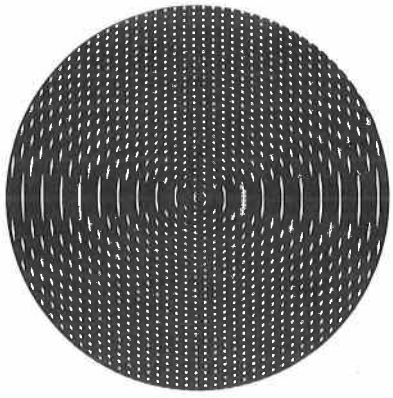
# Fluid Mechanics

**Second Edition**

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**Fluid Mechanics**

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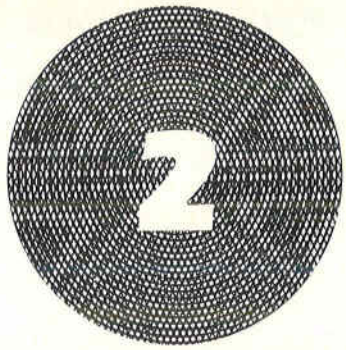

# Pressure Distribution in a Fluid

## 2.1 PRESSURE AND PRESSURE GRADIENT

In Fig. 1.1 we saw that a fluid at rest cannot support shear stress and thus Mohr's circle reduces to a point. In other words, the normal stress on any plane through a fluid element at rest is equal to a unique value called the *fluid pressure* $p$, taken positive for compression by common convention. This is such an important concept that we shall review it with another approach.

Figure 2.1 shows a small wedge of fluid at rest of size $\Delta x$ by $\Delta z$ by $\Delta s$ and depth $b$ into the paper. There is no shear by definition, but we postulate that the pressures $p_x$, $p_z$, and $p_n$ may be different on each face. The weight of the element may also be important. Summation of forces must equal zero (no acceleration) in both the $x$ and $z$ directions

$$\begin{aligned} \sum F_x &= 0 = p_x b\,\Delta z - p_n b\,\Delta s \sin\theta \\ \sum F_z &= 0 = p_z b\,\Delta x - p_n b\,\Delta s \cos\theta - \tfrac{1}{2}\rho g b\,\Delta x\,\Delta z \end{aligned} \tag{2.1}$$

but the geometry of the wedge is such that

$$\Delta s \sin\theta = \Delta z \qquad \Delta s \cos\theta = \Delta x \tag{2.2}$$

Substitution into Eq. (2.1) and rearrangement gives

$$p_x = p_n \qquad p_z = p_n + \tfrac{1}{2}\rho g\,\Delta z \tag{2.3}$$

These relations illustrate two important principles of the hydrostatic, or shear-free, condition: (1) there is no pressure change in the horizontal direction, and (2) there is a vertical change in pressure proportional to density, gravity, and the depth change. We shall exploit these results to the fullest, starting in Sec. 2.3.

In the limit as the fluid wedge shrinks to a "point," $\Delta z \to 0$ and Eqs. (2.3) become

$$p_x = p_z = p_n = p \tag{2.4}$$



Page 3 of 64 ABIOMED Ex. 1032

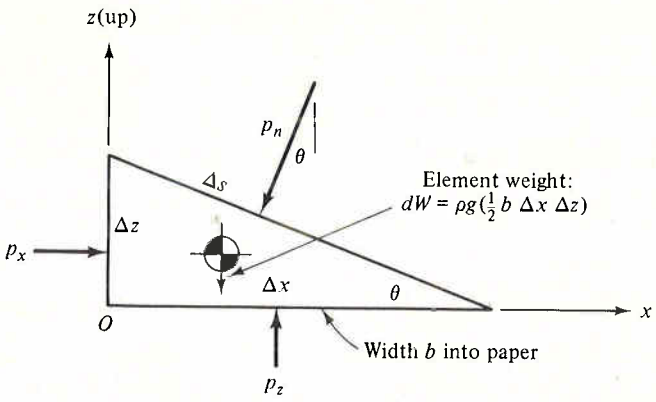


**Fig. 2.1** Equilibrium of a small wedge of fluid at rest.

Since $\theta$ is arbitrary, we conclude that the pressure $p$ at a point in a static fluid is independent of orientation.

What about the pressure at a point in a moving fluid? As discussed in Sec. 1.7, if there are strain rates in a moving fluid, there will be viscous stresses, both shear and normal in general (Sec. 4.3). In that case (Chap. 4) the pressure is defined as the average of the three normal stresses $\sigma_{ii}$ on the element

$$p = -\tfrac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \tag{2.5}$$

The negative sign occurs because a compression stress is considered to be negative whereas $p$ is positive. Equation (2.5) is subtle and rarely needed since the great majority of viscous flows have negligible viscous normal stresses (Chap. 4).

### Pressure Force on a Fluid Element

Pressure (or any other stress, for that matter) causes no net force on a fluid element unless it varies *spatially*.[1] To see this, consider the pressure acting on the two $x$ faces in Fig. 2.2. Let the pressure vary arbitrarily

$$p = p(x, y, z, t) \tag{2.6}$$

The net force in the $x$ direction on the element in Fig. 2.2 is given by

$$dF_x = p\ dy\ dz - \left(p + \frac{\partial p}{\partial x}\,dx\right) dy\ dz = -\frac{\partial p}{\partial x}\,dx\ dy\ dz \tag{2.7}$$

In like manner the net force $dF_y$ involves $-\partial p/\partial y$ and the net force $dF_z$ concerns $-\partial p/\partial z$. The total net-force vector on the element due to pressure is

$$d\mathbf{F}_{\text{press}} = \left(-\mathbf{i}\frac{\partial p}{\partial x} - \mathbf{j}\frac{\partial p}{\partial y} - \mathbf{k}\frac{\partial p}{\partial z}\right) dx\ dy\ dz \tag{2.8}$$

We recognize the term in parentheses as the negative vector gradient of $p$. Denoting $\mathbf{f}$ as the net force per unit element volume, we rewrite Eq. (2.8) as

$$\mathbf{f}_{\text{press}} = -\nabla p \tag{2.9}$$

[1] An interesting application for a large element is in Fig. 3.7.



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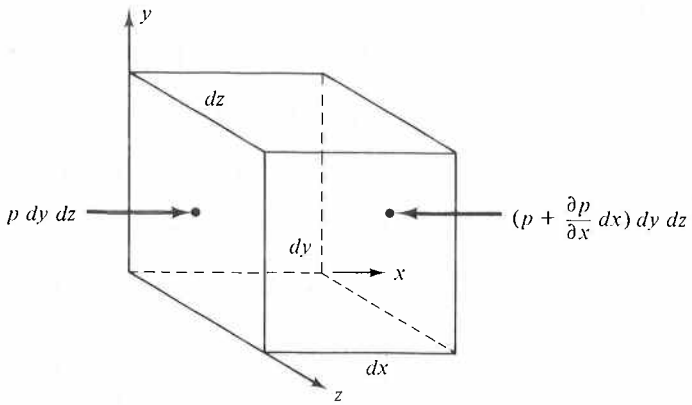


**Fig. 2.2** Net $x$ force on an element due to pressure variation.

Thus it is not the pressure but the pressure *gradient* causing a net force which must be balanced by gravity or acceleration or some other effect in the fluid.

## 2.2 EQUILIBRIUM OF A FLUID ELEMENT

The pressure gradient is a *surface* force which acts on the sides of the element. There may also be a *body* force, due to electromagnetic or gravitational potentials, acting on the entire mass of the element. Here we consider only the gravity force, or weight of the element

$$d\mathbf{F}_{\text{grav}} = \rho \mathbf{g}\, dx\, dy\, dz \tag{2.10}$$

or

$$\mathbf{f}_{\text{grav}} = \rho \mathbf{g}$$

In general there may also be a surface force due to the gradient, if any, of the viscous stresses. For completeness, we write this term here without derivation and consider it more thoroughly in Chap. 4. For an incompressible fluid with constant viscosity, the net viscous force is

$$\mathbf{f}_{\text{VS}} = \mu\left(\frac{\partial^2 \mathbf{V}}{\partial x^2} + \frac{\partial^2 \mathbf{V}}{\partial y^2} + \frac{\partial^2 \mathbf{V}}{\partial z^2}\right) = \mu \nabla^2 \mathbf{V} \tag{2.11}$$

where VS stands for viscous stresses and $\mu$ is the coefficient of viscosity from Chap. 1. Note that the term $\mathbf{g}$ in Eq. (2.10) denotes the acceleration of gravity, a vector acting toward the center of the earth. On earth the average magnitude of $\mathbf{g}$ is $32.174\ \text{ft/s}^2 = 9.807\ \text{m/s}^2$.

