The Physics of Somersaulting and Twisting

Divers, gymnasts, astronauts and cats can do rotational maneuvers in midair that may seem to violate the law of the conservation of angular momentum but in fact do not

by Cliff Frohlich

In the absence of external forces the linear momentum of a body (its mass times its velocity) is conserved. Similarly, in the absence of torques, or rotational forces, the angular momentum of a body (the rotational analogue of linear momentum) is conserved. The concept of angular momentum is deceptively simple, and its misapplication has led to paradox. If a cat is dropped upside down, it turns over in free fall in a fraction of a second and lands on its feet. By the same token, when an expert diver jumps off a springboard, he can start to somersault and twist in midair long after he has left the board. Since the cat and the diver have zero angular momentum, how can they rotate in midair without violating the law of the conservation of angular momentum? Here I want to discuss in detail the physics of somersaulting and twisting in order to flesh out the concept of angular momentum.

Springboard divers routinely execute maneuvers in which their body rotates in space. The basic maneuvers are the somersault and the twist. In the somersault the body rotates head over heels as if the athlete were rotating about an axis extending from his left side to his right side through his waist. In the twist the body spins or pirouettes in midair as if the athlete were rotating about an axis extending from his head to his toes.

Nearly all the most complicated diving maneuvers are either multiple somersaults or combinations of somersaults and twists. A good diver can execute a forward three-and-a-half somersault, which calls for 1,260 degrees of rotation. Another common dive is the forward one-and-a-half somersault with three twists, in which the diver somersaults 540 degrees and twists 1,080 degrees. The best divers have recently begun to perform even more complex feats, such as the forward two-and-a-half somersault with two twists.

The conservation of angular momentum is the fundamental physical principle that governs all maneuvers incorporating somersaults and twists. In such maneuvers air resistance is negligible, so that no torques are applied to the diver once he leaves the board. (The same is of course true of the gymnast once he leaves the mat.) For example, in the forward two-and-a-half somersault with two twists the diver's angular momentum remains fixed at some large nonzero value. This creates a paradox: How can he initiate the twisting rotations in midair without changing his angular momentum?

It is time to precisely define angular momentum, which can be expressed in terms of two other parameters of rotation: angular velocity and moment of inertia. Angular velocity is a vector, represented by a line with an arrowhead, that describes a body's rotational speed and direction. The vector has a length that represents the speed of rotation and an orientation parallel to the axis of rotation. Which end of the vector has the arrowhead attached to it corresponds by convention to whether the rotation about the axis is clockwise or counterclockwise.

For example, when a diver executes a forward double somersault in one second, the magnitude of his average angular velocity is two revolutions per second. The direction of his angular-velocity vector is parallel to the somersaulting axis (and the arrowhead is at the diver's left side). Similarly, when the diver twists, the direction of his angular velocity is along the twisting axis (with the arrowhead at his head if he is twisting to the left). When a diver simultaneously twists and somersaults, his total angular-velocity vector is the vector sum of the angular velocity of twisting and the angular velocity of somersaulting. (The sum of two vectors A and B is found by putting the base of B at the arrowhead of A. The sum is the vector formed by connecting the base of A to

