



US007116710B1

(12) **United States Patent**  
**Jin et al.**

(10) **Patent No.:** **US 7,116,710 B1**  
(45) **Date of Patent:** **Oct. 3, 2006**

(54) **SERIAL CONCATENATION OF  
INTERLEAVED CONVOLUTIONAL CODES  
FORMING TURBO-LIKE CODES**

(75) Inventors: **Hui Jin**, Glen Gardner, NJ (US);  
**Aamod Khandekar**, Pasadena, CA  
(US); **Robert J. McEliece**, Pasadena,  
CA (US)

(73) Assignee: **California Institute of Technology**,  
Pasadena, CA (US)

(\*) Notice: Subject to any disclaimer, the term of this  
patent is extended or adjusted under 35  
U.S.C. 154(b) by 735 days.

(21) Appl. No.: **09/861,102**

(22) Filed: **May 18, 2001**

#### **Related U.S. Application Data**

(60) Provisional application No. 60/205,095, filed on May  
18, 2000.

(51) **Int. Cl.**  
**H04B 1/66** (2006.01)

(52) **U.S. Cl.** ..... **375/240**; 375/262; 375/265;  
375/341; 341/51; 341/102; 714/752

(58) **Field of Classification Search** ..... 375/259,  
375/262, 265, 285, 296, 341, 346, 348; 714/746,  
714/752, 755, 756, 786, 792, 794, 795, 796;  
341/51, 52, 56, 102, 103

See application file for complete search history.

(56) **References Cited**

#### **U.S. PATENT DOCUMENTS**

5,392,299 A 2/1995 Rhines et al.  
5,751,739 A \* 5/1998 Seshadri et al. .... 714/746

5,881,093 A 3/1999 Wang et al.  
6,014,411 A \* 1/2000 Wang ..... 375/259  
6,023,783 A 2/2000 Divsalar et al.  
6,031,874 A 2/2000 Chennakeshu et al.  
6,032,284 A 2/2000 Bliss  
6,044,116 A 3/2000 Wang  
6,396,423 B1 \* 5/2002 Laumen et al. .... 341/95  
6,437,714 B1 \* 8/2002 Kim et al. .... 341/81  
2001/0025358 A1 9/2001 Eidson et al.

#### **OTHER PUBLICATIONS**

Wiberg et al., "Codes and Iterative Decoding on General Graphs",  
1995 Intl. Symposium on Information Theory, Sep. 1995, p. 506.\*  
Appendix A.1 "Structure of Parity Check Matrices of Standardized  
LDPC Codes," Digital Video Broadcasting (DVB) User guidelines  
for the second generation system for Broadcasting, Interactive  
Services, News Gathering and other broadband satellite applications  
(DVB-S2) ETSI TR 102 376 V1.1.1. (2005-02) Technical Report.  
pp. 64.

Benedetto et al., "Bandwidth efficient parallel concatenated coding  
schemes," Electronics Letters 31(24):2067-2069 (Nov. 23, 1995).  
Benedetto et al., "Soft-output decoding algorithms in iterative  
decoding of turbo codes," The Telecommunications and Data  
Acquisition (TDA) Progress Report 42-124 for NASA and California  
Institute of Technology Jet Propulsion Laboratory, Joseph H.  
Yuen, Ed., pp. 63-87 (Feb. 15, 1996).

(Continued)

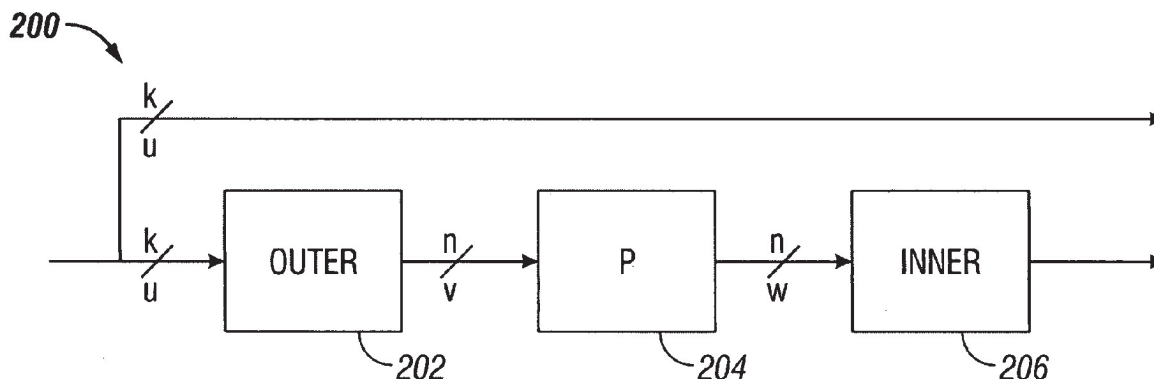
*Primary Examiner*—Dac V. Ha

(74) *Attorney, Agent, or Firm*—Fish & Richardson P.C.

(57) **ABSTRACT**

A serial concatenated coder includes an outer coder and an  
inner coder. The outer coder irregularly repeats bits in a data  
block according to a degree profile and scrambles the  
repeated bits. The scrambled and repeated bits are input to  
an inner coder, which has a rate substantially close to one.

