Contents lists available at ScienceDirect

# Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

# An experimental investigation of spin power losses of a planetary gear set

A. Kahraman<sup>a,\*</sup>, D.R. Hilty<sup>a</sup>, A. Singh<sup>b</sup>

<sup>a</sup> The Ohio State University, Columbus, OH, USA

<sup>b</sup> General Motors Powertrain, Pontiac, MI, USA

#### ARTICLE INFO

Article history: Received 18 August 2014 Received in revised form 2 December 2014 Accepted 3 December 2014 Available online xxxx

Keywords: Planetary gear set Efficiency Spin power loss Drag power loss Gear mesh pocketing Bearing viscous power loss

#### 1. Introduction

#### ABSTRACT

This experimental study investigates the contributions of the key components of load independent (spin) power losses of planetary (epicyclic) gear sets. A dedicated test set up is developed to operate a planetary gear set under unloaded conditions in various hardware configurations within a wide range of speed. Torque provided to the gear set is measured as the spin torque loss. A test matrix is executed specifically to measure total spin loss of the gear set as well as the contributions of its main components, namely drag loss of the sun gear, drag loss of the carrier assembly, pocketing losses at the sun planet and ring planet meshes, viscous planet bearing losses, and planet bearing losses due to centrifugal forces caused by the rotation of the carrier. Variations of the same gear set are tested and results from these tests are compared to estimate and rank order the contributions of these components of power loss. Impact of the rotational speed and oil temperature levels on each component is also quantified.

© 2014 Published by Elsevier Ltd.

Powertrain efficiency has become a major topic in recent years within the transportation, aerospace, and energy industries due to continuously increasing fuel prices and growing concern over sustainability and the environmental impact of fossil fuels. As part of this focus, power losses from transmissions have become one of the major concerns of drive train engineers. On the automotive side, automatic transmissions employ co axial designs where multiple stages of planetary (also called epicyclic) gear sets and wet clutches are used to obtain different gear ratios. As such, the efficiency of the planetary gear sets dictates the overall efficiency of these transmissions. Furthermore, the push towards having transmissions with larger number of forward ratios often results in kinematic configurations that might operate planetary gear sets at higher speeds, presenting additional challenges in terms of drivetrain efficiency.

Power losses of any gear system can be classified in two major groups. One group of losses, often referred to as *mechanical* (load dependent) power losses, are induced by friction at the lubricated gear and bearing contacts and increases with the torque transmitted by the gear system. The second group of losses is formed by load independent mechanisms, often referred to as spin power losses. The term *spin* loss is used loosely to define losses taking place due to rotation of the system without transmitting any torque. In gearboxes that operate in dip lubrication conditions, spin losses represent losses associated with the churning (drag and pocketing) of oil surrounding the gears and bearings. In planetary gear systems, oil is typically delivered to gear and bearing contact interfaces through lubrication paths, designed to spread oil from the rotational center outwards. Under such conditions, gears and bearings are often subject to a certain mixture of oil and air.

\* Corresponding author. Tel.: +1 614 292 4678. *E-mail address:* kahraman.1@osu.edu (A. Kahraman).





CrossMark

 $http://dx.doi.org/10.1016/j.mechmachtheory.2014.12.003\\0094-114X/@ 2014 Published by Elsevier Ltd.$ 

The major sources of spin loss within a planetary gear set comprise (i) viscous drag loss associated with spinning components, namely gears, bearings and the carrier assembly (carrier with planet gears mounted on it), (ii) pumping of lubricant and air from the spaces between meshing gear teeth, and (iii) friction induced losses occurring at planet gear bearings due to centrifugal forces while the gear set free spinning. The drag power losses ( $P_d$ ) are those associated with the interactions of each individual gear and the carrier assembly with the surrounding medium, where windage refers to drag on a component spinning in air, and churning refers to drag associated with lubricant interaction for a component that is fully or partially immersed in oil. The three main sources of drag power loss of a planetary system are sun gear drag ( $P_{ds}$ ), planet carrier assembly drag ( $P_{dc}$ ), and ring gear drag ( $P_{dr}$ ). Any of these com ponents that are stationary would result in zero drag. Each drag power loss term represents the sum of (i) power loss due to oil/air drag on the faces (sides) of the gear or carrier, and (ii) power loss due to oil/air drag on the faces (sides) of the gear or carrier [1 3]. The total drag power loss of the planetary gear set is given as the sum of each source as follows:

$$P_d \quad P_{ds} + P_{dc} + P_{dr}. \tag{1}$$

The pumping power losses ( $P_p$ ) are caused by gears squeezing (or pumping) oil (or oil air mixture) out of the space between the teeth contracts as they roll into mesh [1,4]. These losses occur at each sun gear mesh with planets ( $P_{ps}$ ) as well as at the meshes of the ring gear with planets ( $P_{pr}$ ) such that

$$P_p \qquad N \Big( P_{ps} + P_{pr} \Big) \tag{2}$$

where *N* is the number of planet gears in the gear set.

The planet bearing spin losses ( $P_b$ ) can be described in terms of (i) load dependent (mechanical) losses and (ii) viscous (load in dependent) power losses. Each planet bearing and washer is subject to viscous power loss  $P_{vb}$ . In addition, the rotation of the carrier causes centrifugal forces  $F_b = mr\omega_c^2$  acting on each planet radially. Here, m is the mass of a planet including its bearing, r is the radius of the circle defined by planet centers (sun planet center distance), and  $\omega_c$  is the carrier rotational speed. These centrifugal forces in duce load dependent friction drag at each planet bearing in spite of the fact that no mechanical power is transmitted by the gear set. These radial forces result in certain power losses denoted here as  $P_{bg}$ . In its simplified form, power loss of a cylindrical roller bearings is given as [5,6]

$$P_{gb} = C_{gb}m. \tag{3}$$

Here  $C_{gb} = f_1 d_m r \omega_c^2 \omega_b$  where  $d_m$  being the bearing pitch diameter,  $\omega_b$  being the bearing speed (relative speed of the planet gear with respect to the carrier) and  $f_1$  being an application constant determined through testing. For  $\alpha = \omega_b/\omega_c$ ,  $C_{gb} = f_1 \alpha d_m r \omega_c^3$ . With this, total power loss associated with all of the planet bearings of the planetary gear set is given as

$$P_b = N \Big( P_{vb} + C_{gb} m \Big). \tag{4}$$

Summing these three main sources of power losses, the overall spin power loss of an N planet planetary gear set can be written as

$$P \quad P_d + P_p + P_b. \tag{5}$$

The majority of literature concerning power losses in gears pertains to mechanical efficiency of counter shaft (fixed center) gearing applications. Mechanical efficiency models based on elastohydrodynamic lubrication formulations have been proposed in recent years to investigate the impact of lubricant parameters, surface conditions (magnitude and lay of the surface roughness), gear geometry and operating conditions on contact friction and mechanical power losses (e.g. ref. [7 10]). Likewise, there have been a limited number of detailed experimental investigations of spur and helical gear mechanical power losses to provide much needed experimental databases [11]. Furthermore, some of these models [7,8] were compared to these experiments to demonstrate their accuracy. These studies, while significant to describe load dependent power losses of gears, are of limited relevance to this study, which focuses on spin power losses.

