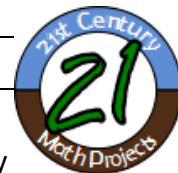




Project Title: Mile Run



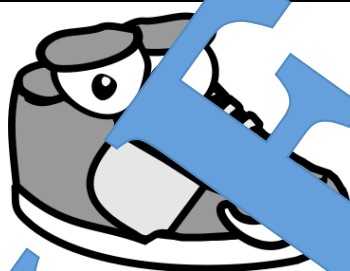
Standard Focus: Data Analysis, Patterns, Algebra & Functions

Time Range: 3-4 Days

Supplies: TI Graphing Technology

Topics of Focus:

- Scatterplots
- Creating and Applying Regression Functions
- Interpolation & Extrapolation of Data



Benchmarks:

Interpreting Functions	F-IF	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
Interpreting Functions	F-IF	6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
Building Functions	F-BF	1. Write a function that describes a relationship between two quantities. ★
Interpreting Categorical and Quantitative Data	S-ID	6a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Fit a linear function or choose a function suggested by the context. Emphasize linear and exponential models.
Interpreting Categorical and Quantitative Data	S-ID	6b. Informally assess the fit of a function to a scatter plot that suggests a linear association.

Procedures:

A.) Students will use Graphing Calculator Technology to make scatterplots using data from the “Mile Run Chart”. (Graphing Calculator Instructions included)

B.) Students will complete the three parts of the Mile Run Project.

The Mile Run



In 1593, the English Parliament declared that 5,280 feet would equal a mile. Ever since, a mile run has become a staple fitness test everywhere -- from militaries to the high school gyms. Although most track events, including all Olympic events, are measured in meters, the mile has stood the test of time and remains sought after running record.

In this project you will investigate world record mile times over history, understanding the difference between interpolation and extrapolation of data. *A TI graphing calculator is required to complete this activity.*

You will need to reference **The Mile Run Chart** which is a chart of records for the Mile Run from 1911 to 1999. As of 2012, the 1999 time of Hicham El Guerrouj is the world record.

Part I. Use the records from 1911 to 1945

1. Using $t=0$ for 1900, enter the year into L1 and the time **in seconds** in L2. Make a scatterplot with the year on the x-axis and the time in seconds on the y-axis.
2. Use your calculator to find the equation of the line of best fit by using the linear regression function.

The line of best fit is:

3. Use the line of best fit to make predictions for the mile time in the years below. Compare the prediction of the best fit line to the actual record in that year.

Year	Prediction from L2 Best fit	Actual Record	(Prediction) - (Actual Record)
1915			
1933			
1937			
1944			

The predictions in the chart are examples of **interpolation**. This is because the years that you were predicting were within the period of time when the data was collected.

4. What is the slope of the line of best fit? _____

5. What does the slope tell you about how the record for the mile run was changing during this period?

6. What is the y-intercept of the line of best fit? _____

7. What is its meaning in terms of the data?

8. Use the equation of the line of best fit to complete the chart below. Use the data in from the Mile Run chart to complete the actual time.

Year	Prediction from Line of Best Fit	Actual Record	(Prediction) – (Actual Record)
1962			
1980			
1985			

The predictions on this chart are examples of **extrapolation**, because the years were outside the period of time when the data was collected.

9. Use the line to predict the year someone will run a four-minute mile.

Part II. Use the records from 1954-1999



1. Using $t=0$ for 1900, in L3, enter the years from 1954-1999, and again enter the time **in seconds** into L4.

2. Use your calculator to find the equation of the line of best fit using the linear regression function.

The line of best fit is: _____

3. What is the slope of the 1954-1999 line of best fit? _____

4. When was the record changing faster, during 1911-1954 or 1954-1999? How do you know?

5. What is the y-intercept and what does it mean in terms of the data?

6. Use the line of best fit to make predictions for mile time in the years below. Compare the prediction of the best fit line to the actual record in that year

Year	Prediction from Line of Best Fit	Actual Record	(Prediction) - (Actual Record)
1957			
1966			
1979			
1985			

7. Use the 1954-1999 equation to predict when a 3:30-mile will be run.

Part III. Use the records from 1911-1999



1. Use augment (L1, L3) STO-> L5 and augment (L2, L4) STO-> L6 to put all of the years together in L5 and all of times in L6. Make a scatterplot, and find the equation of the line of best fit.

