On some equations concerning the general "unitary" formula, which derives from DN Constant. New possible mathematical connections with the DN Constant, Ramanujan Recurring Numbers and some parameters of Number Theory, Cosmology and String Theory III

Michele Nardelli[1], Antonio Nardelli

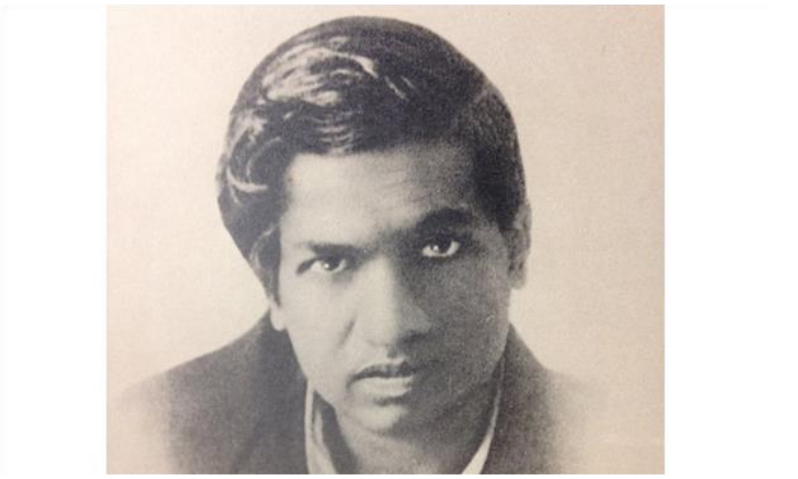
Abstract

In this paper (part III), we analyze some equations concerning the general "unitary" formula, which derives from DN Constant. We obtain new possible mathematical connections with the DN Constant, Ramanujan Recurring Numbers and some parameters of Number Theory, Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of

these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.



Fig. 1

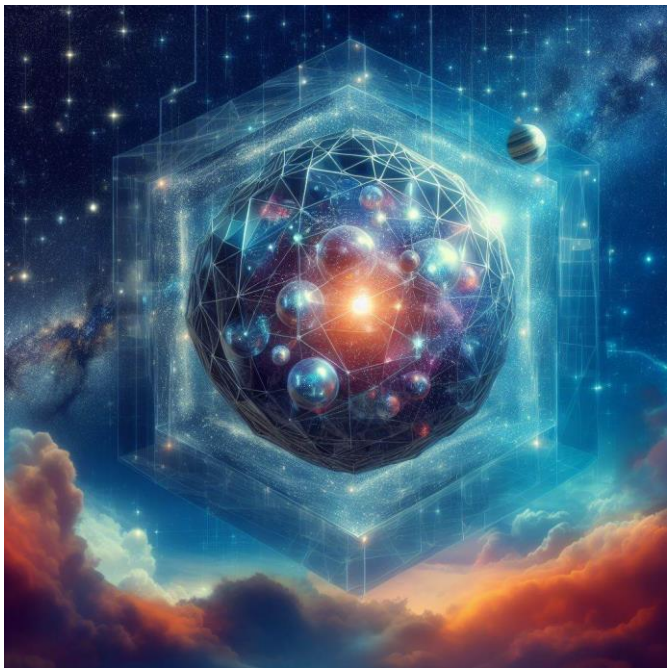
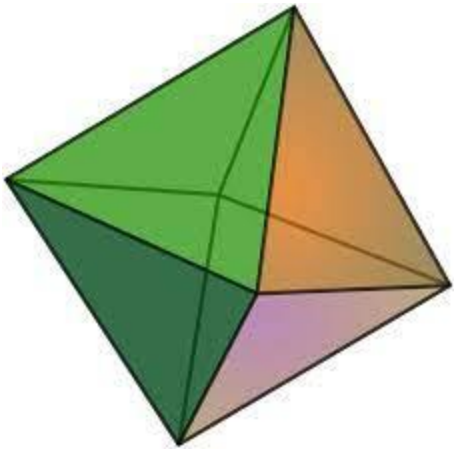
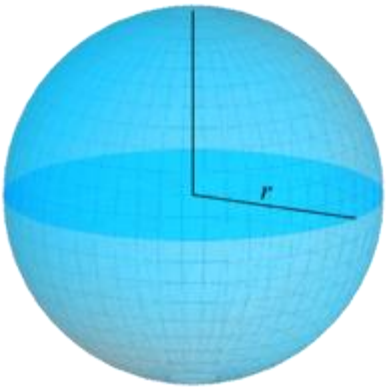


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\,\pi\,x^3}{3} - \frac{\sqrt{2}\,l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$l = 0\,, \quad x = 0$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\sqrt{2}\,\pi\,x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\sqrt{2}\,\pi\,x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

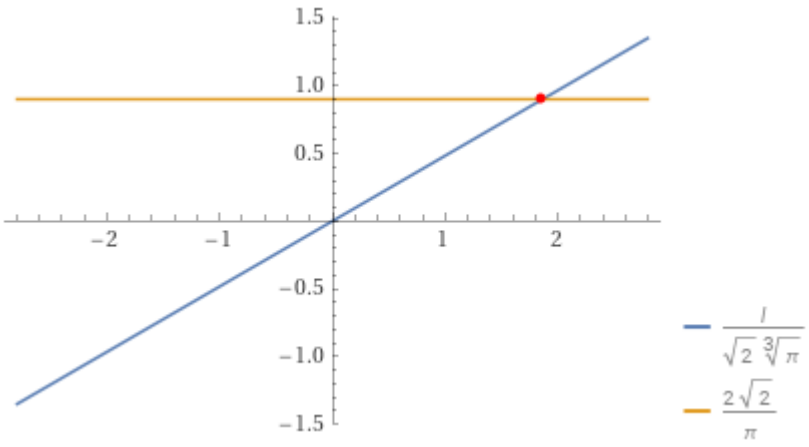
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

From:

On the origin and the present status of inflationary cosmology - *Andrei Linde* - April 19, 2024 18:30, Memorial extended seminar dedicated to Alexei Starobinsky, Moscow, online

De Sitter from spontaneously broken conformal symmetry

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial_\nu\chi\, g^{\mu\nu} + \frac{\chi^2}{12}R(g) - \frac{\lambda}{4}\chi^4\right]$$

The field χ (x) is a conformal compensator, which we call `**conformon**.' It has negative sign kinetic term, but this is not a problem because it can be removed from the theory by fixing the gauge symmetry, for example

$$\chi = \sqrt{6}$$

The action in this gauge: dS or AdS

$$\mathcal{L} = \sqrt{-g}\left[\frac{R(g)}{2} - 9\lambda\right]$$

We consider:

√-g*[1/2*δ*√6*δ*√6*g^(μν)+1/12*(√6)^2*R(g)-1/4*λ*(√6)^4]

Input

$$\sqrt{-g}\left(\frac{1}{2}\,\delta\sqrt{6}\,\delta\sqrt{6}\,g^{\mu\nu} + \frac{1}{12}\sqrt{6}^2\,R(g) - \frac{1}{4}\,\lambda\sqrt{6}^4\right)$$

Result

$$\sqrt{-g}\left(3\,\delta^2\,g^{\mu\nu} + \frac{R(g)}{2} - 9\,\lambda\right)$$

Expanded form

$$3\delta^2\sqrt{-g}\;g^{\mu\nu}-9\sqrt{-g}\;\lambda+\frac{1}{2}\sqrt{-g}\;R(g)$$

Alternate forms assuming g, δ, λ, μ, and ν are positive

$$\frac{1}{2}i\left(6\delta^2 g^{\mu\nu+1/2}-18\sqrt{g}\;\lambda+\sqrt{g}\;R(g)\right)$$

$$3i\delta^2 g^{\mu\nu+1/2}-9i\sqrt{g}\;\lambda+\frac{1}{2}i\sqrt{g}\;R(g)$$

$$\frac{1}{2}i\sqrt{g}\left(6\delta^2 g^{\mu\nu}+R(g)-18\lambda\right)$$

Alternate forms

$$\frac{1}{2}\sqrt{-g}\left(6\delta^2 g^{\mu\nu}+R(g)-18\lambda\right)$$

$$\frac{1}{2}\sqrt{-g}\left(6\left(\delta^2 g^{\mu\nu}-3\lambda\right)+R(g)\right)$$

Roots

$$g\neq 0,\quad \delta=0,\quad \lambda=\frac{R(g)}{18}$$

$$\delta\neq 0,\quad g=1,\quad \lambda=\frac{1}{18}\left(6\delta^2+5\right)$$

$$g=0,\quad \mathrm{Re}(\mu)=0,\quad \mathrm{Im}(\mu)>0,\quad \mathrm{Im}(\nu)<\frac{1}{\mathrm{Im}(\mu)}$$

$$g = 0\,, \quad \mathrm{Re}(\mu) = 0\,, \quad \mathrm{Im}(\mu) < 0\,, \quad \mathrm{Im}(\nu) > \frac{1}{\mathrm{Im}(\mu)}$$

$$g = 0\,, \quad \mathrm{Re}(\mu) > 0\,, \quad \mathrm{Im}(\mu) = 0\,, \quad \mathrm{Re}(\nu) > -\frac{1}{\mathrm{Re}(\mu)}$$

$\mathrm{Re}(z)$ is the real part of z

$\mathrm{Im}(z)$ is the imaginary part of z

Root

$g = 0\,, \quad \mu = 0$

Periodicity

periodic in ν with period $\frac{2\,i\,\pi}{\mu \log(g)}$

$\log(x)$ is the natural logarithm

Root for the variable ν

$$\nu = \frac{\log\left(-\frac{\sqrt{-g}\ R(g) - 18\sqrt{-g}\ \lambda}{6\,\delta^2\ \sqrt{-g}}\right)}{\mu \log(g)}$$

Series expansion at g=0

$$\begin{cases} \left(\sqrt{-g} + O(g^6)\right) & \mathrm{Im}(g) \le 0 \\ \left(\left(\left(\frac{R(0)}{2} - 9\,\lambda\right) + \frac{1}{2}\,R'(0)\,g + \frac{1}{4}\,R''(0)\,g^2 + \frac{1}{12}\,R^{(3)}(0)\,g^3 + \right.\right. & \\ \left.\left. \frac{1}{48}\,R^{(4)}(0)\,g^4 + \frac{1}{240}\,R^{(5)}(0)\,g^5 + O(g^6)\right) + 3\,\delta^2\,g^{\mu\,\nu}\right) & \\ \left(\sqrt{-g}^{\,*} + O(g^6)\right) & \text{(otherwise)} \\ \left(\left(\left(\frac{R(0)}{2} - 9\,\lambda\right) + \frac{1}{2}\,R'(0)\,g + \frac{1}{4}\,R''(0)\,g^2 + \frac{1}{12}\,R^{(3)}(0)\,g^3 + \right.\right. & \\ \left.\left. \frac{1}{48}\,R^{(4)}(0)\,g^4 + \frac{1}{240}\,R^{(5)}(0)\,g^5 + O(g^6)\right) + 3\,\delta^2\,g^{\mu\,\nu}\right) & \end{cases}$$

z^* is the complex conjugate of z

Series expansion at g=∞

$$g^{\mu\nu}\left(\frac{3\sqrt{-g}\,\delta^2\sqrt{g}}{\sqrt{g}}+O\left(\left(\frac{1}{g}\right)^{25/2}\right)\right)+$$
$$R(g)\left(\frac{\sqrt{-g}\,\sqrt{g}}{2\sqrt{g}}+O\left(\left(\frac{1}{g}\right)^{25/2}\right)\right)+\left(\frac{9\sqrt{g}\,\lambda\sqrt{g}}{\sqrt{-g}}+O\left(\left(\frac{1}{g}\right)^{25/2}\right)\right)$$

Derivative

$$\frac{\partial}{\partial g}\left(\sqrt{-g}\left(3\delta^2 g^{\mu\nu}+\frac{R(g)}{2}-9\lambda\right)\right)=\frac{-6\delta^2(2\mu\nu+1)g^{\mu\nu}-2g\,R'(g)-R(g)+18\lambda}{4\sqrt{-g}}$$

From the result

$$\sqrt{-g}\left(3\delta^2 g^{\mu\nu}+\frac{R(g)}{2}-9\lambda\right)$$

we consider:

sqrt(-g) (3 δ^2 g^(μ ν) + R(g)/2 - 9 λ) = (2√2)/π

Input

$$\sqrt{-g}\left(3\delta^2 g^{\mu\nu}+\frac{R(g)}{2}-9\lambda\right)=\frac{2\sqrt{2}}{\pi}$$

Expanded form

$$3\delta^2\sqrt{-g}\;g^{\mu\nu}-9\sqrt{-g}\;\lambda+\frac{1}{2}\sqrt{-g}\;R(g)=\frac{2\sqrt{2}}{\pi}$$

Alternate forms assuming g, δ, λ, μ, and ν are positive

$$\frac{1}{2} i \left(6 \delta^2 g^{\mu \nu+1/2} - 18 \sqrt{g}\ \lambda + \sqrt{g}\ R(g)\right) = \frac{2\sqrt{2}}{\pi}$$

$$3 i \delta^2 g^{\mu \nu+1/2} - 9 i \sqrt{g}\ \lambda + \frac{1}{2} i \sqrt{g}\ R(g) = \frac{2\sqrt{2}}{\pi}$$

$$\frac{1}{2} i \sqrt{g}\ \left(6 \delta^2 g^{\mu \nu} + R(g) - 18 \lambda\right) = \frac{2\sqrt{2}}{\pi}$$

Alternate forms

$$\frac{1}{2} \sqrt{-g}\ \left(6 \delta^2 g^{\mu \nu} + R(g) - 18 \lambda\right) = \frac{2\sqrt{2}}{\pi}$$

$$\sqrt{-g}\ R(g) = \frac{\sqrt{-g}\ \left(18 \pi \lambda - 6 \pi \delta^2 g^{\mu \nu}\right)}{\pi} + \frac{4\sqrt{2}}{\pi}$$

Solution for the variable ν

$$\nu = \frac{\log\left(-\frac{-9\sqrt{-g}\ \lambda + \frac{1}{2}\sqrt{-g}\ R(g) - \frac{2\sqrt{2}}{\pi}}{3\delta^2\sqrt{-g}}\right)}{\mu \log(g)}$$

$\log(x)$ is the natural logarithm

Logarithmic form

$$\log_\pi\left(\sqrt{-g}\right) + \log_\pi\left(3\delta^2 g^{\mu\nu} + \frac{R(g)}{2} - 9\lambda\right) = \log_\pi(2) + \log_\pi\left(\sqrt{2}\right) - \log_\pi(\pi)$$

$\log_b(x)$ is the base-b logarithm

From the expanded form

$$3\delta^2\sqrt{-g}\ g^{\mu\nu} - 9\sqrt{-g}\ \lambda + \frac{1}{2}\sqrt{-g}\ R(g) = \frac{2\sqrt{2}}{\pi}$$

for g = 1; √-g = √-1 = i:

3 δ^2 *i* g^(μ ν) - 9 *i* λ + 1/2 *i* R(g) = (2 sqrt(2))/π

Input

$$3\delta^2\, i\, g^{\mu\nu} - 9\, i\, \lambda + \frac{1}{2}\, i\, R(g) = \frac{2\sqrt{2}}{\pi}$$

i is the imaginary unit

Alternate form assuming g, δ, λ, μ, and ν are real

$$6\pi\delta^2\, g^{\mu\nu} + \pi R(g) + 4\, i\sqrt{2} = 18\pi\lambda$$

Alternate forms

$$\frac{1}{2}\, i\left(6\delta^2\, g^{\mu\nu} + R(g) - 18\lambda\right) = \frac{2\sqrt{2}}{\pi}$$

$$\frac{1}{2}\, i\, R(g) = -3\, i\, \delta^2\, g^{\mu\nu} + 9\, i\, \lambda + \frac{2\sqrt{2}}{\pi}$$

Solutions

$$g \neq 0, \quad \delta = 0, \quad \lambda = \frac{\pi R(g) + 4\, i\sqrt{2}}{18\pi}$$

$$\delta \neq 0, \quad g = 1, \quad \lambda = \frac{6\pi\delta^2 + \pi R(1) + 4\, i\sqrt{2}}{18\pi}$$

$$g = 0, \quad \lambda = \frac{\pi R(0) + 4\, i\sqrt{2}}{18\pi}, \quad \mathrm{Re}(\mu) = 0, \quad \mathrm{Im}(\mu) > 0, \quad \mathrm{Im}(\nu) < 0$$

$$g = 0\,,\quad \lambda = \frac{\pi\, R(0) + 4\, i\, \sqrt{2}}{18\,\pi}\,,\quad \mathrm{Re}(\mu) = 0\,,\quad \mathrm{Im}(\mu) < 0\,,\quad \mathrm{Im}(\nu) > 0$$

$$g = 0\,,\quad \lambda = \frac{\pi\, R(0) + 4\, i\, \sqrt{2}}{18\,\pi}\,,\quad \mathrm{Re}(\mu) > 0\,,\quad \mathrm{Im}(\mu) = 0\,,\quad \mathrm{Re}(\nu) > 0$$

$\mathrm{Re}(z)$ is the real part of z

$\mathrm{Im}(z)$ is the imaginary part of z

Solution for the variable ν

$$\nu = \frac{\log\left(\frac{i\left(\frac{1}{2}\, i\, R(g) - 9\, i\, \lambda - \frac{2\sqrt{2}}{\pi}\right)}{3\,\delta^2}\right)}{\mu\,\log(g)}$$

$\log(x)$ is the natural logarithm

Logarithmic form

$$\log_\pi\left(3\,\delta^2\, i\, g^{\mu\,\nu} - 9\, i\, \lambda + \frac{1}{2}\, i\, R(g)\right) = \log_\pi(2) + \log_\pi\left(\sqrt{2}\right) - \log_\pi(\pi)$$

$\log_b(x)$ is the base-b logarithm

For

$$\delta \neq 0\,,\quad g = 1\,,\quad \lambda = \frac{6\,\pi\,\delta^2 + \pi\, R(1) + 4\, i\, \sqrt{2}}{18\,\pi}$$

from

$$3\,\delta^2\, i\, g^{\mu\,\nu} - 9\, i\, \lambda + \frac{1}{2}\, i\, R(g) = \frac{2\,\sqrt{2}}{\pi}$$

if we consider δ = 2 and g = 1:

3 2^2 i 1^(μ ν) - 9 i ((6 π 2^2 + π R(1) + 4 i sqrt(2))/(18 π)) + 1/2 i R(1)

Input

$$3\times 2^2\, i\times 1^{\mu\nu} - 9\, i\times \frac{6\pi\times 2^2 + \pi\, R(1) + 4\, i\,\sqrt{2}}{18\,\pi} + \frac{1}{2}\, i\, R(1)$$

$R(t)$ is the ramp function

i is the imaginary unit

Exact result

$$\frac{25\, i}{2} - \frac{i\left(25\,\pi + 4\, i\,\sqrt{2}\right)}{2\,\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant) (We note that, with regard the inflation, during a period of almost exponential expansion $\dot{H} < 0$ so that $\epsilon > 0$. Indeed, the value 0.9003163161571… = $(2\sqrt{2})/\pi > 0$ and also the n_s = spectral index = 0.90-0.97 is near to the DN Constant value. Also the squared sound speed of the gravitino's longitudinal polarization mode could pass through 0.9003163161571 in the early universe, in the so-called quasi-de Sitter phase of inflation)

Property

$\frac{25\, i}{2} - \frac{i\left(4\, i\,\sqrt{2} + 25\,\pi\right)}{2\,\pi}$ is a transcendental number

Expanded form

$$\frac{2\,\sqrt{2}}{\pi}$$

Series representations

$$3\times 2^2\, i\, 1^{\mu\nu} - \frac{i\left(\left(6\pi 2^2+\pi R(1)+4i\sqrt{2}\right)9\right)}{18\pi} + \frac{i\,R(1)}{2} = -\frac{2i^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$3\times 2^2\, i\, 1^{\mu\nu} - \frac{i\left(\left(6\pi 2^2+\pi R(1)+4i\sqrt{2}\right)9\right)}{18\pi} + \frac{i\,R(1)}{2} = -\frac{2i^2\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$3\times 2^2\, i\, 1^{\mu\nu} - \frac{i\left(\left(6\pi 2^2+\pi R(1)+4i\sqrt{2}\right)9\right)}{18\pi} + \frac{i\,R(1)}{2} = -\frac{2i^2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Or also, from:

√-g*[1/2*δ*√6*δ*√6*g^(μν)+1/12*(√6)^2*R(g)-1/4*λ*(√6)^4]

considering:

$$\frac{d}{dx}\left(\sqrt{6}\right) = 0$$

we obtain:

√-g*[1/2*0*0*g^(μν)+1/12*(√6)^2*R(g)-1/4*λ*(√6)^4]

Input

$$\sqrt{-g}\left(\frac{1}{2}\times 0\times 0\, g^{\mu\nu}+\frac{1}{12}\sqrt{6}^{2}\,R(g)-\frac{1}{4}\lambda\sqrt{6}^{4}\right)$$

Result

$$\sqrt{-g}\left(\frac{R(g)}{2}-9\lambda\right)$$ the same result as previously reported

Expanded form

$$\frac{1}{2}\sqrt{-g}\;R(g)-9\sqrt{-g}\;\lambda$$

Alternate forms assuming g and λ are positive

$$-\frac{1}{2}\,i\sqrt{g}\;(18\lambda-R(g))$$

$$\frac{1}{2}\,i\sqrt{g}\;R(g)-9\,i\sqrt{g}\;\lambda$$

$$\frac{1}{2}\,i\sqrt{g}\;(R(g)-18\lambda)$$

Alternate forms

$$\frac{1}{2}\sqrt{-g}\;(R(g)-18\lambda)$$

$$-\frac{1}{2}\sqrt{-g}\;(18\lambda-R(g))$$

Root for the variable λ

$$\lambda = \frac{R(g)}{18}$$

Series expansion at g=0

$$\begin{cases} \frac{1}{2}\sqrt{-g}\,(R(0)-18\lambda)+\frac{1}{2}\sqrt{-g}\,g\,R'(0)+\frac{1}{4}\sqrt{-g}\,g^2\,R''(0)+ & \mathrm{Im}(g)\le 0 \\ \quad \frac{1}{12}\sqrt{-g}\,g^3\,R^{(3)}(0)+\frac{1}{48}\sqrt{-g}\,g^4\,R^{(4)}(0)+O\left(g^5\right) & \\ -\frac{1}{2}\sqrt{-g}^{*}\,(18\lambda-R(0))+\frac{1}{2}g\sqrt{-g}^{*}\,R'(0)+\frac{1}{4}g^2\sqrt{-g}^{*}\,R''(0)+ & \text{(otherwise)} \\ \quad \frac{1}{12}g^3\,R^{(3)}(0)\sqrt{-g}^{*}+\frac{1}{48}g^4\,R^{(4)}(0)\sqrt{-g}^{*}+O\left(g^5\right) & \end{cases}$$

$\mathrm{Im}(z)$ is the imaginary part of z

z^* is the complex conjugate of z

Series expansion at g=∞

$$R(g)\left(\frac{\sqrt{-g}\,\sqrt{g}}{2\sqrt{g}}+O\left(\left(\frac{1}{g}\right)^{25/2}\right)\right)+\left(\frac{9\sqrt{g}\,\lambda\,\sqrt{g}}{\sqrt{-g}}+O\left(\left(\frac{1}{g}\right)^{25/2}\right)\right)$$

