# Experimental Self-Plotting of Trajectories

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A ball is rolled repeatedly down an incline and launched in free flight in a vertical plane. It strikes a vertical drawing board set perpendicular to the plane of flight and records each point of striking by means of carbon paper on white. The board is moved farther and farther from point of launching of ball and at same time laterally by equal amounts. Thus a curve which lies in $xy$ plane is made to record points in $yz$ plane. In a short time, good curve of trajectory is obtained and then compared with theoretical curve fitted to origin, highest point, and range. Unexpectedly, the experimental is not sensitive to $g$, so would lead to exactly the same curve on moon or Mars!

AN inexpensive but satisfactory laboratory experiment for plotting the trajectory of a steel ball at various points along its flight may be prepared in a few minutes.[1] It makes use of a disarmingly simple way of causing the ball to record its own $x$ and $y$ coordinates at any point along its path. The assumption is made that the ball may be launched repeatedly from the same point with the same vector velocity $V_0$ (same speed and angle of elevation) by the expedient of rolling the ball time after time down the same incline from the same starting point. Experimental evidence for slight deviations from this basic assumption appears in the results and shows how random errors enter into all gunnery. More expensive means of firing the ball might reduce the scatter of experimental points but not eliminate it. The beauty of the method lies in its utter simplicity, the speed with which satisfactory records are obtained, and the appeal to clear reasoning.

The student learns early in his study of physics that the path of a projectile in a uniform gravitational field free from significant drag is a parabola. He may be convinced of the reasoning behind this conclusion, but there are all too few experiments designed to let him test it. A search of some 50 laboratory manuals shows only two or three experiments that attempt to provide any check for the shape of the path. A near approach is that of rolling a ball on an inclined drawing board, as in Experiment M-96,[2] an experiment which offers a simple way to "dilute gravity." In contrast with the ball rolling on the inclined board, this experiment described herein uses a freely flying ball under the full acceleration due to gravity.

The launching equipment consists of an inclined, grooved stick $SO$ with a curve at the bottom by which the ball may be projected repeatedly at some desired angle $\theta$ after it has attained a desired speed $V_0$ (Fig. 1). The curved incline looks much like a hockey stick which, in fact, it might actually be. The long shank provides a track for acceleration of the ball $B$ from a fixed (but adjustable) stop $S$ as it rolls down. The curve at the bottom permits launching the ball into free flight from the end point $O$ which may be considered as the origin of a system of coordinates with the trajectory described in the vertical $xy$ plane. The last portion of the track is straight so that the angle of elevation $\theta$ may be adjusted and measured. The ball moves in the $xy$ plane with only minor deviations due to wobble of the ball in the groove, vibration of the stick, etc. Any $x$ coordinate of the path and its corresponding $y$ coordinate are self-recorded

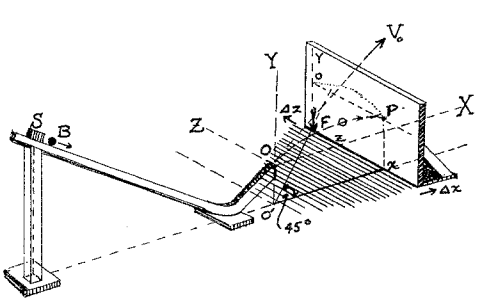



FIG. 1. Apparatus for point-by-point recording of trajectory.

[1] One of several experiments presented at the Annual Meeting of AAPT, New York, January 28, 1960.
[2] *Demonstrations Experiments in Physics* (McGraw-Hill Book Company, Inc., New York, 1938).



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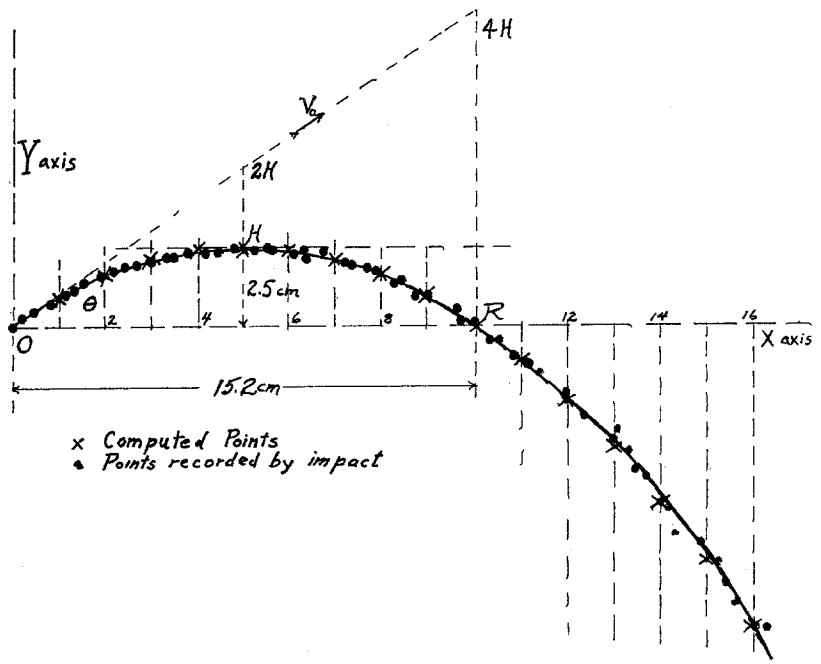



FIG. 2. Sample run, unretouched, with parabola fitted to *O*, *H*, and *R*.

when the ball strikes a drawing board set in the vertical *xz* plane *perpendicular* to the plane of flight. A piece of carbon paper over a sheet of white paper is mounted on the board to record a spot, such as *P*, for each impact or trial.

