# THE ANALYSIS AND DESIGN OF LINEAR CIRCUITS





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# THE ANALYSIS AND DESIGN OF LINEAR CIRCUITS

# SECOND EDITION

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# **RESISTANCE CIRCUITS**

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**CHAPTER 1** • Introduction

CHAPTER 2 • Basic circuit analysis

**CHAPTER 3** • Circuit analysis techniques

**CHAPTER 4** • Active circuits

## COURSE OBJECTIVES

 $\left\{ 0 \right\}$ 

Design

Analysis Given a resistance circuit with dc input signals, select an analysis technique and find the circuit's response in terms of output signals or input–output relationships.

Given a dc signal or power processing function stated in terms of output signals or input–output relationships, design one or more circuits that implement the specified function within stated constraints.

**Evaluation** Given several resistance circuits that reportedly perform the same dc signal or power processing function, verify the claim and rank order the circuits using stated evaluation criteria.

# CHAPTER

# INTRODUCTION

The electromotive action manifests itself in the form of two effects which I believe must be distinguished from the beginning by a precise definition. I will call the first of these "electric tension," the second "electric current."

> André-Marie Ampère, 1820, French Mathematician/Physicist

- 1-1 About This Book
- **1–2** Symbols and Units
- 1-3 Circuit Variables
  - Summary
  - En Route Objectives and Associated Problems
  - Chapter-integrating Problems

**This** book deals with the analysis and design of linear electric circuits. A circuit is an interconnection of electric devices that processes energy or information. Understanding circuits is important because energy and information are the underlying technological commodities in electrical engineering. The study of circuits provides a foundation for areas of electrical engineering such as electronics, power systems, communication systems, and control systems.

4 CHAPTER 1

INTRODUCTION

This chapter describes the structure of this book, introduces basic notation, and defines the primary physical variables in electric circuits—voltage and current. André Ampère (1775–1836) was the first to recognize the importance of distinguishing between the electrical effects now called voltage and current. He also invented the galvanometer, the forerunner of today's voltmeter and ammeter. A natural genius who had mastered all the thenknown mathematics by age 12, he is best known for defining the mathematical relationship between electric current and magnetism. This relationship, now known as Ampère's law, is one of the basic concepts of modern electromagnetics.

The first section of this chapter describes the pedagogical framework and terminology that must be understood to use this book effectively. It describes how the learning objectives are structured to help the student develop the problem-solving abilities needed to analyze and design circuits. The second section provides some of the standard scientific notation and conventions used throughout the book. The last section introduces electric voltage, current, and power—the physical variables used throughout the book to describe the signal-processing and energy-transfer capabilities of linear circuits.

# ABOUT THIS BOOK

The basic purpose of this book is to introduce the analysis and design of linear circuits. Circuits are important in electrical engineering because they process electrical signals that carry energy and information. For the present we can define a **circuit** as an interconnection of electrical devices, and a **signal** as a time-varying electrical entity. For example, the information stored on a compact disk is recovered in the CD-ROM player as electronic signals that are processed by circuits to generate audio and video outputs. In an electrical power system some form of stored energy is converted to electrical form and transferred to loads, where the energy is converted into the form required by the customer. The CD-ROM player and the electrical power system both involve circuits that process and transfer electrical signals carrying energy and information.

In this text we are primarily interested in **linear circuits**. An important feature of a linear circuit is that the amplitude of the output signal is proportional to the input signal amplitude. The proportionality property of linear circuits greatly simplifies the process of circuit analysis and design. Most circuits are only linear within a restricted range of signal levels. When driven outside this range they become nonlinear, and proportionality no longer applies. Although we will treat a few examples of nonlinear circuits, our attention is focused on circuits operating within their linear range.

Our study also deals with interface circuits. For the purposes of this book, we define an **interface** as a pair of accessible terminals at which signals may be observed or specified. The interface idea is particularly important with integrated circuit (IC) technology. Integrated circuits involve many thousands of interconnections, but only a small number are accessi-

#### ABOUT THIS BOOK

ble to the user. Designing systems using integrated circuits involves interconnecting complex circuits that have only a few accessible terminals. This often involves relatively simple circuits whose purpose is to change signal levels or formats. Such interface circuits are intentionally introduced to ensure that the appropriate signal conditions exist at the connections between complex integrated circuits.