The line of best fit is: _____

2. What are the slope and y-intercept? What do they mean in terms of the data?

3. List all three lines of best fit together on this table.

Year Range	Equation of Best Fit
1911-1945	
1954-1999	
1911-1999	

4. Which of the three lines shows the record decreasing the fastest?

5. Which of the three lines show the record decreasing the slowest?

6. You can use the TABLE and TRACE feature on your calculator to complete the chart below.

Year	1911 - 1945 Line of Best Fit Prediction	1954 - 1999 Line of Best Fit Prediction	1911 - 1999 Line of Best Fit Prediction	Actual Record
1913				
1933				
1945				
1957				
1980				

7. Considering the intervals (1911-1945, 1954-1999, or 1911-1999) was the equation that gave the best prediction an example of interpolation or extrapolation? What about the equation that gave the worst prediction?

I = interpolation and E = extrapolation

Year	Best (I or E)	Worst (I or E)
1913		
1933		
1945		
1957		
1980		

Crazy Extrapolations

Use the 1911-1999 line to estimate the time for the mile run record during the following years. (Remember $t=0$ is 1900). You may need to do some quick Googling.

1. The year Columbus first sailed to the Americas. _____
2. The year Julius Caesar was assassinated. _____
3. The year the original *Planet of the Apes* movie was set. _____
4. The year the Civil War ended. _____

EXTENSION How reliable are predictions using interpolation and extrapolation? Why is extrapolation sometimes misleading? Support your views with examples from this exercise.



The Mile Run Chart



Time	Athlete	Country	Year	Location
4:15.4	John Paul Jones	United States	1911	Cambridge, Massachusetts
4:14.4	John Paul Jones	United States	1913	Cambridge, Massachusetts
4:12.6	Norman Taber	United States	1915	Cambridge, Massachusetts
4:10.4	Paavo Nurmi	Finland	1923	Stockholm, Sweden
4:09.2	Jules Ladoumegue	France	1931	Paris, France
4:07.6	Jack Lovelock	New Zealand	1933	Princeton, New Jersey
4:06.8	Glenn Cunningham	United States	1934	Princeton, New Jersey
4:06.4	Sydney Wooderson	England	1937	London, England
4:06.2	Gundar Hägg	Sweden	1942	Goteborg, Sweden
4:06.2	Arne Andersson	Sweden	1942	Stockholm, Sweden
4:04.6	Gundar Hägg	Sweden	1942	Stockholm, Sweden
4:02.6	Arne Andersson	Sweden	1943	Stockholm, Sweden
4:01.6	Arne Andersson	Sweden	1943	Malmö, Sweden
4:01.4	Gundar Hägg	Sweden	1945	Malmö, Sweden
3:59.4	Roger Bannister	England	1954	Oxford, England
3:58.0	John Landy	Australia	1954	Turku, Finland
3:57.2	Derek Ibbotson	England	1957	London, England
3:54.5	Herb Elliott	Australia	1958	Dublin, Ireland
3:54.4	Peter Snell	New Zealand	1962	Wanganui, New Zealand
3:54.1	Peter Snell	New Zealand	1964	Auckland, New Zealand
3:53.6	Michel Jazy	France	1965	Rennes, France
3:51.3	Jim Ryun	United States	1966	Berkeley, California
3:51.1	Jim Ryun	United States	1967	Bakersfield, California
3:51.0	Filbert Bayi	Tanzania	1975	Kingston, Jamaica
3:49.4	John Walker	New Zealand	1975	Goteborg, Sweden
3:49.0	Steve Ovett	England	1979	Oslo, Norway
3:48.8	Steve Ovett	England	1980	Oslo, Norway
3:48.53	Sebastian Coe	England	1981	Zurich, Switzerland
3:48.40	Steve Ovett	England	1981	Koblenz, West Germany
3:47.33	Sebastian Coe	England	1981	Brussels, Belgium
3:46.31	Steve Cram	England	1985	Oslo, Norway
3:44.39	Noureddine Morceli	Algeria	1993	Rieti, Italy
3:43.13	Hicham El Guerrouj	Morocco	1999	Rome, Italy

