VARIOUS DETERMINATIONS OVER A CENTURY OF THE HEIGHT OF MOUNT EVEREST

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TO APPRECIATE the problem of the height of Everest, now brought a long stage nearer to finality by recent observation work of the Survey of India,[4] it is necessary to look back over a century of effort under circumstances of progressively lessening difficulty.

It was in the mid-nineteenth century that Himalayan snow peaks first came under serious observation for height determination; and when first the problem of terrestrial refraction was encountered on an unprecedented scale. With noteworthy vision Sir Andrew Waugh, successor to Sir George Everest, wrote in 1845—as quoted by Mr. Gulatee—"The lofty snow peaks situated north of Nepal are the most stupendous pinnacles of the globe. Their heights and relative positions should form permanent objects in the geodetical operations." Within the next ten years a large crop of observations was reaped and came up for computation.

Just how much was known by Waugh about refraction on terrestrial rays over 100 miles long from plains to peaks—or by anyone else at that time—is uncertain. Waugh had probably read Puissant [1] whose work had been published in 1842. Therein one may read, paragraph 214, sub-paragraph 3: "Dans le pratique de la géodésie, la trajectoire lumineuse est une courbe toujours assez peu étendue pour pouvoir être confondue avec son cercle osculateur. . . ."

And in the next sub-paragraph: "Il est donc vrai de dire que la réfraction terrestre est proportionelle à l'angle formé par les verticales des extrémités de la courbe de réfraction, et qu'elle est sensiblement la même, à une époque donnée, aux deux stations comparées, *si toutefois leur différence de niveau est très petite.*"

In both the above the usual idea of the coefficient of refraction, k, is still in general use; but there are wise limitations which have often been forgotten. In the penultimate sub-paragraph Puissant continues: "Delambre a remarqué en France que n a environ pour valeur moyenne 0·07876, ou simplement 0·08 . . . par des temps brumeux 0·15 . . . 0·06 à 0·08 en été, et 0·08 à 0·10 en hiver. . . . D'autres observateurs ont même trouvé 0·5 par les temps de pluie. . . ." Later on in paragraph 217 Puissant develops a theoretical expression varying as air pressure and as inverse square of absolute temperature of air; and with a factor involving the Lapse Rate of which he says: ". . . et nous la supposerons constante, quoique en réalité elle varie d'un lieu à un autre, et peut-être aussi dans un même lieu en différentes saisons."

Puissant says nothing about diurnal variation of refraction (nor of minimum refraction) though this is the most important variation in the case of rays in the lower atmosphere. Certainly it had been well known to Colonel Everest at least ten years earlier, for in his reconnaissance for the Great Arc of triangulation at the crossing of the featureless plains north of Agra he was making use of *maximum* refraction. He wrote instructions [16] to Boileau: "The extent of view at *time of maximum refraction*; this varies from midnight to sunrise and is generally about 3 o'clock. . . ."

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Returning to Puissant, at end of paragraph 216, he says:

". . . éléments météorologiques qu'il est indispensable de recueillir en même temps que les distances zénithales." Unfortunately this last counsel has seldom been followed; and no temperatures were recorded with the vertical angles to Everest about 1850. In spite of the theory, there seemed then, and even much later, a curious tendency to assign values to k appropriate to sea level, though it is quite certain that it should be smaller at considerable elevations. In Europe, 0·08 was much in favour and A. R. Clarke proposed ·075 for rays over land for Great Britain without allowance for height.

Now long rays to high snow peaks are fugitive things, often interrupted by clouds, particularly as the day wears on. Waugh accordingly ordered his observers to be ready to begin such observations at first light, so that no opportunity be lost. Fortunately observations at early afternoon hours were secured in most cases. Waugh had results of observations of vertical angles to Mount Everest from six stations in the Bengal plains, at average height 230 feet, of lengths ranging from 108 to 119 miles. Closer approach had been denied by the Maharaja in spite of Waugh's repeated urgent requests to enter Nepal—an embargo which lasted during British administration in India. On his own system Waugh derived values for k ranging from 0·0727 to 0·0753; and thence he obtained individual values of the peak height ranging over 36 feet, with the world-renowned mean value of 29,002 feet. At that time there were not the spirit-levelling connections which later reduced the station heights by an average amount of 8 feet.

These values of k used by Waugh are 15 per cent. too large. A proper value is about 0·064 (*not* 0·05 as stated by Mr. Gulatee at p. 2 of his paper). But at that time it appears that there was more confidence in empirical than in formula values, even though the former were based on low-lying rays—not comparable with rays attaining a height of 29,000 feet. Indeed, it would have not been surprising if Waugh had used 0·08. I cannot think that criticism can rightly be directed against him about refraction. Rather let us admire his drive in getting these observations made and computed.