The total vector resultant of these three forces, pressure, gravity, and viscous stress, must either keep the element in equilibrium or cause it to move with acceleration $\mathbf{a}$. From Newton's law, Eq. (1.2), we have

$$\rho \mathbf{a} = \sum \mathbf{f} = \mathbf{f}_{\text{press}} + \mathbf{f}_{\text{grav}} + \mathbf{f}_{\text{visc}} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{V} \tag{2.12}$$

This is one form of the differential momentum equation for a fluid element and will be studied further in Chap. 4. Vector addition is implied by Eq. (2.12): the acceleration reflects the local balance of forces and is not necessarily parallel to the local-velocity vector, which reflects the direction of motion at that instant.



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The present chapter is concerned with cases where the velocity and acceleration are known, leaving one to solve for the pressure variation in the fluid. Later chapters will take up the more general problem where pressure, velocity, and acceleration are all unknown. Rewrite Eq. (2.12) as

$$\nabla p = \rho(\mathbf{g} - \mathbf{a}) + \mu \nabla^2 \mathbf{V} = \mathbf{B}(x, y, z, t) \tag{2.13}$$

where **B** is a short notation for the vector sum on the right-hand side. If **V** and $\mathbf{a} = d\mathbf{V}/dt$ are known functions of space and time and the density and viscosity are known, we can solve Eq. (2.13) for $p(x, y, z, t)$ by direct integration. By components, Eq. (2.13) is equivalent to three simultaneous first-order differential equations

$$\frac{\partial p}{\partial x} = B_x(x, y, z, t) \qquad \frac{\partial p}{\partial y} = B_y(x, y, z, t) \qquad \frac{\partial p}{\partial z} = B_z(x, y, z, t) \tag{2.14}$$

Since the right-hand sides are known functions, they can be integrated systematically to obtain the distribution $p(x, y, z, t)$ except for an unknown function of time, which remains because we have no relation for $\partial p/\partial t$. This extra function is found from a condition of known time variation $p_0(t)$ at some point $(x_0, y_0, z_0)$. If the flow is steady (independent of time), the unknown function is a constant and is found from knowledge of a single known pressure $p_0$ at a point $(x_0, y_0, z_0)$. If this sounds complicated, it is not: we shall illustrate with many examples. Finding the pressure distribution from a known velocity distribution is one of the easiest problems in fluid mechanics, which is why we put it in Chap. 2.

Examining Eq. (2.13), we can single out at least four special cases:

1. Flow at rest or at constant velocity: the acceleration and viscous terms vanish identically, and $p$ depends only upon gravity and density. This is the *hydrostatic* condition. See Sec. 2.3.
2. Rigid-body translation and rotation: the viscous term vanishes identically, and $p$ depends only upon the term $\rho(\mathbf{g} - \mathbf{a})$. See Sec. 2.9.
3. Irrotational motion ($\mathbf{V} \times \mathbf{V} \equiv 0$): the viscous term vanishes identically, and an exact integral called *Bernoulli's equation* can be found for the pressure distribution. See Sec. 4.9.
4. Arbitrary viscous motion: nothing helpful happens, no general rules apply, but still the integration is quite straightforward. See Sec. 6.4.

Let us consider cases 1 and 2 here.

### Absolute, Gage, and Vacuum Pressure

Before embarking on examples, we should note that engineers are likely to specify a pressure measurement in two ways, either *absolute* pressure with respect to a zero pressure reference, or else with respect to local atmospheric pressure. In the latter case the measured pressure may be higher or lower than the local atmosphere. If higher, the difference is called *gage* pressure; if lower, the difference is called *vacuum*



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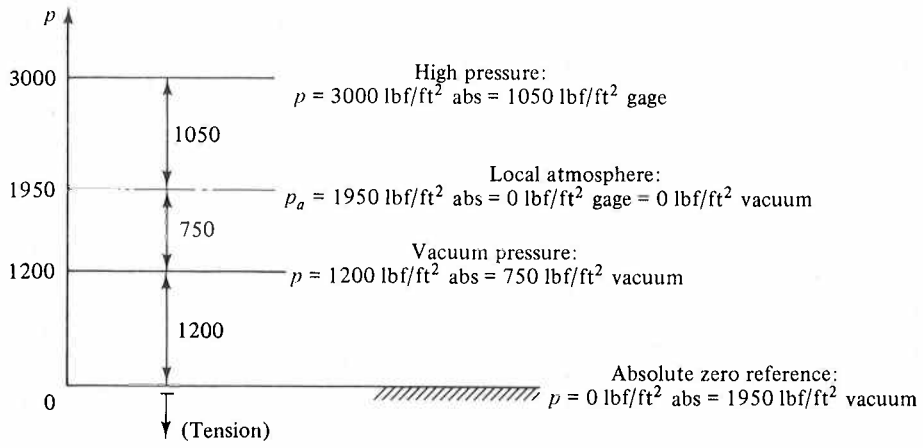


**Fig. 2.3** Illustration of absolute, gage, and vacuum pressure readings.

pressure. This state of affairs is illustrated in Fig. 2.3. The local atmospheric pressure is, say, 1950 lbf/ft$^2$, which might reflect a storm condition in a sea-level city or normal conditions in a city at an altitude of 2300 ft. We can write this as $p_a =$ 1950 lbf/ft$^2$ absolute or 0 lbf/ft$^2$ gage or 0 lbf/ft$^2$ vacuum. Suppose a laboratory pressure gage reads an absolute pressure of 3000 lbf/ft$^2$. This is higher than local atmospheric pressure, and the value may be reported as 3000 − 1950 = 1050 lbf/ft$^2$ gage pressure. (It may then be *your* problem later to determine what $p_a$ was on that day.) Suppose another gage reads 1200 lbf/ft$^2$ absolute. Locally this would be a vacuum pressure and might be reported as 1950 − 1200 = 750 lbf/ft$^2$ vacuum. (Again you might later need to know $p_a$ to carry on your analysis.) Gage pressure is the opposite of vacuum pressure: 750 lbf/ft$^2$ vacuum = −750 lbf/ft$^2$ gage, but negative values are not normally reported. From time to time we shall specify gage or vacuum pressure in problems to keep you on your toes.

## 2.3 HYDROSTATIC PRESSURE DISTRIBUTIONS

If the fluid is at rest or at constant velocity, $\mathbf{a} = 0$ and $\nabla^2\mathbf{V} = 0$. Equation (2.13) for the pressure distribution reduces to

$$\nabla p = \rho \mathbf{g} \tag{2.15}$$

This is a *hydrostatic* distribution and is correct for all fluids at rest, regardless of their viscosity, because the viscous term vanishes identically.

Recall from vector analysis that the vector $\nabla p$ expresses the magnitude and direction of the maximum spatial rate of increase of the scalar property $p$. As a result, $\nabla p$ is perpendicular everywhere to surfaces of constant $p$. Thus Eq. (2.15) states that a fluid in hydrostatic equilibrium will align its constant-pressure surfaces everywhere normal to the local-gravity vector. The maximum pressure increase will be in the direction of gravity, i.e., "down." If the fluid is a liquid, its free surface, being at atmospheric pressure, will be normal to local gravity, or "horizontal." You probably knew all this before, but Eq. (2.15) is the proof of it.


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In our customary coordinate system $z$ is "up." Thus the local-gravity vector for small-scale problems is

$$\mathbf{g} = -g\mathbf{k} \tag{2.16}$$

where $g$ is the magnitude of local gravity, for example, 9.807 m/s$^2$. For these coordinates Eq. (2.15) has the components

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g \tag{2.17}$$

the first two of which tell us that $p$ is independent of $x$ and $y$. Hence $\partial p/\partial z$ can be replaced by the total derivative $dp/dz$, and the hydrostatic condition reduces to

$$\frac{dp}{dz} = -\rho g$$

or

$$p_2 - p_1 = -\int_1^2 \rho g \, dz \tag{2.18}$$

Equation (2.18) is the solution to the hydrostatic problem. The integration requires an assumption about the density and gravity distribution. Gases and liquids are usually treated differently.

