}-3\right)^4\left(11+3\sqrt{13}\right)^2\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}-\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}\right)^{24}\left(2\sqrt{2}-\sqrt{3\sqrt{13}-3}\right)^2$$

Decimal approximation:

$4.882163457879678755563875977822549737616669982002112 6\ldots \times 10^{-8}$

$4.88216345\ldots*10^{-8}$

Alternate forms:

$$\text{root of } 4\,294\,967\,296\,x^8-2\,889\,976\,119\,296\,x^7+1\,354\,743\,414\,784\,x^6+258\,104\,885\,248\,x^5+591\,623\,225\,344\,x^4-638\,322\,728\,960\,x^3+15\,021\,202\,432\,x^2-20\,483\,456\,x+1 \text{ near } x=4.88216\times10^{-8}$$

$$\frac{\left(\sqrt{13}-3\right)^4\left(11+3\sqrt{13}\right)^2\left(2\sqrt{2}-\sqrt{3\left(\sqrt{13}-1\right)}\right)^2\left(\sqrt{\sqrt{13}-3}-\sqrt{5+\sqrt{13}}\right)^{24}}{140\,737\,488\,355\,328}$$

$$\frac{\left(\sqrt{13}-3\right)^4\left(11+3\sqrt{13}\right)^2\left(\sqrt{\sqrt{13}-3}-\sqrt{5+\sqrt{13}}\right)^{24}}{17\,592\,186\,044\,416}+$$

$$\frac{3\left(\sqrt{13}-3\right)^4\left(\sqrt{13}-1\right)\left(11+3\sqrt{13}\right)^2\left(\sqrt{\sqrt{13}-3}-\sqrt{5+\sqrt{13}}\right)^{24}}{140\,737\,488\,355\,328}-$$

$$\frac{\left(\sqrt{13}-3\right)^4\sqrt{\frac{3}{2}\left(\sqrt{13}-1\right)}\left(11+3\sqrt{13}\right)^2\left(\sqrt{\sqrt{13}-3}-\sqrt{5+\sqrt{13}}\right)^{24}}{17\,592\,186\,044\,416}$$

Minimal polynomial:

$$4\,294\,967\,296\,x^8-2\,889\,976\,119\,296\,x^7+1\,354\,743\,414\,784\,x^6+258\,104\,885\,248\,x^5+591\,623\,225\,344\,x^4-638\,322\,728\,960\,x^3+15\,021\,202\,432\,x^2-20\,483\,456\,x+1$$

From which:
(29-1)*10^3(4.88216345*10^-8)^2

where 29 is a Lucas number

Input interpretation:

$(29-1)\times 10^3\,(4.88216345\times 10^{-8})^2$

Result:

$6.6739455867044527\times 10^{-11}$

$6.6739455867044527*10^{-11}$ result practically equal to the value of the Gravitational Constant $6.674 * 10^{-11}$

That is:

28*10^3(4.88216345*10^-8)^2

Input interpretation:

$28\times 10^3\,(4.88216345\times 10^{-8})^2$

Result:

$6.6739455867044527\times 10^{-11}$

$6.6739455867044527*10^{-11}$ as above

With regard 28, we remember that (from Wikipedia), in nuclear physics, a **magic number** is a number of nucleons (either protons or neutrons, separately) such that they are arranged into complete shells within the atomic nucleus. The seven most widely recognized magic numbers as of 2019 are

2, 8, 20, 28, 50, 82, and 126 (sequence A018226 in the OEIS)

((((348+96sqrt13-9(2906+806sqrt13)^0.5)/8)^0.5-((340+96sqrt13-9(2906+806sqrt13)^0.5)/8)^0.5)))^2

Input:

$$\left(\sqrt{\frac{1}{8}\left(348+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}\right)}-\sqrt{\frac{1}{8}\left(340+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}\right)}\right)^2$$

Exact result:

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(348+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}\right)}-\frac{1}{2}\sqrt{\frac{1}{2}\left(340+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}\right)}\right)^2$$

Decimal approximation:

0.98850770225057349372043734086347001315754799813175213602 7...

0.98850770225…

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019

Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

We know that α' is the Regge slope (string tension). With regard the Omega mesons, the values are:

ω	6	$m_{u/d} = 0 - 60$	$0.910 - 0.918$	
ω/ω_3	$5+3$	$m_{u/d} = 255 - 390$	$0.988 - 1.18$	
ω/ω_3	$5+3$	$m_{u/d} = 240 - 345$	$0.937 - 1.000$	
Ψ	3	$m_c = 1500$	0.979	-0.09

the reciprocal of this result is 1/1.0002460865…. = 0,99975397399 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d} = 255 - 390 \mid 0.988 - 1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243 = \phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or 'glueballs').

Alternate forms:

root of $x^8 - 688x^7 - 89x^6 - 952x^5 + 3457x^4 - 952x^3 - 89x^2 - 688x + 1$ near $x = 0.988508$

$$\frac{1}{16}\left(\sqrt{2\left(340+96\sqrt{13}-9\sqrt{2906+806\sqrt{13}}\right)}-\sqrt{6\left(116+32\sqrt{13}-3\sqrt{2906+806\sqrt{13}}\right)}\right)^2$$

$$\frac{1}{8}\left(\sqrt{340+96\sqrt{13}-9\sqrt{2\left(1453+403\sqrt{13}\right)}}-\sqrt{3\left(116+32\sqrt{13}-3\sqrt{2\left(1453+403\sqrt{13}\right)}\right)}\right)^2$$

Minimal polynomial:

$$x^8-688x^7-89x^6-952x^5+3457x^4-952x^3-89x^2-688x+1$$

Thence:

((Sqrt13-3)/2)^4 [((5+sqrt13)/8)^0.5-((-3+sqrt13)/8)^0.5]^24 * [((11+3sqrt13)/2)^2 (((2sqrt2-(-3+3sqrt13)^0.5)/(4sqrt2)))^2] * 0.98850770225057349372043734086347001315754799813175213602 7

Input interpretation:

$$\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}\left(\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\right)\times$$

0.98850770225057349372043734086347001315754799813175213602 7

Result:

$4.82605618176035579371582909580554474121955947911800 89\ldots \times 10^{-8}$

$4.82605618\ldots*10^{-8}$

From which:

(11+3)*ln^2((((Sqrt13-3)/2)^4 [((5+sqrt13)/8)^0.5-((-3+sqrt13)/8)^0.5]^24 * [((11+3sqrt13)/2)^2 (((2sqrt2-(-3+3sqrt13)^0.5)/(4sqrt2)))^2] * 0.98850770225057349372043734086347))+123

Where 11, 3 and 123 are Lucas numbers

Input interpretation:

$$(11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}\left(\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\right)\times 0.98850770225057349372043734086347\right)+123$$

$\log(x)$ is the natural logarithm

Result:

4096.335162662817861768390694214 69...

4096.3351626… ≈ 4096 = 64^2

Alternative representations:

$$(11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}0.98850770225057349372043734086347000\right)+123=$$

$$123+14\log_e^2\left(0.98850770225057349372043734086347000\left(-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{-\sqrt{-3+3\sqrt{13}}+2\sqrt{2}}{4\sqrt{2}}\right)^2\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right)$$

$$(11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}0.98850770225057349372043734086347000\right)+123=$$

$$123+14\left(\log(a)\log_a\left(0.98850770225057349372043734086347000\left(-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{-\sqrt{-3+3\sqrt{13}}+2\sqrt{2}}{4\sqrt{2}}\right)^2\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right)\right)^2$$

$$(11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}0.98850770225057349372043734086347000\right)+123=$$

$$123+14\left(-\mathrm{Li}_1\left(1-0.98850770225057349372043734086347000\left(-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{-\sqrt{-3+3\sqrt{13}}+2\sqrt{2}}{4\sqrt{2}}\right)^2\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right)\right)$$

And:

Input interpretation:

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}\left(\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\right)\times 0.98850770225057349372043734086347\right)+123\right)}$$

$\log(x)$ is the natural logarithm

Result:

64.002618404740425602227541516679 9...

64.0026184… ≈ 64

All 2nd roots of 4096.335162662817861768390694214 69:

$64.002618404740425602227541516679\,e^{0} \approx 64.003$ (real, principal root)

$64.002618404740425602227541516679\,e^{i\pi} \approx -64.003$ (real root)

Alternative representations:

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}0.98850770225057349372043734086347000\right)+123\right)=}$$

$$\sqrt{\left(123+14\log_e^2\left(0.98850770225057349372043734086347000\left(-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{-\sqrt{-3+3\sqrt{13}}+2\sqrt{2}}{4\sqrt{2}}\right)^2\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right)\right)}$$

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}0.98850770225057349372043734086347000\right)+123\right)=}$$

$$\sqrt{\left(123+14\left(\log(a)\log_a\left(0.98850770225057349372043734086347000\left(-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{-\sqrt{-3+3\sqrt{13}}+2\sqrt{2}}{4\sqrt{2}}\right)^2\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right)\right)^2\right)\right)}$$

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right.\right.}$$
$$\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}$$
$$\left.\left.0.9885077022505734937204373408634700000\right)+123\right)=$$
$$\sqrt{\left(123+14\left(-\mathrm{Li}_1\left(1-0.9885077022505734937204373408634700000\right.\right.\right.}$$
$$\left(-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}$$
$$\left(\frac{-\sqrt{-3+3\sqrt{13}}+2\sqrt{2}}{4\sqrt{2}}\right)^2$$
$$\left.\left.\left.\left.\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right)\right)^2\right)$$

Series representations:

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right.\right.}$$
$$\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}$$
$$\left.\left.0.9885077022505734937204373408634700000\right)+123\right)=\sqrt{\left(123+\right.}$$
$$14\left(\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+\frac{1}{\sqrt{2}^2}0.000965339552979075677461364590686982422\right.\right.$$
$$\left(-3+\sqrt{13}\right)^4\left(11+3\sqrt{13}\right)^2$$
$$\left(-\frac{\sqrt{-3+\sqrt{13}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{13}}}{2\sqrt{2}}\right)^{24}$$
$$\left.\left.\left.\left.\left(2\sqrt{2}-\sqrt{-3+3\sqrt{13}}\right)^2\right)^k\right)^2\right)$$