Some studies were conducted to investigate spin power losses of fixed center spur and helical gears. These studies exclude bearing losses as fixed center gear systems can be studied separately from bearing losses. Studies such as references [12 15] presented empirical or computational fluid mechanics based models of windage power losses for single gears in air. Some others [16 18] empirically studied churning effects of a single disk or a gear rotating while partially and fully immersed in oil, thus capturing drag effects caused by dip lubrication. Meanwhile, others [10,11,19 21] aimed at determining spin losses of counter shaft spur or helical gear pairs in mesh with viscous drag and pocketing losses lumped together. Further studies by Perchesky and Whidtbrott [22] and Diab et al. [23] presented models describing pocketing behavior, and Ariura et al. [24], Seetherman and Kahraman [1 3], Talbot et al. [4], and Changenet and Velex [25], presented models capable of separately characterizing pocketing and drag losses for fixed center gear pairs in mesh. These single gear or gear pair spin loss studies form a solid foundation for characterizing spin power losses present in fixed center gearing applications. Their applicability to planetary gear systems, however, is yet to be demonstrated as a planetary gear set possesses unique kinematic and mechanical features including planet bearings, rotating planet carrier, more com plex lubrication schemes, and multi mesh external and internal gearing interactions.

## Page 2 of 14

Most planetary gear sets employ cylindrical roller bearings to support the planet gears. These bearings can rotate at high speeds, generating sizable power losses due to viscous effects and contact friction. As these bearings are an integral part of the planetary system, their power losses also need to be studied. The large majority of literature on roller bearings has been confined to studies of dynamics, load distribution, and fatigue life. Certain subsets of these studies [26–29] focused specifically on planet cylindrical roller bearings using planet gear bores as outer raceways. More recently, Talbot el al [30] proposed a mechanical power loss model of cylindrical roller bearings under combined radial force and overturning moment loading, by incorporating a bearing load distribution model with an elastohydrodynamic rolling power loss model.

Most of the planetary gear train efficiency studies available in the literature (e.g. refs. [31 34]) employed a gear train kinematics point of view. These studies aimed at determining the overall mechanical efficiencies of planetary drive trains using speed and torque equations. They were effective in comparing different transmission designs in terms of their approximate efficiency outcome for designing systems with minimal power recirculation, but they all assumed a constant mechanical efficiency value  $\eta$  for each gear mesh in contact with no account of mechanical losses of planet bearings or any of the spin power losses. These assumed that efficiency values had no physical backing, and failed to include the speed, load, surface roughness and lubrication effects on efficiency. In one of the early attempts in prediction of power losses of planetary gears sets, Anderson et al. [35] modeled power losses in planetary gears using empirical equations derived for counter shaft spur gears. Gear mesh pumping and churning losses were not taken into account. Also, the experiments conducted did not isolate any source of power loss, and so it could not be determined whether each source of power loss was accurately modeled.

A recent study by Talbot et al. [36] presented the results of an extensive experimental planetary gear set efficiency study, including the effects of load, speed, oil temperature, number of planets and gear tooth roughness amplitudes on power losses. By performing measurements loaded and unloaded experiments at the same speed values, they were able to separate spin and mechanical losses. Most recently, the same authors [37] proposed a planetary gear set efficiency prediction methodology including both spin and mechanical losses. Their methodology employed a number of published physics based component power loss model, namely, the elastohydrodynamic lubrication based mechanical gear loss model of Li and Kahraman [9], mechanical bearing loss model of Talbot et al. [30], drag loss model of Seetharaman and Kahraman [1], the gear mesh pocketing model of Talbot et al. [4] as well as the bearing viscous loss equations of Harris and Kotzalas [5].

This existing body of work falls short of bringing an understanding of spin power losses taking place in planetary gear sets under unloaded conditions. In automotive transmissions, losses taking place under unloaded or lightly loaded conditions are of special interest for two reasons. First of all, automatic transmissions and transfer cases might operate certain decks of planetary gears under no load according to specific kinematic configurations defining each gear ratio or power flow condition [38]. Secondly, the duty cycles used to evaluate the fuel economy of a vehicle typically contain ample high speed, low torque driving conditions representative of highway driving conditions. The purpose of this study is to experimentally investigate spin power losses of plane tary gear systems. A set of experiments is carried out on a single set of unloaded, helical planetary gears using different hardware and gearing configurations. The data from these experiments are then used to isolate and quantify the components of spin power loss included in Eqs. (1) (5) and their contributions to the overall spin power loss of the planetary gear set operating throughout the ranges of speed and temperature seen in typical automotive transmissions. The specific objectives of this study are as follows:

• Develop a test set up capable of spinning a single set of planetary gears at high carrier speeds under no load with controlled lubri cation condition and accurately measure power loss.



Fig. 1. Efficiency test machine with the planetary test gearbox.

- Develop and implement a repeatable and realistic test methodology that utilizes various hardware and gearing configurations to eliminate, change, or isolate key components of the spin power loss of a planetary gear set.
- Present spin power loss data and comparisons that demonstrate trends in spin power loss with respect to speed and temperature for all hardware configurations.
- Isolate and rank order the main components of the spin power loss and demonstrate the magnitudes and contributions of these components to the overall spin power loss throughout ranges of the carrier speed and lubricant temperature.

#### 2. Experimental set-up

A planetary gearbox was designed, fabricated and incorporated with an existing high speed test bed used in several earlier studies [2,11,12]. The gearbox was intended to support a planetary gear set for operation under unloaded, high speed conditions. Fig. 1 shows the test machine with the planetary gear test fixture. It comprises a high speed AC motor drive and a belt drive connecting the AC motor to a high speed spindle. At the end of the spindle, a precision torque sensor and a flexible coupling are placed to measure the torque provided to any gearbox mounted on the test bed. The test machine was designed to rotate any test gearbox at speeds up to 10,000 rpm. An external lubrication system provides lubricant to the test gearbox at a desired flow rate and temperature within the range of 30 to 120°C.