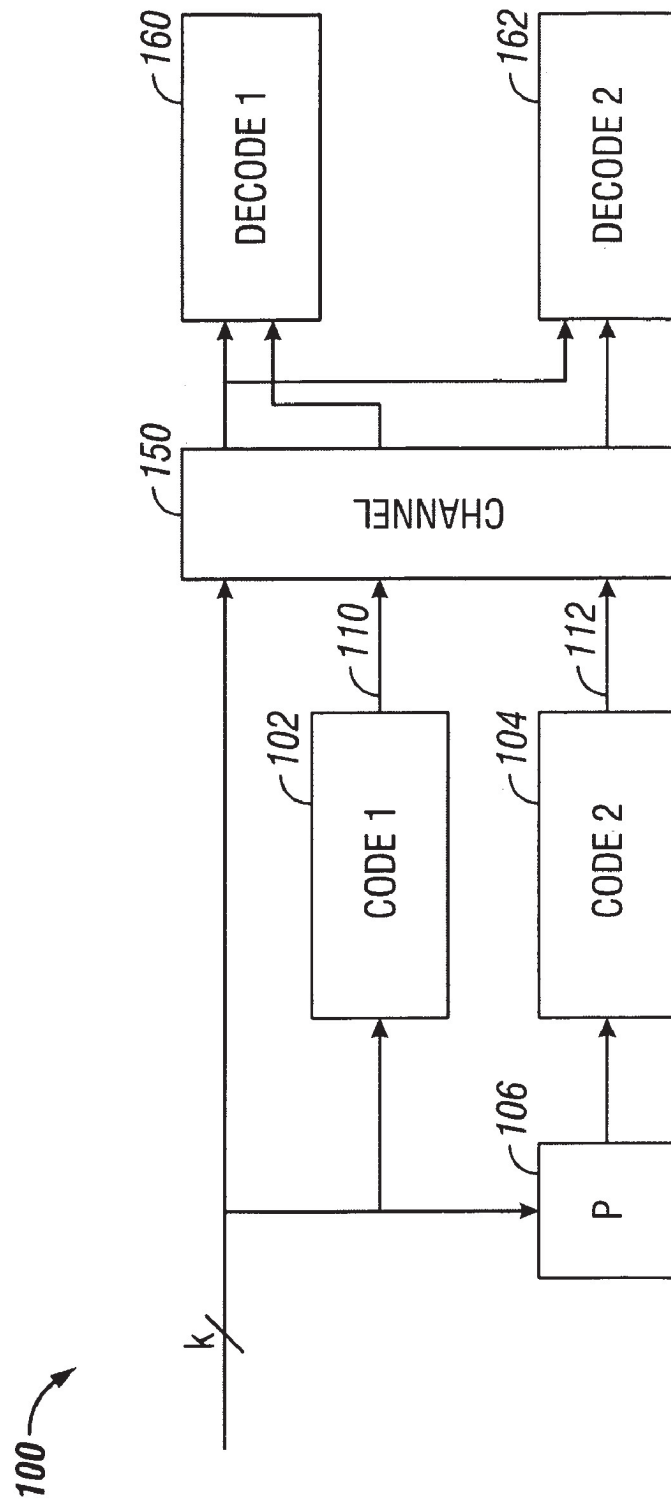
**33 Claims, 5 Drawing Sheets**



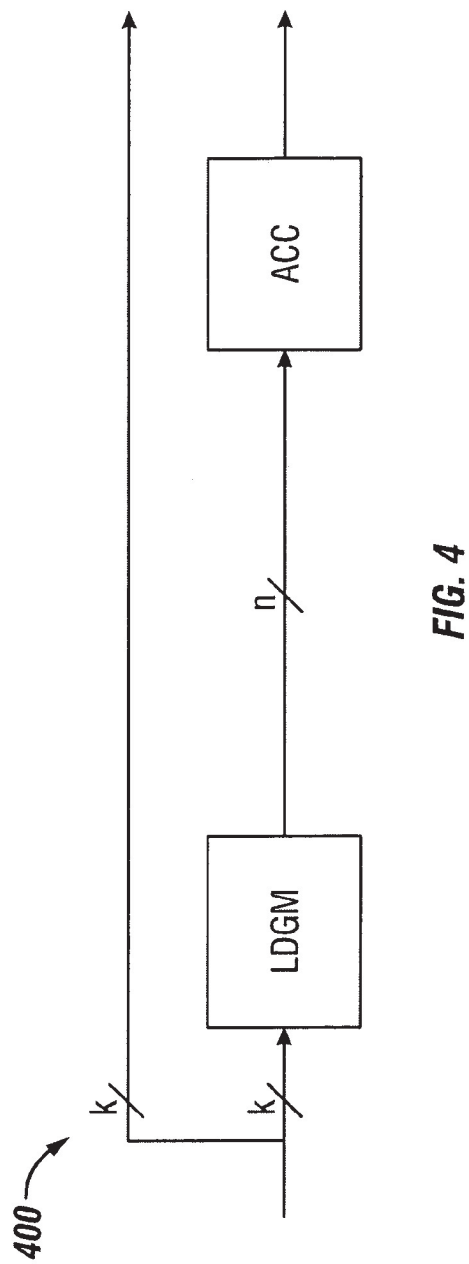
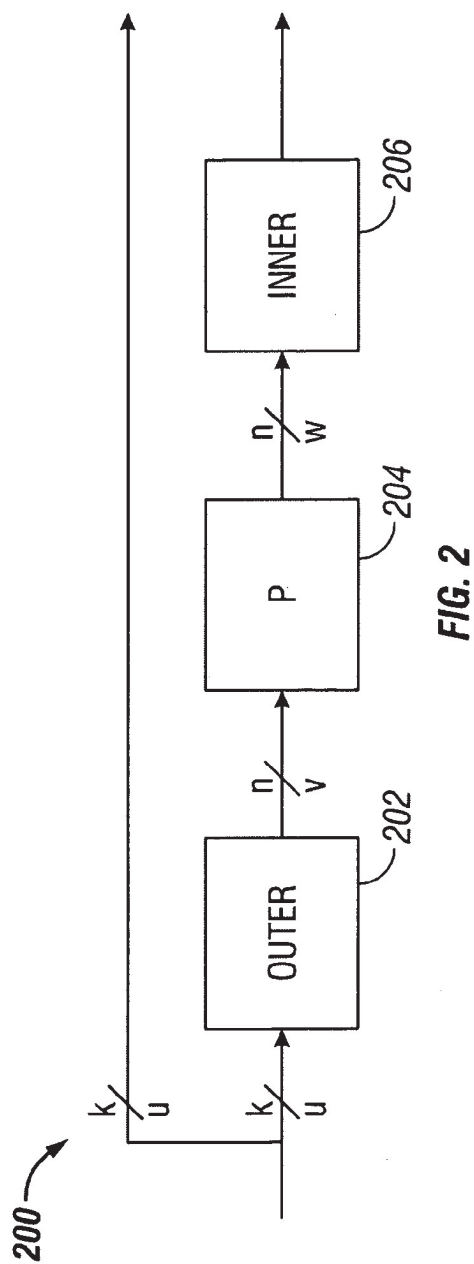
## OTHER PUBLICATIONS