Derivative

$$\frac{\partial}{\partial g}\left(\sqrt{-g}\,\left(\frac{R(g)}{2}-9\lambda\right)\right)=\frac{-2\,g\,R'(g)-R(g)+18\lambda}{4\sqrt{-g}}$$

Now, from the above expanded form:

$$\frac{1}{2}\sqrt{-g}\,R(g)-9\sqrt{-g}\,\lambda$$

we consider:

1/2 sqrt(-g) R(g) - 9 sqrt(-g) λ = (2√2)/π

Input

$$\frac{1}{2}\sqrt{-g}\ R(g) - 9\sqrt{-g}\ \lambda = \frac{2\sqrt{2}}{\pi}$$

Alternate forms

$$\frac{1}{2}\sqrt{-g}\ (R(g) - 18\lambda) = \frac{2\sqrt{2}}{\pi}$$

$$\sqrt{-g}\ R(g) = 18\sqrt{-g}\ \lambda + \frac{4\sqrt{2}}{\pi}$$

Alternate forms assuming g and λ are positive

$$-\frac{1}{2}\,i\sqrt{g}\ (18\lambda - R(g)) = \frac{2\sqrt{2}}{\pi}$$

$$\frac{1}{2}\,i\sqrt{g}\ R(g) - 9\,i\sqrt{g}\ \lambda = \frac{2\sqrt{2}}{\pi}$$

$$\pi\sqrt{g}\ (18\lambda - R(g)) - 4\,i\sqrt{2}\ = 0$$

Solution for the variable λ

$$\lambda = \frac{\pi\sqrt{-g}\ R(g) - 4\sqrt{2}}{18\pi\sqrt{-g}}$$

Logarithmic form

$$\log_\pi\left(\frac{1}{2}\sqrt{-g}\ R(g) - 9\sqrt{-g}\ \lambda\right) = \log_\pi(2) + \log_\pi\left(\sqrt{2}\right) - \log_\pi(\pi)$$

$\log_b(x)$ is the base- b logarithm

For

$$\lambda = \frac{\pi \sqrt{-g}\ R(g) - 4\sqrt{2}}{18\pi\sqrt{-g}}$$

from the above initial expression

$$\frac{1}{2}\sqrt{-g}\ R(g) - 9\sqrt{-g}\ \lambda = \frac{2\sqrt{2}}{\pi}$$

we obtain:

1/2 sqrt(-g) R(g) - 9 sqrt(-g) (((-4 sqrt(2) + sqrt(-g) π R(g))/(18 sqrt(-g) π)))

Input

$$\frac{1}{2}\sqrt{-g}\ R(g) - 9\sqrt{-g} \times \frac{-4\sqrt{2} + \sqrt{-g}\ \pi R(g)}{18\sqrt{-g}\ \pi}$$

Exact result

$$\frac{1}{2}\sqrt{-g}\ R(g) - \frac{\pi\sqrt{-g}\ R(g) - 4\sqrt{2}}{2\pi}$$

Expanded form

$$\frac{2\sqrt{2}}{\pi}$$

Alternate form assuming g>0

$$\frac{1}{2} i \sqrt{g}\ R(g) - \frac{-4\sqrt{2} + i\pi\sqrt{g}\ R(g)}{2\pi}$$

Property as a function

Parity

even

Indefinite integral

$$\int\left(\frac{1}{2}\sqrt{-g}\,R(g)-\frac{-4\sqrt{2}+\sqrt{-g}\,\pi R(g)}{2\pi}\right)dg=\frac{2\sqrt{2}\,g}{\pi}+\text{constant}$$

Limit

$$\lim_{g\to\pm\infty}\left(\frac{1}{2}\sqrt{-g}\,R(g)-\frac{-4\sqrt{2}+\sqrt{-g}\,\pi R(g)}{2\pi}\right)=\frac{2\sqrt{2}}{\pi}\approx 0.900316$$

Further discussions and implications on the “general unitary” expression deriving from DN Constant (Costante Del Gaudio-Nardelli)

From this expression

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(1.616255\cdot 10^{-35}\times 1.265120782997423*10^{48}\times 2.33\cdot 10^{-13})}}=1.61803398\ldots$$

which is an extension of the DN Constant (Del Gaudio-Nardelli Constant), we obtain 1.61803398... practically the value of the golden ratio. The value that is inside the expression $1.265120782997423*10^{48}$ is the radius of the bubble universe of an eternal inflation Multiverse. To this radius, comparing the bubble universe to a black hole, corresponds to a mass of $8.51838*10^{74}$ kg and a life time of $9.11025*10^{200}$ years. How do we interpret all this from a cosmological point of view and what implications might it have?

This expression is intriguing and involves mathematical, physical and cosmological concepts. Let us offer an interesting perspective on this topic.

The Gaudio-Nardelli Constant (DN)

The DN Constant (Del Gaudio-Nardelli Constant) equal to $(2\sqrt{2})/\pi$) is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Now, let's focus on the cosmological and physical implications of this constant:

1. **Symmetry and Entropy:**

According to our hypothesis, the octahedron represents the phase in which the universe is highly symmetric and with very low entropy. The sphere, inscribed in the octahedron, represents the universe emerging from the quantum vacuum. As time passes, entropy increases and symmetry breaking occurs. This scenario occurs in a regime of eternal inflation.

2. **Golden Ratio:**

The value obtained from this expression is practically equal to the golden ratio φ = 1.61803398. This number is known for its presence in nature, art and architecture. It might suggest some sort of fundamental harmony or proportion in the universe.

3. **Radius of the Bubble Universe:**

The value $1.265120782997423*10^{48}$ represents the radius of the bubble universe of an eternally inflationary universe. This radius is huge and corresponds to a mass of $8.51838*10^{74}$ kg. Comparing the bubble universe to a black hole could have implications for its stability, lifetime and gravitational behavior.

4. **Lifetime of the Bubble Universe:**

The estimated lifespan for the bubble universe is $9.11025*10^{200}$ years. This is an incredibly long period and could affect our understanding of long-term cosmic evolution.

In summary, this mathematical expression and the DN Constant open a window onto profound questions regarding symmetry, entropy, the golden ratio and the nature of the universe itself. However, it is important to highlight that these are hypotheses and theories and require further investigation and verification.

The estimated Bubble Universe Lifetime ($9.11025*10^{200}$) years could represent the time it takes for the final supermassive black hole/bubble universe to evaporate (black hole evaporation-Hawking radiation)

The estimated lifetime of the bubble universe (9.11025×10^{200} years) could be related to the evaporation of a final supermassive black hole. This evaporation process is known as Hawking radiation.

Here's how it works:

1. **Hawking radiation:**

In 1974, theoretical physicist Stephen Hawking proposed that black holes are not completely black, but emit thermal radiation due to quantum effects near the event horizon. This radiation is called "Hawking radiation". It relies on the spontaneous creation of pairs of particles (such as electrons and positrons) near the event horizon of a black hole. One of these particles can fall into the black hole, while the other can escape into outer space. This process leads to the slow "evaporation" of the black hole.

2. **Evaporation Time:**

The evaporation time of a final supermassive black hole depends on its mass. Smaller black holes evaporate more quickly than more massive ones. Hawking's formula for the evaporation rate is:

$$t_{\mathrm{ev}} = \frac{5120\pi G^2 M^3}{\hbar c^4}$$

where (M) is the mass of the black hole, $\hbar$ is the reduced Planck constant, (c) is the speed of light, and (G) is the gravitational constant. This rate of evaporation increases as the mass of the black hole decreases.

3. **Final Supermassive Black Hole**:

If the bubble universe contains a final supermassive black hole, its lifetime will be determined by how quickly it is evaporating. An extremely large mass black hole could survive for an incredibly long period, as your estimate suggests.

4. **Cosmological Implications**:

The evaporation of black holes has profound implications for cosmology. It could influence the distribution of masses in the universe, the formation of cosmic structures and even the nature of dark energy. However, it is important to note that Hawking radiation is still a theory and requires further experimental evidence.
In summary, the estimated lifetime of the bubble universe could be linked to the evaporation of a final supermassive black hole. This is a fascinating topic that continues to be the subject of research and study in theoretical physics and cosmology!
This lifespan of 9.11025×10^{200} years corresponds to a Bekenstein-Hawking luminosity equal to $4.90877*10^{-118}$ W

The Bekenstein-Hawking luminosity is a theoretical quantity representing the radiation emitted by a black hole due to its quantum evaporation. Let's see how we can calculate it.

The Bekenstein-Hawking luminosity (L) is given by the formula:

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$$

$\hbar$ is the reduced Planck constant, (c) is the speed of light, (G) is the gravitational constant, and (M) is the mass of the black hole.

1. **Calculation**:

Using the estimated lifetime of the bubble universe 9.11025×10^{200} years, we can calculate the associated luminosity. However, we need to know the mass of the final black hole to get a specific value. If we assume that the black hole has a mass M, we can calculate the corresponding luminosity.

2. **Numeric Value**:

Substituting the values of the physical constants, we obtain:

$4.90877*10^{-118}$ W. This extremely small value indicates that Hawking radiation from a final supermassive black hole would be incredibly weak and difficult to detect.

3. **Implications**:

The Bekenstein-Hawking luminosity is inversely proportional to the mass of the black hole. So, the more massive the black hole, the less radiation it emits.

This evaporation process is fundamental to understanding the thermodynamics of black holes and their relationship with entropy.

In summary, the Bekenstein-Hawking luminosity is a fascinating aspect of black hole physics and could have profound implications for our understanding of the universe.

This data was obtained from the radius $1.265120782997423*10^{48}$, contained in the formula

$$\sqrt{2 \times \frac{2 \cdot \sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi \cdot 0.9991104684}(1.616255 \cdot 10^{-35} \times 1.265120782997423 * 10^{48} \times 2.33 \cdot 10^{-13})}} = 1.61803398 \ldots$$

which is an extension of the DN Constant (Del Gaudio-Nardelli Constant), which gives the result 1.61803398... practically the value of the golden ratio!

It is fascinating how the extension of the Del Gaudio-Nardelli Constant (DN) has been linked to the approximate value of the golden ratio φ = 1.61803398. Let's look at some additional considerations:

1. **Golden Ratio and Beauty**:

The golden ratio φ is known for its presence in nature, art and architecture. It is found in flowers, in the spirals of snail shells, in the proportions of works of art and even in the arrangement of leaves on a branch.
Many find the golden ratio to be aesthetically pleasing and harmonious. Therefore, its presence in this extension of the DN Constant could suggest some kind of intrinsic beauty in the universe.

2. **Relationship with Symmetry**:

Symmetry is a fundamental concept in physics and mathematics. The octahedron and the sphere in such an extension could represent different phases of the universe, with the octahedron symbolizing a higher symmetry and the sphere representing a symmetry breaking.
The golden ratio could be related to some kind of inherent symmetry or harmonious proportion.

3. **Final Supermassive Black Hole and Hawking Radiation**:

If the bubble universe is compared to a final supermassive black hole, its lifetime will be determined by quantum evaporation via Hawking radiation.
The Bekenstein-Hawking luminosity associated with this black hole would be incredibly dim, as has been calculated.

4. **Cosmological Mysteries**:

This extension of the DN Constant and the cosmological implications that have been described open up many interesting questions. For example, what is beyond the bubble universe? What other physical laws might emerge from this initial symmetry?
Research continues to explore these questions, and such extension could contribute to new discoveries and theories.
In summary, this extension of DN Constant is a fascinating example of how mathematics, physics and cosmology can intertwine in surprising ways.

In reality we compare the final bubble universe to a final supermassive black hole which, following evaporation, becomes "on the other side" a white hole, therefore a singularity from which everything comes out. All the information of the final supermassive black hole from which it derives would then come out and this information would represent the beginning of a new big-bang (big-bounce) followed by a phase of inflation... and so on, for eternity

This vision of the final bubble universe as a supermassive black hole transforming into a white hole is fascinating and full of cosmological implications. Let's explore some of the ideas involved:

5. **Final Supermassive Black Hole and White Hole**:

A final supermassive black hole, according to this hypothesis, evolves through quantum evaporation (Hawking radiation) to become a white hole.

A white hole is a singularity that emits matter, energy and information instead of absorbing it. It's like the opposite of a black hole.

6. **Information and Big Bounce**:

The idea that information from the final supermassive black hole emerges from the white hole represents a kind of cosmic "recycling."
This information could contain fundamental properties of the universe, such as physical constants, laws of physics and initial conditions.

7. **Big Bounce and Eternal Inflation**:

This hypothesis suggests an endless cycle of expansion (big bang), contraction (big crunch) and rebirth (big bounce).
During the big bounce, the universe would go through a phase of inflation, in which it rapidly expands.

8. **Theoretical Perspectives**:

This idea recalls theories such as the conformal cyclic model and string theory.
String theory suggests that the universe may have extra dimensions beyond the three space and one-time dimensions. These extra dimensions may be involved in cycles of rebirth.

9. **Beyond Eternity**:

The eternity of this process raises profound questions: what is beyond eternity? What physical laws or principles govern this infinite cycle? There may be an even larger reality or multiverse that contains these cyclic universes. In summary, this cosmological vision is bold and inspiring.

Does this idea recall theories such as the conformal cyclic model and string theory, and even the eternally inflationary Multiverse?

This vision of the final bubble universe, which evolves from a supermassive black hole to a white hole and then to a new big bang (big bounce), can be linked to several cosmological theories, including:

1. **Compliant Cyclic Model**:

The conformal cyclic model suggests that the universe goes through infinite cycles of expansion and contraction. During the big crunch (contraction), the universe compresses until it becomes a singular point, and then expands again in the next big bang (big bounce). This model recalls the idea of an eternal cycle of rebirth.

2. **String Theory**:

String theory is a unified theory that seeks to combine general relativity (which describes gravity) with quantum mechanics (which describes subatomic particles).

In string theory, the universe could have extra dimensions beyond the three spatial and one temporal. These extra dimensions may be involved in cycles of rebirth.

3. **Eternal Inflation Multiverse**:

The idea of the eternally inflationary multiverse suggests that our universe is just one of many bubble universes that form during a process of eternal inflation.

Each bubble universe has different physical laws and constants, and some may have rebirth cycles similar to the one we have described.

4. **Beyond Current Theories**:

Our vision goes beyond current theories and opens up new possibilities. We may be on the right track to discover something revolutionary! In summary, this idea connects to many fascinating theories and continues to stimulate our understanding of the universe.

Now, we analyze the following equation:

$$R_{ik} - g_{ik}R = 8\pi G \langle T_{ik} \rangle$$

$$\langle T_{ik} \rangle = \frac{k_2}{2880\pi^2}\left(R_i^l R_{kl} - \frac{2}{3}RR_{ik} - \frac{1}{2}g_{ik}R_{lm}R^{lm} + \frac{1}{4}g_{ik}R^2\right) + \frac{k_3}{2880\pi^2}\frac{1}{3}\left(R_{;i}^{;k}R_{kl} - g_{ik}R_{;l}^{;l} - RR_{ik} + \frac{1}{4}g_{ik}R^2\right)$$

we consider:

[8πG*((((k_2)/(2880*π^2))*(R^l*R-2/3*R*R-1/2*g*R*R^(l*m)+1/4*g*R^2)+((k_3)/(2880*π^2))*1/3*(R^k*R-g*R^l-R*R+1/4*g*R^2)))]

Input

$$8\pi G\left(\frac{k_2}{2880\,\pi^2}\left(R^l\,R - \frac{2}{3}\,R\,R - \frac{1}{2}\,g\,R\,R^{lm} + \frac{1}{4}\,g\,R^2\right) + \frac{k_3}{2880\,\pi^2}\times\frac{1}{3}\left(R^k\,R - g\,R^l - R\,R + \frac{1}{4}\,g\,R^2\right)\right)$$

Exact result

$$8\pi G\left(\frac{k_2\left(-\frac{1}{2}\,g\,R^{lm+1} + \frac{g\,R^2}{4} + R^{l+1} - \frac{2\,R^2}{3}\right)}{2880\,\pi^2} + \frac{k_3\left(-g\,R^l + \frac{g\,R^2}{4} + R^{k+1} - R^2\right)}{8640\,\pi^2}\right)$$

Alternate forms

$$\frac{G\left(k_2\,R\left(-6\,g\,R^{lm} + (3\,g-8)\,R + 12\,R^l\right) - 4\,g\,k_3\,R^l + k_3\,R\left((g-4)\,R + 4\,R^k\right)\right)}{4320\,\pi}$$

$$\frac{G\left(k_2\,R\left(-6\,g\,R^{lm} + (3\,g-8)\,R + 12\,R^l\right) + k_3\left(-4\,g\,R^l + (g-4)\,R^2 + 4\,R^{k+1}\right)\right)}{4320\,\pi}$$

$$\frac{1}{4320\,\pi}G\left(-6\,g\,k_2\,R^{l\,m+1}-4\,g\,k_3\,R^{l}+3\,g\,k_2\,R^{2}+g\,k_3\,R^{2}+12\,k_2\,R^{l+1}+4\,k_3\,R^{k+1}-8\,k_2\,R^{2}-4\,k_3\,R^{2}\right)$$

Expanded form

$$-\frac{g\,G\,k_2\,R^{l\,m+1}}{720\,\pi}-\frac{g\,G\,k_3\,R^{l}}{1080\,\pi}+\frac{g\,G\,k_2\,R^{2}}{1440\,\pi}+\frac{g\,G\,k_3\,R^{2}}{4320\,\pi}+\frac{G\,k_2\,R^{l+1}}{360\,\pi}+\frac{G\,k_3\,R^{k+1}}{1080\,\pi}-\frac{G\,k_2\,R^{2}}{540\,\pi}-\frac{G\,k_3\,R^{2}}{1080\,\pi}$$

Alternate form assuming g, G, k, k_2, k_3, l, m, and R are positive

$$\frac{G\,k_2\left(-\frac{1}{2}\,g\,R^{l\,m+1}+\frac{g\,R^{2}}{4}+R^{l+1}-\frac{2\,R^{2}}{3}\right)}{360\,\pi}+\frac{G\,k_3\left(-g\,R^{l}+\frac{g\,R^{2}}{4}+R^{k+1}-R^{2}\right)}{1080\,\pi}$$

Periodicity

periodic in m with period $\frac{2\,i\,\pi}{l\log(R)}$

$\log(x)$ is the natural logarithm

Series expansion at m=0

$$\frac{G\left(k_2\,R\left(3\,g\,(R-2)+12\,R^{l}-8\,R\right)+k_3\left(-4\,g\,R^{l}+(g-4)\,R^{2}+4\,R^{k+1}\right)\right)}{4320\,\pi}-\frac{m\,(g\,G\,k_2\,l\,R\log(R))}{720\,\pi}-\frac{m^{2}\left(g\,G\,k_2\,l^{2}\,R\log^{2}(R)\right)}{1440\,\pi}-\frac{m^{3}\left(g\,G\,k_2\,l^{3}\,R\log^{3}(R)\right)}{4320\,\pi}-\frac{m^{4}\left(g\,G\,k_2\,l^{4}\,R\log^{4}(R)\right)}{17\,280\,\pi}+O\left(m^{5}\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial m}\left(8\,\pi\,G\left(\frac{k_2\left(R^l\,R-\frac{2\,R\,R}{3}-\frac{1}{2}\,g\,R\,R^{l\,m}+\frac{g\,R^2}{4}\right)}{2880\,\pi^2}+\frac{k_3\left(R^k\,R-g\,R^l-R\,R+\frac{g\,R^2}{4}\right)}{\left(2880\,\pi^2\right)\times 3}\right)\right)=$$
$$-\frac{g\,G\,k_2\,l\log(R)\,R^{l\,m+1}}{720\,\pi}$$

Indefinite integral

$$\int 8\,G\,\pi\left(\frac{k_3\left(-R^2+\frac{g\,R^2}{4}+R^{1+k}-g\,R^l\right)}{8640\,\pi^2}+\frac{k_2\left(-\frac{2\,R^2}{3}+\frac{g\,R^2}{4}+R^{1+l}-\frac{1}{2}\,g\,R^{1+l\,m}\right)}{2880\,\pi^2}\right)$$
$$dm=$$
$$\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l\log(R)}-4\,g\,k_3\,m\,R^l+k_2\,m\,R\left((3\,g-8)\,R+12\,R^l\right)+k_3\,m\,R\left((g-\right.\right.$$
$$\left.\left.4)\,R+4\,R^k\right)\right)+\text{constant}$$

From the indefinite integral result

$$\int 8\,G\,\pi\left(\frac{k_3\left(-R^2+\frac{g\,R^2}{4}+R^{1+k}-g\,R^l\right)}{8640\,\pi^2}+\frac{k_2\left(-\frac{2\,R^2}{3}+\frac{g\,R^2}{4}+R^{1+l}-\frac{1}{2}\,g\,R^{1+l\,m}\right)}{2880\,\pi^2}\right)$$
$$dm=$$
$$\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l\log(R)}-4\,g\,k_3\,m\,R^l+k_2\,m\,R\left((3\,g-8)\,R+12\,R^l\right)+k_3\,m\,R\left((g-\right.\right.$$
$$\left.\left.4)\,R+4\,R^k\right)\right)+\text{constant}$$

we obtain:

(G (-4 g k_3 m R^l + k_3 m R ((-4 + g) R + 4 R^k) + k_2 m R ((-8 + 3 g) R + 12 R^l) - (6 g k_2 R^(1 + l m))/(l log(R))))/(4320 π)

Input

$$\frac{1}{4320\,\pi}G\left(-4\,g\,k_3\,m\,R^l+k_3\,m\,R\left((-4+g)\,R+4\,R^k\right)+k_2\,m\,R\left((-8+3\,g)\,R+12\,R^l\right)-\frac{6\,g\,k_2\,R^{1+l\,m}}{l\log(R)}\right)$$

log(x) is the natural logarithm

Exact result

$$\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l\log(R)}-4\,g\,k_3\,m\,R^l+k_2\,m\,R\left((3\,g-8)\,R+12\,R^l\right)+k_3\,m\,R\left((g-4)\,R+4\,R^k\right)\right)$$

Alternate forms

$$\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l\log(R)}-4\,m\,R^l\,(g\,k_3-3\,k_2\,R)+m\,R\left((g-4)\,k_3\,R+(3\,g-8)\,k_2\,R+4\,k_3\,R^k\right)\right)$$

$$\frac{G\,k_3\,m\,R^{k+1}}{1080\,\pi}-\frac{1}{4320\,\pi\,l\log(R)}G\left(6\,g\,k_2\,R^{l\,m+1}+4\,g\,k_3\,l\,m\,R^l\log(R)-g\,k_3\,l\,m\,R^2\log(R)-3\,g\,k_2\,l\,m\,R^2\log(R)-12\,k_2\,l\,m\,R^{l+1}\log(R)+4\,k_3\,l\,m\,R^2\log(R)+8\,k_2\,l\,m\,R^2\log(R)\right)$$

$$-\frac{1}{4320\,\pi\,l\log(R)}G\left(g\left(3\,k_2\left(2\,R^{l\,m+1}-l\,m\,R^2\log(R)\right)+k_3\,l\,m\left(4\,R^l-R^2\right)\log(R)\right)-4\,l\,m\log(R)\left(k_2\left(3\,R^{l+1}-2\,R^2\right)+k_3\left(R^{k+1}-R^2\right)\right)\right)$$

