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PREFACE

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In 1968 the Advanced Research Projects Agency (ARPA) of the U.S. Department of Defense began implementation of a computer-communication network which permits the interconnection of heterogeneous computers at geographically distributed centres throughout the United States. This network has come to be known as the ARPANET and has grown from the initial four node configuration in 1969 to almost forty nodes (including satellite nodes in Hawaii, Norway, and London) in late 1973. The major goal of ARPANET is to achieve resource sharing among the network users. The resources to be shared include not only programs, but also unique facilities such as the powerful ILLIAC IV computer and large global weather data bases that are economically feasible when widely shared.

The ARPANET employs a distributed store-and-forward packet-switching approach that is much better suited for computer-communications networks than the more conventional circuit-switching approach. Reasons favouring packet switching include lower cost, higher capacity, greater reliability and minimal delay. All of these factors are discussed in these Proceedings.

Since the initial ARPA experiment and success, a number of packet-switched networks have been planned and designed and some are well on their way towards fully operational status. These networks include: COST-11 being developed by a multinational European effort, which when completed in 1975, would link together major computer science centres in England, France, Switzerland, and Italy; CYCLADES, a French network linking centres in Paris, Rennes, Toulouse and Grenoble, planned for initial operation in early 1974; the Experimental Packet Switching System (EPSS) of the British Post Office which has reached the advanced design stage, and which when completed will represent the first major packet-switched service offered by a common carrier; and SITA, a fully operational, special purpose network for European airlines, developed and operated by Societe Internationale de Telecommunications Aeronautique.

With so many diverse networks being designed, we, the organizers of the Institute, felt that it was important to bring together most of the networks groups for the purpose of learning each other's design approaches and philosophies and to evaluate each other's methods to determine their advantages and drawbacks. Thus the programme of the Institute focussed upon the major problem areas in the design and operation of these networks. Topics included: Software and Hardware Design, Analytical Techniques, Network Design, Satellite Transmission, Economic Considerations, and Descriptions of Existing and Planned Networks.

Topological Design Considerations in Computer Communication
Networks

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INTRODUCTION

In designing a computer-communication network, a large number of constraints must be considered so as to produce a reasonable network topology. Time delays, throughputs, cost, and reliability are some of the major factors connected with producing an optimum design. Each of these factors encompasses a large amount of detailed analysis which must be completed in order to check a network design.

Since network design is such a complex task, it is important to provide the designer with simple criteria for evaluation. Such criteria permit the network designer to develop an intuitive feeling for his designs, and to be aware of the effects of modifications on its parameters.

This paper uses a linear graph model of computer-communications networks to establish a lower bound on delay and vulnerability¹ for such networks. The networks which are analyzed have the property that their graphs are regular. The lower bound on delay is characterized by measuring the average minimum path length in these regular graphs. The vulnerability of these same networks is shown to be equal to the valence of one

of the vertices for the trivalent graphs, and it is conjectured that this is true for higher valences.

Both these criteria would appear to be useful in that designers may measure their network against these so-called "ideal" networks. This paper will only describe the bounds. Subsequent papers will provide algorithms for construction of graphs which satisfy these bounds and proofs of the vulnerability criteria.

BACKGROUND

Many factors can influence the design of computer-communication networks. Constraints such as acceptable upper bounds on cost and delay and lower bounds on throughput and reliability can strongly influence the final design. In packet-switched store-and-forward networks^{2,3}, topology can play a major role in satisfying the delay and reliability constraints.

Packets leaving a source node are routed to intermediate nodes until they finally arrive at their destination node. At each node, a packet undergoes several forms of delay. These include processing delay, queueing delay (if other packets are also routed over the same channel), transmission delay, and propagation delay on the channel. Typically, a packet carries a checksum with it; thus, a packet arriving at a node cannot be forwarded until the last bit of the packet has arrived and the checksum has been verified. It seems apparent that minimizing the average number of nodes and communication lines a packet has to pass through to reach its destination can reduce delay; of course, an alternative routing strategy which is sensitive to local congestion may not always route the packet along the shortest path⁴.

Reliability of networks can be characterized in several different ways depending upon the degree of refinement one requires in an analysis. Once a network has been designed, there are a variety of computational approaches to analyzing reliability which are neatly summarized in Frank and Chou². According to Frank and Chou, present analytical approaches are computationally intractable for large networks and a combination of analysis and simulation is required to obtain estimates of parameters which characterize the reliability of a network and its components. Very little is known, a priori, about the reliability of networks except that duplication of components and selected over-connection seem to improve it.