We state the following conclusions about a hydrostatic condition:

> Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases with depth in the fluid.

An illustration of this is shown in Fig. 2.4. The free surface of the container is atmospheric and forms a horizontal plane. Points $a$, $b$, $c$, and $d$ are at equal depth in a horizontal plane and are interconnected by the same fluid, water; therefore all points have the same pressure. The same is true of points $A$, $B$, and $C$ on the bottom, which all have the same higher pressure than at $a$, $b$, $c$, and $d$. However, point $D$, although at the same depth as $A$, $B$, and $C$, has a different pressure because it lies beneath a different fluid, mercury.

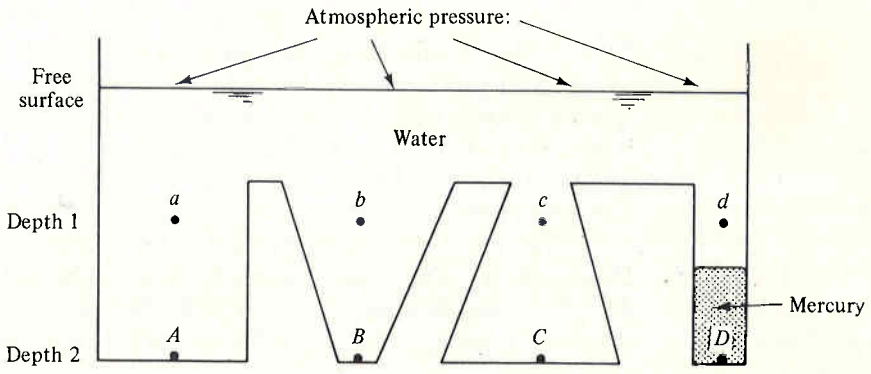


**Fig. 2.4** Hydrostatic-pressure distribution. Points $a$, $b$, $c$, and $d$ are at equal depths in water and therefore have identical pressures. Points $A$, $B$, and $C$ are also at equal depths in water and have identical pressures higher than $a$, $b$, $c$, and $d$. Point $D$ has a different pressure from $A$, $B$, and $C$ because it is not connected to them by a water path.



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### Effect of Variable Gravity

For a spherical planet of uniform density, the acceleration of gravity varies inversely as the square of the radius from its center

$$g = g_0\left(\frac{r_0}{r}\right)^2 \tag{2.19}$$

where $r_0$ is the planet radius and $g_0$ is the surface value of $g$. For earth, $r_0 \approx 3960$ statute miles $\approx 6400$ km. In typical engineering problems the deviation from $r_0$ extends from the deepest ocean, about 11 km, to the atmospheric height of supersonic transport operation, about 20 km. This gives a maximum variation in $g$ of $(6400/6420)^2$ or 0.6 percent. We therefore neglect the variation of $g$ in most problems.

### Hydrostatic Pressure in Liquids

Liquids are so nearly incompressible that we can neglect their density variation in hydrostatics. In Example 1.11 we saw that water density increases only 4.6 percent at the deepest part of the ocean. Its effect on hydrostatics would be about half of this, or 2.3 percent. Thus we assume constant density in liquid hydrostatic calculations, for which Eq. (2.18) integrates to

Liquids: $$p_2 - p_1 = -\rho g(z_2 - z_1) \tag{2.20}$$

or $$z_1 - z_2 = \frac{p_2}{\rho g} - \frac{p_1}{\rho g}$$

We use the first form in most problems. The quantity $\rho g$ is called the *specific weight* of the fluid, with dimensions of weight per unit volume; some values are tabulated in Table 2.1. The quantity $p/\rho g$ is a length called the *pressure head* of the fluid.

For lakes and oceans, the coordinate system is usually chosen as in Fig. 2.5, with $z = 0$ at the free surface, where $p$ equals the surface atmospheric pressure $p_a$. When

**Table 2.1**
**SPECIFIC WEIGHT OF SOME COMMON FLUIDS**

| Fluid | Specific weight $\rho g$ at 68°F = 20°C | |
|---|---|---|
| | lbf/ft$^3$ | N/m$^3$ |
| Air (at 1 atm) | 0.0752 | 11.8 |
| Ethyl alcohol | 49.2 | 7,733 |
| SAE 30 oil | 57.3 | 8,996 |
| Water | 62.4 | 9,790 |
| Seawater | 64.0 | 10,050 |
| Glycerin | 78.7 | 12,360 |
| Carbon tetrachloride | 99.1 | 15,570 |
| Mercury | 846 | 133,100 |


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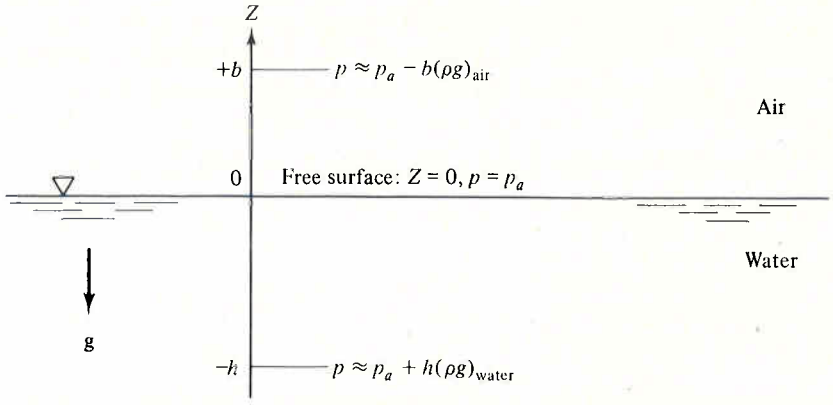


**Fig. 2.5** Hydrostatic-pressure distribution in oceans and atmospheres.

we introduce the reference value $(p_1, z_1) = (p_a, 0)$, Eq. (2.20) becomes, for $p$ at any (negative) depth $z$,

Lakes and oceans: $$p = p_a - \rho g z \tag{2.21}$$

where $\rho g$ is the average specific weight of the lake or ocean. As we shall see, Eq. (2.21) holds in the atmosphere also with an accuracy of 2 percent for heights $z$ up to 1000 ft.

**EXAMPLE 2.1** Newfound Lake, a freshwater lake near Bristol, New Hampshire, has a maximum depth of 60 m, and mean atmospheric pressure is 91 kPa. Estimate the absolute pressure at this maximum depth in kilopascals.

*Solution* From Table 2.1, take $\rho g \approx 9790\ \text{N/m}^3$. With $p_a = 91\ \text{kPa}$ and $z = -60\ \text{m}$, Eq. (2.21) predicts that the pressure at this depth will be

$$p = 91\ \text{kN/m}^2 - (9790\ \text{N/m}^3)(-60\ \text{m})\frac{1\ \text{kN}}{1000\ \text{N}}$$
$$= 91\ \text{kPa} + 587\ \text{kN/m}^2 = 678\ \text{kPa} \qquad \textit{Ans.}$$

By omitting $p_a$ we could state the result as $p = 587\ \text{kPa}$ (gage).

## The Mercury Barometer

The simplest practical application of the hydrostatic formula (2.20) is the barometer (Fig. 2.6), which measures atmospheric pressure. A tube is filled with mercury and inverted while submerged in a reservoir. This causes a near vacuum in the closed upper end because mercury has an extremely small vapor pressure at room temperatures (0.16 Pa at 20°C). Since atmospheric pressure forces a mercury column to rise a distance $h$ into the tube, the upper mercury surface is at zero pressure.

