Why Your Friends Have More Friends than You Do[1]

Scott L. Feld
State University of New York at Stony Brook

It is reasonable to suppose that individuals use the number of friends that their friends have as one basis for determining whether they, themselves, have an adequate number of friends. This article shows that, if individuals compare themselves with their friends, it is likely that most of them will feel relatively inadequate. Data on friendship drawn from James Coleman's (1961) classic study *The Adolescent Society* are used to illustrate the phenomenon that most people have fewer friends than their friends have. The logic underlying the phenomenon is mathematically explored, showing that the mean number of friends of friends is always greater than the mean number of friends of individuals. Further analysis shows that the proportion of individuals who have fewer friends than the mean number of friends their own friends have is affected by the exact arrangement of friendships in a social network. This disproportionate experiencing of friends with many friends is related to a set of abstractly similar "class size paradoxes" that includes such diverse phenomena as the tendencies for college students to experience the mean class size as larger than it actually is and for people to experience beaches and parks as more crowded than they usually are.

Friendship is not only a source of satisfaction and security; it is also a way that individuals evaluate themselves and others. People expect themselves and others to have friends and wonder about the normality of those individuals who appear to have few or no friends. There has been little study of how people determine what is an adequate number of friends, but it is reasonable to suppose that individuals use the number of friends that their friends have as one basis of comparison. This article shows that, if individuals make this type of comparison, it is likely that

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most of them will feel relatively inadequate. I use data on friendship drawn from James Coleman's (1961) classic study *The Adolescent Society* to illustrate the phenomenon that most people have fewer friends than their friends have.

I will explore mathematically the logic underlying the phenomenon, showing that the mean number of friends of friends is always greater than the mean number of friends of individuals. Further analysis shows that the proportion of individuals who have fewer friends than the mean number of friends their own friends have is affected by the exact arrangement of friendships in a social network. While it is not a mathematical necessity that each individual will have fewer friends than the mean of her or his own friends, it is likely that most people will find themselves in this situation.

The basic logic can be described simply. If there are some people with many friendship ties and others with few, those with many ties show up disproportionately in sets of friends. For example, those with 40 friends show up in each of 40 individual friendship networks and thus can make 40 people feel relatively deprived, while those with only one friend show up in only one friendship network and can make only that one person feel relatively advantaged. Thus, it is inevitable that individual friendship networks disproportionately include those with the most friends.

EMPIRICAL EXAMPLES

Friendship is usually thought to be a symmetric relationship, as indicated by the common phrase, "They are friends." One way to operationalize friendship is to consider a friendship to be one that is so regarded by both of the individuals. In *The Adolescent Society,* Coleman (1961) collected data on friendships among the students in 12 high schools. Individuals were asked to name their friends, and pairs of individuals who named one another were given particular attention. It is these "friendships" that will be used as examples.

To illustrate the phenomenon under study here, consider the set of relationships depicted in figure 1, found among eight girls in "Marketville," one of the high schools included in the study. The names are fictitious.

In this example, Betty's only friend, Sue, has more friends than Betty has; Jane's two friends, Dale and Alice, average more friends than Jane has; Dale's three friends, Sue, Alice, and Jane, average more friends than Dale; and so forth. Of the eight girls, five (Betty, Jane, Pam, Dale, and Tina) have fewer friends than the average among their friends, while only two (Sue and Alice) have more friends than the average among their friends; one (Carol) has as many as the average among her friends. Table

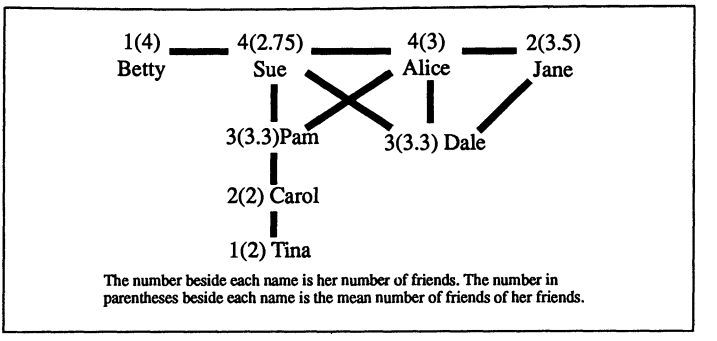


FIG. 1.—Friendships among eight girls at Marketville High School

1 shows each girl's number of friends in the first column and, in the third column, the mean number of friends her friends have. Twice as many (5:2) have fewer than average as have more than the average among their friends.