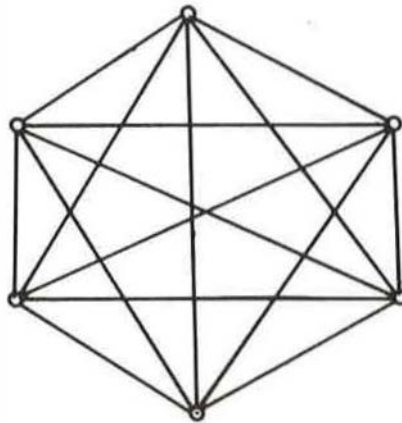
There are several possible strategies for reducing delay and increasing reliability of a network, but in this paper we

concentrate on the effects of topology. Specifically, we examine the relationship between average path length and the number of nodes connected directly to a node (valence of the node).

This paper presents a study of networks which are represented as linear graphs, and it is assumed the reader is familiar with elementary notions of graph theory⁵. The analysis will be directed to packet-switching networks, but most of the work presented should be applicable to networks which use other mechanisms for routing and switching data. The nodes or vertices of the graph will be the packet-switching computers, and the edges or links will represent the transmission lines.

It is possible for networks of arbitrary size to maintain a maximum and average path length of either one or two. If every node in an N -node network is connected to every other node, then we have a network where each node has valence $N-1$ and the shortest and average shortest path lengths are one. Figure 1 shows an example for $N=6$. The reliability of this network is very high, since every node is connected to every other node; however, the number of edges is $\frac{N(N-1)}{2}$, and thus the cost of transmission lines

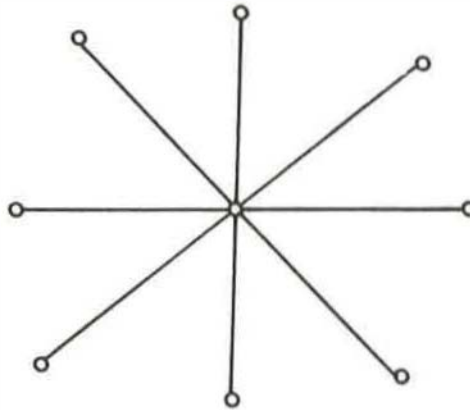
would increase with the square of the number of nodes. Such a scheme is completely impractical for even moderate values of N .



Fully connected 6 node graph
Figure 1

To keep the maximal shortest path length constant at two, one can join all nodes through a central node as shown in the star network of Figure 2. Here transmission line costs increase linearly with N , but reliability and capacity of the central node

pose serious problems.



9 Node star net
Figure 2

Such extreme cases are impractical, and hence intermediate schemes with average valence between 2 and $N-1$ are of prime interest. The next portion of this paper examines the relationship between valence, average path length, and reliability. This relationship will allow an examination of the trade-off between average path length and number of transmission lines in the network; thus, we will develop a measure of the cost of a network as a function of its size under a suitable delay constraint. Some comments will also be made on the relationship of reliability and valence.

AVERAGE PATH LENGTH IN A CLASS OF GRAPHS

This section examines the network topology which produces minimum average path length, and presents an expression which is a lower bound on average path length under reasonable constraints. There is a short discussion of graphs which satisfy this lower bound, but the actual construction techniques are discussed in another paper⁶.

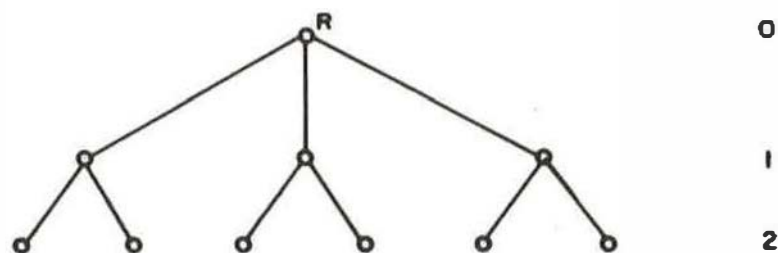
The analysis is started by considering a tree with

$$1 + v \sum_{j=0}^{m-1} (v-1)^j \quad (1)$$

nodes in which each node has either valence V or valence 1. A

tree with 10 nodes and $V=3$ is illustrated in Figure 3. One node is chosen as the root node and is labelled R in Figure 3. The root node R is considered to be at distance zero from itself in the tree, the V nodes adjacent to R are at distance one from R, the $V(V-1)$ nodes adjacent to those at distance one are at distance 2, etc. For ease of expression nodes at distance m from R will be referred to as "nodes at level m ". The levels are shown on the right side of Figure 3. A tree such as the one just described is called a complete tree since new nodes can only be added by starting a new level. In order to find the average path length from R to all nodes in the tree, it is necessary to sum all paths and then divide by $N-1$. Since there are V paths of length 1, $V(V-1)$ paths of length 2, $V(V-1)^2$ paths of length 3, etc., the average path length of a complete tree with m levels is