From Fig. 2.6, Eq. (2.20) applies with $p_1 = 0$ at $z_1 = h$ and $p_2 = p_a$ at $z_2 = 0$:

$$p_a - 0 = -\rho_M g(0 - h)$$

or $$h = \frac{p_a}{\rho_M g} \tag{2.22}$$



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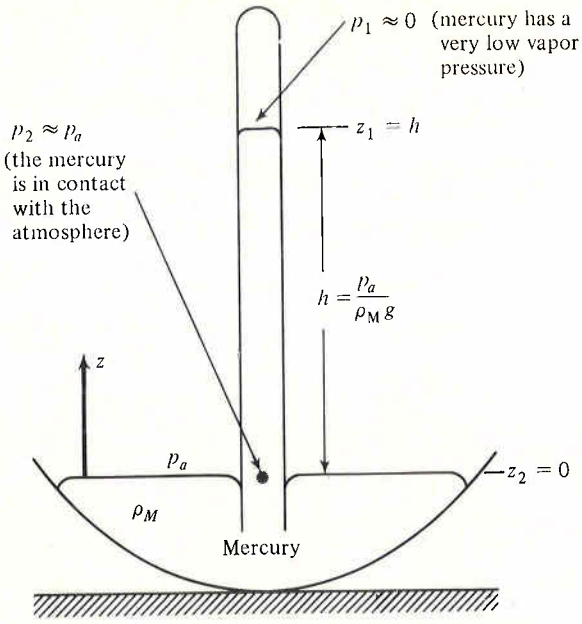


**Fig. 2.6** The mercury barometer: the height of the mercury column is a measure of atmospheric pressure.

At sea-level standard, with $p_a = 101{,}350$ Pa and $\rho_M g = 133{,}100$ N/m$^2$ from Table 2.1, the barometric height is $h = 101{,}350/133{,}100 = 0.761$ m or 761 mm. In the United States the weather service reports this as an atmospheric "pressure" of 29.96 in Hg (inches of mercury). Mercury is used because it is the heaviest common liquid. A water barometer would be 34 ft high.

### Hydrostatic Pressure in Gases

Gases are compressible, with density nearly proportional to pressure. Thus density must be considered as a variable in Eq. (2.18) if the integration carries over large pressure changes. It is sufficiently accurate to introduce the perfect-gas law $p = \rho RT$ in Eq. (2.18)

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g$$

Separate the variables and integrate between points 1 and 2:

$$\int_1^2 \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_1^2 \frac{dz}{T} \tag{2.23}$$

The integral over $z$ requires an assumption about the temperature variation $T(z)$. One common approximation is the *isothermal atmosphere*, where $T = T_0$:

$$p_2 = p_1 \exp\left[ -\frac{g(z_2 - z_1)}{RT_0} \right] \tag{2.24}$$

The quantity in brackets is dimensionless. (Think that over; it must be dimensionless, right?) Equation (2.24) is a fair approximation for the earth, but actually the


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earth's mean atmospheric temperature drops off nearly linearly with $z$ up to an altitude of about 36,000 ft (11,000 m):

$$T \approx T_0 - Bz \tag{2.25}$$

where $T_0$ is sea-level temperature (absolute) and $B$ is the *lapse rate*, both of which vary somewhat from day to day. By international agreement [1] the following standard values are assumed to apply from 0 to 36,000 ft:

$$T_0 = 518.69°\text{R} = 288.16\ \text{K} = 15°\text{C}$$
$$B = 0.003566\ °\text{R/ft} = 0.00650\ \text{K/m} \tag{2.26}$$

This lower portion of the atmosphere is called the *troposphere*. Introducing Eq. (2.25) into (2.23) and integrating, we obtain the more accurate relation

$$p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/RB} \qquad \text{where } \frac{g}{RB} = 5.26 \text{ (air)} \tag{2.27}$$

in the troposphere, with $z = 0$ at sea level. The exponent $g/RB$ is dimensionless (again it must be) and has the standard value of 5.26 for air, with $R = 1717\ \text{ft}^2/(\text{s}^2 \cdot °\text{R})$.

The U.S. Standard Atmosphere [1] is sketched in Fig. 2.7. The pressure is seen to be nearly zero at $z = 30$ km. For tabulated properties see Appendix Table A.6.

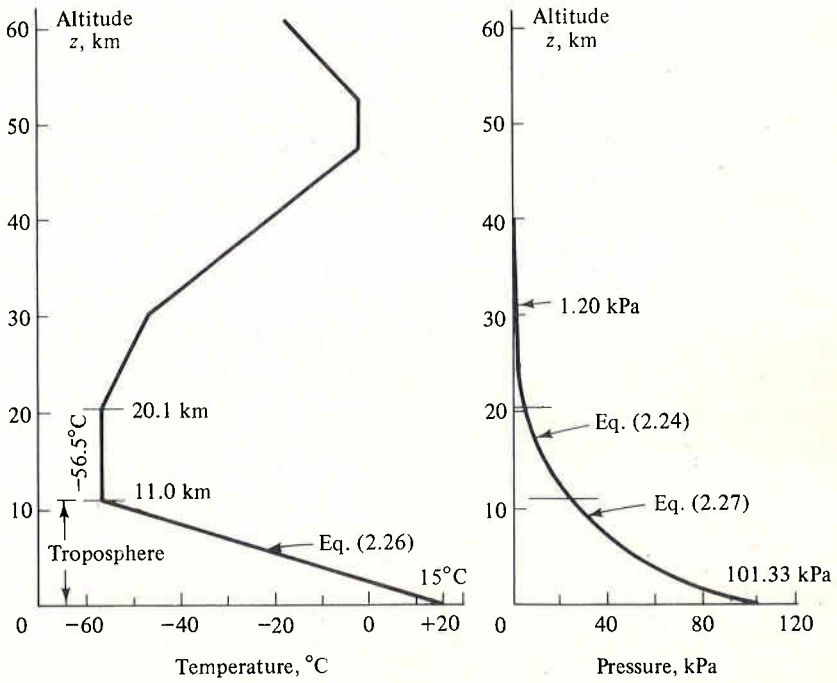


**Fig. 2.7** Temperature and pressure distribution in the U.S. Standard Atmosphere. (*From Ref.* 1.)



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**EXAMPLE 2.2** If standard atmospheric pressure is 101,300 Pa, compute the standard pressure at an altitude of 3000 m (*a*) by the exact formula and compare with (*b*) an isothermal assumption and (*c*) a constant-density formula.

**Solution**

*Part (a)* From the exact relation (2.27)

$$p = p_a\left[1 - \frac{(0.00650\ \text{K/m})(3000\ \text{m})}{(288.16\ \text{K})}\right]^{5.26} = p_a(0.9323)^{5.26}$$

$$= 101{,}300(0.6917) = 70{,}070\ \text{Pa} = 70.07\ \text{kPa} \qquad \textit{Ans. (a)}$$

*Part (b)* For the isothermal assumption, Eq. (2.24) applies with $p_1 = p_a$ and $z_1 = 0$:

$$p \approx p_a \exp\left\{-\frac{(9.807\ \text{m/s}^2)(3000\ \text{m})}{[287\ \text{m}^2/(\text{s}^2 \cdot \text{K})](288.16\ \text{K})}\right\}$$

$$= p_a \exp(-0.3557) = 101{,}300(0.7006) = 70.98\ \text{kPa} \qquad \textit{Ans. (b)}$$

This is 1.3 percent higher than the exact result (*a*).

*Part (c)* The simple linear approximation (2.21) is surprisingly accurate even over the distance of 3000 m. Taking $\rho g \approx 11.8\ \text{N/m}^3$ from Table 2.1, we get

$$p \approx p_a - \rho g z = 101{,}300 - (11.8\ \text{N/m}^3)(3000\ \text{m})$$

$$= 101{,}300 - 35{,}400 = 65{,}900\ \text{Pa} = 65.90\ \text{kPa} \qquad \textit{Ans. (c)}$$

This is only 6.0 percent low compared with the exact result (*a*).

The error involved in using the linear approximation (2.21) can be evaluated by expanding the exact formula (2.27) into a series

$$\left(1 - \frac{Bz}{T_0}\right)^n = 1 - n\frac{Bz}{T_0} + \frac{n(n-1)}{2!}\left(\frac{Bz}{T_0}\right)^2 - \cdots \tag{2.28}$$

where $n = g/RB$. Introducing these first three terms of the series into Eq. (2.27) and rearranging, we obtain

$$p = p_a - \rho_a g z\left(1 - \frac{n-1}{2}\frac{Bz}{T_0} + \cdots\right) \tag{2.29}$$

Thus the error in using the linear formula (2.21) is small if the second term in parentheses in (2.29) is small compared with unity. This is true if

$$z \ll \frac{2T_0}{(n-1)B} = 20{,}800\ \text{m} \tag{2.30}$$

We thus expect errors of less than 5 percent if $z$ or $\delta z$ is less than 1000 m. This explains why the error in part (*c*) of Example 2.2 was rather small.