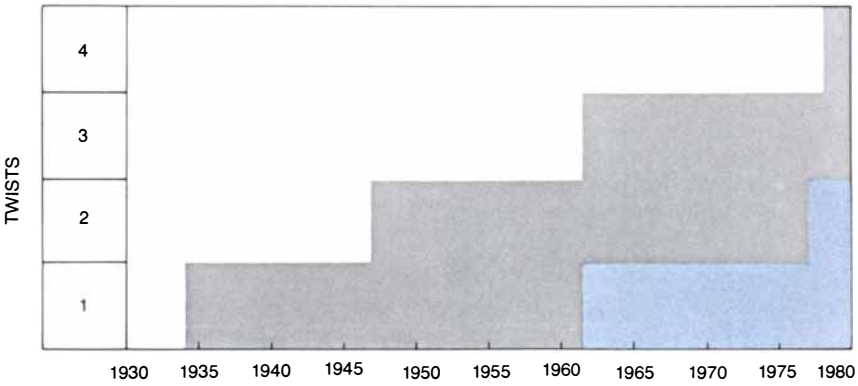
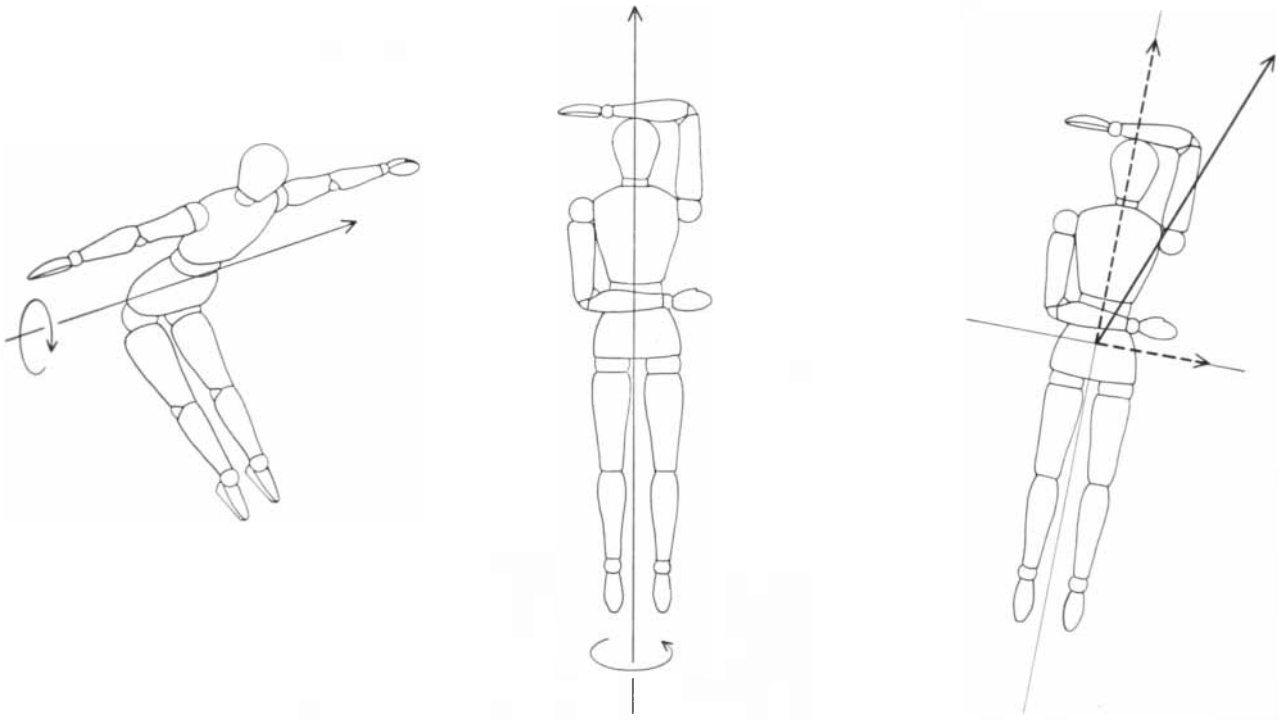
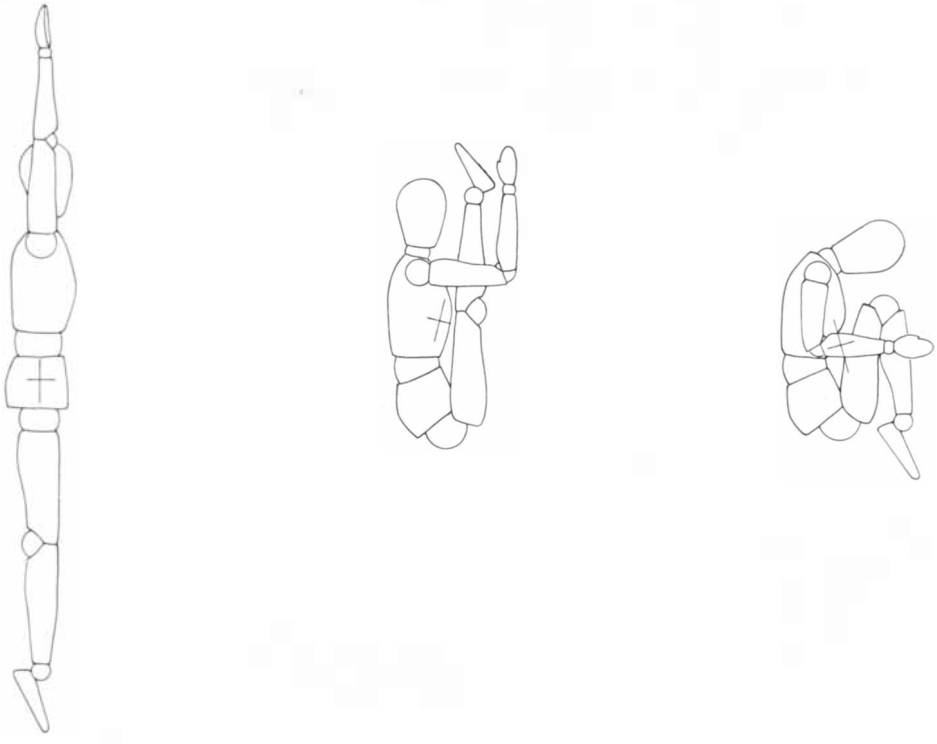


CHART OF THE FORWARD TWISTING SOMERSAULTS that are allowed in intercollegiate three-meter springboard diving competition indicates that maneuvers combining somersaulting and twisting are relatively new. The gray region represents forward one-and-a-half somersault and the colored region forward two-and-a-half somersault. Twists are at the left.

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SOMERSAULTS AND TWISTS are the basic maneuvers that make up most of the complicated dives executed in competition. In the somersault (*left*) the body rotates head over heels as if the diver were rotating about an axis extending from his left side to his right side through his waist. In the twist (*middle*) the body spins in midair as if the diver were rotating about an axis from his head to his toes. When a diver simultaneously twists and somersaults (*right*), his total angular-velocity vector (*solid arrow*) is the vector sum of the angular velocity of somersaulting (*short broken arrow*) and the angular velocity of twisting (*long broken arrow*). Twisting angular-velocity vector is three times longer than the somersaulting angular-velocity vector because the diver is twisting three times faster than he is somersaulting.