#### COURSE OBJECTIVES

This book has a set of learning objectives and related homework that can be used to verify your progress. The overarching course objectives are defined at three levels: analysis, design, and evaluation. The analysis level involves determining the output signals of a given circuit with known input signals. Analysis has the important feature that a unique solution exists in linear circuits. Circuit analysis will occupy the bulk of our attention, since it provides the foundation for understanding the interaction of signals and circuits. The design level involves devising circuits that perform a prescribed signal-processing function. In contrast to analysis, a design problem may have no solution or several solutions. The latter possibility leads to the evaluation level: namely, rank-order circuits that perform the same function using factors such as cost, power consumption, and part counts. In real life the engineer's role is a blend of analysis, design, and evaluation, and the boundaries between these levels are often blurred. By explicitly stating our analysis, design, and evaluation objectives, we intend to help to prepare you for that role.

This book is divided into three parts: (1) resistance circuits (Chapters 1 through 4), (2) dynamic circuits (Chapters 5 through 11), and (3) applications (Chapters 12 through 16). The course objectives are listed at the beginning of the three parts of the book. The course-integrating problems given at the end of each part are designed to test your mastery of the course objectives.

#### **EN ROUTE OBJECTIVES**

The course objectives are built on a foundation of intermediate milestones called en route objectives (EROs). The EROs are listed at the end of each chapter together with homework problems designed to test your understanding of basic concepts and techniques. Although they are listed at the end of each chapter, the EROs are an integral part of the chapter and are keyed to one or more sections of the text. Collectively, the en route objectives specify the basic knowledge and understanding needed to achieve the course objectives. At the very end of each chapter there are a number of chapter-integrating problems. These problems combine several en route objectives and offer an opportunity to deal with more complex problems before attempting the broad course-integrating problems described previously. The primary purpose of the course- and chapter-integrating problems is indicated by the symbols **A** for analysis, **D** for design, and **E** for evaluation.

This book contains many worked examples to help you master the en route objectives. These examples describe the steps needed to obtain the final answer. They usually treat analysis problems, although design examples and application notes are also included. There are in-text exercises that include only the problem statement and the final answer. These exercises provide immediate feedback on your ability to apply the ideas discussed in a particular section. You should work out the solution in detail, and then check your results with the answers given in the exercise.

## 12 SYMBOLS AND UNITS

Throughout this text we will use the international system (SI) of units. The SI system includes six fundamental units: meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), and candela (cd). All the other units of measure can be derived from these six.

Like all disciplines, electrical engineering has its own terminology and symbology. The symbols used to represent some of the more important physical quantities and their units are listed in Table 1–1. It is not our purpose to define these quantities here, nor to offer this list as an item for memorization. Rather, the purpose of this table is merely to list in one place all the electrical quantities used in this book.

Numerical values in engineering range over many orders of magnitude. Consequently, the system of standard decimal prefixes in Table 1–2 is used. These prefixes on a unit abbreviation symbol indicate the power of 10 that is applied to the numerical value of the quantity.

#### Exercise 1-1

Given the pattern in the statement  $1 \text{ k}\Omega = 1 \text{ kilohm} = 1 \times 10^3 \text{ ohms}$ , fill in the blanks in the following statements using the standard decimal prefixes.

- (a) \_\_\_\_ = \_\_\_ =  $5 \times 10^{-3}$  watts
- (b) 10.0 dB = \_\_\_\_\_ = \_\_\_\_
- (c)  $3.6 \text{ ps} = \_\_\_= \_$ (d)  $\_\_= 0.03 \text{ microfarads} =$
- (a) = 0.05 incrotatads =
- (e) \_\_\_\_\_ = \_\_\_\_ gigahertz =  $6.6 \times 10^9$  Hertz

#### Answers:

- (a) 5.0 mW = 5 milliwatts
- (b) 10.0 decibels = 1.0 bel
- (c) 3.6 picoseconds =  $3.6 \times 10^{-12}$  seconds
- (d) 30 nF or 0.03  $\mu F = 30.0 \times 10^{-9} \; Farads$
- (e) 6.6 GHz = 6.6 gigahertz

# 1=3 CIRCUIT VARIABLES

The underlying physical variables in the study of electronic systems are **charge** and **energy**. The idea of electrical charge explains the very strong electrical forces that occur in nature. To explain both attraction and repulsion, we say that there are two kinds of charge—positive and negative.