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right.\right.}$$
$$\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}$$
$$\left.\left.0.98850770225057349372043734086347000\right)+123\right)=$$
$$\sqrt{\left(122+14\log^2\left(\frac{1}{\sqrt{2}^2}0.00096533955297907567746136459068698242 2\right.\right.}$$
$$\left(-3+\sqrt{13}\right)^4\left(11+3\sqrt{13}\right)^2$$
$$\left.\left.\left(-\frac{\sqrt{-3+\sqrt{13}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{13}}}{2\sqrt{2}}\right)^{24}\left(2\sqrt{2}-\sqrt{-3+3\sqrt{13}}\right)^2\right)\right)$$
$$\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(122+14\log^2\left(\frac{1}{\sqrt{2}^2}0.00096533955297907567746136459068698242 2\right.\right.$$
$$\left(-3+\sqrt{13}\right)^4\left(11+3\sqrt{13}\right)^2\left(-\frac{\sqrt{-3+\sqrt{13}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{13}}}{2\sqrt{2}}\right)^{24}$$
$$\left.\left.\left(2\sqrt{2}-\sqrt{-3+3\sqrt{13}}\right)^2\right)\right)^{-k}$$

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\right.\right.}$$
$$\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}$$
$$\left.\left.0.98850770225057349372043734086347000\right)+123\right)=$$
$$\sqrt{\left(122+14\log^2\left(\frac{1}{\sqrt{2}^2}0.00096533955297907567746136459068698242 2\right.\right.}$$
$$\left(-3+\sqrt{13}\right)^4\left(11+3\sqrt{13}\right)^2\left(-\frac{\sqrt{-3+\sqrt{13}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{13}}}{2\sqrt{2}}\right)^{24}$$
$$\left.\left.\left(2\sqrt{2}-\sqrt{-3+3\sqrt{13}}\right)^2\right)\right)\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k$$
$$\left(122+14\log^2\left(\frac{1}{\sqrt{2}^2}0.00096533955297907567746136459068698242 2\right.\right.$$
$$\left(-3+\sqrt{13}\right)^4\left(11+3\sqrt{13}\right)^2\left(-\frac{\sqrt{-3+\sqrt{13}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{13}}}{2\sqrt{2}}\right)^{24}$$
$$\left.\left.\left(2\sqrt{2}-\sqrt{-3+3\sqrt{13}}\right)^2\right)\right)^{-k}\left(-\frac{1}{2}\right)_k$$

Integral representation:

$$\sqrt{\left((11+3)\log^2\left(\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\frac{1}{2}\left(11+3\sqrt{13}\right)\right)^2\left(\frac{2\sqrt{2}-\sqrt{-3+3\sqrt{13}}}{4\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}-\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}\right)^{24}\right.\right.}$$

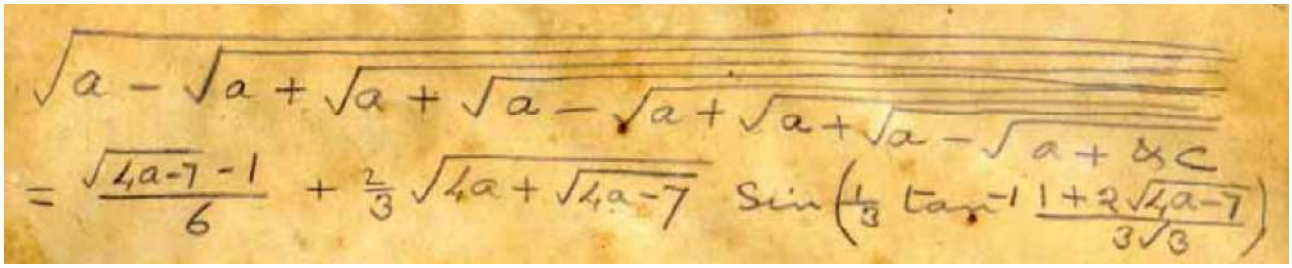
$$\left.\left.0.98850770225057349372043734086347000\right)+123\right)=\sqrt{\left(123+14\left(\int_1\frac{0.00096533955297907567746136459068698242\left(-3+\sqrt{13}\right)^4\left(11+3\sqrt{13}\right)^2\left(-\frac{\sqrt{-3+\sqrt{13}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{13}}}{2\sqrt{2}}\right)^{24}\left(2\sqrt{2}-\sqrt{-3+3\sqrt{13}}\right)^2}{\sqrt{2}^2}\right.\right.}$$

$$\left.\left.\frac{1}{t}\,dt\right)^2\right)$$

From

Manuscript Book II of *Srinivasa Ramanujan*

We have that:



For a = 256, we obtain:

(sqrt(4*256-7)-1)/6 + 2/3 sqrt(4*256+(4*256-7)^0.5) sin (1/3 tan^-1 ((((1+2*(4*256-7)^0.5)/(3sqrt3)))))

Input:

$$\frac{1}{6}\left(\sqrt{4\times256-7}-1\right)+\frac{2}{3}\sqrt{4\times256+\sqrt{4\times256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times256-7}}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(3\sqrt{113}-1\right)+\frac{2}{3}\sqrt{1024+3\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

15.475565815779841358783199305334268849083905246952630291 90...

(result in radians)

15.475565815….. result very near to the Bekenstein-Hawking black hole entropy 15.6730

Alternate forms:

$$\frac{1}{6}\left(-1+3\sqrt{113}+4\sqrt{1024+3\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{9}\left(\sqrt{3}+6\sqrt{339}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(-1+3\sqrt{113}+4\sqrt{1024+3\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)\right)$$

$$-\frac{1}{6}+\frac{\sqrt{113}}{2}+\frac{2}{3}\sqrt{1024+3\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)$$

Alternative representations:

$$\frac{1}{6}\left(\sqrt{4\times256-7}-1\right)+\frac{1}{3}\left(\sqrt{4\times256+\sqrt{4\times256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times256-7}}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(-1+\sqrt{1017}\right)+\frac{2}{3}\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{1017}}{3\sqrt{3}}\right)\right)\sqrt{1024+\sqrt{1017}}$$

$$\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(-1+\sqrt{1017}\right)-\frac{2}{3}\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{1017}}{3\sqrt{3}}\right)\right)\sqrt{1024+\sqrt{1017}}$$

$$\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(-1+\sqrt{1017}\right)+\frac{2\left(-e^{-1/3\,i\tan^{-1}\left(\left(1+2\sqrt{1017}\right)/\left(3\sqrt{3}\right)\right)}+e^{1/3\,i\tan^{-1}\left(\left(1+2\sqrt{1017}\right)/\left(3\sqrt{3}\right)\right)}\right)\sqrt{1024+\sqrt{1017}}}{3\,(2\,i)}$$

Series representations:

$$\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{113}}{2}+\frac{2}{3}\sqrt{1024+3\sqrt{113}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{113}}{2}+\frac{2}{3}\sqrt{1024+3\sqrt{113}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}$$

$$\frac{1}{6}\left(\sqrt{4\times 256-7}-1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{113}}{2}+\frac{1}{9}\sqrt{\left(1024+3\sqrt{113}\right)\pi}\,\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,\frac{36^{s}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}$$

Integral representations:

$$\frac{1}{6}\left(\sqrt{4\times 256-7}-1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{113}}{2}+\frac{2}{9}\sqrt{1024+3\sqrt{113}}\,\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\int_{0}^{1}\cos\left(\frac{1}{3}\,t\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)dt$$

$$\frac{1}{6}\left(\sqrt{4\times 256-7}-1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{113}}{2}-\frac{1}{18}\,i\sqrt{\frac{1024+3\sqrt{113}}{\pi}}\,\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)^{2}/(36s)}}{s^{3/2}}\,ds\quad\text{for}\ \gamma>0$$

$$\frac{1}{6}\left(\sqrt{4\times 256-7}-1\right)+\frac{1}{3}\left(\sqrt{4\times 256+\sqrt{4\times 256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{113}}{2}-\frac{1}{3}i\sqrt{\frac{1024+3\sqrt{113}}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{1}{6}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \text{ for } 0<\gamma<1$$

From which:

1/6 (3 sqrt(113) - 1) + 2/3 sqrt(x/4 + 3 sqrt(113)) sin(1/3 tan^(-1)((1 + 6 sqrt(113))/(3 sqrt(3)))) = 15.47556581577984135878

Input interpretation:

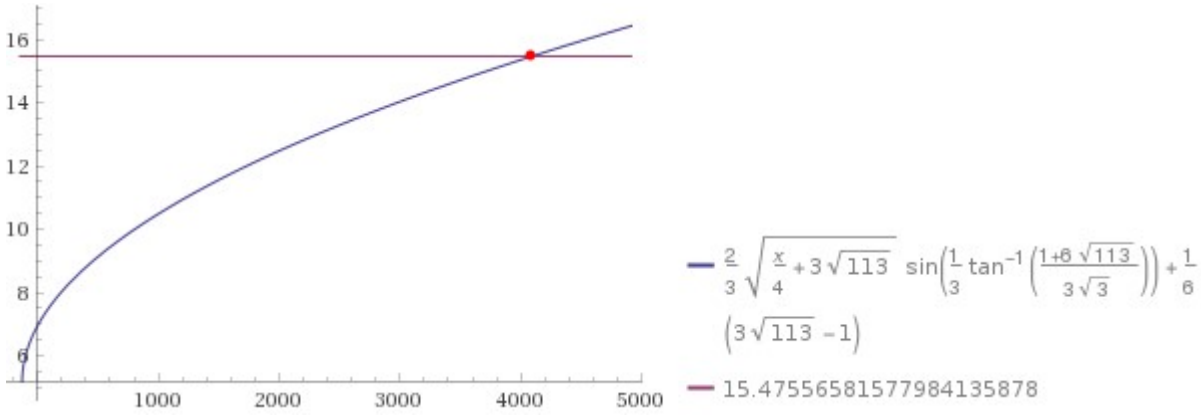
$$\frac{1}{6}\left(3\sqrt{113}-1\right)+\frac{2}{3}\sqrt{\frac{x}{4}+3\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)=15.47556581577984135878$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

$$\frac{2}{3}\sqrt{\frac{x}{4}+3\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+\frac{1}{6}\left(3\sqrt{113}-1\right)=15.47556581577984135878$$

