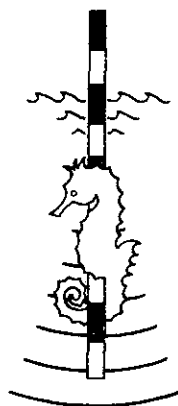


Ex. PGS 1029

THE HYDROGRAPHIC JOURNAL



incorporating
THE CORPORATE MEMBERS DIRECTORY
and
THE HYDROGRAPHIC JOURNAL INDEX

No. 77

JULY 1995

Ex. PGS 1029

The Hydrographic Journal

Number 77

July 1995

The Hydrographic Society:
The Hydrographic Society is a
UK Registered Charity
Number 285013.
A Company Limited by
Guarantee.
Registered in England and
Wales Number 1638304.
Registered Office:
University of East London,
Longbridge Road, Dagenham,
Essex RM8 2AS.

Journal Subscription:
Subscriptions for
Non-Members:
£40 per calendar year
post paid
£10 per single copy
post paid
N.B. despatch by deferred air
mail outside the UK.

Articles

3. The Royal Naval Hydrographic Service 1795-1995
by R.O. Morris
11. A Rigorous and Integrated Approach to Hydrophone
and Source Positioning during Multi-Streamer
Offshore Seismic Exploration
by V. Gikas, P.A. Cross and A. Asiama-Akuamoa
27. Limnology and Hydrographic Survey: The Example
of Lake Malawi
by C.G.C. Martin

Regular Features

33. News from Industry
39. Literature
42. Letters to the Editor
45. The Chairman's Column
by KWK
46. Rhumb Lines - *Personal Views by Sinbad*
48. Reflections
by M. Boreham
50. Review
60. Principal Officers and Council Members

Obituary

43. John Barry Dixon

Special Features

53. Memorandum of Association of the Hydrographic
Society
 54. Articles of Association of the Hydrographic Society
 58. Regulations for the Establishment and Conduct of
National Branches of the Society
 59. Regulations for the Establishment and Conduct of
Regions of the Society
- The Corporate Members Directory 1995-96
The Hydrographic Journal Index 1972-1995

Information Bulletin

- i. Special Announcements
- i. Membership News
- ii. From the Regions
- iv. News from FIG Commission 4
- iv. News from the International Hydrographic Bureau
- v. General Notices
- vii. Employment Wanted
- vii. Calendar
- viii. Society Information

Editor:
J.A. Kitching

**Institute of Marine Studies,
University of Plymouth,
Plymouth PL4 8AA,
United Kingdom,
Tel: 01752 232423
Fax: 01752 232406**

Editorial Board:
E.M. Bradley
D. Goodfellow
J.A. Kitching
Branch representatives:
M.A.B. Jones (*Australasian*)
H.N.B. Roels (*Benelux*)

The opinions expressed in the Journal
are those of the individuals concerned
and are not to be taken as the views of
the Hydrographic Society.

© 1995 - The Hydrographic Society.

The Hydrographic Society hereby grant
permission to photocopy, free of royalty,
provided that such copies are for the
personal use of the copier or for the
personal use of a library client. Such
permission does not extend to copying
for general distribution, advertising or
promotional purposes, resale, or the
creation of new collective works.

Printed by:
Peter Howell & Co.
The Printing Press,
21 Clare Place, Coxside,
Plymouth PL4 0JW,
United Kingdom.
Tel: 01752 250580
Fax: 01752 223855

A Rigorous and Integrated Approach to Hydrophone and Source Positioning during Multi-Streamer Offshore Seismic Exploration

V. Gikas*, P.A. Cross* and A. Asiama-Akuamoa**

Abstract

This paper describes a rigorous and integrated approach for positioning sources and hydrophones within a seismic spread that may contain multi-vessel and multi-streamer configurations. Any number of observations relating to any point(s) within the spread can be accommodated. Quantification and analysis of error propagation within the spread are provided. Test results based on the implementation of the algorithm on a UNIX platform are discussed.

Résumé

Cet article donne la description d'une approche rigoureuse et cohérente des sources de positionnement et des hydrophones dans un déploiement sismique que les configurations multi-vaissaux et multi-streamer peuvent englober. On peut utiliser un nombre indifférent d'observations relatives à n'importe quel(s) point(s) compris dans le déploiement. La qualification et l'analyse de la propagation d'erreurs à l'intérieur du déploiement y est donnée et les résultats des tests basés sur l'exécution de l'algorithme sur une plateforme UNIX y sont discutés.

Resumen

Este artículo describe un enfoque riguroso y armonioso sobre las fuentes de posicionamiento e hidrófonos dentro de un despliegue sísmico que las configuraciones multi-buque y multi-streamer pueden contener. Se puede usar cualquier número de observaciones referentes a un punto(s) cualquiera del interior del despliegue. El artículo proporciona la cuantificación y el análisis de la propagación de errores dentro del despliegue y discute los resultados de las pruebas, basados en la implementación del algoritmo en una plataforma UNIX.

1. Introduction

The basic configuration of an offshore seismic exploration survey is as follows. One or more vessels sail in approximately straight lines whilst towing a number of 'streamers' (typically up to 6 kilometres long) and 'seismic sources'. The streamers carry a number of hydrophones (typically 50-100 per kilometre) and are towed just below the surface of the water [Morgan, 1992]. At a specified distance interval (typically every 20-25 metres) one of the guns is fired resulting in seismic waves which travel through the water and penetrate the subsurface. The times of arrival of the reflected and/or refracted signals are then measured by the hydrophones. The surveying problem is to determine the position of the guns and hydrophones at the instants of firing and reception respectively. In principle the position of the vessel is of no interest - except, of course, for navigation.

In recent years the problem has become increasingly complex, mainly due to an expansion of the type and quantity of survey data collected. In a typical configuration, Fig. 1.10, measurements will include compass orientations at points along the streamer (typically 4-7 per kilometre), laser ranges from the vessel to a variety of floats (for instance those carrying the guns and those at the front of the streamer), underwater acoustic measurements (of the distance) between a number of points at the front and back of the system (referred to as the 'front-end' and 'rear-end' acoustic networks), the position of the tailbuoy and the position of the vessel (both typically, but not necessarily, by

DGPS). More complicated systems may also include acoustics throughout the length of the streamer and additional navigation devices on the vessel. Moreover, in the case of several vessels operating simultaneously, between vessel measurements would also be made.

The most common approach currently applied to the positioning problem is to treat each epoch, and each measuring system, more or less independently. So both the laser and acoustic measurements are used to transfer the position of the vessel to the floats, while the front-end acoustics relate the floats to the guns and front-end of the streamer, and then the compasses determine the streamers shape. The rear-end acoustics and the tailbuoy positioning serve to provide some control of the orientation and stretch of the streamers. Typically the process will involve some sort of curve fitting operation for the compasses [Ridyard, 1989], and several independent 'network adjustments' for the acoustic and laser networks. It is possible that the process will involve 'iterating' several times through the various data types in order to 'best fit' (in some rather general sense) all of the measurements.

Although this approach is probably perfectly satisfactory from an accuracy point of view it suffers from two major disadvantages. Firstly it is highly 'case dependent', i.e. relatively small changes to the configuration or measurement set may lead to major changes in the processing software - something that is especially difficult in real-time (or quasi real-time) quality control. Secondly, and probably most importantly, it is extremely difficult to analyse the error

*University of Newcastle upon Tyne, UK.

**QCTools Ltd, Houston, USA.

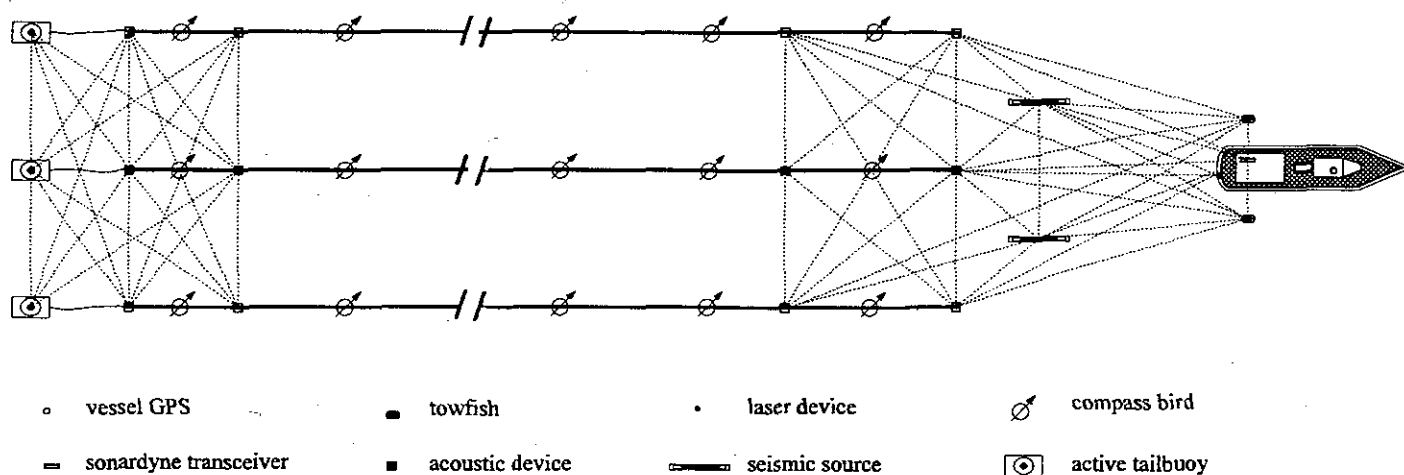


Fig. 1.10: Typical dual source triple streamer configuration

propagation through such a process - hence it is almost impossible to describe the precision and reliability of the final gun and hydrophone positions. This aspect is becoming increasingly important as clients require proof (often in real-time) that the survey specifications are being met.