QUANTITY	SYMBOL	UNIT	UNIT ABBREVIATION
Time	1	second	5
Frequency	f	hertz	Hz
Radian frequency	ω	radian/second	rad/s
Phase angle	θ,φ	degree or radian	° or rad
Energy	w	joule	J
Power	Р	watt	w
Charge	9	coulomb	С
Current	i	ampere	A
Electric field	3	volt/meter	V/m
Voltage	v	volt	v
Impedance	Z	ohm	Ω
Admittance	Y	siemens	S
Resistance	R	ohm	Ω
Conductance	G	siemens	S
Reactance	X	ohm	Ω
Susceptance	В	siemens	S
Inductance, self	L	henry	н
Inductance, mutual	M	henry	н
Capacitance	C	farad	F
Magnetic flux	ф	weber	wb
Flux linkages	λ	weber-turns	wb-t
Power ratio	$\log_{10}(p_2/p_1)$	bel	В

#### TABLE SOME IMPORTANT QUANTITIES, THEIR SYMBOLS, AND UNIT ABBREVIATIONS

Like charges repel, while unlike charges attract one another. The symbol q is used to represent charge. If the amount of charge is varying with time, we emphasize the fact by writing q(t). In the international system (SI), charge is measured in **coulombs** (abbreviated C). The smallest quantity of charge in nature is an electron's charge ( $q_E = 1.6 \times 10^{-19}$ C). Thus, there are  $1/q_E = 6.25 \times 10^{18}$  electrons in 1 coulomb of charge.

Electrical charge is a rather cumbersome variable to measure in practice. Moreover, in most situations the charges are moving, so we find it more convenient to measure the amount of charge passing a given point per unit time. If q(t) is the cumulative charge passing through a point, we define a signal variable *i* called **current** as follows:

$$i = \frac{dq}{dt} \tag{1-1}$$

Current is a measure of the flow of electrical charge. It is the time rate of charge of charge passing a given point in a circuit. The physical dimen-

#### INTRODUCTION

MULTIPLIER	PREFIX	ABBREVIATION
10'8	exa	E
1015	peta	Р
1012	tera	Т
10 <sup>9</sup>	giga	G
106	mega	М
103	kilo	k
10-1	deci	d
10 <sup>-2</sup>	centi	с
10 <sup>-3</sup>	milli	m
10-6	micro	μ
10-9	nano	n
10-12	pico	р
10 <sup>-15</sup>	femto	f
10-18	atto	а

TABLE 12 STANDARD DECIMAL PREFIXES

8 CHAPTER

sions of current are coulombs per second. In the SI system, the unit of current is the **ampere** (abbreviated A). That is,

#### 1 coulomb/second = 1 ampere

Since there are two types of electrical charge, there is a bookkeeping problem associated with the direction assigned to the current. In engineering it is customary to define the direction of current as the direction of the net flow of positive charge.

A second signal variable called **voltage** is related to the change in energy that would be experienced by a charge as it passes through a circuit. The symbol w is commonly used to represent energy. In the SI system of units, energy carries the units of **joules** (abbreviated J). If a small charge dq was to experience a change in energy dw in passing from point A to point B in a circuit, then the voltage v between A and B is defined as the change in energy per unit charge. We can express this definition in differential form as

$$v = \frac{dw}{dq} \qquad (1-2)$$

Voltage does not depend on the path followed by the charge dq in moving from point A to point B. Furthermore, there can be a voltage between two points even if there is no charge motion, since voltage is a measure of how much energy dw would be involved if a charge dq was moved. The dimensions of voltage are joules per coulomb. The unit of voltage in the SI system is the **volt** (abbreviated V). That is,

#### 1 joule/coulomb = 1 volt

of change of energy:

The general definition of physical variable called **power** is the time rate

$$p = \frac{dw}{dt} \tag{1-3}$$

)

The dimensions of power are joules per second, which in the SI system is called a **watt** (abbreviated W). In electrical circuits it is useful to relate power to the signal variables current and voltage. Using the chain rule, Eq. (1-3) can be written as

$$p = \left(\frac{dw}{dq}\right) \left(\frac{dq}{dt}\right) \tag{1-4}$$

Now using Eqs. (1-1) and (1-2), we obtain

$$p = vi \qquad (1-5)$$

The electrical power associated with a situation is determined by the product of voltage and current. The total energy transferred during the period from  $t_1$  to  $t_2$  is found by solving for dw in Eq. (1–3) and then integrating

$$w_{\rm T} = \int_{w_1}^{w_2} dw = \int_{t_1}^{t_2} p \, dt \qquad (1 - 6)$$

## EXAMPLE 1-1

The electron beam in the cathode-ray tube shown in Figure 1–1 carries  $10^{14}$  electrons per second and is accelerated by a voltage of 50 kV. Find the power in the electron beam.