The example epicycylic gear set identified for this study was used earlier by Ligata et al. [39,40] for planet load sharing studies, Inalpolat and Kahraman [41,42] for planetary sideband modulation studies and finally by Talbot et al. [36] for planetary gear set mechanical efficiency studies. In all earlier investigations, the gear set was operated in a back to back power circulation scheme to allow testing under loaded gearing configurations. In those studies, torque was input through the sun gear, while the planet carrier was the output member. The ring gear was held stationary through external splines. In this kinematic configuration, with the design parameters of the test gear set defined in Table 1, a carrier to sun gear speed ratio of 1:2.712 was achieved. In this study, a kinematic configuration was chosen with the carrier as the input member and the ring gear as the fixed member. In this configuration, a carrier speed of 3000 rpm is considered, corresponding to a sun gear speed of about 8136 rpm and a planet bearing speed (relative to its carrier) of about 14,400 rpm. These maximum speed values were deemed sufficient to study spin losses of planetary gear sets used for automotive applications.

A cross sectional side view of the gearbox is shown in Fig. 2(a) to specify its support flange and base plate, which were designed for the gearbox to be compatible with the existing test machine. Also illustrated in this figure is the lubricant reservoir housing around the gearbox. Focusing on critical details of the gearbox design, Fig. 2(b) shows a three dimensional cross sectional view with all key components labeled. The input shaft shown in this figure was supported by two rolling element bearings (SKF 6207 deep groove ball bearing). A flange in the middle of the shaft was press fit into the planet carrier and fastened with a set of bolts such that (i) the carrier is the input member and (ii) it was supported rigidly in both radial and axial directions. Both rolling element bearings were supported on their outer races by a two piece housing. The front side of the housing was mounted on the vertical support flange to achieve a position such that when the sliding test bed was moved to forward position, the splined end of the input shaft engaged the flexible coupling on the machine. The two sides of the housing were piloted radially by the ring gear adaptor, which was sandwiched between the front and back housing details via a set of bolts. The inner surface of the ring adaptor had internal splines designed to hold the ring gear stationary.

Each planet gear was mounted to the carrier through a pin, two rows of caged needle bearings and two thrust washers as labeled in Fig. 2(b) and shown in Fig. 3(a). Fig. 3(b) shows an assembled carrier with six planets. Since the carrier was designed to be taken apart to replace planet components, two retaining rings were devised, one on each side of the carrier, to prevent axial movement of planet pins. In addition, one of these retainer rings prevented any rotations of the pins with respect to the carrier while the other retaining ring acted as a lubricant catcher to increase the oil flow to the planet bearings.

The sun gear was held in position axially between the carrier and the back housing detail by a pair of thrust bearings (SKF AXK 90120). As the gears are of helical type, the axial thrust force acting on the sun gear was taken by these thrust bearings shown in Fig. 2(b). No radial constraints were applied to the sun gear. Therefore, an ideal radial floating condition could be achieved to prevent undesirable load sharing issues [40,41]. Fig. 3(c) shows a view of the ring, sun, and planet gears.

# Table 1 Basic design parameters of the test e gear set used in this study [39].

Parameter	Sun		Planet		Ring
Number of teeth	73		26		125
Normal module [mm]	1.81		1.81		1.81
Pressure angle [deg.]		23		23	
Helix angle [deg.]		13.1		13.1	
Center distance [mm]		92.1			
Active face width [mm]		25		25	

## Page 4 of 14



Fig. 2. (a) Cross-sectional side view and (b) isometric cross-sectional view of the test gearbox showing its components.

Lubrication of the contacts of a planetary gear set is more complex than that of counter shaft gearing. Neither the dip lubrication method nor the jet lubrication method is optimal for planetary gears. Since the planets rotate in most cases, application of dip lubrication at a certain oil level is risky since planet meshes are forced to operate above the oil level for certain portions of carrier ro tation. Likewise, it is not practical to design oil manifolds that rotate with the carrier such that the nozzles of jet lubrication follow the meshes of the rotating planets. The most common planetary lubrication system used in various systems such as automatic transmis sions has been the application of the oil from the rotational center of the planetary gear set. This way, the centrifugal forces push the oil radially outward, directing it to gear mesh and bearing locations through carefully designed paths. The gearbox was lubricated using a typical automatic transmission fluid (ATF). The lubricant was supplied and extracted from the gearbox by a temperature controlled external lubrication system with a large reserve. This system is capable of holding the temperature of the fluid within 5 degrees of desired set temperature. Temperature of the oil was measured within the reservoir and at the inlet to the gearbox.

Large openings machined in front and back sides of the housing, as shown in Fig. 2(b), ensured that the oil pumped through the gear set drained out of the gearbox so that any dip lubrication conditions are prevented. The lubricant drained from the gearbox was accumulated in a large reservoir that contained the gearbox, as shown in Fig. 2(a). Oil collected in the bottom of the reservoir was pumped by a sump pump back to the external lubrication system.

## Page 5 of 14





Fig. 3. (a) A test planet gear with its pin, bearings and thrust washers, (b) the carrier assembly, and (c) the gears of the test gear set.

#### 2.1. Various test hardware options

Per the scope of this study, the power losses caused by viscous air/oil drag on the sun gear and planet carrier,  $P_{ds}$  and  $P_{dc}$ , air/oil pumping at each sun planet and ring planet meshes,  $P_{ps}$  and  $P_{pr}$ , and the viscous and load dependent friction within the planet bear ings,  $P_{vb}$  and  $P_{gb}$ , were considered in this study. The ring gear did not have any drag loss ( $P_{dr} = 0$ ) in this case since it was stationary. With that, the total spin power loss of an N planet planetary gear set in Eq. (5) reduces to

$$P \quad P_{ds} + P_{dc} + N \Big( P_{ps} + P_{pr} \Big) + N \Big( P_{\nu b} + P_{gb} \Big). \tag{6}$$

Variations to the baseline gear set configuration were designed and implemented to isolate various components of power losses. These variations included running tests with missing or modified hardware. The intent of testing with these variations was to collect data under conditions when some of the components of the power loss specified in Eq. (6) were changed or absent. These conditions and associated hardware variations are described below.