There is hence a need to develop a completely general (for flexibility purposes) and rigorous (for error propagation purposes) approach to the positioning of guns and hydrophones during seismic exploration and this paper is an attempt to address that need. It describes the mathematical basis, implementation and testing of a Kalman filter that can in principle, handle any geometrical configuration (i.e. any number of vessels, streamers and guns) and any set of observations.

Kalman filters have, in the past, not proved popular with the offshore positioning community and most offshore operators currently prefer simple and independent 'epoch by epoch' least squares computations. For this reason a brief review of the advantages of using a Kalman filter is included before describing the models used in detail.

1.1. Kalman filtering versus simple least squares

Kalman filtering has the following specific advantages over simple 'epoch by epoch' least squares and it is in order to exploit these fully that Kalman filtering was selected as the basic stochastic process behind the unified solution presented in this paper.

1. Simple least squares treats each epoch independently. This means that it does not use knowledge of the motion of the system. Often, and especially in seismic work, it is possible to make a very accurate prediction of where the network will be at any epoch using just the previous position and the estimated configuration motion. Not using this 'knowledge of motion' is effectively discarding information and leads to poorer quality results than those obtainable from a properly tuned Kalman filter. In the past (and sometimes today) poorly tuned filters were used and in this case results might be worse - simply because the system motion may have not been well determined and/or not used properly in the estimation process. So simple least squares is a safe option - but it does not have the potential accuracy of Kalman filtering. The challenge,

of course, is to tune the filter properly in real time - and the fact that some have failed to do this in the past has led to Kalman filtering gaining a poor reputation in some circles.

2. The use of a Kalman filter for a highly complex seismic configuration enables a rigorous computation of precision and reliability measures such as error ellipses and marginally detectable errors respectively [Cross *et al.*, 1994]. If a step-by-step approach is adopted (such as curve fitting the compass data followed by fitting the results to the acoustics and then to the navigation data) it is almost impossible to compute these measures.
3. Due to its ability to predict the network, a Kalman filter is a far more powerful tool than simple least squares for quality control. Much smaller outliers and biases can be found by Kalman filtering than by simple least squares. It is, however, recommended that, where possible, simple least squares also be carried out at every epoch in order to identify (and correct or remove) the larger outliers. This is because Kalman filtering can be rather time-consuming from a computational point of view and any initial cleaning that can be done by other methods will increase its efficiency.
4. Kalman filtering is able to solve for small biases that will remain in the data if only epoch by epoch processing is used - such as drifts in gyros and (C-O)s in terrestrial (shore-based) ranging systems. These look like noise in simple least squares and can easily go undetected. A lot can be learnt by looking at the time variation of the data. Of course, in principle this could be done in simple least squares by analysing time series of residuals but it would be hard to do this in real time - and hard to feed back any findings into the system.
5. Because it can determine and use the system motion, Kalman filtering is able to use observations that do not completely define the system - i.e. GPS data from just two satellites could be used to update a vessel position. Of course, long periods of such data would lead to a significantly degraded result.
6. A Kalman filter can accept data as and when it is measured. With simple least squares, data has to be

reduced to a specified epoch. Therefore, a Kalman filter can cope well with data arriving as a more or less continuous stream.

7. The Kalman filter regime is highly suited to the mixing of varied data types [Celik and Cross, 1994], when poor satellite geometry leads to poor positions in a DGPS-only solution, the introduction of data from a gyro carried by the vessel can make a major improvement. It would not be possible to combine these data types in simple least squares because, for an individual epoch the gyro does not give any positional information.

2. Streamer modelling

Since the compasses and other measuring devices are not co-located with the hydrophones it is necessary, in any approach, to have a mathematical model that describes the shape of each streamer. Moreover, because of the numerous hydrodynamic forces acting on the cable in the underwater environment, the cable shape is likely to be significantly distorted from a nominal straight line – so a simple linear model is very unlikely to be sufficient. To estimate this distorted shape two alternatives can be considered.

In the first approach a physical model of the hydrodynamic forces acting on the cable could be used to derive equations which describe the streamer shape. It is known that tension forces due to the vessel pull, and drag forces due to the resistance of the cable through the water, determine its three dimensional shape. Any change in the vessel's speed and any fluctuation in the sea waves, or those generated by the vessel, the wind load or the water currents, would mean changes in the towing tension and drag forces respectively. Such a model can only be applied when these external forces acting on the cable are known with a reasonable accuracy [Krail and Brysk, 1989]. It should be stressed, however, that, even if these quantities were known, a system of several streamers and floats would lead to models that would be too complicated and inflexible for the construction and implementation of a practically useful positioning algorithm. It is therefore unlikely that, although they have been used for vessel motion, [Cross and Pritchett, 1986], hydrodynamic models will be adopted for positioning purposes in the foreseeable future.

The other way to tackle this problem is to consider an 'empirical' numerical approach in which the solution to the problem is deduced by adopting a 'model curve' that best fits the observed data.

2.1 Curve-fitting procedures

Several numerical methods can be adopted to obtain the streamer shape. The simplest one is to consider the cable as a straight line which follows exactly the ship's track. Although this approach would be very simple in practice, significant differences from the final expected position may result, not only because of the angle between the ship's track and the cable's baseline, but also because of the 'deformed' shape of the cable.

A more efficient way to address this problem might be to use a mathematical function such as a cubic spline. However, even though a cubic spline gives a curve which is continuous and continuously differentiable, and one which is capable of fitting the data very closely, it is not the best solution of the problem. This is because its coefficients vary along the length of the cable (i.e. the streamer shape is not represented by a single function) and its incorporation into a single operational system, which is the aim of this study, is

extremely difficult. Moreover, because the cubic spline is technically capable of representing faithfully each compass reading, it is hyper-sensitive to compass errors leading to the possibility of a completely unrealistic final curve.

Alternative curve fitting models include least squares polynomial approximation and the use of harmonic functions. This work concentrates on the former. Using least squares polynomials leads to a curve which describes the complete streamer's shape using only one set of coefficients, and furthermore the resultant curve is continuous and continuously differentiable at every point of the cable [Douglas, 1980 and Owsley, 1981]. As a result, this method can be incorporated much more easily in a unified recurrent process such as a Kalman filter.

Variations of the foregoing are also possible in practice for instance Ridyard, (1989), has suggested the use of a 'rolling quadratic' algorithm in which a series of individual quadratics are used to fit a small group of compasses. This algorithm is clearly very effective and this, and similar approaches have been widely adopted within the industry. Whilst they may be very powerful interpolation devices, and whilst they may be very effective in sorting out outliers and highlighting problems, they cannot be easily adopted in the unified approach presented here. This is because (as was the case of the cubic spline) the approach demands the use of a single function to describe the streamer shape.

Hence in this study a 'n-order' polynomial shape model has been utilised. Such 'single' polynomials are not popular in some sections of the exploration industry so their use needs also to be justified from an accuracy point of view (it is clearly not a sufficient argument to use them just because they are convenient), and a series of tests have been carried out in order to do this. These tests involved the fitting of a series of polynomials, of a variety of orders, to real compass data and comparing the results with those obtained from the universally accepted rolling quadratic method. The mathematics and the results are described in the next section.

2.2 Testing of the polynomial approximation

In these tests the only information used is that derived from the magnetic compasses fixed along the length of the cable. This was done in order to compare the polynomial approach with the accepted method (which treats such data independently of any other). In such a case the final accuracy of a streamer position is a function of raw compass data, the local magnetic declination, individual compass corrections and the algorithm used for processing the data. The polynomial observation equations can be written as:

$$B_i = a_0 + a_1 l_i + a_2 l_i^2 + \dots + a_n l_i^n \quad (1)$$

where:

- B_i : is the compass reading
- l_i : is the offset of the i-th compass from its reference point
- a : is the polynomial coefficient

The solution of this equation system, using a least squares method, gives the values of the polynomial coefficients. If we consider the geometrical configuration to be as shown in Fig. 2.10, we have:

$$\theta_{(rad)} = \text{atan}(dv / du) \cong \text{atan}(dv / dl) \quad (2)$$

which for any θ in $(-1^\circ, 1^\circ)$ becomes:

$$\theta_{(rad)} \cong \tan \theta = dv / du \cong dv / dl \quad (3)$$

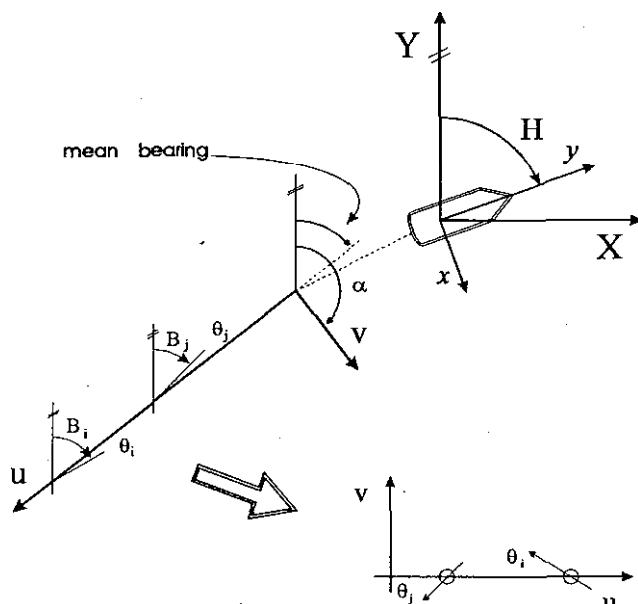


Fig. 2.10: Streamer modelling

Also for:

$$\begin{aligned} B_i > B: B_i &= B + \theta = B + (dv / dl) \\ B_i < B: B_i &= B - \theta = B + (-dv / dl) \end{aligned} \quad (4)$$

Upon substituting equation (1) into equation (4) and integrating we find:

$$v = c_0 l + c_1 l^2 + \dots + c_n l^{n+1} \quad (5)$$

where:

$$c_k = a_k / (k+1), \text{ for } K = 0 \text{ to } n \quad (6)$$

The final coordinates X,Y can then be estimated by rotating these coordinates to the East, North coordinate system using:

$$\begin{aligned} X &= \cos(\alpha)l + \sin(\alpha)v \\ Y &= \sin(\alpha)l - \cos(\alpha)v \end{aligned} \quad (7)$$

As already stated, in order to test the feasibility of the algorithm in terms of correctness and efficiency, the foregoing method has been applied to a subset of real compass data. A full description of the survey configuration and the data set used is given in 4.2 – here it is just mentioned that the streamers were 3.1km long and contained 13 compasses each. The process was carried out for polynomial orders up to eight and for every shot point in the data set. A typical set of results is given in Fig. 2.20. Detailed analysis of hundreds of such sets of curves has led to the conclusion that polynomials of order either five or six fit the data extremely well. The following two general conclusions were also reached.