$$\frac{1}{N-1} \left[V \sum_{j=0}^{m-1} (V-1)^j (j+1) \right]. \quad (2)$$



Complete 3-level Tree
Figure 3

By removing monovalent nodes at the highest numbered level and their associated edges in the complete tree it is possible to arrive at a formula for average path length in a more general tree of this type. If the number of vertices in this tree is N , then the number of vertices removed is

$$1 + V \sum_{j=0}^{m-1} (V-1)^j = N. \quad (3)$$

Then the average path length $P(N, V)$ in this "pruned" tree is

$$P(N, V) = \frac{1}{N-1} \left[v \sum_{j=0}^{m-1} (v-1)^j (j+1) - \left(1 + v \sum_{j=0}^{m-1} (v-1)^j - N \right) m \right], \quad (4)$$

which can be written as

$$P(N, V) = \frac{1}{N-1} \left[v \sum_{j=0}^{m-1} (v-1)^j (j+1-m) + (N-1)m \right]. \quad (5)$$

One might note that

$$1 + v \sum_{j=0}^{m-1} (v-1)^j - N \geq 0. \quad (6)$$

and that m is the smallest integer which satisfies this inequality. Since

$$\sum_{j=0}^{m-1} (v-1)^j = \frac{(v-1)^m - 1}{v-2} \quad (7)$$

then

$$1 + \frac{v}{v-2} \left[(v-1)^m - 1 \right] - N \geq 0. \quad (8)$$

Also, if we use $\{y\}$ to denote the least integer $\geq y$, then

$$m = \left\{ \log_{v-1} \frac{N(v-2) + 2}{v} \right\}, \text{ for } v > 2. \quad (9)$$

If we substitute for m , it is possible to obtain a more explicit relation between the average path length $P(N, V)$, the number of nodes N , and the valence v . It can easily be seen that

$$v \sum_{j=0}^{m-1} (v-1)^j (j+1) = \frac{v}{(v-2)^2} \left[m(v-1)^{m+1} - (m+1)(v-1)^m + 1 \right] \quad (10)$$

and, using (7) and (10), the average path length can be written as

$$\begin{aligned}
P(N, V) &= \frac{1}{N-1} \left[v \left(\frac{m(v-1)^{m+1} - (m+1)(v-1)^m + 1}{(v-2)^2} \right) \right. \\
&\quad \left. - mV \left(\frac{(v-1)^m - 1}{v-2} \right) + (N-1)m \right] \\
&= v \left[\frac{1 - (v-1)^m + m(v-2)}{(v-2)^2(N-1)} \right] + m \\
&= \frac{v}{N-1} \left[\frac{1 - (v-1)^m + m(v-2)}{(v-2)^2} \right] + m \tag{11}
\end{aligned}$$

Expression (11) can be rewritten, using (9), as

$$\begin{aligned}
P(N, V) &= \left\{ \log_{v-1} \frac{N(v-2) + 2}{v} \right\} \\
&\quad + \frac{v}{(N-1)(v-2)^2} \left[1 - (v-1) \left\{ \log_{v-1} \frac{N(v-2) + 2}{v} \right\} \right. \\
&\quad \left. + (v-2) \left\{ \log_{v-1} \frac{N(v-2) + 2}{v} \right\} \right] \tag{12}
\end{aligned}$$

This expression is dominated, for reasonably large N , by the leading logarithmic term. Therefore, the average path length from R to all nodes of the tree in Figure 3 is primarily logarithmic in N except for the case $V=2$ in which it is linear. Curves for $P(N, V)$ for $2 \leq V \leq 6$ and $V+1 \leq N \leq 1000$ are shown in Figure 4, and these graphically illustrate the logarithmic nature of the average path length (and hence the delay) as a function of the number of nodes in the network. Table I presents a few values of average path length for different N and V . The reader should note the drastic reduction in average path length between the cases $V=2$ and $V=3$. But an increase to $V=6$ for $N=900$ only reduces the average path length by a factor of approximately 2, while it increases the number of edges in the graph from 1350 for $V=3$ to 2700 for $V=6$.

