(*Paper No. 2486.*)

"Mental Calculation. A Reminiscence of the late Mr. G. P. Bidder, Past-President."

By W. POLE, F.R.S., Hon. Sec. Inst. C.E.

IN the year 1855 Mr. Bidder gave to the Institution[1] a most interesting description of the wonderful powers of Mental Calculation with which he was endowed, and he made the Address specially valuable by showing how the methods he used might be taught in ordinary education, and employed in calculations generally. He not only gave examples of his modes of working complicated sums in ordinary arithmetic (as applied to weights and measures, time, and money), but also solved more difficult problems, involving square and cube roots, prime numbers, compound interest and so on, including some special calculations of a mechanical and engineering nature.

In the course of the Address he alluded to still higher feats as possible, particularly to the determination of logarithms, which he appeared to consider the *ne plus ultra* of mental calculation. He said, alluding to the two chief processes by which he worked:—

> "Were my powers of registration at all equal to the powers of reasoning or execution, I should have no difficulty, in an inconceivably short space of time, in composing a voluminous table of logarithms; but the power of registration limits the power of calculation, and it is only with great labour and stress of mind, that mental calculation can be carried on beyond a certain extent."

The object of the present Paper is to put on record the fact, that Mr. Bidder subsequently succeeded in performing this astonishing feat, which at the time he spoke he had thought beyond his power. He appears to have considered, in fact, that he had, in the above words, given himself a sort of challenge, and after much further study, he declared his competency to calculate, mentally, and "in an inconceivably short space of time," the

[1] Minutes of Proceedings, vol. xv. p. 255. The memorandum on which the present Paper is based was prepared during Mr. Bidder's lifetime, and with his approval; but something happened to delay its publication, and it has only recently come to light again.

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.

logarithm of any number, with the accuracy of the large published tables, extending to seven or eight places of decimals.

The enormous difficulty of this will be evident if any one will consider how he would set to work to calculate, mentally and accurately, the value of x in the equation say—

$$10^x = 369353,$$

or how he would mentally sum up a large number of terms of the series—

$$0{\cdot}86858896 \left\{\frac{n-1}{n+1} + \frac{1}{3}\left(\frac{n-1}{n+1}\right)^3 + \frac{1}{5}\left(\frac{n-1}{n+1}\right)^5 + \text{\&c.}\right\},$$

n being a large number whose logarithm is to be found.

Mr. Bidder had often been in communication with the Author of this Paper on the subject of mental calculation, and towards the end of his life he communicated this discovery, and requested the Author to aid him in testing its accuracy. For this purpose the following series of prime numbers was selected by the Author, with the aid of a mathematical friend:—

71, 97, 659, 877, 1297, 6719, 8963, 9973, 115,249, 175,349 290,011, 350,107, 229,847, 369,353.

Each of these was given, separately, to Mr. Bidder, and the logarithm in answer was returned very quickly. The time occupied on each varied from half a minute to four minutes, being generally about two minutes.

The logarithms were given in eight places, but the Author had only Hutton's table of seven places to check them with. The majority were stated at first quite correctly; but in some cases errors occurred (generally of one figure only), which were discovered and corrected on simply announcing that the answer was not exact. The experiment was at any rate amply conclusive as to the real efficiency of the mental power to perform the calculations for any numbers.

After the tests were over Mr. Bidder was good enough to explain fully the manner in which the calculations were made; and as this is quite a unique incident in mathematical history, it may be worth putting on record. Indeed, Mr. Bidder wished it to be published as an addendum to his lecture, and gave it to the Author with that view.

In general terms, it may be stated that his mode of calculation was not based on any of the formulas proper for calculating logarithms in the first instance; for probably no amount of genius and skill would suffice for reducing them to manageable form for mental calculation, when the given number was large. Neither

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.

was it a matter of simple memory, for it would be equally impossible for any one to recollect the logarithms, to eight places, of hundreds of thousands of numbers.

It was, however, founded on a power of memory, of a more limited range. Mr. Bidder's process may be said briefly to be this: having stored in his memory the logarithms of a few simple numbers, he was able, by his wonderful skill in dealing with figures, to make use of them, mentally, to calculate accurately the logarithms of any other numbers however large. A brief description will give an idea of the manner in which this was done.

In the first place, as to the mnemonic bases used. He knew by heart the logarithms of a great many *small prime numbers*, nearly all, he said, under 100, and some few above.