- (i) *Case of no sun related power losses.* This variation required operation without a sun gear, hence, removing sun drag ( $P_{ds}$ ) and sun planet mesh pumping losses ( $P_{ps}$ ) from Eq. (6). While the planetary gear set can be rotated with no sun gear, the additional space created in the absence of the sun gear impacts the lubricant flow paths, potentially influencing the other components of power loss. In order to avoid changes in the lubricant flow, a non rotating dummy disk made out of a polymer material was designed to occupy the space of the sun gear.
- (ii) Case of no pumping losses. It was shown in Ref. [1 4] that losses associated with the pumping of medium from the gear mesh cavities were strongly related to the affective face widths of the gears. In order to create a condition when all the power loss components in Eq. (6) were present except the pumping losses P<sub>ps</sub> and P<sub>pr</sub>, a set of planet gears with significantly reduced face widths were designed and fabricated. Fig. 4(a) shows a baseline full face width planet gear with a 25 mm face width while Fig. 4(b) shows a reduced face width version with only 3.175 mm face width.
- (iii) Case of reduced centrifugal effects. As stated earlier, the term P<sub>gb</sub> in Eq. (6) was intended to represent bearing losses caused by the centrifugal forces of planets rotating with the carrier. In order to create an operating condition where these forces were significantly less, another variation of planet gears with reduced mass was designed and procured. As shown in Fig. 4(c), these had

## Page 6 of 14



Fig. 4. (a) A baseline planet gear, (b) a reduced face width planet gear and (c) a reduced mass planet gear.

the same geometrical features as the baseline planets but had a much lower mass. These gears were made of PEEK plastic resin. This material had a density of  $1300 \text{ kg/m}^3$ , compared to  $7800 \text{ kg/m}^3$  of steel. Therefore, the material maintained the shape mimicking steel gears throughout the testing temperature range while only contributing a fraction of the weight of steel gears. Since the bores of planets serve as the outer raceways for the planet bearings, a thin (3.2 mm) steel insert was pressed into each plastic planet. This ensured that the interactions between the bearings and washers with the planet remained the same as the baseline conditions. The mass of a plastic gear, shown in Fig. 4(c), was only 0.06 kg, slightly over one third of the 0.17 kg baseline planet gears.

In addition to above variations requiring different hardware, the existing hardware could also be used in different configurations to represent different power loss combinations. One such configuration was obtained by using the current carrier with only three planets instead of six. In this case, while the drag components of losses were retained, the planet related losses were reduced to half. Another configuration involved the gear set operated without the sun and ring gears. In this case, no meshing action took place, so the velocity of the planets relative to the carrier was zero. Therefore, the only active power loss component was expected to be  $P_{dc}$ , which also in cluded the losses associated with the rolling element bearings holding the carrier shaft and the oil seal.

#### 2.2. Test procedure, data acquisition and test matrix

Special care was taken to ensure that the same hardware and gearing alignments were used for every test. This was done to ensure that any small amount of power loss caused by hardware manufacturing errors was consistent throughout all tests where possible. The same sun and ring gears were used for every test in which a sun or ring gear was present. Planet gears, washers, and bearings were replaced on a few occasions between testing, but an extensive gear run in procedure was followed after each hardware change to ensure that the friction properties within the gearbox did not drastically change during testing due to gears running in. Each set of planet hardware was numbered to identify with which pin locations it would occupy. One tooth on each planet was marked as a ref erence tooth, and planets were assembled in the gearbox such that these reference teeth were positioned radially outwards. Likewise marked reference teeth on the sun and ring gears were aligned initially with the first planet. This way, any run out or eccentricity re lated effects would be repeated in each test in an attempt to reduce variability. For the tests with three planets, the planets, washers, and bearings housed in the first, third, and fifth planet locations in the carrier were removed. The planet gear pins associated with the removed planets remained installed in the test machine to ensure consistent lubrication system behavior between tests run with six and three planet configurations.

A high precision non contact digital torque sensor (full scale range of 50 Nm with an installed linearity of 0.03% of full scale) was used to measure the torque *T* provided to the planetary gear set, representing the torque loss of the gear set. Due to the high accuracy required in tests, the torque meter was recalibrated after each test. The gear set was put through a run in cycle before the actual tests gears to ensure that the bearings operate under stable run in surface roughnesses.

## Page 7 of 14

Each test was conducted by starting up the test machine at the desired test speed and recording (i) input torque to the gearbox *T*, (ii) planet carrier speed  $\omega_c$ , and (iii) the temperatures at the fluid system reservoir and the inlet to the shaft lubrication system. All data was then averaged over the test period to arrive at a set of representative values of *T*,  $\omega_c$ , and reservoir and shaft inlet temperatures for each test. Each test was run for a period of 10 min. The total spin power loss of the gearbox within the tested configuration was then computed as  $P = \omega_c T$ .

A test matrix shown in Table 2 was defined to generate data for four different variations. Test A used a 6 planet carrier with base line, full face width planets. Test B again used baseline planets, but in a 3 planet carrier arrangement. Test C was done using a 6 planet arrangement with reduced face width steel planets while Test D was performed by using the 3 planet arrangement with the reduced mass plastic gears. For tests A to C, three variations were considered: (1) complete gear set with both sun and ring gear active, (2) gear set without the sun gear (with the dummy disk in place of the sun gear) and (3) gear set without both sun and ring gears. With this notation, Test 3C, for instance, represents tests with no sun or ring gear and with a 6 planet carrier housing reduced face width planet gears. Meanwhile, Test D was only conducted using variation (2) with no sun gear. Accordingly, measured spin power loss values for each test configuration represented the following variations of Eq. (6):

$$P_{1A} = P_{ds} + P_{dc} + 6(P_{ps} + P_{pr}) + 6(P_{vb} + C_{gb}m_{AB}),$$
(7a)

$$P_{2A} = P_{dc} + 6P_{pr} + 6(P_{vb} + C_{gb}m_{AB})$$
,  $P_{3A} = P_{dc}$ , (7b, c)

$$P_{1B} \quad P_{ds} + P_{dc} + 3(P_{ps} + P_{pr}) + 3(P_{vb} + C_{gb}m_{AB}),$$
(7d)

$$P_{2B} P_{dc} + 3P_{pr} + 3(P_{vb} + C_{gb}m_{AB}) , \qquad P_{3B} P_{dc}, \qquad (7e, f)$$

$$P_{1C} \cong P_{ds} + P_{dc} + 6 \left( P_{\nu b} + C_{gb} m_C \right), \tag{7g}$$

$$P_{2C} \cong P_{dc} + 6 \left( P_{vb} + C_{gb} m_C \right) , \qquad P_{3C} = P_{dc}, \qquad (7h,i)$$

$$P_{2D} = P_{dc} + 3P_{pr} + 3\left(P_{\nu b} + C_{gb}m_D\right).$$
(7j)

Here, the load dependent bearing power loss is described by the relation  $P_{gb} = C_{gb}m$  as explained in Section 1, where  $m_{AB}$  is the mass of a full face width steel planet gear as shown in Fig. 4(a),  $m_C$  is the mass of a reduced face width steel planet gear as shown in Fig. 4(b) and  $m_D$  is the mass of a reduced mass plastic planet gear as shown in Fig. 4(c). These 10 variations of spin loss measure ments were then used to estimate the contributions of different power loss components defined in Eq. (6).

