

Reservoir Geomechanics

In situ stress and rock mechanics applied to reservoir processes

Mark D. Zoback
Professor of Geophysics

Week 3 – Lecture 5
Rock Strength – Chapter 4 Part I



STANFORD UNIVERSITY



Overview

4	Rock failure in compression, tension and shear	84
	Rock strength in compression	87
	Compressive strength criteria	92
	Strength and pore pressure	104
	Rock strength anisotropy	105
	Estimating rock strength from geophysical log data	107
	Shear-enhanced compaction	118
	Tensile rock failure	121
	Shear failure and the frictional strength of rocks	123
	The critically stressed crust	127
	Limits on <i>in situ</i> stress from the frictional strength of faults	130
	Stress polygon	137

Propagation of Hydraulic Fractures
The Vertical Growth of Hydraulic Fractures

**Next
Lecture**



Outline

Section 1

- Compressive Strength
- Strength Criterion

Section 2

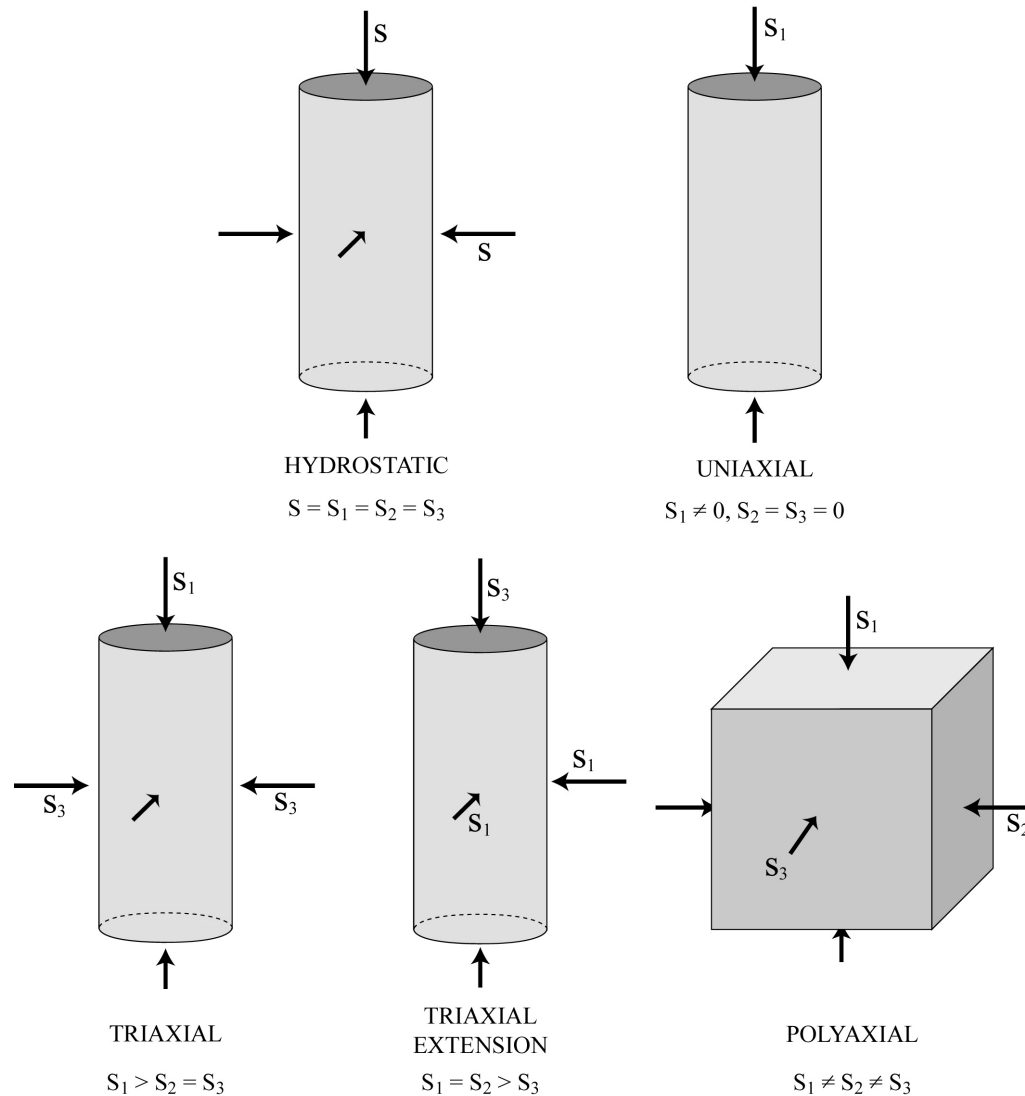
- Strength Anisotropy
- Shear Enhanced Compaction
- Strength from Logs

Section 3

- Tensile Strength
- Hydraulic Fracture Propagation
- Vertical Growth of Hydraulic Fractures



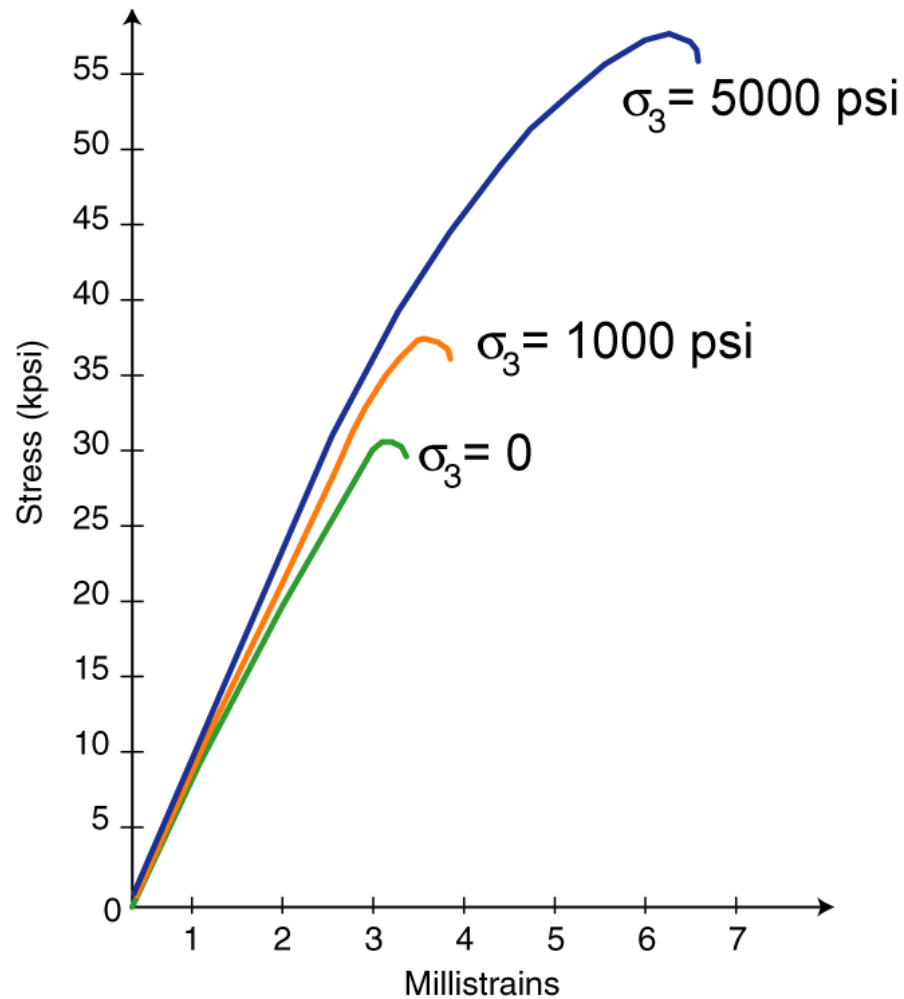
Types of Rock Mechanics Tests





Stress-Strain Curves for Rand Quartzite

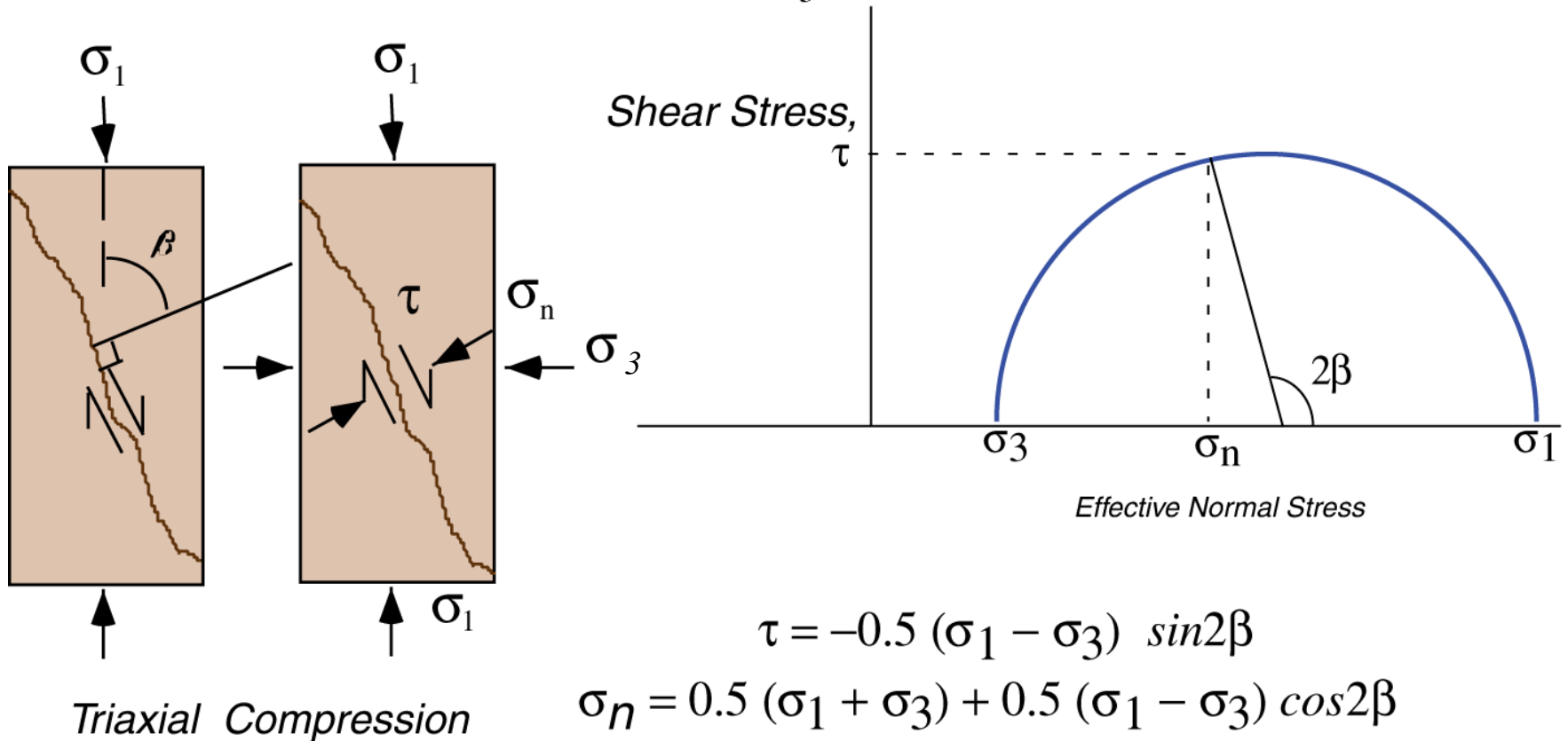
Strength Depends on Confining Pressure





Mohr Circles in Two Dimensions

Describes the stress on a plane at failure. Assumes σ_1 , only depends on the magnitude of σ_3 . Applicable to "triaxial" rock mechanics tests ($\sigma_2 = \sigma_3$).



Equations 4.1 & 4.2 – pg.89

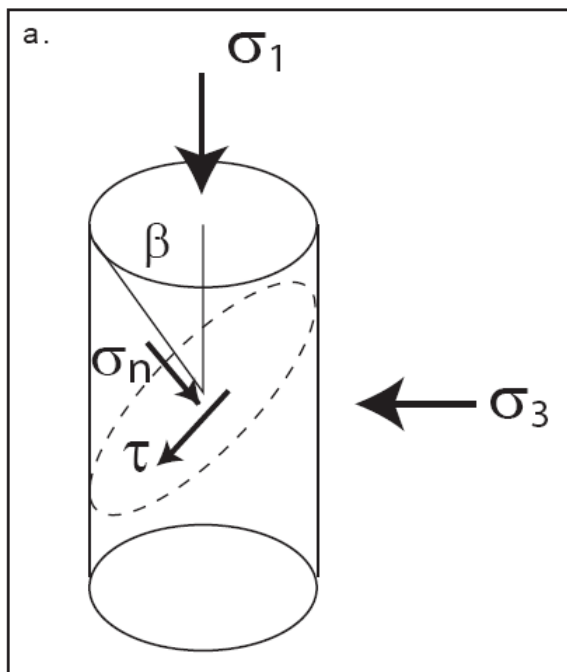


Mohr Failure in Two Dimensions

Assumes σ_1 , only depends on the magnitude of σ_3

Applicable to "triaxial" rock mechanics tests

($\sigma_2 = \sigma_3$)



$$\tau = S_0 + \mu_i \sigma_n$$

$$C_0 = 2 S_0 [(\mu_i^2 + 1)^{1/2} + \mu_i]$$

Equations 4.3 & 4.4 – pg.89

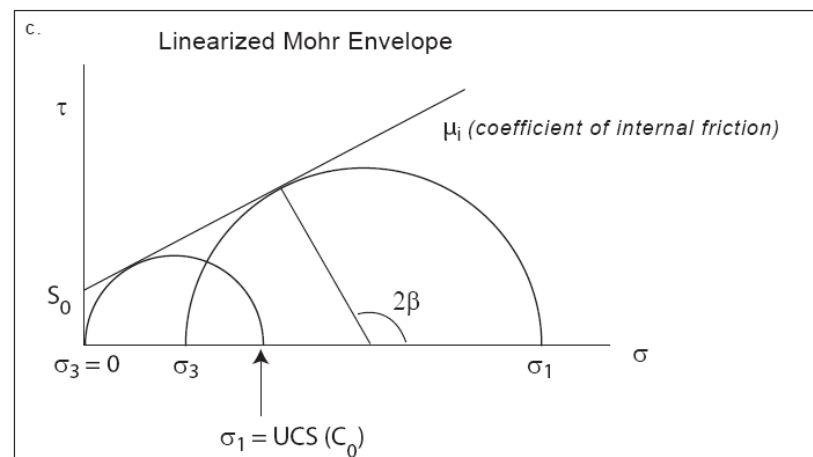
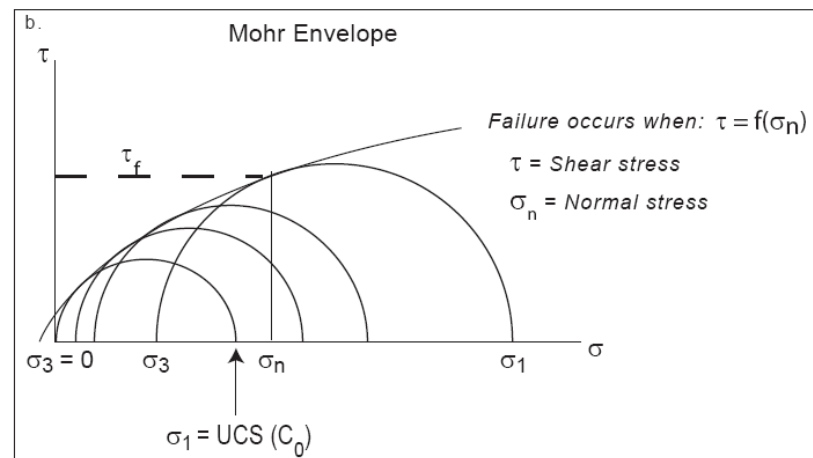