Plot:



Alternate forms:

$$\frac{1}{6}\left(2\sqrt{x+12\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{9}\left(\sqrt{3}+6\sqrt{339}\right)\right)\right)+3\sqrt{113}-1\right)=$$
$$15.475565815779841358 78$$

$$\frac{1}{6}\left(2\sqrt{x+12\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+3\sqrt{113}-1\right)=$$
$$15.475565815779841358 78$$

$$\frac{2}{3}\sqrt{\frac{x}{4}+3\sqrt{113}}\ \text{root of } 4\,290\,801\,664\,x^{12}-12\,872\,404\,992\,x^{10}+14\,481\,455\,616\,x^{8}-7\,773\,539\,072\,x^{6}+2\,156\,853\,360\,x^{4}-299\,706\,336\,x^{2}+16\,540\,489 \text{ near } x=0.476719+\frac{1}{6}\left(3\sqrt{113}-1\right)=15.475565815779841358 78$$

Alternate form assuming x is positive:

$$1.000000000000000000 00\sqrt{x+12\sqrt{113}}=64.988935594859651408 6$$

Expanded form:

Solution:

$x \approx 4095.999999999999999 7$

4095.9999….. ≈ 4096 = 64^2

and:

Input interpretation:

$$\frac{1}{6}\left(3\sqrt{113}-1\right)+\frac{2}{3}\sqrt{\left(\frac{x}{2}\right)^2+3\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)=$$
$$15.475565815779841358 78$$

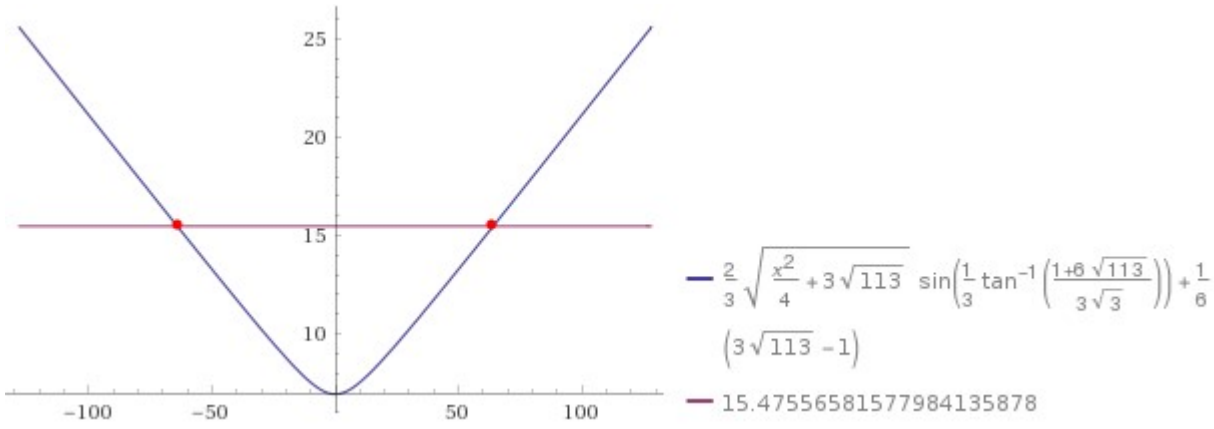
$\tan^{-1}(x)$ is the inverse tangent function

Result:

$$\frac{2}{3}\sqrt{\frac{x^2}{4}+3\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+\frac{1}{6}\left(3\sqrt{113}-1\right)=$$
$$15.475565815779841358 78$$

15.4755658…. as above

Plot:



Alternate forms:

$$\frac{1}{6}\left(2\sqrt{x^2+12\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{9}\left(\sqrt{3}+6\sqrt{339}\right)\right)\right)+3\sqrt{113}-1\right)=$$
$$15.47556581577984135878$$

$$\frac{1}{6}\left(2\sqrt{x^2+12\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+3\sqrt{113}-1\right)=$$
$$15.47556581577984135878$$

$$\frac{2}{3}\sqrt{\frac{x^2}{4}+3\sqrt{113}}\ \text{root of}\ 4\,290\,801\,664\,x^{12}-12\,872\,404\,992\,x^{10}+14\,481\,455\,616\,x^8-7\,773\,539\,072\,x^6+2\,156\,853\,360\,x^4-299\,706\,336\,x^2+16\,540\,489\ \text{near}\ x=0.476719+\frac{1}{6}\left(3\sqrt{113}-1\right)=15.47556581577984135878$$

Alternate form assuming x is positive:

$$1.000000000000000000\sqrt{x^2+12\sqrt{113}}=64.9889355948596514086$$

Expanded form:

$$\frac{2}{3}\sqrt{\frac{x^2}{4}+3\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+\frac{\sqrt{113}}{2}-\frac{1}{6}=$$
$$15.475565815779841358 78$$

Solutions:

$x \approx -63.99999999999999997$

$x \approx 63.99999999999999997$

63.9999…. ≈ 64

We have also:

1/2 ((((sqrt(4*256-7)-1)/6 + 2/3 sqrt(4*256+(4*256-7)^0.5) sin (1/3 tan^-1 ((((1+2*(4*256-7)^0.5)/(3sqrt3)))))))^3 – 125

Input:

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times256-7}-1\right)+\frac{2}{3}\sqrt{4\times256+\sqrt{4\times256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times256-7}}{3\sqrt{3}}\right)\right)\right)^3-125$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{2}\left(\frac{1}{6}\left(3\sqrt{113}-1\right)+\frac{2}{3}\sqrt{1024+3\sqrt{113}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)\right)^3-125$$

(result in radians)

Decimal approximation:

1728.145904500282897500563956571936363133214973844563784373...

(result in radians)

1728.1459….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic

curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$\frac{1}{432}\left(-1+3\sqrt{113}+4\sqrt{1024+3\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)\right)^3-125$$

$$\frac{1}{2}\left(\frac{2}{3}\sqrt{1024+3\sqrt{113}}\ \text{root of } 4\,290\,801\,664\,x^{12}-12\,872\,404\,992\,x^{10}+14\,481\,455\,616\,x^8-7\,773\,539\,072\,x^6+2\,156\,853\,360\,x^4-299\,706\,336\,x^2+16\,540\,489 \text{ near } x=0.476719+\frac{1}{6}\left(3\sqrt{113}-1\right)\right)^3-125$$

$$\frac{1}{108}\left(-20\,421+6885\sqrt{113}+15\,342\sqrt{1024+3\sqrt{113}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+18\sqrt{113\left(1024+3\sqrt{113}\right)}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)+42\cos\left(\frac{2}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)-18\,414\sqrt{113}\,\cos\left(\frac{2}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)\right)$$

Alternative representations:

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times256-7}-1\right)+\frac{2}{3}\sqrt{4\times256+\sqrt{4\times256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times256-7}}{3\sqrt{3}}\right)\right)\right)^3-125=-125+\frac{1}{2}\left(\frac{1}{6}\left(-1+\sqrt{1017}\right)+\frac{2}{3}\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{1017}}{3\sqrt{3}}\right)\right)\sqrt{1024+\sqrt{1017}}\right)^3$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{2}{3}\sqrt{4\times 256+\sqrt{4\times 256-7}}\right.$$
$$\left.\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)^3-125=-125+$$
$$\frac{1}{2}\left(\frac{1}{6}\left(-1+\sqrt{1017}\right)-\frac{2}{3}\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{1017}}{3\sqrt{3}}\right)\right)\sqrt{1024+\sqrt{1017}}\ \right)^3$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\right.$$
$$\left.\frac{2}{3}\sqrt{4\times 256+\sqrt{4\times 256-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)^3-$$
$$125=-125+\frac{1}{2}\left(\frac{1}{6}\left(-1+\sqrt{1017}\right)+\frac{1}{3\,(2\,i)}2\left(-e^{-1/3\,i\tan^{-1}\left(\left(1+2\sqrt{1017}\right)/\left(3\sqrt{3}\right)\right)}+\right.\right.$$
$$\left.\left.e^{1/3\,i\tan^{-1}\left(\left(1+2\sqrt{1017}\right)/\left(3\sqrt{3}\right)\right)}\right)\sqrt{1024+\sqrt{1017}}\ \right)^3$$

i is the imaginary unit

Series representations:

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{2}{3}\sqrt{4\times 256+\sqrt{4\times 256-7}}\right.$$
$$\left.\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)^3-125=-125+$$
$$\frac{1}{432}\left(-1+3\sqrt{113}\ +4\sqrt{1024+3\sqrt{113}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}\right)^3$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times 256-7}\ -1\right)+\frac{2}{3}\sqrt{4\times 256+\sqrt{4\times 256-7}}\right.$$
$$\left.\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times 256-7}}{3\sqrt{3}}\right)\right)\right)^3-125=-125+$$
$$\frac{1}{432}\left(-1+3\sqrt{113}\ +4\sqrt{1024+3\sqrt{113}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}\right)^3$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\sqrt{4\times256-7}-1\right)+\right.$$
$$\left.\frac{2}{3}\sqrt{4\times256+\sqrt{4\times256-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{4\times256-7}}{3\sqrt{3}}\right)\right)\right)^3-$$
$$125=-125+\frac{1}{2}\left(\frac{1}{6}\left(-1+3\sqrt{113}\right)+\frac{2}{3}\sqrt{1024+3\sqrt{113}}\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{1+6\sqrt{113}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}\right)^3$$

From

UCB-PTH-86/27 - LBL-22076 - ROM2F-86/015
Group Theory from "Quarks" at the Ends of Strings
Neil Marcus and Augusto Sagnotti

The calculation of the graphs proceeds in the standard manner [10,12]. The result is simplified by the use of complex coordinates [13]. The amplitude is given by

$$A = 2^{BD/2} \int d[\text{teich}] \frac{1}{S} \frac{1}{V_{\text{ckv}}} \sqrt{\frac{\det' \Delta^1}{N_2}} \left(\frac{\det' \Delta}{\sqrt{g}} \right)^{-13} \quad . \tag{12}$$

Here D is the number of ψ fields; B is the number of boundaries; Δ^1 is the Laplacian acting on the ghost c^μ; N_2 is the determinant of the zero modes of the Laplacian acting on symmetric tensors; V_{ckv} is the volume of the group generated by the conformal killing vectors and S is the symmetry factor of the graph. The only tricky point is that the Klein bottle has an extra symmetry $z \to z + \tau/2$, corresponding to shifting the diagram vertically by one half of its height. The answers are very simple:

$$\begin{aligned} A_t &= \int_M \frac{d\tau \wedge d\bar{\tau}}{2i} (\operatorname{Im} \tau)^{-14} \frac{1}{4} \Delta(\tau) \Delta(\bar{\tau}) \\ A_a &= 2^{(D-13)} \int_0^\infty d\tau_2 \frac{1}{4} \Delta(\tau = i\tau_2) \\ A_K &= 2^{13} \int_0^\infty d\tau_2 \frac{1}{4} \Delta(\tau = i\tau_2) \\ A_M &= -2^{D/2} \int_0^\infty d\tau_2 \frac{1}{2} \Delta(\tau = \frac{1}{2} + i\tau_2) \quad . \end{aligned} \tag{13}$$

Here

$$\Delta(q) \equiv \frac{1}{q \prod (1 - q^n)^{24}} \quad ,$$

and we use the standard, if somewhat confusing, notation that $\Delta(\tau)$ should be interpreted as $\Delta(q \equiv e^{2\pi i \tau})$.