The complete network of all of the girls in Marketville shows the same pattern. Figure 2 reproduces the entire sociogram of mutual choices. Of the 146 girls who have any mutual friends, 80 have fewer friends than the mean among their friends while 41 have more; 25 have the same as the mean among their friends. Thus, nearly twice as many have fewer as have more than the mean among their friends. The same pattern

TABLE 1

A SUMMARY OF THE NUMBERS OF FRIENDS AND THE MEAN NUMBERS OF FRIENDS OF FRIENDS FOR EACH OF THE GIRLS IN FIGURE 1

	Number of Friends (x_i)	Total Number of Friends of Her Friends (Σx_j)	Mean Number of Friends of Her Friends ($\Sigma x_j/x_i$)
Betty	1	4	4
Sue	4	11	2.75
Alice	4	12	3
Jane	2	7	3.5
Pam	3	10	3.3
Dale	3	10	3.3
Carol	2	4	2
Tina	1	2	2
Total	20	60	23.92
Mean	2.5*	3†	2.99*

* For eight girls.
† For 20 friends.

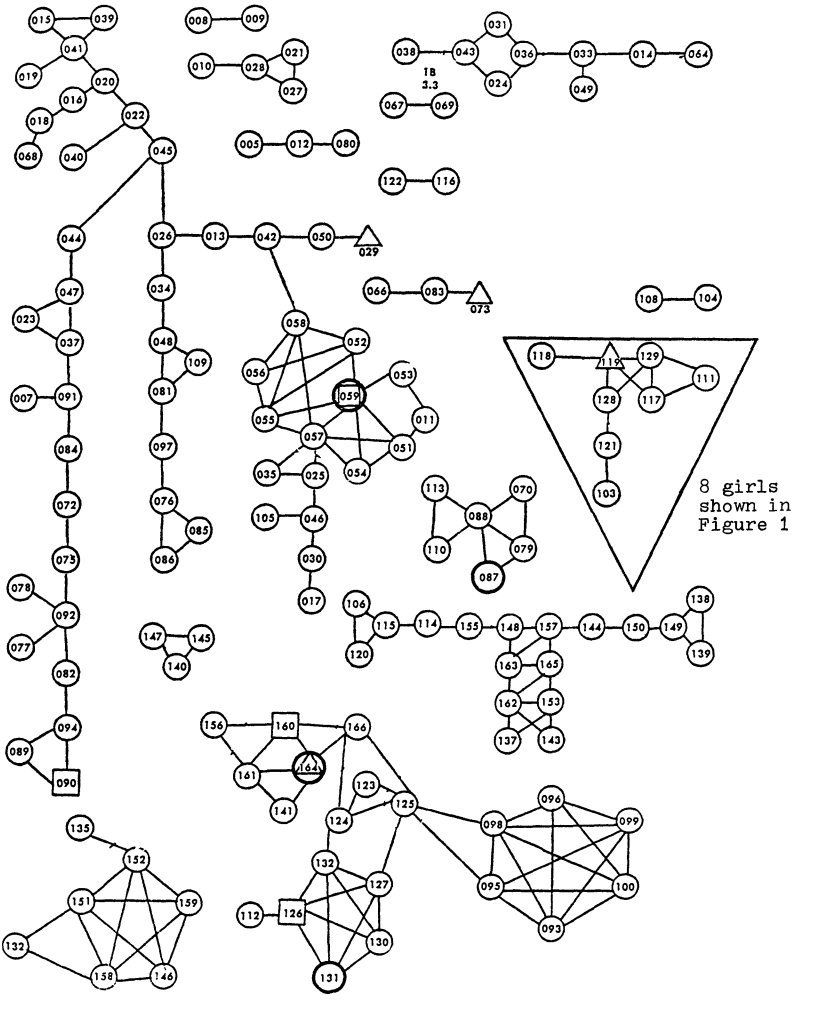


FIG. 2.—Network of reciprocated friendships among Marketville girls; the triangle at right indicates friendships illustrated in fig. 1. (From *The Adolescent Society* by James S. Coleman. © 1961 by the Free Press, a division of Macmillan, Inc. Used with permission.)