Figure 4.2 a,b,c – pg.88



Practical Guide to Determination of C_0 and μ_i

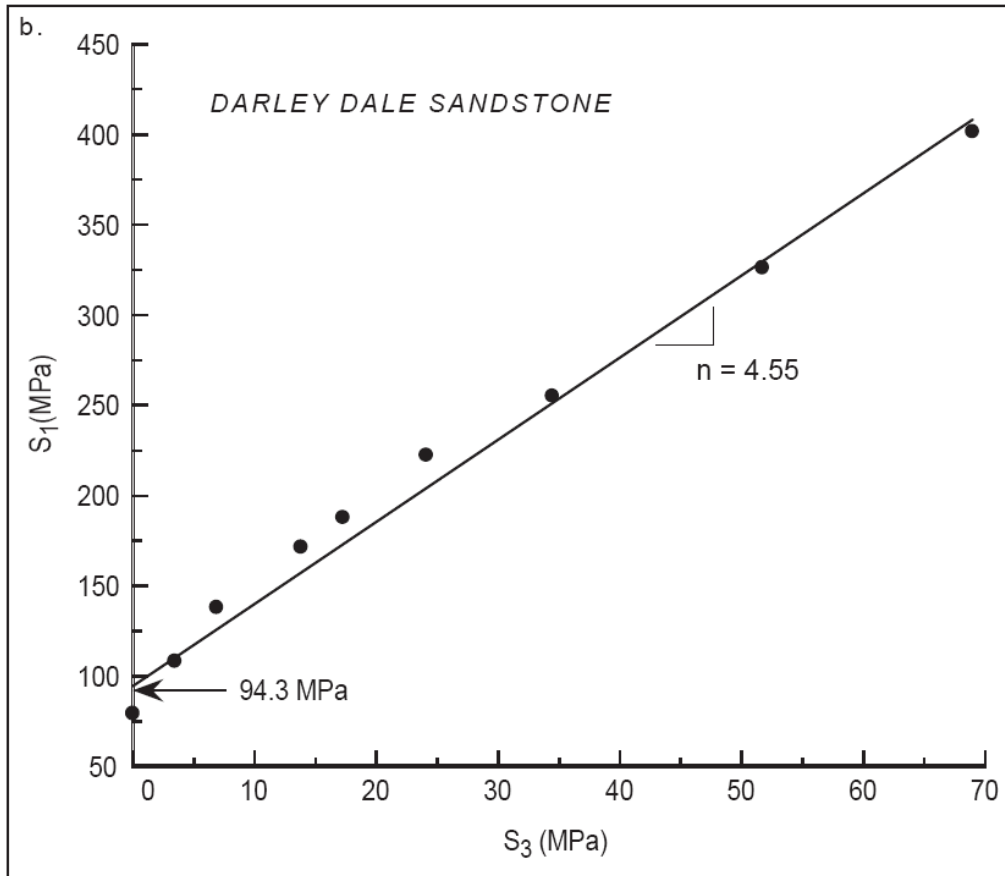


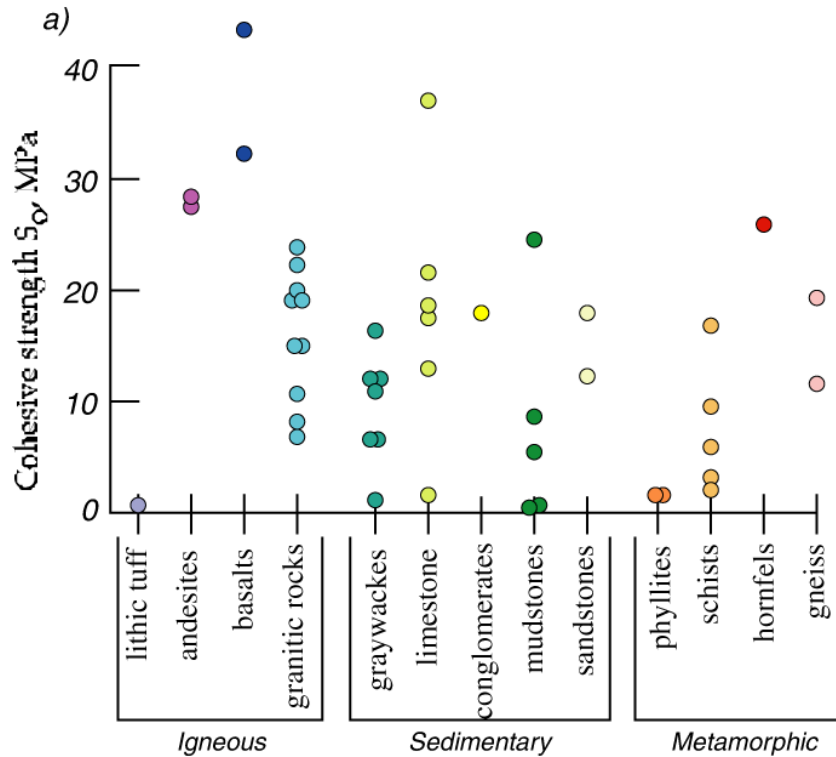
Figure 4.3 b – pg.90

$$\mu_i = \frac{n-1}{2\sqrt{n}}$$

Equation 4.5 – pg.89

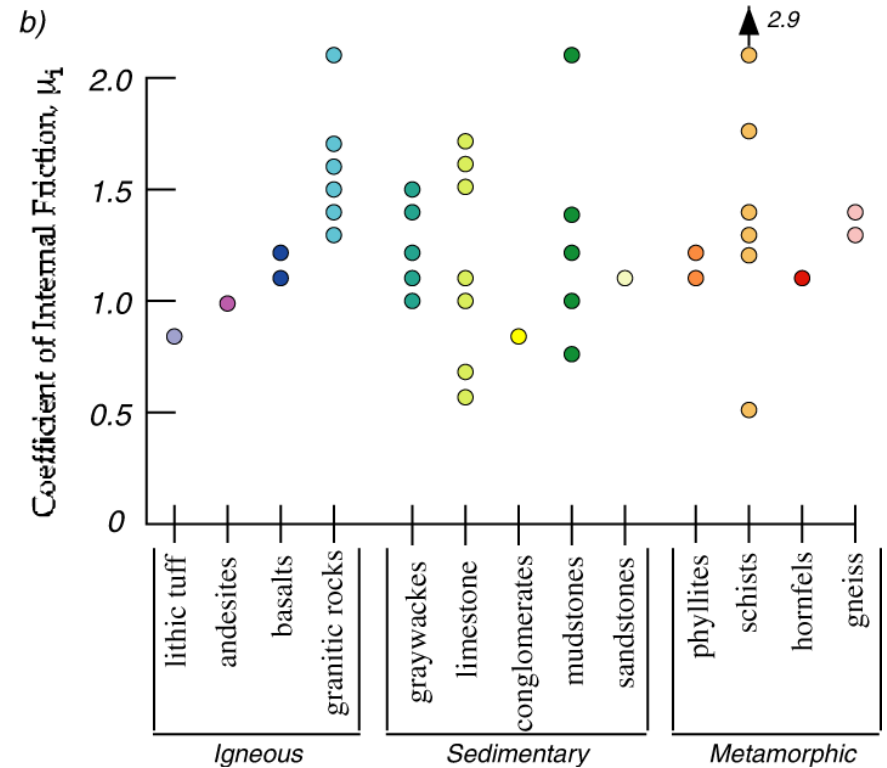


Strong Rocks/Weak Rocks



Weak rocks have low cohesion

Weak rocks have high internal friction





More Complex Failure Criterion that Describe Rock Strength in Compression

Over the years, comprehensive laboratory studies have yielded a variety of failure criterion to describe rock strength in compression which are summarized below. However, to quote Mark Twain,

The efforts of many researchers have already cast much darkness on the subject, and it is likely that, if they continue, we will soon know nothing about it at all.

This statement, reflective of Twain's inherent cynicism, is unfortunately applicable of the degree to which concepts about rock failure based on laboratory rock mechanics has made the subject of rock strength sufficiently complex that it can almost never be practically applied in case studies. Thus, the most important thing to keep in mind is that

Strong rock is strong, weak rock isn't

Our first goal is to capture the essential rock strength. Using advanced failure criterion to describe rock strength is a worthy, but secondary, objective.



Strength Criteria in Which the Stress at Failure, σ_1 , Depends Only on σ_3

Linearized Mohr-Coulomb criterion (Jaeger and Cook, 1979)

$$\sigma_1 = q\sigma_3 + C_0 \quad q = (\sqrt{\mu^2 + 1} + \mu)^2 \quad \tan \Phi = \mu$$

Equations 4.6 - 4.8 – pg.93

Empirical criterion of Hoek and Brown (1980)

$$\sigma_1 = \sigma_3 + C_0 \sqrt{m \frac{\sigma_3}{C_0} + s} \quad \text{Equation 4.9 – pg.98}$$

where m and s are constants that depend on the properties of the rock and on the extent to which it was broken before being subjected to the failure.



Among Failure Criterion that are Functions of Three Principal Stresses

Modified Lade criterion (Ewy, 1998) – A personal favorite

$$\left(\frac{(I_1')^3}{I_3'} - 27 \right) \left(\frac{(I_1')^m}{\rho_a} \right) = \eta$$

$$I_1' = (\sigma_1 + \bar{S}) + (\sigma_2 + \bar{S}) + (\sigma_3 + \bar{S})$$

$$I_3' = (\sigma_1 + \bar{S})(\sigma_2 + \bar{S})(\sigma_3 + \bar{S})$$

$$\bar{S} = S_o / \tan \Phi$$

$$\eta = 4\mu^2 \frac{9\sqrt{\mu^2 + 1} - 7\mu}{\sqrt{\mu^2 + 1} - \mu}$$

Equations 4.13 - 4.17 – pg.100



Failure Envelopes in Stress Space

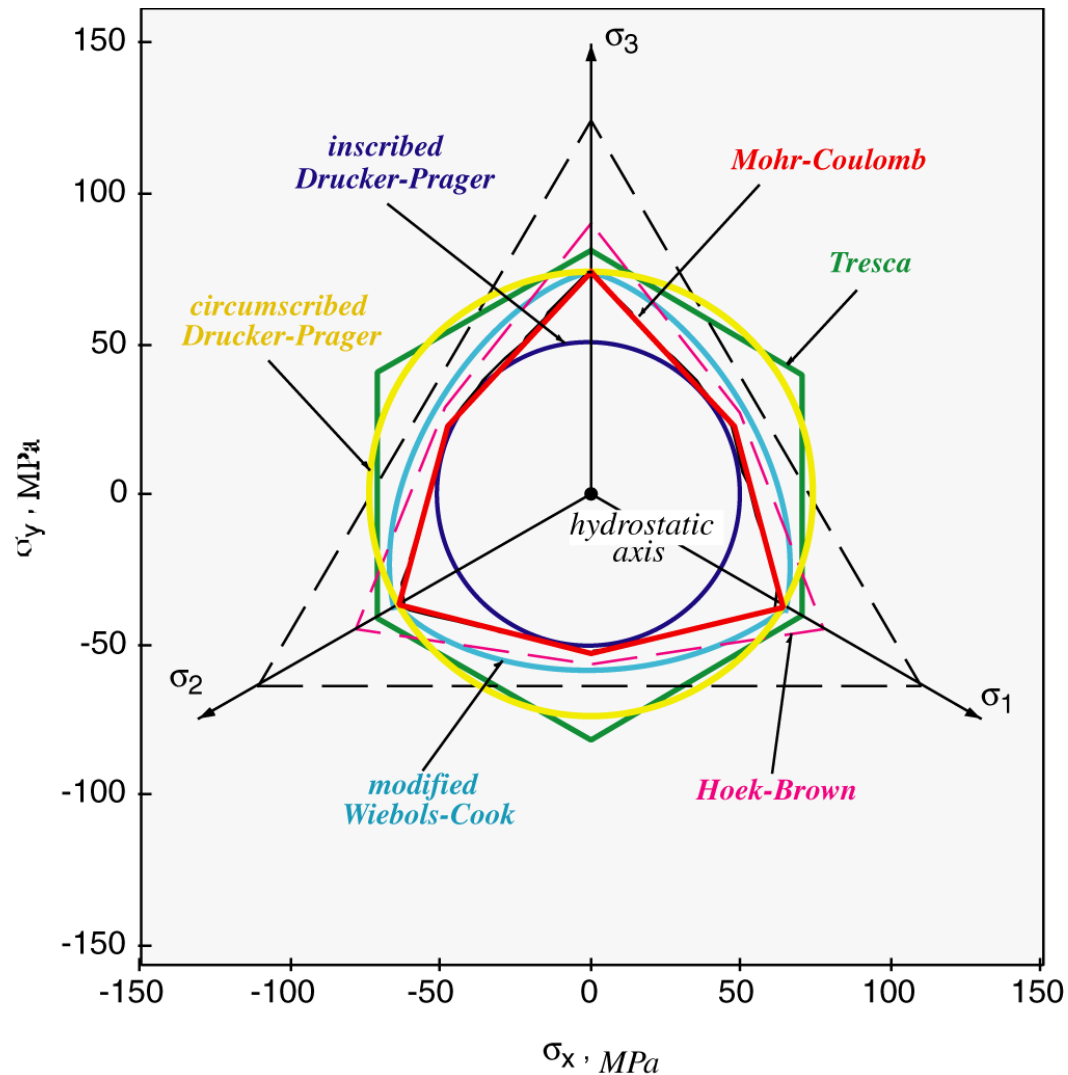


Figure 4.6 – pg.94



Rock Strength is a Function of Simple Effective Stress

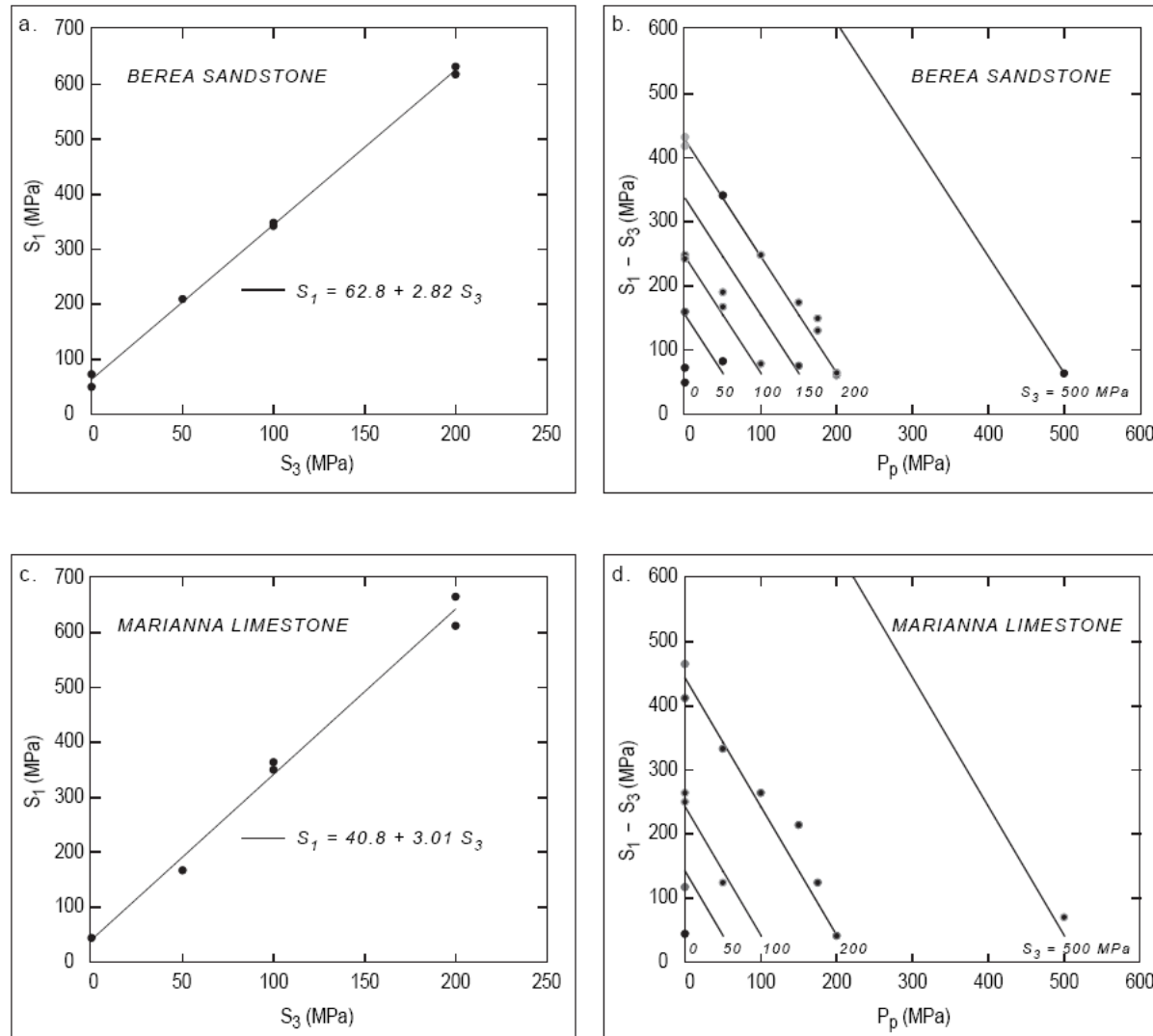


Figure 4.11 a-d – pg.105



Outline

Section 1

- Compressive Strength
- Strength Criterion

Section 2

- Strength Anisotropy
- Shear Enhanced Compaction
- Strength from Logs

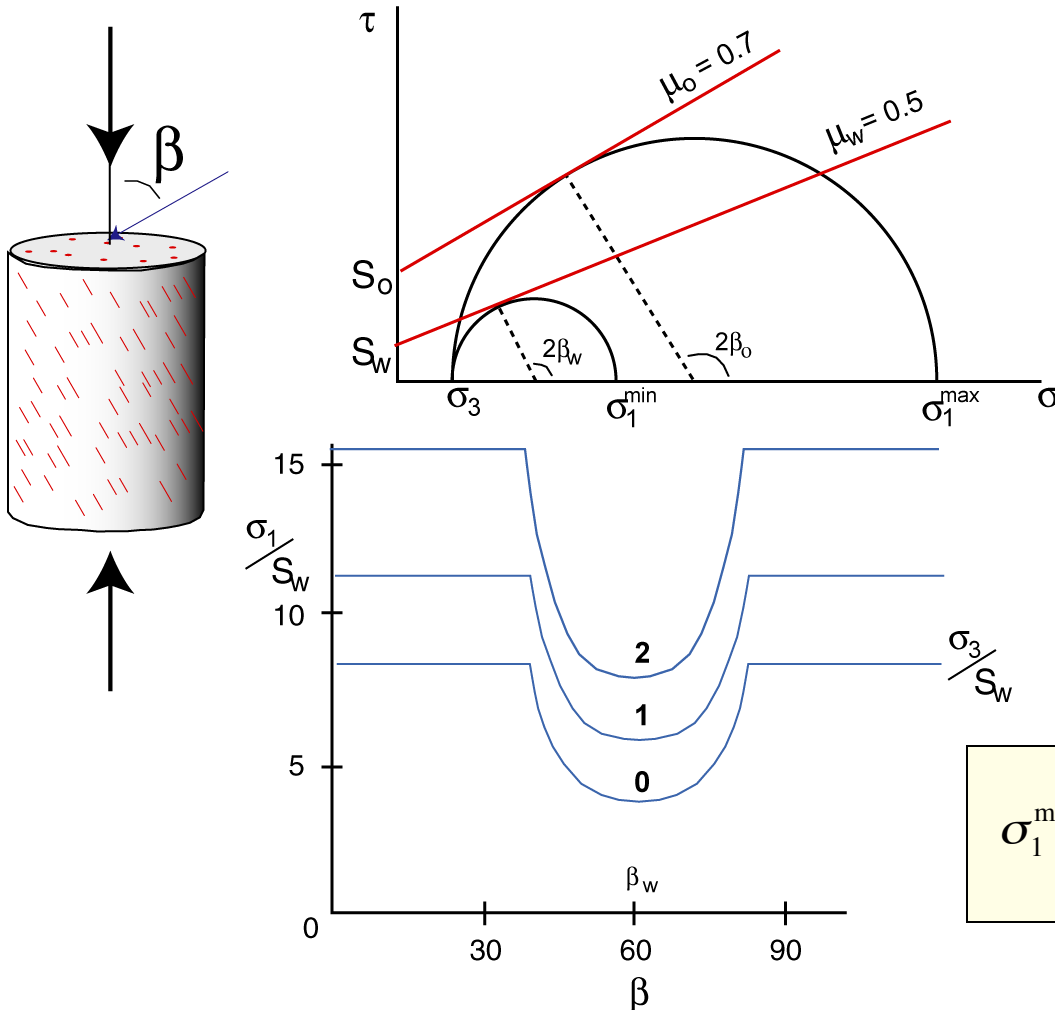
Section 3

- Tensile Strength
- Hydraulic Fracture Propagation
- Vertical Growth of Hydraulic Fractures



Strength Anisotropy

Parallel Planes of Weakness (Bedding/Foliation)



$$\sigma_1 = \sigma_3 = \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cot \beta_w) \sin 2\beta}$$