- Polynomials of order four or less do not describe faithfully the observations. In such cases the differences between the actual compass readings and those predicted by the polynomial can (in a few cases) exceed half a degree. This might be important given that, in practice, cable compass resolution (but not accuracy) can be as high as 0.1°.
- Polynomials of order greater than six can sometimes generate curves characterised by steep changes of

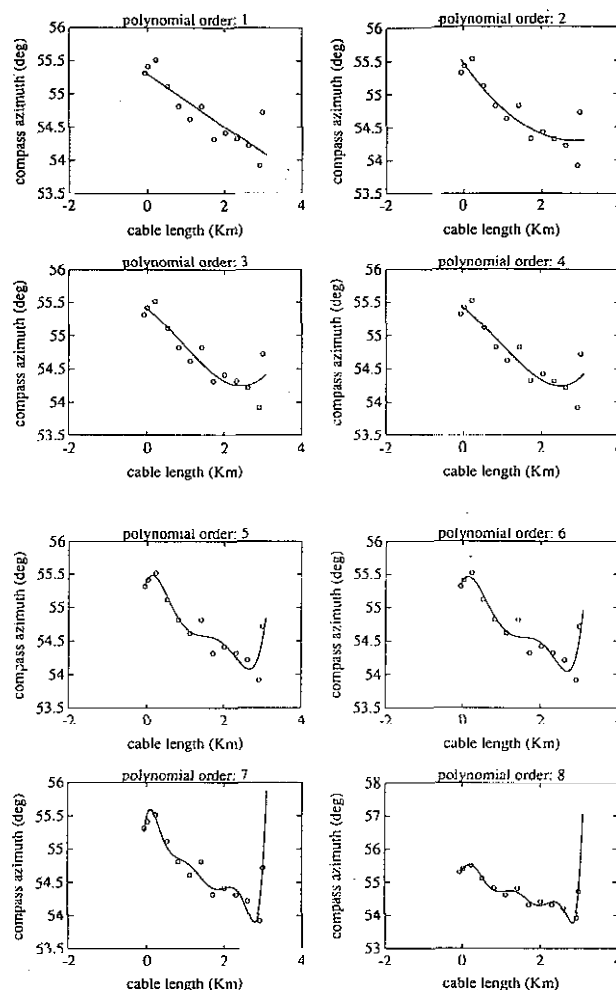


Fig. 2.20: Streamer modelling - Polynomial approximation

gradient, which may effect significantly the fidelity of the final coordinates. This phenomenon is particularly noticeable for compasses close to the tailbuoy.

After the polynomial coefficients had been determined, the Eastings and Northings with respect to the streamer reference point were obtained using equation (7). The differences between these coordinates and those obtained using a 'rolling quadratic' algorithm were then computed. An examination of these indicate that if a fifth or sixth order polynomial is used the maximum resultant differences are of the order of one metre – even for the groups of hydrophones in the far end of the cable.

From these tests it is evident that the use of a polynomial approximation is a highly realistic approach to the problem. Moreover, the method has the advantage of being easily incorporated into a Kalman filter model for real time positioning and quality control from mixed data sources. The n-order polynomial has hence been adopted as the streamer model in the mathematical system developed in this paper.

3. An integrated Kalman filter algorithm

It has already been stated that a rigorous filtering approach has been adopted in this study. What follows is a review of the basic models needed for such an approach and details of the implementation of these models for the positioning of seismic spreads.

3.1 Filtering techniques

The Kalman filter [Kalman, 1960] is probably the best known of the commonly used recursive algorithms for the estimation of the parameters of time-varying systems. It has constituted the framework for a unified and concise treatment of a broad range of filtering problems from electronic engineering to surveying and geodesy. As well as having the many practical advantages already referred to, Kalman filter estimates have the advantage of being least squares estimators, which can be shown [Cross, 1983], to be the best (in the minimum variance sense) within the complete class of linear unbiased estimators.

The implementation of the filter requires the specification of two mathematical models. The *measurement model* or primary model relates the state vector parameters to the observations, and the *dynamic model* or secondary model relates the parameters at epoch t_{i-1} to those at later epoch t_i .

If the state vector is denoted by x and the observations by L then the linearised form of a non-linear measurement model $F(x) = L$ is given by:

$$A_i x_i = b_i + v_i \quad (8)$$

where:

- A_i : is the design matrix given by $\partial F_i / \partial x_i$
- x_i : is the correction to the provisional value of the filtered state vector, $\hat{x}_{i(+)}$
- b_i : is the 'observed - computed' vector, given by $L_i - F_i(\hat{x}_{i(-)})$
- v_i : is the state vector residuals.

The filtered state $\hat{x}_{i(+)}$ is computed iteratively until there is no significant change in the provisional state $\hat{x}_{i(+)}$.

The dynamic model represents the behaviour of the system as it varies with time. The discrete linearised form is given by:

$$x_i = M_{i-1} x_{i-1} + y_{i-1} \quad (9)$$

where:

- M_{i-1} : is the transition matrix from time t_{i-1} to time t_i
- y_{i-1} : is the dynamic model noise from time t_{i-1} to time t_i

The operational equations for, and the recursive nature of, the Kalman filter are well known [Cross, 1983; Brown and Hwang, 1992]. Here emphasis is placed on the contents of the two models - once they have been specified then their implementation within a Kalman filter is, in principle, trivial.

It is, however, worth pointing out that the Kalman filter is not the only optimal (in the least squares sense) mathematical procedure. Another, slightly less well known, set of equations known as the Bayes filter can be used to produce absolutely identical results to those of the Kalman filter. The only difference between them is in the manner in which the so-called *gain matrix* is computed. The computation of the gain matrix in the Bayes filter involves an inversion of a matrix whose size is equal to the number of parameters of the state vector, whilst, in the Kalman form an inversion of a matrix whose size is equal to the number of observations is required. In this study the Bayes equations have actually been implemented since in a typical 3D marine spread a large number of observations contribute to a relatively small number of states. Many modern Kalman filtering software packages include routines for both sets of equations and make automatic (and invisible to the user) decisions as to which to use.

Despite sometimes using the Bayes equations, the words Kalman filter nevertheless appear to be almost universally used to describe the process.

Before describing the models in detail it is necessary to make some remarks about the coordinate systems that are used.

3.2 Coordinate systems

Within a typical seismic configuration there are several sub-systems that are able to move independently of each other, and of the vessel. These include every single float (gun array or any auxiliary reference station) and each streamer [Houtenbos, 1989]. Each sub-system must therefore have its own parameters and coordinate system - which must, in turn, be linked by the mathematical model in order to determine the complete configuration. Before defining the various state vector parameters for each one of the configuration subsystems it is necessary to describe their different coordinate systems.

An earth fixed geodetic system, involving latitude and longitude or a map projection system, is used to describe the final positions of all of the points of interest. The vessel and tailbuoys absolute positions, derived by GPS/DGPS or a radio positioning system, will, of course, naturally be in this system but it is not especially convenient for describing the rest of the spread.

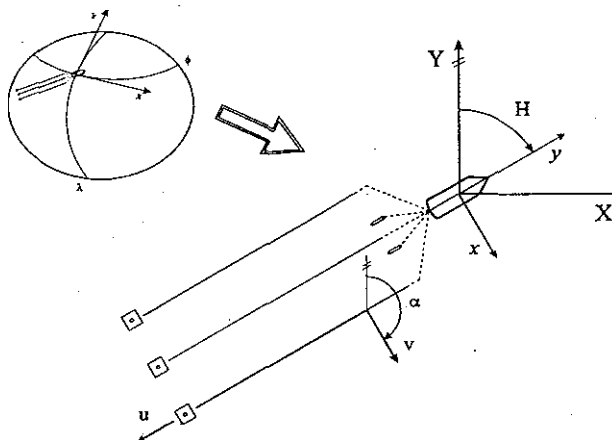


Fig. 3.10: Coordinate systems

For this it is more convenient to use a local topographic coordinate system. This system has its origin at the vessel navigation reference point with the X-axis aligned with the east direction and Y-axis aligned northwards. When necessary the Z-axis is defined as being perpendicular to the XY plane (i.e. upwards) such that the resultant coordinate system is right handed, as in Fig. 3.10. It is obvious that this system moves with respect to a geodetic earth system as the vessel's position changes. Also it is clear that, given the relatively short distances (a few kilometres) involved within the network, there will be minimal error in working with the computed distances and azimuths in the XY (horizontal) plane and then using a direct geodetic formulation to determine the coordinates (latitude and longitude) of the points of interest - i.e. the earth is effectively considered to be flat within the region of the seismic spread.