## 2.4 APPLICATION TO MANOMETRY

From the hydrostatic formula (2.20), a change in elevation $z_2 - z_1$ of a liquid is equivalent to a change in pressure $(p_2 - p_1)/\rho g$. Thus a static column of one or more liquids or gases can be used to measure pressure differences between two



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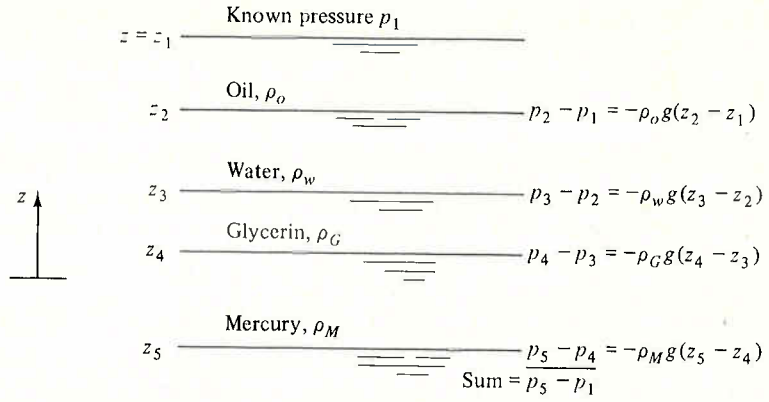


**Fig. 2.8** Evaluating pressure changes through a column of multiple fluids.

points. Such a device is called a *manometer*. If multiple fluids are used, we must change the density in the formula as we move from one fluid to another. Figure 2.8 illustrates the use of the formula with a column of multiple fluids. The pressure change through each fluid is calculated separately. If we wish to know the total change $p_5 - p_1$, we add the successive changes $p_2 - p_1$, $p_3 - p_2$, $p_4 - p_3$, and $p_5 - p_4$. The intermediate values of $p$ cancel, and we have, for the example of Fig. 2.8,

$$p_5 - p_1 = -\rho_0 g(z_2 - z_1) - \rho_w g(z_3 - z_2) - \rho_G g(z_4 - z_3) - \rho_M g(z_5 - z_4) \tag{2.31}$$

No additional simplification is possible on the right-hand side because of the different densities. Notice that we have placed the fluids in order from the lightest on top to the heaviest at bottom. This is the only stable configuration. If we attempt to layer them in any other manner, the fluids will overturn and seek the stable arrangement.

Figure 2.9 shows a simple open manometer for measuring $p_A$ in a closed chamber relative to atmospheric pressure $p_a$, in other words, measuring the gage pressure. The chamber fluid $\rho_1$ is combined with a second fluid $\rho_2$, perhaps for two reasons: (1) to protect the environment from a corrosive chamber fluid or (2) a heavier fluid $\rho_2$ will keep $z_2$ small and the open tube can be shorter. The basic

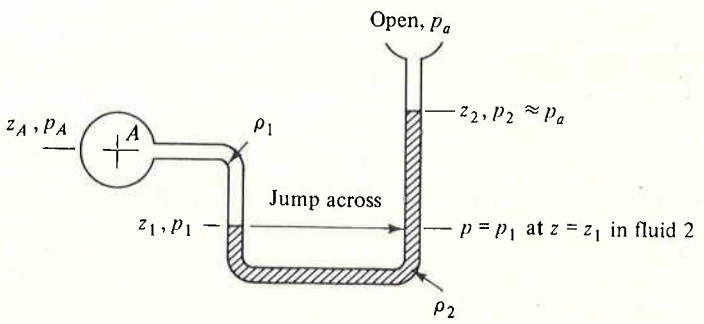


**Fig. 2.9** Simple open manometer for measuring $p_A$ relative to atmospheric pressure.



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hydrostatic relation (2.20) is applied in two steps, first from $z_A$ to $z_1$ and then from $z_1$ to $z_2$:

$$p_A - p_1 = -\rho_1 g(z_A - z_1) \qquad p_1 - p_2 = -\rho_2 g(z_1 - z_2) \tag{2.32}$$

The small amount of air between $z_2$ and the open end is neglected and we assume that $p_2 \approx p_a$. When we add the two relations above, $p_1$ cancels and we obtain the desired result

$$p_A = p_a - \rho_1 g(z_A - z_1) - \rho_2 g(z_1 - z_2) \tag{2.33}$$

Notice that the pressure at level $z_1$ is equal to $p_1$ on both sides of the U-tube in Fig. 2.9. This is because a continuous length of the same fluid connects the two equal levels. The hydrostatic relation (2.20) requires this as a form of Pascal's law:

> Any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure.

This rule can be used to facilitate calculations in multiple-U-tube problems. We can jump across the U-tube if we stay within the same fluid.

**EXAMPLE 2.3** (*a*) Find the pressure $p_A$ in Fig. 2.9 if $p_a = 2116$ lbf/ft$^2$, $z_A = 7$ in, $z_1 = 4$ in, $z_2 = 13$ in, fluid 1 is water, and fluid 2 is mercury. (*b*) What would the height $z_2$ be for the same pressure $p_A$ if the mercury were replaced by glycerin?

*Solution*

*Part (a)* The pressure relation has already been derived in Eq. (2.33). From Table 2.1, $\rho_1 g = 62.4$ and $\rho_2 g = 846$ lbf/ft$^3$. We need $z_A$, $z_1$, and $z_2$ in feet to produce $p$ in pounds force per square foot. Substitute into (2.33)

$$\begin{aligned} p_A &= 2116 - 62.4(\tfrac{7}{12} - \tfrac{4}{12}) - 846(\tfrac{4}{12} - \tfrac{13}{12}) \\ &= 2116 - 16 + 635 = 2735 \text{ lbf/ft}^2 \end{aligned} \qquad \textit{Ans. (a)}$$

*Part (b)* The specific weight of glycerin is 78.7 lbf/ft$^3$. The balance in Eq. (2.33) would now be

$$2735 = 2116 - 16 - 78.7(\tfrac{4}{12} - z_2)$$

or

$$z_2 = \frac{635}{78.7} + \frac{4}{12} = 8.4 \text{ ft} = 101 \text{ in} \qquad \textit{Ans. (b)}$$

This would require a much taller U-tube and is not a good idea in the long run because glycerin tends to mix with water over a period of time.

Instead of attempting to remember manometer formulations, it is simpler to remember Eq. (2.20)[1] and derive the pressure-change relation for each new situation. Figure 2.10 illustrates a multiple-fluid manometer problem for finding the

[1] The author prefers as a memory device to keep the minus sign with $\rho g$ so that the subscripts on $p$ and $z$ will be the same on both sides.



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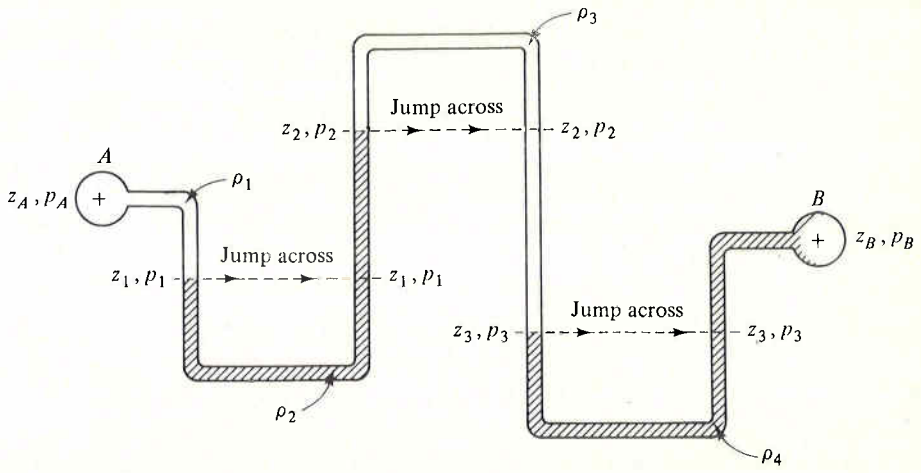


**Fig. 2.10** A complicated multiple-fluid manometer to relate $p_A$ to $p_B$. This system is not especially practical but makes a good homework or examination problem.

difference in pressure between the two chambers $A$ and $B$. Repeated application of Eq. (2.20) results in

$$\begin{aligned} p_A - p_1 &= -\rho_1 g(z_A - z_1) \qquad p_1 - p_2 = -\rho_2 g(z_1 - z_2) \\ p_2 - p_3 &= -\rho_3 g(z_2 - z_3) \qquad p_3 - p_B = -\rho_4 g(z_3 - z_B) \end{aligned} \tag{2.34}$$

Summing these four gives the desired pressure difference $p_A - p_B$.