- Benedetto et al., "Serial Concatenation of Interleaved Codes: Performance Analysis, Design, and Iterative Decoding," The Telecommunications and Data Acquisition (TDA) Progress Report 42-126 for NASA and California Institute of Technology Jet Propulsion Laboratory, Joseph H. Yuen, Ed., pp. 1-26 (Aug. 15, 1996).
- Benedetto et al., "A Soft-Input Soft-Output Maximum A Posteriori (MAP) Module to Decode Parallel and Serial Concatenated Codes," The Telecommunications and Data Acquisition (TDA) Progress Report 42-127 for NASA and California Institute of Technology Jet Propulsion Laboratory, Joseph H. Yuen, Ed., pp. 1-20 (Nov. 15, 1996).
- Benedetto et al., "Parallel Concatenated Trellis Coded Modulation," ICC '96, IEEE, pp. 974-978, (Jun. 1996).
- Benedetto, S. et al., "A Soft-Input Soft-Output APP Module for Iterative Decoding of Concatenated Codes," IEEE Communications Letters 1(1):22-24 (Jan. 1997).
- Benedetto et al., "Serial Concatenation of interleaved codes: performance analysis, design, and iterative decoding," Proceedings from the IEEE 1997 International Symposium on Information Theory (ISIT), Ulm, Germany, p. 106, Jun. 29-Jul. 4, 1997.
- Benedetto et al., "Serial Concatenated Trellis Coded Modulation with Iterative Decoding," Proceedings from IEEE 1997 International Symposium on Information Theory (ISIT), Ulm, Germany, p. 8, Jun. 29-Jul. 4, 1997.
- Benedetto et al., "Design of Serially Concatenated Interleaved Codes," ICC 97, Montreal, Canada, pp. 710-714, (Jun. 1997).
- Berrou et al., "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes," ICC pp. 1064-1070 (1993).
- Digital Video Broadcasting (DVB) User guidelines for the second generation system for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications (DVB-S2) ETSI TR 102 376 V1.1.1. (Feb. 2005) Technical Report, pp. 1-104 (Feb. 15, 2005).
- Divsalar et al., "Coding Theorems for 'Turbo-Like' Codes," Proceedings of the 36<sup>th</sup> Annual Allerton Conference on Communication, Control, and Computing, Sep. 23-25 1998, Allerton House, Monticello, Illinois, pp. 201-210 (1998).
- Divsalar, D. et al., "Multiple Turbo Codes for Deep-Space Communications," The Telecommunications and Data Acquisition (TDA) Progress Report 42-121 for NASA and California Institute of Technology Jet Propulsion Laboratory, Joseph H. Yuen, Ed., pp. 60-77 (May 15, 1995).
- Divsalar, D. et al., "On the Design of Turbo Codes," The Telecommunications and Data Acquisition (TDA) Progress Report 42-123 for NASA and California Institute of Technology Jet Propulsion Laboratory, Joseph H. Yuen, Ed., pp. 99-131 (Nov. 15, 1995).
- Divsalar, D. et al., "Low-rate turbo codes for Deep Space Communications," Proceedings from the 1995 IEEE International Symposium on Information Theory, Sep. 17-22, 1995, Whistler, British Columbia, Canada, p. 35.
- Divsalar, D. et al., "Turbo Codes for PCS Applications," ICC 95, IEEE, Seattle, WA, pp. 54-59 (Jun. 1995).
- Divsalar, D. et al., "Multiple Turbo Codes," MILCOM 95, San Diego, CA pp. 279-285 (Nov. 5-6, 1995).
- Divsalar et al., "Effective free distance of turbo codes," Electronics Letters 32(5): 445-446 (Feb. 29, 1996).
- Divsalar, D. et al., "Hybrid concatenated codes and Iterative Decoding," Proceedings from the IEEE 1997 International Symposium on Information Theory (ISIT), Ulm, Germany, p. 10 (Jun. 29-Jul. 4, 1997).
- Divsalar, D. et al., "Serial Turbo Trellis Coded Modulation with Rate-1 Inner Code," Proceedings from the IEEE 2000 International Symposium on Information Theory (ISIT), Italy, pp. 1-14 (Jun. 2000).
- Jin et al., "Irregular Repeat - Accumulate Codes," 2<sup>nd</sup> International Symposium on Turbo Codes & Related Topics, Sep. 4-7, 2000, Brest, France, 25 slides, (presented on Sep. 4, 2000).
- Jin et al., "Irregular Repeat-Accumulate Codes," 2<sup>nd</sup> International Symposium on Turbo Codes & Related Topics, Sep. 4-7, 2000, Brest, France, pp. 1-8 (2000).
- Richardson, et al., "Design of capacity approaching irregular low density parity check codes," IEEE Trans. Inform. Theory 47: 619-637 (Feb. 2001).
- Richardson, T. and R. Urbanke, "Efficient encoding of low-density parity check codes," IEEE Trans. Inform. Theory 47: 638-656 (Feb. 2001).