About a score of years later General J. T. Walker was investigating refraction in the Punjab plains [5] where extreme and even negative refraction was encountered —diurnal variation of refraction of the kind that makes the observer feel as though in a saucer at dawn, later on seeing objects sinking below the horizon as receding ships at sea! It remained to Sir Sidney Burrard [7] to re-open the question of Himalayan heights about 1905. He was very conscious of the refraction difficulty and soon set Mr. H. G. Shaw [10] observing vertical angles in the neighbourhood of Dehra Dun between stations Nojli (887 feet), Mussoorie (6930 feet), Nag Tiba (9915 feet) and thence to peaks of the snowy range distant about 100, 60 and 50 miles respectively. Since Waugh's time, Everest had been observed from six stations in the Darjeeling Hills, of average height 10,600 feet, lying on an average 95 miles from the peak. For k Burrard used 0·0645 for rays from the Bengal plains and 0·05 for those from the Darjeeling Hills; and his computed heights for Everest ranged over 43 feet with average 29,141. It will be seen below that these values of k were very proper.

The formulae for heights in triangulation from vertical angles do not imply either of the "drastic assumptions" imputed by Mr. Gulatee (p. 5). When there are reciprocal angles, as between stations, the height increment derived (apart from refraction errors) refers to the circle to which the terminal verticals are normal and which touches the horizontal plane at one of the stations. When observations are

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made to an unvisited point the radius of the circle is at choice. A wrong choice will lead to a faulty estimate of the curvature effect.

When observations at several stations are to be associated the problem is complicated if there are significant differential deflections at these stations. Spheroidal heights may be obtained after due correction of observed geoidal vertical angles and geoidal station heights to their spheroidal counterparts. In the vast majority of practical cases no such treatment is needed; the Everest case was exceptional, as sensible geoidal differences exist between the observation stations, and the effects were aggravated by the extreme length of all the rays.

Burrard was very conscious of the deflections of the vertical, a subject which he brought into prominence by two important papers [2, 9] He held, however, that the deflection data available were too scant for tracing the geoid. Indeed the era of the geoid had not yet arrived. Practically any derived height, other than those from spirit-levelling, had reference to a locally positioned spheroid with the dimensions of the Everest spheroid. A few years later, about 1910, I began studying refraction, finding a mine of material to work on in Mr. Shaw's observations [10] of 1905–9, which Burrard's foresight had provided. Results were published in 1913 [11]. In a lecture before the Indian Science Congress at Madras in February 1922 [12], I gave the local spheroidal height of Everest as 29,149 feet. This was derived, taking account of all then known deflection data (see above); the computations being made in the Dehra Dun Computing Office, where they may still be found. The geoidal data had been somewhat increased from what Burrard had but were still scanty. Refraction was derived by formula for which temperature had to be assessed from nearby meteorological reporting stations at corresponding season and hour, as it had not been recorded—as Puissant had laid down (see above). I estimated the geoidal rise from the Bengal plains to Everest at 70 feet, and gave with diffidence the geoidal height 29,080 feet. Some years later Bomford [15] proposed to increase the geoidal rise by 30 feet and so to reduce the mountain height to 29,050. This figure is also given in a note which I wrote for Sir Sidney Burrard in [8] at p. 62.

The study of refraction enabled me to publish in [13] a table, numbered 5 Sur., giving standard horizontal refraction coefficient, and the rule, stated by Mr. Gulatee at end of his paragraph 2,4 for using the value at height one-third of the way along a sloping ray. For this a lapse rate of 3° F./1000 feet was used. Applied to the Everest observations, with standard conditions of temperature and pressure, one finds for the rays from the Bengal plains 0·063; and for those from the Darjeeling Hills 0·053. These may be compared with Burrard's 0·0645 and 0·05. The small differences could be due to departures from standard conditions of 4° and 8° F. respectively. With no exaggeration one may say that the refraction problem for these long rays to Everest had been solved. The only outstanding difficulty was the geoidal shape, for which many more observations were required. These have been provided now by the 1952–54 observations.4

In the late nineteen-forties the political barrier to entry to Nepal was removed. The Survey of India, for the first time free to do so and now concerned with control of new irrigation schemes, executed some good topographical triangulation in Nepal and this passed within 40 miles of Everest. A meridian connection reached from the Ladnia–Harpur side of the old geodetic triangulation to Namche Bazar only 17·5 miles from Everest. The stage was set for new observations for the height of Everest, with the difficulty of geoidal rise reduced ten-fold. The short-sided triangulation brought up what were practically geoidal heights and, apart

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from ordinary triangulation operations, only the change of geoidal slope between it and the peak needed assessment.