Some of the available observations are made relative to devices fixed on the vessel. For this reason it is necessary to define another coordinate system that is attached to the vessel. The origin of this coordinate system coincides with the navigation reference point. Its y axis is aligned with the

vessel's bow-stern direction. Its x axis (starboard) is in the horizontal plane and perpendicular to the y axis whilst z axis is defined to be perpendicular to xy plane (upwards) – see Fig. 3.10.

Finally, in order to estimate the position of any point on each of the streamers, taking into account its distance from the streamer reference point 1 as a parameter, it is necessary to introduce another local coordinate frame for each streamer in the spread. A set of three dimensional coordinate systems (u, v, z) is therefore defined. Each has its origin at the head of the first active section of the streamer, or any other point of known offset, its u axis aligned with the base course of the cable (as results from the Kalman filter computations) and its v axis perpendicular to the u -axis and pointing to the starboard side. The z axis is defined such that the resultant coordinate system is right-handed – see Fig. 3.10.

3.3 Kalman filter functional models

3.3.1 State vector

In order to implement a Kalman filter, we must first define, in general terms, the minimum number of individual and (determinable) parameters (or unknowns) necessary to describe the complete system – this is known as the state vector. In the case of an offshore seismic survey the unknowns consist of those which describe the vessel's position and the motion and those which describe the position and motion of each subsystem. In the following, the unknown parameters are classified by subsystem.

Vessel unknowns

The unknown parameters that describe the vessel position and motion are defined to be the instantaneous values of the following elements:

- ϕ, λ : the geodetic 'ellipsoidal' coordinates of the ship reference point
- $\dot{\phi}, \dot{\lambda}$: the instantaneous velocity of this point.
- c : the crab angle, i.e. angle between course made good and vessel's heading (Figure 3.20).

Note that for many navigation applications it would also be necessary to define the acceleration of the vessel in the

state vector but the almost straight line motion associated with seismic surveying makes this unnecessary in this case.

Float unknowns

The unknown parameters for any tow points attached to the vessel are also included in the state vector. Tow point positions are defined as position vectors expressed in X, Y coordinates along with their velocity components \dot{X}, \dot{Y} with respect to the local topographic coordinate system. It should be stressed here that to date the filter has only been implemented in the XY (horizontal). The (known) Z coordinates of all components, are taken into account by making geometrical 'corrections' to the observations, i.e. observations are corrected to the values they would have had the whole system been in the XY plane. Also, it is important to note that the unknown coordinates X, Y refer to the centres of the floating arrays. It will usually be necessary to correct observations to these centre points.

Streamer unknowns

The streamer unknown parameters must clearly refer directly to the streamer model. For the purposes of this study a polynomial model has been adopted. Hence, the u, v coordinates of any point on a streamer are given by the following equations:

$$u = 1 \quad (10)$$

$$v = c_2 l^2 + c_3 l^3 + \dots + c_n l^n \quad (11)$$

Testing of the integrated algorithm using real data showed that (perhaps not surprisingly) coefficient c_0 must be null since, by definition, v is zero at the head of the cable (i.e. when $l = 0$). Also the c_1 coefficient (which is directly related to the overall orientation of the streamer) is redundant in the state vector since the orientation of the u, v system, the direction angle α , in Figs. 3.10 and 3.20, is considered to be an unknown in the system. Therefore, the streamer parameters consist of the polynomial coefficients c_i , the direction angle α of the v axis and the streamer's coordinate reference system origin X, Y along with its velocity components \dot{X}, \dot{Y} .

VESSEL	1	ϕ
		λ
		$\dot{\phi}$
		$\dot{\lambda}$
		c

FLOAT	1	ORIGIN	X_{f1} Y_{f1} \dot{X}_{f1} \dot{Y}_{f1}
	\vdots		
	m1	ORIGIN	X_{fm1} Y_{fm1} \dot{X}_{fm1} \dot{Y}_{fm1}

STREAMER	1	ORIGIN	X_{s1} Y_{s1} \dot{X}_{s1} \dot{Y}_{s1}
		ORIENTATION	α_{s1}
		POLYNOMIAL COEFFICIENTS	c_{s12} \vdots c_{s1n}
	\vdots		
	m2	ORIGIN	X_{sm2} Y_{sm2} \dot{X}_{sm2} \dot{Y}_{sm2}
		ORIENTATION	α_{sm2}
		POLYNOMIAL COEFFICIENTS	c_{sm22} \vdots c_{sm2n}

Table 3.10: State vector elements

The complete state vector is summarised in Table 3.10. It is evident that the number of states to be estimated for every shotpoint depends on the number of floats and streamers that are utilised throughout the spread as well as on the polynomial order of the streamer model. Hence, for a configuration consisting of m_1 floats, m_2 streamers and for a n -order polynomial, the state vector dimension will be equal to $5 + 4(m_1 + m_2) + nm_2$, which for a typical spread of two sources and three streamers becomes equal to forty elements. The Kalman filter algorithms can easily provide a solution for a state vector of this size since within typical modern seismic configurations the total number of available observations is well over a hundred per shotpoint.

It is worth noting at this stage that the tailbuoy positions do not form part of the state vector. This is because a tailbuoy is treated as a simple extension of the streamer. It would be quite possible to include the tailbuoys in the system as independent points but they would not then be able to fulfil their primary role of providing overall orientation and scale control for the cables.

3.3.2 Observations

As has already been stated, in a modern marine seismic survey several measurement devices are employed to position the various points of interest throughout the spread. The most commonly used devices include magnetic compasses, laser systems, long, short or ultra short baseline acoustic devices, terrestrial radio ranging systems (e.g. Syledis, Hyperfix, ARGO) and differential navigation systems as GPS [Chevron Training Course, 1992; Kelland, 1994]. Regardless whether the actual observations are measured time or phase differences between any two devices in the network or between any device and any shore or satellite station, basic observation types in this study reduce to slope ranges, bearings and bearing differences and the absolute geodetic position of certain nodes in the network. In particular observations equations are formed for the following measurements:

- Vessel geodetic position.
- Vessel gyro.
- Slant acoustic and laser ranges between vessel, sources, streamers and miscellaneous hardware.
- Directions between vessel, sources, streamers and other auxiliary assemblies.
- Float and tailbuoy absolute position.
- Compass bearings along the streamer.

3.3.3 Observation equations

Once the different observation types have been specified and the state vector parameters have been explicitly defined, the measurement functional model can be set up in the form of the observation equations.

Observation equations are simply mathematical representations of the underlying physical and geometric relationships between the measured quantities and the parameters. Note that this stage of the process is crucial in the sense that any mistakes in the formulation of these equations, even seemingly small, will lead to an incorrect design matrix (the matrix A in equation (8)) and small errors in the final solution that, in general will not be easy to detect. Bearing in mind that for the purposes of this study local topographic coordinates have been selected for the computational model, the observation equations for each measurement discussed in 3.3.2 can now be presented.

Vessel geodetic position

Since the vessel's geodetic position is itself an unknown, the observation equations can be written as:

$$\begin{aligned}\varphi_v &= \varphi + v_\varphi \\ \lambda_v &= \lambda + v_\lambda\end{aligned}\quad (12)$$

where:

- φ_v, λ_v : are the unknown ellipsoidal coordinates
- φ, λ : are the observed ellipsoidal coordinates of the vessel
- v_φ, v_λ : are the measurement residuals

However, it should be noted that, if the sensor that provides the vessel's position is located sufficiently far away of the navigation reference point it will be necessary to correct the observation to this point using the general formula that given by equations (18), (19) and (21).

Vessel gyro

The output from the vessel's gyro is essentially the azimuth of the vessel and it can be related, through Fig. 3.20, to the velocity of the vessel via the crab angle, c , as follows:

$$\tan^{-1} \left[\frac{\dot{E}}{\dot{N}} \right] + c = H + v_H \quad (13)$$

or

$$\tan^{-1} \left[\frac{v \cos(\varphi) \dot{\lambda}_v}{\rho \dot{\varphi}_v} \right] + c = H + v_H \quad (14)$$

where:

- H : is the observed gyro measurement.
- \dot{E} : is the instantaneous easterly velocity of the vessel
- \dot{N} : is the instantaneous northerly velocity of the vessel
- c : is the crab angle of the vessel
- v_H : is the measurement residual of the gyro

and:

$$v = \frac{a}{[1 - e^2 \sin^2(\varphi)]^{1/2}}, \quad \rho = \frac{a(1 - e^2)}{[1 - e^2 \sin^2(\varphi)]^{3/2}}, \quad e^2 = \frac{a^2 - b^2}{a^2} \quad (15)$$

with:

- a : is the semi-major axis of the reference ellipsoid
- b : is the semi-minor axis of the reference ellipsoid

Slant ranges and bearings

The standard observation equations for a measured distance and azimuth between any two nodes, i and j , in the network are given by the following expressions:

$$\sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2} = D_{ij} + v_{D_{ij}} \quad (16)$$

$$\tan^{-1} \left[\frac{X_j - X_i}{Y_j - Y_i} \right] = A_{ij} + v_{A_{ij}} \quad (17)$$

where:

- X_i, X_j : are the easting components of stations i and j
- Y_i, Y_j : are the northing components of stations i and j
- Z_i, Z_j : are the distances of stations i and j from the XY plane
- D_{ij} : is the measured distance between stations i and j
- A_{ij} : is the measured or reduced azimuth between stations i and j

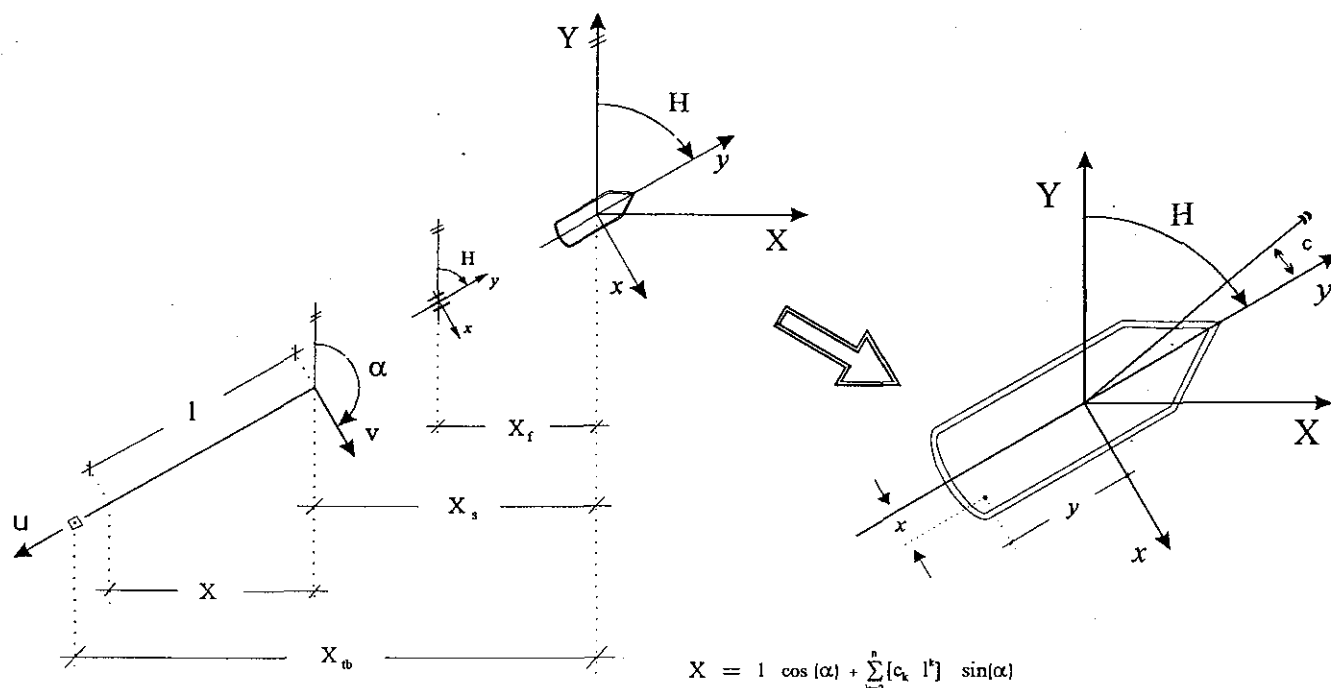


Fig. 3.20: Slant ranges and bearings observation equations

The Cartesian coordinates of the stations i and j are expressed in a different form in accordance to the subsystem they refer. Three different cases are considered here (Fig. 3.20).

If a station i is a point on the vessel (sonardyne, laser device, etc.) then:

$$\begin{aligned} X_i &= x \cos(H) + y \sin(H) \\ Y_i &= -x \sin(H) + y \cos(H) \end{aligned} \quad (18)$$

where:

x, y : are the coordinates of the device fixed on the vessel
 H : is the vessel's heading

Given that the vessel's heading, H , is not considered to be an unknown in the system it must be substituted in equation (18) by:

$$H = \tan^{-1} \left[\frac{v \cos(\phi_v) \dot{\lambda}}{\rho \dot{\phi}_v} \right] + c \quad (19)$$

so that the observation relates only to unknown parameters.

Similarly, if a station i is a device on a float array (gun, towfish, buoy etc.) the observation should first be corrected to the centre of the array using equations (18) and (19). In such a case it would be usual to assume that the orientation of any float structure coincides with that of the vessel gyro. This is justified by the relatively small dimensions of the arrays and the short distances involved within the network. In this case, the coordinates x, y in equation (18) would refer to the nominal offsets of the device measured from the centre of the float.

Finally, station i could be any point on a streamer. In this case, and in order to express X_i, Y_i coordinates as a function only of unknowns, we must refer directly to the streamer unknown parameters. Therefore, for the chosen streamer model of these equations are formed as follows:

$$\begin{aligned} X_i &= X_s + l \cos(\alpha) + \sum_{k=2}^n [c_k l^k] \sin(\alpha) \\ Y_i &= Y_s - l \sin(\alpha) + \sum_{k=2}^n [c_k l^k] \cos(\alpha) \end{aligned} \quad (20)$$

where:

X_s, Y_s : are the Cartesian coordinates of the streamer's reference point in the XY coordinate system
 α : is the instantaneous orientation of the streamer coordinate system u, v
 c_k : are the polynomial coefficients
 n : is the polynomial order
 l : is the offset of station i from the streamer's reference point

In fact ten different equations can be written, for both distance and azimuth observations, depending upon the subsystems to which stations i and j refer. Hence, and for example, the observation equation for a measured azimuth from the vessel's laser to a laser reflector fixed on a streamer is given by the following equation:

$$\tan^{-1} \left[\frac{X_s + 1 \cos(\alpha) + \sum_{k=2}^n [c_k l^k] \sin(\alpha) - \left\{ x \cos \left[\tan^{-1} \left(\frac{v \cos(\phi_v) \dot{\lambda}_v}{\rho \phi_v} \right) + c \right] + y \sin \left[\tan^{-1} \left(\frac{v \cos(\phi_v) \dot{\lambda}_v}{\rho \phi_v} \right) + c \right] \right\}}{Y_s - 1 \sin(\alpha) + \sum_{k=2}^n [c_k l^k] \cos(\alpha) - \left\{ -x \sin \left[\tan^{-1} \left(\frac{v \cos(\phi_v) \dot{\lambda}_v}{\rho \phi_v} \right) + c \right] + y \cos \left[\tan^{-1} \left(\frac{v \cos(\phi_v) \dot{\lambda}_v}{\rho \phi_v} \right) + c \right] \right\}} \right] = \Lambda_{ij} + v_{\Lambda_i}$$

It is important to note, that, in the case of any observed direction the measurement should be first reduced to an azimuth (bearing) and then be corrected for grid convergence before the observation equation is formed.

Float and tailbuoy geodetic position

For any floating body towed by the vessel, except for the tailbuoys, geodetic position observation equations can be formed (Fig. 3.20) as follows:

$$\begin{aligned} \phi_v + \frac{Y_f}{\rho} &= \phi_f + v_{\phi_f} \\ \lambda_v + \frac{X_f}{v \cos(\phi_v)} &= \lambda_f + v_{\lambda_f} \end{aligned} \quad (21)$$

where:

- X_f, Y_f : are the unknown Cartesian coordinates of the float
- ϕ_v, λ_v : are the unknown ellipsoidal coordinates of the vessel
- ϕ_f, λ_f : are the observed geodetic coordinates of the float
- $v_{\phi_f}, v_{\lambda_f}$: are the measurement residuals

Note that these equations make the (entirely reasonable) assumption that the radius of curvature in the plane of the meridian ρ , and the prime vertical v , throughout the spread are equal to those for the reference navigation point.

Tailbuoy position observation equations differ slightly from the equations given above because, as explained in paragraph 3.3.1, tailbuoy coordinates are not parameters of the system, i.e. they are not in the state vector. A streamer's parameters are therefore required in order to obtain its tailbuoy position. Substituting equation (20) into equation (21) leads to:

$$\begin{aligned} \phi_v + \frac{Y_s - 1 \sin(\alpha) + \sum_{k=2}^n [c_k l^k] \cos(\alpha)}{\rho} &= \phi_{tb} + v_{\phi_{tb}} \\ \lambda_v + \frac{X_s + 1 \cos(\alpha) + \sum_{k=2}^n [c_k l^k] \sin(\alpha)}{v \cos(\phi_v)} &= \lambda_{tb} + v_{\lambda_{tb}} \end{aligned} \quad (22)$$

where:

- ϕ_{tb}, λ_{tb} : are the tailbuoy measured geodetic coordinates
- $v_{\phi_{tb}}, v_{\lambda_{tb}}$: are the measurement residuals.