Linear Regression

on a TI-83 or TI-84

Adapted from: (<http://school.lardbucket.org/math/calculator/regression>)

Entering Data

Suppose you want to calculate a regression with the data in the table below:

X	Y
1	2
3	8
4	10
6	14

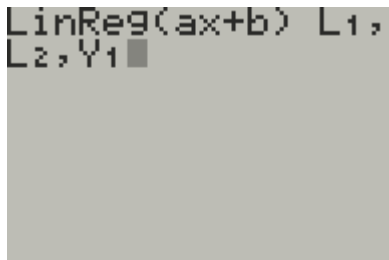
1. To enter your data into your calculator, get to the list editor. To do that, press the **[STAT]** button, then pick the **EDIT** option. (**Press [ENTER].**)
2. You should then see a table. Across the top should be L1, L2, and L3. If not, see below. If there is any data under L1 or L2, move the selection so you have the list highlighted, press the **[CLEAR]** button, and then the right arrow to clear the list.
3. When both lists are cleared, select the empty data in L1 by moving your selection over the "____". Start entering your X-values under L1. Make sure you enter the data in order.
4. When you are done entering the X-values, move to the L2 column and enter your Y-values. Make sure they are aligned. The data should line up across the screen like it does in the table above, giving you something like this picture:

L1	L2	L3
1	2	---
3	8	---
4	10	---
6	14	---
---	---	---
L2(5)		

If your lists don't show L1, L2, and L3 across the top, reset your stats list editor. To do this, press **[STATS]**, select option 5, "SetUpEditor", and press **[ENTER]** twice.

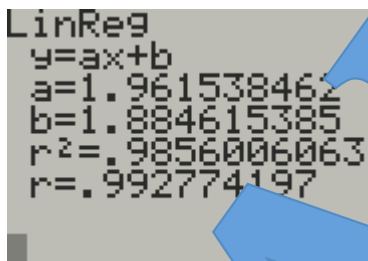
Performing a Linear Regression

1. Press the **[STAT]** button. Use the right arrow to select "**CALC**". Scroll down to the type of regression you would like to use. You can use a linear regression (**LinReg(ax+b)**). Press **[ENTER]**.
2. Type "L1" by pressing **[2ND]** and the number **1**. Then press the comma button **,**. Next, hit **[2ND]** and then the number **2** to enter "L2". Then press the comma button **,**. Your screen should now show something like "**LinReg(a+bx) L1,L2,**".
3. Press **[VARS]**, press the right arrow to select "**Y VARS**", and press **[ENTER]** to select "**Function...**". Choose the function you would like to store your equation in and press **[ENTER]**. (You will likely want to use "Y1".) Your screen should look something like the picture below.



```
LinReg(ax+b) L1,  
L2,Y1
```

4. You can now press **[ENTER]**, and the calculator will calculate your regression. Where a = the slope of the line; b = the intercept of the line; r^2 and r are used for correlation, but will not be used in this assignment. The screen will now show the results of the regression.