#### SOLUTION:

Since current is the rate of charge flow, we can find the net current by multiplying the charge of the electron  $q_E$  by the rate of electron flow  $dn_E/dt$ .

$$i = q_{\rm E} \frac{dn_{\rm E}}{dt} = (1.6 \times 10^{-19}) (10^{14}) = 1.6 \times 10^{-5} \,{\rm A}$$

Therefore, the beam power is

$$p = vi = (50 \times 10^3)(1.6 \times 10^{-5}) = 0.8 \text{ W}$$

#### (FIGURE 1 - 1)

## EXAMPLE 1-2

The current through a circuit element is 50 mA. Find the total charge and the number of electrons transferred during a period of 100 ns.

#### SOLUTION:

The relationship between current and charge is given in Eq. (1–1) as

1.1

$$i = \frac{dq}{dt}$$

Since the current i is given, we calculate the charge transferred by solving this equation for dq and then integrating

$$q_{\rm T} = \int_{q_1}^{q_2} dq = \int_0^{10^{-7}} i \, dt$$
$$= \int_0^{10^{-7}} 50 \times 10^{-3} \, dt = 50 \times 10^{-10} \rm{C} = 5 \, \rm{nC}$$

There are  $1/q_{\rm E} = 6.25 \times 10^{18}$  electrons/coulomb, so the number of electrons transferred is

$$n_{\rm E} = (5 \times 10^{-9} \,{\rm C})(6.25 \times 10^{18} \,{\rm electrons/C}) = 31.2 \times 10^{9} \,{\rm electrons}$$

#### Exercise 1-2

A device dissipates 100 W of power. How much energy is delivered to it in 10 seconds?

Answer: 1 kJ

#### Exercise 1-3

The graph in Figure 1–2(a) shows the charge q(t) flowing past a point in a wire as a function of time.

- (a) Find the current i(t) at t = 1, 2.5, 3.5, 4.5, and 5.5 ms.
- (b) Sketch the variation of i(t) versus time.

#### Answers:

- (a) -10 nA, +40 nA, 0 nA, -20 nA, 0 nA.
- (b) The variations in i(t) are shown in Figure 1–2(b).

#### THE PASSIVE SIGN CONVENTION

We have defined three circuit variables (current, voltage, and power) using two basic variables (charge and energy). Charge and energy, like mass, length, and time, are basic concepts of physics that provide the scientific foundation for electrical engineering. However, engineering problems rarely involve charge and energy directly, but are usually stated in terms of voltage, current, and power. The reason for this is simple: The circuit vari-



#### CIRCUIT VARIABLES

ables are much easier to measure and therefore are the most useful working variables in engineering practice.

At this point, it is important to stress the physical differences between current and voltage variables. Current is a measure of the time rate of charge passing a point in a circuit. We think of current as a *through variable*, since it describes the flow of electrical charge through a point in a circuit. On the other hand, voltage is not measured at a single point, but rather between two points or across an electrical device. Consequently, we think of voltage as an *across variable* that inherently involves two points.

The arrow and the plus and minus symbols in Figure 1–3 are *reference* marks that define the positive directions for the current and voltage associated with an electrical device. These reference marks do not represent an assertion about what is happening physically in the circuit. The response of an electrical circuit is determined by physical laws, and not by the reference marks assigned to the circuit variables.

The reference marks are benchmarks assigned at the beginning of the analysis. When the actual direction and reference direction agree, the answers found by circuit analysis will have positive algebraic signs. When they disagree, the algebraic signs of the answers will be negative. For example, if circuit analysis reveals that the current variable in Figure 1–3 is positive [i.e., i(t) > 0], then the sign of this answer, together with the assigned reference direction, indicates that the current passes through point A in Figure 1–3 from left to right. Conversely, when analysis reveals that the current variable is negative, then this result, combined with the assigned reference direction, tells us that the current passes through point A from right to left. In summary, the algebraic sign of the answer together with arbitrarily assigned reference marks tell us the actual directions of a voltage or current variable.