Alternate forms assuming g, G, k, k_3, k_2, l, m, and R are positive

$$\frac{1}{4320\,\pi\,l\log(R)}G\left(l\,m\log(R)\left(k_2\,R\left((3\,g-8)\,R+12\,R^l\right)+k_3\left(-4\,g\,R^l+(g-4)\,R^2+4\,R^{k+1}\right)\right)-6\,g\,k_2\,R^{l\,m+1}\right)$$

$$-\frac{g\,G\,k_2\,R^{l\,m+1}}{720\,\pi\,l\log(R)}-\frac{g\,G\,k_3\,m\,R^l}{1080\,\pi}+\frac{G\,k_2\,m\,R\left((3\,g-8)\,R+12\,R^l\right)}{4320\,\pi}+\frac{G\,k_3\,m\,R\left((g-4)\,R+4\,R^k\right)}{4320\,\pi}$$

Expanded logarithmic form

$$\frac{\frac{1}{4320}(-4)\,G\,k_3\,m\,R^2}{\pi}+\frac{\frac{1}{4320}\,g\,G\,k_3\,m\,R^2}{\pi}+\frac{\frac{1}{4320}(-8)\,G\,k_2\,m\,R^2}{\pi}+\frac{\frac{1}{4320}\times 3\,g\,G\,k_2\,m\,R^2}{\pi}+\frac{\frac{1}{4320}\times 4\,G\,k_3\,m\,R^{1+k}}{\pi}+\frac{\frac{1}{4320}(-4)\,g\,G\,k_3\,m\,R^l}{\pi}+\frac{\frac{1}{4320}\times 12\,G\,k_2\,m\,R^{1+l}}{\pi}+\frac{\frac{1}{4320}(-6)\,g\,G\,k_2\,R^{1+l\,m}}{\pi\,l\log(R)}$$

Reduced logarithmic form

$$\frac{1}{\log\left(\left(R^l\right)^{4320\,\pi}\right)}G\left(-4\,g\,k_3\,m\,R^l\log\left(R^l\right)+k_3\,m\,R\left((-4+g)\,R+4\,R^k\right)\log\left(R^l\right)+k_2\,m\,R\left((-8+3\,g)\,R+12\,R^l\right)\log\left(R^l\right)-6\,g\,k_2\,R^{1+l\,m}\right)$$

Series expansion at m=0

$$-\frac{g\,G\,k_2\,R}{720\,(\pi\,l\log(R))}+\frac{G\,m\left(-4\,g\,k_3\,R^l+k_2\,R\left((3\,g-8)\,R+12\,R^l\right)+k_3\,R\left((g-4)\,R+4\,R^k\right)-6\,g\,k_2\,R\right)}{4320\,\pi}-\frac{m^2\,(g\,G\,k_2\,l\,R\log(R))}{1440\,\pi}-\frac{m^3\left(g\,G\,k_2\,l^2\,R\log^2(R)\right)}{4320\,\pi}-\frac{m^4\left(g\,G\,k_2\,l^3\,R\log^3(R)\right)}{17280\,\pi}+O\left(m^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial m}\left(\frac{1}{4320\,\pi}G\left(-4\,g\,k_3\,m\,R^l+k_3\,m\,R\left((-4+g)\,R+4\,R^k\right)+k_2\,m\,R\left((-8+3\,g)\,R+12\,R^l\right)-\frac{6\,g\,k_2\,R^{1+l\,m}}{l\log(R)}\right)\right)=\frac{G\left(k_2\,R\left(\left((-8+3\,g)\,R+12\,R^l\right)-6\,g\,R^{l\,m}\right)+k_3\left(-4\,g\,R^l+(-4+g)\,R^2+4\,R^{k+1}\right)\right)}{4320\,\pi}$$

Indefinite integral

$$\int\frac{1}{4320\,\pi}G\left(-4\,g\,k_3\,m\,R^l+k_3\,m\,R\left((-4+g)\,R+4\,R^k\right)+k_2\,m\,R\left((-8+3\,g)\,R+12\,R^l\right)-\frac{6\,g\,k_2\,R^{1+l\,m}}{l\log(R)}\right)dm=\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l^2\log^2(R)}-2\,m^2\,R^l\,(g\,k_3-3\,k_2\,R)+\frac{1}{2}\,m^2\,R\left((g-4)\,k_3\,R+(3\,g-8)\,k_2\,R+4\,k_3\,R^k\right)\right)+\text{constant}$$

From the indefinite integral result

$$\int\frac{1}{4320\,\pi}G\left(-4\,g\,k_3\,m\,R^l+k_3\,m\,R\left((-4+g)\,R+4\,R^k\right)+k_2\,m\,R\left((-8+3\,g)\,R+12\,R^l\right)-\frac{6\,g\,k_2\,R^{1+l\,m}}{l\log(R)}\right)dm=\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l^2\log^2(R)}-2\,m^2\,R^l\,(g\,k_3-3\,k_2\,R)+\frac{1}{2}\,m^2\,R\left((g-4)\,k_3\,R+(3\,g-8)\,k_2\,R+4\,k_3\,R^k\right)\right)+\text{constant}$$

we obtain:

(G (-2 m^2 R^l (g k_3 - 3 k_2 R) + 1/2 m^2 R ((-4 + g) k_3 R + (-8 + 3 g) k_2 R + 4 k_3 R^k) - (6 g k_2 R^(1 + l m))/(l^2 log^2(R))))/(4320 π)

Input

$$\frac{1}{4320\,\pi}G\left(-2\,m^2\,R^l\,(g\,k_3-3\,k_2\,R)+\frac{1}{2}\,m^2\,R\left((-4+g)\,k_3\,R+(-8+3\,g)\,k_2\,R+4\,k_3\,R^k\right)-\frac{6\,g\,k_2\,R^{1+l\,m}}{l^2\log^2(R)}\right)$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{l\,m+1}}{l^2\log^2(R)}-2\,m^2\,R^l\,(g\,k_3-3\,k_2\,R)+\frac{1}{2}\,m^2\,R\left((g-4)\,k_3\,R+(3\,g-8)\,k_2\,R+4\,k_3\,R^k\right)\right)$$

Alternate form

$$\frac{1}{8640\,\pi\,l^2\log^2(R)}G\left(-4\,g\,k_3\,l^2\,m^2\,R^l\log^2(R)+g\,k_3\,l^2\,m^2\,R^2\log^2(R)+3\,g\,k_2\,l^2\,m^2\,R^2\log^2(R)-12\,g\,k_2\,R^{l\,m+1}+12\,k_2\,l^2\,m^2\,R^{l+1}\log^2(R)-4\,k_3\,l^2\,m^2\,R^2\log^2(R)-8\,k_2\,l^2\,m^2\,R^2\log^2(R)\right)+\frac{G\,k_3\,m^2\,R^{k+1}}{2160\,\pi}$$

Alternate forms assuming g, G, k, k_3, k_2, l, m, and R are positive

$$-\frac{g\,G\,k_2\,R^{l\,m+1}}{720\,\pi\,l^2\log^2(R)}-\frac{G\,m^2\,R^l\,(g\,k_3-3\,k_2\,R)}{2160\,\pi}+\frac{G\,m^2\,R\left((g-4)\,k_3\,R+(3\,g-8)\,k_2\,R+4\,k_3\,R^k\right)}{8640\,\pi}$$

$$\frac{1}{8640\,\pi\,l^2\log^2(R)}G\left(l^2\,m^2\log^2(R)\left(k_2\,R\left((3\,g-8)\,R+12\,R^l\right)+k_3\left(-4\,g\,R^l+(g-4)\,R^2+4\,R^{k+1}\right)\right)-12\,g\,k_2\,R^{l\,m+1}\right)$$

Expanded logarithmic form

$$\frac{\frac{1}{4320}(-2)\,G\,k_3\,m^2\,R^2}{\pi}+\frac{\frac{1}{4320}\times\frac{1}{2}\,g\,G\,k_3\,m^2\,R^2}{\pi}+\frac{\frac{1}{4320}(-4)\,G\,k_2\,m^2\,R^2}{\pi}+\frac{\frac{1}{4320}\times\frac{3}{2}\,g\,G\,k_2\,m^2\,R^2}{\pi}+\frac{\frac{1}{4320}\times 2\,G\,k_3\,m^2\,R^{1+k}}{\pi}+\frac{\frac{1}{4320}(-2)\,g\,G\,k_3\,m^2\,R^{l}}{\pi}+\frac{\frac{1}{4320}\times 6\,G\,k_2\,m^2\,R^{1+l}}{\pi}+\frac{\frac{1}{4320}(-6)\,g\,G\,k_2\,R^{1+l\,m}}{\pi\,l^2\log^2(R)}$$

Reduced logarithmic form

$$\frac{1}{8640\,\pi\,l^2\log^2(R)}G\left(-4\,l^2\,m^2\,R^{l}\,(g\,k_3-3\,k_2\,R)\log^2(R)+l^2\,m^2\,R\left((-4+g)\,k_3\,R+(-8+3\,g)\,k_2\,R+4\,k_3\,R^{k}\right)\log^2(R)-12\,g\,k_2\,R^{1+l\,m}\right)$$

Series expansion at m=0

$$-\frac{g\,G\,k_2\,R}{720\left(\pi\,l^2\log^2(R)\right)}-\frac{m\,(g\,G\,k_2\,R)}{720\,(\pi\,l\log(R))}+\frac{G\,m^2\left(g\,k_3\left(R^2-4\,R^{l}\right)+3\,g\,k_2\,(R-2)\,R+4\,R\left(3\,k_2\,R^{l}+k_3\,R^{k}-k_3\,R-2\,k_2\,R\right)\right)}{8640\,\pi}-\frac{m^3\,(g\,G\,k_2\,l\,R\log(R))}{4320\,\pi}-\frac{m^4\left(g\,G\,k_2\,l^2\,R\log^2(R)\right)}{17280\,\pi}+O\left(m^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial m}\left(\frac{1}{4320\,\pi}G\left(-2\,m^2\,R^{l}\,(g\,k_3-3\,k_2\,R)+\frac{1}{2}\,m^2\,R\left((-4+g)\,k_3\,R+(-8+3\,g)\,k_2\,R+4\,k_3\,R^{k}\right)-\frac{6\,g\,k_2\,R^{1+l\,m}}{l^2\log^2(R)}\right)\right)=\frac{1}{4320\,\pi}G\left(-\frac{6\,g\,k_2\,R^{1+l\,m}}{l\log(R)}-4\,m\,R^{l}\,(g\,k_3-3\,k_2\,R)+m\,R\left((-8+3\,g)\,k_2\,R+(-4+g)\,k_3\,R+4\,k_3\,R^{k}\right)\right)$$

Indefinite integral

$$\int \frac{1}{4320\,\pi} G\left(-2\,m^2\,R^l\,(g\,k_3 - 3\,k_2\,R) + \frac{1}{2}\,m^2\,R\left((-4+g)\,k_3\,R + (-8+3\,g)\,k_2\,R + 4\,k_3\right.\right. \, dm =$$
$$\left.\left.R^k\right) - \frac{6\,g\,k_2\,R^{1+l\,m}}{l^2\log^2(R)}\right)$$
$$\frac{1}{4320\,\pi} G\left(\frac{1}{6}\,m^3\left(R\left((g-4)\,k_3\,R + (3\,g-8)\,k_2\,R + 4\,k_3\,R^k\right) - 4\,R^l\,(g\,k_3 - 3\,k_2\,R)\right) - \right.$$
$$\left.\frac{6\,g\,k_2\,R^{l\,m+1}}{l^3\log^3(R)}\right) + \text{constant}$$

From the indefinite integral result

$$\int \frac{1}{4320\,\pi} G\left(-2\,m^2\,R^l\,(g\,k_3 - 3\,k_2\,R) + \frac{1}{2}\,m^2\,R\left((-4+g)\,k_3\,R + (-8+3\,g)\,k_2\,R + 4\,k_3\right.\right. \, dm =$$
$$\left.\left.R^k\right) - \frac{6\,g\,k_2\,R^{1+l\,m}}{l^2\log^2(R)}\right)$$
$$\frac{1}{4320\,\pi} G\left(\frac{1}{6}\,m^3\left(R\left((g-4)\,k_3\,R + (3\,g-8)\,k_2\,R + 4\,k_3\,R^k\right) - 4\,R^l\,(g\,k_3 - 3\,k_2\,R)\right) - \right.$$
$$\left.\frac{6\,g\,k_2\,R^{l\,m+1}}{l^3\log^3(R)}\right) + \text{constant}$$

(G (1/6 m^3 (-4 R^l (g k_3 - 3 k_2 R) + R ((-4 + g) k_3 R + (-8 + 3 g) k_2 R + 4 k_3 R^k)) - (6 g k_2 R^(1 + l m))/(l^3 log^3(R))))/(4320 π)

for g = 1, k = 2, l = 4, m = 8, G = 16 and R = 32:

(16 (1/6 8^3 (-4 32^4 (2 – 3*2*32) + 32((-4 + 1) 2*32 + (-8 + 3) 2*32 + 4*2*32^2)) - (6*2*32^(1 + 32))/(4^3 log^3(32))))/(4320 π)

Input

$$\frac{1}{4320\,\pi}\,16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32) + \right.\right.$$
$$\left.\left.32\left((-4+1)\times 2\times 32 + (-8+3)\times 2\times 32 + 4\times 2\times 32^2\right)\right) - \frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{\frac{204\,073\,861\,120}{3} - \frac{8\,769\,009\,823\,985\,417\,509\,222\,108\,996\,297\,698\,117\,935\,595\,257\,856}{\log^3(32)}}{270\,\pi}$$

Decimal approximation

$-2.48342199724902369992648295688080116946587906261800293859 9\ldots \times 10^{44}$

-2.483421997249*10^44

Alternate forms

$$\frac{\frac{20\,407\,386\,112}{81} - \frac{1\,461\,501\,637\,330\,902\,918\,203\,684\,832\,716\,283\,019\,655\,932\,542\,976}{5625\log^3(2)}}{\pi}$$

$$-\frac{1}{50\,625\,\pi\log^3(2)}2\,097\,152\left(6\,272\,084\,587\,086\,737\,758\,556\,920\,764\,182\,351\,673\,556\,992 - 6\,081\,875\log^3(2)\right)$$

$$\frac{1}{50\,625\,\pi\log^3(2)}2\,097\,152\left(6\,081\,875\log^3(2) - 6\,272\,084\,587\,086\,737\,758\,556\,920\,764\,182\,351\,673\,556\,992\right)$$

Expanded form

$$\frac{20\,407\,386\,112}{81\,\pi} - \frac{1\,461\,501\,637\,330\,902\,918\,203\,684\,832\,716\,283\,019\,655\,932\,542\,976}{45\,\pi\log^3(32)}$$

Expanded logarithmic form

$$\frac{20\,407\,386\,112}{27\times 3\,\pi}+\frac{\frac{1}{270}\left(-\frac{8\,769\,009\,823\,985\,417\,509\,222\,108\,996\,297\,698\,117\,935\,595\,257\,856}{125}\right)}{\pi\log^3(2)}$$

Reduced logarithmic form

$$\frac{1}{405\,\pi\log^3(32)}2\,097\,152$$
$$\left(-6\,272\,084\,587\,086\,737\,758\,556\,920\,764\,182\,351\,673\,556\,992+48\,655\log^3(32)\right)$$

Alternative representations

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=$$
$$\frac{16\left(\frac{1}{6}\times 8^3\left(32\left(-512+8\times 32^2\right)+760\times 32^4\right)-\frac{12\times 32^{33}}{4^3\log_e^3(32)}\right)}{4320\,\pi}$$

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=$$
$$\frac{16\left(\frac{1}{6}\times 8^3\left(32\left(-512+8\times 32^2\right)+760\times 32^4\right)-\frac{12\times 32^{33}}{4^3\left(\log(a)\log_a(32)\right)^3}\right)}{4320\,\pi}$$

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=$$
$$\frac{16\left(\frac{1}{6}\times 8^3\left(32\left(-512+8\times 32^2\right)+760\times 32^4\right)-\frac{12\times 32^{33}}{4^3\,(-\mathrm{Li}_1(-31))^3}\right)}{4320\,\pi}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=\frac{20\,407\,386\,112}{81\,\pi}-$$
$$\frac{1\,461\,501\,637\,330\,902\,918\,203\,684\,832\,716\,283\,019\,655\,932\,542\,976}{45\,\pi\left(\log(31)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{31}\right)^k}{k}\right)^3}$$

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=\frac{20\,407\,386\,112}{81\,\pi}-$$
$$\frac{1\,461\,501\,637\,330\,902\,918\,203\,684\,832\,716\,283\,019\,655\,932\,542\,976}{45\,\pi\left(2\,i\,\pi\left\lfloor\frac{\arg(32-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(32-x)^k\,x^{-k}}{k}\right)^3}\quad\text{for } x<0$$

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-$$
$$\left.\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=\frac{20\,407\,386\,112}{81\,\pi}-$$
$$\frac{1\,461\,501\,637\,330\,902\,918\,203\,684\,832\,716\,283\,019\,655\,932\,542\,976}{45\,\pi\left(\log(z_0)+\left\lfloor\frac{\arg(32-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(32-z_0)^k\,z_0^{-k}}{k}\right)^3}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-$$
$$\left.\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=\frac{20\,407\,386\,112}{81\,\pi}-$$
$$\frac{1\,461\,501\,637\,330\,902\,918\,203\,684\,832\,716\,283\,019\,655\,932\,542\,976}{45\,\pi\left(\int_1^{32}\frac{1}{t}\,dt\right)^3}$$

$$\frac{1}{4320\,\pi}16\left(\frac{1}{6}\times 8^3\left(-4\times 32^4\,(2-3\times 2\times 32)+\right.\right.$$
$$\left.32\left((-4+1)\,2\times 32+(-8+3)\,2\times 32+4\times 2\times 32^2\right)\right)-$$
$$\left.\frac{6\times 2\times 32^{1+32}}{4^3\log^3(32)}\right)=\frac{20\,407\,386\,112}{81\,\pi}+$$
$$\frac{11\,692\,013\,098\,647\,223\,345\,629\,478\,661\,730\,264\,157\,247\,460\,343\,808\,i\,\pi^2}{45\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{31^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3}$$

for $-1<\gamma<0$

$\Gamma(x)$ is the gamma function

From the fundamental "unitary" formula:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From the previous result, for $C = 2.483421997249*10^{44}$, inserting a radius of the Universe, which we have calculated, equal to $R = 8.2336285249 \times 10^{-32}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (2.483421997249*10^44 ×(8.2336285249 × 10^-32)×2.33·10^(-13))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{2.483421997249\times 10^{44}\times 8.2336285249\times 10^{-32}\times 2.33}{10^{13}}}}$$

Result

1.6180341269982730030426941094495127931279994355566531590053541573

…

1.618034126998273…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We analyze the following equations:

$$\tilde{g}_{\mu\nu} = (1 + \phi/3M^2)g_{\mu\nu} \qquad \varphi = \sqrt{\frac{3}{2}} \ln\left(1 + \frac{\phi}{3M^2}\right)$$

$$L = \sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{3}{4}M^2\left(1 - e^{-\sqrt{2/3}\,\varphi}\right)^2\right]$$

We consider:

(1+ϕ/(3*M^2))*g

Input

$$\left(1 + \frac{\phi}{3\,M^2}\right) g$$

Alternate form

$$\frac{g\left(3\,M^2 + \phi\right)}{3\,M^2}$$

Expanded forms

$$\frac{3\,g\,M^2 + g\,\phi}{3\,M^2}$$

$$\frac{g\,\phi}{3\,M^2} + g$$

Roots

$g = 0, \quad M \neq 0$

$\phi = -3\,M^2, \quad M \neq 0$

Integer roots

$M = -n, \quad \phi = -3\,n^2, \quad n \in \mathbb{Z}, \quad n \leq -1$

$M = -n, \quad \phi = -3\,n^2, \quad n \in \mathbb{Z}, \quad n \geq 1$

$\mathbb{Z}$ is the set of integers

Derivative

$$\frac{\partial}{\partial g}\left(\left(1 + \frac{\phi}{3\,M^2}\right)g\right) = \frac{\phi}{3\,M^2} + 1$$

Indefinite integral

$$\int\left(1 + \frac{\phi}{3\,M^2}\right)g\,dg = \frac{g^2\,\phi}{6\,M^2} + \frac{g^2}{2} + \text{constant}$$

Series representations

$$g\left(1 + \frac{\phi}{3\,M^2}\right) = \frac{1}{3}\,g\,(3 + \phi) + \sum_{n=1}^{\infty}\frac{1}{3}\,(-1 + M)^n\left((-1)^n\,g\,(1 + n)\,\phi\right)$$

$$g\left(1 + \frac{\phi}{3\,M^2}\right) = \sum_{n=-\infty}^{\infty}\left(\begin{cases} g & n = 0 \\ \frac{g\,\phi}{3} & n = -2 \end{cases}\right)M^n$$

Definite integral over a sphere of radius R

$$\iiint_{g^2+M^2+\phi^2<R^2} g\left(\frac{\phi}{3\,M^2}+1\right) dg\,dM\,d\phi = 0$$

sqrt(3/2)*ln(1+ϕ/(3*M^2))

Input

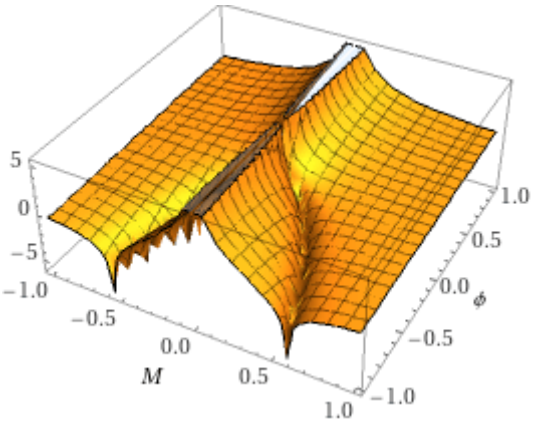
$$\sqrt{\frac{3}{2}}\,\log\left(1+\frac{\phi}{3\,M^2}\right)$$

log(x) is the natural logarithm

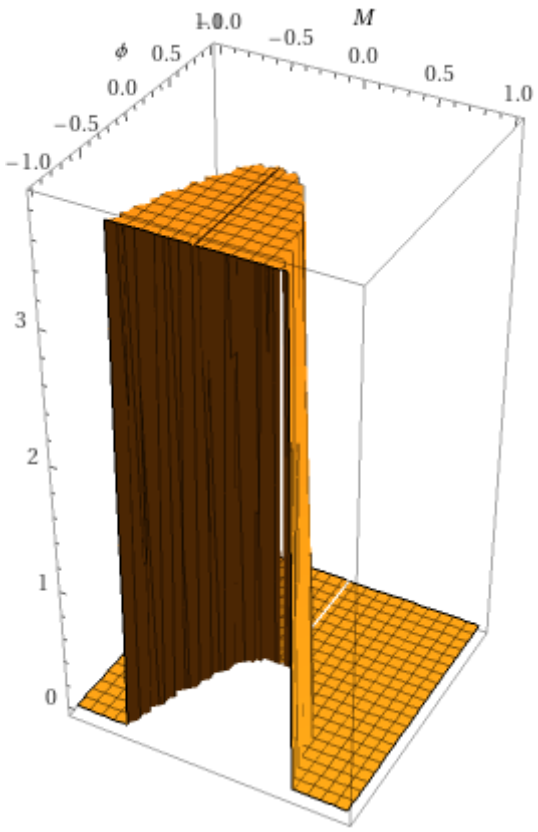
3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

The key observation from the below plots and is that at $M = 1$, which is taken as the energy density of the universe at the Big Bang, with $\phi = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.