This analysis presents a derivation of the average path length for trees similar to the tree of Figure 3. It is certainly the minimum for a tree joining N nodes, since any method of connecting the nodes would either produce the same configuration or would violate the valence constraint. An important related question is whether it is possible to construct a graph which has valence V and for which expression (12) is also the minimum average

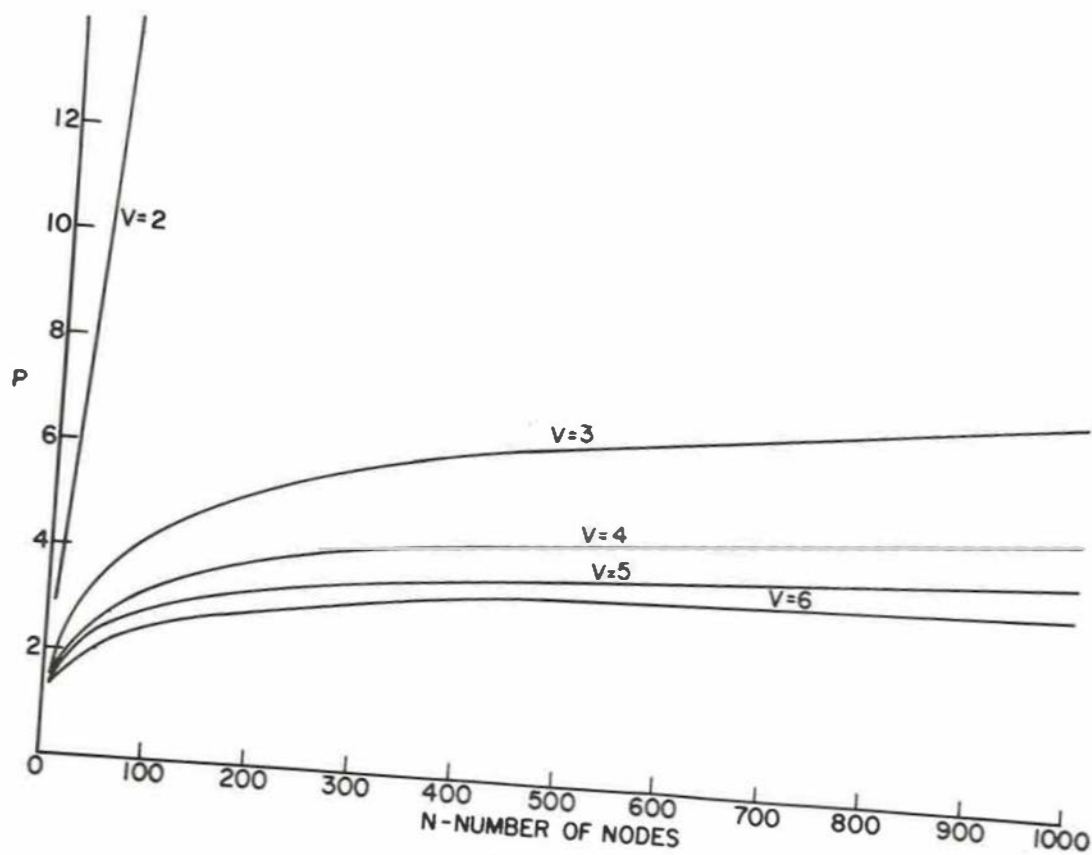


Figure 4 Plot of $P(N, v)$

path length. Such a graph could possibly be constructed if the nodes of valence unity of a tree such as the one in Figure 3 could be joined to make a V -valent graph such that, by choosing every node in turn as the root node, we could redraw the graph as a tree having a complete set of nodes at each of levels $0, 1, 2, \dots, m-1$. Since all distance sets would be identical for each root node, the minimum average path length from each node would be the same, and therefore the minimum average path length for the entire graph would be expression (12). The maximum path length in such graphs is just m ; so equation (9) shows that this quantity also grows logarithmically. Even if graphs with this property cannot be constructed for particular N and V , expression (12) still represents a desirable lower bound on average path length, and thus presents information about the behaviour of average path length, and hence delay, as a function of valence.

		V				
		2	3	4	5	6
N	8	2.29	1.57	1.43	1.29	1.14
	20	5.26	2.37	1.95	1.74	1.68
	50	12.76	3.41	2.59	2.39	2.14
	100	25.25	4.27	3.27	2.70	2.58
	250	62.75	5.55	4.07	3.45	3.08
	500	125.25	6.52	4.57	3.88	3.54
	900	225.25	7.32	5.20	4.38	3.75

Table I
Some Typical Values of $P(N, V)$

Fortunately, we are able to construct graphs which attain this lower bound for many values of N and V . The analysis of this problem in graph construction and the complete enumeration of many of the cases is discussed in another paper⁶. So far, graphs are known for $V=3$, and $N=4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 26, 28, 30, 34$. It is also known that graphs do not exist for certain values of N ⁶. An example of such a minimum average path length graph is shown in Figure 5. This is the graph for $V=3$ and $N=10$, and is the well known Petersen graph.