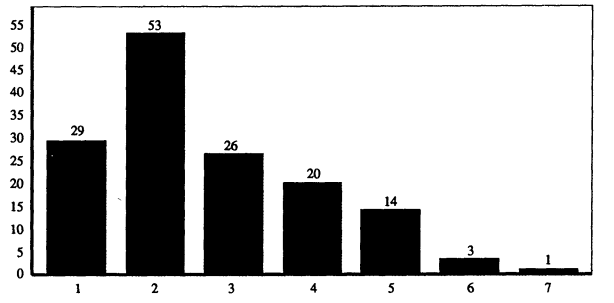
appears among the boys in Marketville and among the girls and boys of the other high schools reported in the Coleman study.[2]

TWO DISTRIBUTIONS: FRIENDS OF INDIVIDUALS AND FRIENDS OF FRIENDS

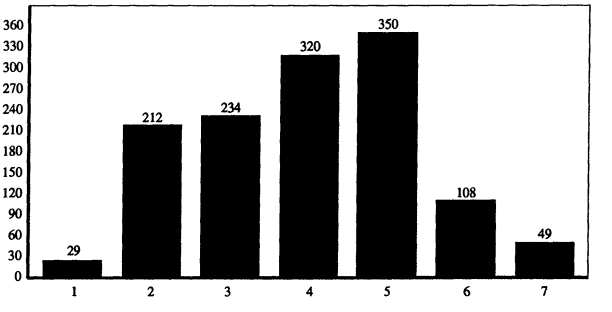
The phenomenon of people finding that their friends have more friends than they do can be partially understood by recognizing the difference between the distribution of numbers of friends of individuals and the distribution of the numbers of friends of friends. The distribution of friends of individuals is just the usual distribution of numbers of friends that we would usually examine, but the distribution of friends of friends includes some of the same individuals over and over. This complexity may become more understandable in reconsidering the case of the eight girls in figure 1. In the situation described in figure 1 and table 1, the distribution of friends of individuals would be for the eight girls having a total of 20 friends, with a mean of 2.5 friends per individual. However, the distribution of friends of friends includes more cases. For example, Jane's friends, Dale and Alice, contribute their numbers of friends to this distribution, and Sue's friends, Dale, Alice, Pam, and Betty, also contribute their numbers to this distribution. Note that Dale and Alice contribute to this distribution more than once; in fact, each friend contributes to the distribution of the numbers of friends of friends as many times as she has friends. There are a total of 20 friends (obviously counting some of the eight girls more than once) having a total of 60 friends, with a mean of 3.0 friends per friend. When each individual compares him- or herself with the average number of friends of his or her friends, the comparison is with a sample from the numbers of friends of friends, which is a different distribution from that of numbers of friends among individuals.

For the entire set of girls in Marketville (from fig. 2), the distribution of friends among individuals is skewed to the right, as shown in figure 3*a*, which indicates that a few individuals have many friends; this appears to

[2] Nearly identical results are obtained when individuals compare themselves with the "median" among their friends. The phenomenon that most individuals would feel relatively deprived if they compared themselves with the mean of their friends is essentially replicated if they compare themselves with the median (which is equivalent to determining whether a majority of their friends have more friends than they have). This can be seen in the example of fig. 1, where exactly the same girls who are below (above) the mean of their friends are also below (above) the median of their friends. Although comparison with the majority of one's friends may be as important as comparing with the mean, the present discussion is limited to discussion of the mean to simplify the discussion—also, the mathematics concerning the mean is somewhat more straightforward than that of the median.



a) The mean is 2.7.



b) The mean is 3.4

FIG. 3.—(*a*) Distribution of numbers of friends for Marketville girls; (*b*) distribution of number of friends' friends for Marketville girls.

be a typical distribution of numbers of friends among individuals. The distribution of friends among friends is a weighted version of the original distribution, weighting those with many friends especially heavily; this weighting counteracts the original skew, as shown in figure 3*b*. The important characteristic of the distribution of numbers of friends of friends is that it inevitably has a higher mean than the distribution of friends of individuals.