Each test configuration was run at two different oil temperatures of 40 °C and 90 °C. The contributions of various loss mechanisms may be different at the two test temperatures since ATF experiences significant changes in viscosity with respect to temperature [11]. Each test variation at each temperature level was run at 6 discrete input (carrier) speeds ranging from  $\omega_c = 500$  to 3000 rpm (in in crements of 500 rpm). With 10 hardware variations (1A to 2D in Table 2), two temperature levels and 6 speed values at each temperature value, a total of 120 pieces of data were defined in this test matrix, excluding the repeatability tests.

In order to demonstrate the repeatability of the measurements, test group 1A in Table 2 was chosen as the repeatability test con dition, and several 1A tests were performed at various stages of the test program. These tests were staggered throughout the entire

Table 2	
Spin power loss planetary gear configuration test matrix.	

Test	Number of Planets, <i>n</i>	Sun gear	Ring gear	Planet gears
1A	6	Yes	Yes	Baseline
2A		No	Yes	Baseline
3A		No	No	Baseline
1B	3	Yes	Yes	Baseline
2B		No	Yes	Baseline
3B		No	No	Baseline
1C	6	Yes	Yes	Reduced face width
2C		No	Yes	Reduced face width
3C		No	No	Reduced face width
2D	3	No	Yes	Reduced mass

## Page 8 of 14

A. Kahraman et al. / Mechanism and Machine Theory 86 (2015) 48-61



Fig. 5. Measured power losses from Test 1A repeatability tests at 90 °C.

test program to ensure that no tangible changes took place to the test conditions, the measurement system, or the test articles that would have an effect on *P* measurements. Fig. 5 compares the *P* values measured through five different 1A tests at various input speed values at 90 °C showing acceptable levels of repeatability.

The spin power loss values  $P_{1A}$  through  $P_{2D}$  in Eqs. (7a) (7j) represent a certain sub set or variations of the total planet gear set spin power loss expression given in Eq. (6). A direct measurement of every component of the spin power loss without others present is not possible as these components cannot be separated physically. For instance, one cannot measure bearing viscous loss  $P_{vb}$  without



Fig. 6. Measured power losses (a)P<sub>1A</sub>, P<sub>2A</sub>, and P<sub>3A</sub>, (b) P<sub>1B</sub>, P<sub>2B</sub>, and P<sub>3B</sub>, (c)P<sub>1C</sub>, P<sub>2C</sub>, and P<sub>3C</sub>, and (d) P<sub>2B</sub> and P<sub>2D</sub> as functions of  $\omega_c$  at 40 °C.

56

having the centrifugal loss  $P_{gb}$ . Likewise, pocketing losses  $P_{ps}$  and  $P_{pr}$  cannot be isolated from the drag losses  $P_{ds}$  and  $P_{dc}$ . However, each component of power loss in Eq. (6) can be estimated indirectly by comparing the results from each test variation  $P_{1A}$  to  $P_{2D}$ . Here, the term "estimation" is used in place of "measurement" to indicate that the results are a composite of two or more individual experiments, rather than being direct measurements.

#### 3. Experimental results

The measured spin power loss results from the tests listed in Table 2, denoted  $P_{1A}$  through  $P_{2D}$  measured from tests 1A through 2D, are shown in Fig. 6 at an oil inlet temperature of 40 °C. In this figure, individual graphs are provided to compare ( $P_{1A}$ ,  $P_{2A}$ ,  $P_{3A}$ ), ( $P_{1B}$ ,  $P_{2B}$ ,  $P_{3B}$ ) and ( $P_{1C}$ ,  $P_{2C}$ ,  $P_{3C}$ ) as well as a separate plot for  $P_{2D}$ . In Fig. 6(a), the contributions from  $P_{ps}$  and  $P_{ds}$  can be seen as the difference between  $P_{1A}$  values from the example gear set under baseline condition 1A (with six full face width steel planets and sun and ring gear), and  $P_{2A}$  from test 2A with no sun gear (no sun drag and pocketing losses). In this figure,  $P_{1A}$  is about 3.85 kW at  $\omega_c = 3000$  rpm while  $P_{2A}$  reaches only about 2.92 kW at the same speed. The contributions of  $P_{pr}$ ,  $P_{vb}$ , and  $P_{gb}$  with the mass of the baseline planet gear  $m_{AB}$  can be viewed as the difference between tests  $P_{2A}$  and  $P_{3A}$ .  $P_{3A}$  values represent the test condition 3A, where the sun and ring gears are removed (only the contribution of  $P_{dc}$  is present). At  $\omega_c = 3000$  rpm,  $P_{3A}$  is measured to be only 0.35 kW. The corresponding tests 1B, 2B and 3B with the 3 planet carrier yield results (Fig. 6(b)) considerably less than those for the 6 planet carrier. For instance,  $P_{1B}$  is about 1.48 kW lower than  $P_{1A}$  at  $\omega_c = 3000$  rpm. This decrease represents the losses associated with the three missing planets. Values of  $P_{2B}$  and  $P_{3B}$  are again smaller compared to  $P_{1B}$ , as expected.

The tests with reduced face width planets (tests 1C, 2C) in Fig. 6(c) reveal lower loss values than those from full face width planet tests (1A and 2A) in Fig. 6(a). For instance, at 3000 rpm,  $P_{1C}$  is about 0.88 kW lower than  $P_{1A}$  at 40 °C. This reduction in power loss can be attributable to the pocketing loss term  $6(P_{ps} + P_{pr})$  in Eq. (6). Finally, in Fig. 6(d), measured  $P_{2D}$  (representing test 2D with reduced mass gears and no sun gear) values are presented. They are compared to the corresponding  $P_{2B}$  values in this figure to illustrate the influence of losses associated with the centrifugal loads due to the planet mass. Here  $P_{2B}$  is only 0.05 kW higher than  $P_{2D}$  at  $\omega_c = 3000$  rpm, suggesting that the centrifugal effects are secondary in this case.