MOMENT OF INERTIA of a body about an axis, which by definition is the body's tendency to resist changes in angular velocity about that axis, is larger the more the mass of the body is concentrated away from the axis. The moment of inertia of a man rotating about his somersaulting axis (*left*) is largest when his body is straight. The moment of inertia is smaller when his body is piked, or bent at the waist (*middle*). The moment of inertia is smallest when his body is tucked, or bent at both the waist and the knees (*right*). The somersaulting moment of inertia is 19.8 kilograms times meters squared (kg.m.^{2}) when the man's body is straight, 5.9 when his body is piked and 3.8 when his body is tucked. The moment of inertia about his twisting axis when he is straight is smaller: 1.1 kilograms times meters squared.

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the arrowhead of *B*. Vector addition is commutative: *A* plus *B* equals *B* plus *A*.)

The moment of inertia of a rigid body about an axis is the body's tendency to resist changes in angular velocity about that axis, just as the mass of a body is its tendency to resist changes in linear velocity. It is obvious that massive and extended bodies have a larger moment of inertia than lighter and smaller ones. In fact, the contribution of each particle in a body to the total moment of inertia about an axis equals the mass of the particle times the square of its distance from the axis of rotation.

For example, a typical male diver with his body straight and his arms at his sides has a moment of inertia of 14 kilograms times meters squared (kg.m.^{2}) about his somersaulting axis but a moment of inertia of only one kilogram times meter squared about his twisting axis. Although the mass of each particle of the diver remains the same for both somersaulting and twisting, the moment of inertia is larger about the somersaulting axis because on the average each particle is farther from that axis than it is from the twisting one.

Like angular velocity, angular momentum is a vector. The total angular momentum of an athlete executing a twisting somersault is the vector sum of the angular momentum about his somersaulting axis and the angular momentum about his twisting axis. His somersaulting angular momentum is simply the product of his somersaulting angular velocity and his moment of inertia about his somersaulting axis. In addition his twisting angular momentum is simply the product of his twisting angular velocity and his moment of inertia about his twisting axis. The total angular-momentum vector of an athlete remains constant in direction and magnitude in the absence of torques, just as the linear momentum of a body remains constant in the absence of external forces. This constancy is the essence of the law of the conservation of angular momentum. When the diver is still on the springboard, he can acquire angular momentum if it exerts torques on his body, but once he is in the air his angular momentum remains constant.

The analogy between angular momentum (moment of inertia times angular velocity) and linear momentum (mass times linear velocity) is not perfect. Because the mass of a body is invariant a constant linear momentum means a constant linear velocity. Because the moment of inertia is not invariant but subject to change, however, a constant angular momentum does not mean a constant angular velocity. The angular velocity and the moment of inertia can vary inversely as long as their product remains constant.

A diver is not a rigid object, so that it is possible for him to vary the moment of inertia about his somersaulting axis by tucking up or straightening out his body and to vary the moment of inertia about his twisting axis by pulling in or extending his arms. For example, in the backward somersault the diver leaves the springboard with his body straight as he begins to somersault backward. If he keeps his body straight, he will somersault about one and a half times before entering the water. If he tucks himself up into a ball by pulling in his arms and legs, he will spin much faster and somersault at least two and a half times. As he tucks up, his angular momentum remains constant in magnitude and direction, but his angular velocity increases by a factor of about four because the moment of inertia about his somersaulting axis decreases by a factor of about four. In the backward somersault the angular-velocity vector changes not in direction but in magnitude. The vector remains parallel to the angular-momentum vector throughout the dive.

The angular-velocity vector does not, however, remain parallel to the angular-momentum vector in all cases. If the rotation is about an axis that is not one of the symmetry axes of the body (such as the somersaulting axis or the twisting axis), then the angular-velocity vector and the angular-momentum vector will point in different directions. Suppose a diver is twisting and somersaulting simultaneously. His total angular-momentum vector is the vector sum of his somersaulting angular velocity times his somersaulting moment of inertia and his twisting angular momentum times his twisting moment of inertia. Since the moments of inertia are not equal, the angular-momentum vector and the angular-velocity vector point in different directions. Of course, the direction and the magnitude of the angular-momentum vector remain constant throughout the dive regardless of how the diver twists, squirms or flails. The lack of parallelism of angular velocity and angular momentum brings out another difference between rotational motion and linear motion. In the linear case the velocity vector and the momentum vector are always parallel.

It is possible for a diver to change in midair both his somersaulting angular momentum and his twisting angular momentum as long as their sum, the total angular momentum, remains constant in magnitude and direction. In a twisting somersault the diver starts to execute an ordinary somersault, in which his angular-velocity and angular-momentum vectors lie along the somersaulting axis. To twist his body he simultaneously "throws" his right arm down and his left arm up in the plane of his body. That causes his body to rotate clockwise a few degrees (as seen from the front) so that his somersaulting axis

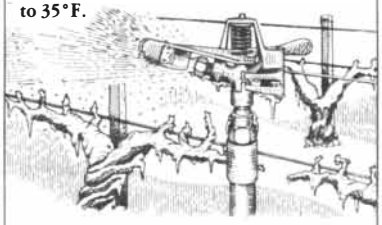


Spring Frost Protection

The most beautiful time of year in the Sonoma Valley is the spring, and with it comes the awakening of the vineyard – and the many concerns of the vintner.