\* cited by examiner



**FIG. 1**  
(Prior Art)





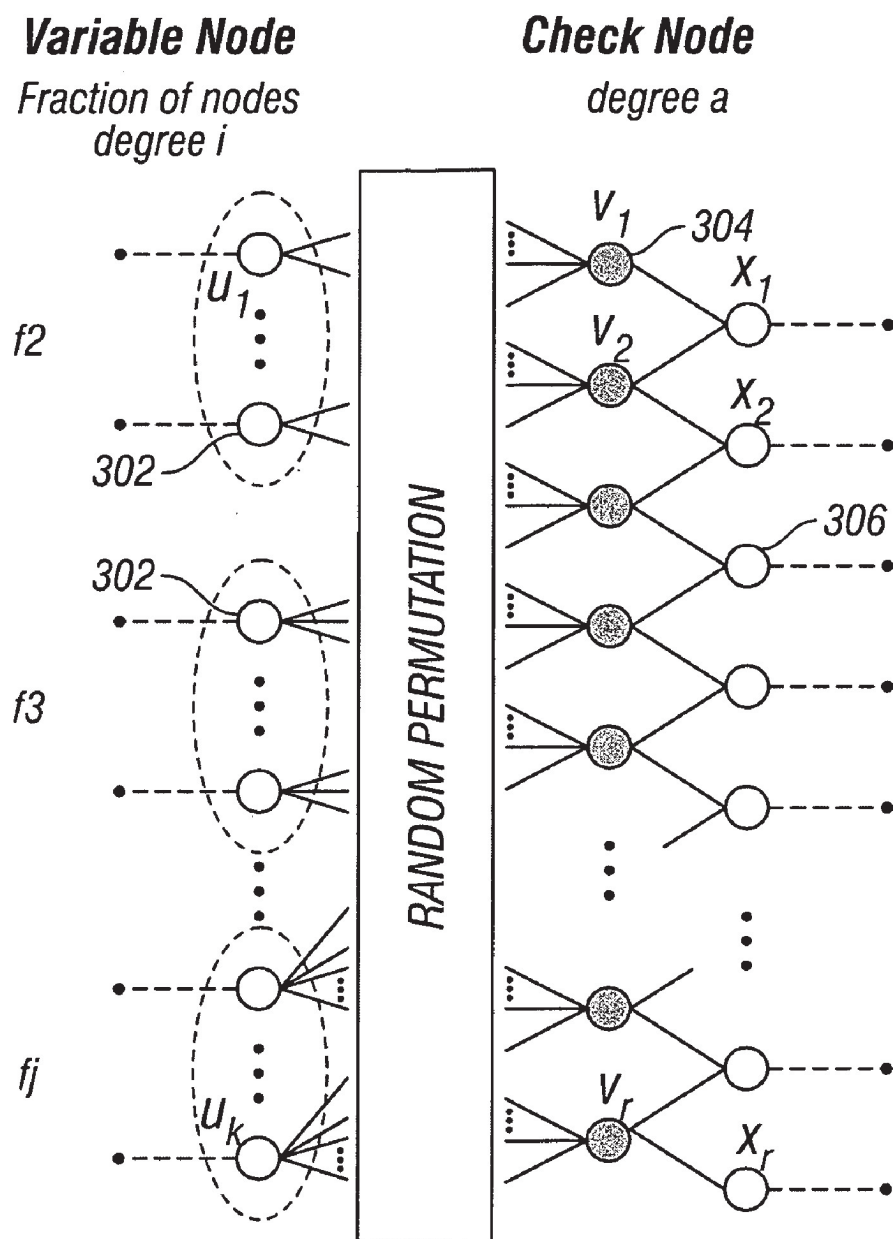


FIG. 3

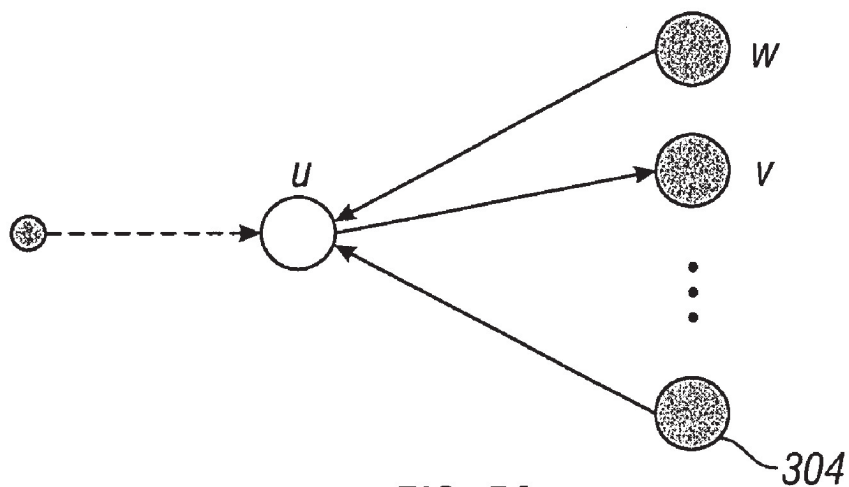


FIG. 5A

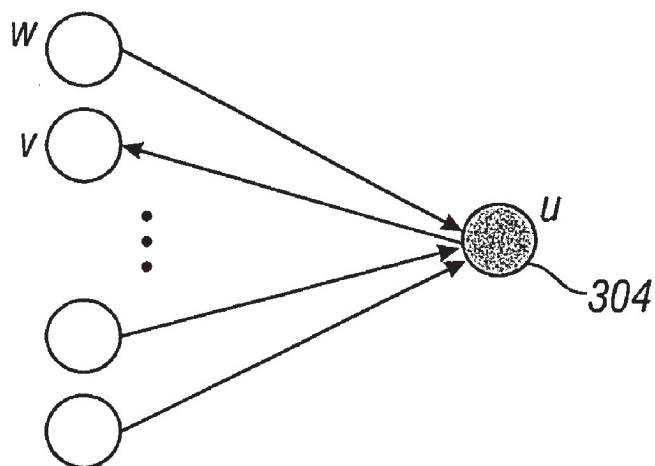


FIG. 5B

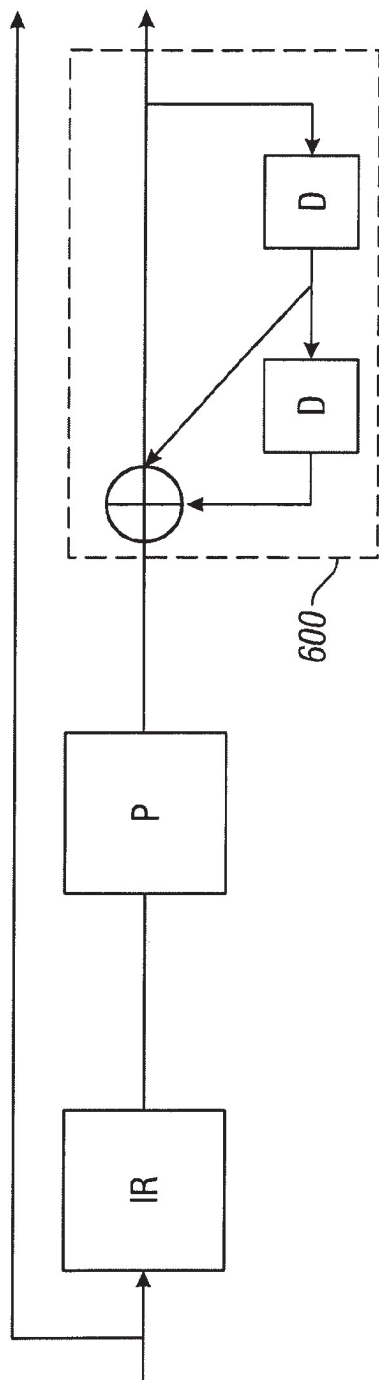


FIG. 6

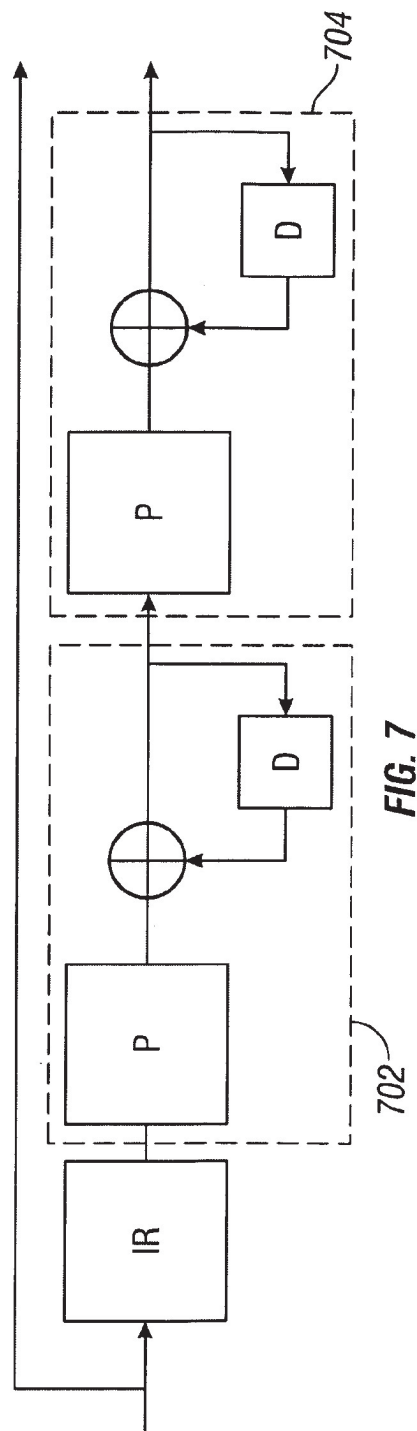


FIG. 7

1

# SERIAL CONCATENATION OF INTERLEAVED CONVOLUTIONAL CODES FORMING TURBO-LIKE CODES

## CROSS-REFERENCE TO RELATED APPLICATIONS

This application claims priority to U.S. Provisional Application Ser. No. 60/205,095, filed on May 18, 2000, and to U.S. application Ser. No. 09/922,852, filed on Aug. 18, 2000 and entitled Interleaved Serial Concatenation Forming Turbo-Like Codes.

## GOVERNMENT LICENSE RIGHTS

The U.S. Government has a paid-up license in this invention and the right in limited circumstances to require the patent owner to license others on reasonable terms as provided for by the terms of Grant No. CCR-9804793 awarded by the National Science Foundation.

## BACKGROUND

Properties of a channel affect the amount of data that can be handled by the channel. The so-called "Shannon limit" defines the theoretical limit of the amount of data that a channel can carry.

Different techniques have been used to increase the data rate that can be handled by a channel. "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes," by Berrou et al. ICC, pp 1064-1070, (1993), described a new "turbo code" technique that has revolutionized the field of error correcting codes. Turbo codes have sufficient randomness to allow reliable communication over the channel at a high data rate near capacity. However, they still retain sufficient structure to allow practical encoding and decoding algorithms. Still, the technique for encoding and decoding turbo codes can be relatively complex.

A standard turbo code **100** is shown in FIG. 1. A block of  $k$  information bits is input directly to a first coder **102**. A  $k$  bit interleaver **106** also receives the  $k$  bits and interleaves them prior to applying them to a second coder **104**. The second coder produces an output that has more bits than its input, that is, it is a coder with rate that is less than 1. The coders **102**, **104** are typically recursive convolutional coders.

Three different items are sent over the channel **150**: the original  $k$  bits, first encoded bits **110**, and second encoded bits **112**. At the decoding end, two decoders are used: a first constituent decoder **160** and a second constituent decoder **162**. Each receives both the original  $k$  bits, and one of the encoded portions **110**, **112**. Each decoder sends likelihood estimates of the decoded bits to the other decoders. The estimates are used to decode the uncoded information bits as corrupted by the noisy channel.