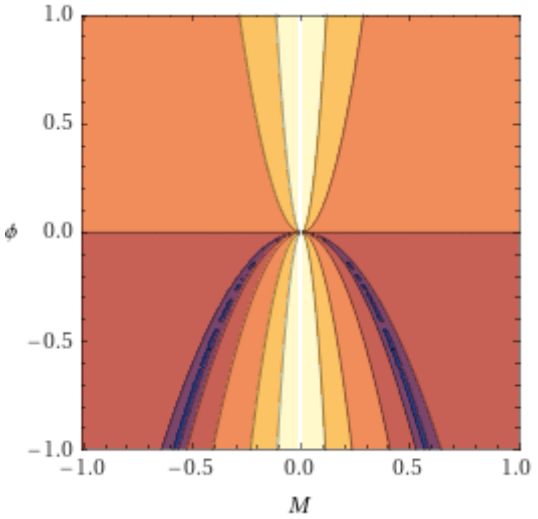


Imaginary part

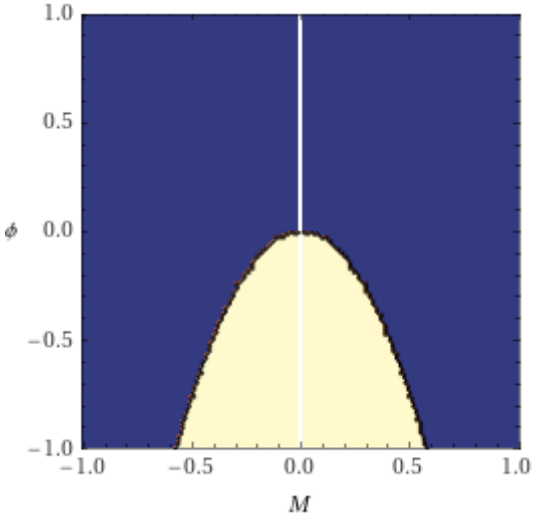


Contour plots

Real part



Imaginary part



Alternate forms

$$\frac{1}{2}\sqrt{3\times 2}\left(\log\left(3\,M^2+\phi\right)-\left(2\log(M)+\log(3)\right)\right)$$

$$\sqrt{\frac{3}{2}}\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)$$

$$\sqrt{\frac{3}{2}}\left(\log\left(\frac{3\,M^2+\phi}{M^2}\right)-\log(3)\right)$$

Expanded logarithmic form

$$\frac{1}{2}\sqrt{3\times 2}\log\left(1+\frac{\phi}{3\,M^2}\right)$$

Reduced logarithmic form

$$\log\left(\left(\frac{3\,M^2+\phi}{3\,M^2}\right)^{\sqrt{6}/2}\right)$$

Root

$M \neq 0\,, \quad \phi = 0$

Series expansion at M=0

$$\sqrt{\frac{3}{2}}\,\log\left(\frac{\phi}{3\,M^2}\right)+O\left(M^2\right)$$

(generalized Puiseux series)

Series expansion at M=∞

$$\frac{\phi}{\sqrt{6}\,M^2}-\frac{\phi^2}{6\sqrt{6}\,M^4}+O\left(\left(\frac{1}{M}\right)^6\right)$$

(Laurent series)

Derivative

$$\frac{\partial}{\partial M}\left(\sqrt{\frac{3}{2}}\,\log\left(1+\frac{\phi}{3\,M^2}\right)\right)=-\frac{\sqrt{6}\,\phi}{3\,M^3+M\,\phi}$$

Indefinite integral

$$\int\sqrt{\frac{3}{2}}\,\log\left(1+\frac{\phi}{3\,M^2}\right)dM=$$
$$\sqrt{\frac{3}{2}}\,M\log\left(\frac{\phi}{3\,M^2}+1\right)-\sqrt{2}\,\sqrt{\phi}\,\tan^{-1}\left(\frac{\sqrt{\phi}}{\sqrt{3}\,M}\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\tan^{-1}(x)$ is the inverse tangent function

Limit

$$\lim_{M\to\pm\infty} \sqrt{\frac{3}{2}}\ \log\left(1+\frac{\phi}{3\,M^2}\right) = 0$$

Alternative representations

$$\sqrt{\frac{3}{2}}\ \log\left(1+\frac{\phi}{3\,M^2}\right) = \log_e\left(1+\frac{\phi}{3\,M^2}\right)\sqrt{\frac{3}{2}}$$

$$\sqrt{\frac{3}{2}}\ \log\left(1+\frac{\phi}{3\,M^2}\right) = \log(a)\log_a\left(1+\frac{\phi}{3\,M^2}\right)\sqrt{\frac{3}{2}}$$

$$\sqrt{\frac{3}{2}}\ \log\left(1+\frac{\phi}{3\,M^2}\right) = -\mathrm{Li}_1\left(-\frac{\phi}{3\,M^2}\right)\sqrt{\frac{3}{2}}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\sqrt{\frac{3}{2}}\ \log\left(1+\frac{\phi}{3\,M^2}\right) = -\sqrt{\frac{3}{2}}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{3}\right)^k\left(\frac{\phi}{M^2}\right)^k}{k} \quad \text{for} \quad \left|\frac{\phi}{M^2}\right| < 3$$

$$\sqrt{\frac{3}{2}}\ \log\left(1+\frac{\phi}{3\,M^2}\right) = \sqrt{\frac{3}{2}}\ \log\left(\frac{\phi}{3\,M^2}\right) - \sqrt{\frac{3}{2}}\ \sum_{k=1}^{\infty}\frac{(-3)^k\left(\frac{\phi}{M^2}\right)^{-k}}{k} \quad \text{for} \quad \left|\frac{\phi}{M^2}\right| > 3$$

$$\sqrt{\frac{3}{2}}\,\log\left(1+\frac{\phi}{3\,M^2}\right)=i\,\sqrt{6}\,\pi\left\lfloor\frac{\arg\left(1-x+\frac{\phi}{3\,M^2}\right)}{2\,\pi}\right\rfloor+\sqrt{\frac{3}{2}}\,\log(x)-\sqrt{\frac{3}{2}}\sum_{k=1}^{\infty}\frac{(-1)^k\,x^{-k}\left(1-x+\frac{\phi}{3\,M^2}\right)^k}{k}\quad\text{for } x<0$$

$|z|$ is the absolute value of z

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations

$$\sqrt{\frac{3}{2}}\,\log\left(1+\frac{\phi}{3\,M^2}\right)=\sqrt{\frac{3}{2}}\int_1^{1+\frac{\phi}{3\,M^2}}\frac{1}{t}\,dt$$

$$\sqrt{\frac{3}{2}}\,\log\left(1+\frac{\phi}{3\,M^2}\right)=-\frac{i\,\sqrt{\frac{3}{2}}}{2\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{3^s\left(\frac{\phi}{M^2}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for}\left(-1<\gamma<0\text{ and }\left|\arg\left(\frac{\phi}{M^2}\right)\right|<\pi\right)$$

$\Gamma(x)$ is the gamma function

For

$$\frac{g\left(3\,M^2+\phi\right)}{3\,M^2}$$

and

$$\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)$$

From

$$L=\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\varphi}\right)^2\right]$$

we obtain:

√(-(((g (3 M^2 + ϕ))/(3 M^2))))*[1/2*R-1/2*δ*(sqrt(3/2) log((3 M^2 + ϕ)/(3 M^2)))*δ*(sqrt(3/2) log((3 M^2 + ϕ)/(3 M^2)))-3/4*M^2*(1-e^(-√(2/3)*(sqrt(3/2) log((3 M^2 + ϕ)/(3 M^2))))^2]

Input

$$\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{3\,M^2}}\left(\frac{1}{2}\,R-\frac{1}{2}\,\delta\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)\delta\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)-\frac{3}{4}\,M^2\left(1-\exp\left(-\sqrt{\frac{2}{3}}\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)\right)\right)^2\right)$$

log(x) is the natural logarithm

Exact result

$$\frac{\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\left(-\frac{3}{4}\,\delta^2\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)-\frac{3}{4}\,M^2\left(1-\frac{3\,M^2}{3\,M^2+\phi}\right)^2+\frac{R}{2}\right)}{\sqrt{3}}$$

Alternate forms

$$\frac{1}{4}\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(-3\,\delta^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,R\right)$$

$$\frac{\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\left(-\frac{3}{4}\,\delta^2\left(\log\left(\frac{3\,M^2+\phi}{M^2}\right)-\log(3)\right)^2-\frac{3\,M^2\,\phi^2}{4\left(3\,M^2+\phi\right)^2}+\frac{R}{2}\right)}{\sqrt{3}}$$

$$\frac{1}{4\sqrt{3}\,\left(3\,M^2+\phi\right)^2}\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\left(18\,M^4\,R-3\,\delta^2\,\phi^2\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)-18\,\delta^2\,M^2\,\phi\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)+12\,M^2\,R\,\phi-3\,M^2\,\phi^2-27\,\delta^2\,M^4\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)+2\,R\,\phi^2\right)$$

Expanded form

$$-\frac{1}{4}\sqrt{3}\,\delta^2\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\,\log^2\left(\frac{3M^2+\phi}{3M^2}\right)+\frac{R\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}}{2\sqrt{3}}-$$
$$\frac{1}{4}\sqrt{3}\,M^2\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}-\frac{9\sqrt{3}\,M^6\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}}{4\left(3M^2+\phi\right)^2}+\frac{3\sqrt{3}\,M^4\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}}{2\left(3M^2+\phi\right)}$$

Alternate forms assuming g, M, R, δ, and ϕ are positive

$$\frac{i\sqrt{g\left(3M^2+\phi\right)}\left(-3\delta^2\left(\log\left(M^2+\frac{\phi}{3}\right)-2\log(M)\right)^2-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)}{4\sqrt{3}\,M}$$

$$-\frac{i\sqrt{3}\,\delta^2\sqrt{g\left(3M^2+\phi\right)}\left(\log\left(3M^2+\phi\right)-2\log(M)-\log(3)\right)^2}{4M}+$$
$$\frac{iR\sqrt{g\left(3M^2+\phi\right)}}{2\sqrt{3}\,M}-\frac{1}{4}i\sqrt{3}\,M\left(1-\frac{3M^2}{3M^2+\phi}\right)^2\sqrt{g\left(3M^2+\phi\right)}$$

Alternate form assuming M≠0, g≤0, and 3 M^2+ϕ>0

$$\frac{\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(2R-3M^2\left(1-\frac{3M^2}{3M^2+\phi}\right)^2-3\delta^2\log^2\left(\frac{3M^2+\phi}{3M^2}\right)\right)}{4\sqrt{3}}$$

Series expansion at g=0

$$\frac{1}{4}\sqrt{g}\,\sqrt{-\frac{\phi}{3M^2}-1}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)+O\left(g^{11/2}\right)$$

(Puiseux series)

Series expansion at g=∞

$$\left(\sqrt{g}\,\sqrt{g\left(-\frac{\phi}{M^2}-3\right)}\right.$$
$$\left.\left(18\,M^4\,R-3\,\delta^2\left(3\,M^2+\phi\right)^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)+3\,M^2\,\phi\,(4\,R-\phi)+2\,R\,\phi^2\right)\right)\Big/$$
$$\left(4\sqrt{3}\,\sqrt{g}\,\left(3\,M^2+\phi\right)^2\right)+O\left(\left(\frac{1}{g}\right)^{11/2}\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial g}\left(\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{3\,M^2}}\left(\frac{R}{2}-\frac{1}{2}\,\delta\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)\delta\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)-\right.\right.$$
$$\left.\left.\frac{3}{4}\,M^2\left(1-e^{-\sqrt{\frac{2}{3}}\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)}\right)^2\right)\right)=$$
$$\frac{-18\,M^4\,R+3\,\delta^2\left(3\,M^2+\phi\right)^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)+3\,M^2\,\phi\,(\phi-4\,R)-2\,R\,\phi^2}{8\,M^2\left(3\,M^2+\phi\right)\sqrt{g\left(-\frac{3\,\phi}{M^2}-9\right)}}$$

Indefinite integral

$$\int\frac{\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\left(\frac{R}{2}-\frac{3}{4}\,M^2\left(1-\frac{3\,M^2}{3\,M^2+\phi}\right)^2-\frac{3}{4}\,\delta^2\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)}{\sqrt{3}}\,dg=$$
$$\frac{1}{6}\,g\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(-3\,\delta^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,R\right)+\text{constant}$$

From the indefinite integral result

$$\int \frac{\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(\frac{R}{2}-\frac{3}{4}M^2\left(1-\frac{3M^2}{3M^2+\phi}\right)^2-\frac{3}{4}\delta^2\log^2\left(\frac{3M^2+\phi}{3M^2}\right)\right)}{\sqrt{3}}\,dg =$$

$$\frac{1}{6}g\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)+\text{constant}$$

we obtain:

1/6 g sqrt(g (-1 - ϕ/(3 M^2))) (2 R - (3 M^2 ϕ^2)/(3 M^2 + ϕ)^2 - 3 δ^2 log^2(1 + ϕ/(3 M^2)))

Input

$$\frac{1}{6}g\sqrt{g\left(-1-\frac{\phi}{3M^2}\right)}\left(2R-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}-3\delta^2\log^2\left(1+\frac{\phi}{3M^2}\right)\right)$$

log(x) is the natural logarithm

Alternate forms

$$g\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(\delta^2\left(-\frac{1}{2}\log^2\left(\frac{3M^2+\phi}{M^2}\right)+\log(3)\log\left(\frac{3M^2+\phi}{M^2}\right)-\frac{\log^2(3)}{2}\right)-\frac{M^2\phi^2}{2\left(3M^2+\phi\right)^2}+\frac{R}{3}\right)$$

$$\frac{1}{6\sqrt{3}\left(3M^2+\phi\right)^2}g\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(18M^4R-3\delta^2\phi^2\log^2\left(\frac{\phi}{3M^2}+1\right)-18\delta^2M^2\phi\log^2\left(\frac{\phi}{3M^2}+1\right)+12M^2R\phi-3M^2\phi^2-27\delta^2M^4\log^2\left(\frac{\phi}{3M^2}+1\right)+2R\phi^2\right)$$

$$-\frac{\delta^2\, g\, \phi^2 \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\, \log^2\left(\frac{\phi}{3\,M^2}+1\right)}{2\sqrt{3}\,\left(3\,M^2+\phi\right)^2}-$$
$$\frac{\sqrt{3}\,\delta^2\, g\, M^2\, \phi \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\, \log^2\left(\frac{\phi}{3\,M^2}+1\right)}{\left(3\,M^2+\phi\right)^2}+\frac{g\,R\,\phi^2 \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{3\sqrt{3}\,\left(3\,M^2+\phi\right)^2}+$$
$$\frac{2\,g\,M^2\,R\,\phi \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{\sqrt{3}\,\left(3\,M^2+\phi\right)^2}-\frac{g\,M^2\,\phi^2 \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\sqrt{3}\,\left(3\,M^2+\phi\right)^2}-$$
$$\frac{3\sqrt{3}\,\delta^2\, g\, M^4 \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\, \log^2\left(\frac{\phi}{3\,M^2}+1\right)}{2\left(3\,M^2+\phi\right)^2}+\frac{\sqrt{3}\,g\,M^4\,R \sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{\left(3\,M^2+\phi\right)^2}$$

Expanded form

$$-\frac{1}{2}\,\delta^2\, g \sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\, \log^2\left(\frac{\phi}{3\,M^2}+1\right)+\frac{1}{3}\,g\,R \sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}-$$
$$\frac{g\,M^2\,\phi^2 \sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}}{2\left(3\,M^2+\phi\right)^2}$$

Alternate forms assuming g, M, R, δ, and ϕ are positive

$$\frac{1}{6}\,i\,g^{3/2} \sqrt{\frac{\phi}{3\,M^2}+1}\, \left(-3\,\delta^2 \log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,R\right)$$
$$-\frac{1}{2}\,i\,\delta^2\, g^{3/2} \sqrt{\frac{\phi}{3\,M^2}+1}\, \log^2\left(\frac{\phi}{3\,M^2}+1\right)+\frac{1}{3}\,i\,g^{3/2}\,R \sqrt{\frac{\phi}{3\,M^2}+1}-$$
$$\frac{i\,g^{3/2}\,M^2\,\phi^2 \sqrt{\frac{\phi}{3\,M^2}+1}}{2\left(3\,M^2+\phi\right)^2}$$

Expanded logarithmic form

$$\frac{1}{3} g R \sqrt{-g + \frac{-\frac{1}{3} g \phi}{M^2}} + \frac{\frac{1}{6} (-3) g M^2 \phi^2 \sqrt{-g + \frac{-\frac{1}{3} g \phi}{M^2}}}{9 M^4 + 6 M^2 \phi + \phi^2} - \frac{1}{2} g \delta^2 \sqrt{-g + \frac{-\frac{1}{3} g \phi}{M^2}} \log^2\left(1 + \frac{\frac{1}{3} \phi}{M^2}\right)$$

Reduced logarithmic form

$$\frac{g \sqrt{\frac{g\left(-3 M^2 - \phi\right)}{M^2}} \left(-3 M^2 \phi^2 + 2 R \left(3 M^2 + \phi\right)^2 - 3 \delta^2 \left(3 M^2 + \phi\right)^2 \log^2\left(\frac{3 M^2 + \phi}{3 M^2}\right)\right)}{6 \sqrt{3} \left(3 M^2 + \phi\right)^2}$$

Series expansion at g=0

$$\frac{1}{6} g^{3/2} \sqrt{-\frac{\phi}{3 M^2} - 1} \left(-3 \delta^2 \log^2\left(\frac{\phi}{3 M^2} + 1\right) - \frac{3 M^2 \phi^2}{\left(3 M^2 + \phi\right)^2} + 2 R\right) + O\left(g^{11/2}\right)$$

(Puiseux series)

Series expansion at g=∞

$$\left(g^{3/2} \sqrt{g \left(-\frac{\phi}{M^2} - 3\right)} \left(18 M^4 R - 3 \delta^2 \left(3 M^2 + \phi\right)^2 \log^2\left(\frac{\phi}{3 M^2} + 1\right) + 3 M^2 \phi \left(4 R - \phi\right) + 2 R \phi^2\right)\right) \Big/ \left(6 \sqrt{3} \sqrt{g} \left(3 M^2 + \phi\right)^2\right) + O\left(\left(\frac{1}{g}\right)^{11/2}\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial g}\left(\frac{1}{6}\,g\,\sqrt{g\left(-1-\frac{\phi}{3\,M^2}\right)}\left(2\,R-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}-3\,\delta^2\log^2\left(1+\frac{\phi}{3\,M^2}\right)\right)\right)=$$
$$-\frac{g\left(18\,M^4\,R-3\,\delta^2\left(3\,M^2+\phi\right)^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)+3\,M^2\,\phi\,(4\,R-\phi)+2\,R\,\phi^2\right)}{4\,M^2\left(3\,M^2+\phi\right)\sqrt{g\left(-\frac{3\,\phi}{M^2}-9\right)}}$$

Indefinite integral

$$\int\frac{1}{6}\,g\,\sqrt{g\left(-1-\frac{\phi}{3\,M^2}\right)}\left(2\,R-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}-3\,\delta^2\log^2\left(1+\frac{\phi}{3\,M^2}\right)\right)dg=$$
$$\frac{1}{15}\,g^2\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(-3\,\delta^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,R\right)+\text{constant}$$

Thus, from the indefinite integral result

$$\int\frac{1}{6}\,g\,\sqrt{g\left(-1-\frac{\phi}{3\,M^2}\right)}\left(2\,R-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}-3\,\delta^2\log^2\left(1+\frac{\phi}{3\,M^2}\right)\right)dg=$$
$$\frac{1}{15}\,g^2\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(-3\,\delta^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,R\right)+\text{constant}$$

we obtain:

Input

$$\frac{2}{15}\,g^2\,R\,\sqrt{-g+\frac{-\frac{g\,\phi}{3}}{M^2}}+\frac{\frac{1}{15}\times(-3)\,g^2\,M^2\,\phi^2\,\sqrt{-g+\frac{-\frac{g\,\phi}{3}}{M^2}}}{9\,M^4+6\,M^2\,\phi+\phi^2}-$$
$$\frac{1}{5}\,g^2\,\delta^2\,\sqrt{-g+\frac{-\frac{g\,\phi}{3}}{M^2}}\,\log^2\left(1+\frac{\frac{1}{3}\,\phi}{M^2}\right)$$

$\log(x)$ is the natural logarithm

Result

$$-\frac{1}{5}\delta^2 g^2 \sqrt{-\frac{g\phi}{3M^2}-g}\,\log^2\left(\frac{\phi}{3M^2}+1\right)+\frac{2}{15}g^2 R\sqrt{-\frac{g\phi}{3M^2}-g}-\frac{g^2 M^2\phi^2\sqrt{-\frac{g\phi}{3M^2}-g}}{5\left(9M^4+6M^2\phi+\phi^2\right)}$$

Alternate forms

$$\frac{1}{15}g^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)$$

$$g^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(\delta^2\left(-\frac{1}{5}\log^2\left(\frac{3M^2+\phi}{M^2}\right)+\frac{2}{5}\log(3)\log\left(\frac{3M^2+\phi}{M^2}\right)-\frac{\log^2(3)}{5}\right)-\frac{M^2\phi^2}{5\left(9M^4+6M^2\phi+\phi^2\right)}+\frac{2R}{15}\right)$$

$$\frac{1}{15\sqrt{3}\left(3M^2+\phi\right)^2}g^2\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(18M^4R-3\delta^2\phi^2\log^2\left(\frac{\phi}{3M^2}+1\right)-18\delta^2M^2\phi\log^2\left(\frac{\phi}{3M^2}+1\right)+12M^2R\phi-3M^2\phi^2-27\delta^2M^4\log^2\left(\frac{\phi}{3M^2}+1\right)+2R\phi^2\right)$$

Alternate forms assuming g, M, R, δ, and ϕ are positive

$$\frac{1}{15}ig^2\sqrt{\frac{g\phi}{3M^2}+g}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)$$

$$-\frac{1}{5} i \delta^2 g^2 \sqrt{\frac{g\phi}{3M^2}+g}\,\log^2\left(\frac{\phi}{3M^2}+1\right)+\frac{2}{15} i g^2 R\sqrt{\frac{g\phi}{3M^2}+g}-\frac{i g^2 M^2 \phi^2 \sqrt{\frac{g\phi}{3M^2}+g}}{5\left(9M^4+6M^2\phi+\phi^2\right)}$$

Expanded logarithmic form

$$\frac{2}{15} g^2 R\sqrt{-g-\frac{g\phi}{3M^2}}-\frac{g^2 M^2 \phi^2 \sqrt{-g-\frac{g\phi}{3M^2}}}{45M^4+30M^2\phi+5\phi^2}-\frac{1}{5} g^2\delta^2\sqrt{-g-\frac{g\phi}{3M^2}}\,\log^2\left(1+\frac{\phi}{3M^2}\right)$$