In this situation, if individuals compared their numbers of friends with the mean number of friends of their own friends, and their friends were a representative sample of friends (mean number of friends of friends was 3.4), then 74% of the individuals would find themselves to be relatively deprived.[3]

[3] In a discussion of this phenomenon, Guillermo Owen and Bernard Grofman noted that, even if individuals are accurately informed of the distribution of friends among individuals, a majority of the individuals will generally be below the mean, because the median is below the mean in distributions that are skewed to the right. In this

VARIANCE AFFECTS THE MEAN NUMBER OF FRIENDS OF FRIENDS

In general, there is a simple relationship between the original distribution of friends among individuals and the distribution of friends of friends. If the original distribution has n individuals with x_i ties apiece, the mean can be determined as $\Sigma x_i/n$. However, the distribution of friends has Σx_i cases (for all of the friends) and they have a total of Σx_i^2 friends, since each individual is counted as many times as she or he has friends, x_i, and that individual has Σx_i friends. Thus, the mean number of friends among the friends is $(\Sigma x_i^2)/(\Sigma x_i)$. This can be shown to be a simple function of the mean and variance in the original distribution of ties.[4] That is: mean number of friends of friends = $(\Sigma x_i^2)/(\Sigma x_i)$ = mean(x) + variance(x)/mean(x).

The expression above shows that the mean among friends is always at least as great as the mean among individuals, and the mean among friends increases with the variance among individuals, with a given mean among individuals. The mean among friends is much greater than the mean among individuals if there is much variation in the population.

The Arrangement of Individual Friends

While the mean of the distribution of number of friends of friends is completely determined by the distribution of friends of individuals, the particular samples of friends of friends may vary among individuals. If each individual's friends are approximately representative of all friends (in terms of their numbers of friends), then individuals comparing themselves with their own friends are essentially comparing themselves with the overall distribution among friends. The calculations in the previous section indicate that the mean number of friends of friends is higher

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case, 56% of the individuals were below the mean number of friends of individuals; the higher mean of numbers of friends' friends increases the proportion of individuals who experience themselves as below the mean of their friends' friends—74% of the individuals are below that mean.

[4] The mean number of friends' friends is just the total number of friends' friends divided by the number of friends. To determine the total number of friends' friends, consider that each individual is a friend x_i times and has x_i friends, so that individual contributes x_i friends' friends x_i times, a total of x_i^2 friends' friends. Thus, the total number of friends' friends (the numerator) is simply this quantity summed over all individuals, Σx_i^2. The total number of friends (the denominator) is simply the number of friends of each individual, x_i, summed over all individuals, Σx_i. Thus, the mean number of friends' friends is just $(\Sigma x_i^2)/(\Sigma x_i)$. Some relatively simple algebra shows that this can be expressed as a function of the mean $(\Sigma x_i/n)$ and the variance $(\Sigma x_i^2/n - \text{mean}^2)$. Thus, the mean number of friends' friends is:

$$(\Sigma x_i^2)/(\Sigma x_i) = \text{mean}(x) + \text{variance}(x)/\text{mean}(x).$$

than the mean number of friends of individuals; consequently, a higher proportion of individuals will be below the mean number of friends of friends than below the mean number of friends of individuals (the more appropriate comparison).[5] However, individuals may have unrepresentative sets of friends, and the following two sections consider some of the possible consequences of such unrepresentativeness.

First, it is important to recognize yet another distribution and another mean. Refer again to the situation in figure 1 as summarized in table 1. The eight Marketville girls have a total of 20 friendships, with a mean of 2.5. The friends have a total of 60 friends, with a mean of 3.0. At the same time, each girl has a mean among her friends, and the means for all the girls have a mean of 2.99. This last mean differs from the mean number of friends of friends (only slightly in this case) because the two-step averaging process weights each of the friends differently: each of the means of Sue's four friends are averaged, and that average counts equally with Betty's average based on her only friend. Thus, the particular arrangement of the friendships affects this last average.

Correlations between Individuals and Their Friends

There may be situations in which individuals are disproportionately friends with others with similar friendship volumes (similars attract) and other situations in which people are disproportionately friends with those with different friendship volumes (e.g., where some individuals "collect" several otherwise-isolated individuals as their friends). The implications of these types of correlations can be seen in some hypothetical examples. Figure 4 shows four possible ways that the same distribution of friendship volumes can be arranged in networks; this distribution includes four individuals having three friends each and 12 individuals having one friend each.