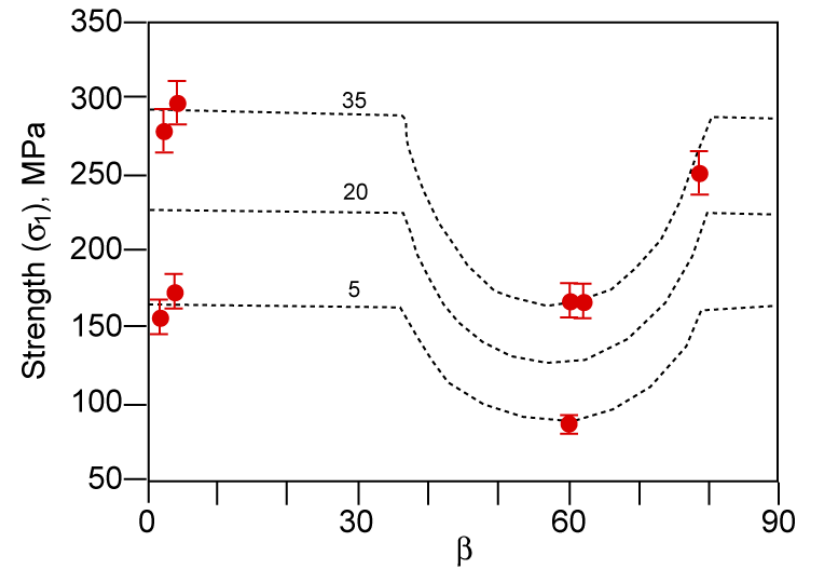
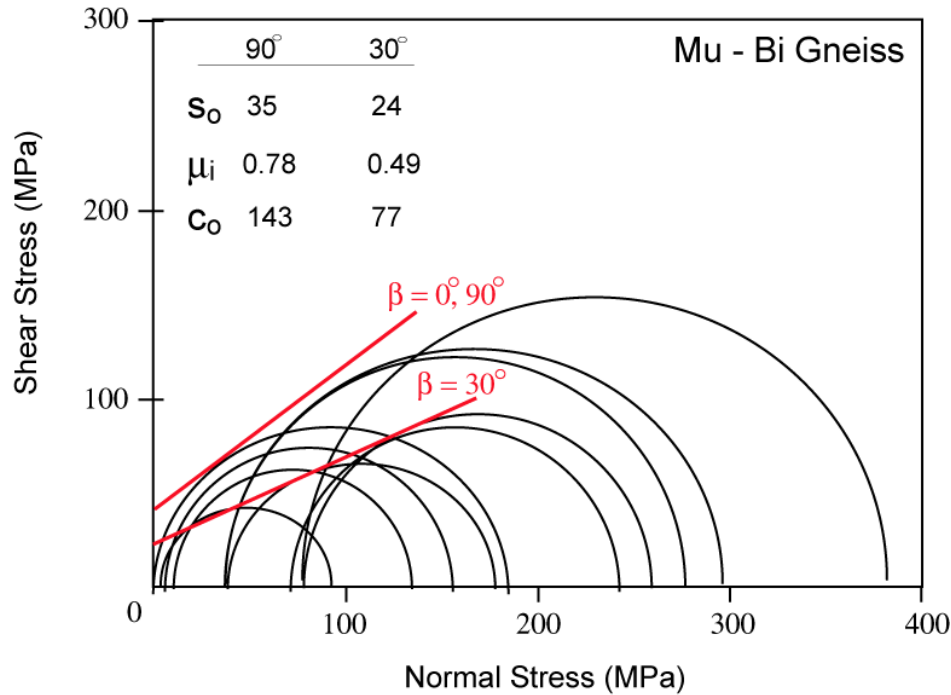
$$\text{if } \tan 2\beta_w = -\frac{1}{\mu_w}$$

$$\sigma_1^{\min} = \sigma_3 + 2(S_w + \mu_w \sigma_3) \left[(\mu_w^2 + 1)^{\frac{1}{2}} + \mu_w \right]$$

Equations 4.33 - 4.34 – pg.107

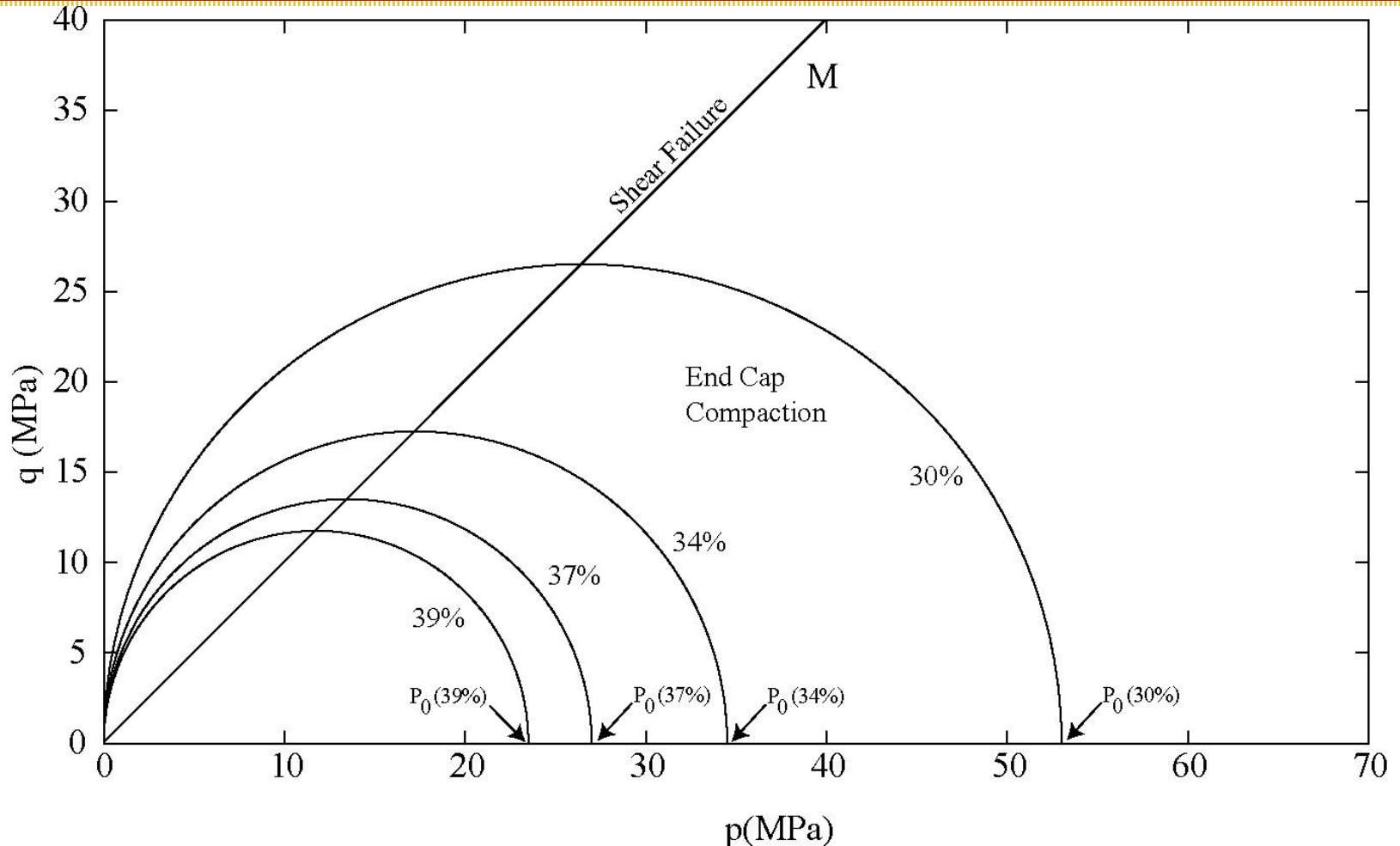


Highly Foliated Gneiss





Shear Enhanced Compaction (End Cap)



Cam-Clay Model: Elliptical End Caps

Figure 4.19 – pg.119



Shear Enhanced Compaction (End Cap)

$$p = \frac{1}{3} J_1 = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)$$

Equation 4.35 – pg.118

$$p = \frac{1}{3} (S_1 + S_2 + S_3) - P_P$$

$$\underline{q} = \sqrt{3J_{2D}}$$

Equation 4.36 – pg.118

$$\underline{q}^2 = \frac{1}{2} [(S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_1 - S_3)^2]$$

$$M^2 p^2 - M^2 p_0 p + \underline{q}^2 = 0$$

Equation 4.37 – pg.119



Shear Enhanced Compaction (End Cap)

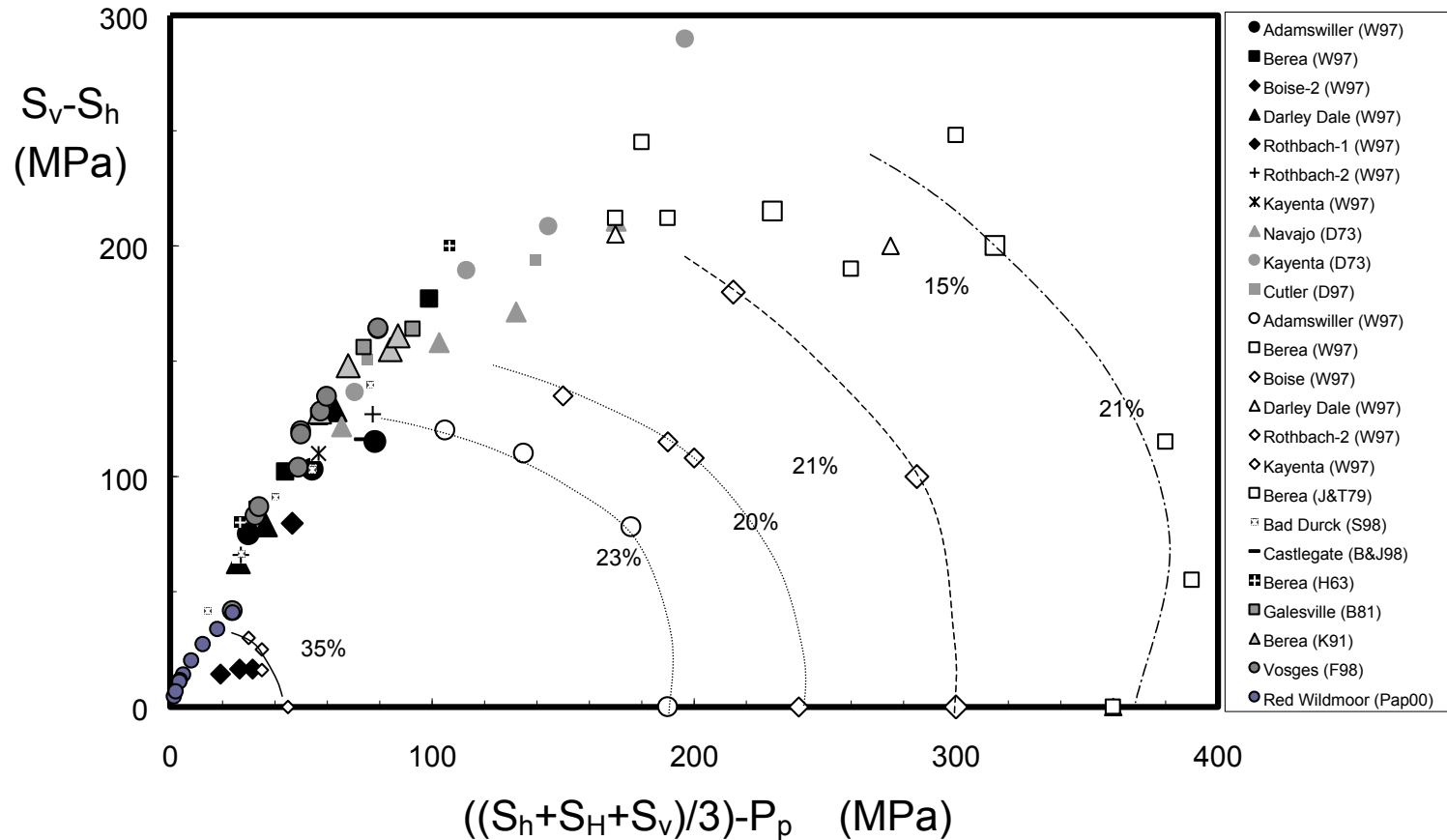


Figure 4.20 – pg.120



End Caps and Lab Tests

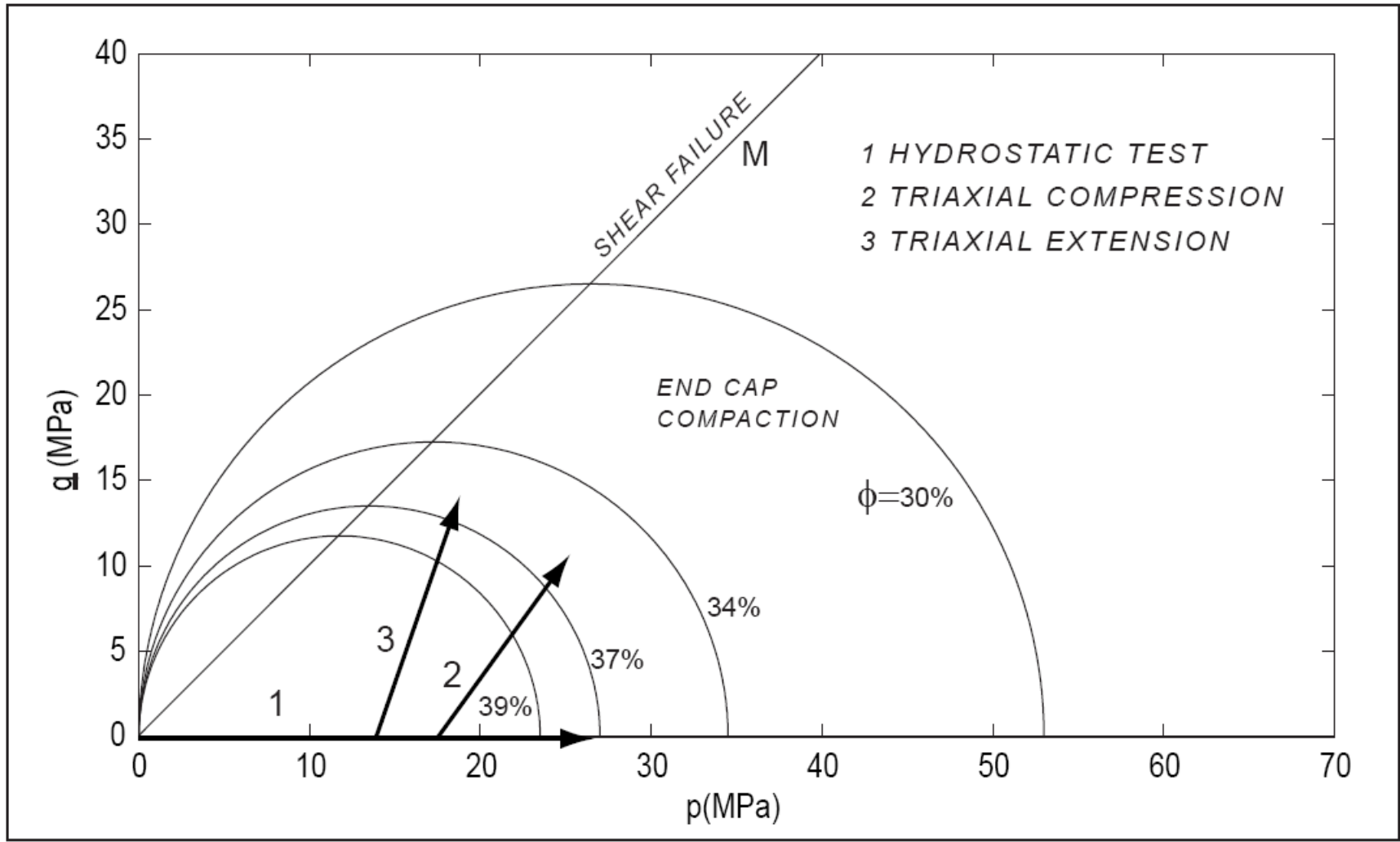


Figure 4.19 – pg.119



Deformation Analysis in Reservoir Space (DARS)

To understand the deformation mechanisms of a producing reservoir utilizing relatively simple laboratory tests and in situ measurements

DARS is a formalism for estimating the evolution of porosity, permeability and the potential for induced normal faulting in a producing reservoir