Provided with this great opportunity Mr. Gulatee planned observations on a broader basis than would be necessary for the sole object of Everest height—for which he is to be commended—and valuable extra geodetic information has been obtained. The circuit formed by the loop of new topo triangulation and the old triangulation between the sides Ladnia–Harpur and Sandakphu–Phallut was now provided with four Laplace points and three extra bases (quality not stated). The upshot was circuit-closing errors of 2 seconds in azimuth, 1/40,000 in scale and 12 feet in position, in a distance of 300 miles. He added extension triangulation of eight stations, flanking the topo triangulation. These extra stations lay at elevations from 8670 to 14,762 feet and at distances from 29 to 47 miles from Everest. At all of them, as at several of the meridian portion, Ladnia–Namche Bazar, and on the eastern branch to Darjeeling, he provided deflections in both components, from astrolabe observations. To the spirit-levelling connections at Darjeeling and the next station to Harpur was added a new connection at Chatra h.s., some 35 miles south of Mayam, the nearest of the eight extra stations from which Everest was to be observed. One may regret that longitudes were not observed with an impersonal instrument.

In all this Mr. Gulatee had for guidance the experience gained in some 8000 miles of geoidal sections, observed in India 1932–39, along which astrolabe observations were made at intervals 10–20 miles. Admittedly the Nepal terrain was difficult. The observers deserve full praise.

From three of the extra stations, horizontal angles were taken to Everest—the largest subtended angle being about 42°. These led to a fix some 40 feet S. and 40 feet W. of the old accepted position. Fixation of a snow peak offers difficulties in that there is no precise mark to intersect. What may appear the highest point viewed in one azimuth by a ray inclined some 5° will not be so regarded on another azimuth. Moreover the snow itself is mobile with wind and time. The new fix brings Everest some 56 feet closer to the new observing stations, and thus lessens the computed height by some 5 feet.

Section 5 of Mr. Gulatee's paper is headed "Outline of the New Method"; then three main sources of uncertainty in "the older determinations" are stated and an illustrative diagram follows. There is, however, nothing new in the method. This with the handicap of scanty deflection data, was used for my 1922 lecture.[12] The diagram shows no new concept not included in one to be found in an Appendix[6] of Burrard's, written in 1910.

What is new is the great simplification in the problem due to the withdrawal of the embargo on entering Nepal and accordingly the observations which it has recently been possible to make. There is nothing new in the type of these observations.

With the resulting deflections it is a straightforward matter to trace the geoidal rise, as in section 7 of.[4] Now only extrapolation beyond Namche Bazar for 17·5 miles is uncertain—compared with 90 miles formerly—and in this short distance an error as much as 10 seconds in deflection at Everest would change geoidal level by only 2 feet. Mr. Gulatee has gone to the trouble, of debatable value, of computing Hayford deflections at a number of points to assist inter- and extra-polation of additional deflections. Of these he says that they "produce greater sinuosity in the deflection curves"; and these cancel out almost entirely in the integrated height effect. Later, in the last two sub-paragraphs of this section 7, he gives computed

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results of geoidal elevations at Everest—due to simple topography 333 feet; to compensated topography 31 feet; the actual value derived from multiple deflections being 92 feet.

From the observed and the interpolated deflections he obtains the geoidal rises between each of the observing stations and the peak, and these range from 30 to 52 feet. He also finds the rise between Ladnia and Everest to be 109 feet. A chart of geoidal contours at 5-foot intervals covering region 26° to 28° lat. and 85° to 88° long. is given.

Section 2 of Mr. Gulatee's paper is headed "Refraction." In this is set out an account of refraction in which there is no advance on what can be found in Survey of India publications prior to 1930, *e.g.* pp. 109–14 of.[14] Mr. Gulatee quotes a formula from Bomford's 'Geodesy,' which is the essential basis of the refraction table 5 Sur. of [13]; but he has elected to use a lapse rate of 3°·2 F./1000 feet rather than the 3°·0 mentioned by Bomford and used for 5 Sur. The effect of this is to reduce k by 1·3 per cent.; and on the longest of the new rays to Everest, that from Mayam h.s., this increases the computed height by 2 feet. One can hardly fail to notice that a change in the other sense to 2°·7/1000 feet (which would agree with the density law given by Brunt in his 'Physics of the Atmosphere') would reduce the scatter of the new height determinations from 16 feet to 11 feet.

In his section 9, Mr. Gulatee adopts the role of critic of some of his predecessors in the Survey of India, especially of Burrard. Burrard opened up new fields in geodesy and he most certainly knew very well the significance of the height 29,141 feet which he had determined for Mount Everest. Burrard was not talking about geoidal heights, nor was anyone else at that time. Any thinking person knew what was meant and also recognized that there was no practical alternative until extensive geoidal observations had been made and computed. Burrard did more than anyone to bring deflections into the geodetic picture and laid foundations for the geoidal determinations which were initiated in the Survey of India prior to Mr. Gulatee's recruitment.