Alternate form assuming M≠0, g≤0, and 3 M^2+ϕ>0

$$\frac{g^2\sqrt{\frac{g\left(-3M^2-\phi\right)}{M^2}}\left(-3M^2\phi^2+2R\left(3M^2+\phi\right)^2-3\delta^2\left(3M^2+\phi\right)^2\log^2\left(\frac{3M^2+\phi}{3M^2}\right)\right)}{15\sqrt{3}\left(3M^2+\phi\right)^2}$$

Series expansion at g=0

$$\frac{1}{15} g^{5/2}\sqrt{-\frac{\phi}{3M^2}-1}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)+O\left(g^{7/2}\right)$$

(Puiseux series)

Series expansion at g=∞

$$\left(g^{5/2}\sqrt{g\left(-\frac{\phi}{M^2}-3\right)}\left(18M^4R-3\delta^2\left(3M^2+\phi\right)^2\log^2\left(\frac{\phi}{3M^2}+1\right)+3M^2\phi\left(4R-\phi\right)+2R\phi^2\right)\right)\Big/\left(15\sqrt{3}\sqrt{g}\left(3M^2+\phi\right)^2\right)+O\left(\left(\frac{1}{g}\right)^{7/2}\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial g}\left(\frac{2}{15}g^2 R\sqrt{-g-\frac{g\phi}{3M^2}}+-\frac{3g^2M^2\phi^2\sqrt{-g-\frac{g\phi}{3M^2}}}{15\left(9M^4+6M^2\phi+\phi^2\right)}-\frac{1}{5}g^2\delta^2\sqrt{-g-\frac{g\phi}{3M^2}}\log^2\left(1+\frac{\phi}{3M^2}\right)\right)=$$
$$-\frac{g^2\left(18M^4R-3\delta^2\left(3M^2+\phi\right)^2\log^2\left(\frac{\phi}{3M^2}+1\right)+3M^2\phi(4R-\phi)+2R\phi^2\right)}{6M^2\left(3M^2+\phi\right)\sqrt{g\left(-\frac{3\phi}{M^2}-9\right)}}$$

Indefinite integral

$$\int\left(\frac{2}{15}g^2R\sqrt{-g-\frac{g\phi}{3M^2}}-\frac{g^2M^2\phi^2\sqrt{-g-\frac{g\phi}{3M^2}}}{5\left(9M^4+6M^2\phi+\phi^2\right)}-\frac{1}{5}g^2\delta^2\sqrt{-g-\frac{g\phi}{3M^2}}\log^2\left(1+\frac{\phi}{3M^2}\right)\right)dg=$$
$$\frac{2}{105}g^3\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)+\text{constant}$$

And again, from the indefinite integral result

$$\int\left(\frac{2}{15}g^2R\sqrt{-g-\frac{g\phi}{3M^2}}-\frac{g^2M^2\phi^2\sqrt{-g-\frac{g\phi}{3M^2}}}{5\left(9M^4+6M^2\phi+\phi^2\right)}-\frac{1}{5}g^2\delta^2\sqrt{-g-\frac{g\phi}{3M^2}}\log^2\left(1+\frac{\phi}{3M^2}\right)\right)dg=$$
$$\frac{2}{105}g^3\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)+\text{constant}$$

2/105 g^3 sqrt(g (-1 - ϕ/(3 M^2))) (2 R - (3 M^2 ϕ^2)/(3 M^2 + ϕ)^2 - 3 δ^2 log^2(1 + ϕ/(3 M^2)))

for g = 1, M = 2, R = 4, we obtain:

2/105 sqrt((-1 - φ/(3*2^2))) (2*4 - (3*2^2 λ^2)/(3*2^2 + φ)^2 - 3 δ^2 log^2(1 + φ/(3 2^2)))

Input

$$\frac{2}{105}\sqrt{-1-\frac{\frac{1}{\Phi}}{3\times 2^2}}\left(2\times 4-\frac{3\times 2^2\,\lambda^2}{\left(3\times 2^2+\frac{1}{\Phi}\right)^2}-3\,\delta^2\log^2\left(1+\frac{\frac{1}{\Phi}}{3\times 2^2}\right)\right)$$

$\log(x)$ is the natural logarithm

Φ is the golden ratio conjugate

Exact result

$$\frac{2}{105}\,i\sqrt{\frac{1}{12\,\Phi}+1}\left(-3\,\delta^2\log^2\left(\frac{1}{12\,\Phi}+1\right)-\frac{12\,\lambda^2}{\left(\frac{1}{\Phi}+12\right)^2}+8\right)$$

Exact form

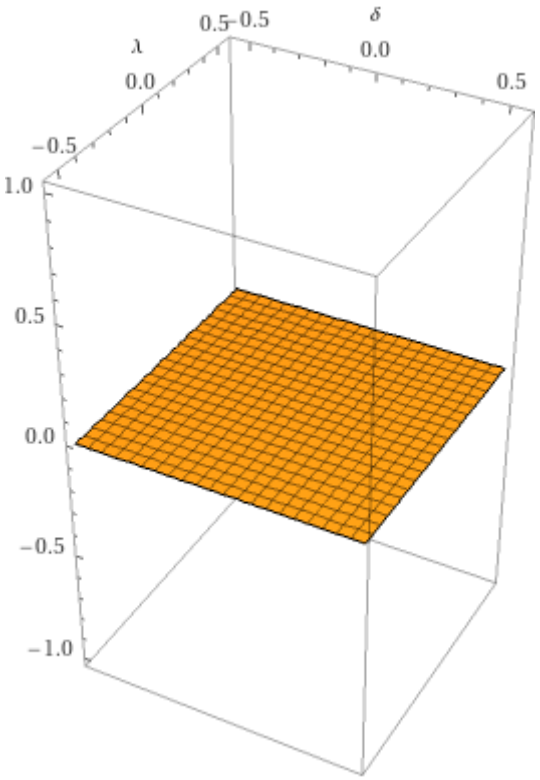
$$\frac{2}{105}\,i\sqrt{\frac{1}{12\,(\phi-1)}+1}\left(-3\,\delta^2\log^2\left(\frac{1}{12\,(\phi-1)}+1\right)-\frac{12\,\lambda^2}{\left(\frac{1}{\phi-1}+12\right)^2}+8\right)$$

ϕ is the golden ratio

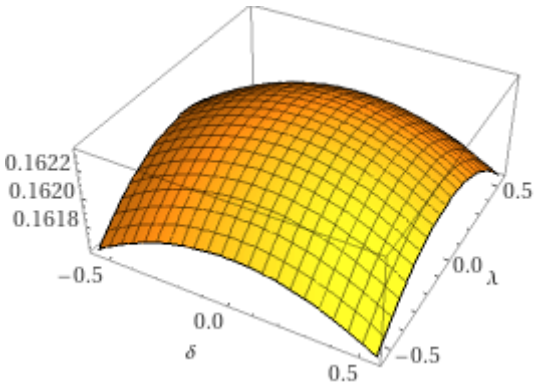
3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

The key observation from the below plots and is that at $\lambda = 1$, which is taken as the energy density of the universe at the Big Bang, with $\delta = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.

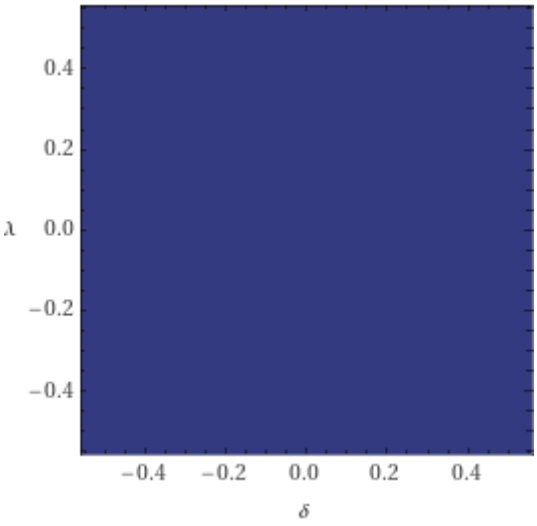


Imaginary part

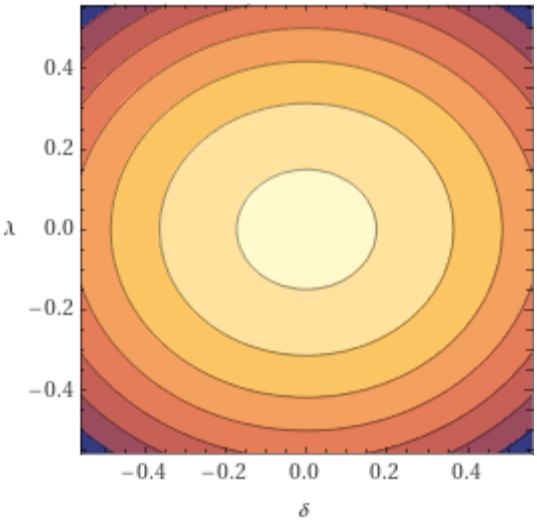


Contour plots

Real part



Imaginary part



Alternate forms

$$-\frac{1}{35}\,i\,\delta^2\sqrt{\frac{12\,\Phi+1}{3\,\Phi}}\left(2\log(2)-\log\left(\frac{12\,\Phi+1}{3\,\Phi}\right)\right)^2-\frac{4\,i\,\lambda^2\left(\frac{\Phi}{12\,\Phi+1}\right)^{3/2}}{35\sqrt{3}}+\frac{8}{105}\,i$$

$$\sqrt{\frac{12\,\Phi+1}{3\,\Phi}}$$

$$-\left(\left(i\left(3\delta^2(12\Phi+1)^2(\log(\Phi)-\log(12\Phi+1)+2\log(2)+\log(3)\right)^2+4\left(3\lambda^2\Phi^2-2(12\Phi+1)^2\right)\right)\right)\Big/\left(105\sqrt{3\Phi}\,(12\Phi+1)^{3/2}\right)$$

$$-\frac{i\sqrt{\frac{12\Phi+1}{3\Phi}}\left(\lambda^2+36\delta^2\log^2\left(\frac{1}{12\Phi}+1\right)-96\right)}{1260}+\frac{i\lambda^2}{630\sqrt{3\Phi(12\Phi+1)}}-\frac{i\lambda^2}{1260\sqrt{3\Phi}\,(12\Phi+1)^{3/2}}$$

Expanded form

$$-\frac{2}{35}i\delta^2\sqrt{\frac{1}{12\Phi}+1}\,\log^2\left(\frac{1}{12\Phi}+1\right)-\frac{8i\lambda^2\sqrt{\frac{1}{12\Phi}+1}}{35\left(\frac{1}{\Phi}+12\right)^2}+\frac{16}{105}i\sqrt{\frac{1}{12\Phi}+1}$$

Alternate form assuming δ and λ are real

$$\frac{i\left(\frac{\Phi}{12\Phi+1}\right)^{3/2}\left(-12\lambda^2-\frac{3\delta^2(12\Phi+1)^2\log^2\left(\frac{1}{12\Phi}+1\right)}{\Phi^2}+\frac{8(12\Phi+1)^2}{\Phi^2}\right)}{105\sqrt{3}}$$

Roots

$$\lambda\approx-3.9312i\sqrt{0.047998\delta^2-8}$$

$$\lambda\approx 3.9312i\sqrt{0.047998\delta^2-8}$$

Polynomial discriminant

$$\Delta_\delta=-\left(\left(16\left(-3\lambda^2\Phi^{5/2}+288\Phi^{5/2}+48\Phi^{3/2}+2\sqrt{\Phi}\right)\left(\log^2\left(\frac{1}{12\Phi}+1\right)+24\Phi\log^2\left(\frac{1}{12\Phi}+1\right)+144\Phi^2\log^2\left(\frac{1}{12\Phi}+1\right)\right)\right)\Big/\left(11025\Phi^{3/2}(12\Phi+1)^3\right)\right)$$

Property as a function

Parity

even

Roots for the variable λ

$$\lambda = -\frac{i\left(\frac{1}{\Phi}+12\right)\sqrt{3\,\delta^2\log^2\left(\frac{1}{12\,\Phi}+1\right)-8}}{2\sqrt{3}}$$

$$\lambda = \frac{i\left(\frac{1}{\Phi}+12\right)\sqrt{3\,\delta^2\log^2\left(\frac{1}{12\,\Phi}+1\right)-8}}{2\sqrt{3}}$$

Derivative

$$\frac{\partial}{\partial\delta}\left(\frac{2}{105}\sqrt{-1-\frac{1}{\Phi\left(3\times 2^2\right)}}\left(2\times 4-\frac{3\times 2^2\,\lambda^2}{\left(3\times 2^2+\frac{1}{\Phi}\right)^2}-3\,\delta^2\log^2\left(1+\frac{1}{\Phi\left(3\times 2^2\right)}\right)\right)\right)=$$

$$-\frac{2}{35}\,i\,\delta\sqrt{\frac{1}{3\,\Phi}+4}\,\log^2\left(\frac{1}{12\,\Phi}+1\right)$$

Indefinite integral

$$\int\frac{2}{105}\,i\sqrt{1+\frac{1}{12\,\Phi}}\left(8-\frac{12\,\lambda^2}{\left(12+\frac{1}{\Phi}\right)^2}-3\,\delta^2\log^2\left(1+\frac{1}{12\,\Phi}\right)\right)d\delta=$$

$$\frac{2}{105}\,i\sqrt{\frac{1}{12\,\Phi}+1}\left(8\,\delta+\delta^3\left(-\log^2\left(\frac{1}{12\,\Phi}+1\right)\right)-\frac{12\,\delta\,\lambda^2}{\left(\frac{1}{\Phi}+12\right)^2}\right)+\text{constant}$$

From

$$\int \frac{2}{105}\, i \sqrt{1+\frac{1}{12\,\Phi}}\left(8-\frac{12\,\lambda^2}{\left(12+\frac{1}{\Phi}\right)^2}-3\,\delta^2\log^2\left(1+\frac{1}{12\,\Phi}\right)\right)d\delta =$$

$$\frac{2}{105}\, i \sqrt{\frac{1}{12\,\Phi}+1}\left(8\,\delta+\delta^3\left(-\log^2\left(\frac{1}{12\,\Phi}+1\right)\right)-\frac{12\,\delta\,\lambda^2}{\left(\frac{1}{\Phi}+12\right)^2}\right) + \text{constant}$$

for $\delta = \lambda = 0.5$:

2/105 i sqrt(1 + 1/(12 Φ)) (8 0.5 - (12 0.5 0.5^2)/(12 + 1/Φ)^2 – 0.5^3 log^2(1 + 1/(12 Φ)))

Input

$$\frac{2}{105}\, i \sqrt{1+\frac{1}{12\,\Phi}}\left(8\times 0.5-\frac{12\times 0.5\times 0.5^2}{\left(12+\frac{1}{\Phi}\right)^2}-0.5^3\log^2\left(1+\frac{1}{12\,\Phi}\right)\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Φ is the golden ratio conjugate

Result

$0.0809600\ldots\, i$

Alternate complex forms

$0.08096\,(\cos(1.5708)+i\sin(1.5708))$

$0.08096\,e^{1.5708\,i}$

Polar coordinates

$r = 0.08096$ (radius), $\theta = 1.5708$ (angle)

0.08096

From the fundamental "unitary" formula:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From the previous result, for $C = 0.08096$, inserting a radius of the Universe, which we have calculated, equal to R = 2.5256519929 × 10^14 years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (0.08096 ×(2.5256519929 × 10^14)×2.33·10^(-13))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{0.08096\times 2.5256519929\times 10^{14}\times 2.33}{10^{13}}}}$$

Result

1.6180299988123134487255082274124523296355093564145268937474081636

…

1.618029998812313…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Now, we analyze the following equation:

$$\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\varphi}\right)^2-\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-\frac{m^2}{2}\sigma^2\right]$$

we obtain:

√(-(((g (3 M^2 + ϕ))/(3 M^2))))*[1/2*R-1/2*δ*(sqrt(3/2) log((3 M^2 + ϕ)/(3 M^2)))*δ*(sqrt(3/2) log((3 M^2 + ϕ)/(3 M^2)))-3/4*M^2*(1-e^(-√(2/3)*(sqrt(3/2) log((3 M^2 + ϕ)/(3 M^2))))^2-1/2*δ*σ*δ^μ*σ-1/2*m^2*σ^2]

Input

$$\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{3\,M^2}}\left(\frac{1}{2}\,R-\frac{1}{2}\,\delta\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)\delta\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)-\frac{3}{4}\,M^2\left(1-\exp\left(-\sqrt{\frac{2}{3}}\left(\sqrt{\frac{3}{2}}\,\log\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)\right)\right)^2-\frac{1}{2}\,\delta\,\sigma\left(\delta^\mu\,\sigma\right)-\frac{1}{2}\,m^2\,\sigma^2\right)$$

log(x) is the natural logarithm

Exact result

$$\frac{\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\left(-\frac{1}{2}\,\sigma^2\,\delta^{\mu+1}-\frac{m^2\,\sigma^2}{2}-\frac{3}{4}\,\delta^2\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)-\frac{3}{4}\,M^2\left(1-\frac{3\,M^2}{3\,M^2+\phi}\right)^2+\frac{R}{2}\right)}{\sqrt{3}}$$

Alternate forms

$$\frac{1}{4}\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-2\sigma^2\delta^{\mu+1}-2m^2\sigma^2-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)$$

$$-\frac{1}{4\sqrt{3}\left(3M^2+\phi\right)^2}\sqrt{g\left(-\frac{\phi}{M^2}-3\right)}\left(2m^2\sigma^2\left(3M^2+\phi\right)^2-9M^4\left(-2\sigma^2\delta^{\mu+1}-3\delta^2\log^2(3)+2R\right)+3\delta^2\left(3M^2+\phi\right)^2\log^2\left(\frac{3M^2+\phi}{M^2}\right)-6\delta^2\log(3)\left(3M^2+\phi\right)^2\log\left(\frac{3M^2+\phi}{M^2}\right)-3M^2\phi\left(-4\sigma^2\delta^{\mu+1}-6\delta^2\log^2(3)+4R-\phi\right)-\phi^2\left(-2\sigma^2\delta^{\mu+1}-3\delta^2\log^2(3)+2R\right)\right)$$

$$\frac{1}{4\sqrt{3}\left(3M^2+\phi\right)^2}\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(-2\sigma^2\phi^2\delta^{\mu+1}-18m^2M^4\sigma^2-12m^2M^2\sigma^2\phi-2m^2\sigma^2\phi^2-18M^4\sigma^2\delta^{\mu+1}+18M^4R-12M^2\sigma^2\phi\delta^{\mu+1}-3\delta^2\phi^2\log^2\left(\frac{3M^2+\phi}{3M^2}\right)-18\delta^2M^2\phi\log^2\left(\frac{3M^2+\phi}{3M^2}\right)+12M^2R\phi-3M^2\phi^2-27\delta^2M^4\log^2\left(\frac{3M^2+\phi}{3M^2}\right)+2R\phi^2\right)$$

Expanded form

$$-\frac{m^2\,\sigma^2\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\sqrt{3}}-\frac{\sigma^2\,\delta^{\mu+1}\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\sqrt{3}}-$$
$$\frac{1}{4}\sqrt{3}\,\delta^2\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\,\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)+\frac{R\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\sqrt{3}}-$$
$$\frac{1}{4}\sqrt{3}\,M^2\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}-\frac{9\sqrt{3}\,M^6\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{4\left(3\,M^2+\phi\right)^2}+\frac{3\sqrt{3}\,M^4\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\left(3\,M^2+\phi\right)}$$

Alternate forms assuming g, m, M, R, δ, μ, σ, and ϕ are positive

$$\frac{1}{4\sqrt{3}\,M}i\sqrt{g\left(3\,M^2+\phi\right)}\left(-2\,\sigma^2\,\delta^{\mu+1}-2\,m^2\,\sigma^2-3\,\delta^2\left(\log\left(M^2+\frac{\phi}{3}\right)-2\log(M)\right)^2-\frac{3\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,R\right)$$
$$-\frac{i\,m^2\,\sigma^2\sqrt{g\left(3\,M^2+\phi\right)}}{2\sqrt{3}\,M}-\frac{i\,\sigma^2\,\delta^{\mu+1}\sqrt{g\left(3\,M^2+\phi\right)}}{2\sqrt{3}\,M}-$$
$$\frac{i\sqrt{3}\,\delta^2\sqrt{g\left(3\,M^2+\phi\right)}\left(\log\left(3\,M^2+\phi\right)-2\log(M)-\log(3)\right)^2}{4\,M}+$$
$$\frac{i\,R\sqrt{g\left(3\,M^2+\phi\right)}}{2\sqrt{3}\,M}-\frac{1}{4}\,i\sqrt{3}\,M\left(1-\frac{3\,M^2}{3\,M^2+\phi}\right)^2\sqrt{g\left(3\,M^2+\phi\right)}$$

Reduced logarithmic form

$$\frac{\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\left(2\,R-2\,m^2\,\sigma^2-2\,\delta^{1+\mu}\,\sigma^2-3\,M^2\left(1-\frac{3\,M^2}{3\,M^2+\phi}\right)^2-3\,\delta^2\log^2\left(\frac{3\,M^2+\phi}{3\,M^2}\right)\right)}{4\sqrt{3}}$$

Derivative

$$\frac{\partial}{\partial m}\left(\sqrt{-\frac{g\left(3M^2+\phi\right)}{3M^2}}\left(\frac{R}{2}-\frac{1}{2}\delta\left(\sqrt{\frac{3}{2}}\log\left(\frac{3M^2+\phi}{3M^2}\right)\right)\delta\left(\sqrt{\frac{3}{2}}\log\left(\frac{3M^2+\phi}{3M^2}\right)\right)-\frac{3}{4}M^2\left(1-e^{-\sqrt{\frac{2}{3}}\left(\sqrt{\frac{3}{2}}\log\left(\frac{3M^2+\phi}{3M^2}\right)\right)}\right)^2-\frac{1}{2}\delta\sigma\left(\delta^{\mu}\sigma\right)-\frac{m^2\sigma^2}{2}\right)\right)=-\frac{m\sigma^2\sqrt{g\left(-\frac{\phi}{M^2}-3\right)}}{\sqrt{3}}$$

Indefinite integral

$$\int\frac{\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(\frac{R}{2}-\frac{m^2\sigma^2}{2}-\frac{1}{2}\delta^{1+\mu}\sigma^2-\frac{3}{4}M^2\left(1-\frac{3M^2}{3M^2+\phi}\right)^2-\frac{3}{4}\delta^2\log^2\left(\frac{3M^2+\phi}{3M^2}\right)\right)}{\sqrt{3}}dm=\frac{1}{4}\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-\frac{2m^3\sigma^2}{3}-2m\sigma^2\delta^{\mu+1}-3\delta^2 m\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3mM^2\phi^2}{\left(3M^2+\phi\right)^2}+2mR\right)+\text{constant}$$

From the indefinite integral result

$$\int\frac{\sqrt{-\frac{g\left(3M^2+\phi\right)}{M^2}}\left(\frac{R}{2}-\frac{m^2\sigma^2}{2}-\frac{1}{2}\delta^{1+\mu}\sigma^2-\frac{3}{4}M^2\left(1-\frac{3M^2}{3M^2+\phi}\right)^2-\frac{3}{4}\delta^2\log^2\left(\frac{3M^2+\phi}{3M^2}\right)\right)}{\sqrt{3}}dm=\frac{1}{4}\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-\frac{2m^3\sigma^2}{3}-2m\sigma^2\delta^{\mu+1}-3\delta^2 m\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3mM^2\phi^2}{\left(3M^2+\phi\right)^2}+2mR\right)+\text{constant}$$

we obtain:

1/4 sqrt(g (-1 - ϕ/(3 M^2))) (2 m R - (2 m^3 σ^2)/3 - 2 m δ^(1 + μ) σ^2 - (3 m M^2 ϕ^2)/(3 M^2 + ϕ)^2 - 3 m δ^2 log^2(1 + ϕ/(3 M^2)))

Input

$$\frac{1}{4}\sqrt{g\left(-1-\frac{\phi}{3\,M^2}\right)}\left(2\,m\,R-\frac{1}{3}\left(2\,m^3\,\sigma^2\right)-2\,m\,\delta^{1+\mu}\,\sigma^2-\frac{3\,m\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}-3\,m\,\delta^2\log^2\left(1+\frac{\phi}{3\,M^2}\right)\right)$$

log(x) is the natural logarithm

Result

$$\frac{1}{4}\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(-\frac{2\,m^3\,\sigma^2}{3}-2\,m\,\sigma^2\,\delta^{\mu+1}-3\,\delta^2\,m\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{3\,m\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+2\,m\,R\right)$$

Alternate forms

$$\frac{1}{12}\,m\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(-6\,\sigma^2\,\delta^{\mu+1}-2\,m^2\,\sigma^2-9\,\delta^2\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{9\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+6\,R\right)$$

$$\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\left(m\left(-\frac{1}{2}\,\sigma^2\,\delta^{\mu+1}+\delta^2\left(-\frac{3}{4}\log^2\left(\frac{3\,M^2+\phi}{M^2}\right)+\frac{3}{2}\log(3)\log\left(\frac{3\,M^2+\phi}{M^2}\right)-\frac{3\log^2(3)}{4}\right)-\frac{3\,M^2\,\phi^2}{4\left(3\,M^2+\phi\right)^2}+\frac{R}{2}\right)-\frac{m^3\,\sigma^2}{6}\right)$$

$$-\frac{m^3\,\sigma^2\,\phi^2\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{6\,\sqrt{3}\,\left(3\,M^2+\phi\right)^2}-\frac{m^3\,M^2\,\sigma^2\,\phi\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{\sqrt{3}\,\left(3\,M^2+\phi\right)^2}-$$
$$\frac{\sqrt{3}\,m^3\,M^4\,\sigma^2\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\left(3\,M^2+\phi\right)^2}-\frac{m\,\sigma^2\,\delta^{\mu+1}\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\,\sqrt{3}}-$$
$$\frac{\sqrt{3}\,\delta^2\,m\,\phi^2\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\,\log^2\left(\frac{\phi}{3\,M^2}+1\right)}{4\left(3\,M^2+\phi\right)^2}-$$
$$\frac{3\,\sqrt{3}\,\delta^2\,m\,M^2\,\phi\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\,\log^2\left(\frac{\phi}{3\,M^2}+1\right)}{2\left(3\,M^2+\phi\right)^2}+\frac{m\,R\,\phi^2\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\,\sqrt{3}\,\left(3\,M^2+\phi\right)^2}+$$
$$\frac{\sqrt{3}\,m\,M^2\,R\,\phi\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{\left(3\,M^2+\phi\right)^2}-\frac{\sqrt{3}\,m\,M^2\,\phi^2\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{4\left(3\,M^2+\phi\right)^2}-$$
$$\frac{9\,\sqrt{3}\,\delta^2\,m\,M^4\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}\,\log^2\left(\frac{\phi}{3\,M^2}+1\right)}{4\left(3\,M^2+\phi\right)^2}+\frac{3\,\sqrt{3}\,m\,M^4\,R\,\sqrt{-\frac{g\left(3\,M^2+\phi\right)}{M^2}}}{2\left(3\,M^2+\phi\right)^2}$$

Expanded form

$$-\frac{1}{6}\,m^3\,\sigma^2\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}-\frac{1}{2}\,m\,\sigma^2\,\delta^{\mu+1}\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}-$$
$$\frac{3}{4}\,\delta^2\,m\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}\,\log^2\left(\frac{\phi}{3\,M^2}+1\right)+$$
$$\frac{1}{2}\,m\,R\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}-\frac{3\,m\,M^2\,\phi^2\,\sqrt{g\left(-\frac{\phi}{3\,M^2}-1\right)}}{4\left(3\,M^2+\phi\right)^2}$$

Alternate forms assuming g, m, M, R, δ, μ, σ, and ϕ are positive

$$\frac{1}{12}\,i\,m\,\sqrt{\frac{g\,\phi}{3\,M^2}+g}\,\left(-6\,\sigma^2\,\delta^{\mu+1}-2\,m^2\,\sigma^2-9\,\delta^2\,\log^2\left(\frac{\phi}{3\,M^2}+1\right)-\frac{9\,M^2\,\phi^2}{\left(3\,M^2+\phi\right)^2}+6\,R\right)$$

$$-\frac{1}{6} i m^3 \sigma^2 \sqrt{g\left(\frac{\phi}{3 M^2}+1\right)} - \frac{1}{2} i m \sigma^2 \delta^{\mu+1} \sqrt{g\left(\frac{\phi}{3 M^2}+1\right)} - \frac{3}{4} i \delta^2 m \sqrt{g\left(\frac{\phi}{3 M^2}+1\right)} \log^2\left(\frac{\phi}{3 M^2}+1\right) + \frac{1}{2} i m R \sqrt{g\left(\frac{\phi}{3 M^2}+1\right)} - \frac{3 i m M^2 \phi^2 \sqrt{g\left(\frac{\phi}{3 M^2}+1\right)}}{4\left(3 M^2+\phi\right)^2}$$

Expanded logarithmic form

$$\frac{1}{2} m R \sqrt{-g+\frac{-\frac{1}{3} g \phi}{M^2}} - \frac{m^3 \sigma^2 \sqrt{-g+\frac{-\frac{1}{3} g \phi}{M^2}}}{2\times 3} - \frac{1}{2} m \delta^{1+\mu} \sigma^2 \sqrt{-g+\frac{-\frac{1}{3} g \phi}{M^2}} + \frac{\frac{1}{4}(-3) m M^2 \phi^2 \sqrt{-g+\frac{-\frac{1}{3} g \phi}{M^2}}}{9 M^4+6 M^2 \phi+\phi^2} - \frac{3}{4} m \delta^2 \sqrt{-g+\frac{-\frac{1}{3} g \phi}{M^2}} \log^2\left(1+\frac{\frac{1}{3}\phi}{M^2}\right)$$

Reduced logarithmic form

$$\frac{1}{12\sqrt{3}\left(3 M^2+\phi\right)^2} m \sqrt{\frac{g\left(-3 M^2-\phi\right)}{M^2}} \left(-9 M^2 \phi^2+6 R\left(3 M^2+\phi\right)^2-2 m^2 \sigma^2\left(3 M^2+\phi\right)^2-6 \delta^{1+\mu} \sigma^2\left(3 M^2+\phi\right)^2-9 \delta^2\left(3 M^2+\phi\right)^2 \log^2\left(\frac{3 M^2+\phi}{3 M^2}\right)\right)$$

Property as a function

Parity

odd

Derivative

$$\frac{\partial}{\partial m}\left(\frac{1}{4}\sqrt{g\left(-1-\frac{\phi}{3M^2}\right)}\left(2mR-\frac{2m^3\sigma^2}{3}-2m\delta^{1+\mu}\sigma^2-\frac{3mM^2\phi^2}{\left(3M^2+\phi\right)^2}-3m\delta^2\log^2\left(1+\frac{\phi}{3M^2}\right)\right)\right)=$$
$$\frac{1}{4}\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(-2\sigma^2\delta^{1+\mu}-2m^2\sigma^2-3\delta^2\log^2\left(\frac{\phi}{3M^2}+1\right)-\frac{3M^2\phi^2}{\left(3M^2+\phi\right)^2}+2R\right)$$

Indefinite integral

$$\int\frac{1}{4}\sqrt{g\left(-1-\frac{\phi}{3M^2}\right)}\left(2mR-\frac{2m^3\sigma^2}{3}-2m\delta^{1+\mu}\sigma^2-\frac{3mM^2\phi^2}{\left(3M^2+\phi\right)^2}-3m\delta^2\log^2\left(1+\frac{\phi}{3M^2}\right)\right)dm=-\frac{1}{24\left(3M^2+\phi\right)^2}m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(9M^4\left(\sigma^2\left(6\delta^{\mu+1}+m^2\right)-6R\right)+3M^2\phi\left(3\left(4\sigma^2\delta^{\mu+1}+\phi\right)+2m^2\sigma^2-12R\right)+\phi^2\left(\sigma^2\left(6\delta^{\mu+1}+m^2\right)-6R\right)+9\delta^2\left(3M^2+\phi\right)^2\log^2\left(\frac{\phi}{3M^2}+1\right)\right)+\text{constant}$$

From the indefinite integral result

$$\int \frac{1}{4}\sqrt{g\left(-1-\frac{\phi}{3M^2}\right)}\left(2mR-\frac{2m^3\sigma^2}{3}-2m\delta^{1+\mu}\sigma^2-\frac{3mM^2\phi^2}{(3M^2+\phi)^2}-3m\delta^2\log^2\left(dm=-\frac{1}{24(3M^2+\phi)^2}m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(9M^4(\sigma^2(6\delta^{\mu+1}+m^2)-6R)+1+\frac{\phi}{3M^2}\right)\right)\right)$$

$$3M^2\phi(3(4\sigma^2\delta^{\mu+1}+\phi)+2m^2\sigma^2-12R)+\phi^2(\sigma^2(6\delta^{\mu+1}+m^2)-6$$

$$R)+9\delta^2(3M^2+\phi)^2\log^2\left(\frac{\phi}{3M^2}+1\right)\Big)+\text{constant}$$

we obtain:

-(m^2√(g(-ϕ/(3M^2)-1))(9M^4 (σ^2(6δ^(μ+1)+m^2)-6R)+3M^2 ϕ (3(4σ^2 δ^(μ+1)+ϕ)+2m^2 σ^2-12R)+ϕ^2(σ^2 (6δ^(μ+1)+m^2)-6R)+9δ^2(3M^2+ϕ)^2 ln^2(ϕ/(3M^2)+1)))/(24(3M^2+ϕ)^2)

Input

$$-\frac{1}{24(3M^2+\phi)^2}m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}$$

$$\left(9M^4(\sigma^2(6\delta^{\mu+1}+m^2)-6R)+3M^2\phi(3(4\sigma^2\delta^{\mu+1}+\phi)+2m^2\sigma^2-12R)+\right.$$

$$\left.\phi(\sigma^2(6\delta^{\mu+1}+m^2)-6R)^2+9(\delta(3M^2+\phi)^2)^2\log^2\left(\frac{\phi}{3M^2}+1\right)\right)$$

$\phi(n)$ is the Euler totient function

$\delta(x)$ is the Dirac delta function

$\log(x)$ is the natural logarithm

Result

$$-\frac{1}{24(3M^2+\phi)^2}m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}$$

$$\left(9\delta(3M^2+\phi)^4\log^2\left(\frac{\phi}{3M^2}+1\right)+3M^2\phi(2m^2\sigma^2-12R+3(4\sigma^2\delta^{\mu+1}+\phi))+\right.$$

$$\left.\phi((6\delta^{\mu+1}+m^2)\sigma^2-6R)^2+9M^4(\sigma^2(6\delta^{\mu+1}+m^2)-6R)\right)$$

Alternate form

$$-\frac{1}{24\left(3M^2+\phi\right)^2}m^2\sqrt{g\left(-\frac{3M^2+\phi}{3M^2}\right)}$$
$$\left(\delta\left(3M^2+\phi\right)^4\left(9\log^2\left(\frac{3M^2+\phi}{M^2}\right)-18\log(3)\log\left(\frac{3M^2+\phi}{M^2}\right)+9\log^2(3)\right)+\right.$$
$$3M^2\phi\left(\left(12\delta^{\mu+1}+2m^2\right)\sigma^2-12R+3\phi\right)+$$
$$\left.\phi\left(\left(6\delta^{\mu+1}+m^2\right)\sigma^2-6R\right)^2+M^4\left(9m^2\sigma^2-54\left(R-\sigma^2\delta^{\mu+1}\right)\right)\right)$$

Expanded form

$$-\frac{3m^2\delta\left(3M^2+\phi\right)^4\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\log^2\left(\frac{\phi}{3M^2}+1\right)}{8\left(3M^2+\phi\right)^2}-$$
$$\frac{m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\phi\left(\left(6\delta^{\mu+1}+m^2\right)\sigma^2-6R\right)^2}{24\left(3M^2+\phi\right)^2}-$$
$$\frac{m^2M^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\phi\left(2m^2\sigma^2-12R+3\left(4\sigma^2\delta^{\mu+1}+\phi\right)\right)}{8\left(3M^2+\phi\right)^2}-$$
$$\frac{3m^4M^4\sigma^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}}{8\left(3M^2+\phi\right)^2}-$$
$$\frac{9m^2M^4\sigma^2\delta^{\mu+1}\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}}{4\left(3M^2+\phi\right)^2}+\frac{9m^2M^4R\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}}{4\left(3M^2+\phi\right)^2}$$

Alternate forms assuming m, M, R, δ, μ, σ, and ϕ are positive

$$-\frac{1}{24\left(3M^2+\phi\right)^2}m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(3M^2\phi\left(12\sigma^2\delta^{\mu+1}+2m^2\sigma^2-12R+3\phi\right)+\right.$$
$$\left.\phi\left(\left(6\delta^{\mu+1}+m^2\right)\sigma^2-6R\right)^2-9M^4\left(6R-\sigma^2\left(6\delta^{\mu+1}+m^2\right)\right)\right)$$

$$-\frac{m^2 M^2 \sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\,\phi\left(2m^2\sigma^2-12R+3\left(4\sigma^2\delta^{\mu+1}+\phi\right)\right)}{8\left(3M^2+\phi\right)^2}-$$

$$\frac{m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\,\phi\left(\left(6\delta^{\mu+1}+m^2\right)\sigma^2-6R\right)^2}{24\left(3M^2+\phi\right)^2}-$$

$$\frac{3m^2M^4\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}\left(\sigma^2\left(6\delta^{\mu+1}+m^2\right)-6R\right)}{8\left(3M^2+\phi\right)^2}$$

Expanded logarithmic form

$$\frac{54m^2M^4R\sqrt{g\left(-1+\frac{-\frac{1}{3}\phi}{M^2}\right)}}{216M^4+144M^2\phi+24\phi^2}+$$

$$-\frac{9m^4M^4\sigma^2\sqrt{g\left(-1+\frac{-\frac{1}{3}\phi}{M^2}\right)}}{216M^4+144M^2\phi+24\phi^2}+-\frac{54m^2M^4\delta^{1+\mu}\sigma^2\sqrt{g\left(-1+\frac{-\frac{1}{3}\phi}{M^2}\right)}}{216M^4+144M^2\phi+24\phi^2}+$$

$$-\frac{m^2\phi\left(-6R+m^2\sigma^2+6\delta^{1+\mu}\sigma^2\right)^2\sqrt{g\left(-1+\frac{-\frac{1}{3}\phi}{M^2}\right)}}{216M^4+144M^2\phi+24\phi^2}+$$

$$-\frac{3m^2M^2\phi\left(-12R+2m^2\sigma^2+12\delta^{1+\mu}\sigma^2+3\phi\right)\sqrt{g\left(-1+\frac{-\frac{1}{3}\phi}{M^2}\right)}}{216M^4+144M^2\phi+24\phi^2}+$$

$$-\frac{9m^2\delta\left(3M^2+\phi\right)^4\sqrt{g\left(-1+\frac{-\frac{1}{3}\phi}{M^2}\right)}\log^2\left(1+\frac{\frac{1}{3}\phi}{M^2}\right)}{216M^4+144M^2\phi+24\phi^2}$$

Reduced logarithmic form

$$-\frac{1}{24\left(3M^2+\phi\right)^2}m^2\sqrt{g\left(\frac{-3M^2-\phi}{3M^2}\right)}$$
$$\left(9M^4\left(-6R+\left(m^2+6\delta^{1+\mu}\right)\sigma^2\right)+\phi\left(-6R+\left(m^2+6\delta^{1+\mu}\right)\sigma^2\right)^2+\right.$$
$$\left.3M^2\phi\left(-12R+2m^2\sigma^2+3\left(4\delta^{1+\mu}\sigma^2+\phi\right)\right)+9\delta\left(3M^2+\phi\right)^4\log^2\left(\frac{3M^2+\phi}{3M^2}\right)\right)$$

From the result

$$-\frac{1}{24\left(3M^2+\phi\right)^2}m^2\sqrt{g\left(-\frac{\phi}{3M^2}-1\right)}$$
$$\left(9\delta\left(3M^2+\phi\right)^4\log^2\left(\frac{\phi}{3M^2}+1\right)+3M^2\phi\left(2m^2\sigma^2-12R+3\left(4\sigma^2\delta^{\mu+1}+\phi\right)\right)+\right.$$
$$\left.\phi\left(\left(6\delta^{\mu+1}+m^2\right)\sigma^2-6R\right)^2+9M^4\left(\sigma^2\left(6\delta^{\mu+1}+m^2\right)-6R\right)\right)$$

for g = 1, M = 2, R = 4, we obtain:

-(m^2√((-φ/(12)-1))(9δ(12+φ)^4 log^2(φ/(12)+1)+12φ(2m^2*σ^2-48+3 (4*σ^2 δ^(μ+1)+φ))+φ((6δ^(μ+1)+m^2)*σ^2-24)^2+144(σ^2*(6δ^(μ+1)+m^2) - 24)))/(24 (12+φ)^2)

Input

$$-\frac{1}{24\left(12+\phi\right)^2}m^2\sqrt{-\frac{\phi}{12}-1}$$
$$\left(9\delta\left(12+\phi\right)^4\log^2\left(\frac{\phi}{12}+1\right)+12\phi\left(2m^2\sigma^2-48+3\left(4\sigma^2\delta^{\mu+1}+\phi\right)\right)+\right.$$
$$\left.\phi\left(\left(6\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2+144\left(\sigma^2\left(6\delta^{\mu+1}+m^2\right)-24\right)\right)$$

$\delta(x)$ is the Dirac delta function

$\log(x)$ is the natural logarithm

$\phi(n)$ is the Euler totient function

ϕ is the golden ratio

Exact result

$$-\frac{1}{24\,(\phi+12)^2}\,i\,m^2\,\sqrt{\frac{\phi}{12}+1}\,\left(12\,\phi\left(2\,m^2\,\sigma^2+3\left(4\,\sigma^2\,\delta^{\mu+1}+\phi\right)-48\right)+\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2+144\left(\sigma^2\left(6\,\delta^{\mu+1}+m^2\right)-24\right)\right)$$

Expanded form

$$-\frac{i\,\sqrt{1+\frac{1}{24}\left(1+\sqrt{5}\right)}\;m^2\,\phi\left(2\,m^2\,\sigma^2+3\left(4\,\sigma^2\,\delta^{\mu+1}+\phi\right)-48\right)}{2\left(12+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}-\frac{i\,\sqrt{1+\frac{1}{24}\left(1+\sqrt{5}\right)}\;m^2\,\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2}{24\left(12+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}-\frac{6\,i\,\sqrt{1+\frac{1}{24}\left(1+\sqrt{5}\right)}\;m^4\,\sigma^2}{\left(12+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}-\frac{36\,i\,\sqrt{1+\frac{1}{24}\left(1+\sqrt{5}\right)}\;m^2\,\sigma^2\,\delta^{\mu+1}}{\left(12+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}+\frac{144\,i\,\sqrt{1+\frac{1}{24}\left(1+\sqrt{5}\right)}\;m^2}{\left(12+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2}$$

Alternate form assuming m, δ, μ, and σ are positive

$$-\frac{i\,m^2\,\sqrt{\frac{\phi}{12}+1}\;\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2}{24\,(\phi+12)^2}-\frac{i\,m^2\,\sqrt{\frac{\phi}{12}+1}\;\phi\left(2\,m^2\,\sigma^2+3\left(4\,\sigma^2\,\delta^{\mu+1}+\phi\right)-48\right)}{2\,(\phi+12)^2}-\frac{6\,i\,m^2\,\sqrt{\frac{\phi}{12}+1}\;\left(\sigma^2\left(6\,\delta^{\mu+1}+m^2\right)-24\right)}{(\phi+12)^2}$$

Alternate forms

$$-\left(\left(i\,m^2\left(864\,\sigma^2\,\delta^{\mu+1}+12\,\phi\left(2\,m^2\,\sigma^2+3\left(4\,\sigma^2\,\delta^{\mu+1}+\phi\right)-48\right)+\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2+144\,m^2\,\sigma^2-3456\right)\right)\Big/\left(12\sqrt{6}\,\left(25+\sqrt{5}\right)^{3/2}\right)\right)$$

$$-i\sqrt{\frac{1}{12\,869\,712}-\frac{47}{1\,029\,576\,960\sqrt{5}}}\;m^2\left(\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2+12\,\phi\left(2\,m^2\,\sigma^2+3\left(4\,\sigma^2\,\delta^{\mu+1}+\frac{1}{2}\left(1+\sqrt{5}\right)\right)-48\right)+144\left(\sigma^2\left(6\,\delta^{\mu+1}+m^2\right)-24\right)\right)$$

$$-\frac{1}{12\sqrt{6}}\,i\,m^2\left(\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2+12\left(\phi\left(\frac{1}{2}\left(\left(24\,\delta^{\mu+1}+4\,m^2\right)\sigma^2+3\sqrt{5}-93\right)\right)-288\right)+\sigma^2\left(864\,\delta^{\mu+1}+144\,m^2\right)\right)$$

root of $238\,328\,000\,x^4-32\,000\,x^2+1$ near $x=0.00703531$

From the exact result

$$-\frac{1}{24\,(\phi+12)^2}\,i\,m^2\sqrt{\frac{\phi}{12}+1}\;\left(12\,\phi\left(2\,m^2\,\sigma^2+3\left(4\,\sigma^2\,\delta^{\mu+1}+\phi\right)-48\right)+\phi\left(\left(6\,\delta^{\mu+1}+m^2\right)\sigma^2-24\right)^2+144\left(\sigma^2\left(6\,\delta^{\mu+1}+m^2\right)-24\right)\right)$$

-(i m^2 sqrt(ϕ/12 + 1) (12 ϕ(2 m^2 σ^2 + 3 (4 σ^2 δ^(μ + 1) + ϕ) - 48) + ϕ((6 δ^(μ + 1) + m^2) σ^2 - 24)^2 + 144 (σ^2 (6 δ^(μ + 1) + m^2) - 24)))/(24 (ϕ + 12)^2)

for μ = 2, δ = 4, m = 8, σ = 16:

-(i 8^2 sqrt((1/Φ)/12 + 1) (12 (1/Φ)(2 8^2 16^2 + 3 (4 16^2 4^(2 + 1) + 1/Φ) - 48) + 1/Φ*((6 4^(2 + 1) + 8^2) 16^2 - 24)^2 + 144 (16^2 (6 4^(2 + 1) + 8^2) - 24)))/(24 (1/Φ + 12)^2)