Cam-Clay Model: Elliptical End Caps Fit to Hydrostatic Compression Data

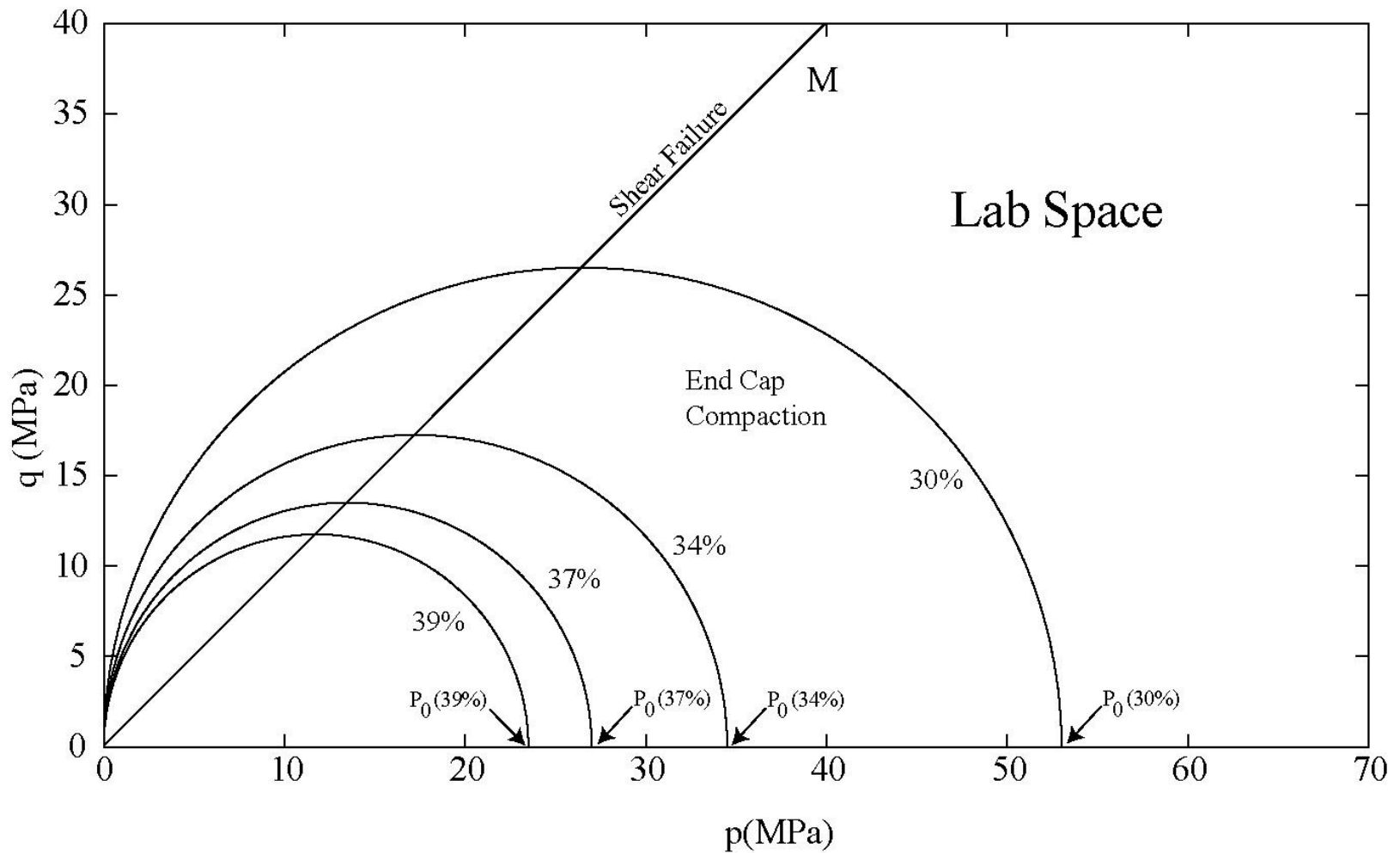
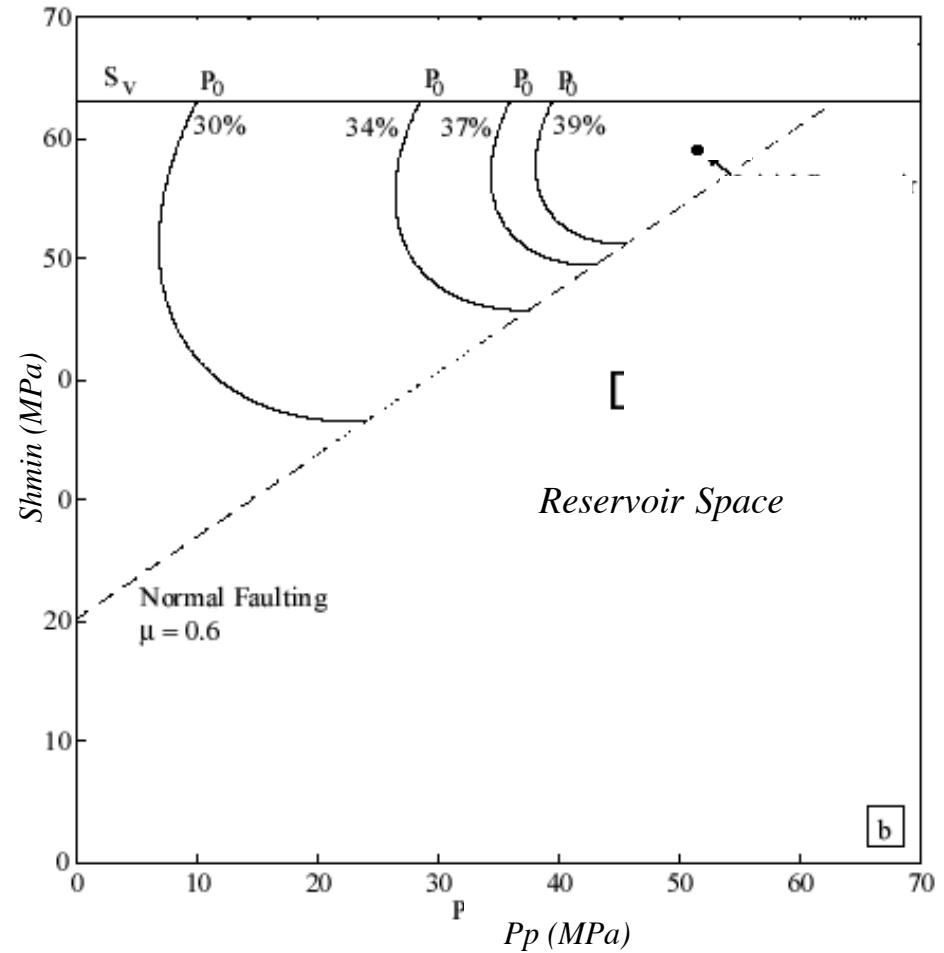
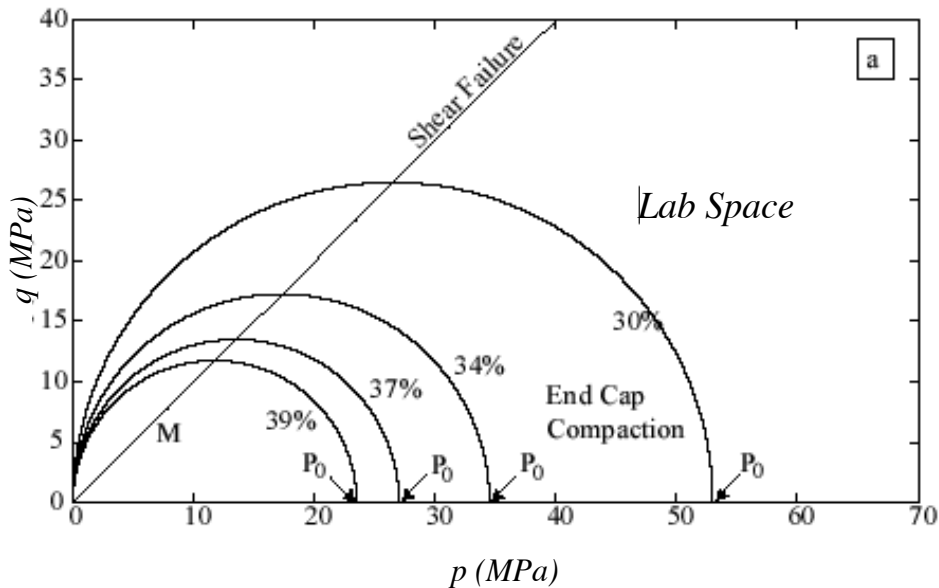


Figure 4.19 – pg.119



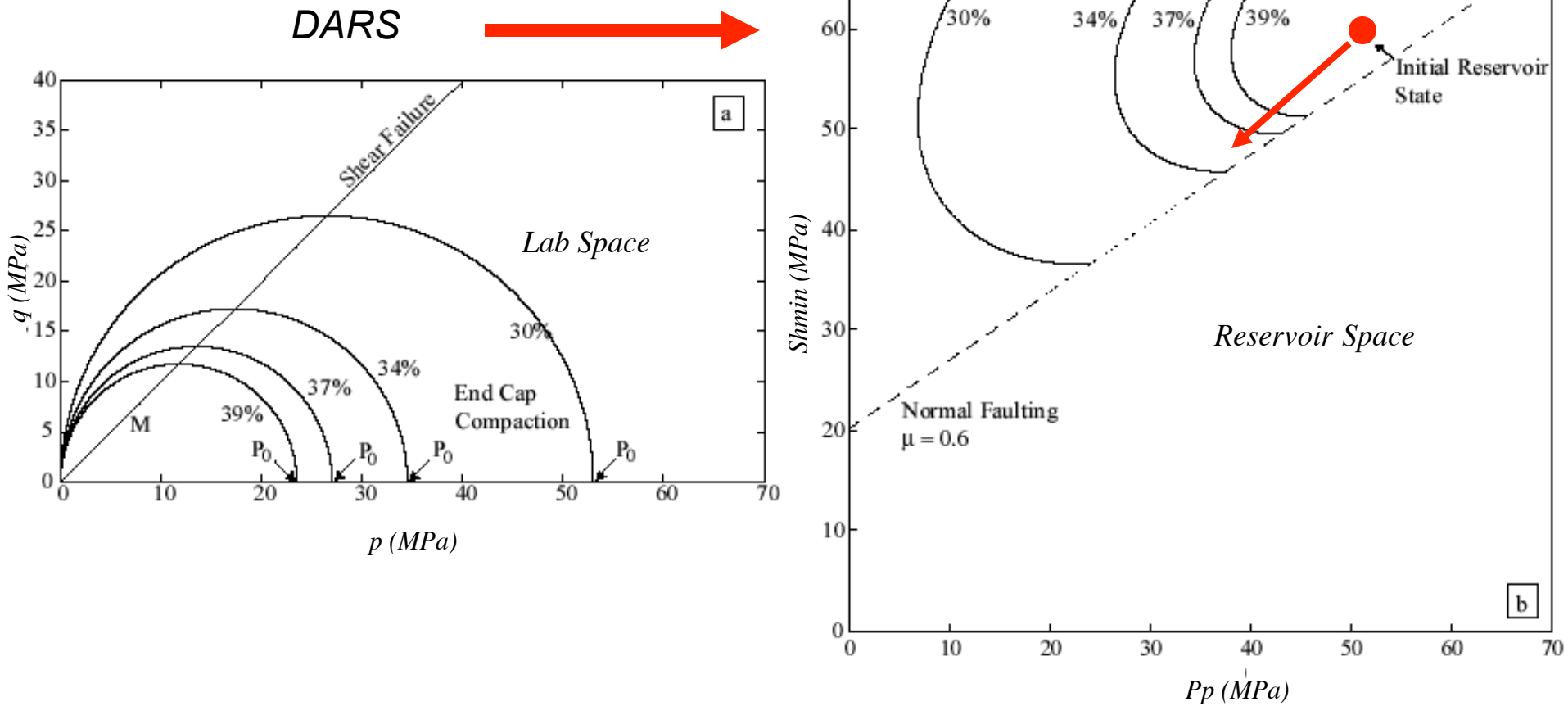
Deformation Analysis in Reservoir Space (DARS)

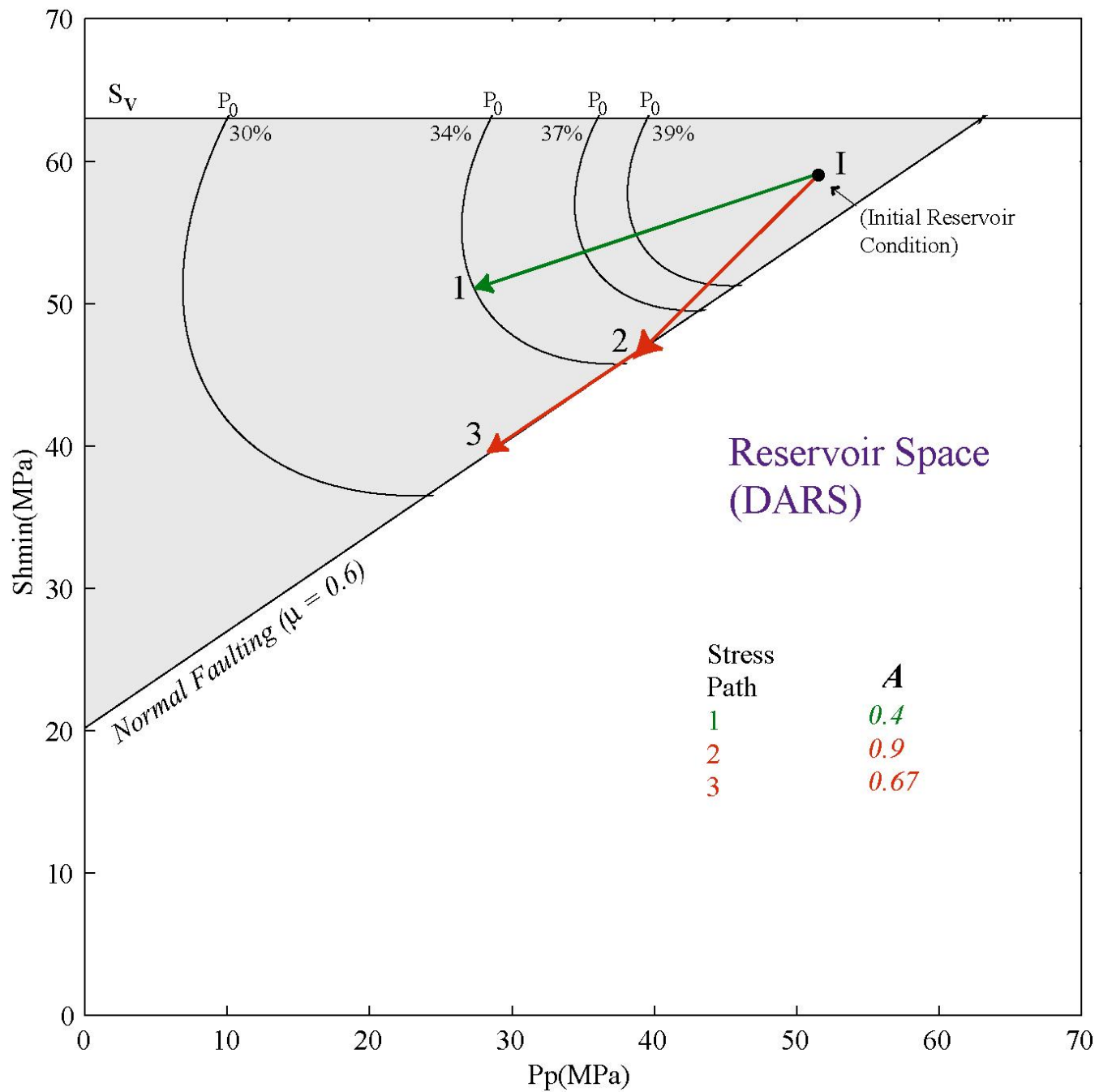
DARS





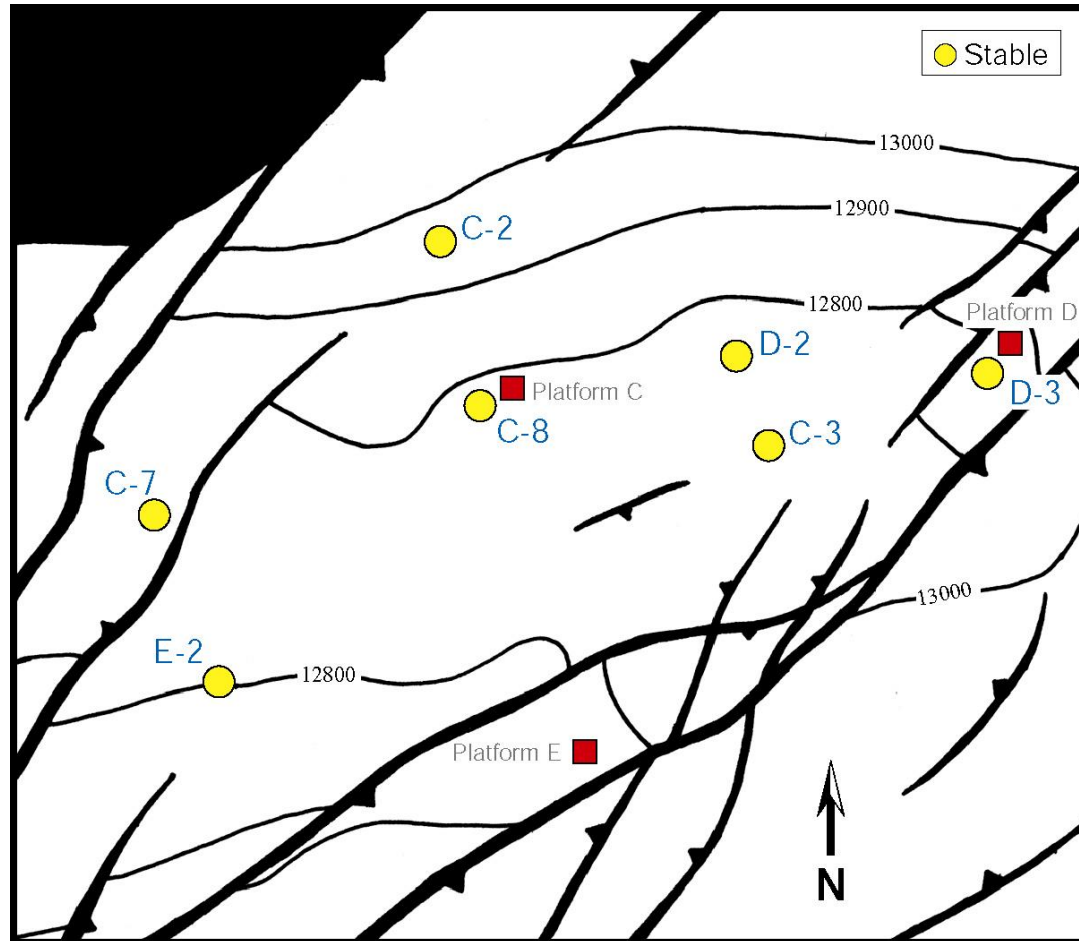
Deformation Analysis in Reservoir Space (DARS)