In Figure 1–3, the current reference arrow enters the device at the terminal marked with the plus voltage reference mark. This orientation is called the **passive sign convention**. Under this convention, the power p(t) is positive when the device absorbs power and is negative when it delivers power to the rest of the circuit. Since  $p(t) = v(t) \times i(t)$ , a device absorbs power when the voltage and current variables have the same algebraic sign and delivers power when they have opposite signs. Certain devices, such as heaters (a toaster, for example), can only absorb power. The voltage and current variables associated with these devices must always have the same algebraic sign. On the other hand, a battery absorbs power [p(t) > 0] when it is being charged and delivers power [p(t) < 0] when it is discharging. Thus, the voltage and current variables for a battery can have the same or the opposite algebraic signs.

The passive sign convention is used throughout this book. It is also the convention used by circuit simulation computer programs, such as SPICE and MICRO-CAP.<sup>1</sup> To interpret correctly the results of circuit analysis, it



**(FIGURE 1 – 3)** Voltage and current reference marks for a two-terminal device.

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We discuss computer-aided circuit analysis in subsequent chapters. MICRO-CAP is a trademark of Spectrum Software, Inc.



#### GROUND

Since voltage is defined between two points, it is often useful to define a common voltage reference point called **ground**. The voltages at all other points in a circuit are then defined with respect to this common reference point. We indicate the voltage reference point using the ground symbol shown in Figure 1–4. Under this convention we sometimes refer to the variables  $v_A(t)$ ,  $v_B(t)$ , and  $v_C(t)$  as the voltages at points A, B, and C, respectively. This terminology appears to contradict the fact that voltage is an across variable that involves two points. However, the terminology means that the variables  $v_A(t)$ ,  $v_B(t)$ ,  $v_B(t)$ , and  $v_C(t)$  are the voltages defined between points A, B, and C, and the common voltage reference point at point G.

Using a common reference point for across variables is not an idea unique to electrical circuits. For example, the elevation of a mountain is the number of feet or meters between the top of the mountain and a common reference point at sea level. If a geographic point lies below sea level, then its elevation is assigned a negative algebraic sign. So it is with voltages. If circuit analysis reveals that the voltage variable at point A is negative [i.e.,  $v_A(t) < 0$ ], then this fact together with the reference marks in Figure 1–4 indicate that the potential at point A is less than the ground potential.

#### EXAMPLE 1-3

Figure 1–5 shows a circuit formed by interconnecting five devices, each of which has two terminals. A voltage and current variable has been assigned to each device using the passive sign convention. The working variables for each device are observed to be as follows:

	DEVICE 1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5
v	+100 V	?	+25 V	+75 V	-75 V
i	?	+5 mA	+5 mA	?	+5 mA
р	-1 W	+0.5 W	?	0.75 W	?

(a) Find the missing variable for each device and state whether the device is absorbing or delivering power.

(b) Check your work by showing that the sum of the device powers is zero.



bol indicates a common voltage reference point.

#### **CIRCUIT VARIABLES**

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#### FIGURE 1-5



#### SOLUTION:

(a) We use p = vi to solve for the missing variable since two of the three circuit variables are given for each device.

Device 1:  $i_1 = p_1/v_1 = -1/100 = -10 \text{ mA } [p(t) < 0, \text{ delivering power}]$ Device 2:  $v_2 = p_2/i_2 = 0.5/0.005 = 100 \text{ V } [p(t) > 0, \text{ absorbing power}]$ Device 3:  $p_3 = v_3i_3 = 25 \times 0.005 = 0.125 \text{ W } [p(t) > 0, \text{ absorbing power}]$ Device 4:  $i_4 = p_4/v_4 = 0.75/75 = 10 \text{ mA } [p(t) > 0, \text{ absorbing power}]$ Device 5:  $p_5 = v_5i_5 = -75 \times 0.005 = -0.375 \text{ W } [p(t) < 0, \text{ delivering power}]$ 

(b) Summing the device powers yields

$$p_1 + p_2 + p_3 + p_4 + p_5 = -1 + 0.5 + 0.125 + 0.75 - 0.375$$
$$= +1.375 - 1.375 = 0$$

This example shows that the sum of the power absorbed by devices is equal in magnitude to the sum of the power supplied by devices. A power balance always exists in the types of circuits treated in this book and can be used as an overall check of circuit analysis calculations.

#### Exercise 1-4

The working variables of a set of two-terminal electrical devices are observed to be as follows:

	DEVICE 1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5
v	+10 V	?	-15 V	+5 V	?
i	-3 A	-3 A	+10 mA	?	-12 mA
p	?	+40 W	?	+10 mW	-120 mW

Using the passive sign convention, find the magnitude and sign of the unknown variable and state whether the device is absorbing or delivering power.