Before obtaining the partition function, it is instructive to examine the behaviour of the amplitudes as τ_2 becomes infinite. After changing variables to q, the dangerous point is $q = 0$. The torus remains separate. The other graphs give

$$A_{\text{real}} = 2^{(D-13)} \int_0^1 \frac{dq}{8\pi} \Delta(q) + 2^{13} \int_0^1 \frac{dq}{8\pi} \Delta(q) + 2^{D/2} \int_{-1}^0 \frac{dq}{4\pi} \Delta(q) \quad . \tag{14}$$

If one chooses $D = 26$, giving the group SO(8192), this becomes

$$A_{\text{real}} = 2^{13} \int_{-1}^1 \frac{dq}{4\pi} \Delta(q) \quad . \tag{15}$$

From (15) , for

$$\Delta(q) \equiv \frac{1}{q \prod (1 - q^n)^{24}}$$

$$\Delta(q \equiv e^{2\pi i \tau}).$$

1/(exp(2Pi)*product (1-exp(2Pi)^n)^24 for n = 2, we obtain:

1/[(exp(2Pi)) ((1-exp(2Pi)^2)^24)]

Input:

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}}$$

Exact result:

$$\frac{e^{-2\pi}}{\left(1-e^{4\pi}\right)^{24}}$$

Decimal approximation:

$1.955031108847853448796366294494669808598166319840922\ldots \times 10^{-134}$

1.9550311…*10^{-134}

Property:

$\frac{e^{-2\pi}}{\left(1-e^{4\pi}\right)^{24}}$ is a transcendental number

Alternate forms:

$$\frac{e^{-2\pi}}{\left(e^{4\pi}-1\right)^{24}}$$

$$\frac{e^{-2\pi}}{\left(e^{\pi}-1\right)^{24}\left(1+e^{\pi}\right)^{24}\left(1+e^{2\pi}\right)^{24}}$$

$$e^{-2\pi} + \frac{1}{281\,474\,976\,710\,656\,(-1+e^{\pi})^{24}} - \frac{19}{140\,737\,488\,355\,328\,(-1+e^{\pi})^{23}} + \frac{363}{9295}$$
$$\frac{}{140\,737\,488\,355\,328\,(-1+e^{\pi})^{22}} - \frac{}{281\,474\,976\,710\,656\,(-1+e^{\pi})^{21}} + \frac{717\,487}{695\,403}$$
$$\frac{}{2\,251\,799\,813\,685\,248\,(-1+e^{\pi})^{20}} - \frac{}{281\,474\,976\,710\,656\,(-1+e^{\pi})^{19}} + \frac{72\,190\,517}{402\,986\,581}$$
$$\frac{}{4\,503\,599\,627\,370\,496\,(-1+e^{\pi})^{18}} - \frac{}{4\,503\,599\,627\,370\,496\,(-1+e^{\pi})^{17}} + \frac{31\,600\,958\,565}{69\,041\,790\,017}$$
$$\frac{}{72\,057\,594\,037\,927\,936\,(-1+e^{\pi})^{16}} - \frac{}{36\,028\,797\,018\,963\,968\,(-1+e^{\pi})^{15}} + \frac{544\,452\,058\,163}{488\,968\,971\,963}$$
$$\frac{}{72\,057\,594\,037\,927\,936\,(-1+e^{\pi})^{14}} - \frac{}{18\,014\,398\,509\,481\,984\,(-1+e^{\pi})^{13}} + \frac{12\,904\,424\,034\,527}{1\,229\,643\,194\,285}$$
$$\frac{}{144\,115\,188\,075\,855\,872\,(-1+e^{\pi})^{12}} - \frac{}{4\,503\,599\,627\,370\,496\,(-1+e^{\pi})^{11}} + \frac{223\,025\,165\,069\,751}{590\,160\,722\,630\,947}$$
$$\frac{}{288\,230\,376\,151\,711\,744\,(-1+e^{\pi})^{10}} - \frac{}{288\,230\,376\,151\,711\,744\,(-1+e^{\pi})^{9}} + \frac{46\,846\,820\,580\,453\,871}{54\,655\,156\,220\,538\,729}$$
$$\frac{}{9\,223\,372\,036\,854\,775\,808\,(-1+e^{\pi})^{8}} - \frac{}{4\,611\,686\,018\,427\,387\,904\,(-1+e^{\pi})^{7}} + \frac{7\,519\,462\,478\,478\,295}{500\,964\,876\,751\,869\,847}$$
$$\frac{}{288\,230\,376\,151\,711\,744\,(-1+e^{\pi})^{6}} - \frac{}{9\,223\,372\,036\,854\,775\,808\,(-1+e^{\pi})^{5}} + \frac{7\,908\,871\,750\,199\,401\,947}{926\,365\,882\,350\,132\,313}$$
$$\frac{}{73\,786\,976\,294\,838\,206\,464\,(-1+e^{\pi})^{4}} - \frac{}{4\,611\,686\,018\,427\,387\,904\,(-1+e^{\pi})^{3}} + \frac{52\,845\,090\,211\,660\,526\,293}{89\,738\,578\,359\,079\,629\,525}$$
$$\frac{}{147\,573\,952\,589\,676\,412\,928\,(-1+e^{\pi})^{2}} - \frac{}{147\,573\,952\,589\,676\,412\,928\,(-1+e^{\pi})} +$$
$$\frac{1}{281\,474\,976\,710\,656\,(1+e^{\pi})^{24}} + \frac{19}{140\,737\,488\,355\,328\,(1+e^{\pi})^{23}} + \frac{363}{9295}$$
$$\frac{}{140\,737\,488\,355\,328\,(1+e^{\pi})^{22}} + \frac{}{281\,474\,976\,710\,656\,(1+e^{\pi})^{21}} + \frac{717\,487}{695\,403}$$
$$\frac{}{2\,251\,799\,813\,685\,248\,(1+e^{\pi})^{20}} + \frac{}{281\,474\,976\,710\,656\,(1+e^{\pi})^{19}} + \frac{72\,190\,517}{402\,986\,581}$$
$$\frac{}{4\,503\,599\,627\,370\,496\,(1+e^{\pi})^{18}} + \frac{}{4\,503\,599\,627\,370\,496\,(1+e^{\pi})^{17}} + \frac{31\,600\,958\,565}{69\,041\,790\,017}$$
$$\frac{}{72\,057\,594\,037\,927\,936\,(1+e^{\pi})^{16}} + \frac{}{36\,028\,797\,018\,963\,968\,(1+e^{\pi})^{15}} + \frac{544\,452\,058\,163}{488\,968\,971\,963}$$
$$\frac{}{72\,057\,594\,037\,927\,936\,(1+e^{\pi})^{14}} + \frac{}{18\,014\,398\,509\,481\,984\,(1+e^{\pi})^{13}} + \frac{12\,904\,424\,034\,527}{1\,229\,643\,194\,285}$$
$$\frac{}{144\,115\,188\,075\,855\,872\,(1+e^{\pi})^{12}} + \frac{}{4\,503\,599\,627\,370\,496\,(1+e^{\pi})^{11}} + \frac{223\,025\,165\,069\,751}{590\,160\,722\,630\,947}$$
$$\frac{}{288\,230\,376\,151\,711\,744\,(1+e^{\pi})^{10}} + \frac{}{288\,230\,376\,151\,711\,744\,(1+e^{\pi})^{9}} + \frac{46\,846\,820\,580\,453\,871}{54\,655\,156\,220\,538\,729}$$
$$\frac{}{9\,223\,372\,036\,854\,775\,808\,(1+e^{\pi})^{8}} + \frac{}{4\,611\,686\,018\,427\,387\,904\,(1+e^{\pi})^{7}} + \frac{7\,519\,462\,478\,478\,295}{500\,964\,876\,751\,869\,847}$$
$$\frac{}{288\,230\,376\,151\,711\,744\,(1+e^{\pi})^{6}} + \frac{}{9\,223\,372\,036\,854\,775\,808\,(1+e^{\pi})^{5}} + \frac{7\,908\,871\,750\,199\,401\,947}{926\,365\,882\,350\,132\,313}$$
$$\frac{}{73\,786\,976\,294\,838\,206\,464\,(1+e^{\pi})^{4}} + \frac{}{4\,611\,686\,018\,427\,387\,904\,(1+e^{\pi})^{3}} + \frac{52\,845\,090\,211\,660\,526\,293}{89\,738\,578\,359\,079\,629\,525}$$
$$\frac{}{147\,573\,952\,589\,676\,412\,928\,(1+e^{\pi})^{2}} + \frac{}{147\,573\,952\,589\,676\,412\,928\,(1+e^{\pi})} -$$
$$\frac{1}{16\,777\,216\,(1+e^{2\pi})^{24}} - \frac{13}{16\,777\,216\,(1+e^{2\pi})^{23}} - \frac{11}{2\,097\,152\,(1+e^{2\pi})^{22}} - \frac{413}{12\,079}$$
$$\frac{}{16\,777\,216\,(1+e^{2\pi})^{21}} - \frac{}{134\,217\,728\,(1+e^{2\pi})^{20}} - \frac{}{134\,217\,728\,(1+e^{2\pi})^{19}} - \frac{192\,053}{6977}$$
$$\frac{}{268\,435\,456\,(1+e^{2\pi})^{18}} - \frac{}{4\,194\,304\,(1+e^{2\pi})^{17}} - \frac{}{4\,294\,967\,296\,(1+e^{2\pi})^{16}} - \frac{29\,057\,573}{15\,033\,173}$$
$$\frac{}{4\,294\,967\,296\,(1+e^{2\pi})^{15}} - \frac{}{4\,294\,967\,296\,(1+e^{2\pi})^{14}} - \frac{}{4\,294\,967\,296\,(1+e^{2\pi})^{13}} - \frac{52\,197\,833}{87\,960\,053}$$
$$\frac{}{8\,589\,934\,592\,(1+e^{2\pi})^{12}} - \frac{}{8\,589\,934\,592\,(1+e^{2\pi})^{11}} - \frac{}{17\,179\,869\,184\,(1+e^{2\pi})^{10}} - \frac{280\,226\,581}{424\,650\,931}$$
$$\frac{}{4\,294\,967\,296\,(1+e^{2\pi})^{9}} - \frac{}{549\,755\,813\,888\,(1+e^{2\pi})^{8}} - \frac{428\,618\,123}{1\,230\,994\,787}$$
$$\frac{}{549\,755\,813\,888\,(1+e^{2\pi})^{7}} - \frac{}{274\,877\,906\,944\,(1+e^{2\pi})^{6}} - \frac{95\,901\,844\,939}{73\,718\,750\,239}$$
$$\frac{}{549\,755\,813\,888\,(1+e^{2\pi})^{5}} - \frac{}{4\,398\,046\,511\,104\,(1+e^{2\pi})^{4}} - \frac{149\,089\,381\,939}{60\,582\,962\,507}$$
$$\frac{}{4\,398\,046\,511\,104\,(1+e^{2\pi})^{3}} - \frac{}{8\,796\,093\,022\,208\,(1+e^{2\pi})^{2}} - \frac{}{2\,(1+e^{2\pi})} - \frac{1\,684\,433\,835\,077}{3\,883\,457\,090\,629}$$