## SUMMARY

A coding system according to an embodiment is configured to receive a portion of a signal to be encoded, for example, a data block including a fixed number of bits. The coding system includes an outer coder, which repeats and scrambles bits in the data block. The data block is apportioned into two or more sub-blocks, and bits in different sub-blocks are repeated a different number of times according to a selected degree profile. The outer coder may include a repeater with a variable rate and an interleaver. Alternatively, the outer coder may be a low-density generator matrix (LDGM) coder.

2

The repeated and scrambled bits are input to an inner coder that has a rate substantially close to one. The inner coder may include one or more accumulators that perform recursive modulo two addition operations on the input bit stream.

The encoded data output from the inner coder may be transmitted on a channel and decoded in linear time at a destination using iterative decoding techniques. The decoding techniques may be based on a Tanner graph representation of the code.

## BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a schematic diagram of a prior "turbo code" system.

FIG. 2 is a schematic diagram of a coder according to an embodiment.

FIG. 3 is a Tanner graph for an irregular repeat and accumulate (IRA) coder.

FIG. 4 is a schematic diagram of an IRA coder according to an embodiment.

FIG. 5A illustrates a message from a variable node to a check node on the Tanner graph of FIG. 3.

FIG. 5B illustrates a message from a check node to a variable node on the Tanner graph of FIG. 3.

FIG. 6 is a schematic diagram of a coder according to an alternate embodiment.

FIG. 7 is a schematic diagram of a coder according to another alternate embodiment.

## DETAILED DESCRIPTION

FIG. 2 illustrates a coder **200** according to an embodiment. The coder **200** may include an outer coder **202**, an interleaver **204**, and inner coder **206**. The coder may be used to format blocks of data for transmission, introducing redundancy into the stream of data to protect the data from loss due to transmission errors. The encoded data may then be decoded at a destination in linear time at rates that may approach the channel capacity.

The outer coder **202** receives the uncoded data. The data may be partitioned into blocks of fixed size, say  $k$  bits. The outer coder may be an  $(n,k)$  binary linear block coder, where  $n > k$ . The coder accepts as input a block  $u$  of  $k$  data bits and produces an output block  $v$  of  $n$  data bits. The mathematical relationship between  $u$  and  $v$  is  $v = T_0 u$ , where  $T_0$  is an  $n \times k$  matrix, and the rate of the coder is  $k/n$ .