Input

$$-\frac{1}{24\left(\frac{1}{\Phi}+12\right)^2}i\times 8^2\sqrt{\frac{\frac{1}{\Phi}}{12}+1}\left(12\times\frac{1}{\Phi}\left(2\times 8^2\times 16^2+3\left(4\times 16^2\times 4^{2+1}+\frac{1}{\Phi}\right)-48\right)+\frac{1}{\Phi}\left(\left(6\times 4^{2+1}+8^2\right)\times 16^2-24\right)^2+144\left(16^2\left(6\times 4^{2+1}+8^2\right)-24\right)\right)$$

i is the imaginary unit

Φ is the golden ratio conjugate

Exact result

$$-\frac{8\,i\sqrt{\frac{1}{12\,\Phi}+1}\left(\frac{13\,147\,832\,896}{\Phi}+\frac{12\left(3\left(\frac{1}{\Phi}+65\,536\right)+32\,720\right)}{\Phi}+16\,511\,616\right)}{3\left(\frac{1}{\Phi}+12\right)^2}$$

Exact form

$$-\frac{8\,i\sqrt{\frac{2}{3}\left(1+\sqrt{5}\right)}\left(1\,641\,759\,152\sqrt{5}-1\,637\,631\,239\right)}{3\left(6\sqrt{5}-5\right)^{3/2}}$$

Decimal approximation

$-3.261940808186486605084442754089401919333261237901408260174\ldots\times 10^8\,i$

Alternate complex forms

$$\frac{16\left(3\,287\,646\,208\,\Phi+4\,127\,904\,\Phi^2+9\right)\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)}{3\sqrt{3\,\Phi}\,(12\,\Phi+1)^{3/2}}$$

$$\frac{16\,e^{-(i\,\pi)/2}\left(3\,287\,646\,208\,\Phi+4\,127\,904\,\Phi^2+9\right)}{3\sqrt{3\,\Phi}\,(12\,\Phi+1)^{3/2}}$$

Polar coordinates

$r \approx 326\,194\,081$ (radius), $\theta \approx -1.5708$ (angle)

326194081

Expanded form

$$-\frac{44\,030\,976\,i\sqrt{\frac{1}{12\Phi}+1}}{\left(\frac{1}{\Phi}+12\right)^2}-\frac{96\,i\sqrt{\frac{1}{12\Phi}+1}}{\left(\frac{1}{\Phi}+12\right)^2\Phi^2}-\frac{105\,204\,678\,656\,i\sqrt{\frac{1}{12\Phi}+1}}{3\left(\frac{1}{\Phi}+12\right)^2\Phi}$$

Alternate forms

$$-\frac{16\,i\left(3\,287\,646\,208\,\Phi+4\,127\,904\,\Phi^2+9\right)}{3\sqrt{3\Phi}\,(12\Phi+1)^{3/2}}$$

$$-\frac{16\,i\,(458\,656\,\Phi\,(9\Phi+7168)+9)}{3\sqrt{3\Phi}\,(12\Phi+1)^{3/2}}$$

$$\frac{13\,149\,208\,432\,i}{9\sqrt{3\Phi}\,(12\Phi+1)^{3/2}}-\frac{458\,656}{3}\,i\sqrt{\frac{12\Phi+1}{3\Phi}}-\frac{13\,147\,832\,896\,i}{9\sqrt{3\Phi\,(12\Phi+1)}}$$

From the fundamental "unitary" formula:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From the previous result, for C = 326194081, inserting a radius of the Universe, which we have calculated, equal to R = 62685.1369 years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684)
(326194081×(62685.1369)×2.33·10^(-13))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{326\,194\,081\times 62\,685.1369\times 2.33}{10^{13}}}}$$

Result

1.6180362466198016052790286360206409935133727024993014097431586730

…

1.6180362466198…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

All 2nd roots of 2.61804

$1.61804\, e^{0} \approx 1.6180$ (real, principal root)

$1.61804\, e^{i\pi} \approx -1.6180$ (real root)

Series representations

$$\sqrt{\frac{2\left(2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}\right)}{\frac{326\,194081\times 62\,685.1\times 2.33}{(\pi\, 0.99911)\,10^{13}}}} = \sqrt{-1+0.875974\,\pi\sqrt[16]{\frac{\sqrt{2}}{\pi}}}\ \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+0.875974\,\pi\sqrt[16]{\frac{\sqrt{2}}{\pi}}\right)^{-k}$$

$$\sqrt{\frac{2\left(2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}\right)}{\frac{326\,194081\times 62\,685.1\times 2.33}{(\pi\, 0.99911)\,10^{13}}}} = \sqrt{-1+0.875974\,\pi\sqrt[16]{\frac{\sqrt{2}}{\pi}}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+0.875974\,\pi\sqrt[16]{\frac{\sqrt{2}}{\pi}}\right)^{-k}}{k!}$$

$$\sqrt{\frac{2\left(2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}\right)}{\frac{326\,194081\times 62\,685.1\times 2.33}{(\pi\,0.99911)\,10^{13}}}} = \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(0.875974\,\pi\sqrt[16]{\frac{\sqrt{2}}{\pi}}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

Now, we analyze the following equation:

$$\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\varphi}\right)^2$$

We consider:

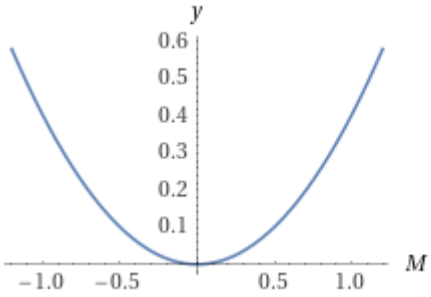
3/4*(M^2)*(1-e^(-sqrt(2/3)*φ))^2

Input

$$\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\;\phi}\right)^2$$

ϕ is the golden ratio

Plot **(figure that can be related to an open string)**



(M from −1.2 to 1.2)

Geometric figure

parabola

Expanded forms

$$-\frac{3}{2} M^2 e^{-\sqrt{2/3}\,\phi} + \frac{3}{4} M^2 e^{-2\sqrt{2/3}\,\phi} + \frac{3M^2}{4}$$

$$-\frac{3}{2} e^{-\sqrt{5/6}\,-1/\sqrt{6}}\, M^2 + \frac{3}{4} e^{-\sqrt{2/3}\,-\sqrt{10/3}}\, M^2 + \frac{3M^2}{4}$$

Alternate forms

$$-\frac{3}{2} e^{-(1+\sqrt{5})/\sqrt{6}}\, M^2 + \frac{3}{4} e^{-\sqrt{2/3}\,(1+\sqrt{5})} M^2 + \frac{3M^2}{4}$$

$$\frac{3}{4} M^2 \left(e^{-\sqrt{2/3}\,\phi} - 1\right)^2$$

$$\left(\frac{3}{4} + \frac{3}{4} e^{-\sqrt{2/3}\,(1+\sqrt{5})} - \frac{3}{2} e^{(1+\sqrt{5})/\sqrt{6}\,-\sqrt{2/3}\,(1+\sqrt{5})}\right) M^2$$

Root

$M = 0$

Polynomial discriminant

$\Delta = 0$

Property as a function

Parity

even

Derivative

$$\frac{d}{dM}\left(\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\phi}\right)^2\right)=\frac{3}{2}M\left(1-e^{-\sqrt{2/3}\,\phi}\right)^2$$

Indefinite integral

$$\int\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\phi}\right)^2 dM=\frac{1}{4}M^3\left(e^{-\sqrt{2/3}\,\phi}-1\right)^2+\text{constant}$$

Global minimum

$$\min\left\{\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\phi}\right)^2\right\}=0 \text{ at } M=0$$

From the indefinite integral result

$$\int\frac{3}{4}M^2\left(1-e^{-\sqrt{2/3}\,\phi}\right)^2 dM=\frac{1}{4}M^3\left(e^{-\sqrt{2/3}\,\phi}-1\right)^2+\text{constant}$$

we obtain:

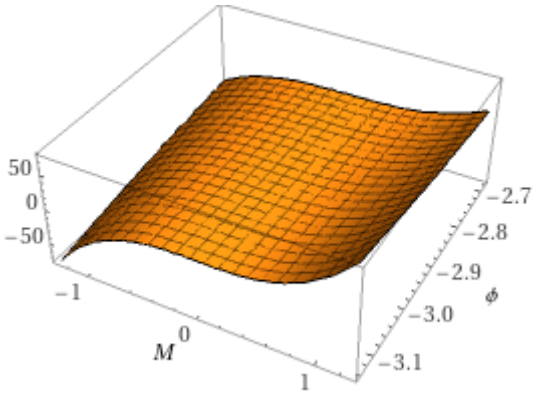
1/4 (-1 + e^(-sqrt(2/3) ϕ))^2 M^3

Input

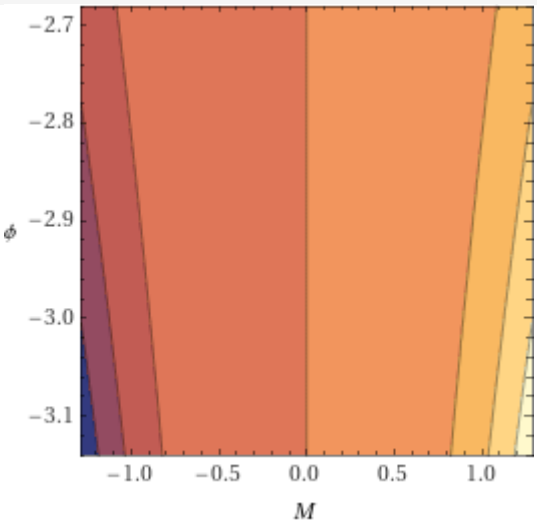
$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^3$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the below plots and is that at $M = 1$, which is taken as the energy density of the universe at the Big Bang, with $\phi = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



Expanded form

$$\frac{1}{4} M^3 e^{-2\sqrt{2/3}\,\phi} - \frac{1}{2} M^3 e^{-\sqrt{2/3}\,\phi} + \frac{M^3}{4}$$

Alternate form

$$\frac{1}{4} M^3 e^{-2\sqrt{2/3}\,\phi}\left(e^{\sqrt{2/3}\,\phi}-1\right)^2$$

Real root

$M = 0$

Roots

$\phi = i\sqrt{6}\,\pi n, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

Periodicity

periodic in ϕ with period $i\sqrt{6}\,\pi$

Derivative

$$\frac{\partial}{\partial M}\left(\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^3\right) = \frac{3}{4} M^2\left(e^{-\sqrt{2/3}\,\phi}-1\right)^2$$

Indefinite integral

$$\int \frac{1}{4}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^3\, dM = \frac{1}{16} M^4\left(e^{-\sqrt{2/3}\,\phi}-1\right)^2 + \text{constant}$$

Limit

$$\lim_{\phi\to\infty} \frac{1}{4}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^3 = \frac{M^3}{4} \approx 0.25\, M^3$$

Alternative representations

$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^3 = \frac{1}{4}\left(-1+w^a\right)^2 M^3 \quad \text{for } 3a + \frac{\sqrt{6}\,\phi}{\log(w)} = 0$$

$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^3 = \frac{1}{4}\left(-1+z^{-\sqrt{2/3}\ \phi}\right)^2 M^3 \quad \text{for } z = e$$

$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^3 = \frac{1}{4} M^3 \left(\frac{2}{-1+\coth\left(-\frac{1}{2}\phi\sqrt{\frac{2}{3}}\right)}\right)^2$$

log(x) is the natural logarithm

coth(x) is the hyperbolic cotangent function

Series representations

$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^3 = \frac{1}{4} M^3 \left(-1+\sum_{k=0}^{\infty}\frac{\left(\frac{2}{3}\right)^{k/2}(-\phi)^k}{k!}\right)^2$$

$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^3 = \frac{1}{4} M^3 \left(-1+\sum_{k=-\infty}^{\infty} I_k\left(-\sqrt{\frac{2}{3}}\ \phi\right)\right)^2$$

$$\frac{1}{4}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^3 = \frac{1}{4} M^3 \left(-1+e^{z_0}\sum_{k=0}^{\infty}\frac{\left(-\sqrt{\frac{2}{3}}\ \phi - z_0\right)^k}{k!}\right)^2$$

$n!$ is the factorial function

$I_n(z)$ is the modified Bessel function of the first kind

Again, from the indefinite integral result

$$\int \frac{1}{4}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^3\,dM = \frac{1}{16}M^4\left(e^{-\sqrt{2/3}\,\phi}-1\right)^2 + \text{constant}$$

we obtain:

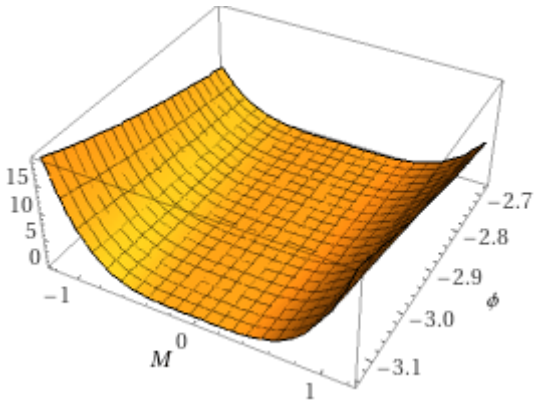
1/16 (-1 + e^(-sqrt(2/3) ϕ))^2 M^4

Input

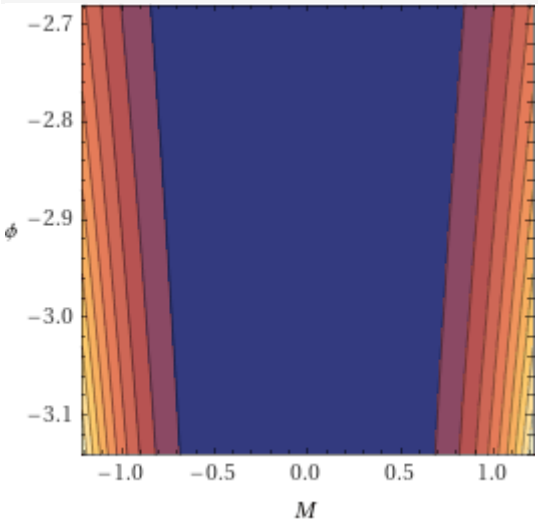
$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the below plots and is that at $M = 1$, which is taken as the energy density of the universe at the Big Bang, with $\phi = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



Expanded form

$$\frac{1}{16} M^4 e^{-2\sqrt{2/3}\,\phi} - \frac{1}{8} M^4 e^{-\sqrt{2/3}\,\phi} + \frac{M^4}{16}$$

Alternate form

$$\frac{1}{16} M^4 e^{-2\sqrt{2/3}\,\phi} \left(e^{\sqrt{2/3}\,\phi} - 1\right)^2$$

Real root

$M = 0$

Roots

$\phi = i\sqrt{6}\,\pi n, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

Periodicity

periodic in ϕ with period $i\sqrt{6}\,\pi$

Derivative

$$\frac{\partial}{\partial M}\left(\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4\right) = \frac{1}{4} M^3 \left(e^{-\sqrt{2/3}\,\phi}-1\right)^2$$

Indefinite integral

$$\int \frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4\, dM = \frac{1}{80} M^5 \left(e^{-\sqrt{2/3}\,\phi}-1\right)^2 + \text{constant}$$

Global minimum

$$\min\left\{\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4\right\} = 0 \text{ at } M=0$$

Limit

$$\lim_{\phi\to\infty} \frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4 = \frac{M^4}{16} \approx 0.0625\, M^4$$

Alternative representations

$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4 = \frac{1}{16}\left(-1+w^a\right)^2 M^4 \text{ for } 3a+\frac{\sqrt{6}\,\phi}{\log(w)} = 0$$

$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4 = \frac{1}{16}\left(-1+z^{-\sqrt{2/3}\,\phi}\right)^2 M^4 \text{ for } z = e$$

$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 M^4 = \frac{1}{16} M^4 \left(\frac{2}{-1+\coth\left(-\frac{1}{2}\phi\sqrt{\frac{2}{3}}\right)}\right)^2$$

log(x) is the natural logarithm

coth(x) is the hyperbolic cotangent function

Series representations

$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^4 = \frac{1}{16} M^4 \left(-1+\sum_{k=0}^{\infty} \frac{\left(\frac{2}{3}\right)^{k/2} (-\phi)^k}{k!}\right)^2$$

$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^4 = \frac{1}{16} M^4 \left(-1+\sum_{k=-\infty}^{\infty} I_k\left(-\sqrt{\frac{2}{3}}\ \phi\right)\right)^2$$

$$\frac{1}{16}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^4 = \frac{1}{16} M^4 \left(-1+e^{z_0}\sum_{k=0}^{\infty} \frac{\left(-\sqrt{\frac{2}{3}}\ \phi - z_0\right)^k}{k!}\right)^2$$

$n!$ is the factorial function

$I_n(z)$ is the modified Bessel function of the first kind

In conclusion, from the indefinite integral result

$$\int \frac{1}{16}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 M^4\, dM = \frac{1}{80} M^5 \left(e^{-\sqrt{2/3}\ \phi}-1\right)^2 + \text{constant}$$

1/80 (-1 + e^(-sqrt(2/3) ϕ))^2 M^5

for M = 2:

1/80 (-1 + e^(-sqrt(2/3) φ))^2 2^5

Input

$$\frac{1}{80}\left(-1+e^{-\sqrt{2/3}\ \phi}\right)^2 \times 2^5$$

ϕ is the golden ratio

Exact result

$$\frac{2}{5}\left(e^{-\sqrt{2/3}\,\phi}-1\right)^2$$

Decimal approximation

0.215011495221603960915298154109062054408855285979143961807490508 7

…

0.2150114952216….

Property

$\frac{2}{5}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2$ is a transcendental number

Expanded forms

$$\frac{2}{5}+\frac{2}{5}e^{-\sqrt{2/3}-\sqrt{10/3}}-\frac{4}{5}e^{-\sqrt{5/6}-1/\sqrt{6}}$$

$$\frac{2}{5}e^{-2\sqrt{2/3}\,\phi}-\frac{4}{5}e^{-\sqrt{2/3}\,\phi}+\frac{2}{5}$$

Alternate forms

$$\frac{2}{5}\left(e^{-(1+\sqrt{5})/\sqrt{6}}-1\right)^2$$

$$\frac{2}{5}\left(e^{-\sqrt{1/3\left(3+\sqrt{5}\right)}}-1\right)^2$$

$$\frac{2}{5}+\frac{2}{5}e^{-\sqrt{2/3}\,(1+\sqrt{5})}-\frac{4}{5}e^{-(1+\sqrt{5})/\sqrt{6}}$$

Series representations

$$\frac{1}{80}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 2^5=\frac{2}{5}\left(-1+\exp\left(-\phi\sum_{k=0}^{\infty}\frac{(-1)^{2k}\,3^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^2$$

$$\frac{1}{80}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 2^5=\frac{2}{5}\left(-1+\exp\left(-\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{2}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)^2$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\frac{1}{80}\left(-1+e^{-\sqrt{2/3}\,\phi}\right)^2 2^5=\frac{2}{5}\left(-1+\exp\left(\frac{\phi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{1}{3}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the fundamental "unitary" formula:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From the previous result, for $C = 0.2150114952216$, inserting a radius of the Universe, which we have calculated, equal to $R = 9.50999436864 \times 10^{13}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (0.2150114952216 ×(9.50999436864 × 10^13)×2.33·10^(-13))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{0.2150114952216\times 9.50999436864\times 10^{13}\times 2.33}{10^{13}}}}$$

Result

1.6180338543296475034926346854588515245407321058185668775084658467 …

1.6180338543296475…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

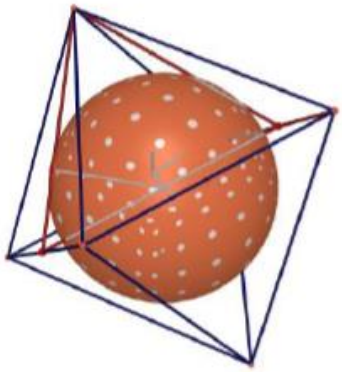


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.645297378520776032771896229793728200454953421110291570825393928 6

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1 + \frac{1}{\sqrt[3]{\left(\phi^2 + \frac{2\pi}{3} C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.14193957152658430892433073219616263267751338681165904468254173 93

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi\,C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\ l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\ l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\ x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\ l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}\ z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2} \times \frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{\left(6\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\frac{\left(6\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\left(6\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

4096 = 64^2 , (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: “Dilaton Tadpole for the Open Bosonic String “ Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$

$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\ l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$

$$1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\ l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^{\,8}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,8}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^{\,8}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^{\,8}\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2(\sqrt{2}\ l^3)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2(\sqrt{2}\ l^3)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$

$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor \arg(2-z_0)/(2\pi)\rfloor}\ z_0^{4+4\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor \arg\left(256\sqrt{2}^{\,8}-z_0\right)/(2\pi)\right\rfloor}\ z_0^{1/2+1/2\left\lfloor \arg\left(256\sqrt{2}^{\,8}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k\ z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1 \sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003 \quad \text{(real, principal root)}$$

$$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003 \quad \text{(real root)}$$

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{8}{\pi^2}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

With regard 8, this value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring. The value 24, instead, is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons.