**EXAMPLE 2.4** Find the pressure difference $p_A - p_B$ in Fig. 2.10 if $z_A = 1.6$ m, $z_1 = 0.7$ m, $z_2 = 2.1$ m, $z_3 = 0.9$ m, $z_B = 1.8$ m, fluids 1 and 3 are water, and fluids 2 and 4 are mercury.

*Solution* The specific weights of water and mercury are 9790 and 133,100 N/m$^3$, respectively. The pressure relations have already been derived in Eq. (2.34). Summing them and substituting numerical values, we have

$$\begin{aligned} p_A - p_B &= -9790(1.6 - 0.7) - 133{,}100(0.7 - 2.1) - 9790(2.1 - 0.9) \\ &\quad - 133{,}100(0.9 - 1.8) = -8811 + 186{,}340 - 11{,}748 + 119{,}790 \\ &= 285{,}571 \text{ Pa} = 286 \text{ kPa} \end{aligned} \qquad \textit{Ans.}$$

The intermediate six-figure result of 285,571 Pa is utterly fatuous since the measurements cannot be made that accurately.

In making these manometer calculations we have neglected the capillary-height changes due to surface tension, which were discussed in Example 1.13. These effects cancel out if there is a fluid interface, or *meniscus*, on both sides of the U-tube, as in Fig. 2.9. Otherwise, as in the right-hand U-tube of Fig. 2.10, a capillary correction can be made or the effect can be made negligible by using large-bore tubes.

## 2.5 HYDROSTATIC FORCES ON PLANE SURFACES

A common problem in the design of structures which interact with fluids is the computation of the hydrostatic force on a plane surface. If we neglect density



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changes in the fluid, Eq. (2.20) applies and the pressure on any submerged surface varies linearly with depth. For a plane surface, the linear stress distribution is exactly analogous to combined bending and compression of a beam in strength-of-materials theory. The hydrostatic problem thus reduces to simple formulas involving the centroid and moments of inertia of the plate cross-sectional area.

Figure 2.11 shows a plane panel of arbitrary shape completely submerged in a liquid. The panel plane makes an arbitrary angle $\theta$ with the horizontal free surface, so that the depth varies over the panel surface. If $h$ is the depth to any element area $dA$ of the plate, from Eq. (2.20) the pressure there is $p = p_a + \rho g h$.

To derive formulas involving the plate shape, establish an $xy$ coordinate system in the plane of the plate with the origin at its centroid, plus a dummy coordinate $\xi$ down from the surface in the plane of the plate. Then the total hydrostatic force on one side of the plate is given by

$$F = \int p\, dA = \int (p_a + \rho g h)\, dA = p_a A + \rho g \int h\, dA \tag{2.35}$$

The remaining integral is evaluated by noticing from Fig. 2.11 that $h = \xi \sin\theta$ and, by definition, the centroidal slant distance from the surface to the plate is

$$\xi_{CG} = \frac{1}{A} \int \xi\, dA \tag{2.36}$$

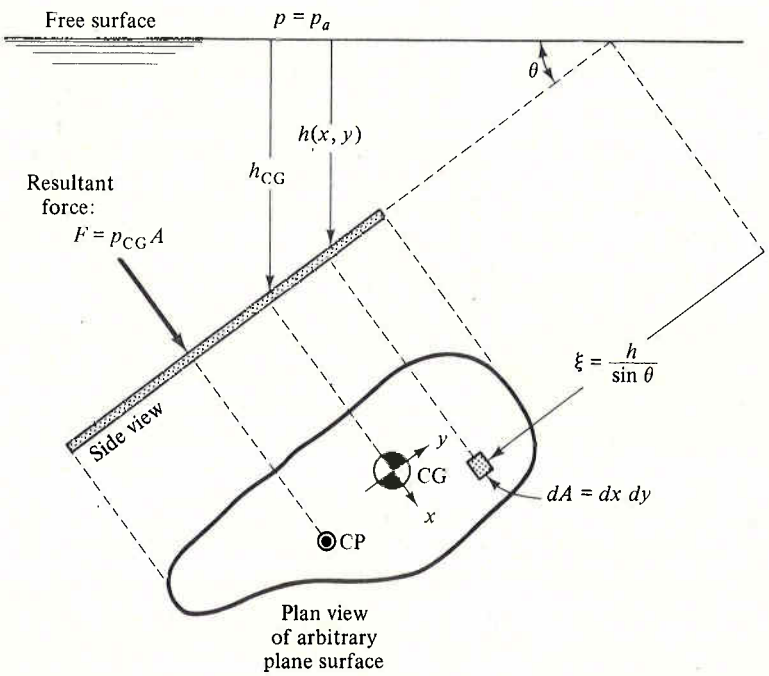


**Fig. 2.11** Hydrostatic force and center of pressure on an arbitrary plane surface of area $A$ inclined at an angle $\theta$ below the free surface.



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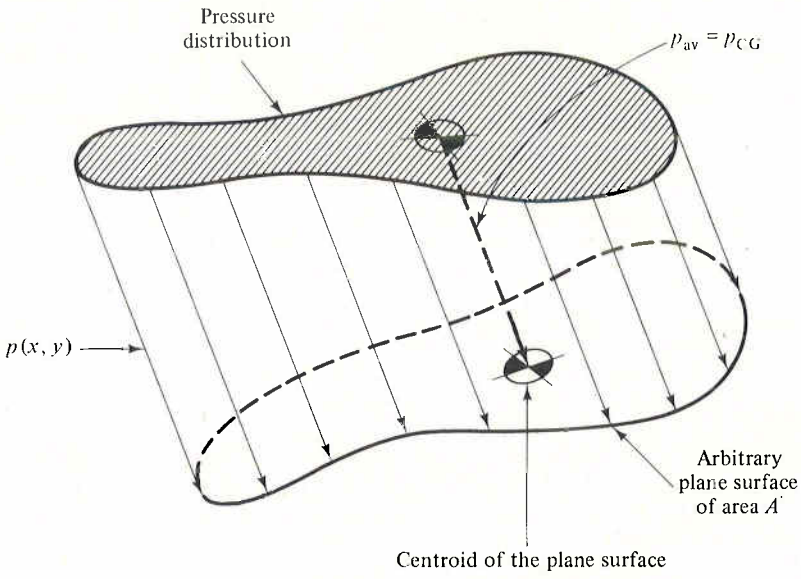


**Fig. 2.12** The hydrostatic-pressure force on a plane surface is equal, regardless of its shape, to the resultant of the three-dimensional linear pressure distribution on that surface, $F = p_{CG}A$.

Therefore, since $\theta$ is constant along the plate, Eq. (2.35) becomes

$$F = p_a A + \rho g \sin\theta \int \xi \, dA = p_a A + \rho g \sin\theta \, \xi_{CG} A \tag{2.37}$$

Finally, unravel this by noticing that $\xi_{CG} \sin\theta = h_{CG}$, the depth straight down from the surface to the plate centroid. Thus

$$F = p_a A + \rho g h_{CG} A = (p_a + \rho g h_{CG})A = p_{CG} A \tag{2.38}$$

The force on one side of any plane submerged surface in a uniform fluid equals the pressure at the plate centroid times the plate area, independent of the shape of the plate or the angle $\theta$ at which it is slanted.

Equation (2.38) can be visualized physically in Fig. 2.12 as the resultant of a linear stress distribution over the plate area. This simulates combined compression and bending of a beam of the same cross section. It follows that the "bending" portion of the stress causes no force if its "neutral axis" passes through the plate centroid of area. Thus the remaining "compression" part must equal the centroid stress times the plate area. This is the result of Eq. (2.38).

However, to balance the bending-moment portion of the stress, the resultant force $F$ does not act through the centroid but below it toward the high-pressure side. Its line of action passes through the *center of pressure* CP of the plate, as sketched in Fig. 2.11. To find the coordinates $(x_{CP}, y_{CP})$, we sum moments of the elemental force $p\,dA$ about the centroid and equate to the moment of the resultant $F$. To compute $y_{CP}$, we equate

$$F y_{CP} = \int yp \, dA = \int y(p_a + \rho g \xi \sin\theta) \, dA = \rho g \sin\theta \int y\xi \, dA \tag{2.39}$$



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The term $\int p_a y\, dA$ vanishes by definition of centroidal axes. Introducing $\xi = \xi_{CG} - y$, we obtain

$$Fy_{CP} = \rho g \sin\theta\left(\xi_{CG}\int y\, dA - \int y^2\, dA\right) = -\rho g \sin\theta\, I_{xx} \tag{2.40}$$

where again $\int y\, dA = 0$ and $I_{xx}$ is the area moment of inertia of the plate area about its centroidal $x$ axis, computed in the plane of the plate. Substituting for $F$ gives the result

$$y_{CP} = -\rho g \sin\theta \frac{I_{xx}}{p_{CG}A} \tag{2.41}$$

The negative sign in Eq. (2.41) shows that $y_{CP}$ is below the centroid at a deeper level and, unlike $F$, depends upon the angle $\theta$. If we move the plate deeper, $y_{CP}$ approaches the centroid because every term in Eq. (2.41) remains constant except $p_{CG}$, which increases.

The determination of $x_{CP}$ is exactly similar:

$$\begin{aligned} Fx_{CP} &= \int xp\, dA = \int x[p_a + \rho g(\xi_{CG} - y)\sin\theta]\, dA \\ &= -\rho g \sin\theta \int xy\, dA = -\rho g \sin\theta\, I_{xy} \end{aligned} \tag{2.42}$$

where $I_{xy}$ is the product of inertia of the plate, again computed in the plane of the plate. Substituting for $F$ gives

$$x_{CP} = -\rho g \sin\theta \frac{I_{xy}}{p_{CG}A} \tag{2.43}$$

For positive $I_{xy}$, $x_{CP}$ is negative because the dominant pressure force acts in the third, or lower left, quadrant of the panel. If $I_{xy} = 0$, usually implying symmetry, $x_{CP} = 0$ and the center of pressure lies directly below the centroid on the $y$ axis.

### Gage-Pressure Formulas

In most cases the ambient pressure $p_a$ is neglected because it acts on both sides of the plate; e.g., the other side of the plate is inside a ship or on the dry side of a gate or dam. In this case $p_{CG} = \rho g h_{CG}$, and the center of pressure becomes independent of specific weight

$$F = \rho g h_{CG} A \qquad y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} \qquad x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} \tag{2.44}$$

Figure 2.13 gives the area and moments of inertia of several common cross sections for use with these formulas.



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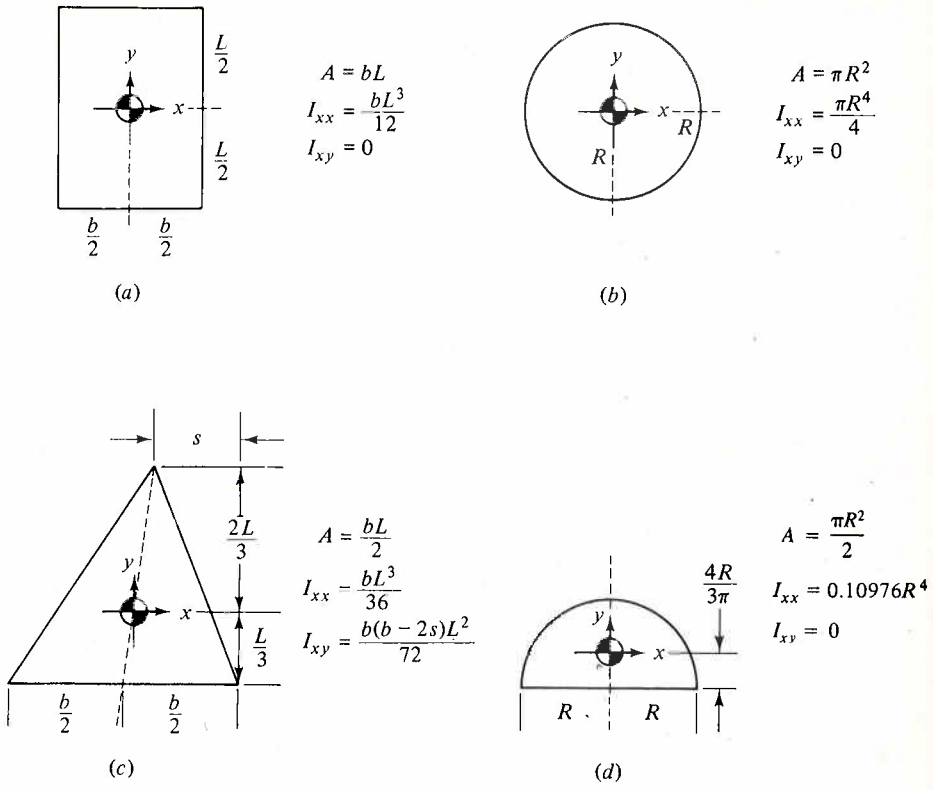


**Fig. 2.13** Centroidal moments of inertia for various cross sections: (*a*) rectangle, (*b*) circle, (*c*) triangle, and (*d*) semicircle.

**EXAMPLE 2.5** The gate in Fig. E2.5*a* is 5 ft wide, is hinged at point *B*, and rests against a smooth wall at point *A*. Compute (*a*) the force on the gate due to seawater pressure; (*b*) the horizontal force *P* exerted by the wall at point *A*; and (*c*) the reactions at the hinge *B*.

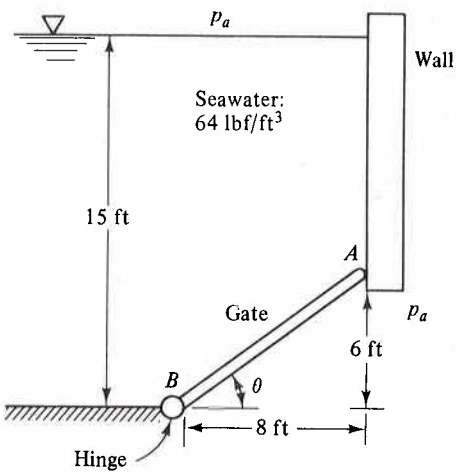


**Fig. E2.5a**



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*Solution*

*Part (a)* By geometry the gate is 10 ft long from $A$ to $B$, and its centroid is halfway between, or at elevation 3 ft above point $B$. The depth $h_{CG}$ is thus $15 - 3 = 12$ ft. The gate area is $5(10) = 50$ ft$^2$. Neglect $p_a$ as acting on both sides of the gate. From Eq. (2.38) the hydrostatic force on the gate is

$$F = p_{CG}A = \rho g h_{CG} A = (64 \text{ lbf/ft}^3)(12 \text{ ft})(50 \text{ ft}^2) = 38{,}400 \text{ lbf} \qquad \textit{Ans. (a)}$$

*Part (b)* First we must find the center of pressure of $F$. A free-body diagram of the gate is shown in Fig. E2.5*b*. The gate is a rectangle, and hence

$$I_{xy} = 0 \qquad \text{and} \qquad I_{xx} = \frac{bL^3}{12} = \frac{(5 \text{ ft})(10 \text{ ft})^3}{12} = 417 \text{ ft}^4$$

The distance $l$ from the CG to the CP is given by Eq. (2.44) since $p_a$ is neglected

$$l = -y_{CP} = +\frac{I_{xx} \sin\theta}{h_{CG}A} = \frac{(417 \text{ ft}^4)(\frac{6}{10})}{(12 \text{ ft})(50 \text{ ft}^2)} = 0.417 \text{ ft}$$

The distance from point $B$ to force $F$ is thus $10 - l - 5 = 4.583$ ft. Summing moments counterclockwise about $B$ gives

$$PL \sin\theta - F(5 - l) = P(6 \text{ ft}) - (38{,}400 \text{ lbf})(4.583 \text{ ft}) = 0$$

or

$$P = 29{,}300 \text{ lbf} \qquad \textit{Ans. (b)}$$

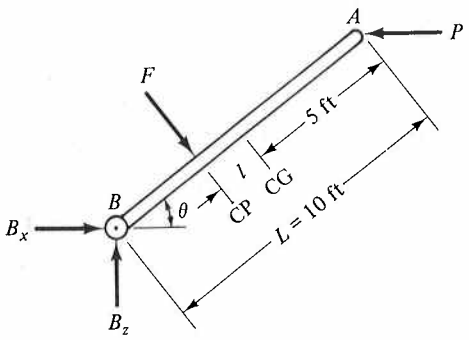


**Fig. E2..5b**

*Part (c)* With $F$ and $P$ known, the reactions $B_x$ and $B_z$ are found by summing forces on the gate

$$\sum F_x = 0 = B_x + F \sin\theta - P = B_x + 38{,}400(0.6) - 29{,}300$$

or

$$B_x = 6300 \text{ lbf}$$

$$\sum F_z = 0 = B_z - F \cos\theta = B_z - 38{,}400(0.8)$$

or

$$B_z = 30{,}700 \text{ lbf} \qquad \textit{Ans. (c)}$$

This example should have reviewed your knowledge of statics.