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FIGURE 1-6

#### Answers:

Device 1: p = -30 W (delivering power) Device 2: v = -13.3 V (absorbing power) Device 3: p = -150 mW (delivering power) Device 4: i = +2 mA (absorbing power) Device 5: v = +10 V (absorbing power)

#### Exercise 1-5

Figure 1–6 shows a circuit with six two-terminal devices and their assigned voltage and current variables. The working variables for each element are observed to be as follows:

	DEVICE 1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5	DEVICE 6
v	+7.5 V	+5 V	2	+7.5 V	7	+12.5 V
i.	-3 A	?	+0.5 A	+1.5 A	+2.5 A	+1 A
p	?	+10 W	+1.25 W	?	-12.5 W	?

- (a) Using the passive sign convention, find the missing variable for each device and state whether the device is absorbing or delivering power.
- (b) Use the power balance to check your work.



#### Answers:

(a) Device 1: p<sub>1</sub> = -22.5 W (delivering power) Device 2: i<sub>2</sub> = +2 A (absorbing power) Device 3: v<sub>3</sub> = +2.5 V (absorbing power) Device 4: p<sub>4</sub> = +11.25 W (absorbing power) Device 5: v<sub>5</sub> = -5 V (delivering power) Device 6: p<sub>6</sub> = +12.5 W (absorbing power)
(b) 35-35 = 0

#### PROBLEMS

# SUMMARY

- Circuits are important in electrical engineering because they process signals that carry energy and information. A **circuit** is an interconnection of electrical devices. A **signal** is an electrical current or voltage that carries energy or information. An **interface** is a pair of accessible terminals at which signals may be observed or specified.
- This book defines overall course objectives at the analysis, design, and evaluation levels. In **circuit analysis** the circuit and input signals are given and the object is to find the output signals. The object of **circuit design** is to devise one or more circuits that produce prescribed output signals for given inputs signals. The **evaluation** problem involves appraising alternative circuit designs using criteria such as cost, power consumption, and parts count.
- Charge (q) and energy (w) are the basic physical variables involved in electrical phenomena. Current (i), voltage (v), and power (p) are the derived variables used in circuit analysis and design. In the SI system, charge is measured in coulombs (C), energy in joules (J), current in amperes (A), voltage in volts (V), and power in watts (W).
- **Current** is defined as dq/dt and is a measure of the flow of electrical charge. Voltage is defined as dw/dq and is a measure of the energy required to move a small charge from one point to another. Power is defined as dw/dt and is a measure of the rate at which energy is being transferred. Power is related to current and voltage as p = vi.
- The **reference marks** (arrows and plus/minus signs) assigned to a device are reference directions, not indications of the way a circuit responds. The actual direction of the response is determined by comparing the reference direction and the algebraic sign of the answer found by circuit analysis using physical laws.
- Under the **passive sign convention**, the current reference arrow is directed toward the terminal with the positive voltage reference mark. Under this convention, the device power is positive when it absorbs power and is negative when it delivers power. When current and voltage have the same (opposite) algebraic signs, the device is absorbing (delivering) power.

# EN ROUTE OBJECTIVES AND ASSOCIATED PROBLEMS

## ERO 1-1 electrical symbols and units (sect. 1-2)

Given an electrical quantity described in terms of words, scientific notation, or decimal prefix notation, convert the quantity to an alternate description.

- 1–1 Write the following statements in symbolic form:
  - (a) twelve milliamperes
  - (b) four hundred fifty five kilohertz

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- (a) 66 kHz (b) 10 krad/s
- (c) 2.02 MW
- (d) 2.02 J
- (e) 1.1 MΩ
- **1–3** Express the following quantities using appropriate engineering prefixes (i.e., state the numeric to the nearest standard prefix).
  - (a) 2,200,000 volts
  - **(b)**  $2300 \times 10^{-9}$  farads
  - (c) 6200 ohms
  - (d)  $7.52 \times 10^5$  joules
  - (e) 0.000235 henrys
- 1-4 Commercial electrical power companies measure energy consumption in kilowatt-hours, denoted kWh. One kilowatt-hour is the amount of energy transferred by 1 kW of power in a time period of 1 hour. During a one-month time period, a power company billing statement reports a user's total energy usage to be 2128 kWh. Determine the number of joules used during the billing period.
- 1–5 Fill in the blanks in the following statements.
  - (a) To convert capacitance from microfarads to picofarads, multiply by \_\_\_\_\_.
  - (b) To convert resistance from kilohms to megohms, multiply by
  - (c) To convert current from amperes to milliamperes, multiply by