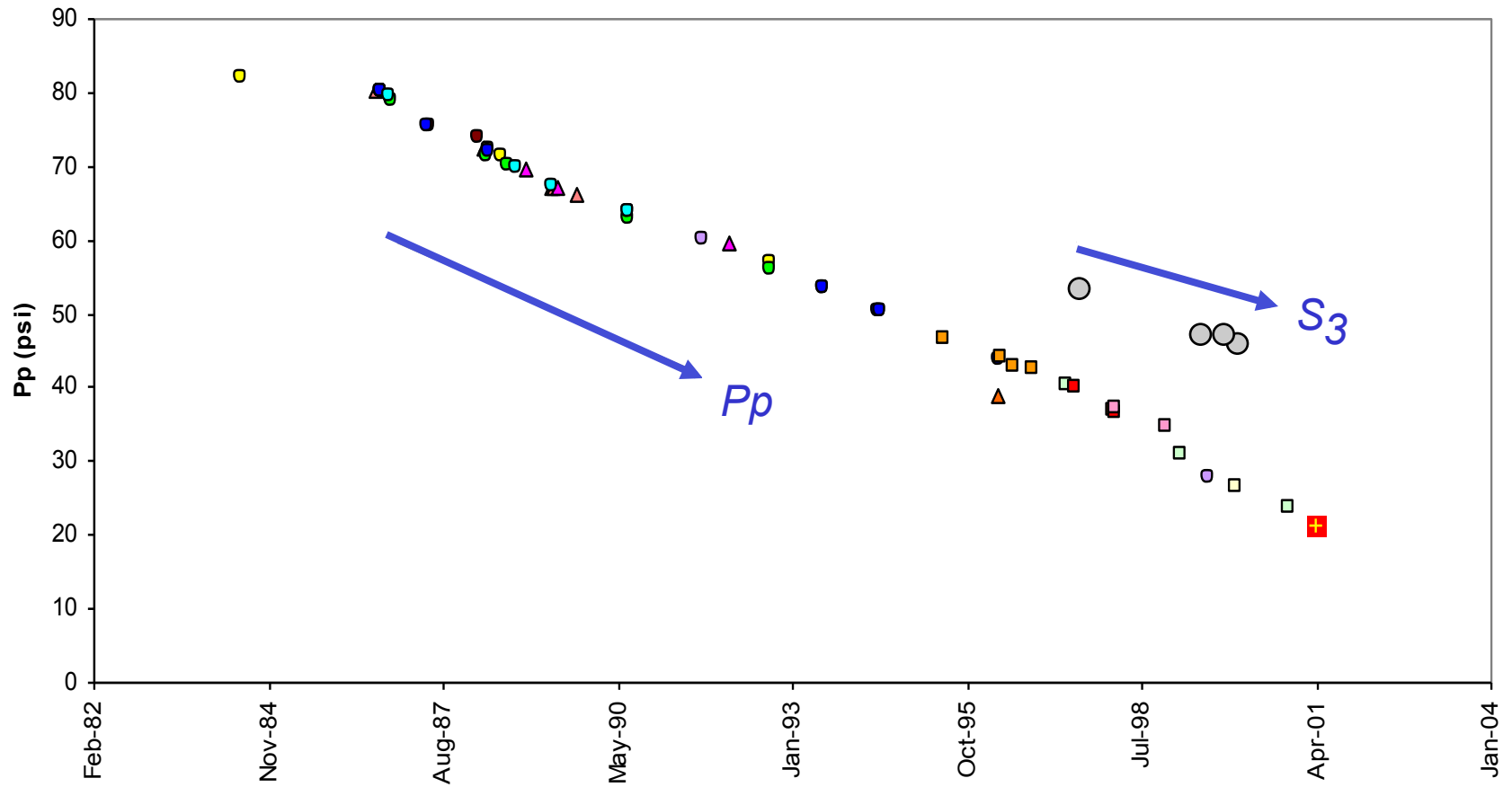


Gulf of Mexico Field X

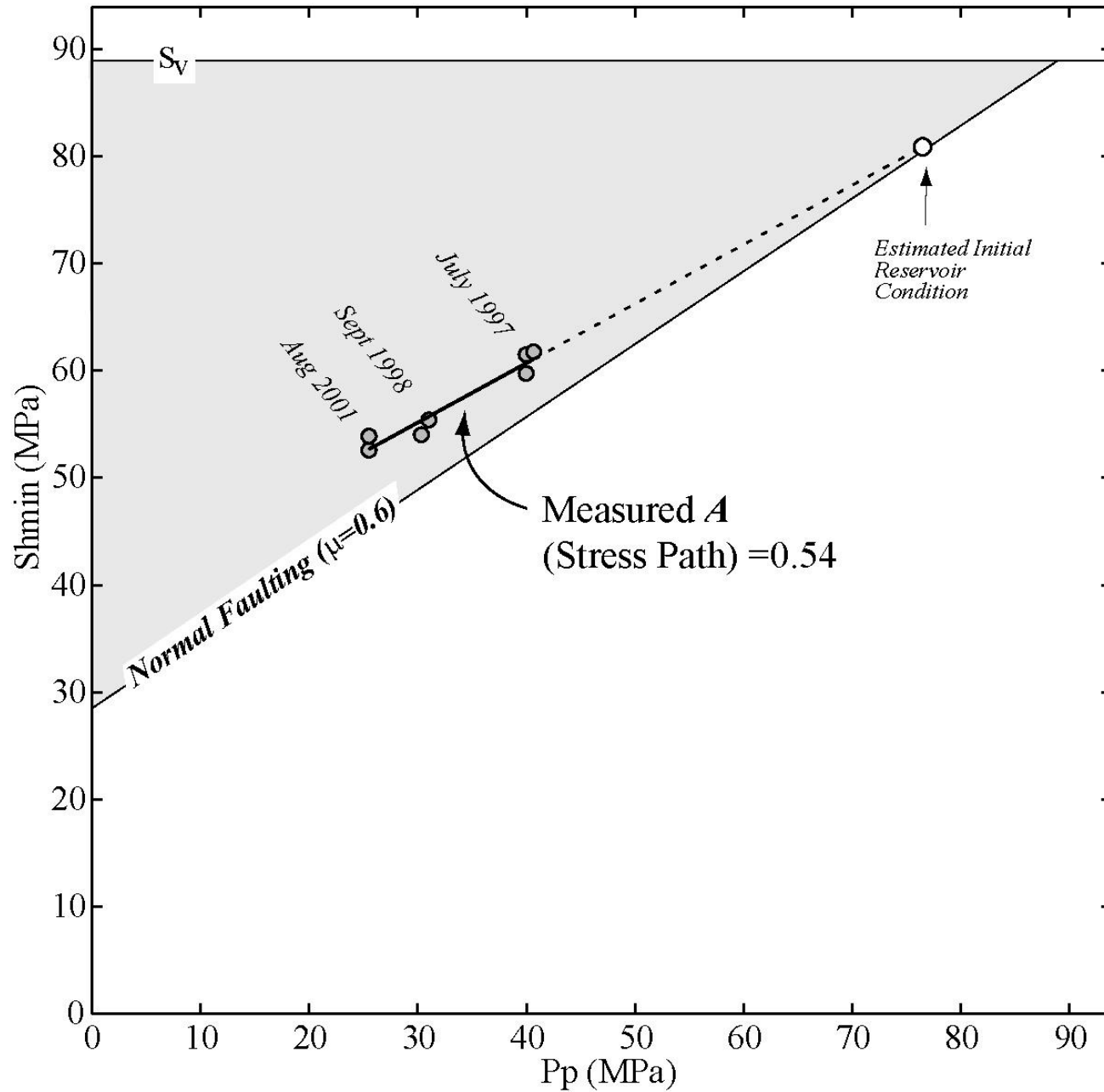




Field X



Evolution of Stress State for GOM Field X





Estimating Rock Strength From Geophysical Logs

- Why?
- What?
- How Well Does it Work?
- Be Careful Out There!

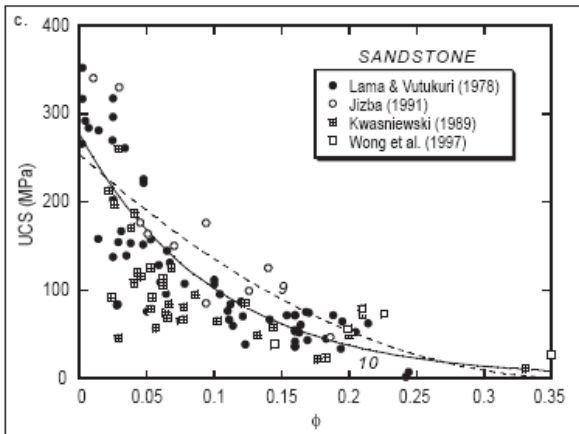
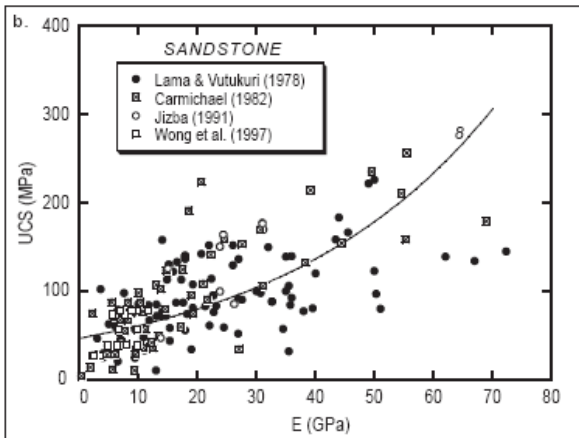
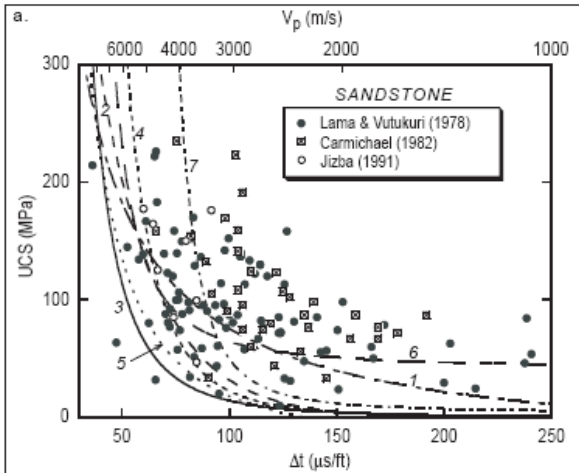


Figure 4.14 – pg.110

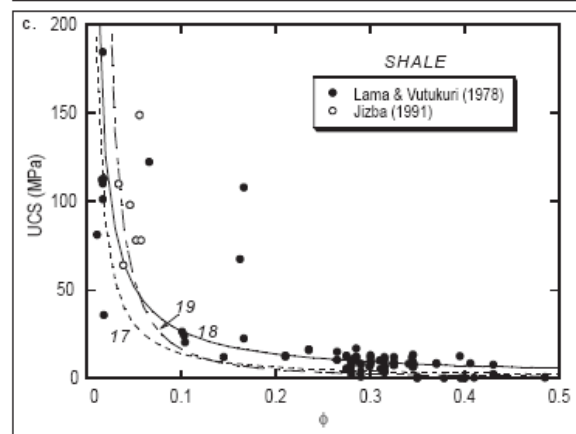
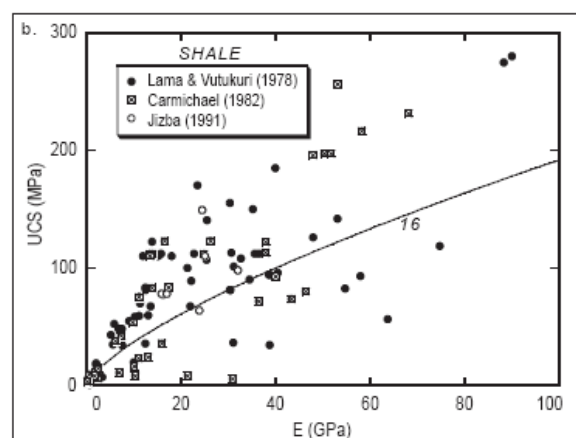
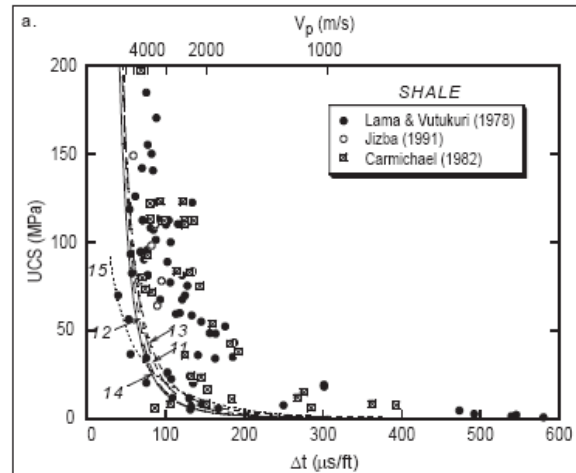


Figure 4.15 – pg.111

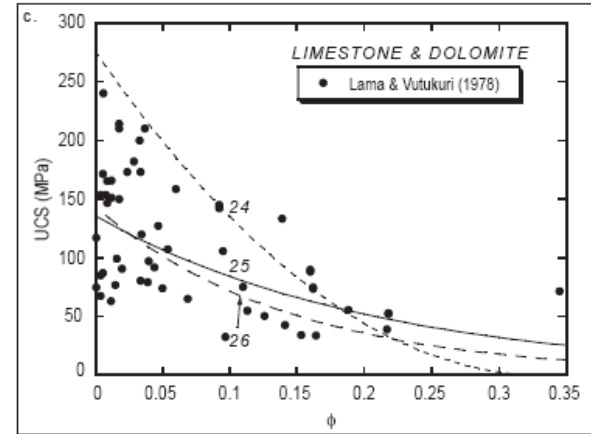
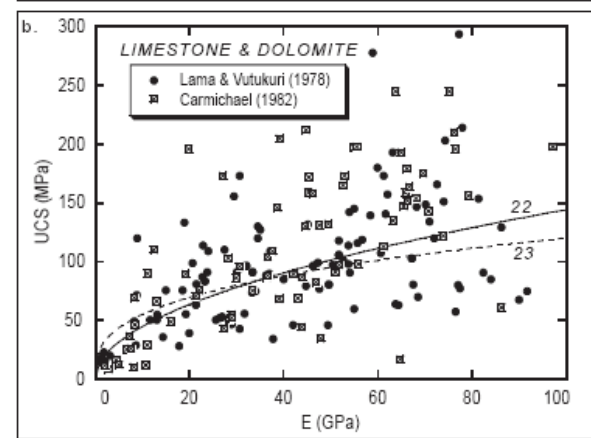
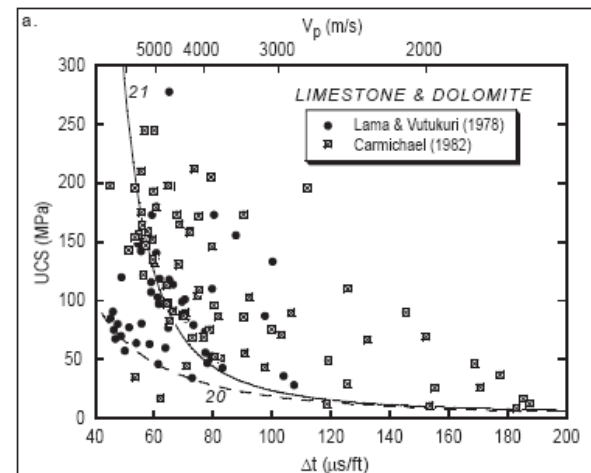


Figure 4.16 – pg.112



Sandstone

Eq. No.	UCS, MPa	Region Where Developed	General Comments	Reference
1	$0.035 V_p - 31.5$	Thuringia, Germany	-	(Freyburg 1972)
2	$1200 \exp(-0.036\Delta t)$	Bowen Basin, Australia	Fine grained, both consolidated and unconsolidated sandstones with wide porosity range	(McNally 1987)
3	$1.4138 \times 10^7 \Delta t^{-3}$	Gulf Coast	Weak and unconsolidated sandstones	Unpublished
4	$3.3 \times 10^{-20} \rho^2 V_p^4 [(1+\nu)/(1-\nu)]^2 (1-2\nu) [1 + 0.78 V_{clay}]$	Gulf Coast	Applicable to sandstones with UCS >30 MPa	(Fjaer, Holt et al. 1992)
5	$1.745 \times 10^{-9} \rho V_p^2 - 21$	Cook Inlet, Alaska	Coarse grained sands and conglomerates	(Moos, Zoback et al. 1999)
6	$42.1 \exp(1.9 \times 10^{-11} \rho V_p^2)$	Australia	Consolidated sandstones with $0.05 < \phi < 0.12$ and UCS >80 MPa	Unpublished
7	$3.87 \exp(1.14 \times 10^{-10} \rho V_p^2)$	Gulf of Mexico	-	Unpublished
8	$46.2 \exp(0.000027E)$	-	-	Unpublished
9	$A (1-B\phi)^2$	Sedimentary basins worldwide	Very clean, well consolidated sandstones with $\phi < 0.30$	(Vernik, Bruno et al. 1993)
10	$277 \exp(-10\phi)$	-	Sandstones with $2 < \text{UCS} < 360 \text{ MPa}$ and $0.002 < \phi < 0.33$	Unpublished

Units used: V_p (m/s), Δt ($\mu\text{s}/\text{ft}$), ρ (kg/m^3), V_{clay} (fraction), E (MPa), ϕ (fraction)

Table 4.1 – pg.113



Shale

	UCS, MPa	Region Where Developed	General Comments	Reference
11	$0.77 (304.8/\Delta t)^{2.93}$	North Sea	Mostly high porosity Tertiary shales	(Horsrud 2001)
12	$0.43 (304.8/\Delta t)^{3.2}$	Gulf of Mexico	Pliocene and younger	Unpublished
13	$1.35 (304.8/\Delta t)^{2.6}$	Globally	-	Unpublished
14	$0.5 (304.8/\Delta t)^3$	Gulf of Mexico	-	Unpublished
15	$10 (304.8/\Delta t - 1)$	North Sea	Mostly high porosity Tertiary shales	(Lal 1999)
16	$0.0528 E^{0.712}$	-	Strong and compacted shales	Unpublished
17	$1.001 \phi^{-1.143}$	-	Low porosity ($\phi < 0.1$), high strength shales	(Lashkaripour and Dusseault 1993)
18	$2.922 \phi^{-0.96}$	North Sea	Mostly high porosity Tertiary shales	(Horsrud 2001)
19	$0.286 \phi^{-1.762}$	-	High porosity ($\phi > 0.27$) shales	Unpublished

Units used: Δt ($\mu\text{s}/\text{ft}$), E (MPa), ϕ (fraction)

Table 4.2 – pg.114



Carbonates

	UCS, MPa	Region Where Developed	General Comments	Reference
20	$(7682/\Delta t)^{1.82} / 145$	-	-	(Militzer 1973)
21	$10^{(2.44 + 109.14/\Delta t)} / 145$	-	-	(Golubev and Rabinovich 1976)
22	$0.4067 E^{0.51}$	-	Limestone with $10 < \text{UCS} < 300$ MPa	Unpublished
23	$2.4 E^{0.34}$	-	Dolomite with $60 < \text{UCS} < 100$ MPa	Unpublished
24	$C (1 - D\phi)^2$	Korobcheyev deposit, Russia	C is reference strength for zero porosity ($250 < C < 300$ MPa). D ranges between 2 and 5 depending on pore shape	(Rzhevsky and Novick 1971)
25	$143.8 \exp(-6.95\phi)$	Middle East	Low to moderate porosity ($0.05 < \phi < 0.2$) and high UCS ($30 < \text{UCS} < 150$ MPa)	Unpublished
26	$135.9 \exp(-4.8\phi)$	-	Representing low to moderate porosity ($0 < \phi < 0.2$) and high UCS ($10 < \text{UCS} < 300$ MPa)	Unpublished

Units used: Δt ($\mu\text{s}/\text{ft}$), E (MPa), ϕ (fraction)

Table 4.3 – pg.115



Coefficient of Internal Friction

	Φ , degree	General Comments	Reference
27	$\sin^{-1} ((V_p - 1000) / (V_p + 1000))$	Applicable to shale	(Lal 1999)
28	$70 - 0.417GR$	Applicable to shaly sedimentary rocks with $60 < GR < 120$	Unpublished

Units used: V_p (m/s), GR (API)