The 1952–54 observations have given a value of 109 feet for the geoidal rise between Ladnia and Mount Everest. When this is subtracted from Burrard's spheroidal height the result is 29,032; which accords closely with Mr. Gulatee's value 29,028, said to be for time of minimum snow. This latter height is smaller by 5 feet on account of the new position assigned to Mount Everest (see above). All this confirms the opinion, held since 1925, that the only outstanding difficulty was assessment of geoidal rise. And yet Mr. Gulatee tells us "the estimation of geoidal rise between the stations of observation and Mount Everest involves an extra step but it has to be done." This point was brought out strongly in [12] and again in [3] and estimates of geoidal rise were made using the scanty available deflection data. Mr. Gulatee has had just the same opportunities of doing this as anyone else for over twenty-five years, and did nothing until all serious difficulties were removed by access to Nepal being allowed.

Mr. Gulatee now earns the thanks of geodesists for providing adequate deflection data penetrating the Himalaya 100 miles. It is indeed unfortunate that in presenting the results of important work, he has signally failed to acknowledge how very essentially he has depended on earlier workers. He has also been guilty of plagiarism. His criticisms of that most distinguished geodesist, the late Sir Sidney Burrard, can most kindly be attributed to ignorance.

It is agreeable in conclusion to accord hearty praise to the triangulators and

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astrolabe observers whose work has rendered possible the closer approach to the true geoidal height of the "world's most stupendous pinnacle." The height observations were made in months December to March, at which season precise work at altitudes approaching 15,000 feet must involve physical hardship.

Summary of altitude results

(*a*) Waugh's value, 29,002 feet, is a spheroidal height. It is too low owing to over-correction for refraction.

(*b*) Burrard's value, 29,141 feet, is a good spheroidal height. The corresponding geoidal height, 109 feet lower, is *29,032 feet.*

(*c*) The present writer's 29,080 feet[12, 3], was the first attempt at a geoidal height. The reductions took account of any known deflections and the correction, – 69 feet, was the geoidal rise inferred from the geoid in meridians 78° and 88° and, more recently 75°. With the recently found value, – 109 feet,[4] it reduces to *29,040 feet* geoidal height.

(*d*) Mr. Gulatee's *29,028 feet* is a geoidal height, at season of minimum snow, in years 1952–54.

It is not to be expected that the height of a snow peak will be invariable over different years and seasons.

REFERENCES

[1] L. Puissant, 'Traité de géodésie' (3me. edition), Tome premier. Paris. 1842.
[2] S. G. Burrard, "On the intensity and direction of gravity in India." *Proc. roy. Soc.* Ser. A. **76** (1905) 313–15, and *Phil. Trans.* Ser. A. **205** (1905) 28, 318.
[3] J. de Graaff-Hunter, "Heights and names of Mount Everest and other peaks." *Occ. Notes R. Astr. Soc.* **3** (1953) 15.

SURVEY OF INDIA PUBLICATIONS

[4] B. L. Gulatee, "The height of Mount Everest: a new determination (1952–54)." *Technical Paper* No. 8, 1954.
[5] J. T. Walker, Appendix 3 of 'Account of the Great Trigonometrical Survey of India." Vol. II, 1879.
[6] S. G. Burrard, Appendix 8 of 'Account of the Great Trigonometrical Survey of India.' Vol. XIX, 1910.
[7] S. G. Burrard and H. H. Hayden, 'A sketch of the geography and geology of the Himalayan mountains and Tibet.' Delhi, 1907–8.
[8] S. G. Burrard and A. M. Heron, Revision of the above. Delhi, 1932–33.
[9] S. G. Burrard, "The attraction of the Himalayan mountains upon the plumbline in India." *Prof. Pap. Surv. India* No. 5 (1901).
[10] H. G. Shaw, "Observations of atmospheric refraction 1905–9." *Prof. Pap. Surv. India* No. 11 (1911).
[11] J. de Graaff-Hunter, "Formulae for atmospheric refraction and their application to terrestrial refraction and geodesy." *Prof. Pap. Surv. India* No. 14 (1913).
[12] J. de Graaff-Hunter, "The height of Mount Everest and other peaks." Lecture before the Indian Science Congress, Madras, 1922. Reprinted in *Geod. Rep. Surv. India*, **1** (1924).
[13] J. de Graaff-Hunter, 'Auxiliary Tables.' Part II (Fifth edition) reprinted, Dehra Dun, 1928.
[14] J. de Graaff-Hunter, "Geodesy." *Dep. Pap. Surv. India* No. 12 (1929).
[15] Guy Bomford, *Geod. Rep. Surv. India*, **3** (1929) 133–35.
[16] R. H. Phillimore, Historical Records of the Survey of India. Vol. IV, 'George Everest, 1830–43" (in proof stage, December, 1954).

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