The all-important expedient used in recording the trajectory point by point is as follows: As the board is moved away from the point *O* some distance $\Delta x$, it is also moved laterally an equal distance $\Delta z$. This is easily accomplished by keeping the board always parallel to the *z* axis with its edge or some fiducial point *F* always on a line *O'F* which makes angles of 45° with the *x* and *z* axes. It is evident, therefore, that the distances *x* and *z* are always equal, hence the *zy* curve made by successive spots on the graph is identical with an *xy* curve. For ease in setting the board, it is helpful to lay off this 45° line and a series of parallel guidelines perpendicular to the *x* axis. A little time spent in doing this in advance will pay dividends in the quality of results obtained.

In a very few minutes, a student may make a rather complete record of shots along the trajectory, say, 10 shots each for each of 15 *x* positions of the board, changing the board only when a sufficient number of single shots have been made. Or he may choose to move the board a small $\Delta x$ and equal $\Delta z$ between shots, as was done in the record shown in Fig. 2. He will thus obtain a curve well populated with points through which he then draws an empirical curve as the "best fit" for the data obtained. Details for finding the theoretical curve are given in the following. The former method places emphasis on the *scatter* of the shots for the same *x* distance,caused by minor variations in the initial velocity and angle of propulsion. The scatter of points, of course, gets greater as the distance of flight from point *O* to target increases, as any hunter knows. The student may seek ways to reduce this scatter. For example, a metal *V* groove proves to be better than the rounded wooden groove which allows more lateral wobble. Security of the incline and of the vertical board are, of course, important.

The experimenter may easily determine in advance whether his trajectory will stay "in bounds" on the paper prepared to receive the plot. He simply sets his drawing board *parallel* to the motion, with the *y* axis at *O*, and he watches to see if the flight of the ball lies satisfactorily within the scope of his paper. This simple operation, with appropriate adjustment of the stop *S* and the angle of elevation $\theta$ may be done without making spurious marks on the record, inasmuch as the flying ball does not touch the board when it is so oriented.

A sample run of shots recorded in less than 15 minutes on an ordinary sheet of typewriter

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paper is shown unretouched in Fig. 2, together with a parabola drawn as the "best fit," and a computed set of points (×) for a curve made to fit at $O$, the initial point, $H$, the top of the trajectory, and $R$, the range or value of $x$ when $y$ is again zero. The student may derive a useful (but unfamiliar) equation by which to do the necessary plotting of theoretical points. Starting with the familiar parametric equations of projectile motion,

$$x = V_0 t \cos\theta, \quad \text{and} \quad y = V_0 t \sin\theta - \tfrac{1}{2} g t^2,$$

we are led to the equation for the range $R$,

$$R = (2 V_0 \sin\theta \cos\theta)/g.$$

When $t$ and $V_0$ are eliminated from these three equations, $g$ conveniently drops out, too. We then have the purely geometrical equation of a parabola,

$$y = x[1 - x/R] \tan\theta. \qquad (1)$$

This is an equation highly suitable for quick computation of points on the path when $\theta$ and $R$ are known. Thus, when $x = R$, we find $\tan\theta = 4H/R$, where $H$ is the maximum height of the path above the point of launching. The value of $\theta$ thus obtained may be compared with $\theta$ as measured from the position of the launching stick or as measured by drawing a tangent to the trajectory at $O$. The path between $O$ and $R$ may then be divided into, say, 10 equal increments of $x$, extrapolating with additional equal parts beyond $R$ in case the trajectory falls below its initial level, as it does in Fig. 2. The $y$ value for each $x$ value may then be calculated and graphed from the measured values of $R$ and $\theta$.

In an age when conditions on other planets seem to be of interest, including such items as the surface gravity to be expected, it is arresting to find a projectile experiment which is *not* sensitive to space travel. If this experiment were performed on the moon or Mars, with the ball rolling down the same incline over the same course, it would produce the *same* curve, not just a similar curve. The reason for this is not hard to find, but the fact itself may well stir the experimenter's imagination. If this is first posed to him as a "thought experiment" and if he correctly diagnoses this interesting conclusion, he probably has that valuable talent known as physical intuition.

Finally, this experiment as outlined does not lead directly to a determination of $g$ or of $V_0$, as there is no explicit measurement of the time of flight of the ball. One may, of course, assume the local value of $g$ and work out the initial velocity and the time of flight. For example, in the actual record shown in Fig. 2, $\theta$ was found to be 34°, $V_0$ was 127 cm/sec, and the time of flight from $O$ to $R$ was 0.14 sec. Some 58 individual points were recorded in less than 15 min, most of the time being spent in resetting the board between trials. Increments of $x$ and $z$ about 0.4 cm were used. After the 58 runs had been made, the carbon paper was removed and the solid curve was drawn. Thereafter, $OR$ was laid off into 10 equal parts, and six more parts were found to the right of $R$. The value of $y$ was computed for each of these 16 points, and the ×'s marking them were found to lie in all cases less than the diameter of the impact marks from the experimental curve. The whole operation, including all graphing and computations, required less than one hour.

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