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Kinematics of a Rigid Body

Rigid body

A body is **rigid** if its particles do not move relative to one another. Thus, the body has a fixed shape and all its parts have a fixed position relative to one another. No real body is absolutely rigid. For instance, consider a ball of hardened steel, such as found in a ball bearing. On a microscopic scale this body is certainly not rigid — the atoms in the steel are in a state of continuous thermal agitation; they randomly move back and forth about their equilibrium positions. On a macroscopic scale we can ignore these atomic motions because, on the average, they do not change the shape of the body. But even so, the ball is not quite rigid — if subjected to a very heavy load, the ball will suffer a noticeable deformation. In this chapter and in the following chapter, we will ignore such deformations produced by the forces acting on bodies. We will study the motion of bodies under the assumption that rigidity is a good approximation.

11.1 Motion of a Rigid Body

A rigid body can simultaneously have two kinds of motion: it can change its position in space and it can change its orientation in space. Change in position is translational motion; as we saw in Chapter 9, this motion can be conveniently described by the motion of the center of mass. Change in orientation is **rotational motion**; it can be proved mathematically that any change of orientation of a rigid body can be regarded as a rotation about some axis.¹

Rotational motion

¹ This is Euler's theorem.



Fig. 11.1 A hammer in free fall under the influence of gravity. The center of mass of the hammer, marked by the colored line, moves with constant vertical acceleration g, just like a particle in free fall. (Courtesy Harold E. Edgerton, MIT.)

As an example consider the motion of a hammer thrown up in the air (Figure 11.1). The orientation of the hammer changes relative to fixed coordinates attached to the ground. Instantaneously, the body rotates about a horizontal axis, say, a horizontal axis that passes through the center of mass (in Figure 11.1 this axis sticks out of the plane of the page and is marked with a dot). The complete motion can be described as a simultaneous rotation of the hammer about this axis and a translation of the axis along a parabolic path. It is worth keeping in mind that the choice of axis is somewhat arbitrary: instead of using a horizontal axis passing through the center of mass of the hammer, we could just as well use a horizontal axis passing through any other point, say, through the end of the handle. The hammer rotates in relation to both of these axes; the only difference lies in their translational motion — the axis through the end of the hammer undergoes a much more complicated translational motion than the axis through the center of mass. This emphasizes that there are two ways in which rotation is relative: first, it is necessary to specify the fixed reference frame with respect to which the motion occurs; and second, it is necessary to specify some point of the body through which the axis of rotation is supposed to pass (by hypothesis, this point then does not rotate). The freedom in the choice of axis of rotation will come in handy in the discussion of the motion of a rolling wheel in Section 12.5. However, it is often most convenient to suppose that the axis passes either through the center of mass or through some point which is held fixed by rigid mechanical constraints (e.g., a point on a fixed pivot or an axle).

In the above example of the thrown hammer the axis of rotation always remains horizontal. In the general motion of a rigid body, the axis of rotation can have any direction and can also change its direction. To describe such a complicated motion, it is convenient to separate the rotation into three components along three perpendicular axes. According to nautical and aeronautical terminology, the three components of the rotational motion are called **roll**, **pitch**, and **yaw** (Figure 11.2). However, in the following sections we will not deal with

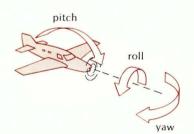


Fig. 11.2 The three independent rotational motions of an aircraft.