Series representations:

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}} = \frac{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-2\pi}}{\left(-1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)^{24}\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{\pi}\right)^{24}\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)^{24}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}} = \frac{\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-2\pi}}{\left(-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)^{24}\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{\pi}\right)^{24}\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)^{24}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}} = e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)} \Big/ \left(\left(-1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^{24}\left(1+e^{\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^{24}\left(1+e^{2\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^{24}\left(1+e^{4\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^{24}\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^{24}\right)$$

Integral representations:

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}} = \frac{e^{-6\int_0^{\infty}\sin^4(t)/t^4\,dt}}{\left(-1+e^{12\int_0^{\infty}\sin^4(t)/t^4\,dt}\right)^{24}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}} = \frac{e^{-16/3\int_0^{\infty}\sin^3(t)/t^3\,dt}}{\left(-1+e^{32/3\int_0^{\infty}\sin^3(t)/t^3\,dt}\right)^{24}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^2(2\pi)\right)^{24}} = \frac{e^{-2\left(\frac{3\sqrt{3}}{4}+24\int_0^{\frac{1}{4}}\sqrt{t-t^2}\,dt\right)}}{\left(1-e^{4\left(\frac{3\sqrt{3}}{4}+24\int_0^{\frac{1}{4}}\sqrt{t-t^2}\,dt\right)}\right)^{24}}$$

For n = 1:

1/[(exp(2Pi)) (1-exp(2Pi)^24)]

Input:

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)}$$

Exact result:

$$\frac{e^{-2\pi}}{1-e^{48\pi}}$$

Decimal approximation:

$$-6.042022078324069172398614601097072303451117135785880\ldots \times 10^{-69}$$

-6.042022078…*10^{-69}

Property:

$\frac{e^{-2\pi}}{1-e^{48\pi}}$ is a transcendental number

Alternate forms:

$$\frac{1}{e^{2\pi}-e^{50\pi}}$$

$$-\frac{e^{-2\pi}}{e^{48\pi}-1}$$

$$-\left(e^{-2\pi}\Big/\left(\left(e^{\pi}-1\right)\left(1+e^{\pi}\right)\left(1+e^{2\pi}\right)\left(1-e^{\pi}+e^{2\pi}\right)\left(1+e^{\pi}+e^{2\pi}\right)\left(1+e^{4\pi}\right)\left(1-e^{2\pi}+e^{4\pi}\right)\left(1+e^{8\pi}\right)\left(1-e^{4\pi}+e^{8\pi}\right)\left(1-e^{8\pi}+e^{16\pi}\right)\right)\right)$$

Series representations:

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)} = -\frac{e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{-1+e^{192\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)} = \frac{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-2\pi}}{1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{48\pi}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)} = -\frac{\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-2\pi}}{-1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{48\pi}}$$

Integral representations:

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)} = \frac{e^{-4\int_0^{\infty}1/(1+t^2)\,dt}}{1-e^{96\int_0^{\infty}1/(1+t^2)\,dt}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)} = \frac{e^{-8\int_0^{1}\sqrt{1-t^2}\,dt}}{1-e^{192\int_0^{1}\sqrt{1-t^2}\,dt}}$$

$$\frac{1}{\exp(2\pi)\left(1-\exp^{24}(2\pi)\right)} = \frac{e^{-4\int_0^{1}1/\sqrt{1-t^2}\,dt}}{1-e^{96\int_0^{1}1/\sqrt{1-t^2}\,dt}}$$

Performing the product, we have the following results:

a)

1/(exp(2Pi)*product ((1-exp(2Pi)^n)^24), n = 0.5..1

Input interpretation:

$$\frac{1}{\exp(2\pi)\prod_{n=0.5}^{1}\left(1-\exp^{n}(2\pi)\right)^{24}}$$

Result:

9.69747×10^{-36}

$9.69747*10^{-36}$

b)

1/(exp(2Pi)*product ((1-exp(2Pi)^n)^24), n = 0.618..1

Input interpretation:

$$\frac{1}{\exp(2\,\pi)\prod_{n=0.618}^{1}(1-\exp^{n}(2\,\pi))^{24}}$$

Result:

1.03568×10^{-43}

1.03568*10^-43

Or:

c)

1/(exp(2Pi)*product ((1-exp(2Pi)^n)^24), n = 0.652619..1

Input interpretation:

$$\frac{1}{\exp(2\,\pi)\prod_{n=0.652619}^{1}(1-\exp^{n}(2\,\pi))^{24}}$$

Result:

5.07317×10^{-46}

5.07317*10^-46 result practically equal to 5.072985×10^{-46} F that is the electrical capacity of Planck

Indeed:

$$C_{\mathrm{P}}=\frac{q_{\mathrm{P}}}{V_{\mathrm{P}}}=\frac{l_P}{k_e}=\sqrt{\frac{4\pi\varepsilon_0^2\hbar G}{c^3}}$$

sqrt((((((4*Pi*(8.85418781762e-12)^2*(1.054571817e-34)*(6.67430e-11))) / (299792458^3)))))

Input interpretation:

$$\sqrt{\frac{4\pi (8.85418781762 \times 10^{-12})^2 \times 1.054571817 \times 10^{-34} \times 6.67430 \times 10^{-11}}{299\,792\,458^3}}$$

Result:

$5.072984670523948053018306379909870242152474712974606 1\ldots \times 10^{-46}$

$5.07298467\ldots*10^{-46}$

From

1/(exp(2Pi)*product ((1-exp(2Pi)^n)^24), n = 0.65578..1

where 0.65578 = 1 – 1.65578 (note that the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578)

we obtain:

Input interpretation:

$$\frac{1}{\exp(2\pi) \prod_{n=0.65578}^{1} (1 - \exp^n(2\pi))^{24}}$$

Result:

3.12474×10^{-46}

$3.12474*10^{-46}$

We have also:

1/(exp(2Pi)*product ((1-exp(2Pi)^n)^24), n = 1..0.5

Input interpretation:

$$\frac{1}{\exp(2\pi) \prod_{n=1}^{0.5} (1 - \exp^n(2\pi))^{24}}$$

Result:

$e^{-2\pi} \approx 0.00186744$

0.00186744

We note that:

1/((((1/(exp(2Pi)*product ((1-exp(2Pi)^n)^24), n = 1..0.5)))))

Input interpretation:

$$\frac{1}{\frac{1}{\exp(2\pi)\prod_{n=1}^{0.5}(1-\exp^n(2\pi))^{24}}}$$

Result:

$e^{2\pi} \approx 535.492$

535.492

From

$$A_{rest} = 2^{13}\int_{-1}^{1}\frac{dq}{4\pi}\Delta(q) \quad .$$

we obtain:

a)

2^13 * integrate (1/(4Pi))*535.492)q, q = 0..1

Definite integral:

$$8192\int_0^1 42.6131\, q\, dq = 174543.$$

174543

or:

b)

2^13 * integrate ((1/(4Pi))*0.00186744)q, q = 0..1

Definite integral:

$$8192 \int_0^1 0.000148606\, q\, dq = 0.608691$$

0.608691

And:

1/(((2^13 * integrate ((1/(4Pi))*0.00186744)q, q = 0..1)))

Input interpretation:

$$\frac{1}{2^{13} \int_0^1 \left(\frac{1}{4\pi} \times 0.00186744\right) q\, dq}$$

Result:

1.64287

$1.64287 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Computation result:

$$\frac{1}{2^{13} \int_0^1 \frac{0.00186744\, q}{4\pi}\, dq} = 1.64287$$

From the previous expression, we obtain also:

2^13 * integrate ((1/(4Pi))*5.07317*10^-46)q, q = 0..1

Definite integral:

$$8192 \int_0^1 4.0371 \times 10^{-47}\, q\, dq = 1.6536 \times 10^{-43}$$

$1.6536 * 10^{-43}$

and

2^13 * integrate ((1/(4Pi))*3.12474*10^-46)q, q = 0..1

Definite integral:

$$8192 \int_0^1 2.48659 \times 10^{-47}\, q\, dq = 1.01851 \times 10^{-43}$$

1.01851 * 10^{-43}

The two results can be interpreted also:

1.6536 * 10^{-43}

Input interpretation:

1.6536×10^{-43} seconds

Comparisons as time:

≈ time after Big Bang at which gravity separated from electroweak and strong forces ($\approx 1 \times 10^{-43}$ s)
≈ 3.1 × time for light to travel a Planck length ($1\, t_P$)
≈ 3.1 × Planck time ($\approx 5.4 \times 10^{-44}$ s)

Interpretations:
time

Corresponding quantity:

Frequency ν from $\nu = 1/T$:
6.047×10^{42} Hz (hertz)

1.01851 * 10^{-43}

Input interpretation:

1.01851×10^{-43} seconds

Comparisons as time:

≈ time after Big Bang at which gravity separated from electroweak and strong forces ($\approx 1 \times 10^{-43}$ s)

≈ 1.9 × time for light to travel a Planck length ($1\ t_P$)

≈ 1.9 × Planck time ($\approx 5.4 \times 10^{-44}$ s)

Interpretations:

time

Corresponding quantity:

Frequency ν from $\nu = 1/T$:

9.8183×10^{42} Hz (hertz)

Thence the two results, are about equal to the time after Big Bang at which gravity separated from electroweak and strong forces

Now, we have that:

In the superstring one would choose $D = 10$, giving the group SO(32)

Defining $qe^{2\pi i\phi} = e^{2\pi i l}$ and $N = 2^{D/2}$, and using the transformation property

$$\Delta\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^{-12}\Delta(\tau) \quad ,$$

the four graphs become

$$(2\pi)^{13}\int_0^1 \frac{dq}{2q}\log^{-14}(q)\int_0^1 d\phi \cdot \left(\frac{1}{2}\Delta(qe^{2\pi i\phi})\Delta(qe^{-2\pi i\phi}) + \frac{1}{2}\Delta(q^2) + \frac{1}{2}N^2\Delta(\sqrt{q}) - \frac{1}{2}N\Delta(-\sqrt{q})\right) \quad . \tag{17}$$

The partition function is the factor inside the brackets, with the ϕ integral enforcing the constraint $N = \tilde{N}$. The first two contributions arise from the closed sector and the last two from the open sector. Each pair of graphs possesses a geometrical sym-

$$\Delta(q) \equiv \frac{1}{q\prod(1-q^n)^{24}}$$

$$\Delta(q \equiv e^{2\pi i\tau}).$$

That are equal to

$$\frac{1}{\exp(2\pi)\prod_{n=1}^{0.5}(1-\exp^n(2\pi))^{24}}$$

$e^{-2\pi} \approx 0.00186744$

0.00186744

Further:

$e^{2\pi} \approx 535.492$

535.492

From

$$(2\pi)^{13}\int_0^1 \frac{dq}{2q}\log^{-14}(q)\int_0^1 d\phi \cdot \left(\frac{1}{2}\Delta(qe^{2\pi i\phi})\Delta(qe^{-2\pi i\phi}) + \frac{1}{2}\Delta(q^2) + \frac{1}{2}N^2\Delta(\sqrt{q}) - \frac{1}{2}N\Delta(-\sqrt{q})\right)$$

we obtain:

[1/2 * (0.00186744 * 535.492 * 0.00186744 * 0.00186744) + 1/2 * (0.00186744)^2 + 1/2 * (((2^5)^2 * (0.00186744)^0.5)) – 1/2 * (2^5) * -(0.00186744)^0.5]

Input interpretation:

$$\frac{1}{2}(0.00186744\times 535.492\times 0.00186744\times 0.00186744) + \frac{1}{2}\times 0.00186744^2 + \frac{1}{2}\left((2^5)^2\sqrt{0.00186744}\right) - \frac{1}{2}\times 2^5\times(-1)\sqrt{0.00186744}$$

Result:

22.816935642202027574022562095824204713929197193996759284 61...

22.8169356…

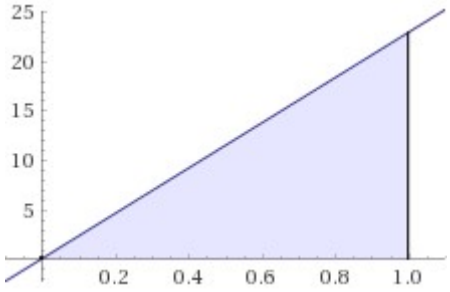
integrate[1/2 * (0.00186744 * 535.492 * 0.00186744 * 0.00186744) + 1/2 * (0.00186744)^2 + 1/2 * (((2^5)^2 * (0.00186744)^0.5)) – 1/2 * (2^5) * -(0.00186744)^0.5] x, x = 0..1

Definite integral:

$$\int_0^1 \left(\frac{1}{2} \times 0.00186744 \times 535.492 \times 0.00186744 \times 0.00186744 + \frac{0.00186744^2}{2} + \frac{1}{2}(2^5)^2 \sqrt{0.00186744} - \frac{1}{2} \times 2^5 (-1) \sqrt{0.00186744}\right) x\, dx = 11.4085$$

11.4085

Visual representation of the integral:



With regard the partition function, we note that in the list of partitions of n (the partition numbers), p(6) = 11 and p(8) = 22 (A000041 sequence OEIS)

We have also:

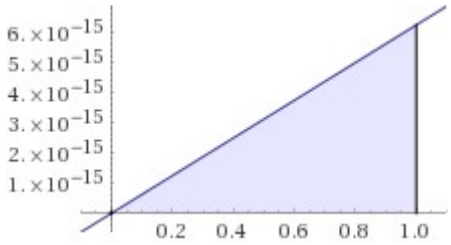
Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1

Definite integral:

$$\int_0^1 \frac{x}{(2\exp(2\pi))\log^{14}(\exp(2\pi))}\, dx = \frac{e^{-2\pi}}{65536\,\pi^{14}} \approx 3.1237 \times 10^{-15}$$

$\log(x)$ is the natural logarithm

Visual representation of the integral:



(2Pi)^13 * integrate[1/2 * (0.00186744 * 535.492 * 0.00186744 * 0.00186744) + 1/2 * (0.00186744)^2 + 1/2 * (((2^5)^2 * (0.00186744)^0.5)) – 1/2 * (2^5) * -(0.00186744)^0.5] x, x = 0..1

Definite integral:

$$8192\,\pi^{13}\int_0^1 22.8169\,x\,dx = 2.71372\times10^{11}$$

Thence, in conclusion:

2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1

Definite integral:

$$271\,372\,000\,000\int_0^1 \frac{e^{-2\pi}\,x}{32\,768\,\pi^{14}}\,dx = 0.000847684$$

0.000847684

We note that:

1/(((2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1)))

Input interpretation:

$$\frac{1}{2.71372\times10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}$$

$\log(x)$ is the natural logarithm

Result:

1179.68

1179.68

We note that the sum of the partition numbers p(15) = 176 and p(22) = 1002 is 1178, that is very near to the result that we have obtained.

From which:

1/(((2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1)))+8+2

Input interpretation:

$$\frac{1}{2.71372\times 10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}+8+2$$

$\log(x)$ is the natural logarithm

Result:

1189.68

1189.68 result practically equal to the rest mass of Sigma baryon 1189.37

[1/(((2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1)))]^1/14

Input interpretation:

$$\sqrt[14]{\frac{1}{2.71372\times 10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}}$$

$\log(x)$ is the natural logarithm

Result:

1.65734

1.65734 result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

And:

1/(((2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1))) + 521 + 29

Input interpretation:

$$\frac{1}{2.71372\times 10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}+521+29$$

$\log(x)$ is the natural logarithm

Result:

1729.68

1729.68

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((1/(((2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1))) + 521 + 29)))^1/15

Input interpretation:

$$\sqrt[15]{\frac{1}{2.71372\times 10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}+521+29}$$

$\log(x)$ is the natural logarithm

Result:

1.64386

1.64386 $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

and again:

1/(((2.71372×10^11 * Integrate [1/(2*(exp(2Pi))) * 1/(ln^14 (exp(2Pi)))]x, x = 0..1)))
+13

Input interpretation:

$$\frac{1}{2.71372\times 10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}+13$$

$\log(x)$ is the natural logarithm

Result:

1192.68

1192.68 result practically equal to the rest mass of Sigma baryon 1192.642

From the previous formula

$$\frac{1}{2.71372\times 10^{11}\int_0^1\left(\frac{1}{2\exp(2\pi)}\times\frac{1}{\log^{14}(\exp(2\pi))}\right)x\,dx}$$

1179.68

We note that from the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 167 , we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(167/15)) / (2*5^(1/4)*sqrt(167))

Input:

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{167}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{167}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{167/15}\,\pi}\sqrt{\frac{\phi}{167}}}{2\sqrt[4]{5}}$$

Decimal approximation:

1174.437518583385207971944208082789598173766564972137224882...

1174.437518…

Property:

$\frac{e^{\sqrt{167/15}\,\pi}\sqrt{\frac{\phi}{167}}}{2\sqrt[4]{5}}$ is a transcendental number

Alternate forms:

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\;e^{\sqrt{167/15}\,\pi}$$

$$\frac{\sqrt{\frac{1}{334}\left(1+\sqrt{5}\right)}\;e^{\sqrt{167/15}\,\pi}}{2\sqrt[4]{5}}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{167}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{167}} = \frac{\exp\left(\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{167}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(167-z_0)^k z_0^{-k}}{k!}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{167}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{167}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\right.$$
$$\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{167}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{167}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$
$$\left(2\sqrt[4]{5}\ \exp\left(i\pi\left\lfloor\frac{\arg(167-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(167-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{167}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{167}} =$$
$$\left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{167}{15}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{167}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{167}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\right.$$
$$\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(167-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}z_0^{-1/2\lfloor\arg(167-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/\left(2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(167-z_0)^k z_0^{-k}}{k!}\right)$$

From

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\ e^{\sqrt{167/15}\,\pi}$$

Adding 5, that is a Fibonacci number, we obtain:

1/2 sqrt((5 + sqrt(5))/1670) e^(sqrt(167/15) π) + 5

Input:

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\ e^{\sqrt{167/15}\,\pi}+5$$

Decimal approximation:

1179.437518583385207971944208082789598173766564972137224882...