Cultivation of the vineyard is begun immediately to loosen and aerate the rain packed soil and to turn under the wild, winter growth.

The greatest concern of the spring season, however, is the everpresent threat of a killing frost. If the budding grape vines are frost-burned, only a partial crop may develop or even no crop at all. My Dad calls this "the season of long days and short nights," because we are constantly alert for the frost alarm that gets us out of bed fast when the temperature dips to 35°F.



Overhead sprinkler systems are then turned on. Newly forming shoots which will eventually become grapes and leaves are showered with a fine mist that continually forms ice around the tender shoots as the temperature drops below 32°F. Water gives off heat when it freezes and the temperature of the shoots remains at 32°F as long as the misting continues. This protects the shoots from frost damage which would occur at 28°F and below.

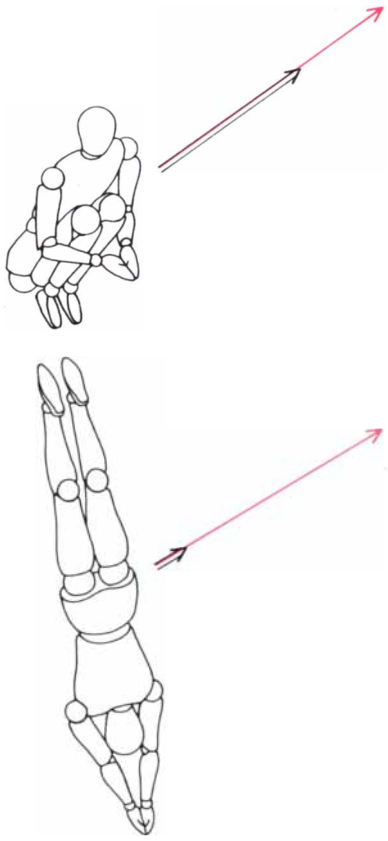


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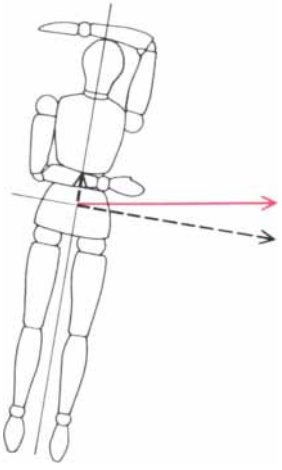
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STRAIGHTENING OUT OF A TUCK will cause the angular velocity (*black arrow*) of a diver to decrease fivefold because his moment of inertia is five times larger in the straight position than in the tucked one. The angular velocity and moment of inertia can vary inversely as long as their product, the angular momentum (*colored arrow*), remains constant.



ANGULAR-MOMENTUM VECTOR (*colored arrow*) of a diver executing a twisting somersault is the vector sum of the somersaulting angular momentum (*long broken arrow*) and the twisting angular momentum (*short broken arrow*). Because the moments of inertia are not the same about the somersaulting and twisting axes angular-momentum vector need not be parallel to angular-velocity vector.

is no longer parallel to his total angular-momentum vector. In other words, his somersaulting angular momentum is in a direction different from that of his initial angular momentum. As a result the diver's body starts to twist, creating exactly enough twisting angular momentum so that the vector sum of his twisting angular momentum and his somersaulting angular momentum is equal to his initial angular momentum.

Any movement of the body, not just the arms, will bring on twisting if it causes the diver's somersaulting axis to move away from the direction of the total angular-momentum vector. To stop the twisting the diver must move his arms or some other part of his body so that the somersaulting axis is once again parallel to the total angular-momentum vector. Such twists are called angular-momentum twists because they can be executed only if the diver has a nonzero angular momentum.

This last point has been a source of considerable confusion. To initiate continuous twisting in midair the diver need only have some initial angular momentum in any direction; he need not start twisting while he is still in contact with the board. Because some investigators did not realize that the angular-momentum and the angular-velocity vectors can be nonparallel they maintained erroneously that it is impossible for a diver to twist unless he starts to do so on the board. Another incorrect explanation was proposed in 1969 by George Eaves of the University of Leeds in his comprehensive book on the physics of diving. Eaves stated that a diver can initiate twisting in midair but can maintain that twisting motion only if he moves his legs continuously with respect to his torso. High-speed films of divers as well as the theoretical considerations I have discussed so far establish that divers do not have to move their legs to keep twisting and that they can start twisting long after they have left the board. In the 1979 U.S. Outdoor Diving Championships nine divers executed dives in which they somersaulted two and a half times and twisted once, initiating the twist only after completing one and a half somersaults.

Much of the confusion about the physics of diving stems from the fact that the twisting somersault is a relatively new maneuver. The forward one-and-a-half somersault with one twist was allowed in competition for the first time in 1934. Investigations of the mechanics of the twisting somersault were scarcely undertaken until the 1950's, when dives with multiple twists became common. Erroneous old hypotheses about the mechanics of twisting have survived in the diving literature of today.