RELIABILITY OF NETWORKS

This section describes some preliminary results on the reliability of networks with minimum average path length. In particular, vulnerability of these networks is examined.

A basic measure of vulnerability¹ is usually defined to be

the number of nodes and/or edges which must be removed from a network or connected graph in order to separate it into two disconnected components. An algorithm has been devised¹ which will compute the vulnerability of a given network, but there are no known results to describe the behaviour of a class of networks. The availability of knowledge of the behaviour of such a class would provide the network designer with an intuitive feeling for what could be achieved.

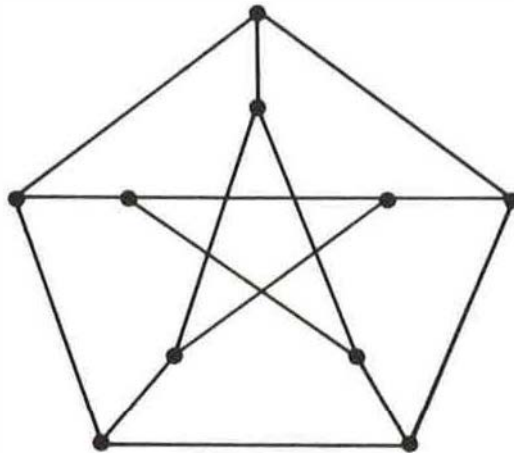


Figure 5
A Minimum Average Path Length Graph
for $N = 10$, $V = 3$, $P(N, V) = 5/3$

The class of graphs with minimum average path length has many properties which make it amenable to vulnerability analysis.

It has been shown⁷ that the class of cubic or trivalent graphs satisfying the minimum average path length constraint are 3 - connected. This means that at least 3 nodes must be removed from the network before two disconnected components can be realized.

If we define connectivity or node-connectivity $K = K(G)$ as the minimum number of nodes in a graph G whose removal results in a trivial graph or a graph of two or more components, define line-connectivity $\lambda = \lambda(G)$ in an analogous fashion, and define $\delta = \delta(G)$ as the minimum valence in a graph, then these three quantities are related by an inequality due to Whitney^{5,8}, namely,

$$K(G) \leq \lambda(G) \leq \delta(G).$$

This inequality implies that, if each node of a graph has uniform valence V and if $K=V$, then $\lambda=V$ also. For the case $V=3$, then $\lambda=3$.

We conjecture that graphs with minimum average path length and valence V are V -connected for $V > 3$. This conjecture seems reasonable, since graphs which satisfy the path length condition would have vertices "closely coupled" by edges.

The question of separating a graph into components by removing a mixed set of points and lines has been studied^{5,9}. The separation of a graph G is characterized by a connectivity pair (a,b) which is an ordered pair of non-negative integers such that there is a set of a nodes and b edges whose removal disconnects the graph, and there is no set of $a-1$ nodes and b edges or a nodes and $b-1$ edges with this property.

For graphs which are V -connected, it has been shown⁷ that there is a unique sequence of connectivity pairs which completely characterizes the vulnerability of the graph. The general expression for this sequence is

$$(v=0), (v-1,1), (v-2,2) \dots (2,v-2), (1,v-1), (0,v).$$

The sum of the two entries in each pair is a constant V , and this means that a combination of nodes and edges whose sum is V will completely disconnect the graph or reduce it to the trivial graph.

CONCLUSIONS

This paper has studied certain properties of linear graphs representing computer-communication networks. In particular, an expression has been derived which provides a lower bound on the average path length of a graph with N nodes and constant valence V for each node. Graphs which satisfy this lower bound have been constructed for many values of N with particular emphasis on the value $V=3$. These graphs with minimum average path length have other properties which make them useful models for networks. Since they have nodes of constant valence, they also provide a convenient model for reliability analysis. In particular, for $V=3$, it has been shown that the graphs are 3-connected and that their complete connectivity, and hence vulnerability, can be specified. It is conjectured that such graphs with higher valence V are V -connected. If this is the case, their vulnerability can also be completely specified.

This analysis describes an achievable lower bound on both path length and vulnerability for constant valence graphs, and provides a convenient norm against which the network designer can compare his designs.

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