The possibilities are arranged from the one with a perfect positive correlation between individuals and their friends (fig. 4*a*) to one with a

[5] Note that, even though the mean number of friends of an individual's friends is unrepresentatively high as an estimate of the mean number of friends of individuals, an individual who understands the nature of this problem can use the information derived from his or her friends to estimate the mean number of friends for all people. The information from friends should be weighted to take account of the frequency with which that particular individual is included in various people's experience, so a friend with x_j friends should be weighted by $1/x_j$ so that friend is not overcounted. Thus, the individual's appropriate estimate of the mean number of friends of individuals is given by $\Sigma x_j(1/x_j)/\Sigma(1/x_j) = x_i/\Sigma(1/x_j)$. This more realistic estimate is always less than the mean number of friends of that individual's friends and so will make the individual feel appropriately better relative to others.

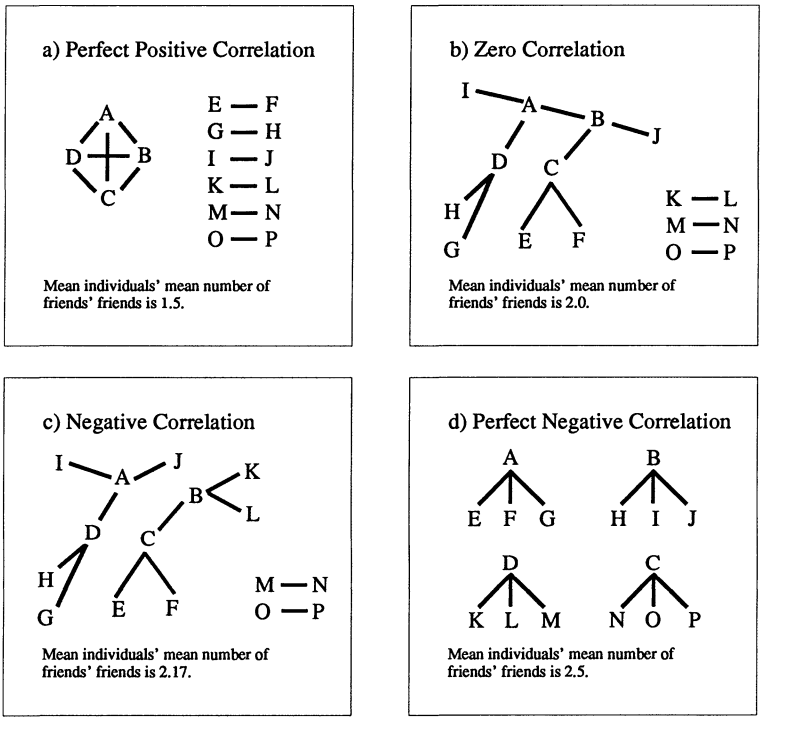


FIG. 4.—Four arrangements of the same distribution of individual numbers of friends (A, B, C, and D have three friends each, and E, F, G, H, I, J, K, L, M, N, O, and P have one friend each).

perfect negative correlation (fig. 4*d*). It should be apparent in all cases that the mean among the 16 individuals is 1.5 friends each and the mean among the 24 friends is 2.0 friends each. However, the mean individual mean number of friends' friends increases from figure 4*a* to figure 4*d;* the more negative the correlation between the individuals and their friends, the greater the mean individual mean number of friends' friends and the greater the proportion of individuals below that mean. For figures 4*b*, 4*c*, and 4*d*, where some individuals differ from their friends, the proportions of those who have fewer friends than the mean of their friends are 60%, 67%, and 75%, respectively.

The mean individual mean number of friends' friends can be calculated as follows:[6] mean individual mean number of friends' friends = $\Sigma(x_i/x_j)/n$, for all i and j who are friends with one another.

[6] It is apparent that for each individual, i, with friends designated by j's, the total number of friends' friends is Σx_j and the mean is $(\Sigma x_j)/x_i = \Sigma(x_j/x_i)$. The total of these

For a given set of friendship volumes, this expression is minimized when $x_i = x_j$ in all cases; in that situation, the mean individual mean number of friends' friends is just the mean number of friends of individuals. On the other hand, the maximum value is achieved when individuals with the fewest friends are friends of those with the most friends—in that case, the mean can be considerably larger than the mean number of friends' friends. It may be as high as:[7] mean(x) + 2 × [variance(x)/mean (x)].