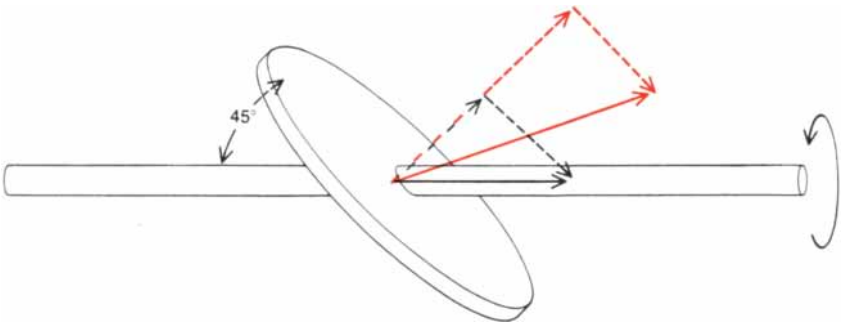
Another source of misunderstanding about the physics of diving is the fact that although twisting somersaults are often initiated in midair, they are sometimes begun when the diver is still in contact with the springboard. A twist resulting entirely from torques applied to the diver by the board is called a torque twist. In backward twisting somersaults the diver is particularly likely to begin his twisting on the board because his "throw" to initiate twisting in midair is less efficient in the backward somersaulting position than in the forward position.

The rate of twisting depends on the angle θ between the somersaulting axis and the angular-momentum vector during the twist. In the backward somersault the diver's body is nearly straight. Throwing his arms to initiate a twist makes the angle θ about 11 degrees, which causes his body to twist at a rate of about three twists per somersault. In the forward somersault the diver's body is not straight but piked, or bent at the waist. When he throws his arms, the angle θ will be larger than 11 degrees because his moment of inertia in the piked position is smaller than that in the straight position. For an angle θ of 20 degrees the diver will twist at a rate of about five and a half twists per somersault when he comes out of the piked position. In the backward somersault he can twist at a comparable rate if he gets some torque from the board with which to start the twisting motion.

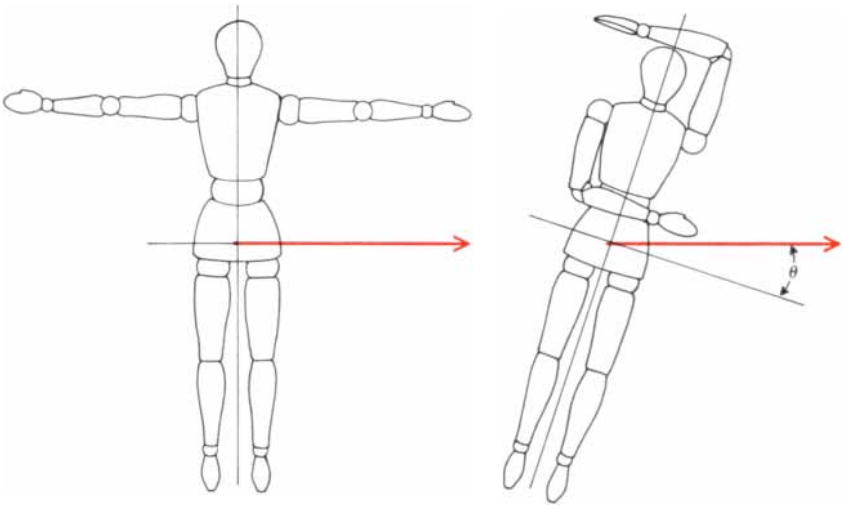
At the beginning of this discussion I posed the question of how with zero angular momentum a cat or a diver can twist in midair. The mechanisms I have outlined so far apply only to bodies with nonzero angular momentum, which is constant but can flow back and forth between different kinds of rotational motion (such as somersaulting and twisting) in the course of a single maneuver. Even with zero angular momentum a cat or a man can accomplish a certain amount of twisting and somersaulting motion because a body need not be motionless to have zero angular momentum.

For example, a man could have zero angular momentum if his arms and upper torso twisted to the right as his legs and lower body twisted to the left. To conserve angular momentum the portion of his body with the smallest moment of inertia would need to twist faster, since angular momentum is equal not to the twisting velocity but to the twisting velocity times the moment of inertia. Since the body of a man is not rigid, he can twist by judiciously varying the relative moments of inertia of his upper body and lower body. Unlike the angular-momentum twists of most twisting somersaults, zero-angular-momentum twists persist only as long as the upper body is in motion in relation to the lower body. If the diver holds his body com-