(d) To convert power from watts to megawatts, multiply by \_\_\_\_

- **1-6** The decibel (dB) is one tenth of a bel. The number of dB in a power ratio is defined to be 10  $\log_{10}(p_2/p_1)$ . Signal power levels are often specified in dBm, which is the number of decibels referenced to a power level of  $p_1 = 1$  milliwatt.
  - (a) Express the following power levels in dBm: 60 W, 10  $\mu$ W, and 8 kW.
  - (b) Express the following power levels in W: 60 dBm, −35 dBm, and 200 dBm.

#### ERO 1-2 CIRCUIT VARIABLES (SECT. 1-3)

Given any two of the three signal variables (i, v, p) or the two basic variables (q, w), find the magnitude and direction (sign) of the unspecified variables.

- 1-7 An ampere-hour (Ah) meter measures the integral, with respect to
- time, of the current in a conductor. During a 12-hour period a certain meter records 3000 Ah.
  - (a) Find the number of coulombs that flowed through the meter during the recording period.
  - (b) Find the average current during the recording period.

- (c) two hundred picoseconds
- (d) two thousand one hundred volts
- 1–2 Write out the meaning of the following symbols in words:
  - (a) 66 kHz
  - (b) 10 krad/s
  - (c) 2.02 MW
  - (d) 2.02 J
  - **(e)** 1.1 MΩ
- 1-3 Express the following quantities using appropriate engineering prefixes (i.e., state the numeric to the nearest standard prefix).
  - (a) 2,200,000 volts
  - **(b)**  $2300 \times 10^{-9}$  farads
  - (c) 6200 ohms
  - (d)  $7.52 \times 10^{5}$  joules
  - (e) 0.000235 henrys
- 1-4 Commercial electrical power companies measure energy consumption in kilowatt-hours, denoted kWh. One kilowatt-hour is the amount of energy transferred by 1 kW of power in a time period of 1 hour. During a one-month time period, a power company billing statement reports a user's total energy usage to be 2128 kWh. Determine the number of joules used during the billing period.
- 1–5 Fill in the blanks in the following statements.
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  - (c) To convert current from amperes to milliamperes, multiply by
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### ERO 1-2 CIRCUIT VARIABLES (SECT. 1-3)

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#### PROBLEMS

**1-8** Figure P1-8 shows a plot of the net positive charge flowing in a wire versus time. Sketch the corresponding current during the same time period.

1-9 The net positive charge flowing through a device varies as q(t) =

 $3t^2$  C. Find the current through the device at t = 0 s, t = 1 s, and t = 3 s.

1-10 The current through a device is given by i(t) = 0.05t A. How many coulombs enter the device between time t = 0 s and t = 5 s?

**1–11** G Figure P1–11 shows a plot of the net positive charge flowing in a wire versus time. Sketch the corresponding current during the same time period.





FIGURE P1-11

1–12 An incandescent lamp absorbs 75 W when connected to a 120-V source.

- (a) Find the current through the lamp.
- (b) Find the cost of operating the lamp for 8 hours when electricity costs 7.2 cents/kW-hr.
- 1–13 The 12-V automobile battery in Figure P1–13 has an output capacity of 200 ampere-hours (Ah) when connected to a headlamp that absorbs 50 watts of power. Assume the battery voltage is constant.
  - (a) Find the current supplied by the battery.
  - (b) How long can the battery power the headlight?

**1–14** A certain semiconductor material carries a current density  $4 \times 10^4$  A per square meter (m<sup>2</sup>) and has  $5 \times 10^{22}$  free electrons/m<sup>3</sup>. Find the average velocity of the electrons flowing in the material. *Hint:* the charge on an electron is  $1.6 \times 10^{-19}$  C.

1–15 A total of 50 ampere-hours are supplied to a 12-V battery during recharging. Determine the number of joules supplied to the battery. Assume battery voltage is constant.

1–16 The voltage between two points is 100 V. How much energy is required to move a charge of 3 µJ from one point to the other?

- 1–17 A two-terminal device has the current versus voltage characteristics shown in Figure P1–17. Assuming the passive sign convention,
  - (a) Determine the device power when it operates at point A and state whether the device is absorbing or delivering power.
  - (b) Repeat part (a) when the device operates at points B and C.
  - (c) Identify the operating point on the curve (it could be a point other than A, B, or C) at which the device delivers the most power.