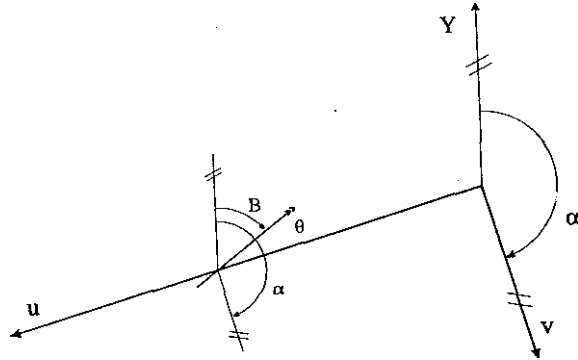


Fig. 3.30: Compass azimuth observations

Compass bearings

To form a compass bearing equation it is necessary to consider the geometry of the configuration as shown in Fig. 3.30. The observation equation for a compass of offset 1, measured from the streamer reference point, is then formed as follows:

$$\alpha - \left[\tan^{-1} \left(\frac{dv}{du} \right) + \frac{\pi}{2} \right] = B_{comp} + v_{B_{comp}} \quad (23)$$

where:

- α : is the instantaneous orientation of the streamer coordinate system u, v
- B_{comp} : is the observed compass bearing
- $v_{B_{comp}}$: is the measurement residual

and:

$$\frac{dv}{du} = \frac{d \left(\sum_{k=2}^n c_k l^k \right)}{dl} = \sum_{k=2}^n [k c_k l^{k-1}] \quad (24)$$

Compass observations should be reduced to the grid before we incorporate them into the Kalman filter process. This is done by correcting them for magnetic declination and grid convergence according to the following equation:

$$B_{comp} = B + mag_decl + grid_conv \quad (25)$$

Therefore, combining equations (23), (24) and (25) we obtain the complete form:

$$\alpha - \left[\tan^{-1} \left(\sum_{k=2}^n [k c_k l^{k-1}] \right) + \frac{\pi}{2} \right] - mag_decl - grid_conv = B + v_B \quad (26)$$

Note that, in principle, magnetic declination could be placed in the state vector and recovered from the measurements along with all of the other parameters. This has not been done in the work reported here but the fact that it is possible is another advantage of this unified approach.

The foregoing observation equations are given in their original, and mostly non-linear, form. To determine the coefficient matrix (A in equation (8)), it is necessary to linearise these equations by applying the Taylor series expansion. Because of the large number of unknown parameters contributing to the system and the complicated nature of some of the equations, many of these differentials are better obtained numerically, and where relevant this has been done in this implementation.

3.4 Kalman filter transition equations

In Kalman filtering it is a basic assumption that the secondary model is able to describe perfectly the system dynamics in the mean sense, i.e. such that model errors are limited to white noise sequences, v_i and y_i for the measurement and dynamic models respectively. Here a simple Taylor's expansion of the state vector elements is used for this purpose:

$$x = x_0 + \dot{x}_0 t + \frac{1}{2} \ddot{x}_0 t^2 + \dots \quad (27)$$

As has already been explained the stable nature of seismic exploration surveys has led to the state vector only including zero and first order terms (no acceleration or higher order terms). Consequently, and using equation (27), the dynamics of a seismic vessel and any other floating body (source, streamer reference point, etc.) is described, for a short period of time, by the following equations.

$$\begin{aligned} X_i &= X_{i-1} + \dot{X}_{i-1} dt + \frac{1}{2} a_x dt^2 \\ Y_i &= Y_{i-1} + \dot{Y}_{i-1} dt + \frac{1}{2} a_y dt^2 \\ \dot{X}_i &= \dot{X}_{i-1} + a_x dt \\ \dot{Y}_i &= \dot{Y}_{i-1} + a_y dt \end{aligned} \quad (28)$$

where a_x and a_y , the average acceleration components for the time interval dt , are treated as white noise [Houtenbos, 1982].

The remaining elements of the state vector, i.e. the crab angle c , the orientation of each streamer coordinate system α and the polynomial coefficients can be modelled in a simpler way because they are not expected to vary significantly with time. Therefore these states are modelled as a linear function of time according to the equation:

$$d_i = d_{i-1} + \dot{d}_{i-1} dt \quad (29)$$

where:

- d : is the state vector element
- \dot{d} : is the rate of change of d

3.5 Stochastic models

In order to implement a Kalman filter two stochastic models have to be specified. These are invariably in the form of covariance matrices and they describe the precision of the observations and of the dynamic model respectively, i.e. they describe the quality of the measurements and how well the model describes reality. Correct specifications of these stochastic models is essential for both the proper 'tuning' of the filter and its capability to produce accurate quality (precision and reliability) measures [Salzman, 1993]. The tuning of the filter refers, in essence, to the relative sizes of the elements of the observation and model covariance

matrices. By decreasing the variances (increasing the weights) of the observations the final filter estimates can be made to fit the observations more closely - but with the danger of small observational errors appearing as obviously impossible vessel manoeuvres. Conversely decreasing the variances of the dynamic model leads to too smooth a final answer and one that cannot react quickly to rapid changes in the true track of the vessel and of the hardware being towed. Such situations are well-known in practice and are evidenced, for instance, by a ship's track continuing to be shown as straight long after all on board are well aware that a turn has been made.

In mixed measurement systems it is also necessary to consider carefully the relative sizes of the elements within the covariance matrix of the observations. For instance by selecting the elements appropriately it would be easy, for instance, to make the final results fit the compass data very closely and virtually ignore the acoustics, or vice versa.

Although it is the relative sizes of these covariance matrices that is critical to the fidelity of the filter, it is their absolute sizes that drives the computed covariance matrices of the predicted and final filter estimates. Too small covariance matrices will lead to over-optimistic quality measures, and vice versa. Hence the problem is one of determining both relative and absolute sizes of the elements of the covariance matrices.

The correct practical approach to the solution of this problem is a matter for on-going research in several centres. For instance, one suggested approach [Celik and Cross, 1994] is whereby the standard deviations of the variance observations types are first determined by independent study (e.g. epoch by epoch network adjustment of acoustic ranges and simple curve fitting to the compasses). These values are then considered 'fixed' and the elements of the covariance matrix of the dynamic model are tuned until (on average) the correct number of rejections is made during the analysis of the innovation sequence. Certain model statistics are then used to scale the overall sizes of both matrices. This approach is relatively straight forward in the case of a seismic spread because the system is so well behaved. Much more research is, however, still needed in this area.

4. Implementation and testing of the method

The method has been implemented in a software package written at the University of Newcastle upon Tyne, and this, along with its testing with a real data set will now be discussed.

4.1 Software implementation

To verify the correctness of the mathematical model and the feasibility of the associated algorithms, software has been written to integrate the complete procedure so far discussed. It has been written in the 'C' programming language to run under the UNIX operating system. As well as (of course) computing the state vector at each epoch, the implemented algorithms also provide the position of any float in the spread, and the position of any hydrophone group. Additionally, and since they are a natural bi-product of the Kalman filter process, the associated covariance matrices are computed - leading to the computation of error ellipses and other quality measures not just for the vessel but for any point in the configuration (for instance the propagation of error law can be used to compute error ellipses at individual hydrophone

positions). Of course the vessel and tailbuoys positions and their precisions also form part of the software output. The algorithms are flexible enough to produce a solution simultaneously involving all observations or, for analysis and testing purposes, just some of them (e.g. only compasses). This feature gives an immediate overview of possible trends in the performance and contribution of a particular observation type.

All measurements are tested for outliers before they are accepted by the system. This testing procedure involves an examination of the predicted residual of each observation for every shotpoint. At this stage in the development of the software this check is simply to identify (and probably reject) outlying observations. Rigorous statistical testing [Salzmann, 1993], will be implemented in the future.

4.2 Data description

The survey configuration consisted of a single vessel, dual source and triple streamers equipped with active tailbuoys. Both vessel and tailbuoys were located by means of Syledis radio positioning while vessel positioning was supported by a DGPS system. A total of 240 hydrophone groups were located in every streamer of approximately 3km length. At the front end a total number of 45 acoustic ranges were measured while 29 acoustic ranges were observed at the tail network. In addition to this, 4 laser ranges and 7 directions from the vessel to energy sources and streamers were observed. Also, 13 magnetic azimuths provided the orientation of each of the three cables. Thus, including vessel and tailbuoys positions as well as vessel gyro, a total number of 132 observations were collected at every shot.

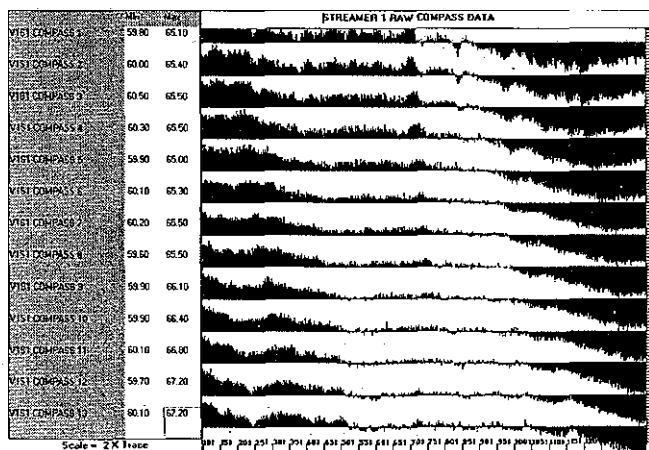


Fig. 4.10: Raw compass data time series plot

In addition to these data files bitmap time series for the raw compass and sonardyne data types have been provided – samples are in Figs. 4.10 and 4.20. This information is mainly used to assist in identifying problems and trends in the performance of these devices. It frequently occurs that some of the compass or acoustic data is of such poor quality that it is impossible to meet the clients specifications and therefore these measurements should be rejected or used with extreme care [van Zeelst, 1991]. Compass performance becomes very unstable in choppy and cross current seas. Moreover, it can be affected by local magnetic anomalies, man made or natural [Gerber, 1987].