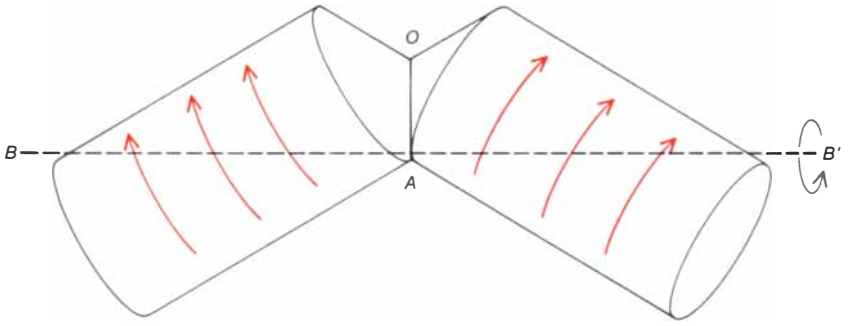
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DISK ON A ROTATING SHAFT AT AN ANGLE shows that the angular-velocity vector (*solid black*) and the angular-momentum vector (*solid color*) need not be parallel. The angular-velocity vector can be broken down into a component along the symmetry axis of the disk (*broken black*) and a component parallel to the face of the disk (*broken black*). Because the moment of inertia of the disk about its symmetry axis is twice the moment of inertia about an axis parallel to the disk's face, the magnitude of the component of the angular momentum about the symmetry axis (*broken color*) is twice the magnitude of the component parallel to the disk's face (*broken color*). As the shaft and the disk rotate, the total angular-momentum vector (*solid color*) sweeps out a cone. That the vector is changing direction continuously means that the disk is always experiencing a torque, which is provided by the bearings of the rotating shaft.



IN ANGULAR-MOMENTUM TWISTS, in which the diver leaves the springboard with a nonzero angular momentum, the rate of twisting (ω_T) equals the rate of somersaulting (ω_S) times the ratio of the somersaulting moment of inertia (I_S) to the twisting moment of inertia (I_T) times the sine of the angle θ between the somersaulting axis (*black line*) and the angular-momentum vector (*red arrow*). In other words, ω_T equals $\omega_S(I_S/I_T)\sin\theta$. This means that to maximize the twisting rate the diver must "throw" his arms and head to maximize θ.



"CAT TWISTS," in which a cat or a man with zero angular momentum flips over in free fall, can be accomplished in principle even if the relative moments of inertia of the upper body and the lower body do not vary in the course of the flip. This kind of twist is best understood if the body is viewed as consisting of two rigid cylinders with cone-shaped ends. The cones are attached to each other so that the cylinders can roll without slipping along their line of contact (*OA*). Each cylinder has angular momentum because it is rotating about its own axis. To conserve total nonzero angular momentum the entire body must rotate in the opposite direction about axis *BB'*. The arrows represent not the angular velocity but the direction of rotation.

pletely rigid, the twisting motion immediately stops.

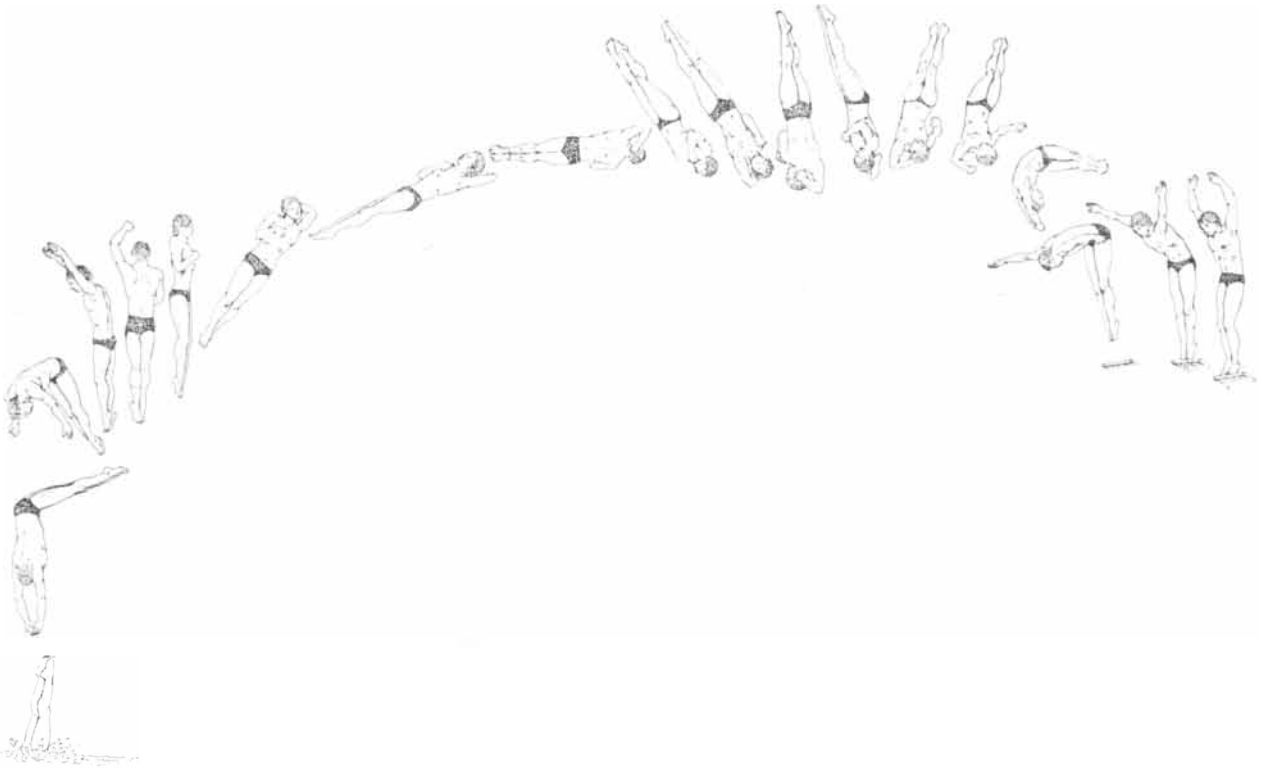
In principle it is also possible for a man or a cat to execute zero-angular-momentum twists ("cat twists") without varying the relative moments of inertia of the upper body and the lower body. This kind of twist is best understood if the body is viewed as consisting of two rigid cylinders attached to each other at the ends [*see bottom illustration at left*]. If the body is not straight but bent so that the two cylinders form an angle of, say, 90 degrees, then twisting is possible. If the upper cylinder twists to the right about its axis, the lower cylinder must swing to the left to conserve the zero angular momentum. Similarly, if the lower cylinder twists to the right, the upper cylinder must swing to the left. When both cylinders twist to the right about their respective axes, the entire body must swing to the left to conserve the zero angular momentum. If the dimensions of the cylinders and the angle between the cylinder axes are correctly chosen, the body will do a half twist to the left as its upper and lower parts twist once to the right. Again the twisting stops if the body is held rigid.