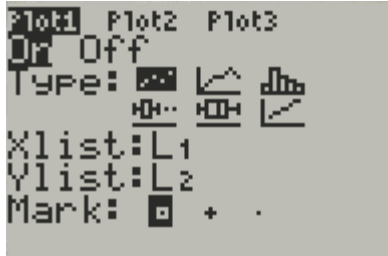


```
LinReg  
y=ax+b  
a=1.961538462  
b=1.884615385  
r^2=.9856006063  
r=.992774197
```

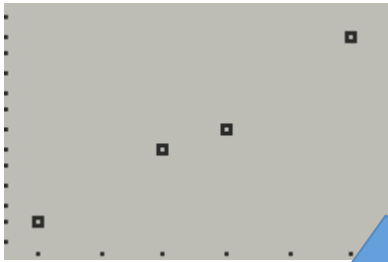
5. The equation for the regression line has now been stored in Y1 for graphing and estimating.

Plotting the Data

1. **To set up plots:** Press **[2ND]** and then **[Y=]** to get to the "STAT PLOTS" screen. Select the first plot and press **[ENTER]**. You should make the selections as seen in the picture below. (**Note: for Parts II and III you will be using L3, L4, L5, L6 and will need to make the appropriate changes.**)



2. To set up the graph display of your data, press **[2ND]** and scroll down to "ZoomStat" and hit **[ENTER]**. Your data should now be displayed. If you used the data above, the plot should look like this:



Thank you for being my Math Friend!

If you liked this

21st Century Math Project

You might like others. [link the logo]



Math it Up.
Boondigggy.

The Mile Run



In 1593, the first recorded mile run was by a man named William Miles. He ran the mile in 15.28 minutes. The mile run has been a part of the first test everywhere from military training to the modern day. The mile run is a test of time and endurance. It is a test of the runner's ability to maintain a steady pace over a long distance. The mile run is a test of the runner's ability to maintain a steady pace over a long distance. The mile run is a test of the runner's ability to maintain a steady pace over a long distance.

In this project, you will investigate world record mile times over history while comparing the difference between interpolation and extrapolation. A TI graphing calculator is required to complete this activity.

You will need to reference **The Mile Run Chart** which is a chart of records for the Mile Run from 1911 to 1999. As of 2012, the 1999 time of Hicham El Guerrouj remains the world record.

Part I. Use the records from 1911 to 1999

1. Using $t=0$ for 1900, enter the year into a list and the time in seconds in L2. Make a scatterplot with the year on the x-axis and the time in seconds on the y-axis.

2. Use your calculator to find the equation of the line of best fit by using the linear regression function.

The line of best fit is: $y(x) = -.431859x + 262.2$

3. Use the line of best fit to predict the mile time in the years below. Compare the prediction of the best fit line to the actual record in that year.

Year	Prediction from Line of Best Fit	Actual Record	(Prediction from Line of Best Fit) - (Actual Record)
1915	255.72	252.6	3.129
1933	247.955	247	.955
1937	246.228	246.4	-.172
1944	243.205	241.6	1.605

The predictions in the chart are examples of **interpolation**. This is because the years that you were predicting were within the period of time when the data was collected.

4. What is the slope of the line of best fit? $-.431859$

5. What does the slope tell you about how the record for the mile run was changing during this period?

It was decreasing at a rate of $-.431859$ seconds per year.

6. What is the y-intercept of the line of best fit? **262.207**

7. What is its meaning in terms of the data?

It is the predicted number of seconds for the record in 1900.

8. Use the equation of the line of best fit to complete the chart below. Use the data in from the Mile Run chart to complete the actual time.

Year	Prediction from Line of Best Fit	Actual Record	(Prediction from Line of Best Fit) - (Actual Record)
1962	235.432	236.464	1.032
1980	227.658	228.800	-1.142
1985	225.499	226.510	-1.011

The predictions on this chart are examples of **extrapolation**, because the years were outside the period of time where data was collected.

9. Use the line to predict when the four-minute mile would have been run.

1951

Part II. Use the records from 1954-1999.

1. Using $t=0$ for 1900, in L3, enter the years from 1954-1999, and again enter the time **in seconds** into L4. Make a scatterplot.