Further Implications of the Exact Arrangement of Friendships

Even with a given correlation between individuals and their friends, there can be variation in the distribution of friendships. The exact arrangement of friendship among individuals will determine the number of individuals with more friends than the mean of their friends (e.g., there could be a few individuals whose friends have many more friends

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means is just the sum of these expressions over all i's and their corresponding j's, and the mean of these means is just this total divided by the number of individuals, n. That is, $\Sigma\Sigma(x_i/x_j)/n$.This can be illustrated with three individuals, A, B, and C, and two ties, A—B—C, and four friendships, AB, BA, BC, and CB. Individual A has one friend, B, who has two friends (a mean of 2); B has two friends, A and C, with one friend each (a mean of 1); and C has one friend, C, with two friends (a mean of 2). The three individuals have a mean of their means of 5/3.

[7] The mean individual's mean number of friends' friends is given by this expression in the case of the "wheel" pattern of friendships, in which one person is friends with everyone else and they are not friends with one another. Some equations show the values of the parameters in the case of a wheel composed of n individuals (1 hub and $n - 1$ spokes). They are: mean number of friends of individuals = $2(n - 1)/n$; variance in number of friends = $(n - 2)^2(n - 1)/(n^2)$; mean number of friends' friends = $n/2$; and mean individual mean number of friends' friends = $[(n - 1)^2 + 1]/n$. It can be seen that the value of the mean individual mean number of friends' friends is the specified function of the mean and variance. For example, in the case of 10 individuals (1 hub and 9 spokes), the mean number of friends of individuals is 1.8 with a variance of 5.76, the mean number of friends' friends is 5, and the mean individual mean number of friends' friends is 8.2. An example with three individuals (1 hub and 2 spokes) is presented in n. 6. Since the wheel appears to be an extreme type of distribution, it is conjectured that this expression gives the *maximum* value of the mean individual's mean number of friends' friends for a specified distribution of friendship volumes. An additional basis for this conjecture is the apparent symmetry. The minimum value of mean individual mean number of friends' friends is achieved when all individuals have the same number of friends as each of their friends, and that mean is just the mean number of friends. When particular friends are "randomly" assigned, the mean individual mean number of friends' friends is equal to the mean number of friends' friends, which is the variance divided by the mean greater than the mean number of friends (as given by the expression in the text). Since the minimum value is just the variance divided by the mean less than the random case, the conjecture is that the maximum value is just the variance divided by the mean more than the random case.

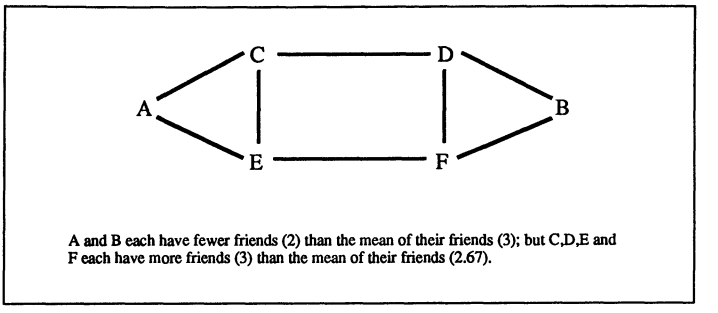


FIG. 5.—An exceptional situation in which a majority of individuals have more friends than the mean of their friends.

than they have, while there are many others whose friends have a few more friends than they have). Thus, while the means of the various distributions are determined, the number of individuals who have fewer friends than the mean of their friends' friends will depend on the exact arrangement of friendships. As shown in figure 5, it is even possible, under very carefully contrived conditions, for a majority of individuals to have more friends than the mean of their friends.