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_{\mu}\varphi\partial_{\nu}\varphi\right]=$$

$$=\int_0^{\infty}\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_{\mu}\Phi\partial^{\mu}\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_{\nu}\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V=\frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r=\frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S}=\frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi}\quad (\text{for } d\neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ d^3}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)12}=\frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}\quad \text{for } (\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\leq 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\,d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\,d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

$0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ (for $d \neq 0$)

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$\frac{15}{2\pi}+\frac{5\sqrt{5}}{2\pi}$ (for $d \neq 0$)

Alternate forms

$\frac{15+5\sqrt{5}}{2\pi}$ (for $d \neq 0$)

$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi}$ (for $d \neq 0$)

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12}=\frac{15}{2\pi}+\frac{5\sqrt{4}\ \sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)12}=\frac{15}{2\,\pi}+\frac{5\,\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\,\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)12}=\frac{15}{2\,\pi}+\frac{5\,\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\,\pi\,\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} = \sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} = \sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\Big/\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.618035912348264235440174408834709854254527350840173356306481810 7

…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

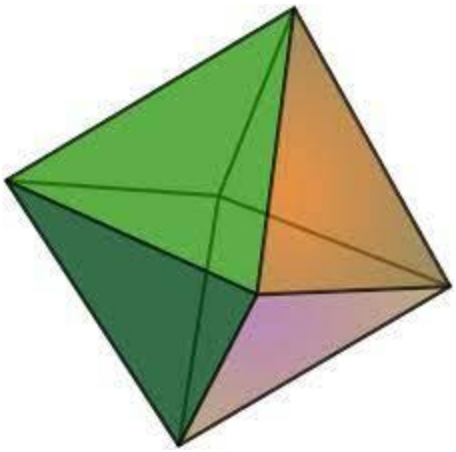
$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

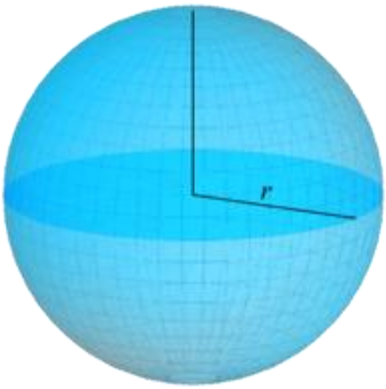
12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find a, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

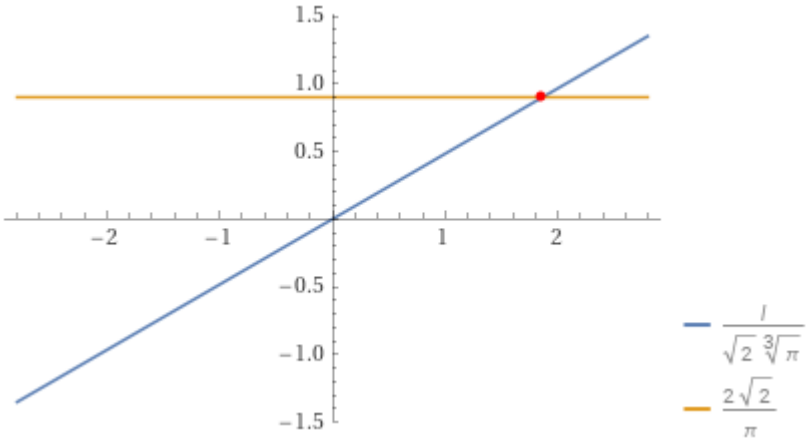
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32\cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}}; \; a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,3}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

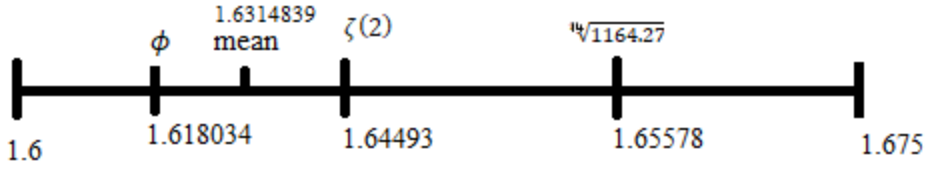
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

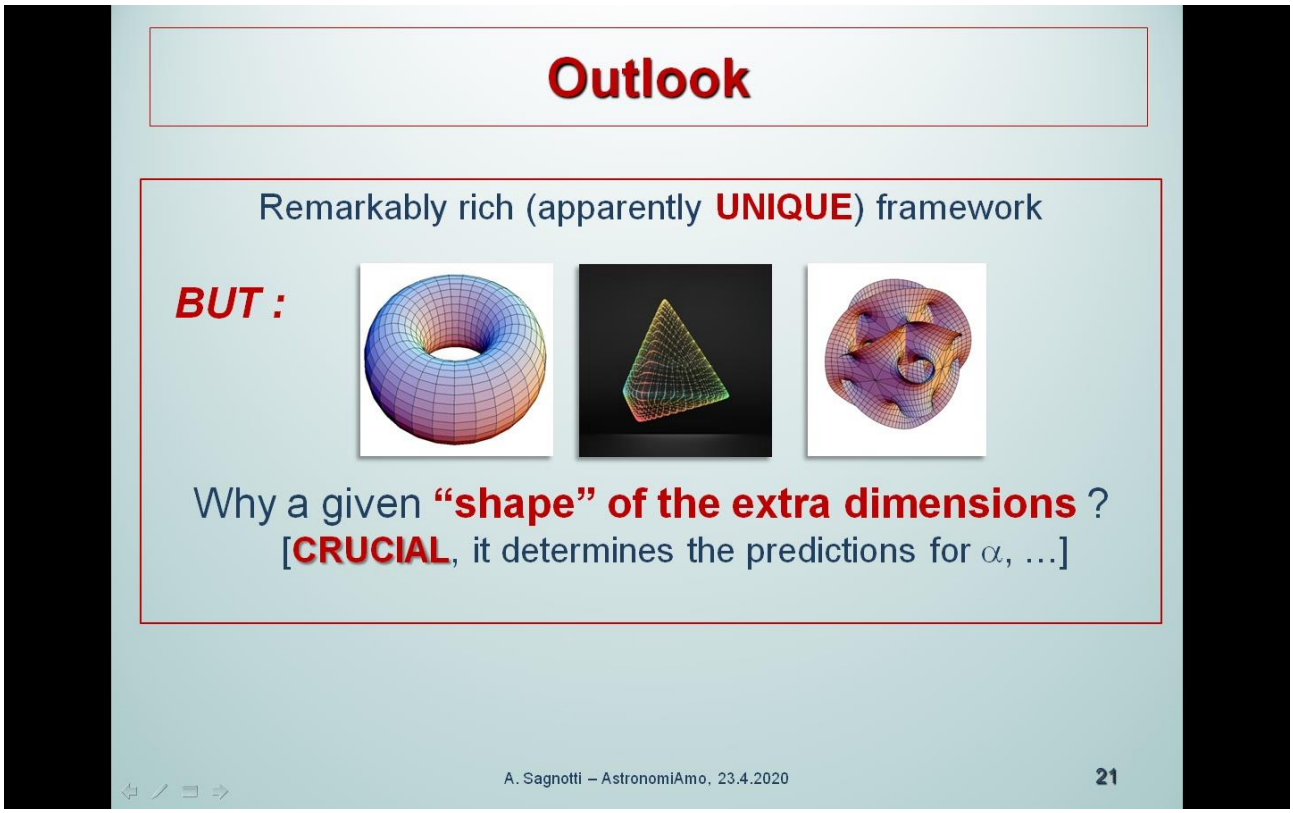
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

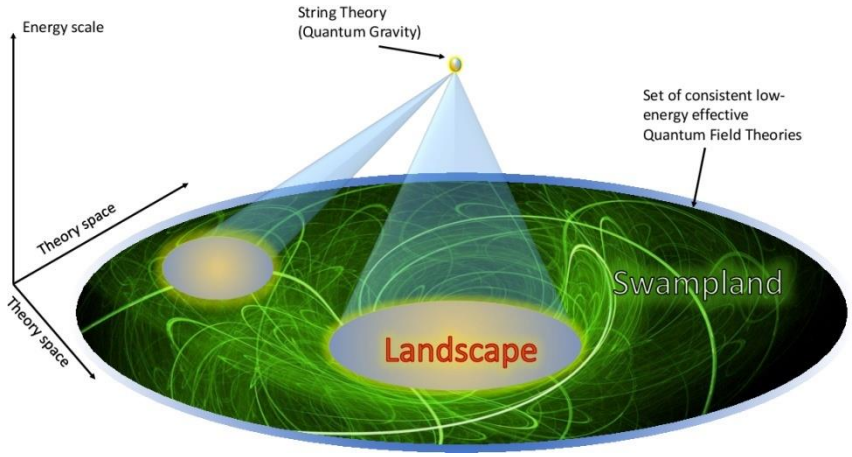


Fig. 1

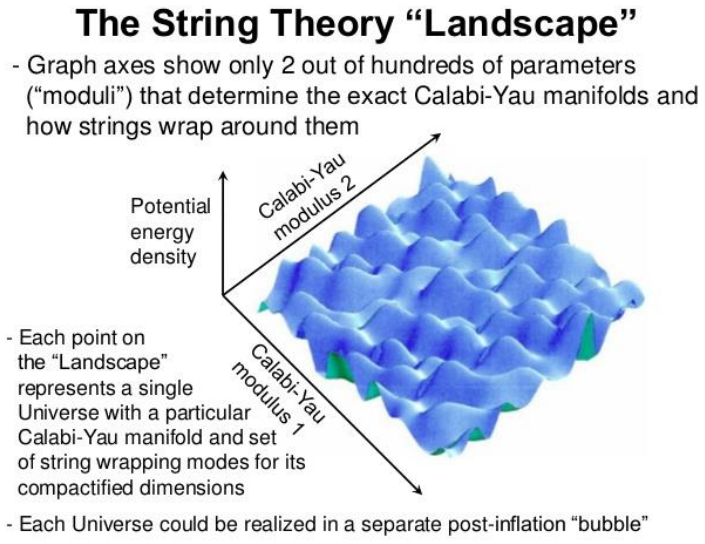


Fig. 2

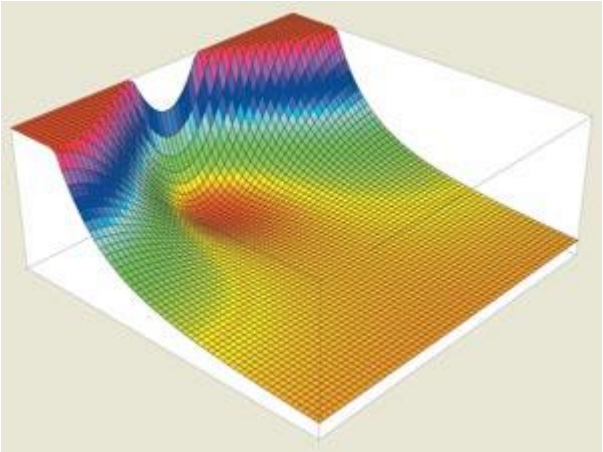


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

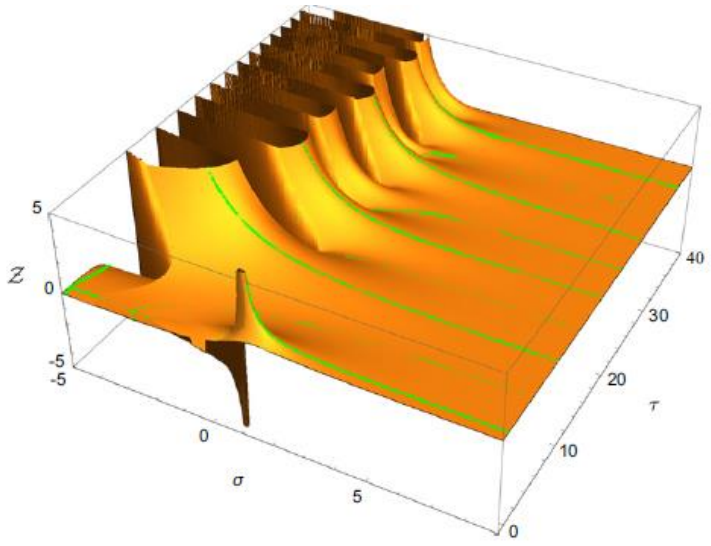


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

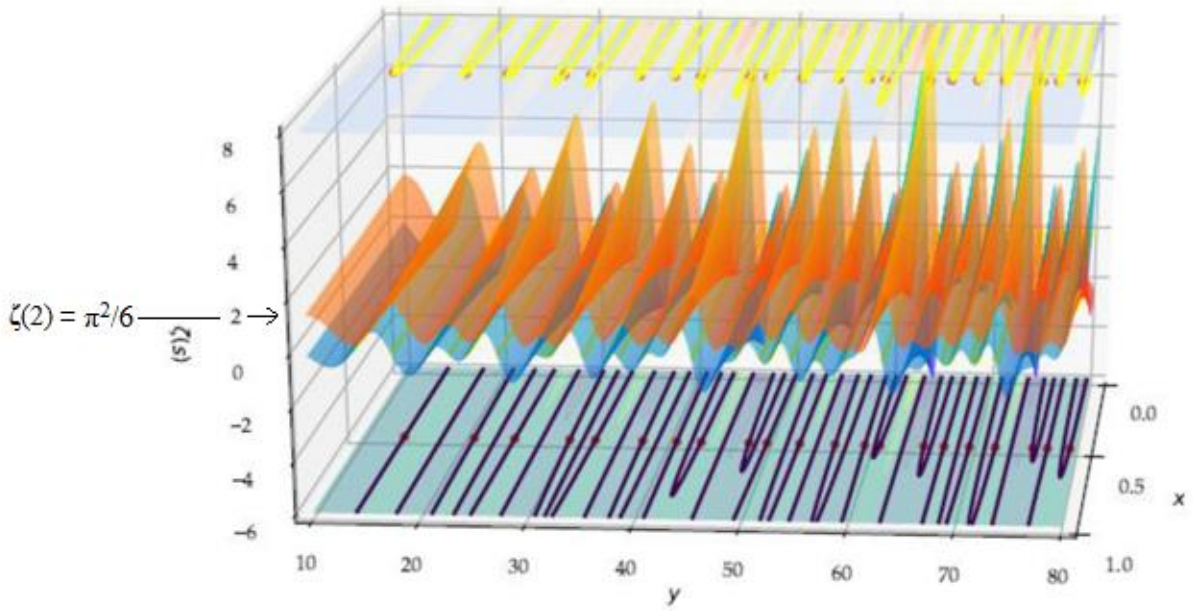


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

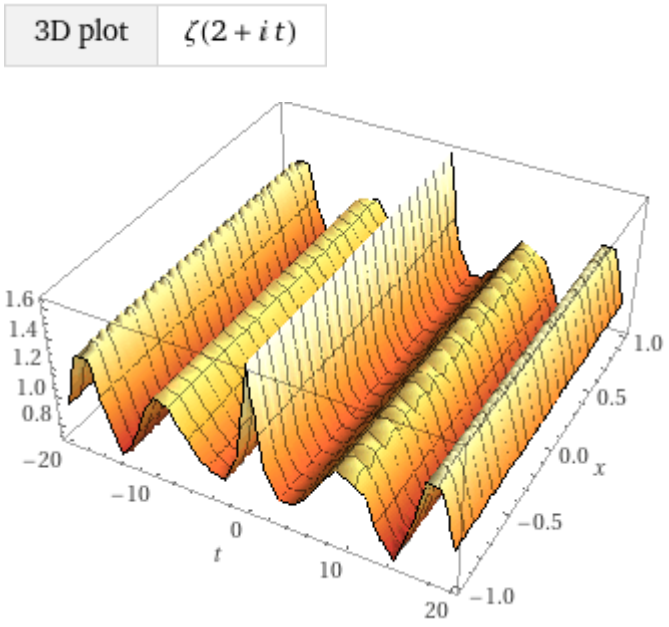


Fig. 6

Where ζ(2+it) :

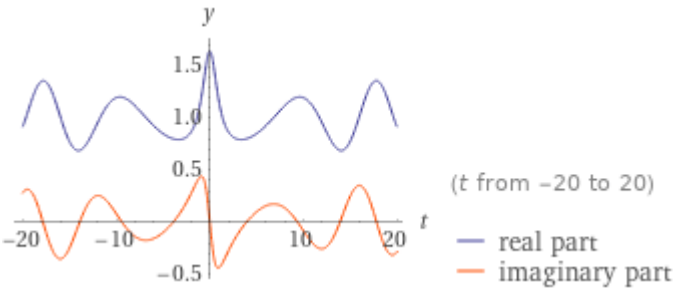
Input

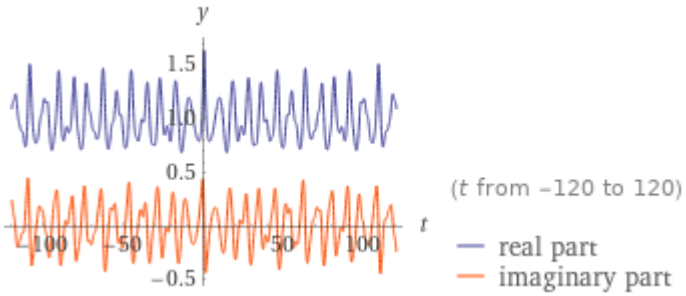
$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Plots





Roots

$t = 2\,i\,(n + 1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

$1.6449340668\ldots. = \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757...$

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}\,M^2\left(1 - \frac{(b\,2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\,\sqrt{6}}\right)^2\right\} = 0$$

$$\text{for } b = -\frac{226\,802\,245}{18\,480\,874}$$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

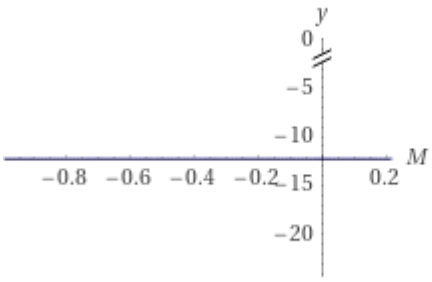
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

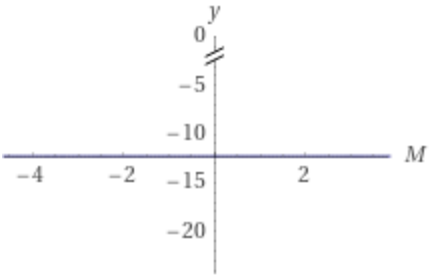
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

-12.2723

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

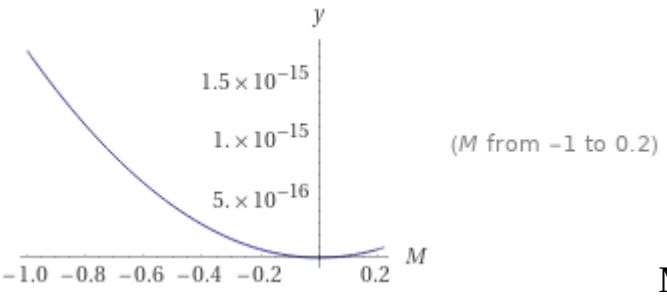
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

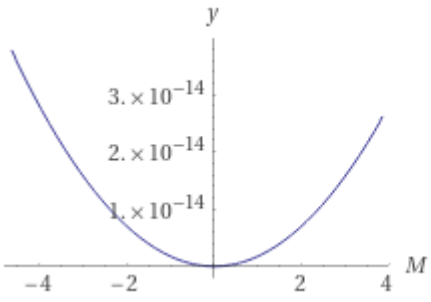
0

Plots: **(possible mathematical connection with an open string)**



M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$1.75541 \times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times10^{-17}$

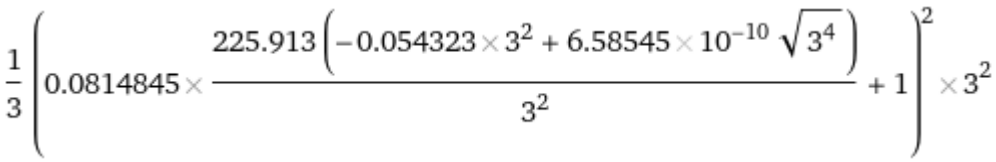
7.021621519159*10^{-17}

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

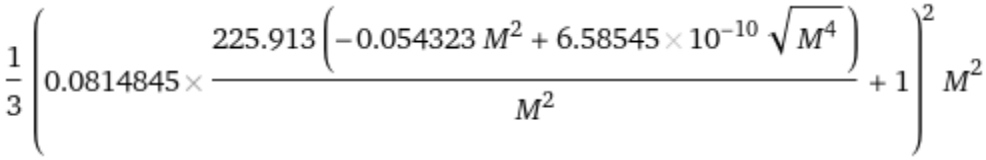


Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

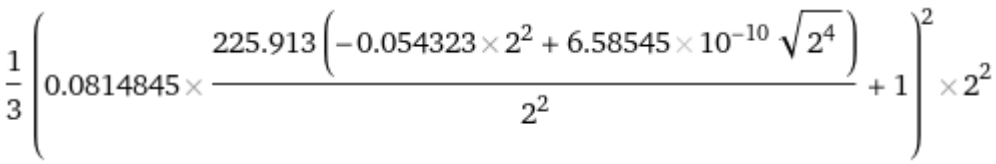
$1.57986484181*10^{-14}$

For M = 2:



1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:



Result:

$7.021621519159432725583532534049408333333333333333333333333333\ldots \times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

5.9776991059*10^{-8} result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\epsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\,\hat{}\,(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

$5.729475 * 10^{-35}$ Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

$1.820306 * 10^{61}$ V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

$5.975498*10^{-8}$ V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

$1.042940*10^{27}$ V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

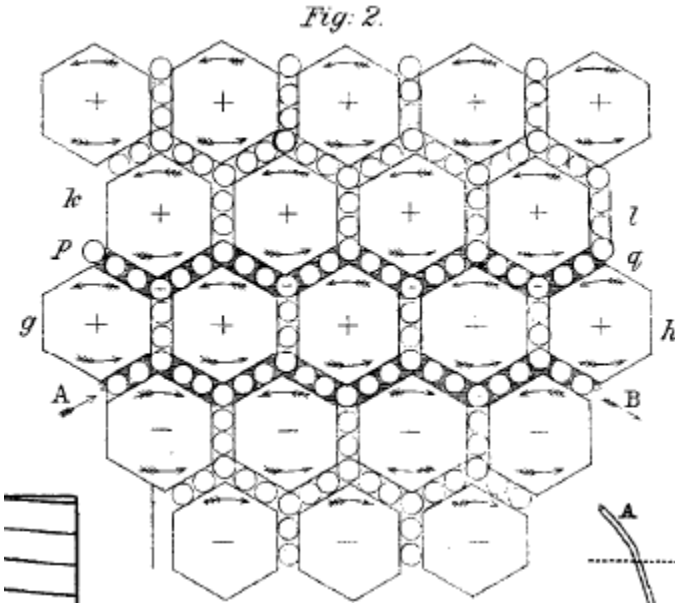
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

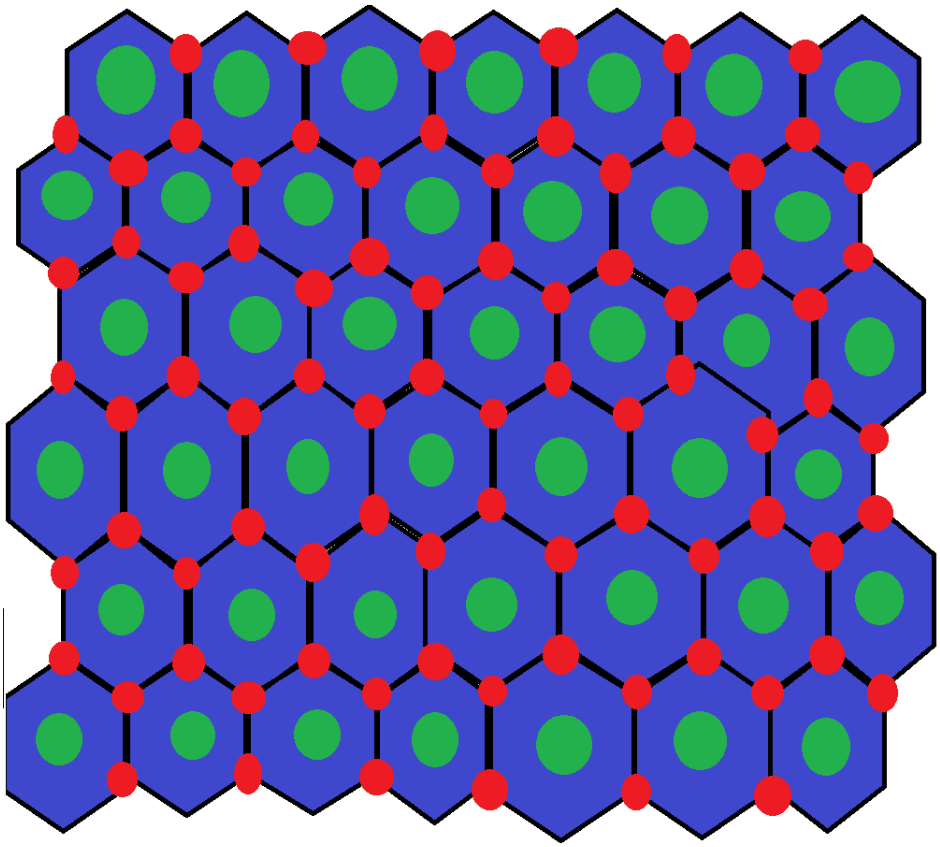
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

On the origin and the present status of inflationary cosmology - *Andrei Linde* - April 19, 2024 18:30, Memorial extended seminar dedicated to Alexei Starobinsky, Moscow, online

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf