

Manual of Photogrammetry



Fourth Edition

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of Göttingen for quantitative evaluation of stereoradiographs.

The analytical method is very versatile and accurate and has important advantages over the analog method, such as increased accuracy of measurements and superior flexibility of procedure. In cases where the final output is to be continuous lines, a computer must be used to convert the point data into plotter commands. Thus the analog method might be preferable in such cases.

The accuracy of different close-range photogrammetric procedures is often determined by experiments and checks. For standardized methods, one may use simple rules-of-thumb gained by experience for judging the accuracy in a specific case. For other methods, more complex means are applied to determine the accuracy of the results. In some methods a rigorous adjustment of the unknowns also includes the calculation of standard errors. In these cases, well-surveyed test fields are used, as in the experiments performed by Hottier (Hottier, 1972, 1974, 1976). For a single stereomodel and analytical data reduction, he found a considerable gain in accuracy as a result of measurement redundancy:

when the number of repeated settings per image point is increased from one to three (gain of about 30% in accuracy);

when the number of targets defining each object point is increased from one to three (gain of about 40%); and

when the number of frames taken from each station is increased from one to three (gain of about 40% in accuracy).

Hottier found no correlation between the gains in accuracy, the geometry of the photographic set-up and the type of camera. He reported (Hottier, 1976) that the maximum gain in accuracy is about 50% and that this is obtainable only through mixing the parameters of measurement redundancy (setting per image point, number of targets defining an object point, and the number of frames per camera station). Hottier's extensive experiments revealed that there is practically no gain beyond 7 or 9 total comparator measurements. The optimum accuracy can be achieved in a number of combinations, for example by choosing one setting per image point, three targets per object point and three frames per camera station. If one chooses one setting per image point, one target per object point, and three frames per camera station, one would expect some 40% gain in accuracy over that of the 1-1-1 case (one comparator setting per image point, one target per object point, and one frame per camera station), according to Hottier's extensive experimentations.

In many special applications of photogrammetry a digital model of the object being mapped is often a more desirable output than the con-

ventional contour map, on account of its flexibility, whether one is interested in measuring the surface geometry or in evaluating the deformations and movement of objects. A digital model provides the user with almost unlimited flexibility in the use of the electronic computer for design and analysis. Mathematical modeling of a surface and its behavior can be conveniently performed using a digital model. Such parameters as surface area and volume as well as deformation of surface points can all be computed directly from the digital model(s). Analytical methods have been developed to generate contours, cross-sections, profiles and perspective views directly from digital data.

The analytical approach can obviously be used in the rather simple case where the inner and outer orientation parameters of the cameras are known, but its advantages and versatility become much more pronounced in the most general case of photogrammetry where a simultaneous solution incorporates the inner and outer orientation parameters of the cameras (two or more), and the object-space coordinates of object points, all as unknowns. Significant advances have been made recently in perfecting the analytical method and in adapting this to solving problems in close-range photogrammetry. For example, through the use of advanced analytical methods, the photogrammetric reduction of photographs from non-metric cameras has become possible.

16.2.3.2.1 MATHEMATICAL FORMULATION IN CLOSE-RANGE PHOTOGRAMMETRY

16.2.3.2.1.1 For Photography from Metric Cameras. With little or no modifications, many of the computational methods which were developed over the years for aerial mapping (chapter II) and aerotriangulation (chapter IX) can be applied in close-range photogrammetric projects. Wong (1976) gave an overview of the mathematical formulation and digital analysis in close-range photogrammetry.

16.2.3.2.1.2 For Photography from Non-Metric Cameras. Because of the lack of fiducial marks in non-metric cameras, special techniques had to be devised. Among the approaches particularly suitable for non-metric photography is the *Direct Linear Transformation (DLT)* approach developed at the University of Illinois (Abdel-Aziz & Karara, 1971, 1974, Karara & Abdel-Aziz, 1974). The solution is based on the concept of direct transformation from comparator coordinates into object-space coordinates, thus by-passing the traditional intermediate step of transforming image coordinates from a comparator system to a photo-coordinate system. As such, the solution makes no use of fiducial marks. The method is based on the following pairs of equations:

$$x + (x - x_0) (K_1 r^2 + K_2 r^4 + K_3 r^6 + \dots) + (r^2 + 2[x - x_0]^2) P_1 + 2(y - y_0)(x - x_0) P_2 = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1} \quad (16.1)$$

$$y + (y - y_0) (K_1 r^2 + K_2 r^4 + K_3 r^6 + \dots) + 2(x - x_0)(y - y_0) P_1 + (r^2 + 2[y - y_0]^2) P_2 = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

where x and y are the measured comparator coordinates of an image point; x_0 and y_0 define the position of the principal point in the comparator coordinate system; $r^2 = (x^2 + y^2)$; K_1, K_2, K_3, P_1, P_2 are lens distortion coefficients; X, Y, Z are the object-space coordinates of the point imaged; and L_1 to L_{11} are eleven unknown constants.

When originally presented in 1971 (Abdel-Aziz and Karara, 1971) the basic equations of

terms (K_2, K_3, P_1, P_2). Eliminating these terms in equation (16.1), and substituting x' for $(x - x_0)$ and y' for $(y - y_0)$, equation (16.1) can be reduced to (assuming that $x_0 = y_0 = 0$):

$$x + x' K_1 r^2 = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1} \quad (16.2)$$

$$y + y' K_1 r^2 = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

A complete documentation of the computer program for the DLT solution was written by Marzan and Karara (1975).

As explained by Wong (1976) (see chapter II) two different approaches can be used in solving equations (16.2). (a) Direct approach. The following equations can be derived directly from equation (16.2)

$$L_1 X + L_2 Y + L_3 Z + L_4 - x L_9 - x Y L_{10} - x Z L_{11} - x K_1 r^2 A - x = r_x$$

$$L_5 X + L_6 Y + L_7 Z + L_8 - y L_9 - y Y L_{10} - y Z L_{11} - y K_1 r^2 A - y = r_y \quad (16.3)$$

DLT did not involve any camera interior orientation constants, and represented a direct linear transformation between comparator coordinates and object-space coordinates. When the DLT mathematical model was later expanded to encompass interior orientation constants the title DLT was retained.

in which $A = L_9 X + L_{10} Y + L_{11} Z + 1$, and r_x and r_y are the residual errors on the condition equations.

Letting $AK_1 = K_1'$, equation (16.3) can be reduced to the following matrix equation:

$$\begin{bmatrix} r_x \\ r_y \end{bmatrix} + \begin{bmatrix} -X & -Y & -Z & -1 & 0 & 0 & 0 & 0 & xX & xY & xZ & xr^2 \\ 0 & 0 & 0 & 0 & -X & -Y & -Z & -1 & yX & yY & yZ & yr^2 \end{bmatrix} L + \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (16.4)$$

The number of constants of interior orientation taken into consideration in equations (16.1) depends on the degree of comprehensiveness desired. On the basis of experimental investigations, Karara and Abdel-Aziz (1974) concluded that, for all practical purposes, only the term K_1 needs to be taken into account in modeling lens distortions and film deformations. In the cases tested by them, no significant improvement in accuracy was gained by incorporating additional

in which $L = [L_1 \ L_2 \ L_3 \ \dots \ L_{11} \ K_1']^T$.

Each object-point which has known object-space coordinates (X, Y, Z) gives rise to two such equations. A minimum of six control points are needed for a unique solution of the 12 unknowns in the L -matrix. It is a direct solution and no iteration is needed. (b) Iterative approach. Let V_x and V_y represent the small random errors in the measured (comparator) coordinates. The following equations can be derived from equation (16.2) by omitting the effects of V_x and V_y in the lens distortion constant K_1 .

$$V_x - \frac{X}{A} L_1 - \frac{Y}{A} L_2 - \frac{Z}{A} L_3 - \frac{1}{A} L_4 + \frac{xX}{A} L_9 + \frac{xY}{A} L_{10} + \frac{xZ}{A} L_{11} + xr^2 K_1 + \frac{x}{A} = 0$$

$$V_y - \frac{X}{A} L_5 - \frac{Y}{A} L_6 - \frac{Z}{A} L_7 + \frac{1}{A} L_8 + \frac{yX}{A} L_9 + \frac{yY}{A} L_{10} + \frac{yZ}{A} L_{11} + yr^2 K_1 + \frac{y}{A} = 0 \quad (16.5)$$

in which $A = L_9 X + L_{10} Y + L_{11} Z + 1$.

Since equation (16.5) is non-linear, it must be linearized and the subsequent least squares solution must be iterative.

g these terms in x' for $(x - x_0)$ 6.1) can be re- = 0):

$$\frac{L_9 Z + L_4}{L_{11} Z + 1} = \frac{L_7 Z + L_8}{L_{11} Z + 1} \quad (16.2)$$

computer pro-
written by Mar-

see chapter II)
used in solving
each. The fol-
directly from

(16.3)

In the DLT approach object-space control points should be well distributed. One must avoid having all object-space control points in one plane. As much deviation from the planar arrangements as can be allowed by the depth-of-field is highly recommended.

Having computed the coefficients from all the photographs, the object-space coordinates of any object point which is imaged in two or more photographs are then computed.

In the DLT method, the eleven constants (L_1, L_2, \dots, L_{11}) are considered as independent. In 1977 Bopp and Krauss developed an exact solution to the DLT basic equations (referred to by them as the "11-parameter solution"), in which they took into account the dependency between these eleven constants and the nine independent constants necessary to represent the "perfect camera" (Bopp and Krauss, 1977). This was done by introducing two constraints, which are:

$$\begin{aligned} & (L_1^2 + L_2^2 + L_3^2) - (L_5^2 + L_6^2 + L_7^2) \\ & + [(L_5 L_9 + L_6 L_{10} + L_7 L_{11})^2 \\ & - (L_1 L_9 + L_2 L_{10} + L_3 L_{11})^2] \\ & (L_9^2 + L_{10}^2 + L_{11}^2)^{-1} = 0 \end{aligned} \quad (16.6)$$

and

$$\begin{aligned} & (L_1 L_5 + L_2 L_6 + L_3 L_7) \\ & - (L_9^2 + L_{10}^2 + L_{11}^2)^{-1} (L_1 L_9 + L_2 L_{10} + L_3 L_{11}) \\ & (L_5 L_9 + L_6 L_{10} + L_7 L_{11}) = 0. \end{aligned}$$

The constraints are incorporated in the solution as additional observation equations with zero variances.

This method leads to a least squares adjustment with linear fractional observation equations and non-linear constraints. The constants are computed iteratively using three-dimensional, object-space control. With these constants, the object space coordinates of all image-points contained in at least two photographs can be determined in a second step. In contrast to the DLT, the "11-parameter solution" can also be applied to cases where the known interior orientation should be kept.

Another method particularly applicable for non-metric cameras is the self-calibration method developed at the University of New Brunswick (Faig, 1976).

16.2.3.2.2 ON-LINE ANALYTICAL PHOTOGRAMMETRY

On-line analytical photogrammetry is analytical photogrammetry in which the computer is connected directly to the comparator on which measurements are being made. (Use only of stereocomparators will be considered in this section.) The operator usually has control of the progress of computation not only through his use of the stereocomparator but also through a keyboard from which he can give instructions to the computer during the compilation. The theory

for this kind of photogrammetry is identical to that used when measurement is completed before giving the data to the computer. However, because of the great variety of problems encountered in close-range photogrammetry, the more immediate control that the operator has over the computations in on-line analytical photogrammetry as compared with the "batch" method is sometimes advantageous.

16.2.3.2.2.1 Typical Features of Close-Range Photogrammetry. One of the main features of close-range photogrammetry is the absence of standard conditions for photography. For instance, the range of photographic scales is quite different in such applications as photography of human beings and the photography of architectural monuments. The object may be stationary, as in the photography of ships under construction, or moving so rapidly as to rapidly as to require special cameras, as for ballistics. The object may be very small and close, as in the photography of teeth, or vast and at the upper range of distance, as in the photography of open-pit mines.

This absence of standard conditions means, for analytical photogrammetry, that either the theory must be so complex that it can handle all situations, or it must be broken up into a number of separate theories each adapted to particular situations. For example, while the equations for central projection are suitable, with only slight modification, in aerial and terrestrial photogrammetry, they often require great modification to be applicable to problems in close-range photogrammetry. This is particularly true when photographs are taken with special lens systems such as fish-eye or anamorphic lenses.

A properly formulated program for on-line analytical photogrammetry has the capability of dealing efficiently with a variety of different problems. This may be an advantage when different problems must be solved rapidly or when the same kind of problem must be solved with a variety of different initial conditions.

16.2.3.2.2.2 Analytical Formulations. From the operational point of view there are three basic phases of an on-line analytical operation:

- definition of the image geometry;
- reconstruction of the photogrammetric model; and
- detailed photogrammetric compilation of the model.

In defining the image geometry one actually chooses from existing models by determining the type of general conditions, and further specifies the characteristics of the image by its interior orientation and distortion parameters. Obviously, the image geometry essentially affects the process in the following operational phases. The reconstruction of a photogrammetric model results in a good description of relations between the images and the object by means of the parameters of exterior orientation. The reconstruction phase includes a one-step collection of measured data and usually an iterative solution

of the parameters. The compilation phase represents an operation appearing practically continuous even though it is simulated by a fast repetitive cycle of digital computations. These proceed in a stream of densely spaced discrete data points defined by the operator's control of the floating mark in the observed optical model. In general, this computation is based on transformations between the model and image spaces with the use of all previously derived parameters of interior and exterior orientation. Included in the computations are corrections for image distortions.

In general, two different groups of parameters characterize the image-object relationship in on-line analytical systems:

- inner geometry (x', d),
- outer geometry (X, g).

Here, vector $x' = (x', y')^T$ represents the image coordinates, d is a vector of distortion parameters typical of the imaging system, $X = (X, Y, Z)^T$ is a vector of object coordinates, and g is a vector of parameters of exterior orientation. In a symbol form one can write for the imaging process

$$X \xrightarrow{g, d} x' \quad (16.7)$$

and for the reconstruction process

$$(x', x'', d) \xleftarrow{g} X. \quad (16.8)$$

After deriving the unknowns g with the use of suitable control points and with a previous knowledge of d , one can start the routine intersection of individual model points X from stereo-observations x', x''

$$(x', x'') \xrightarrow{d, g} X. \quad (16.9)$$

Image Geometry. The imaging process performs a suitable conversion of data from a three-dimensional object space into a two-dimensional image space. This conversion is always achieved by the use of physical means which represent a projecting system.

To unify various possibilities one can base the photogrammetric reconstruction on scaled projections using straight lines derived from corrected image coordinates. If applicable the corrections should also include the effect of the original curvilinear projection. This approach yields the basic equation of on-line photogrammetry representing the computer-controlled feedback link

$$x' + c = \frac{1}{\mu} \bar{P}^T \Delta X \quad (16.10)$$

where c is a vector of corrections for image distortions, μ is a scale factor which is variable in central projections and constant in parallel projections, and $\Delta X = (\Delta X, \Delta Y, \Delta Z)^T$ are object

coordinates reduced with respect to a suitable reference point. Projection matrix \bar{P} (3,2) represents the orientation of the projection bundle or beam in the object coordinate system, as analyzed by Kratky (1975, 1976).

Model Reconstruction. In off-line computations, the photogrammetric model can be reconstructed from corresponding sets of photo-coordinates x', x'' matched with control coordinates X as expressed in equation (16.8), using a suitable mathematical model for the expected relationship. In on-line analytical systems the same relationship is expressed in a slightly different form. The communication between photo- and object-coordinates is mediated by auxiliary model coordinates $x = (x, y, z)^T$

$$\begin{array}{ccc} & g & \\ (x', x'', d) & \xleftrightarrow{\quad} & X \\ & \begin{array}{c} \leftarrow x \rightarrow \\ g_1 \downarrow g_2 \\ \bar{x} \end{array} & \end{array} \quad (16.8a)$$

The coordinate system (x) of the model becomes "master" for the remaining systems, extended by a graphical output \bar{x} .

Before the feedback ($x \rightarrow x', x''$) is established, photo-coordinates x', x'' are measured and the constants for g are computed. The computation could be arranged in several different ways. Here, we refer to the formulation given by Kratky (1975) and summarize its essential features.

In a single model the individual points are intersected from pairs of conjugated rays. A collinearity equation can be applied to each ray, in a general form $F(g, l) = 0$. Here, vector l represents ideal photo-coordinates. The real measurements x', x'' are not consistent with the equation F and yield residuals u . Vector g is composed of photograph-related constants g_0 representing interior and exterior orientation of both pictures, and by object-related constants g_r giving unknown coordinates for all points which are not control points. Elimination of g_r in the least-squares formulation leads to modified normal equations

$$B_0^T P_0 B_0 g_0 + B_0^T P_0 u = 0 \quad (16.12)$$

where B_0 is the matrix of coefficients related to the vector g_0 and P_0 a weight matrix representing weights introduced by the elimination of g_r . The normal equations are formed sequentially from individual point contributions with the use of (4 × 4)-matrices

$$(P_0)_j = P_j [I - B_r (B_r^T B_r)^{-1} B_r^T]_j. \quad (16.13)$$

Here, $(B_r)_j$ is a (4,3)-coefficient matrix associated with the coordinates of an object-point and P_j is a weight derived from the coordinate difference ΔZ_j between the object point and projection center according to

$$P_j = 1/\Delta Z_j^2.$$

Any constraint $F(g_0, c) = 0$ among the parameters g_0 is linearized to $B_c g_c - c = 0$. Ultimately, the parameters g_0 are computed from the accumulated normal equations

$$g_0 = (B_0^T P_0 B_0)^{-1} B_0^T P_0 u. \quad (16.14)$$

Since the formulation is based on linearized relations the solution must be iteratively updated. If required, the computation is completed by calculating the coordinates of individual intersected points p_j from updated residuals u_j

$$(g_x)_j = - (B_x)_j^T u_j. \quad (16.15)$$

Derived parameters g_0 can be used in the on-line mode with the aid of a proper decomposition $g_0^T = (g_1^T, g_2^T)$ where g_1 represents the return in the feedback link ($x \rightarrow x'$).

In general, the model coordinate system can be defined in an arbitrary manner with respect to the object, but it is advantageous to assume equal photo- and model-scales and introduce an auxiliary coordinate shift C . Thus, the working equations of an on-line analytical system which is physically control-driven *via* the model, can be given by

$$\begin{aligned} x' &= \lambda \bar{P}^T \Delta x - c, \\ X &= C + m x, \\ \bar{x} &= v x \end{aligned} \quad (16.16)$$

where λ is a modified scale factor and v an arbitrary ratio to generate a graphical plot. The first formula in equation (16.16) represents transformations for both images. The exterior orientation is fully returned to the ($x \rightarrow x'$) link except for the scale factor m which is used in the ($x \rightarrow X$) computation. In operations, the floating mark is driven in the directions of the object coordinate system. Other possibilities of arranging the return are discussed in (Kratky, 1976).

Detailed Compilation. The first two operational phases in an on-line analytical process are concerned with the derivation of a valid analytical model for image geometry and, by using it, with the analytical reconstruction of the model for object geometry. In both phases, the on-line function is prepared and checked, but the computations required are performed off-line or in near-real time. Only after the models of both image and object geometries are derived, are they ready to be used in a process of detailed photogrammetric compilation which represents a typical real-time operation. The control of this operation is characterized by equation (16.8a) or in a modified form by

$$\begin{array}{l} \left[\begin{array}{l} d, g_1 \rightarrow (x', x'') \\ g_2 \rightarrow X \\ v \rightarrow \bar{x} \end{array} \right. \end{array}$$

With the computer performing all three computations in a high frequency, the operator retains dynamic control of the system through x

and receives his feedback from stereo-observations of computer-positioned image details for x' and x'' .

16.2.3.2.2.3 Computer Program for General Orientation. Recently, the design of a new analytical plotter has been completed at the National Research Council of Canada. As a part of extensive photogrammetric computer programs supporting the functions of the ANAPLOT system, a program for the general orientation of a single model was developed, with particular emphasis on its use in close-range photogrammetry.

In accordance with the principles outlined above, the orientation problem solved by the program can be easily modified to fit the conditions of a given photogrammetric situation. This is achieved by a direct operator-computer interaction at the time of execution.

The program is capable of handling any photo-scale and a variable number of unknown orientation parameters. The basic solution yields complete exterior orientation of two photographs with a total of 12 unknowns. It could be extended to provide an on-the-job calibration of interior orientation with three additional parameters common to both photographs or with three independent parameters for each of the photographs. The maximum number of unknowns is then 18. The number of parameters can be reduced by choosing a simplified orientation with certain elements preset to given values. An extreme case is represented by a relative orientation with five unknown parameters.

The computer is told which unknowns are to be included in the solution by a string of nine zeros or ones assigned in a standard way to individual elements of orientation. The string is used in the computations as a mask to compress or expand the vectors and matrices of the solution in an appropriate manner.

The problem can also include additional arbitrary conditions on any of the 18 unknowns. Since the constraints have to be entered in a simple way at execution time their formulation is restricted to linear forms. The operator enters the weight of the constraint, its numerical value and the coefficient vector associated with the constrained parameters. In its present form the program can accommodate up to seven constraints of two different types. The operator can either assign specific values to certain parameters and to their linear combinations or require a specific ratio of some computed values.

The reconstruction of a model is usually supported by a number of control points. In the on-line processing, measurements of points are made in an arbitrary sequence. The operator identifies each point by entering its code or number and specifies the corresponding control support with a suitable numeric code. Code "3" means that all three coordinates X , Y , and Z are known, code "2" denotes that X , and Y only are known, code "1" indicates that only the eleva-

tion Z is known and zero is the code that shows no control at all. Obviously, the combination of the control support and of the constraint formulation must provide a minimum information to avoid singular solutions.

The program is written in FORTRAN IV for the PDP 11/45 minicomputer controlling the ANAPLOT system, and is used in the near-real time mode after a conversational initialization of the conditions by the operator. The conversation proceeds from the keyboard of the computer terminal. The computer asks the operator to provide the required information in a series of steps as shown in figure 16-18 which is a facsimile of a computer printout.

In this example, the first instruction to the computer identifies the photographs as F-1 and F-2 and gives the scale factor 1 for the model computation, the principal distance as $f = 150$ mm, and a nine-digit string specifying the type of orientation. Here, zeros indicate which unknowns are to be retained in the solution while code one indicates which parameters are to be omitted from the solution. The string is coded for a standard sequence: $X_c, Y_c, Z_c, \kappa, \phi, \omega, dx', dy', df$, representing three groups of three values each, for the position of the projection center, for the rotation elements and for the elements of interior orientation, respectively. In the example, the full, twelve-parameter orientation is required, with no need to compute the elements of interior orientation. Next a single constraint is defined by determining the photogrammetric base, $b = 100.05$ mm, associated with a weight $P = 100$. The first ten values of the coefficient vector are given and the remaining eight zeros are truncated. The encoded constraint can be interpreted as ${}^2X_c - {}^1X_c = 100.05$. The following information specifies four of the measured

points, 11, 13, 31, 33, as supported by all three object coordinates whereas points 21 and 23 are used without any control support.

16.2.3.2.2.4 Practical Examples. Figures 16-19 through 16-26 give a series of examples illustrating different types of solutions obtained with the same general program from fictitious data. The data were computer-generated at 1:1 scale for two sets of regularly distributed groups of nine points at two different elevation levels. Point numbers 11 to 33 indicate the row-column position of points at the lower level while points 111 to 133 have corresponding positions at a 15 mm higher level.

Example 1 (Full Orientation with no Constraints—figure 16-19), is a straight forward solution with 12 unknowns. It was completed in four iterations. The indicated time figure includes the time used for printing intermediate results and should be reduced by three seconds per line of print in order to assess the real speed of solution. In this instance the time to obtain a solution is about 15 seconds. The resulting linear parameters are expressed in mm and the angular orientation is represented by rotation matrices. The resulting coordinates (X, Y, Z) and the parallax of the model (PY) are also in mm whereas all the photograph-related residuals are printed out in μm . Depending on the availability of control, the listing also shows discrepancies in control points. Since the computation is based on ideal data the resulting fit is perfect and all residuals equal zero.

In example 2 (full orientation with base constraint—figure 16-20), the solution is artificially constrained by enforcing a wrong length of the photogrammetric base. Since the weight used is 100 times larger than that given the measured photo-coordinates the constraint is very strong, as is obvious from $b = {}^2X_c - {}^1X_c = 200.026 - 99.976 = 100.05$ mm. As a result of the wrong length the orientation of the photographs is slightly disturbed and the listing shows residuals up to $9 \mu\text{m}$.

In example 3 (full orientation with coordinate constraints—figure 16-21), both projection centers are falsely offset by $50 \mu\text{m}$ which again affects the quality of the reconstruction resulting in residuals up to $10 \mu\text{m}$.

In example 4 (full orientation with relative constraint—figure 16-22) the constraint is very unnatural and is used only to demonstrate the potential and flexibility of on-line constraining. The absolutely enforced false condition $2\phi_1 + \phi_2 = 0$ can be checked by comparing the rotation elements m_{13} in both resulting matrices, i.e.

$$-2 \times 0.023993 + 0.047986 = 0.$$

Geometric relations are considerably disturbed and photo-residuals reach a maximum value of $161 \mu\text{m}$.

Although artificial in this instance, relative constraints can be useful when one wants a

```

.R MODEL

SPECIFY MODEL - SCALE - F - MASK
F-1 F-2 1, 150000,000000111

ADDITIONAL CONSTRAINTS?
YES
HOW MANY?
1

ENTER WEIGHT - VALUE - MATRIX
100, 100.05, -1,0,0, 0,0,0, 0,0,0, 1

ENTER POINT NUMBER - SPECIFY CONTROL SUPPORT
11 3
13 3
21 0
23 0
31 3
33 3

```

FIGURE 16-18. Information required in the computer program for general orientation.

ITERATION INCREMENTS

XC	YC	ZC	KPPA	PHI	OME
-0.02528	0.04058	0.00151	0.00850	-0.01864	0.02790
0.11565	0.04672	-0.00188	-0.04656	0.05122	-0.05952
-0.00183	0.00357	-0.00150	0.00145	-0.00140	0.00206
0.00053	-0.00257	0.00186	0.00664	-0.00082	-0.00015
0.00000	-0.00000	-0.00001	0.00004	0.00004	0.00003
-0.00000	0.00000	0.00002	-0.00006	-0.00040	-0.00034
0.00000	-0.00000	0.00000	-0.00000	0.00000	0.00000
-0.00000	-0.00000	-0.00000	-0.00002	0.00000	0.00000

PARAMETERS

	X	Y	Z	DX	DY	F
F-1	100.000	199.999	250.000	0.000	0.000	150.000
F-2	200.001	199.999	250.000	0.000	0.000	150.000
	0.999750	0.009997	-0.019997	0.997948	-0.039999	0.050004
	-0.009395	0.999506	0.029994	0.042978	0.997272	-0.060004
	0.020287	-0.029798	0.999350	-0.047468	0.062030	0.996945

MODEL COORDINATES AND DISCREPANCIES..... PHOTO DISCREPANCIES....

PT	X	Y	Z	PY	VPY	VXL	VYL	VXR	VYR
11	100.00	300.00	100.00	0.00	0	0	0	0	0
	0.00	-0.00	0.00						
13	200.00	300.00	100.00	-0.00	0	0	0	0	0
	-0.00	-0.00	0.00						
21	100.00	200.00	100.00	-0.00	0	0	0	0	0
	-0.00	0.00							
23	200.00	200.00	100.00	-0.00	0	0	0	0	0
			0.00						
31	100.00	100.00	100.00	-0.00	0	0	0	0	0
	0.00	0.00	0.00						
33	200.00	100.00	100.00	-0.00	0	0	0	0	0
	-0.00	0.00	0.00						

REDUNDANCY = 9

STANDARD UNIT ERROR = 0.4 MICRONS

STANDARD ERRORS OF UNKNOWNNS

XC	YC	ZC	KPPA	PHI	OME
0.0008	0.0008	0.0004	0.0000	0.0000	0.0000
0.0009	0.0009	0.0004	0.0000	0.0000	0.0000

FIGURE 16-19. Example 1. Full orientation with no constraints.

symmetrical distribution of corresponding parameters in the left and right pictures by setting a condition $p_1 + p_2 = 0$.

In example 5 (full orientation and calibration—figure 16-23), all 18 unknowns are derived in this solution when the required string in the initial specification is reduced to a single zero. A minimum of two additional control points located above the level of remaining points guarantees an on-the-job calibration of both cameras. Since

no wrong values were constrained in the solution, all residuals are equal to zero. A low degree of redundancy in control data causes higher uncertainty in the derived parameters than in some previous examples.

In example 6 (full orientation and calibration with constraint—figure 16-24), the solution from the previous example is degraded by assigning a wrong value of 148 mm to the principal distance of the left camera. Because principal distances

ITERATION INCREMENTS					
XC	YC	ZC	KPPA	PHI	OME
-0.02781	0.03614	0.00094	0.00850	-0.02065	0.02518
0.11581	0.05357	-0.00238	-0.04802	0.05160	-0.05523
0.00055	0.00799	-0.00095	0.00151	0.00049	0.00477
0.00055	-0.00940	0.00234	0.00812	-0.00100	-0.00437
-0.00001	0.00001	-0.00004	-0.00001	0.00002	0.00004
-0.00001	-0.00003	-0.00001	-0.00006	-0.00046	-0.00042
-0.00000	0.00000	-0.00000	-0.00000	-0.00000	-0.00000
-0.00000	-0.00000	-0.00000	-0.00003	0.00000	0.00000

PARAMETERS						
	X	Y	Z	DX	DY	F
F-1	99.976	199.998	249.993	0.000	0.000	150.000
F-2	200.026	199.998	249.992	0.000	0.000	150.000
	0.999747	0.010003	-0.020140	0.997941	-0.039986	0.050148
	-0.009396	0.999506	0.029993	0.042975	0.997272	-0.060010
	0.020430	-0.029796	0.999347	-0.047611	0.062041	0.996937

MODEL COORDINATES AND DISCREPANCIES..... PHOTO DISCREPANCIES....									
PT	X	Y	Z	FY	VPY	VXL	VYL	VXR	VYR
11	100.00	300.00	100.00	-0.01	9	3	5	0	-4
	0.00	0.01	0.00						
13	200.00	300.00	100.00	0.01	-9	-1	-4	-3	4
	-0.00	-0.01	0.00						
21	100.00	200.00	100.00	0.00	0	0	0	0	0
23	200.00	200.00	100.00	-0.00	0	0	0	0	0
31	100.00	100.00	100.00	0.01	-8	3	-5	0	3
	0.00	-0.01	0.00						
33	200.00	100.00	100.00	-0.01	9	0	4	-3	-5
	-0.00	0.01	0.00						

REDUNDANCY = 7
STANDARD UNIT ERROR = 5.7 MICRONS

STANDARD ERRORS OF UNKNOWN

XC	YC	ZC	KPPA	PHI	OME
0.0094	0.0151	0.0053	0.0000	0.0001	0.0001
0.0094	0.0151	0.0053	0.0000	0.0001	0.0001

FIGURE 16-20. Example 2. Full orientation with base constraint.

are treated as negative values in the program, and their initial approximation is -150 mm the constraint must be formulated as $df_1 = 2.0$.

In example 7 (relative orientation—figure 16-25), only six angular parameters are allowed in the computation by choosing a mask "111000111." However, the relative orientation is defined by five elements and thus, one of the parameters must be eliminated by an additional single constraint. In this example, ω_1 was constrained to zero. Consequently, the error-free model is appreciably tilted, as is obvious from the resulting model coordinates. Because no control

support was available, the origin of the model coordinates is at the left projection center by default.

In example 8 (relative orientation—figure 16-26), the solution uses 12 unknowns and, obviously, seven of them must be controlled by suitable constraints. This is done by assigning coordinates to both projection centers and by specifying the correct value for ω_1 . In contrast to the previous example, the resulting model is not tilted and is in addition error free. The printout (figure 16-26) contains additional information on the redundancy, on the standard error of unit

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ITERATION INCREMENTS

XC	YC	ZC	KPPA	PHI	OME
-0.02678	0.03611	0.00124	0.00849	-0.01975	0.02516
0.11650	0.05351	-0.00256	-0.04801	0.05222	-0.05526
-0.00000	0.00801	-0.00110	0.00150	0.00001	0.00479
0.00000	-0.00935	0.00247	0.00815	-0.00149	-0.00433
0.00000	0.00000	-0.00004	0.00000	0.00003	0.00003
-0.00000	-0.00003	-0.00002	-0.00009	-0.00045	-0.00042
-0.00000	0.00000	-0.00000	-0.00000	0.00000	0.00000
-0.00000	-0.00000	-0.00000	-0.00003	0.00001	0.00000

PARAMETERS

	X	Y	Z	DX	DY	F
F-1	100.050	199.996	250.016	0.000	0.000	150.000
F-2	200.050	199.996	249.984	0.000	0.000	150.000
	0.999756	0.009985	-0.019711	0.997935	-0.039974	0.050287
	-0.009391	0.999506	0.029984	0.042972	0.997272	-0.060017
	0.020001	-0.029792	0.999356	-0.047751	0.062054	0.996930

MODEL COORDINATES AND DISCREPANCIES..... PHOTO DISCREPANCIES....

PT	X	Y	Z	PY	VPY	VXL	VYL	VXR	VYR
11	100.00	300.00	100.00	0.00	-2	-6	-10	1	-8
	-0.01	-0.00	-0.01						
13	200.00	300.00	100.00	0.00	0	2	8	-7	9
	-0.01	-0.00	0.01						
21	100.01	200.00	100.01	-0.00	0	0	0	0	0
23	200.01	200.00	99.99	0.00	0	0	0	0	0
31	100.00	100.00	100.00	-0.00	1	-7	9	2	8
	-0.01	0.00	-0.01						
33	200.00	100.00	100.00	-0.00	2	1	-8	-6	-10
	-0.01	0.00	0.01						

REDUNDANCY = 8

STANDARD UNIT ERROR = 10.6 MICRONS

STANDARD ERRORS OF UNKNOWN

XC	YC	ZC	KPPA	PHI	OME
0.0011	0.0279	0.0082	0.0001	0.0000	0.0001
0.0011	0.0280	0.0082	0.0001	0.0000	0.0001

FIGURE 16-21. Example 3. Full orientation with coordinate constraints.

weight and standard errors of the derived parameters.

16.2.3.2.2.5 Conclusions. The versatility of on-line systems is greater than that of any other photogrammetric system and the expansion of functions is mostly a matter of software development. This fact combined with the versatility of on-line analytical systems make them extremely attractive for close-range applications, especially in biomedicine and engineering.

16.2.3.3 SEMI-ANALYTICAL APPROACH

In this approach, a stereoplotter is used to form the stereomodel of the object. The scaling

and absolute orientation of the model are then done analytically. The use of a stereoplotter rather than a comparator and the added flexibility resulting from the analytical phases makes this approach highly attractive.

16.2.4 AREAS OF APPLICATION OF CLOSE-RANGE PHOTOGRAMMETRY

The ever-expanding areas of application of close-range photogrammetry can be grouped into three major areas: architectural photogrammetry, biomedical and bioengineering