**Fig. 7.** Comparison of (a)  $P_{1A}$ , (b)  $P_{1B}$ , (c)  $P_{1C}$ , (d)  $P_{2D}$  as functions of  $\omega_c$  at 40 and 90 °C.

Page 10 of 14

Direct comparisons between the power loss measurements at 40 and 90 °C are made in Fig. 7. The viscosity of the ATF lubricant is a key parameter for all of the components of power loss. A decrease in oil viscosity was shown to decrease both pumping and drag power losses of spur and helical gears [1,2,11] operating under jet or dip lubrication conditions as well as and hypoid gear [43,44]. The ATF used in this study has a kinematic viscosity of 29.5 cSt at 40 °C and only 7.15 cSt at 90 °C [11]. As seen in Fig. 7, this directly impacts the resultant spin loss values. In Fig. 7(a), the  $P_{1A}$  values at 40 °C are considerably higher than those at 90 °C. For instance, at  $\omega_c = 3000$  rpm,  $P_{1A} = 3.85$  kW at 40 °C and only 2.98 kW at 90 °C. This 0.87 kW (25%) difference seen in  $P_{1A}$  is directly attributable to the oil temperature. Similar trends are observed in Fig. 7(b – d) for  $P_{1B}$ ,  $P_{1C}$  and  $P_{2D}$ , respectively, further demonstrating the influence of oil temperature and resultant changes in oil viscosity.

#### 3.1. Determination of spin power loss components

Individual components of spin power loss defined in Eq. (6) can be estimated using the test data presented in Figs. 6 and 7 in view of the set of Eqs. (7a) (7j). Various schemes were used here to estimate (i) the viscous drag losses  $P_{dc}$  and  $P_{ds}$  from the planet carrier and the sun gear, (ii) gear mesh pumping losses  $P_{ps}$  and  $P_{pr}$  from the planet sun and the planet ring gear meshes, and (iii) planet bearing viscous and mechanical friction losses ( $P_{vb}$  and  $P_{gb}$ ).

According to Eqs. (7b,c), (7e,f), (7h,i), the carrier viscous drag loss  $P_{dc}$  should dictate the bulk of the losses in tests 3A, 3B and 3C with the losses associated with the input shaft bearings and the seal also lumped with these measurements. Accordingly, the  $P_{dc}$  values from these tests are given as

$$P_{dc} = P_{3A}.$$
 (8)

The same can be found as  $P_{dc} = P_{3B}$  or  $P_{dc} = P_{3C}$ . The data presented in Fig. 6 indicates that  $P_{dc}$  values estimated by these three separate tests are in close agreement.

The sun gear viscous drag loss P<sub>ds</sub> can be estimated by processing the losses from tests 1A, 2A, 1B and 2B as

$$P_{ds} \quad P_{2A} - P_{1A} + 2(P_{1B} - P_{2B}). \tag{9}$$

Alternatively, results of tests 1C and 2C can be used more directly to estimate the same as  $P_{ds} = P_{1C} - P_{2C}$ . Next, the pumping power loss of a single planet sun gear mesh ( $P_{ps}$ ) can be estimated as

$$P_{ps} = \frac{1}{6} [P_{1A} - P_{2A} - P_{1C} + P_{2C}] \tag{10}$$



Fig. 8. Components of the spin power loss of the 6-planet gear set at (a) 40 °C and (b) 90 °C.

Page 11 of 14

while there are two other ways to estimate the same from other combinations of tests. Likewise, an equation to calculate the pumping power loss of a single planet ring gear mesh  $(P_{pr})$  is given as

$$P_{pr} = \frac{1}{6} [2P_{2B} - 2P_{3B} - P_{2C} + P_{3C}] + \frac{1}{3} \frac{(m_C - m_{AB})}{(m_{AB} - m_D)} [P_{2B} - P_{2D}].$$
(11)

In this equation, the first term would be sufficient to estimate  $P_{pr}$  with the assumption that the difference in masses of the full face width and reduced face width planets ( $m_{AB}$  and  $m_C$ ) is small enough to cause the same levels of bearing power losses. The second term in this equation corrects for any differences in bearing power losses caused by a change in the mass of the planet gears due to the reduction in the face width.

Meanwhile, the viscous (load independent) power loss  $P_{vb}$  of a single bearing can be estimated in two ways, one of which is given below

$$P_{\nu b} = \frac{1}{6} [P_{2C} - P_{3C}] - \frac{1}{6} \frac{m_C}{(m_{AB} - m_D)} [P_{2A} + P_{3A} - 2P_{2D}].$$
(12)

Again, the second term of this equation is to account for the differences in planet masses. Finally, the planet bearing mechanical power loss parameter  $C_{gb}(P_{gb} = C_{gb}m)$  can be found as

$$C_{gb} = \frac{1}{3(m_{AB} - m_D)} [P_{2B} - P_{2D}].$$
(13)

Using Eqs. (8) (13), each of the six power loss sources can be isolated and the magnitudes of their contributions to overall *P* can be quantified. Fig. 8(a) and (b) show a break down of spin power loss components of the planetary gear set estimated by using the scheme outlined above at 40 and 90 °C, respectively. In these figures, individual contributions of components  $P_{ds}$ ,  $P_{dc}$ ,  $6P_{ps}$ ,  $6P_{vb}$  and  $6P_{gb}$  to the total power loss are shown. Here, *P* is calculated for test 1A as the sum of these components rather than using the actual  $P_{1A}$  measurements from test 1A. Several observations are made from these figures:

- The power losses caused by viscous effects at the planet bearings appear to be the most dominant components of the planetary gear set spin loss. This is true at different temperatures as well as throughout the entire carrier speed range. The 6*P*<sub>vb</sub> values constitute as much as 56% of the total power loss at 40 °C and 62% of the power loss at 90 °C. These percentages are rather consistent with speed.
- Load dependent bearing losses caused by centrifugal forces represented by  $6P_{gb}$  are almost nonexistent at lower speeds, but become a more significant contributor at the higher speeds, especially at higher oil inlet temperatures where the elastohydrodynamic film thicknesses along the contacts of the planet bearings become thinner to increase the corresponding mechanical losses.