Secondly, it will be easily understood that these would enable him to calculate the logarithms of any large numbers which were multiples of those he knew. Thus for the logarithm of 63 he had only to add together the logarithms of 7 and 9; for the logarithm of 3567, he had only to sum up the logarithms of 29, 41, and 3, and so on.

In carrying out this process he had an almost miraculous power of seeing, as it were intuitively, what factors would divide any large given number not a prime. Thus if he was given the number 17,861, he would instantly remark it was $= 337 \times 53$; or he would see as quickly that 1659 was $= 79 \times 7 \times 3$. He could not, he said, explain how he did this, it seemed like a natural instinct to him.

These two qualifications, therefore—the knowledge of the logarithms of certain small primes, and the power of reducing large compound numbers to their component factors—constituted the foundation on which Mr. Bidder's operations proceeded. They would suffice for determining the logarithms of a great many numbers by simply adding together the logarithms of their component factors, which he could do mentally with the greatest ease.

But it still remains to be explained how he treated the case of large primes, and this is really the great interest of his process.

His first endeavour was to find some multiple number *very near* the number given, differing from it only in the last place of figures, and if possible only by unity. For example, being given the number 1051, he would easily find the logarithm of 1050 $(= 30 \times 7 \times 5)$ to which he would then have to make an addition for the difference of 1. This addition would be very important (for the accuracy aimed at), as it would affect the last five figures

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.

of the logarithm. The operation is difficult and complicated, and the manner of dealing with it was really the key to Mr. Bidder's success. It is due to him, therefore, to explain it somewhat fully.

His method consisted in the use, as a basis, of the following Table, which he knew by heart (only seven places are given here, he himself used eight):—

For an Addition to any Number n of		There must be added to its Logarithm.
$\frac{n}{100}$	Log. 1·01	= 0·0043214
$\frac{n}{1000}$	Log. 1·001	= 0·00043407
$\frac{n}{10000}$	Log. 1·0001	= 0·0000434
$\frac{n}{100000}$	Log. 1·00001	= 0·0000043

Suppose therefore, as in the above case, there is to be added to the logarithm a sum corresponding to an addition to the number of $\frac{n}{1050}$ $(= 1)$. Mr. Bidder would take the proportion $\frac{1}{1000} : 0{\cdot}0004341 :: \frac{1}{1050} : 0{\cdot}0004134$. This calculation would pass through his mind instantaneously, and thus he would get, by mental addition—

Log. 30 . . .	= 1·4771213
„ 5 . . .	= 0·6989700
„ 7 . . .	= 0·8450980
Addition for the 1	= 0·0004134
Log. of 1051 . .	= 3·0216027

But it would sometimes be necessary to add a larger proportion than above mentioned, say between $\frac{n}{100}$ and $\frac{n}{1000}$. For these he adopted mentally a kind of proportionate sliding scale. Thus, for the number 601, he would see that $\frac{n}{600}$ would require $0{\cdot}000\dot{7}\dot{2}32$, so that—

Log. 600 . . .	= 2·7781513
Addition for the 1	= 0·0007232
Log. of 601 . .	= 2·7788745

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.

He preferred that the differences should be less than $\frac{n}{1000}$, and to attain this result he would often multiply the given number by some other. For example, with the number 8771, instead of taking it as—

$$(73 \times 12) + 1$$

he multiplied it by 13, which gave 11,401 = (609 × 19) + 1 and he found the logarithm thus—

Log. 600 . . .	= 2·7781512
„ 19 . . .	= 1·2787536
Addition for $\frac{0{\cdot}000043428}{1{\cdot}14}$	= 0·0000381
Log. 11401. . .	= 4·0569429
Deduct log. 13. .	= 1·1139433
Log. 877 . . .	= 2·9429996

Similarly to find the logarithm of 97 (which he could not remember at once), he multiplied it by 33, giving 3,201 (= 100 × 2^5 + 1) which he worked in the same way, saying that it was more certain than 96 + 1, and quite as easy to him.

The following statement will illustrate how these principles were applied for each of the test numbers given him:—

71 given by memory;
97 already explained;
$659 = (3 \times 2 \times 11) - 1$;
877 already explained;
$1297 = \frac{(400 \times 107) + 1}{33}$;
$6719 = (64 \times 105) - 1$;
$8963 = (27 \times 4 \times 83) - 1$;
$9973 = \frac{(9 \times 41) + 1}{37}$;
$115249 = (25 \times 461) - 1$;
[and $461 = (23 \times 20) + 1$]
$175349 = (25 \times 7014) - 1$;
[and $7014 = 7000 + 14$]
$229847 = 230000 - 153$;
$290011 = (29 \times 10000) + 11$;
$350107 = (7 \times 50000) + 107$;
$369353 = (9 \times 41000) + 369 - 16$.