However, very few arrangements of friendships have this consequence, and there are no theoretical reasons to expect these exceptional situations. If the mean number of friends' friends and the mean individual mean number of friends' friends are much higher than the mean among individuals, we can expect that a high proportion of individuals will have fewer friends than the mean among their friends, as is true among the Marketville girls and the boys and girls of the other high schools included in *The Adolescent Society* (Coleman 1961).[8]

[8] Further research might explore how various systematic processes in the construction of social networks might lead to particular types of patterns of friendships with particular consequences for the experiences of friends' friends. For example, if friendships are primarily established through one focus or a few foci of activity (Feld 1981), then individuals might have numbers of friends similar to the numbers their own friends have (i.e., people who draw many friends from a large focus of activity will have friends who also have many friends from the same large focus of activity), and the experience of relative deprivation may be minimized. On the other hand, if individuals disproportionately make friends with a few individuals with particular desirable characteristics (see Feld and Elmore 1982), there may be large amounts of variation in friendship volumes that lead to the widespread experience of relative deprivation by individuals. Actual patterns of friendships reflect several underlying processes by which friendships are developed and maintained and can become very complex. Formally, the proportion of individuals who experience relative deprivation is determined by the probability that the mean (median) number of an individual's friends' friends is greater than the individual's own number of friends under a particular specifiable set of conditions.

Asymmetric Relationships

An analogous phenomenon occurs in situations with asymmetric relationships, the type that are directly revealed in sociometric-choice data. In that case, most individuals choose people who are more popular than they are. The logic employed when individuals compare their own popularity with that of the people they choose is identical to that described above. Individuals who are popular are chosen by many others and so can lead many others to feel relatively deprived; individuals who are unpopular are rarely chosen and so can make few people feel advantaged. The distributions of popularity among individuals and among those they choose can be shown to have the same characteristics as the various distributions of numbers of friends described above.

Related Phenomena

The tendency for individuals to experience a biased sample of numbers of friends of others is one of a large set of related phenomena. Feld and Grofman (1977) called one such phenomenon the "class size paradox"; they showed that, if there is any variation in college class sizes, then students experience the average class size as being larger than it is. They experience a higher average class size than exists for the college because many students experience the large classes, while few students experience the small classes. Hemenway (1982) noted the same phenomenon in terms of college class size and remarked on several other similar phenomena; specifically, he suggested that people disproportionately experience the most crowded times in public places (including restaurants, beaches, and highways) and so experience these places as being more crowded than they usually are.[9]

It should be noted that class size paradoxes are often experienced in situations in which they are not seen as paradoxical. For example, most cities are small, but most people live in large cities; while most organizations are small, a disproportionate number of individuals work for large organizations (Granovetter 1984).

Whether paradoxical or not, it is important to recognize that the experiences of class sizes have a reality of their own. The fact that many

[9] A class size paradox arises when individuals disproportionately experience classes containing more people. This idea can be extended to include an "observer class size paradox," whereby individuals observing classes of objects are more likely to observe and therefore be aware of the larger classes of objects. For example, Good (1983) suggested that galaxies with more planets are more likely to be observed than smaller galaxies; consequently, the average size of galaxies that are observed is larger than the average size of galaxies.

individuals experience disproportionately large average class sizes (large college classes, crowded expressways, populous cities, large families, etc.) may be more sociologically and practically significant than the object average; for example, it may not matter so much that roads are usually empty if most people are caught in rush-hour traffic.

Furthermore, the recognition of the different ways that people experience the same objective situation can help us understand some conflicts of interest. For example, Feld and Grofman (1980) consider that college faculty members experience the actual average class size, while their students disproportionately experience the larger classes; as a result, even though faculty and students have similar preferences for smaller classes, students have an interest in minimizing variation in class size, while faculty have an interest in maximizing that variation.

CONCLUSIONS

The term "class size paradox" can be considered a generic term for all phenomena that arise where classes are of varied sizes, members of those classes disproportionately experience the larger classes, and most individuals therefore experience the average class size as larger than it is. Such phenomena are often more than mathematical curiosities; they have implications for how people experience and respond to various aspects of their environments.

The tendency for most people to have fewer friends than their friends have is one sociologically significant class size paradox. Individuals who find themselves associated with people with more friends than they have may conclude that they themselves are below average and somehow inadequate. The analysis presented in this paper indicates that most individuals have friends who have more friends than average and so provide an unfair basis for comparison. Understanding the nature of a class size paradox should help people to understand that their position is relatively much better than their personal experiences have led them to believe.

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