Fig. 9. Comparisons of the total power loss calculated from its components using Eq. (6) to the actual measurements from test 1A at (a) 40 °C and (b) 90 °C.

Page 12 of 14

- The gear mesh pumping losses are also a major contributor to the overall power loss. Here 6P<sub>pr</sub> constitutes as much as 28% of the total power loss at lower speeds for both temperatures, but its contributions decrease at higher speeds. This may indicate that it does not rise as sharply with speed as some of the other forms of power loss. 6P<sub>ps</sub>, on the other hand, is negligible at lower speeds, but increases sharply with speed, attaining as much as about 14% of the total power loss at high speeds for both temperatures. In general pocketing losses at the ring planet meshes are higher than those at the sun planet meshes since the internal gear mesh is more conformal resulting in an increase in the number of pockets formed. In addition, oil to air ratio of the medium pumped out of the internal meshes can be expected to be higher than that of the external meshes since the oil spreads outward from the center of the gear set accumulating along the ring gear.
- The power losses caused by viscous drag,  $P_{ds}$  and  $P_{dc}$ , do not appear to be major contributors to *P*. In fact,  $P_{ds}$  is almost negligible at high speeds for both test temperatures, while  $P_{dc}$  accounts for about 7 9% of the total power loss throughout the range of speed.

In order to show that the methods used to separate sources of power loss, represented by Eqs. (8) (13) are reasonable, power loss values calculated by using Eq. (6) from for a six planet system displayed in Fig. 8 were compared to actual measurements from test 1A test in Fig. 9(a) and (b) at 40 °C and 90 °C, respectively. At both temperatures, total spin loss *P* calculated as sum of its components matches the direct measurements from test 1A (baseline 6 planet gear set) quite well. A difference well below 10% is observed be tween the values calculated by using Eq. (6) as the sum of the components given by Eqs. (8) (13) and actual measurements for the entire speed and temperature ranges. Such a difference can be viewed to be reasonably small, considering that many components of power loss were calculated using data from different tests.

#### 4. Summary and conclusion

An experimental study was performed to investigate the mechanisms of spin power loss of planetary gear sets. A test set up was developed for this purpose with the capability of spinning a single, unloaded planetary gear set in various hardware configurations at desired test speeds while measuring torque provided to the gear set. This torque value, representing the torque loss of the gear set, was used to determine the spin power loss at a given speed value. It has been shown that this test machine as well as the test proce dure implemented is capable of producing repeatable power loss measurements within wide ranges of speed and lubricant temper ature. A test matrix was defined specifically to not only measure total spin loss but also provide test variations that can be used to determine the contributions of the following main components of the power loss: (i) drag loss of the sun gear, (ii) drag loss of the carrier assembly, (iii) pocketing losses at the sun planet meshes, (iv) pocketing losses at the ring planet meshes, (v) viscous planet bearing losses, and (vi) planet bearing losses due to centrifugal forces. For this purpose, planetary gear set versions were developed with different sets of hardware. These included reduced face width planets to eliminate the pocketing losses at the gear meshes and reduced mass planets to alter the bearing losses caused by centrifugal effects. In addition, the test fixtures were designed to allow operation without the sun gear or the ring gear to isolate the losses associated with them.

Based on the results presented in this study, several conclusions can be made in regards to spin power losses of planetary gear sets. First of all, the main premise of this study was that the spin power loss of a planetary gear set could be represented by a set of com ponents consisting of viscous drag power losses on the sun gear and the planet carrier, gear mesh pumping losses in the planet sun meshes and planet ring meshes, and losses in the planet bearings attributed to viscous and mechanical (centrifugal load dependent) friction. Tests have shown that this assumption is indeed valid, i.e. the total power loss can be considered as the sum of these relatively independent components. This is rather significant especially for modeling efforts, indicating that individual models to predict each of these power loss components can be superimposed to obtain the total spin power loss of a planetary gear set. Power losses caused by viscous friction within the planet bearings were shown to be by far the most dominant sources of spin power loss, accounting for about one half of the spin power loss at all temperatures in a 6 planet gear set with the mechanical bearing losses due to centrifugal forces becoming more significant at higher speeds. Gear mesh pumping losses were also shown to be primary contributors to the spin power losses while the contributions of the power losses caused by viscous drag being secondary. The temperature of the lubricant was also seen to influence spin power losses tangibly where an increase in lubricant test temperature was shown to produce a sizeable decrease in overall spin power loss.

#### Acknowledgments

This research study was sponsored through a grant from General Motors Powertrain. (Grant Number: RF60016957).

#### References

- [1] S. Seetharaman, A. Kahraman, Load-independent power losses of a spur gear pair: model formulation, J. Tribol. 131 (2009) 022201.
- [2] S. Seetharaman, A. Kahraman, M.D. Moorhead, T.T. Petry-Johnson, Oil churning power losses of a gear pair: experiments and model validation, J. Tribol. 131 (2009) 022202.
- [3] S. Seetharaman, A. Kahraman, A windage power loss model for spur gear pairs, Tribol. Trans. 53 (4) (2010) 473–484.
- [4] D. Talbot, A. Kahraman, S. Saatharaman, A pocketing power loss model for helical gears, J. Tribol. 136 (2014) 021105.
- [5] T.A. Harris, M. Kotzalas, Roller Bearing Analysis: Essential Concepts of Bearing Technology, 3rd ed. CRC Press, Taylor & Francis Group, Boca Raton, FL, 2007.
- [6] A. Palmgren, Ball and Roller Bearing Engineering, John Wiley & Sons Inc., New York, 1959.
- [7] H. Xu, A. Kahraman, N.E. Anderson, D.G. Maddock, Prediction of mechanical efficiency of parallel-axis gear pairs, J. Mech. Des. 129 (2007) 58-68.
- [8] S. Li, A. Kahraman, A transient mixed elastohydrodynamic lubrication model for spur gear pairs, J. Tribol. 132 (1) (2010) 011501.
- [9] S. Li, A. Kahraman, Prediction of spur gear mechanical power losses using a transient elastohydrodynamic lubrication model, Tribol. Trans. 53 (2010) 554–563.