Table 4.4 – pg.116



Application to the GOM

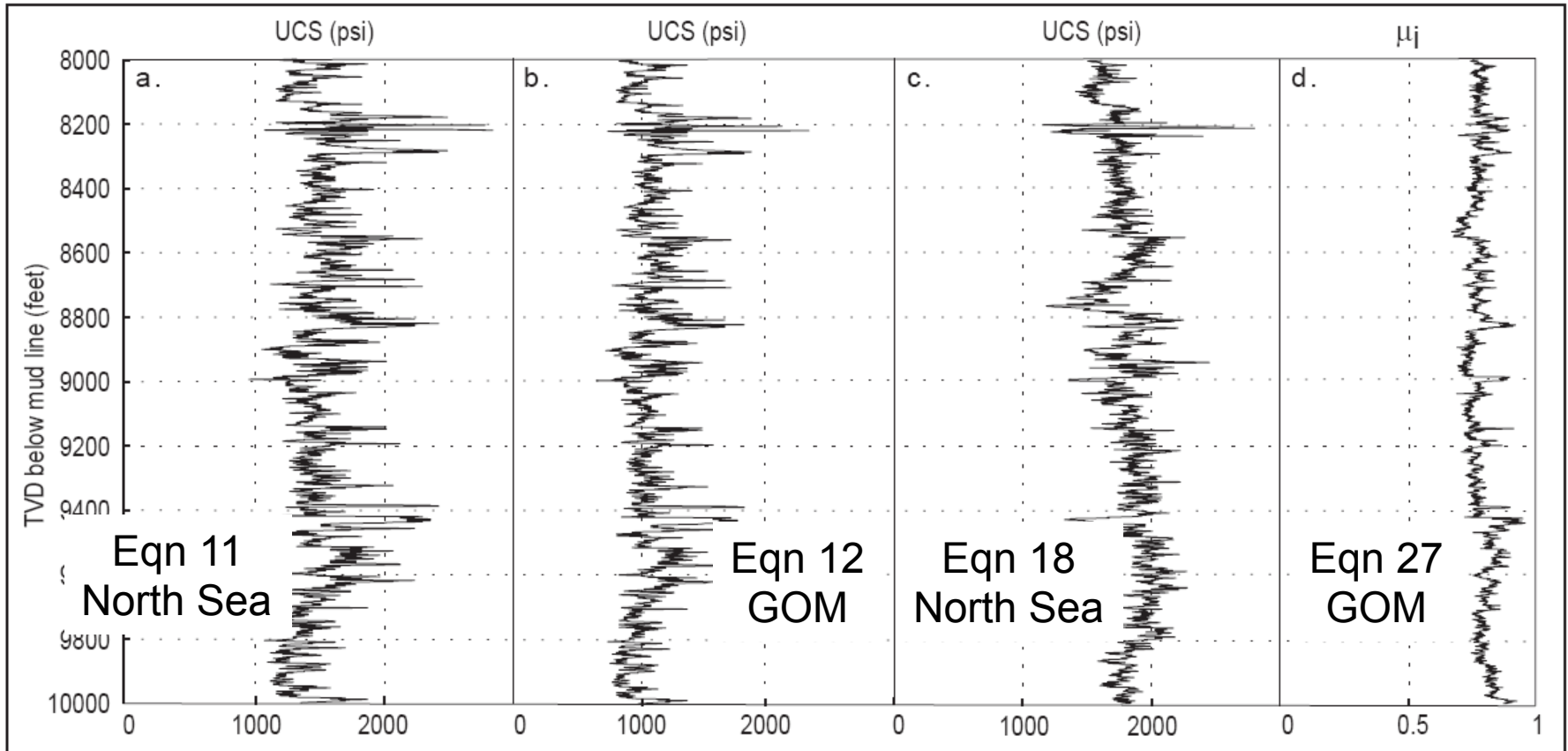


Figure 4.17 – pg.116

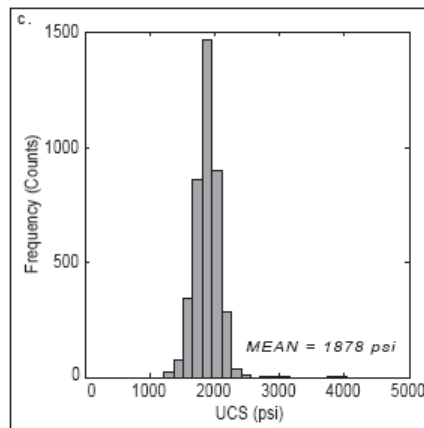
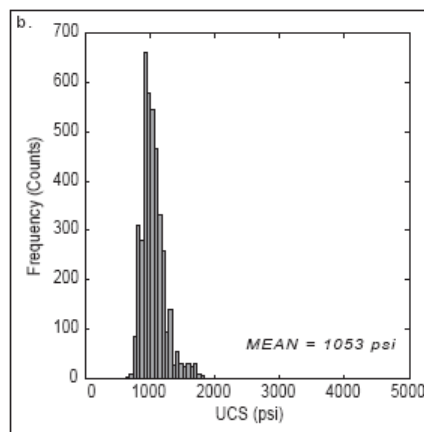
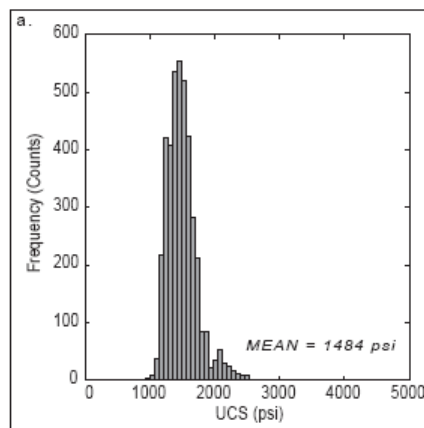
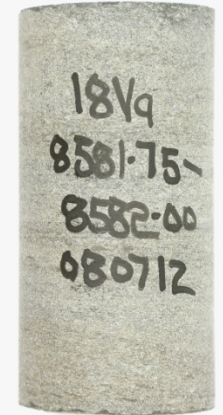


Figure 4.18 – pg.117



Organic Rich Shales

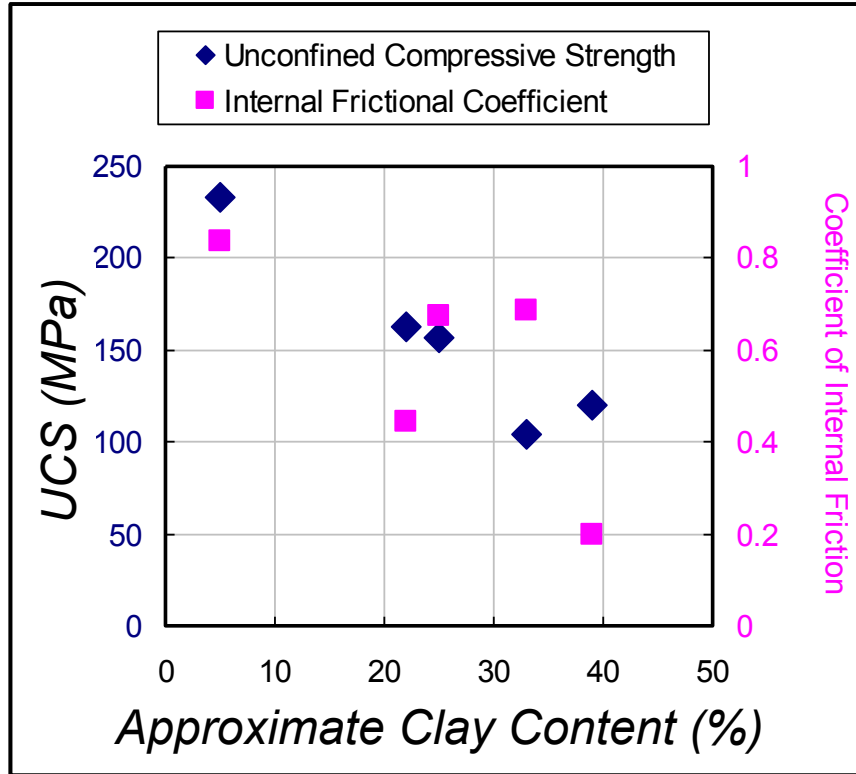
Sample group	Clay	Carbonate	QFP	TOC (wt%)
Barnett-dark	29-43	0-6	48-59	4.1-5.8
Barnett-light	2-7	37-81	16-53	0.4-1.3
Haynesville-dark	36-39	20-23	31-35	3.7-4.1
Haynesville-light	20-22	49-53	23-24	1.7-1.8
Fort St. John	32-39	3-5	54-60	1.6-2.2
Eagle Ford-dark	12-21	46-54	22-29	4.4-5.7
Eagle Ford-light	6-14	63-78	11-18	1.9-2.5



- *Bedding plane and sample cylinder axis is either parallel (horizontal samples) or perpendicular (vertical samples)*
- *3-10 % porosity*
- *All room dry, room temperature experiments*

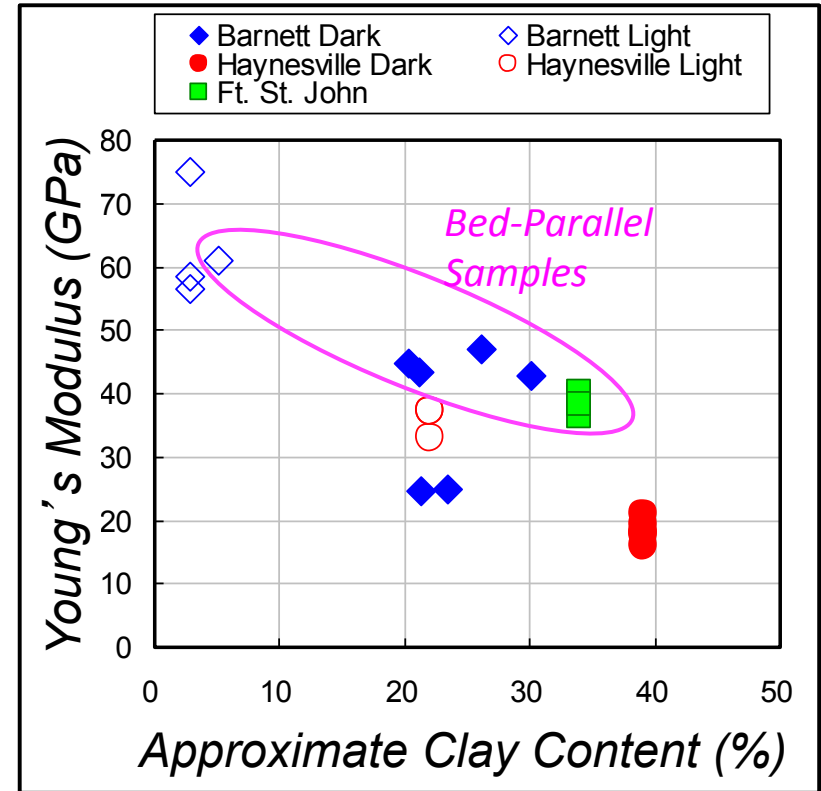


Strength



- Strength decreases with clay content
- Internal friction coefficient decreases from 0.9 to 0.2

Young's Modulus



- Modulus correlate with clay content and porosity
- Bedding parallel samples are systematically stiffer



Outline

Section 1

- Compressive Strength
- Strength Criterion

Section 2

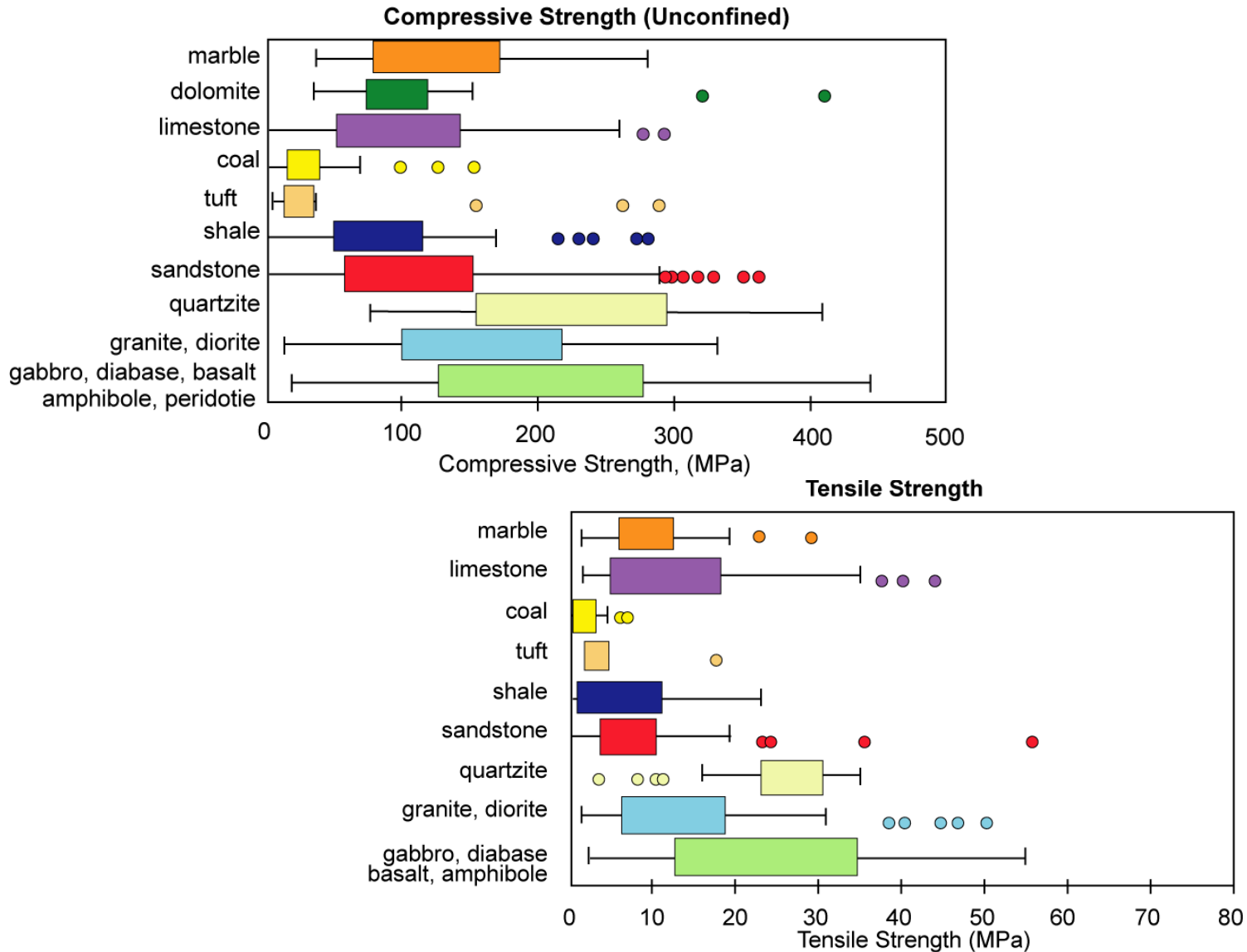
- Strength Anisotropy
- Shear Enhanced Compaction
- Strength from Logs

Section 3

- Tensile Strength
- Hydraulic Fracture Propagation
- Vertical Growth of Hydraulic Fractures

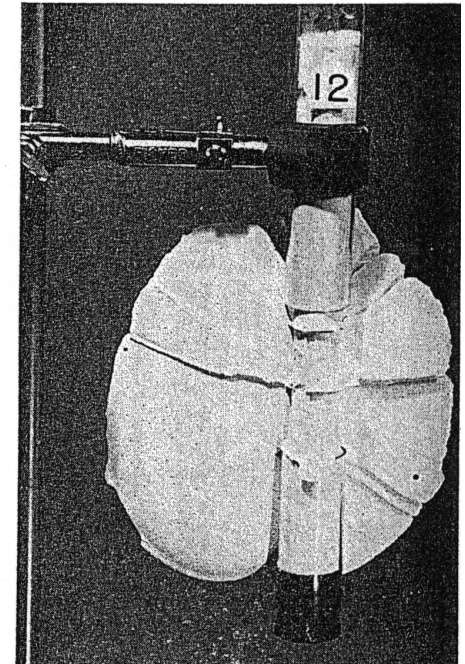
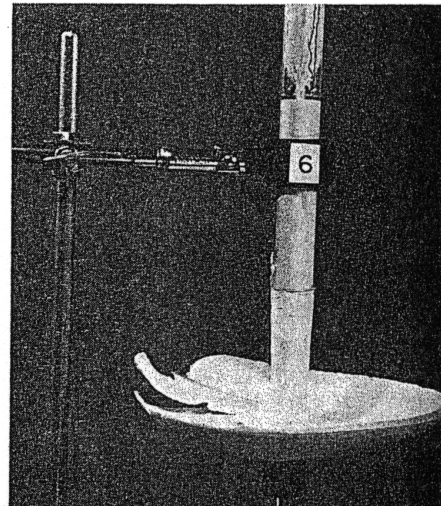
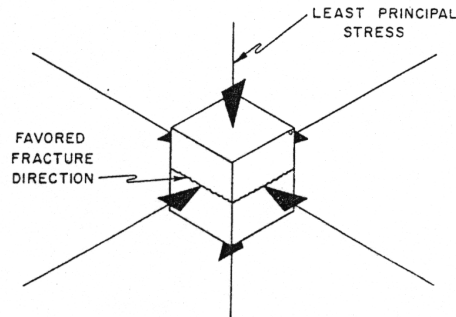
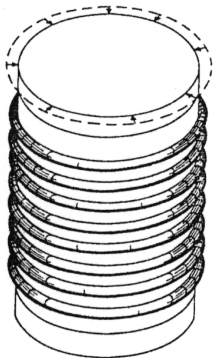
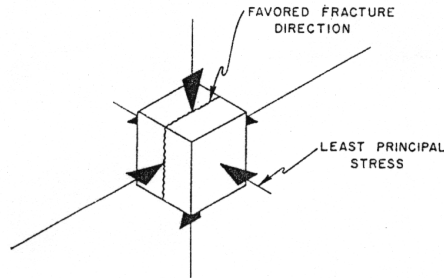
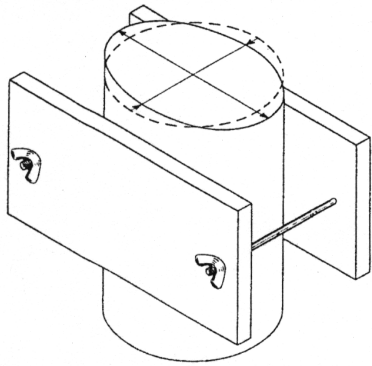