The rate of the coder may be irregular, that is, the value of  $T_0$  is not constant, and may differ for sub-blocks of bits in the data block. In an embodiment, the outer coder **202** is a repeater that repeats the  $k$  bits in a block a number of times  $q$  to produce a block with  $n$  bits, where  $n = qk$ . Since the repeater has an irregular output, different bits in the block may be repeated a different number of times. For example, a fraction of the bits in the block may be repeated two times, a fraction of bits may be repeated three times, and the remainder of bits may be repeated four times. These fractions define a degree sequence, or degree profile, of the code.

The inner coder **206** may be a linear rate-1 coder, which means that the  $n$ -bit output block  $x$  can be written as  $x = T_1 w$ , where  $T_1$  is a nonsingular  $n \times n$  matrix. The inner coder **210** can have a rate that is close to 1, e.g., within 50%, more preferably 10% and perhaps even more preferably within 1% of 1.

In an embodiment, the inner coder **206** is an accumulator, which produces outputs that are the modulo two (mod-2) partial sums of its inputs. The accumulator may be a

3

truncated rate-1 recursive convolutional coder with the transfer function  $1/(1+D)$ . Such an accumulator may be considered a block coder whose input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by the formula

$$y_1 = x_1$$

$$y_2 = x_1 \oplus x_2$$

$$y_3 = x_1 \oplus x_2 \oplus x_3$$

$$y_n = x_1 \oplus x_2 \oplus x_3 \oplus \dots \oplus x_n$$

where " $\oplus$ " denotes mod-2, or exclusive-OR (XOR), addition. An advantage of this system is that only mod-2 addition is necessary for the accumulator. The accumulator may be embodied using only XOR gates, which may simplify the design.

The bits output from the outer coder **202** are scrambled before they are input to the inner coder **206**. This scrambling may be performed by the interleaver **204**, which performs a pseudo-random permutation of an input block  $v$ , yielding an output block  $w$  having the same length as  $v$ .

The serial concatenation of the interleaved irregular repeat code and the accumulate code produces an irregular repeat and accumulate (IRA) code. An IRA code is a linear code, and as such, may be represented as a set of parity checks. The set of parity checks may be represented in a bipartite graph, called the Tanner graph, of the code. FIG. 3 shows a Tanner graph **300** of an IRA code with parameters  $(f_1, \dots, f_r; a)$ , where  $f_i \geq 0$ ,  $\sum_i f_i = 1$  and " $a$ " is a positive integer. The Tanner graph includes two kinds of nodes: variable nodes (open circles) and check nodes (filled circles). There are  $k$  variable nodes **302** on the left, called information nodes. There are  $r$  variable nodes **306** on the right, called parity nodes. There are  $r = (k \sum_i f_i) / a$  check nodes **304** connected between the information nodes and the parity nodes. Each information node **302** is connected to a number of check nodes **304**. The fraction of information nodes connected to exactly  $i$  check nodes is  $f_i$ . For example, in the Tanner graph **300**, each of the  $f_2$  information nodes are connected to two check nodes, corresponding to a repeat of  $q=2$ , and each of the  $f_3$  information nodes are connected to three check nodes, corresponding to  $q=3$ .

Each check node **304** is connected to exactly " $a$ " information nodes **302**. In FIG. 3,  $a=3$ . These connections can be made in many ways, as indicated by the arbitrary permutation of the  $ra$  edges joining information nodes **302** and check nodes **304** in permutation block **310**. These connections correspond to the scrambling performed by the interleaver **204**.

In an alternate embodiment, the outer coder **202** may be a low-density generator matrix (LDGM) coder that performs an irregular repeat of the  $k$  bits in the block, as shown in FIG. 4. As the name implies, an LDGM code has a sparse (low-density) generator matrix. The IRA code produced by the coder **400** is a serial concatenation of the LDGM code and the accumulator code. The interleaver **204** in FIG. 2 may be excluded due to the randomness already present in the structure of the LDGM code.

If the permutation performed in permutation block **310** is fixed, the Tanner graph represents a binary linear block code with  $k$  information bits  $(u_1, \dots, u_k)$  and  $r$  parity bits  $(x_1, \dots, x_r)$ , as follows. Each of the information bits is associated with one of the information nodes **302**, and each of the parity bits is associated with one of the parity nodes **306**. The value of a parity bit is determined uniquely by the condition that the mod-2 sum of the values of the variable nodes connected

4

to each of the check nodes **304** is zero. To see this, set  $x_0=0$ . Then if the values of the bits on the  $ra$  edges coming out of the permutation box are  $(v_1, \dots, v_{ra})$ , then we have the recursive formula

$$x_j = x_{j-1} + \sum_{i=1}^{\lambda} v_{(j-1)\lambda+i}$$

for  $j=1, 2, \dots, r$ . This is in effect the encoding algorithm.

Two types of IRA codes are represented in FIG. 3, a nonsystematic version and a systematic version. The nonsystematic version is an  $(r, k)$  code, in which the codeword corresponding to the information bits  $(u_1, \dots, u_k)$  is  $(x_1, \dots, x_r)$ . The systematic version is a  $(k+r, k)$  code, in which the codeword is  $(u_1, \dots, u_k; x_1, \dots, x_r)$ .

The rate of the nonsystematic code is

$$R_{\text{nssys}} = \frac{a}{\sum_i if_i}$$

The rate of the systematic code is

$$R_{\text{sys}} = \frac{a}{a + \sum_i if_i}$$

For example, regular repeat and accumulate (RA) codes can be considered nonsystematic IRA codes with  $a=1$  and exactly one  $f_i$  equal to 1, say  $f_q=1$ , and the rest zero, in which case  $R_{\text{nssys}}$  simplifies to  $R=1/q$ .