As can be seen from Fig. 4.10 compass performance for the starboard streamer for the dataset used is quite stable for

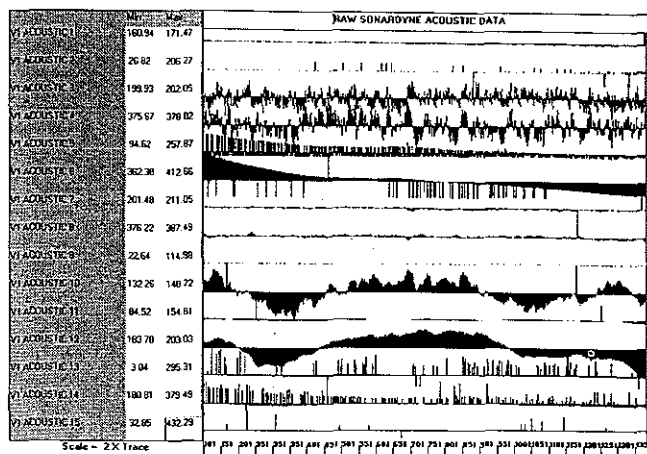


Fig. 4.20: Raw sonardyne acoustic data time series plot

all compasses with insignificant variations in the differences from shot to shot. This conclusion applies for the centre and port streamers as well. On the contrary, some of the raw acoustic ranges seem to be quite noisy (see Fig. 4.20 ranges 5, 6 and 14). The time series are reproduced here just to give a general indication of the quality of the data used in this test. The Kalman filter, if properly tuned, should identify and reject any outlying observations.

4.3 Processing of data

Around 800 continuous shotpoints (spanning a total time period of 110 minutes) from the foregoing data set have been processed and a number of analyses of the results have been performed. Note that all the presented results were obtained using a fifth order polynomial streamer model.

STANDARD DEVIATIONS OF THE OBSERVATIONS	
vessel position	3.0 m
gyro	0.5°
acoustic and laser ranges	2.0 m
laser bearings	0.5°
compass azimuths	0.5°
tailbuoy positions	3.0 m

Table 4.10: Stochastic model of the observations

STANDARD DEVIATIONS OF THE DRIVING NOISE	
vessel acceleration	0.01 m/sec ²
crab angle rate	0.04°/sec
float and streamer reference point acceleration	0.01 m/sec ²
streamer base line orientation rate	0.01°/sec
streamer polynomial coefficients rate	c ₂ 0.5 E-6 m/m ² /sec
	c ₃ 0.5 E-9 m/m ³ /sec
	c ₄ 0.5 E-12 m/m ⁴ /sec
	c ₅ 0.5 E-15 m/m ⁵ /sec

Table 4.20: Stochastic model of the dynamic model

As has been mentioned in 3.5, two stochastic models have to be specified before a Kalman filter can be implemented. Here it has been assumed that, since seismic surveys are carried out in calm seas, standard deviations of 0.01m/sec^2 can be used for the vessel, floats, and streamer reference point accelerations. Also, the correlation between Northings and Eastings accelerations has been assumed to be zero [Houtenbos, 1989]. Tables 4.10 and 4.20 give a full description of the standard deviations used to develop the stochastic models adopted for these tests.

Predicted residuals

A predicted residual is computed from the difference between a measurement at a particular time (usually a shot point) and the measured quantity computed from the predicted state of the system. Predicted residuals (sometimes known as the innovation sequence) are an excellent way to assess the performance of a system. Predicted residuals that are, overall unbiased (i.e. zero mean), and commensurate in size with the expected observation errors, show that the observation and dynamic models are correct, that they are correctly implemented, and that the filter is properly tuned (i.e. correct stochastic models have been used).

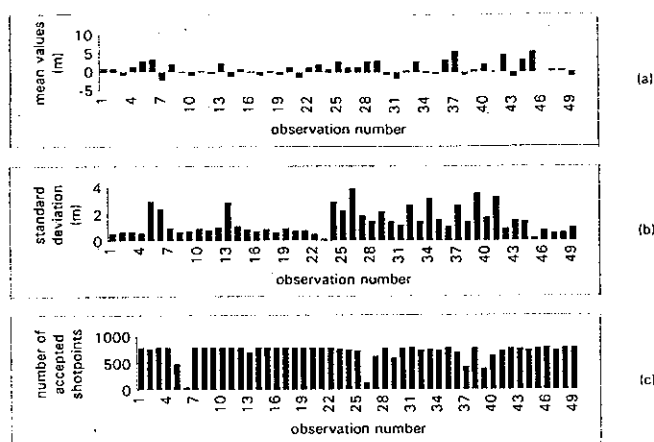
Fig. 4.30 contains the mean values and standard deviations of the observation residuals computed from all 800 shotpoints for the acoustic ranges in the front and tail networks as well as for the compass measurement of all three streamers. Given that, the *a priori* standard deviations of the acoustic ranges and compass azimuths of the measurement model have been set to be 2.0 metre and 0.5 degrees respectively, the results Fig. 4.30 (a), (d), (g) are extremely encouraging. As can be seen from these histograms the mean values for most of the acoustic ranges are less than 2.0 metre, but in any case much less than 3 standard deviations – while all mean values for the compass residuals are less than 0.5 degrees. Examination of Fig. 4.30 in more detail shows that ranges for which a large number of shotpoints has been rejected, present bigger standard deviations. This is to be expected as the contribution of these observations to the final solution is smaller (there are less of them) – see diagrams (b), (c), (e), (f). This phenomenon is most distinct for ranges 5, 6, 13, 26, 37 and 39 in the front end network and for ranges 5, 6 and 8 in the tail network. Moreover, these results can be explained by examining this diagram in combination with Fig. 4.20. Acoustic ranges 5 and 6 of the front network and 5 of the tail network in Fig. 4.30 correspond to ranges 5, 6 and 14 in Fig. 4.20 respectively, which are the most noisy as noted in 4.2. Hence it can be concluded that the filter is correctly identifying the outlying observations. Also it can be seen that the mean values for the compasses in Fig. 4.30 follow an approximately white noise pattern strongly supporting the use of the polynomial cable shape model.

Independent checks

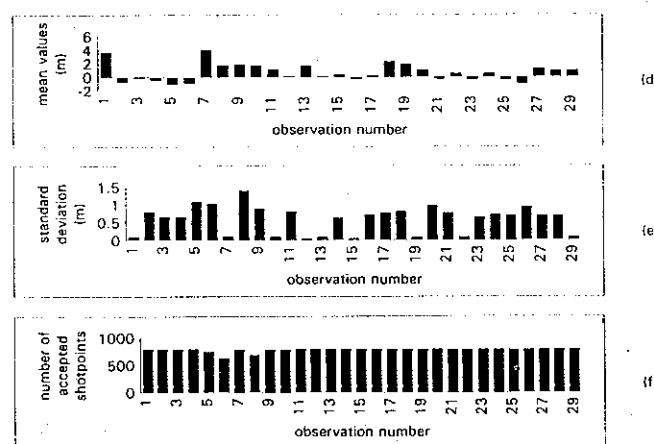
Another way to ascertain the effectiveness of the models is to carry out completely independent checks, i.e. to compare identical quantities computed using completely different data. Two such tests have been carried out.

In the first test, two estimates of the tailbuoy positions were compared. The first estimate came directly from the tailbuoy Syledis measurements and the second came from the Kalman filter using all of the data except those measurements. The results for all three streamers are shown in Fig. 4.40. Note that, to aid interpretation, the differences

FRONT END ACOUSTIC NETWORK



TAIL ACOUSTIC NETWORK



COMPASS AZIMUTHS

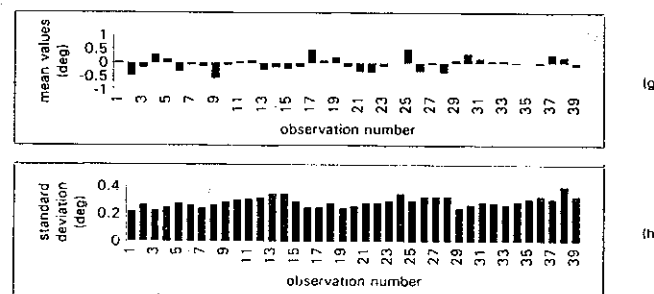


Fig. 4.30: Statistics of the predicted residuals (innovation sequence)

in Eastings and Northings have been rotated into their along-track and cross-track components. Two points are immediately evident: the differences in each direction are of the same magnitude for the three cables, and there is a significant difference in the two directional components. The along-track differences vary up to 10.0-12.0 metres, while the cross-track differences vary up to 20.0-25.0 metres. This disparity is easily understood as the along-track differences are cable-length related and the cross-track differences are cable-orientation related – clearly in this uncontrolled

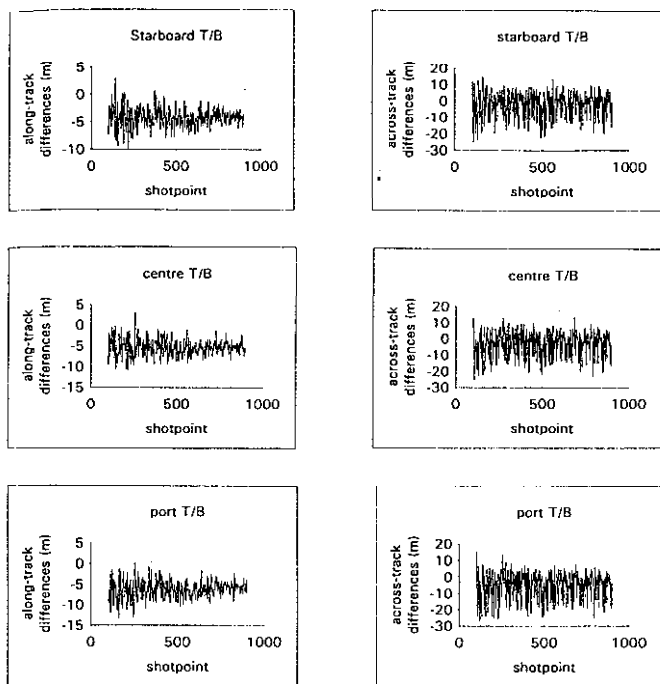


Fig. 4.40: Differences between Syledis (observed) and filter derived tailbuoy location

manner (the filter has no tailbuoy positioning) it is not surprising that the larger errors are orientation related.