Rock Strength Measurement



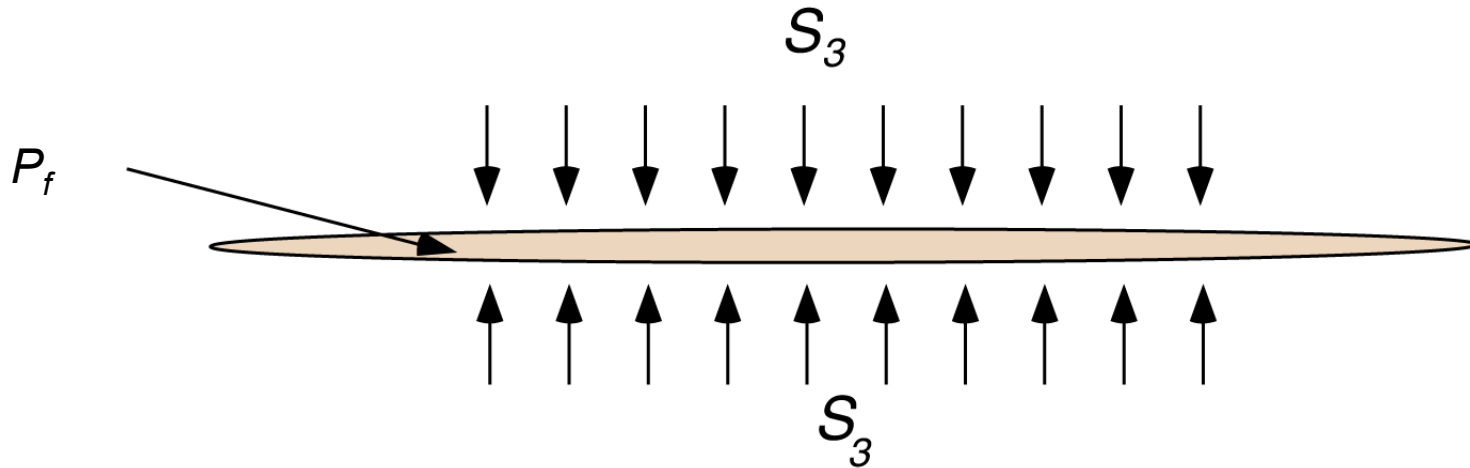


Hydraulic Fractures Propagate Perpendicular to the Least Principal Stress





Propagation of a Mode I Fracture

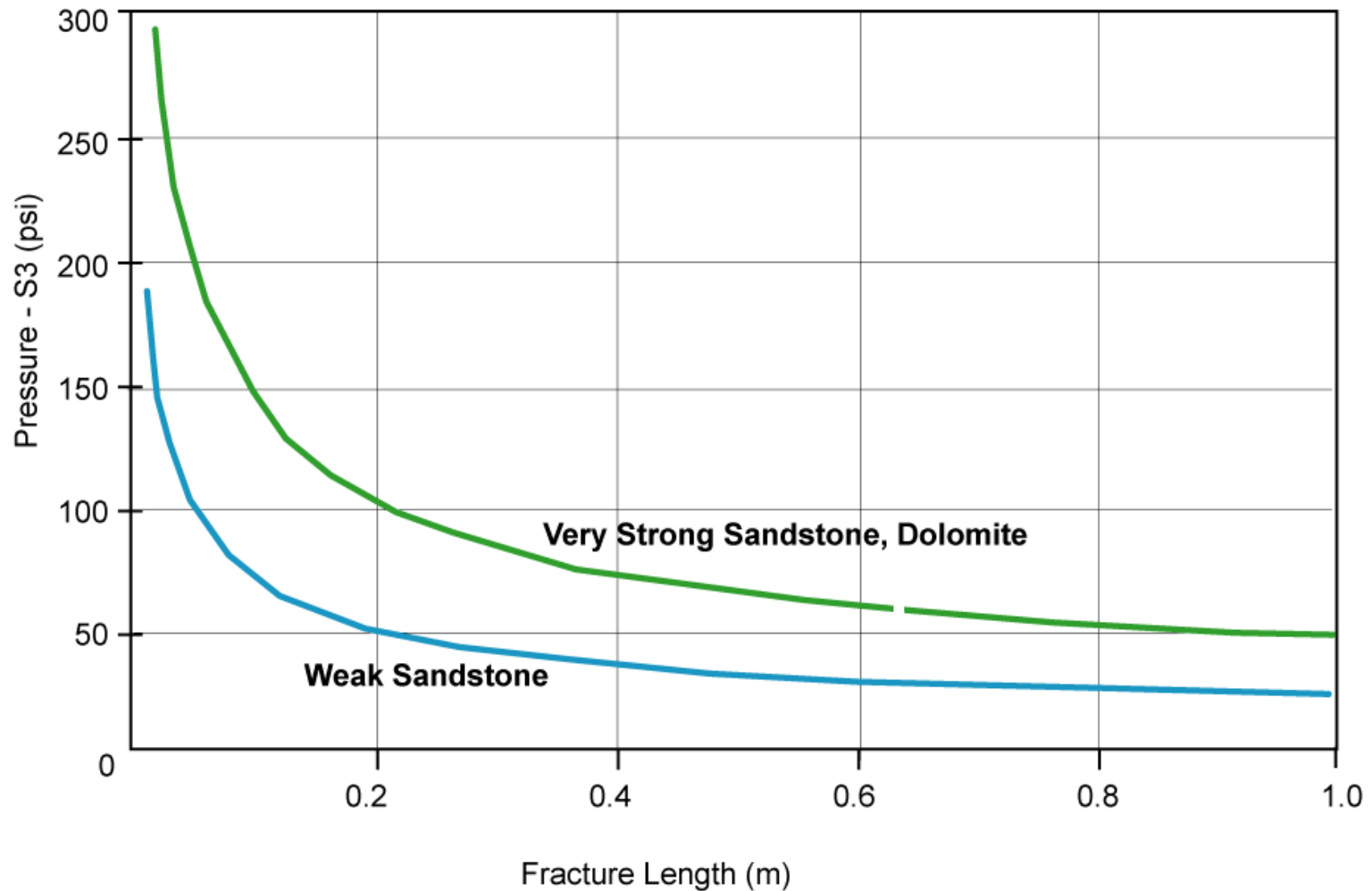


$$K_I = (P_f - S_3) \pi L^{1/2}$$

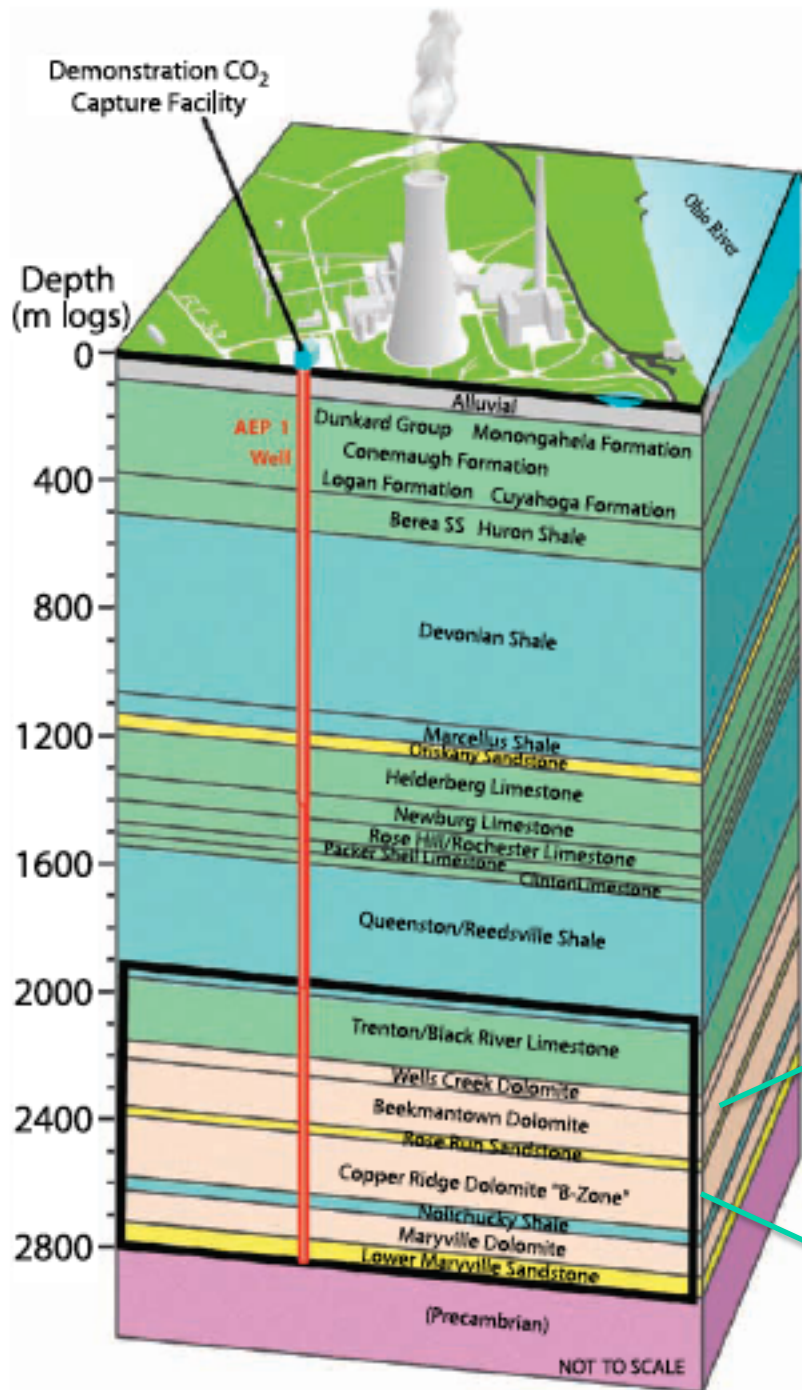
When $L > 1$ m,
 $P_f - S_3$ is negligible



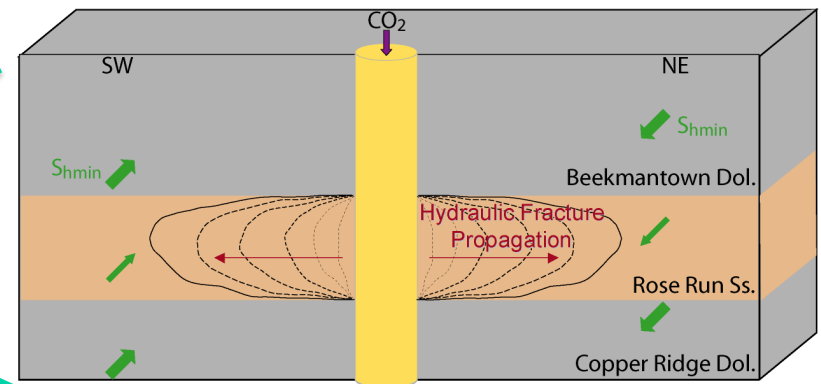
Tensile Strength of Mode I Cracks in Sedimentary Rocks is Irrelevant for Fracture Propagation*



**Once the fracture begins to propagate*

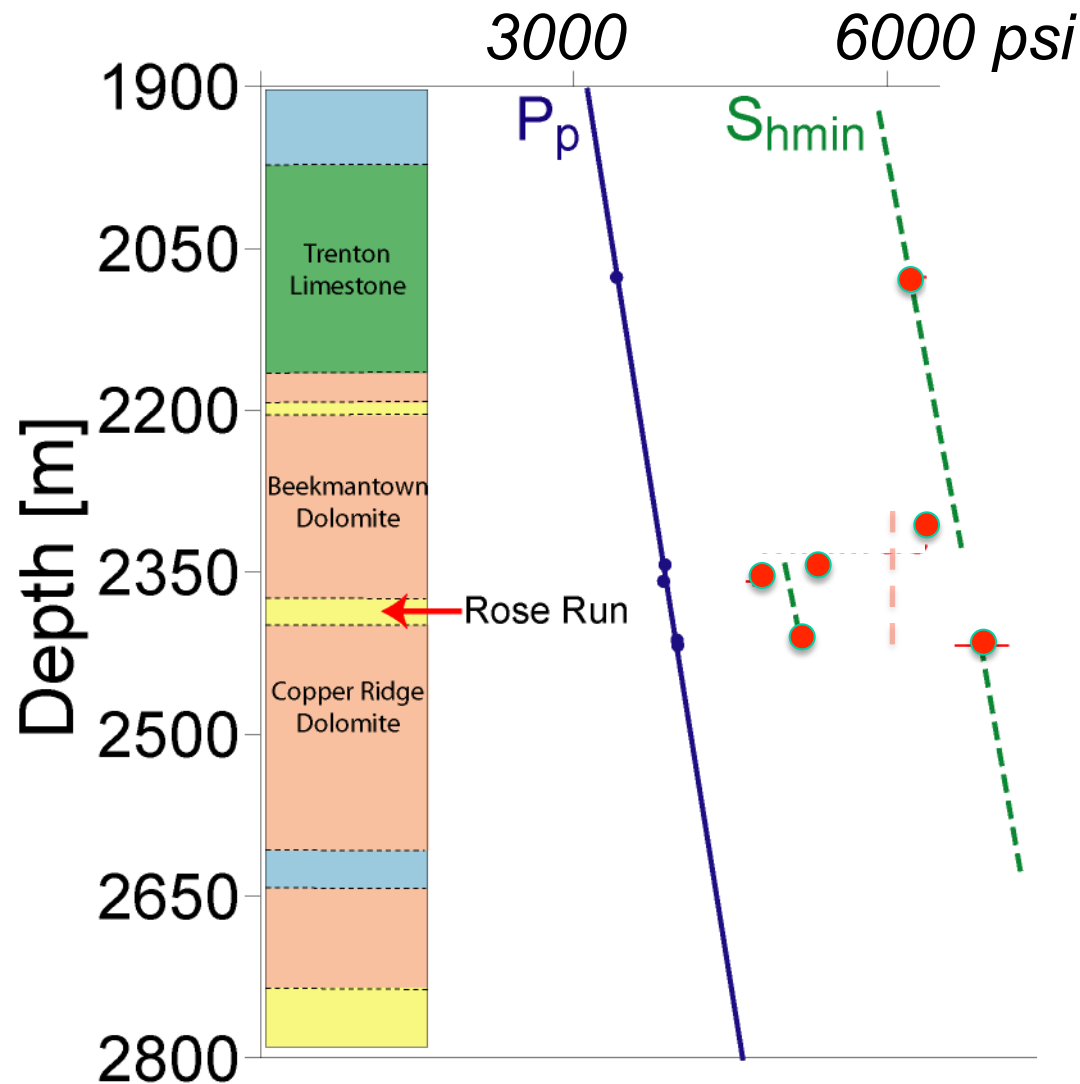


What Controls the Vertical Growth of Hydraulic Fractures?



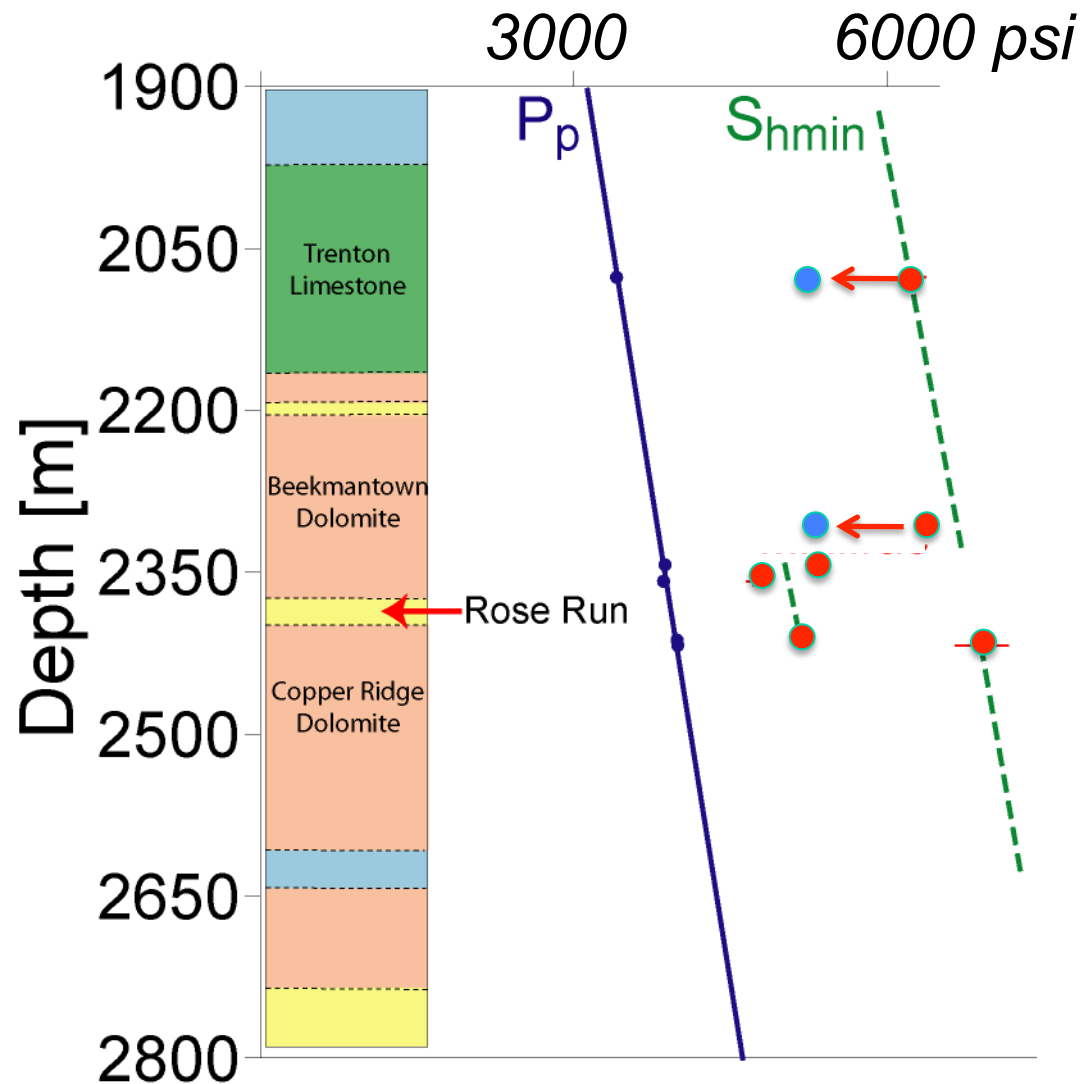


Case 1 – A Strong Contrast Between the Magnitude of S_{hmin} Within the Target Formation Prevents Vertical Propagation



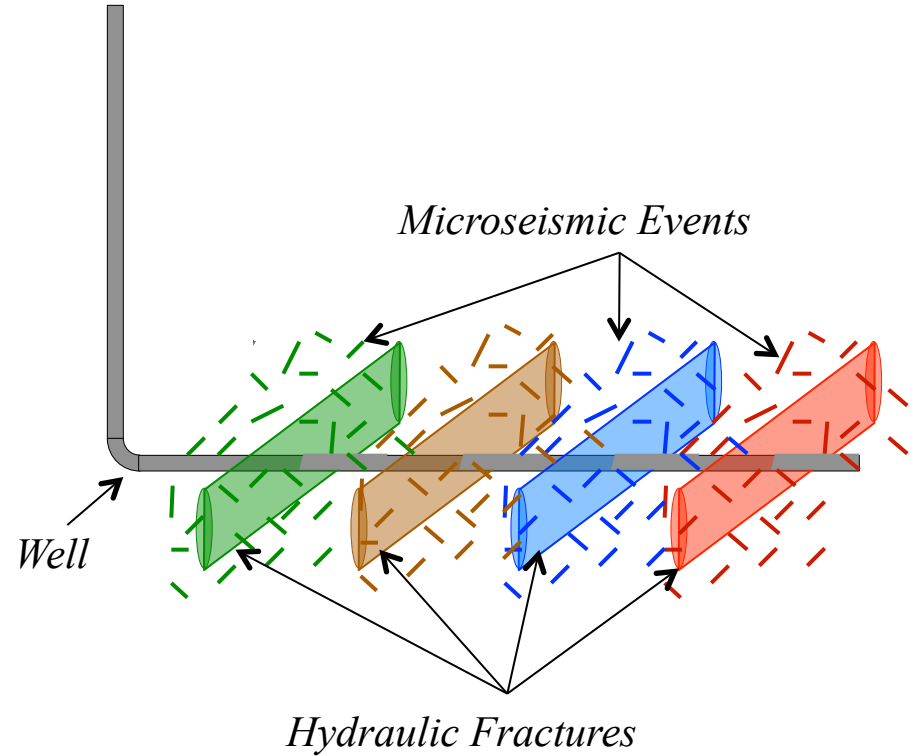
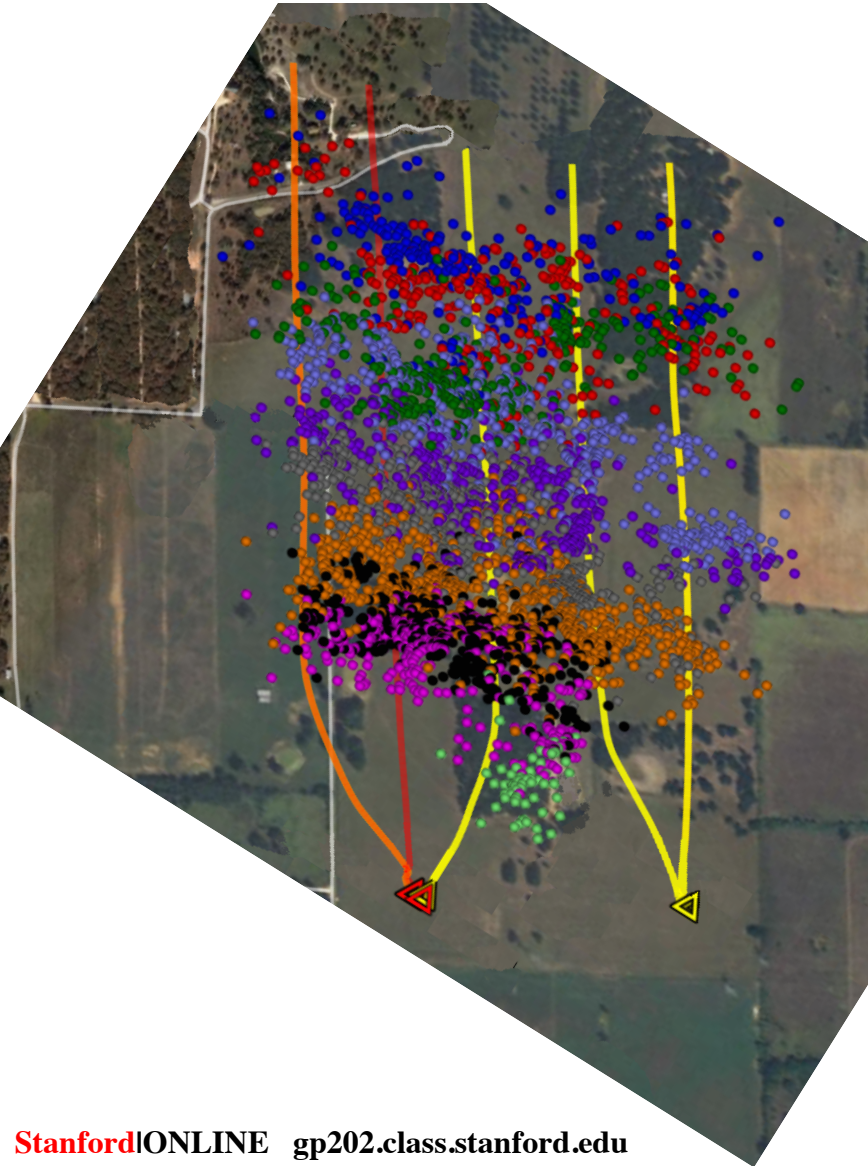


Case 2 – What if S_{hmin} Above the Shale has a Similar Magnitude?