1179.437518…. a value very near to the previous result

Property:

$5+\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\;e^{\sqrt{167/15}\,\pi}$ is a transcendental number

Alternate forms:

$$\frac{16\,700+\sqrt{1670\left(5+\sqrt{5}\right)}\;e^{\sqrt{167/15}\,\pi}}{3340}$$

$$5+\frac{\left(\sqrt{1-\frac{2}{\sqrt{5}}}+\frac{\sqrt{2+\sqrt{5}}}{\sqrt[4]{5}}\right)e^{\sqrt{167/15}\,\pi}}{4\sqrt{167}}$$

$$\frac{20\sqrt{167}+\left(\sqrt{1-\frac{2}{\sqrt{5}}}+\frac{\sqrt{2+\sqrt{5}}}{\sqrt[4]{5}}\right)e^{\sqrt{167/15}\,\pi}}{4\sqrt{167}}$$

Series representations:

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\;e^{\sqrt{167/15}\,\pi}+5=\frac{1}{2}\left(10+\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{167}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sqrt{z_0}\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5+\sqrt{5}}{1670}-z_0\right)^k z_0^{-k}}{k!}\right)\quad\text{for}\ \left(\text{not}\ \left(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\leq 0\right)\right)$$

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\;e^{\sqrt{167/15}\,\pi}+5=$$
$$\frac{1}{2}\left(10+\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{167}{15}-x\right)}{2\pi}\right\rfloor\right)\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{167}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{5+\sqrt{5}}{1670}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$
$$\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+\frac{5+\sqrt{5}}{1670}\right)^k}{k!}\Bigg)\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1670}}\;e^{\sqrt{167/15}\,\pi}+5=$$
$$\frac{1}{2}\left(10+\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{167}{15}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(\frac{167}{15}-z_0\right)/(2\pi)\right\rfloor}\right.\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{167}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5+\sqrt{5}}{1670}-z_0\right)/(2\pi)\right\rfloor}$$
$$\left.z_0^{1/2+1/2\left\lfloor\arg\left(\frac{5+\sqrt{5}}{1670}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5+\sqrt{5}}{1670}-z_0\right)^k z_0^{-k}}{k!}\right)$$

Now, we have that:

In conclusion, the variables we have introduced give all the desired properties for obtaining Chan-Paton factors. One obvious remaining question is the number of ψ^I fields required. A clue may be found by examining the conformal anomaly of the theory. The Liouville term vanishes in $d = 26$ dimensions [15]. The conformally dependent terms in the unmodified effective action can be written as [10]

$$S_{conf} = \frac{d-2}{8\pi\epsilon}\int_M d^2z\sqrt{g} - \frac{d}{8\sqrt{\pi\epsilon}}\int_{\partial M} ds + \frac{d}{8\pi}\int_{\partial M} ds\, k_g \quad . \qquad (18)$$

Of these, the geodesic curvature term vanishes with our boundary condition. The other terms are usually cancelled by appropriate counterterms. However, it is intriguing that the ψ^I fields contribute only to the remaining perimeter term, and do so with the appropriate sign to cancel it. However, the normalization of all the non-logarithmically divergent terms is ambiguous, and one cannot conclude from the functional integral that the required number of ψ^I fields is d, as we would like. It is clear that the value $D = d$, giving SO(8192) for the bosonic string and SO(32) for the superstring, is singled out. It leads to finiteness in the superstring, but

From (18), for g = – 1, d = 26 and k = 2

and

$$(*F_{(D-p)})^{\mu_1\mu_2\cdots\mu_D} = \frac{\epsilon^{\mu_1\mu_2\cdots\mu_D}}{2\sqrt{-g}} F_{\mu_{D-p+1}\mu_{D-p+2}\cdots\mu_D},$$

where $g \equiv det(\text{metric})$, which in our case is the flat metric, so $g = -1$.

We obtain:

$$S_{conf} = \frac{d-2}{8\pi\epsilon}\int_M d^2z\sqrt{g} - \frac{d}{8\sqrt{\pi\epsilon}}\int_{\partial M} ds + \frac{d}{8\pi}\int_{\partial M} ds\, k_g$$

(26-2)/(8Pi*1/12) integrate i dx – 26/(8*sqrt(Pi*1/12)) integrate (dx) + 26/(8Pi) integrate (2dx)

Input:

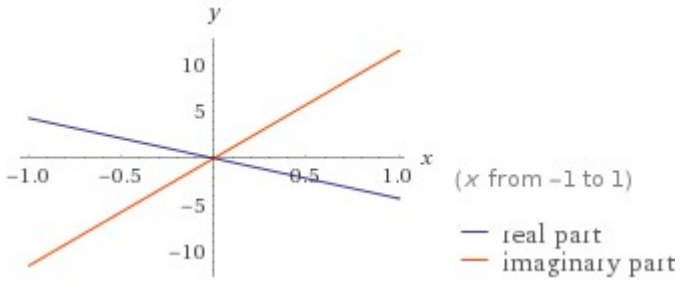
$$\frac{26-2}{8\pi\times\frac{1}{12}}\int i\,dx - \frac{26}{8\sqrt{\pi\times\frac{1}{12}}}\int 1\,dx + \frac{26}{8\pi}\int 2\,dx$$

i is the imaginary unit

Exact result:

$$-\frac{13}{2}\sqrt{\frac{3}{\pi}}\,x+\frac{\left(\frac{13}{2}+36\,i\right)x}{\pi}$$

Plot:



Alternate forms:

$$\frac{\left(-13\sqrt{3\,\pi}+(13+72\,i)\right)x}{2\,\pi}$$

$$-\frac{\left(13\sqrt{3\,\pi}+(-13-72\,i)\right)x}{2\,\pi}$$

$$\left(-\frac{13\sqrt{\frac{3}{\pi}}}{2}+\frac{\frac{13}{2}+36\,i}{\pi}\right)x$$

Indefinite integral:

$$\frac{26-2}{\frac{8\,\pi}{12}}\int i\,dx-\frac{26}{8\sqrt{\frac{\pi}{12}}}\int 1\,dx+\frac{26}{8\,\pi}\int 2\,dx=$$

$$\left(\text{constant}+-\frac{13}{2}\sqrt{\frac{3}{\pi}}\,x\right)+\left(\text{constant}+\frac{36\,i\,x}{\pi}\right)+\left(\text{constant}+\frac{13\,x}{2\,\pi}\right)$$

From

$$-\frac{13}{2}\sqrt{\frac{3}{\pi}}\,x+\frac{\left(\frac{13}{2}+36\,i\right)x}{\pi}$$

for x = 1, we obtain:

Input:

$$-\frac{13}{2}\sqrt{\frac{3}{\pi}}+\frac{\frac{13}{2}+36\,i}{\pi}$$

i is the imaginary unit

Exact result:

$$-\frac{13\sqrt{\frac{3}{\pi}}}{2}+\frac{\frac{13}{2}+36\,i}{\pi}$$

Decimal approximation:

$-4.282818394543319615627511173055824353038537511899026294 0\ldots +$
$11.459155902616464175359630962821034066481094493312864309\ldots\, i$

Polar coordinates:

$r \approx 12.2333$ (radius), $\theta \approx 110.493°$ (angle)

12.2333 (see below)

Alternate forms:

$$\frac{-13\sqrt{3\pi}+(13+72\,i)}{2\pi}$$

$$-\frac{13\sqrt{3\pi}+(-13-72\,i)}{2\pi}$$

Series representations:

$$\frac{1}{2}\sqrt{\frac{3}{\pi}}\,(-13)+\frac{\frac{13}{2}+36\,i}{\pi}=\frac{13}{2\pi}+\frac{36\,i}{\pi}-\frac{13}{2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{3}{\pi}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{1}{2}\sqrt{\frac{3}{\pi}}\,(-13)+\frac{\frac{13}{2}+36\,i}{\pi}=\frac{13}{2\pi}+\frac{36\,i}{\pi}-\frac{13}{2}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{3}{\pi}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \le 0))$

$$\frac{1}{2}\sqrt{\frac{3}{\pi}}\,(-13)+\frac{\frac{13}{2}+36\,i}{\pi}=\frac{13}{2\,\pi}+\frac{36\,i}{\pi}+\frac{13\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-1+\frac{3}{\pi}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\sqrt{\pi}}$$

From ln(196883), we obtain:

Input:

$\log(196\,883)$

$\log(x)$ is the natural logarithm

Decimal approximation:

12.190364922657093458766455576004905429718973818061244670 83...

12.190364922… result equal to the black hole entropy 12.1904 and very near to the previous value 12.2333

Property:

$\log(196\,883)$ is a transcendental number

Alternate form:

$\log(47)+\log(59)+\log(71)$

Alternative representations:

$\log(196\,883)=\log_e(196\,883)$

$\log(196\,883)=\log(a)\log_a(196\,883)$

$\log(196\,883)=-\mathrm{Li}_1(-196\,882)$

Integral representations:

$$\log(196\,883)=\int_1^{196883}\frac{1}{t}\,dt$$

$$\log(196\,883)=-\frac{i}{2\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{196\,882^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for}\quad -1<\gamma<0$$

From:

Integrable Scalar Cosmologies I. Foundations and links with String Theory
P. Fré , A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013

We have that:

It is thus convenient to define the two fields

$$\Phi_t = \sqrt{\frac{d-2}{2(d-1)}}\left(\frac{3}{2}\,\phi - \frac{10-d}{d-2}\,\sigma\right), \tag{6.7}$$

$$\Phi_s = \sqrt{\frac{10-d}{2(d-1)}}\left(\frac{1}{2}\,\phi + 3\,\sigma\right), \tag{6.8}$$

$$\varphi = \sqrt{\frac{d-1}{2(d-2)}}\,\Phi_t\,,$$

and thus in the striking fact that $\gamma_9 = 1$ *for all dimensions* $4 \leq d \leq 10$.

$$V = V_0\left(e^{\sqrt{3}\,\gamma_9\,\Phi_t} + e^{\sqrt{3}\,\gamma_\Lambda\,\Phi_t}\right), \tag{6.18}$$

where

$$\gamma_9 = \sqrt{\frac{d^2 - 14\,d + 184}{24\,(d-4)}}\,, \quad \gamma_\Lambda = -\frac{10}{3}\,\frac{(d-4)(d-10)}{\sqrt{2(d^2 - 14\,d + 184)}} \tag{6.19}$$

For: $\phi = 0.9991104684$; $\sigma = \pm 1$ and d = 6

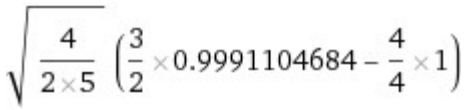
From

$$\Phi_t = \sqrt{\frac{d-2}{2(d-1)}}\left(\frac{3}{2}\,\phi - \frac{10-d}{d-2}\,\sigma\right)$$

we obtain:

sqrt(4/(2*5)) (3/2*0.9991104684 - 4/4*1)

Input interpretation:



Result:

0.3153838822...