It is an open question whether in practice cats and men need to vary the relative moments of their body to execute zero-angular-momentum twists. Thomas R. Kane and M. P. Scher of Stanford University have studied photographs of a twisting cat and have concluded that the relative moments indeed remain the same. Donald McDonald of St. Bartholomew's Hospital in London was unable to reach a conclusion in his studies of cats and divers. Most investigators, including me, think that in practice cats and men usually do vary the relative moments of their upper and lower bodies by adjusting the position of their arms and legs.

A man with zero angular momentum can not only execute "cat twists" to rotate about his twisting axis but also execute rotations about his somersaulting axis. If he moves his legs in a way that gives his lower body angular momentum, his upper body must also move in order to conserve his total zero angular momentum. As in a cat twist, he can perform a series of motions ("tuck drops") with his arms and legs that will reorient his body about his somersaulting axis, although the somersaulting motion will stop if he holds his body rigid.

Most work on man's ability to control his orientation in midair has centered not on springboard divers or gymnasts but on astronauts. A man working in space in a weightless environment must be able to control his body orientation, but can he start in a motionless position and with a few simple movements reorient himself in any direction

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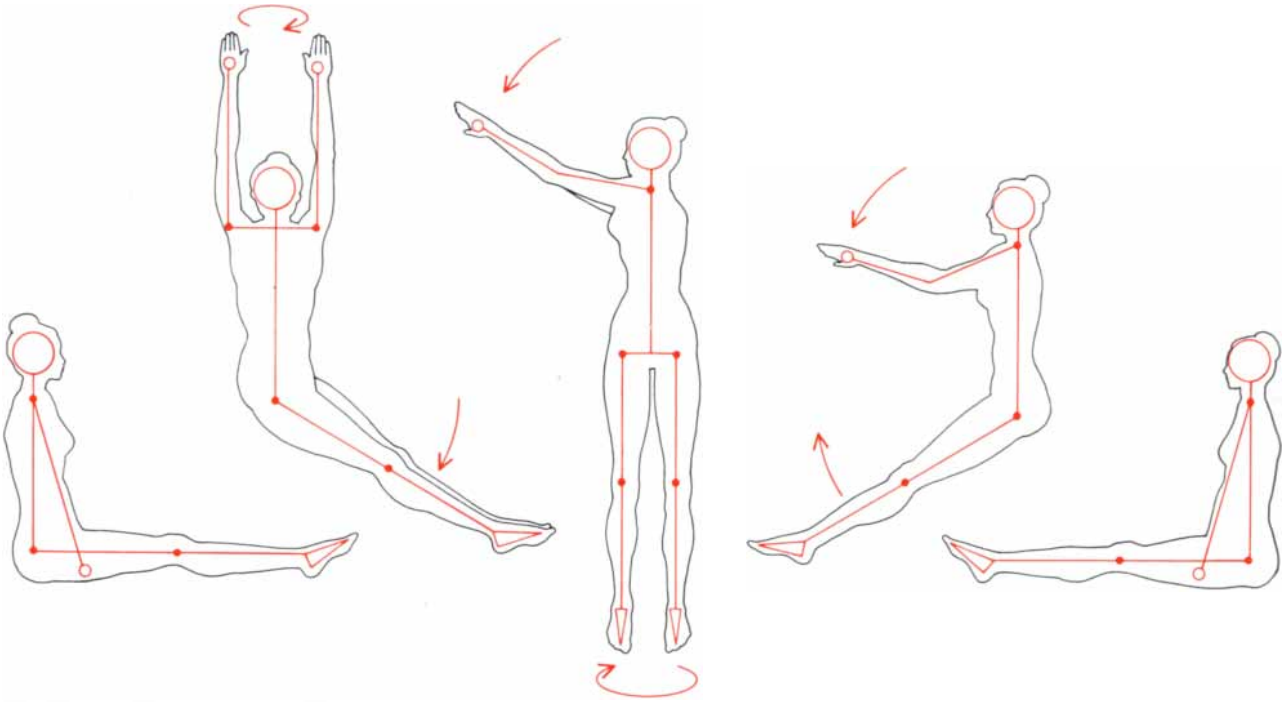


A FORWARD ONE-AND-A-HALF SOMERSAULT with three twists is a maneuver in which the diver leaves the springboard with nonzero angular momentum. Immediately after takeoff the diver is somersaulting without twisting, so that his angular-velocity and angular-momentum vectors are parallel. To initiate twisting he moves his arms and head until his somersaulting axis is no longer parallel to his angular-momentum vector. His body must twist in order to conserve angular momentum. Near the end of the dive the athlete moves his arms so that his somersaulting axis is once again parallel to the angular-momentum vector. That immediately stops the twisting motion. This illustration is based on a motion picture the author made at the 1979 U.S. Outdoor Diving Championships, held in Decatur, Ala.