The IRA code may be represented using an alternate notation. Let  $\lambda_i$  be the fraction of edges between the information nodes **302** and the check nodes **304** that are adjacent to an information node of degree  $i$ , and let  $\rho_i$  be the fraction of such edges that are adjacent to a check node of degree  $i+2$  (i.e., one that is adjacent to  $i$  information nodes). These edge fractions may be used to represent the IRA code rather than the corresponding node fractions. Define  $\lambda(x) = \sum_i \lambda_i x^{i-1}$  and  $\rho(x) = \sum_i \rho_i x^{i-1}$  to be the generating functions of these sequences. The pair  $(\lambda, \rho)$  is called a degree distribution. For  $L(x) = \sum_i f_i x_i$ ,

$$f_i = \frac{\lambda_i / i}{\sum_j \lambda_j / j}$$

$$L(x) = \int_0^x \lambda(t) dt / \int_0^1 \lambda(t) dt$$

The rate of the systematic IRA code given by the degree distribution is given by

$$\text{Rate} = \left( 1 + \frac{\sum_j \rho_j / j}{\sum_j \lambda_j / j} \right)^{-1}$$

"Belief propagation" on the Tanner Graph realization may be used to decode IRA codes. Roughly speaking, the belief



## 5

propagation decoding technique allows the messages passed on an edge to represent posterior densities on the bit associated with the variable node. A probability density on a bit is a pair of non-negative real numbers  $p(0)$ ,  $p(1)$  satisfying  $p(0)+p(1)=1$ , where  $p(0)$  denotes the probability of the bit being 0,  $p(1)$  the probability of it being 1. Such a pair can be represented by its log likelihood ratio,  $m=\log(p(0)/p(1))$ . The outgoing message from a variable node  $u$  to a check node  $v$  represents information about  $u$ , and a message from a check node  $u$  to a variable node  $v$  represents information about  $u$ , as shown in FIGS. 5A and 5B, respectively.

The outgoing message from a node  $u$  to a node  $v$  depends on the incoming messages from all neighbors  $w$  of  $u$  except  $v$ . If  $u$  is a variable message node, this outgoing message is

$$m(u \rightarrow v) = \sum_{w \neq v} m(w \rightarrow u) + m_0(u)$$

where  $m_0(u)$  is the log-likelihood message associated with  $u$ . If  $u$  is a check node, the corresponding formula is

$$\tanh \frac{m(u \rightarrow v)}{2} = \prod_{w \neq v} \tanh \frac{m(w \rightarrow u)}{2}$$

Before decoding, the messages  $m(w \rightarrow u)$  and  $m(u \rightarrow v)$  are initialized to be zero, and  $m_0(u)$  is initialized to be the log-likelihood ratio based on the channel received information. If the channel is memoryless, i.e., each channel output only relies on its input, and  $y$  is the output of the channel code bit  $u$ , then  $m_0(i)=\log(p(u=0|y)/p(u=1|y))$ . After this initialization, the decoding process may run in a fully parallel and local manner. In each iteration, every variable/check node receives messages from its neighbors, and sends back updated messages. Decoding is terminated after a fixed number of iterations or detecting that all the constraints are satisfied. Upon termination, the decoder outputs a decoded sequence based on the messages  $m(u)=\sum w_m(w \rightarrow u)$ .

Thus, on various channels, iterative decoding only differs in the initial messages  $m_0(u)$ . For example, consider three memoryless channel models: a binary erasure channel (BEC); a binary symmetric channel (BSC); and an additive white Gaussian noise (AGWN) channel.

In the BEC, there are two inputs and three outputs. When 0 is transmitted, the receiver can receive either 0 or an erasure E. An erasure E output means that the receiver does not know how to demodulate the output. Similarly, when 1 is transmitted, the receiver can receive either 1 or E. Thus, for the BEC,  $y \in \{0, E, 1\}$ , and

$$m_0(u) = \begin{cases} +\infty & \text{if } y = 0 \\ 0 & \text{if } y = E \\ -\infty & \text{if } y = 1 \end{cases}$$

In the BSC, there are two possible inputs (0,1) and two possible outputs (0, 1). The BSC is characterized by a set of

## 6

conditional probabilities relating all possible outputs to possible inputs. Thus, for the BSC  $y \in \{0, 1\}$ ,

$$m_0(u) = \begin{cases} \log \frac{1-p}{p} & \text{if } y = 0 \\ -\log \frac{1-p}{p} & \text{if } y = 1 \end{cases}$$

and

In the AWGN, the discrete-time input symbols  $X$  take their values in a finite alphabet while channel output symbols  $Y$  can take any values along the real line. There is assumed to be no distortion or other effects other than the addition of white Gaussian noise. In an AWGN with a Binary Phase Shift Keying (BPSK) signaling which maps 0 to the symbol with amplitude  $\sqrt{E_s}$  and 1 to the symbol with amplitude  $-\sqrt{E_s}$ , output  $y \in \mathbb{R}$ , then

$$m_0(u) = 4y\sqrt{E_s}/N_0$$

where  $N_0/2$  is the noise power spectral density.

The selection of a degree profile for use in a particular transmission channel is a design parameter, which may be affected by various attributes of the channel. The criteria for selecting a particular degree profile may include, for example, the type of channel and the data rate on the channel. For example, Table 1 shows degree profiles that have been found to produce good results for an AWGN channel model.

TABLE 1

a	2	3	4
$\lambda_2$	0.139025	0.078194	0.054485
$\lambda_3$	0.2221555	0.128085	0.104315
$\lambda_5$		0.160813	
$\lambda_6$	0.638820	0.036178	0.126755
$\lambda_{10}$			0.229816
$\lambda_{11}$			0.016484
$\lambda_{12}$		0.108828	
$\lambda_{13}$		0.487902	
$\lambda_{14}$			
$\lambda_{16}$			
$\lambda_{27}$			0.450302
$\lambda_{28}$			0.017842
Rate	0.333364	0.333223	0.333218
$\sigma_{GA}$	1.1840	1.2415	1.2615
$\sigma^*$	1.1981	1.2607	1.2780
( $E_b/N_0$ ) * (dB)	0.190	-0.250	-0.371
S.L. (dB)	-0.4953	-0.4958	-0.4958

Table 1 shows degree profiles yielding codes of rate approximately  $1/3$  for the AWGN channel and with  $a=2, 3, 4$ . For each sequence, the Gaussian approximation noise threshold, the actual sum-product decoding threshold and the corresponding energy per bit ( $E_b$ )-noise power ( $N_0$ ) ratio in dB are given. Also listed is the Shannon limit (S.L.).

As the parameter "a" is increased, the performance improves. For example, for  $a=4$ , the best code found has an iterative decoding threshold of  $E_b/N_0=-0.371$  dB, which is only 0.12 dB above the Shannon limit.

The accumulator component of the coder may be replaced by a "double accumulator" 600 as shown in FIG. 6. The double accumulator can be viewed as a truncated rate 1 convolutional coder with transfer function  $1/(1+D+D^2)$ .