- [10] S. Li, A. Vaidyanathan, J. Harianto, A. Kahraman, Influence of design parameters on mechanical power losses of helical gear pairs, J. Adv. Mech. Des. Syst. Manuf. 3 (2) (2009) 146–158.
- [11] T. Petry-Johnson, A. Kahraman, N.E. Anderson, D. Chase, An experimental investigation of power losses of high-speed spur gears, J. Mech. Des. 130 (6) (2008) 062601.
- [12] P.H. Dawson, Windage loss in larger high-speed gears, Proc. Inst. Mech. Eng. Part A Power Process Eng. 198 (1) (1984) 51–59.
- [13] Y. Diab, F. Ville, P. Velex, Investigations on power losses in high speed gears, J. Eng. Tribol. 220 (2006) 191–298.
- [14] P.M. Wild, N. Dijlali, G.W. Vickers, Experimental and computational assessment of windage losses in rotating machinery, J. Fluids Eng. 118 (1996) 116–122.
- [15] K. Al-Shibl, K. Simmons, C.N. Eastwick, Modeling gear windage power loss from an enclosed spur gear, Proc. Inst. Mech. Eng. Part A Power Process Eng. 221 (3) (2007) 331–341.
- [16] J.D. Daily, R.E. Nece, Chamber dimensional effects on induced flow and frictional resistance of enclosed rotating disks, J. Basic Eng. Trans. ASME 82 (1960) 217–232.
- [17] R.W. Mann, C.H. Marston, Friction drag on bladed disks in housings as a function of Reynolds number, axial and radial clearance and blade aspect ratio and solidity, J. Basic Eng. Trans. ASME 83 (4) (1961) 719–723.
- [18] R.J. Boness, Churning losses of discs and gears running partially submerged in oil, Proceedings of the ASME Fifth International Power Transmission and Gearing Conference, Chicago, IL, 1989, pp. 355–359.
- [19] A.S. Terekhov, Basic problems of heat calculation of gear reducers, JSME International Conference on Motion and Power Transmissions, Hiroshima, Japan, 1991, pp. 490–495.
- [20] P. Luke, A. Olver, A study of churning losses in dip-lubricated spur gears, J. Aerosp. Eng. 213 (1999) 337–346.
- [21] B.R. Höhn, K. Michaelis, T. Völlmer, Thermal rating of gear drives: balance between power loss and heat dissipation, AGMA, Fall Technical Meeting, Paper No. 96FTM8, 1996, pp. 1–12.
- [22] M.J. Pechersky, N.J. Wittbrodt, An analysis of fluid flow between meshing spur gear teeth, Proceedings of the ASME Fifth International Power Transmission and Gearing Conference, Chicago, IL, 1989, pp. 335–342.
- [23] Y. Diab, F. Ville, H. Houjoh, P. Sainsot, P. Velex, Experimental and numerical investigations on the air-pumping phenomenon in high-speed spur and helical gears, Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci. 219 (2005) 785–800.
- [24] Y. Ariura, T. Ueno, S. Sunamoto, The lubricant churning loss in spur gear systems, Bull. JSME 16 (1973) 881-890.
- [25] C. Changenet, P. Velex, A model for the prediction of churning losses in geared transmissions-preliminary results, J. Mech. Des. 129 (1) (2006) 128-133.
- [26] A.B. Jones, T.A. Harris, Analysis of rolling element idler gear bearing having a deformable outer-race structure, J. Basic Eng. Trans. ASME (1963) 273–278.
- [27] T.A. Harris, J.L. Broschard, Analysis of an improved planetary gear-transmission bearing, J. Basic Eng. Trans. ASME (1964) 457-462.
- [28] J.Y. Liu, Y.P. Chui, Analysis of planetary bearing in a gear transmission system, J. Lubr. Technol. Trans. ASME (1976) 40-46.
- [29] Y. Chiu, M. Myers, A rational approach for determining permissible speed for needle roller bearings, SAE Tech. Paper No. 982030, 1998.
- [30] D. Talbot, S. Li, A. Kahraman, Prediction of mechanical power loss of cylindrical roller bearings under combined radial and moment loading, J. Mech. Des. 135 (2013) 121007.
- [31] A.M. Krstich, Determination of the general equation of the gear efficiency of planetary gear trains, Int. J. Veh. Des. 8 (3) (1987) 365–374.
- [32] E. Pennestri, F. Freudenstein, The mechanical efficiency of epicyclic gear trains, J. Mech. Des. 115 (1993) 645–651.
- [33] H.I. Hsei, L.W. Tsai, The selection of the most efficient clutching sequence associated with automatic transmission mechanisms, J. Mech. Des. 115 (1998) 645–651.
- [34] J.M. Del Castillo, The analytical expression of efficiency of planetary gear trains, Mech. Mach. Theory 37 (2001) 197–214.
   [35] N.E. Anderson, S.H. Loewenthal, J.D. Black, An analytical method to predict efficiency of aircraft gearboxes, NASA Technical Memorandum, 0499-9320, 1984.
- [36] D. Talbot, A. Kahraman, A. Singh, An experimental investigation of the efficiency of planetary gear sets, J. Mech. Des. 134 (2012) 021003.
- [37] D. Talbot, A. Kahraman, A methodology to predict power losses of planetary gear sets, J. Tribol. (2014) (in review).
- [38] A. Kahraman, H. Ligata, K. Kienzle, D. Zini, A kinematics and power flow analysis methodology for automatic transmission planetary gear trains, J. Mech. Des. 126 (2004) 1071–1081.
- [39] H. Ligata, A. Kahraman, A. Singh, An experimental study of the influence of manufacturing errors on the planetary gear stresses and planet load sharing, J. Mech. Des. 13 (4) (2008) 041701.
- [40] H. Ligata, A. Kahraman, A. Singh, Closed-form planet load sharing formulae for planetary gear sets using translational analogy, J. Mech. Des. 131 (2009) 021007.
   [41] M. Inalpolat, A. Kahraman, A dynamic model to predict modulation sidebands of a planetary gear set having manufacturing errors, J. Sound Vib. 329 (2010) 371–393
- [42] M. Inalpolat, A. Kahraman, A theoretical and experimental investigation of modulation sidebands of planetary gear sets, J. Sound Vib. 323 (2009) 677–696.
- 43] H. Xu, A. Singh, D. Maddock, A. Kahraman, J. Hurley, Thermal mapping of an automotive rear drive axle, SAE Int. J. Engines 4 (1) (2011) 888–901.
- [44] H. Xu, A. Singh, A. Kahraman, J. Hurley, S. Shon, Effects of bearing prelaod, oil volume and operating temperature on axle power losses, J. Mech. Des. 134 (2012) 054501.