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#### FIGURE P1-17



- **1–18** Under the passive sign convention, the current through and voltage across a two-terminal device are i = 20 mA and v = 5 V.
  - (a) Find the device power and state whether the device is absorbing or delivering power.
  - (b) Repeat (a) when i = -20 mA.
- **1–19** The voltage across a two-terminal device is always 15 V. The maximum power the device can dissipate is 0.5 W. Determine the maximum current magnitude allowed by the device power rating.
- **1–20** A laser produces 10-kW bursts of power that last 20 ns. If the burst rate is 50 bursts per second, what is the average power in the laser's output?
- **1–21** Two electrical devices are connected at an interface, as shown in Figure P1–21. Using the reference marks shown in the figure, find the power transferred and state whether the power is transferred from A to B or B to A when

(a) $v = +12$ V and	<b>(b)</b> $v = -33$ V and
i = -2.2  A	i = -1.2  mA
(c) $v = +15$ V and	(d) $v = -37.5$ V and
i = +40  mA	i = -43  mA
00	

**1–22** Figure P1–22 shows an electric circuit with a voltage and current variable assigned to each of the six devices. The device signal variables are observed to be as follows:

	DEVICE1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5	DEVICE 6
v	+15 V	+10 V	?	+15 V	2	7
i	-3 A	?	+0.5 A	+2.5 A	+3.5 A	-1 A
p	?	+30W	+2.5 W	?	-35 W	+10 W





#### FIGURE P1-22



Find the unknown signal variable associated with each device and state whether the device is absorbing or delivering power. Use the power balance to check your work.

**1–23** Figure P1–23 shows an electric circuit with a voltage and current variable assigned to each of the eight devices. The device signal variables are observed to be as follows:

	DEVICE 1	DEVICE 2	DEVICE 3	DEVICE 4	DEVICE 5	DEVICE 6	DEVICE 7	DEVICE 8
v	+12 V	?	?	-12 V	+12 V	+8 V	?	+1 V
ī	+2 A	-4 A	-8 A	-2 A	2 A	?	2 A	?
p	?	+32 W	-160 W	2	?	48 W	+6 W	+2 W

Find the unknown signal variable associated with each device and state whether the device is absorbing or delivering power. Use the power balance to check your work.



#### FIGURE P1-23

- **1–24** The voltage across and current through a two-terminal device are  $v(t) = 5e^{-100t}$  V and  $i(t) = -5e^{-100t}$  mA.
  - (a) Find the power delivered by the device at t = 0, t = 5 ms, and t = 25 ms.

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- (b) Find the total energy delivered by the device during the interval  $0 < t < \infty$ .
- **1–25** The voltage across and current through a two-terminal device are  $v(t) = 120 \cos 1000t$  V and  $i(t) = 4 \sin 1000t$  A.
  - (a) Show that the device absorbs power during some time intervals and delivers power during others.
  - (b) Find the maximum power absorbed by the device and the maximum power delivered by the device. *Hint*:  $\cos x \sin x = \frac{1}{2} \sin 2x$ .

# CHAPTER-INTEGRATING PROBLEMS

## 1-26 POWER RATIO IN dB

In complete darkness the voltage across and current through a twoterminal light detector are +5.6 V and +8 nA. In full sunlight the voltage and current are +0.9 V and +4 mA. Express the light/dark power ratio of the device in decibels (dB). (See Problem 1–6 for the definition of a decibel.)

#### 1-27 INCREMENTAL SENSITIVITY

Incremental sensitivity is an indication of the smallest change that can be detected by a measuring device or system. It is defined as the change in output indication caused by a specified change in the input stimulus. A temperature measuring system produces outputs of 1 mA and 2.5 V when the input temperature is 10°C. At 200°C the outputs are 5 mA and 7.5 V. Determine the sensitivity of the system in mW per °C.

#### 1-28 STORAGE BATTERY EFFICIENCY

The ampere-hour efficiency of a storage battery is the ratio of its ampere-hour output to the ampere-hour input required to recharge the battery. A certain 24-V battery has a rated output of 500 ampere-hours. When the battery is completely drained, a battery charger must deliver 65 A for 8 hours to recharge the battery. Assume the battery voltage is constant.

(a) Determine the ampere-hour efficiency of the battery.

(b) Determine the number of joules required to recharge the battery.