Alternatively, a pair of accumulators may be added, as shown in FIG. 7. There are three component codes: the "outer" code 700, the "middle" code 702, and the "inner"



code 704. The outer code is an irregular repetition code, and the middle and inner codes are both accumulators.

IRA codes may be implemented in a variety of channels, including memoryless channels, such as the BEC, BSC, and AWGN, as well as channels having non-binary input, non-symmetric and fading channels, and/or channels with memory.

A number of embodiments have been described. Nevertheless, it will be understood that various modifications may be made without departing from the spirit and scope of the invention. Accordingly, other embodiments are within the scope of the following claims.

The invention claimed is:

1. A method of encoding a signal, comprising:  
obtaining a block of data in the signal to be encoded;  
partitioning said data block into a plurality of sub-blocks,  
each sub-block including a plurality of data elements;  
first encoding the data block to form a first encoded data  
block, said first encoding including repeating the data  
elements in different sub-blocks a different number of  
times;  
interleaving the repeated data elements in the first  
encoded data block; and  
second encoding said first encoded data block using an  
encoder that has a rate close to one.
2. The method of claim 1, wherein said second encoding  
is via a rate 1 linear transformation.
3. The method of claim 1, wherein said first encoding is  
carried out by a first coder with a variable rate less than one,  
and said second encoding is carried out by a second coder  
with a rate substantially close to one.
4. The method of claim 3, wherein the second coder  
comprises an accumulator.
5. The method of claim 4, wherein the data elements  
comprises bits.
6. The method of claim 5, wherein the first coder com-  
prises a repeater operable to repeat different sub-blocks a  
different number of times in response to a selected degree  
profile.
7. The method of claim 4, wherein the first coder com-  
prises a low-density generator matrix coder and the second  
coder comprises an accumulator.
8. The method of claim 1, wherein the second encoding  
uses a transfer function of  $1/(1+D)$ .
9. The method of claim 1, wherein the second encoding  
uses a transfer function of  $1/(1+D+D^2)$ .
10. The method of claim 1, wherein said second encoding  
utilizes two accumulators.
11. A method of encoding a signal, comprising:  
receiving a block of data in the signal to be encoded, the  
data block including a plurality of bits;  
first encoding the data block such that each bit in the data  
block is repeated and two or more of said plurality of  
bits are repeated a different number of times in order to  
form a first encoded data block; and  
second encoding the first encoded data block in such a  
way that bits in the first encoded data block are accu-  
mulated.
12. The method of claim 11, wherein the said second  
encoding is via a rate 1 linear transformation.
13. The method of claim 11, wherein the first encoding is  
via a low-density generator matrix transformation.
14. The method of claim 11, wherein the signal to be  
encoded comprises a plurality of data blocks of fixed size.

15. A coder comprising:

a first coder having an input configured to receive a stream  
of bits, said first coder operative to repeat said stream  
of bits irregularly and scramble the repeated bits; and  
a second coder operative to further encode bits output  
from the first coder at a rate within 10% of one.

16. The coder of claim 15, wherein the stream of bits  
includes a data block, and wherein the first coder is operative  
to apportion said data block into a plurality of sub-blocks  
and to repeat bits in each sub-block a number of times,  
wherein bits in different sub-blocks are repeated a different  
number of times.

17. The coder of claim 16, wherein the second coder  
comprises a recursive convolutional encoder with a transfer  
function of  $1/(1+D)$ .

18. The coder of claim 16, wherein the second coder  
comprises a recursive convolutional encoder with a transfer  
function of  $1/(1+D+D^2)$ .

19. The coder of claim 15, wherein the first coder com-  
prises a repeater having a variable rate and an interleaver.

20. The coder of claim 15, wherein the first coder com-  
prises a low-density generator matrix coder.

21. The coder of claim 15, wherein the second coder  
comprises a rate 1 linear encoder.

22. The coder of claim 21, wherein the second coder  
comprises an accumulator.

23. The coder of claim 22, wherein the second coder  
further comprises a second accumulator.

24. The coder of claim 15, wherein the second coder  
comprises a coder operative to further encode bits output  
from the first coder at a rate within 1% of one.

25. A coding system comprising:

a first coder having an input configured to receive a stream  
of bits, said first coder operative to repeat said stream  
of bits irregularly and scramble the repeated bits;

a second coder operative to further encode bits output  
from the first coder at a rate within 10% of one in order  
to form an encoded data stream; and

a decoder operative to receive the encoded data stream  
and decode the encoded data stream using an iterative  
decoding technique.

26. The coding system of claim 25, wherein the first coder  
comprises a repeater operative to receive a data block  
including a plurality of bits from said stream of bits and to  
repeat bits in the data block a different number of times  
according to a selected degree profile.

27. The coding system of claim 26, wherein the first coder  
comprises an interleaver.

28. The coding system of claim 25, wherein the first coder  
comprises a low-density generator matrix coder.

29. The coding system of claim 25, wherein the second  
coder comprises a rate 1 accumulator.

30. The coding system of claim 25, wherein the decoder  
is operative to decode the encoded data stream using a  
posterior decoding techniques.

31. The coding system of claim 25, wherein the decoder  
is operative to decode the encoded data stream based on a  
Tanner graph representation.

32. The coding system of claim 25, wherein the decoder  
is operative to decode the encoded data stream in linear time.

33. The coding system of claim 25, wherein the second  
coder comprises a coder operative to further encode bits  
output from the first coder at a rate within 1% of one.

UNITED STATES PATENT AND TRADEMARK OFFICE  
**CERTIFICATE OF CORRECTION**

PATENT NO. : 7,116,710 B1  
APPLICATION NO. : 09/861102  
DATED : October 3, 2006  
INVENTOR(S) : Hui Jin, Aamod Khandekar and Robert J. McEliece

Page 1 of 1

It is certified that error appears in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

At column 1, line 8, please amend the paragraph as follows:

This application claims the priority ~~[[to]]~~ of U.S. Provisional Application Ser. No. 60/205,095, filed on May 18, 2000, and ~~[[to]]~~ is a continuation-in-part of U.S. application Ser. No. 09/922,852, filed on Aug. 18, 2000 and entitled Interleaved Serial Concatenation Forming Turbo-Like Codes.

Signed and Sealed this

Twenty-second Day of July, 2008

A handwritten signature in black ink, appearing to read "Jon W. Dudas". The signature is stylized with a large, looped initial "J" and a cursive "Dudas".

JON W. DUDAS  
*Director of the United States Patent and Trademark Office*