The errors are of course the sum of several components including the unknown behaviour of the tailbuoy tether and errors in the 'check', i.e. the Syledis positions. They are, however, still not large (an actually no larger than expected – see Fig. 4.60 (b) and the later discussion). A maximum 20 metre cross track error over a 3km cable represents a maximum overall orientation error of less than 0.5 degrees – and the mean error is clearly very much smaller than this (of the order of 5 metres – or 0.1 degrees). Overall this test adds further confirmation that the model and its implementation are correct – at the very least it does not provide evidence to the contrary.

In the second 'independent' test the state vector elements were used, at every shotpoint, to compute the coordinates of two acoustic devices: one on the starboard streamer and the other on the port streamer, and both located in the tail network. The computed value of the distance was directly compared with the acoustically observed value given in the data set. Obviously, as in the first test, and in order for the check to be independent, this observation was not included in the filter solution. The resulting differences are shown in Fig. 4.50. They have an approximately zero mean and rarely reach one metre – representing an exceptionally strong argument that the model is correct.

Tests of precision

It has already been mentioned that the greatest single asset of the suggested method is its ability to provide a rigorous measure of precision throughout the seismic spread. Typical 95% error ellipses were computed at a sample of points along the cables under three different circumstances. Note that the input standard deviations chosen for the observations are 0.5 degrees for the compass bearings, 2.0 metres for the acoustic ranges and 3.0 metres for the Syledis positions, and

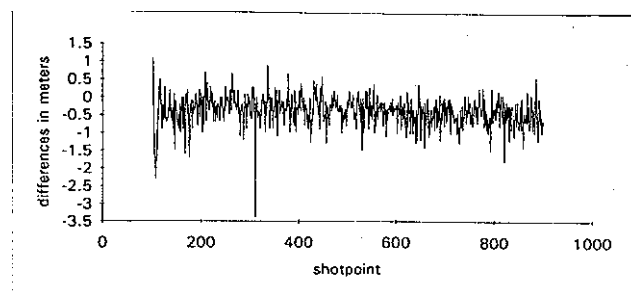


Fig. 4.50: Differences between an acoustic observed range (between two devices on separate streamers in the tail network) and that derived from the Kalman filter (not including the observation)

that these values were based on the survey specifications and on previous experience. The three tested configurations were:

- (a) all measurements,
- (b) no tailbuoy Syledis, and
- (c) no tailbuoy Syledis and no tail end acoustics.

From the three points shown in Fig. 4.60 it is apparent that maximum error occurs, as expected, almost exactly in the cross-track direction while minimum error occurs along the in-line direction. This is irrespective of which observation types or data subsets have been incorporated into the filter. In fact, the differences between the direction of minimum error and the cables' baselines, as computed

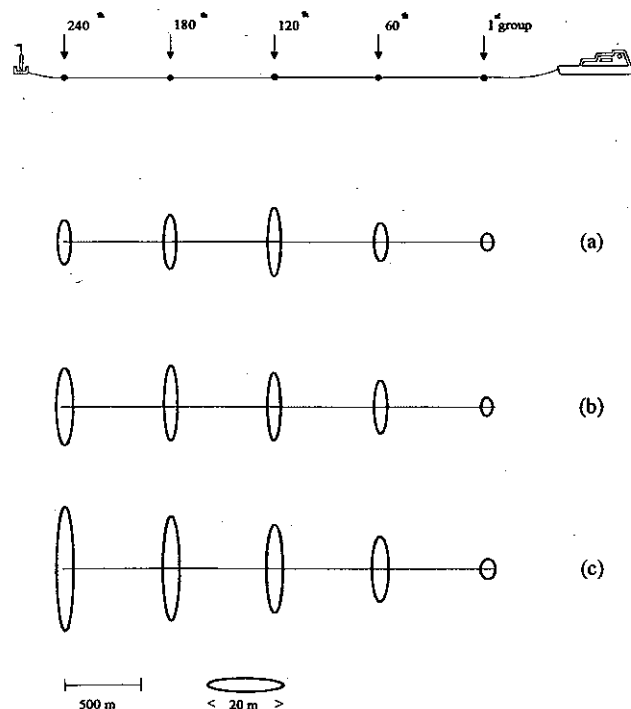


Fig. 4.60: Hydrophone group error ellipses

from the filter, are no more than a few degrees and in most of the cases less than 2.0 degrees. Moreover, from the same plots it can be seen that the standard deviations for the front end of the cable are quite small and of the same magnitude in all directions. This is because the network is very strong in this area due to the large number, and excellent geometry, of the available observations. Also, it is immediately evident that the in-line deviations are of the same magnitude in the three plots.

In plot Fig. 4.60 (a) the cross-track standard deviations range from about 2.0 metres at the front end, reach 10.0 metres for the groups in the middle and between 6.0 and 7.0 metres at the far end of the cables. In plot Fig. 4.60 (b), the standard deviations for the second half of the cables vary between 10.0 and 12.0 metres. Finally, if neither active tailbuoys nor tail acoustics are used, standard deviations can reach 18.0 metres at the tail end. It should also be noted here that the standard deviations of the energy source (gun) positions are similar to those at the front end of the cable.

Of course the general trends of these quality measures could have been predicted from common sense. It is obvious that with tailbuoy positioning the weakest part of the system is in the centre and without it the weakest part is at the tail. The advantage of this approach is that the uncertainties can be quantified so that they can be directly compared with the survey specification for quality control purposes.

5. Conclusions

It is apparent from the results of the tests that the suggested method is a highly realistic approach to the problem of integrated processing of 3D marine seismic data. The significant innovation of the method is centred upon its ability to incorporate all different observation types into one integrated solution by means of a Kalman filter. Moreover, by design, the whole network is conceptually split into several subsystems so that the resultant algorithm can be flexible enough to describe the geometry of any likely practical set-up in a rigorous mathematical manner. The position of any point of interest throughout the spread, and, most importantly, its precision, can hence be computed.

It has also been shown, by both data pre-analysis, and by the results of trials with the final algorithm, that polynomials can be adopted as a realistic mathematical representation of seismic streamer shape.

In order for the filter to operate efficiently, raw data should first be cleaned by removing any outliers. It is suggested that, due to the relatively long cycle time of such complicated filters as the one developed here, it would be extremely useful in practice to carry out separate analyses for the front and tail acoustic networks, as well as for every streamer compass group using simple least squares algorithms to detect outliers. Of course it would be the raw (edited) data that would be fed into the integrated filter – the separate filters only being for data screening.

Acknowledgements

The authors would like to thank Dave Ridyard and Duncan Griffiths of QC Tools for their valuable comments and suggestions, and the European Commission and Athens Academy for their general support of research into streamer modelling at the University of Newcastle upon Tyne.

References

- Brown, R.G. and Hwang, P.Y.C. *Introduction to Random Signal and Applied Kalman Filtering*, John Wiley, 499pp, 1992.
- Celik, R.N. and Cross, P.A. *Implications for Quality Assessment and Quality Control from Integrated Offshore Positioning Systems*, Hydro '94 Proceedings, 13-15 September 1994.
- Chevron Training Course, Texas 14-18 September 1992.
- Cross, P.A. *Advanced Least Squares Applied to Position Fixing*, Working paper no.6, Department of Land Surveying, North East London Polytechnic, 205pp, 1983.
- Cross, P.A. and Pritchett, C.H. *A Kalman Filter for Real-time Positioning During Geophysical Surveys at Sea*, Paper 410P.3, XVIII FIG Congress 19pp, Toronto, 1986.
- Cross, P.A., Hawksbee, D.J., Nicolai, R. *Quality Measures for Differential GPS Positioning*, Hydrographic Journal, No.72, p.17-22, 1994.
- Douglas, C. and Gilbert, *Performance Characteristics of a Three-Dimensional Array Shape Estimator*, Analysis & Technology, Inc., North Stonington, Connecticut 06359, 1980.
- Gerber, K., *Seismic Streamer Positioning with Magnetic Compasses and Syledis*, Practical Experiences, 1987.
- Houtenbos, A.P.E.M., *Prediction, Filtering and Smoothing of Offshore Navigation Data*, Hydrographic Journal, No.25, p.5-16, 1982.
- Houtenbos A.P.E.M., *Integrated Processing of 3D Marine Seismic Positioning Data*, Hydrographic Journal, No.52, p.13-25, 1989.
- Kalman, R.E., *A New Approach to Linear Filtering and Prediction Problems*, ASME Journal of Basic Engineering, Vo.82, p.34-35, 1960.
- Kelland, N.C., *Developments in Integrated Underwater Acoustic Positioning*, Hydrographic Journal, No.71, p.19-27, 1994.
- Krail, P.M. and Brysk, H. *The Shape of a Marine Streamer in Cross Current*, Geophysics, Vol.54, No.3, p.302-308, 1989.
- Morgan, G. James, *Positioning Challenges in Marine 3-D Seismic Surveys*, Ocean Industry, February 1992.
- Owsley, N.L., *Shape Estimation for a Flexible Underwater Cable*, Naval Underwater Systems Centre, New London, Connecticut 06320, 1981.
- Ridyard, D., Memo to Angus Jamienson JTS, 1989.
- Salzmann Martin, *Least Squares Filtering and Testing for Geodetic Navigation Applications*, Netherlands Geodetic Commission, New Series, No.37, 1993.
- van Zeelst, J., *Further Considerations in Positioning of 3D Seismic Surveys*, Hydrographic Journal, No.59, p.12-24, 1991.