0.3153838822…

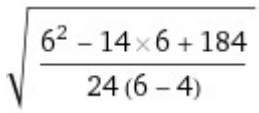
From

$$\gamma_9 = \sqrt{\frac{d^2 - 14\,d + 184}{24\,(d - 4)}}$$

For d = 6:

sqrt[(6^2-14*6+184)/(24(6-4))]

Input:



Result:



Decimal approximation:

1.68325082306034632556056431951160011843398316074660215697 5...

1.683250823….

Alternate form:



From

$$\gamma_\Lambda = -\frac{10}{3}\,\frac{(d-4)(d-10)}{\sqrt{2(d^2-14\,d+184)}}$$

we obtain:

-10/3* [(((6-4)(6-10))) / ((sqrt(2(6^2-14*6+184))))]

Input:

$$-\frac{10}{3}\times\frac{(6-4)\,(6-10)}{\sqrt{2\,(6^2-14\times 6+184)}}$$

Result:

$$\frac{20}{3\sqrt{17}}$$

Decimal approximation:

1.616904166908886490126043080774147852998901657009262915450...

1.6169041669…. result that is a good approximation to the value of the golden ratio 1.618033988749...

Alternate form:

$$\frac{20\sqrt{17}}{51}$$

From

$$V = V_0\left(e^{\sqrt{3}\,\gamma_9\,\Phi_t} + e^{\sqrt{3}\,\gamma_\Lambda\,\Phi_t}\right)$$

We obtain, for $V_0 = 1$:

(((exp(sqrt3*1.683250823*0.3153838822)+exp(sqrt3*1.6169041669*0.3153838822)
)))

Input interpretation:

$$\exp\left(\sqrt{3}\times 1.683250823\times 0.3153838822\right)+\exp\left(\sqrt{3}\times 1.6169041669\times 0.3153838822\right)$$

Result:

4.926772915...

4.926772915…

Series representations:

$$\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)=$$
$$\exp\left(0.509946\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)+\exp\left(0.53087\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)=$$
$$\exp\left(0.509946\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\exp\left(0.53087\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)=$$
$$\exp\left(\frac{0.254973\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)+$$
$$\exp\left(\frac{0.265435\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)$$

From which:

233(((exp(sqrt3*1.683250823*0.3153838822)+exp(sqrt3*1.6169041669*0.3153838822))))-123-0.9568666373

Input interpretation:

$$233\left(\exp\left(\sqrt{3}\times 1.683250823\times 0.3153838822\right)+\exp\left(\sqrt{3}\times 1.6169041669\times 0.3153838822\right)\right)-123-0.9568666373$$

Result:

1023.981223...

1023.981223… ≈ 1024 = 64*16

Series representations:

$$233\left(\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)\right)-123-0.956867=233\left(-0.532004+\exp\left(0.509946\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)+\exp\left(0.53087\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$233\left(\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)\right)-123-0.956867=233\left(-0.532004+\exp\left(0.509946\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\exp\left(0.53087\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$233\left(\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)\right)-123-0.956867=233\left(-0.532004+\exp\left(\frac{0.254973\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)+\exp\left(\frac{0.265435\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)\right)$$

and again:

Input interpretation:

$$8\left(233\left(\exp\left(\sqrt{3}\times 1.683250823\times 0.3153838822\right)+\exp\left(\sqrt{3}\times 1.6169041669\times 0.3153838822\right)\right)-123-0.9568666373\right)$$

Result:

8191.849781...

8191.849781… ≈ 8192 = 2*64^2

Series representations:

$$8\left(233\left(\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)\right)-123-0.956867\right)=1864\left(-0.532004+\exp\left(0.509946\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)+\exp\left(0.53087\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$8\left(233\left(\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)\right)-123-0.956867\right)=1864\left(-0.532004+\exp\left(0.509946\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\exp\left(0.53087\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$8\left(233\left(\exp\left(\sqrt{3}\ 1.68325\times 0.315384\right)+\exp\left(\sqrt{3}\ 1.61690416690000\times 0.315384\right)\right)-123-0.956867\right)=$$
$$1864\left(-0.532004+\exp\left(\frac{0.254973\ \sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\ 2^{-s}\ \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)+\exp\left(\frac{0.265435\ \sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\ 2^{-s}\ \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)\right)$$

Now, we have that:

To begin with, one can notice that independent shifts of Φ_s and Φ_t can turn the general scalar potential resulting from $D9$ and $D3$ branes into the universal form

$$V = V_0\left(e^{\sqrt{3}\,\Phi_t} + e^{\sqrt{3}\left(\frac{1}{2}\,\Phi_t - \frac{\sqrt{3}}{2}\,\Phi_s\right)}\right), \qquad (6.17)$$

for

$$\Phi_s = \sqrt{\frac{10-d}{2(d-1)}}\left(\frac{1}{2}\,\phi + 3\sigma\right)$$

sqrt[(10-6)/(2(6-1))] * (1/2*0.9991104684+3)

Input interpretation:

$$\sqrt{\frac{10-6}{2\,(6-1)}}\left(\frac{1}{2}\times 0.9991104684+3\right)$$

Result:

2.2133130675...

2.2133130675…

From

$$V = V_0 \left(e^{\sqrt{3}\,\Phi_t} + e^{\sqrt{3}\left(\frac{1}{2}\,\Phi_t - \frac{\sqrt{3}}{2}\,\Phi_s\right)} \right)$$

we obtain:

[(exp(sqrt3*0.3153838822) + exp(sqrt3(1/2*0.3153838822-(sqrt3)/2*2.2133130675))]

Input interpretation:

$$\exp\left(\sqrt{3} \times 0.3153838822\right) + \exp\left(\sqrt{3}\left(\frac{1}{2} \times 0.3153838822 + \frac{\sqrt{3}}{2} \times (-2.2133130675)\right)\right)$$

Result:

1.774293182...

1.774293182…

Series representations:

$$\exp\left(\sqrt{3}\ 0.315384\right) + \exp\left(\sqrt{3}\left(\frac{0.315384}{2} - \frac{2.21331306750000\sqrt{3}}{2}\right)\right) =$$
$$\exp\left(0.315384\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right) +$$
$$\exp\left(\sqrt{2}\left(\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)\right)\left(0.157692 - 1.10665653375000\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\exp\left(\sqrt{3}\ 0.315384\right) + \exp\left(\sqrt{3}\left(\frac{0.315384}{2} - \frac{2.21331306750000\sqrt{3}}{2}\right)\right) =$$
$$\exp\left(0.315384\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) +$$
$$\exp\left(\sqrt{2}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)\right)\left(0.157692 - 1.10665653375000\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\exp\left(\sqrt{3}\ 0.315384\right)+\exp\left(\sqrt{3}\left(\frac{0.315384}{2}-\frac{2.21331306750000\ \sqrt{3}}{2}\right)\right)=$$
$$\exp\left(0.315384\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$
$$\exp\left(\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right.$$
$$\left.\left(0.157692-1.10665653375000\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

Thence, we have:

$$V = V_0\left(e^{\sqrt{3}\ \gamma_9\ \Phi_t}+e^{\sqrt{3}\ \gamma_\Lambda\ \Phi_t}\right)$$

= 4.926772915…

$$V = V_0\left(e^{\sqrt{3}\ \Phi_t}+e^{\sqrt{3}\left(\frac{1}{2}\ \Phi_t-\frac{\sqrt{3}}{2}\ \Phi_s\right)}\right)$$

= 1.774293182…

From which:

1+1/ integrate (((4.926772915 -[(exp(sqrt3*0.3153838822)) + exp(sqrt3(1/2*0.3153838822-(sqrt3)/2*2.2133130675))]))x, x = 0..1

Input interpretation:

$$1+1\Big/\left(\int_0^1\left(4.926772915-\left(\exp\left(\sqrt{3}\times 0.3153838822\right)+\exp\left(\sqrt{3}\left(\frac{1}{2}\times 0.3153838822+\frac{\sqrt{3}}{2}\times(-2.2133130675)\right)\right)\right)x\,dx\right)$$

Result:

1.63442

$1.63442 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Computation result:

$$1+1\Big/\left(\int_0^1\left(4.926772915-\left(\exp\left(\sqrt{3}\ 0.3153838822\right)+\exp\left(\sqrt{3}\left(\frac{0.3153838822}{2}-\frac{\sqrt{3}\ 2.2133130675}{2}\right)\right)\right)\right)x\,dx\right)=1.63442$$

Observations

Note that:

$$g_{22}=\sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}}-24+276e^{-\pi\sqrt{22}}-\cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}}+\cdots, \end{aligned}$$

so that

$$64(g_{22}^{24}+g_{22}^{-24})=e^{\pi\sqrt{22}}-24+4372e^{-\pi\sqrt{22}}+\cdots=64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}}=2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}}+\cdots$$

And

$$64(g_{22}^{24}+g_{22}^{-24})=e^{\pi\sqrt{22}}-24+4372e^{-\pi\sqrt{22}}+\cdots=64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

References

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

Vietnam Journal of Mathematics **36**:1(2008) 103–124 - **On Some New Explicit Evaluations of Class Invariants** - *M. S. Mahadeva Naika and K. Sushan Bairy*

Manuscript Book II of *Srinivasa Ramanujan*

UCB-PTH-86/27 - LBL-22076 - ROM2F-86/015
Group Theory from "Quarks" at the Ends of Strings
Neil Marcus and Augusto Sagnotti

Integrable Scalar Cosmologies I. Foundations and links with String Theory
P. Fré , A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013