A FORWARD TWO-AND-A-HALF SOMERSAULT with two twists demonstrates unequivocally that divers can initiate continuous twisting in midair. In this maneuver the diver does more than one full somersault before he starts to twist. To maintain the twisting the diver does not have to move his legs with respect to his torso. Illustration is based on the film of the 1979 U.S. Outdoor Diving Championships.

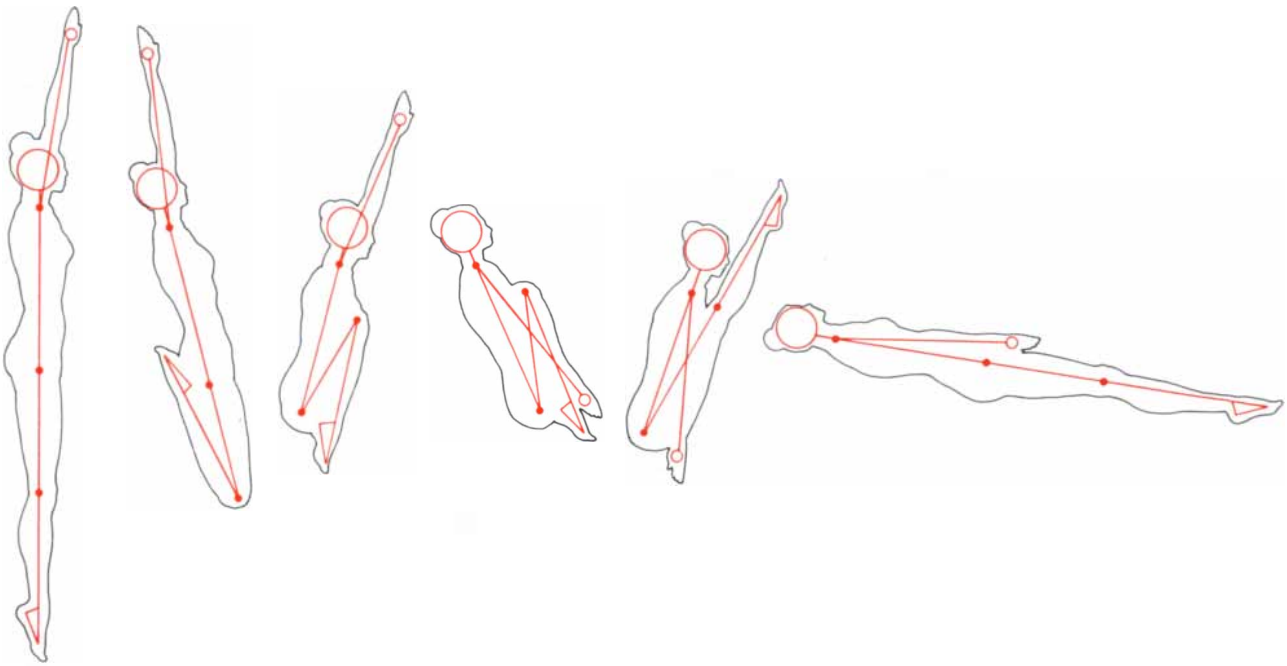
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"SWIVEL HIPS" is a simple twisting maneuver executed by trampoliners that relies on zero-angular-momentum cat twists. By judiciously varying the relative moments of inertia of her upper body and lower body the trampoliner is able to twist half a revolution.

he wants? He can, and the underlying physics is the same for an astronaut as it is for a diver, since both men are moving in the absence of torques. From my discussion of diving it should be clear that an astronaut could reorient himself in any direction he chose by doing cat twists to rotate about his twisting axis and tuck drops to rotate about his somersaulting axis.

The work of Kane and Scher also demonstrates that an astronaut could easily control his motion, although the simple movements they recommend are quite different from the ones divers and gymnasts would execute in order to achieve the same reorientations. For example, they suggest that an astronaut could twist 70 degrees to the left by first moving his right leg forward and his left leg backward, then sweeping his right leg in a circle to the right and the rear and sweeping his left leg in a circle to the left and the front and finally bringing his legs back together. I believe once men actually start working regularly in space they will learn to achieve large reorientations of their bodies by making the same motions divers and gymnasts make.



"TUCK DROP" is a maneuver in which an athlete with zero angular momentum does somersaulting rotations. By repeated tuck drops he can rotate as much as he wants. An astronaut in space could easily reorient himself in any direction with swivel hips and tuck drops.

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