2. Use your calculator to find the equation of the line of best fit using the linear regression function.

The line of best fit is: $y = -.336x + 255.393$

3. What is the slope of the 1954-1999 line of best fit? $-.336$

4. When was the record changing faster, during 1911-1945 or 1954-1999? How do you know?

1911-1945. In 1911-1945, it was changing at .432 per year and in 1954-1999, it was changing at .336 seconds per year.

5. What is the y-intercept and what does it mean in terms of the data?

255.393. It is the time predicted for 1900 by the equation.

6. Use the line of best fit to make predictions for the mile time in the years below. Compare the prediction of the best fit line to the actual record in that year

Year	Prediction from Line of Best Fit	Actual Record	(Prediction from Line of Best Fit) – (Actual Record)
1957	236.241	237.2	-0.959
1966	233.8	231.3	1.917
1979	228.84	229.0	-0.151
1985	226.833	224.31	2.523

7. Use the equation to predict when a 3:30-mile will be run.

2035

Part III. Use the records from 1911-1999.

1. Use augment (L1, L3) STO L5 and augment (L2, L4) STO L6 to put all of the years together in L5 and all of times in L6. Make a scatterplot, and find the equation of the line of best fit.

The line of best fit is: $y = -.413x + 261.249$

2. What are the slope and y-intercept? What do they mean in terms of the data?
Slope is $-.413$, the number of seconds per year that the time decreased. y-intercept is 261.249 , the predicted time for the mile run in 1900.

3. List all three lines of best fit together on this table:

Year Range	Line of Best Fit
1911-1945	$y = -.35x + 261.207$
1954-1999	$y = -.35x + 261.393$
1911-1999	$y = -.413x + 261.249$

4. Which of the three lines shows the record decreasing the fastest?

Fastest 1911-1945

5. Which of the three lines shows the record decreasing the slowest?

Slowest 1945-1999

6. You can use the $\text{VARS} \rightarrow \text{EQ} \rightarrow \text{1-ASK}$ feature on your calculator to complete the chart below.

Year	1911 – 1945 Line of Best Fit Prediction	1954 – 1999 Line of Best Fit Prediction	1911 – 1999 Line of Best Fit Prediction	Actual Record
1913	256.58	257.022	255.875	254.4
1933	247.607	244.299	247.607	247.6
1945	242.773	240.265	242.646	241.4
1957	237.591	236.231	237.685	237.2
1980	227.658	228.499	228.177	228.8

7. Considering the intervals (1911-1945, 1954-1999, or 1911-1999) was the equation that gave the best prediction an example of interpolation or extrapolation? What about the equation that gave the worst prediction?

I = interpolation and E = extrapolation

Year	Best (I or E)	Worst (I or E)
1913	1911-1999 (I)	1954-1999 (E)
1933	1911-1999 (I)	1954-1999 (E)
1945	1954-1999 (E)	1911-1945 (I)
1957	1911-1945 (E)	1954-1999 (I)
1980	1954-1999 (I)	1911-1945 (E)

Crazy Extrapolation

Use the 1911-1999 line to estimate the time of the mile run record during the following years. (Remember $t=0$ is 1900). You may need to do some quick Googling.

1. The year Columbus first sailed to America. (1492)
The predicted time is 1064.88 seconds (7 min, 9 seconds)
2. The year Julius Caesar was assassinated. (44 BC)
The predicted time is 1064.88 seconds (7 min, 9 seconds)
3. The year the original *Planet of the Apes* was set. (3978)
The predicted time is -596.965 seconds (10 min, 1.965 seconds)
4. The year the Civil War ended. (1865)
The predicted time is 1064.88 seconds (7 min, 9 seconds)

EXTENSION: Describe the reliability of predictions using interpolation and extrapolation? Why is extrapolation sometimes misleading? Support your views with examples from this exercise.

In general, interpolation is more reliable than extrapolation. Interpolation is most accurate towards the middle of the data set and may be less accurate at the edges of the data set (see 1945). Also, interpolation is more accurate when the data is fairly close to the data set. It could not have taken over 17 hours to run the 1945 mile run. For do we expect time travel in the year 3978AD.