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.

In these latter cases the calculation of the excess was more complicated, thus—

$$\frac{1}{1000} : 0004341 : : \frac{1{\cdot}07}{3501} : : 0{\cdot}0001328.$$

In the case of 369,353 he first added for a $\frac{1}{1000}$ part and then deducted for the 16—a good example of his readiness in devising the best way of performing the calculation.

It is worth mentioning that Mr. Bidder declared that the logarithms he knew by heart had been all calculated mentally; he had never written them down, nor looked for information into a Table for many years.

Although this great feat is probably far in advance of any other mental calculation on record, it will be seen that, when thus analysed, it is explained on the same principles as are given in Mr. Bidder's lecture of 1855, and which may be classified as follows:—

1. A good memory for retaining certain standard numbers for reference.

2. The power of performing the ordinary simple arithmetical operations of multiplication, division, proportion, &c., on large numbers with great facility, quickness, and accuracy.

3. A remarkable intuitive detection of multiple numbers, and an instantaneous perception of the factors forming them.

4. The power of what Mr. Bidder called "registration," *i.e.*, of keeping a string of successive processes and results clear in the mind, and working accurately upon them.

5. The power of devising instantly the best mode of performing a complicated problem, as regards facility, quickness, and certainty.

All these were undoubtedly the result of natural gifts; the second, third, and fourth, special and phenomenal. The fifth Mr. Bidder considered to be largely improved by practice and experience.

It is obvious that such powers cannot be taught to even the most promising and intelligent pupils; but undoubtedly the processes described may furnish useful hints for improving the power of practical calculation.

Note.

Mr. Bidder's son, Mr. G. P. Bidder, Q.C. (himself an accomplished mathematician), has favoured the Author with the following

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.

remarks on the above memorandum. They show the truth of the assertion, so confidently made by his father, that many of the principles of labour-saving in calculation which he devised could be taught and made useful.

"I remember very well my father's fondness for calculating logarithms, and the facility with which he performed the operation. We often discussed the subject, and in consequence my own attention was drawn to it, and I eventually became able to calculate them mentally without much trouble, though not at a speed comparable with his, nor with such accuracy. Moreover, I always contented myself with six places of decimals.

"Of course the great desideratum is to devise a method which relieves the mind, as far as possible, of the burthen of performing and registering long calculations. The details of my father's method, as described in the Paper, are new to me, and I think it may be interesting to compare it with my own, which, although in its main lines very similar, yet differs somewhat in form; for having obtained from him an idea of the general principles he adopted, I arrived at my own details independently. My method for numbers not large was as follows:—

"I knew by heart the logarithms of 2, 3, 7, and 11, and also the modulus 0·4343. My rule then was to select some multiple of the prime of the form $m + n$, where m is a multiple of 2, 3, 7, and 11, and n was very small, usually = 1.

$$\text{Then } \log_{10}(m+n) = \log_{10} m + \log_{10}\left(1+\frac{n}{m}\right).$$

$$\text{But } \log_{10}\left(1+\frac{n}{m}\right) = 0{\cdot}4343 \log_e\left(1+\frac{n}{m}\right),$$

$$\text{and } \log_e\left(1+\frac{n}{m}\right) = \frac{n}{m} - \tfrac{1}{2}\left(\frac{n}{m}\right)^2 + \tfrac{1}{3}\left(\frac{n}{m}\right)^3, \text{ \&c.}$$

"This I found quite practicable mentally for primes not very large.

"For very large numbers (not having my father's great power of seizing instantaneously upon component factors), I adopted another method, which I can best illustrate by an example, which recently I worked out mentally, namely, to find the logarithm of 724871.

"The method depends on the use of the powers of 1·1, 1·01, 1·001, &c., of which the logarithms are assumed to be known. It is not difficult to make out that—

$$724871 = 72 \times (1{\cdot}001)^6 \times (1{\cdot}0001)^7 \times (1{\cdot}00001)^{4\cdot5}.$$

"Adding the logarithms together, the result is obtained, ·860261. The mental strain is much less than would at first sight appear.

"George P. Bidder."

Downloaded by [UNIVERSITY OF BATH] on [11/09/16]. Copyright © ICE Publishing, all rights reserved.