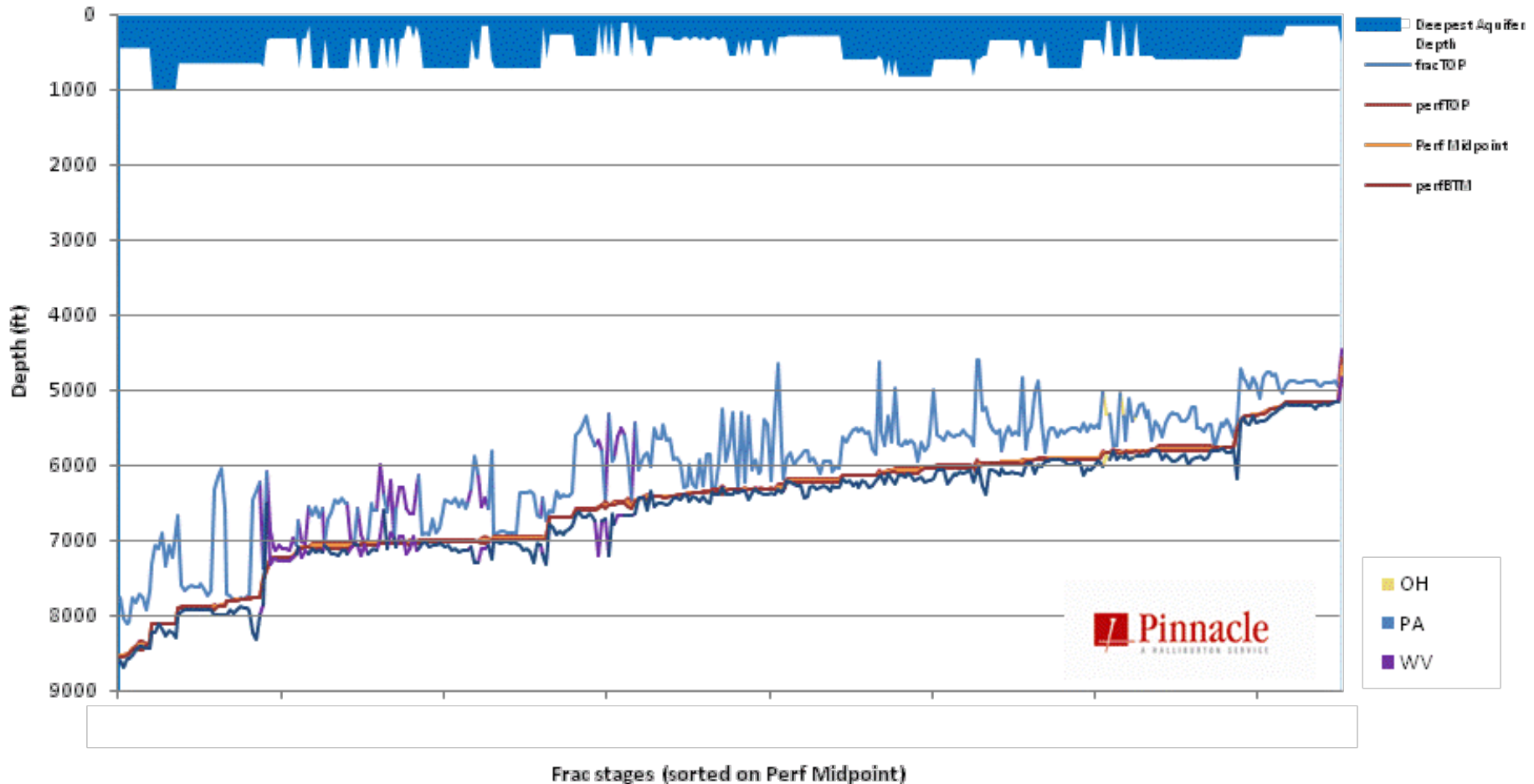
Multi-Stage Hydraulic Fracturing





Tendency for Upward Vertical Hydraulic Fracture Growth in the Marcellus Shale

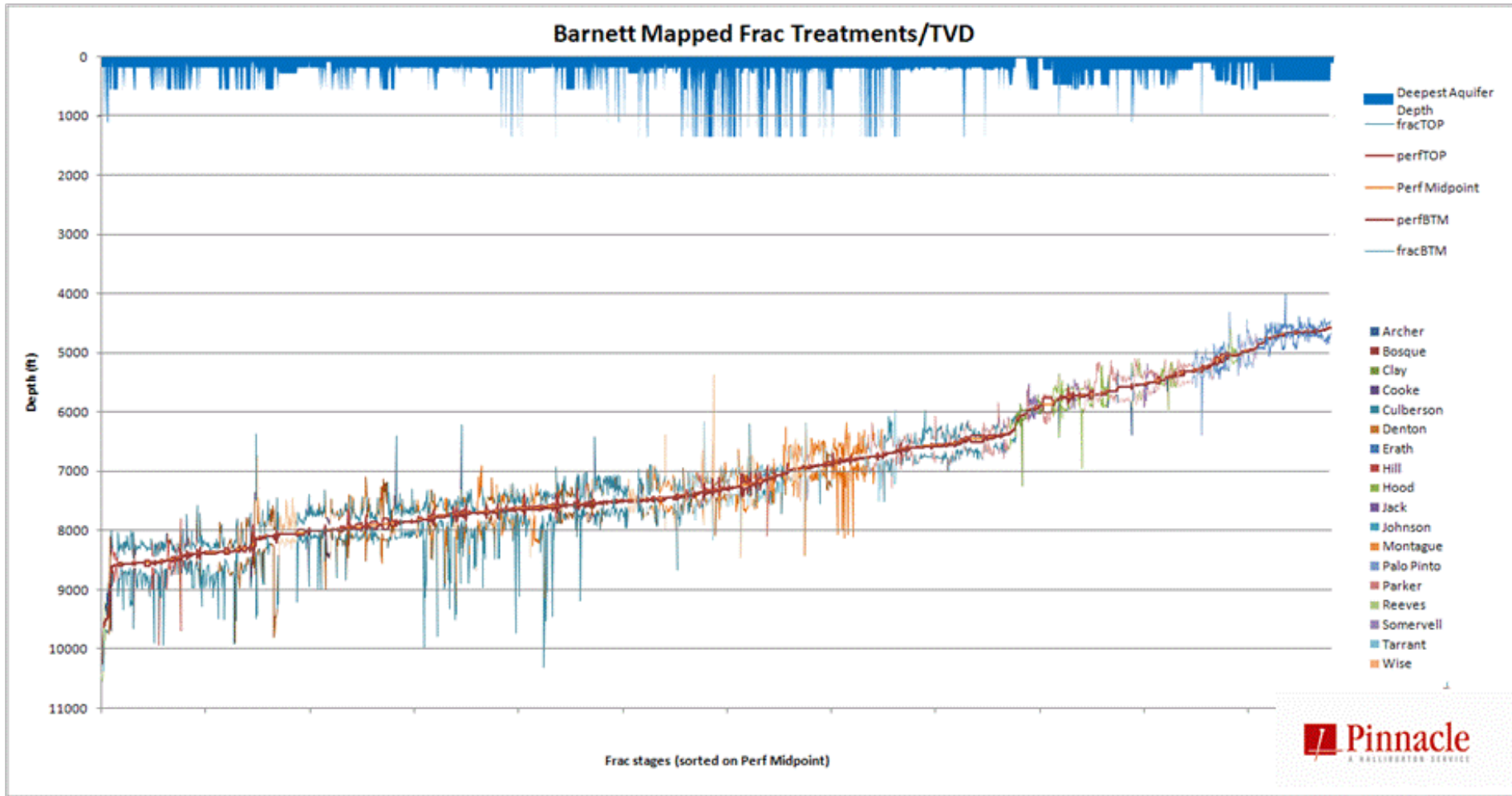
Marcellus Mapped Frac Treatments/TVD



Fisher (2010)

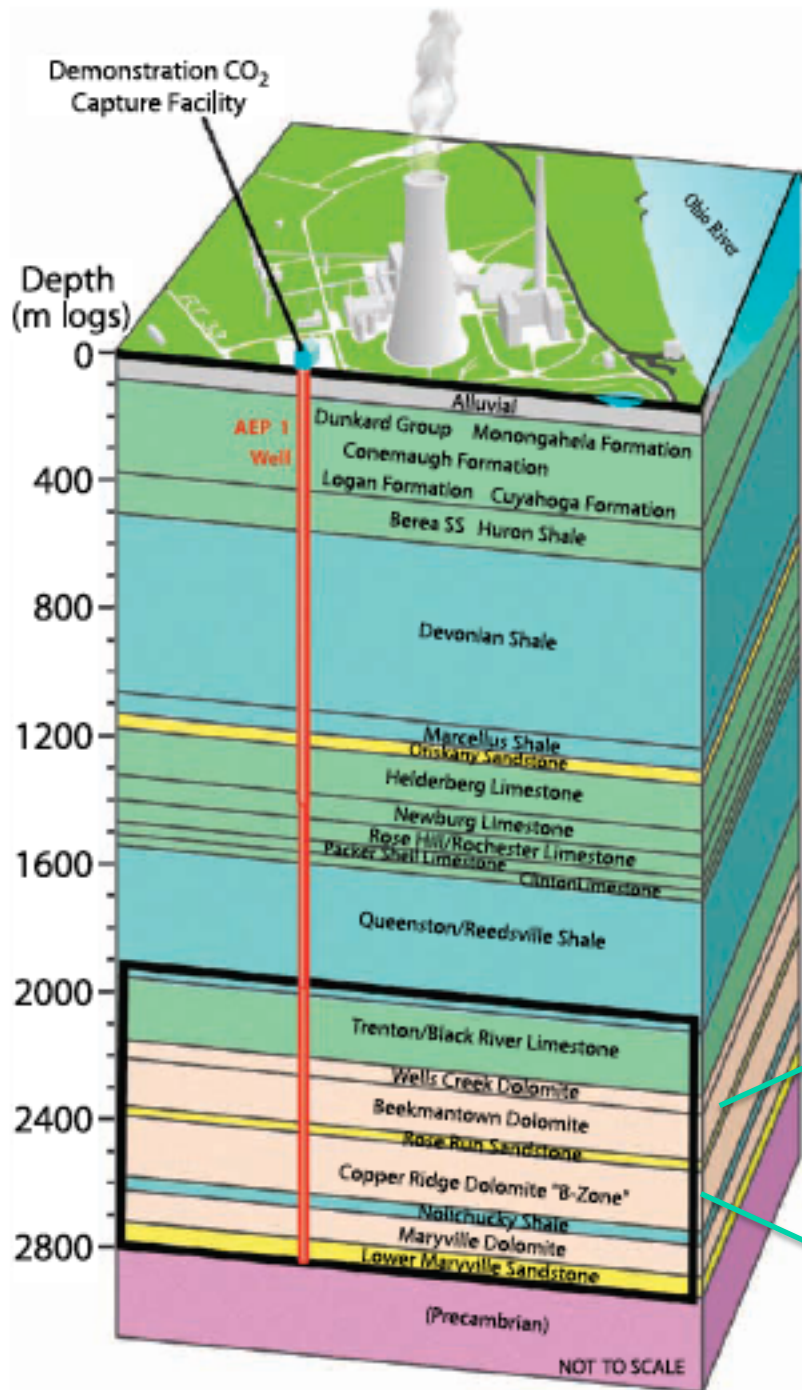


Tendency for Downward Growth of Hydraulic Fractures in the Barnett Shale into the Ellenburger Limestone



Fisher (2010)

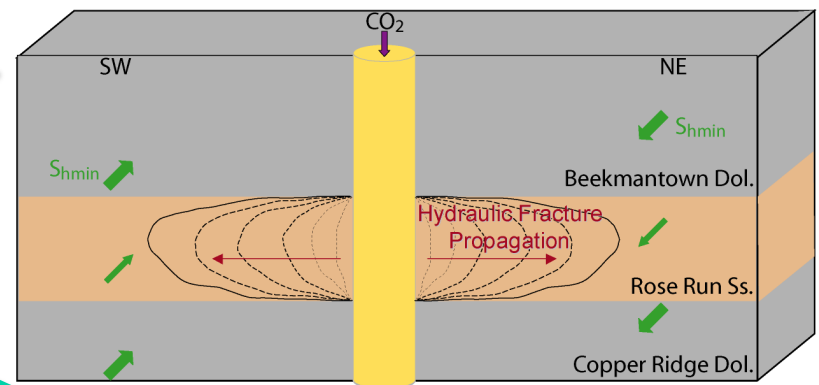
<http://nwis.waterdata.usgs.gov/nwis/inventory>



What Controls the Vertical Growth of Hydraulic Fractures?

The Variation of the S_3 (S_{hmin}) With Depth

Measure It!





Extended Leak Off Test (or Mini-Frac)

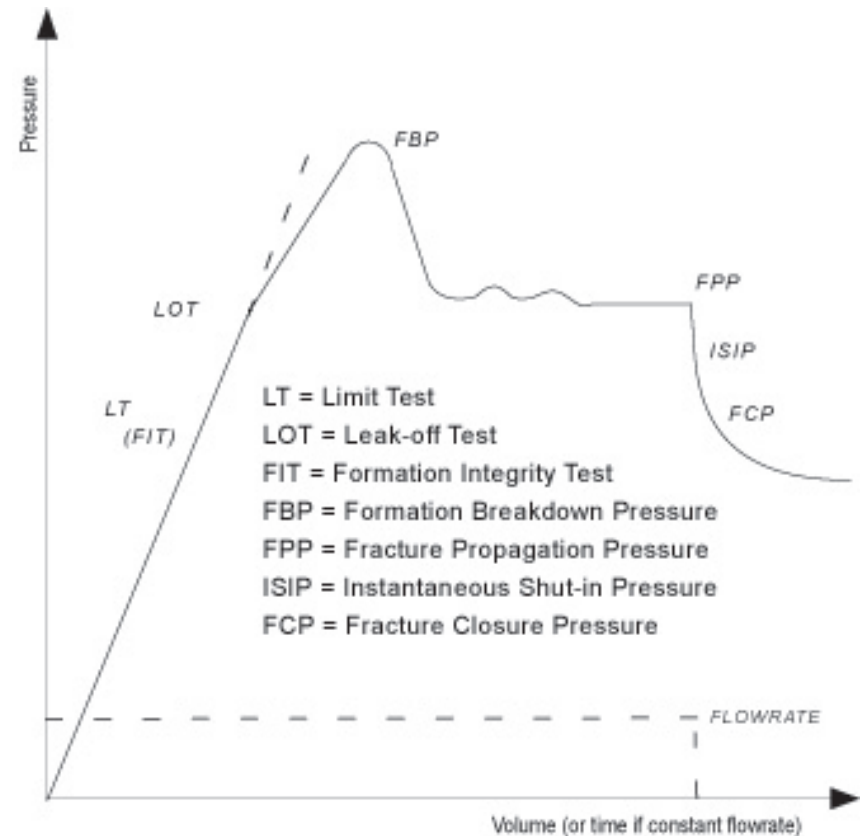
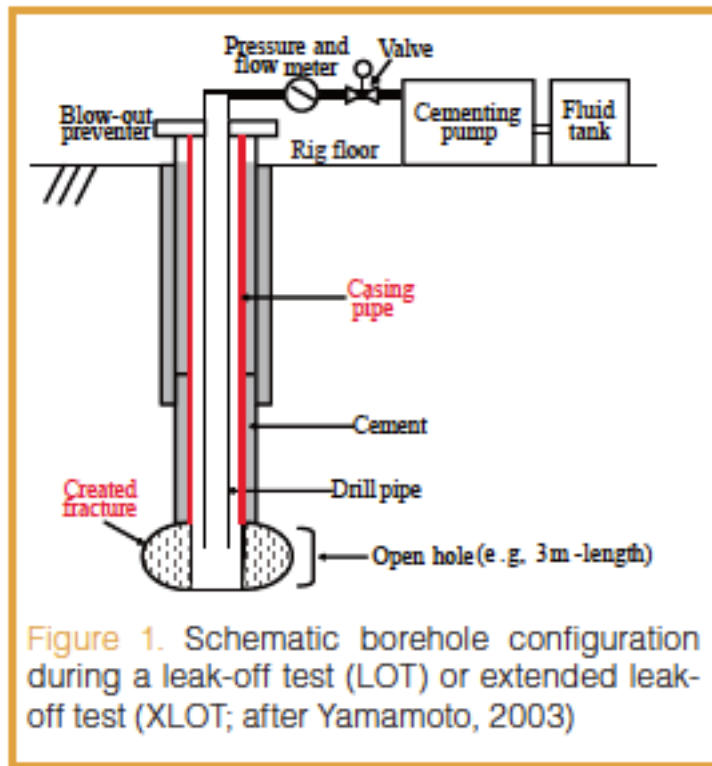


Figure 7.2 – pg. 211



Case 1 – A Strong Contrast Between the Magnitude of S_{hmin} Within the Target Formation Prevents Vertical Propagation