**EXAMPLE 2.6** A tank of oil has a right triangular panel near the bottom, as in Fig. E2.6. Omitting $p_a$, find the (*a*) hydrostatic force and (*b*) CP on the panel.



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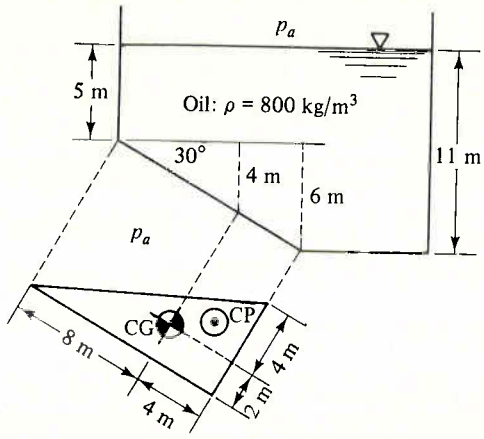


**Fig. E2.6**

*Solution*

*Part (a)* The triangle has properties given in Fig. 2.13*c*. The centroid is one-third up (4 m) and one-third over (2 m) from the lower left corner, as shown. The area is

$$\tfrac{1}{2}(6\text{ m})(12\text{ m}) = 36\text{ m}^2$$

The moments of inertia are

$$I_{xx} = \frac{bL^3}{36} = \frac{(6\text{ m})(12\text{ m})^3}{36} = 288\text{ m}^4$$

and

$$I_{xy} = \frac{b(b-2s)L^2}{72} = \frac{(6\text{ m})[6\text{ m} - 2(6\text{ m})](12\text{ m})^2}{72} = -72\text{ m}^4$$

The depth to the centroid is $h_{CG} = 5 + 4 = 9$ m; thus the hydrostatic force from Eq. (2.44) is

$$F = \rho g h_{CG} A = (800\text{ kg/m}^3)(9.807\text{ m/s}^2)(9\text{ m})(36\text{ m}^2)$$
$$= 2.54 \times 10^6\text{ (kg}\cdot\text{m)/s}^2 = 2.54 \times 10^6\text{ N} = 2.54\text{ MN} \qquad \textit{Ans. (a)}$$

*Part (b)* The CP position is given by Eqs. (2.44):

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(288\text{ m}^4)(\sin 30°)}{(9\text{ m})(36\text{ m}^2)} = -0.444\text{ m}$$

$$x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} = -\frac{(-72\text{ m}^4)(\sin 30°)}{(9\text{ m})(36\text{ m}^2)} = +0.111\text{ m} \qquad \textit{Ans. (b)}$$

The resultant force $F = 2.54$ MN acts through this point, which is down and to the right of the centroid.

## 2.6 HYDROSTATIC FORCES ON CURVED SURFACES

The resultant pressure force on a curved surface is most easily computed by separating into horizontal and vertical components. Consider the arbitrary curved surface sketched in Fig.. 2.14*a*. The incremental pressure forces, being normal to the local area element, vary in direction along the surface and thus cannot be added numerically. We could sum the separate three components of these elemental



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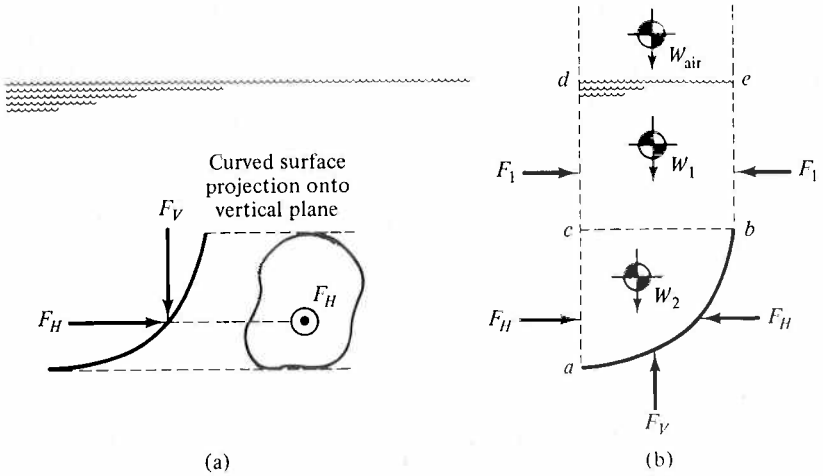


**Fig. 2.14** Computation of hydrostatic force on a curved surface: (*a*) submerged curved surface; (*b*) free-body diagram of fluid above the curved surface.

pressure forces, but it turns out that we need not perform a laborious three-way integration.

Figure 2.14*b* shows a free-body diagram of the column of fluid contained in the vertical projection above the curved surface. The desired forces $F_H$ and $F_V$ are exerted by the surface on the fluid column. Other forces are shown due to fluid weight and horizontal pressure on the vertical sides of this column. The column of fluid must be in static equilibrium. On the upper part of the column, *bcde*, the horizontal components $F_1$ exactly balance and are not relevant to the discussion. On the lower, irregular portion of fluid *abc* adjoining the surface, summation of horizontal forces shows that the desired force $F_H$ due to the curved surface is exactly equal to the force $F_H$ on the vertical left side of the fluid column. This left-side force can be computed by the plane-surface formula, Eq. (2.38), based on a vertical projection of the area of the curved surface. This is a general rule and simplifies the analysis:

> The horizontal component of force on a curved surface equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

If there are two horizontal components, both can be computed by this scheme.

Summation of vertical forces on the fluid free body then shows that

$$F_V = W_1 + W_2 + W_{\text{air}} \tag{2.45}$$

We can state this in words as our second general rule:

> The vertical component of pressure force on a curved surface equals in magnitude and direction the weight of the entire column of fluid, both liquid and atmosphere, above the curved surface.

Thus the calculation of $F_V$ involves little more than finding centers of mass of a column of fluid—perhaps a little integration if the lower portion *abc* has a particularly vexing shape.



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**EXAMPLE 2.7** A dam has a parabolic shape $z/z_0 = (x/x_0)^2$ as shown in Fig. E2.7*a*, with $x_0 = 10$ ft and $z_0 = 24$ ft. The fluid is water, $\rho g = 62.4$ lbf/ft$^3$, and atmospheric pressure may be omitted. Compute the forces $F_H$ and $F_V$ on the dam and the position CP where they act. The width of the dam is 50 ft.

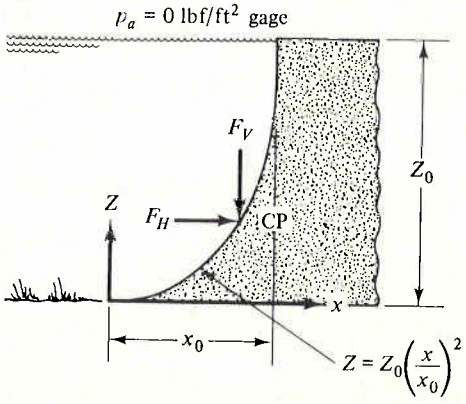


Fig. E2.7a

*Solution* The vertical projection of this curved surface is a rectangle 24 ft high and 50 ft wide, with its centroid halfway down, or $h_{CG} = 12$ ft. The force $F_H$ is thus

$$F_H = \rho g h_{CG} A_{proj} = (62.4 \text{ lbf/ft}^3)(12 \text{ ft})(24 \text{ ft})(50 \text{ ft})$$
$$= 899{,}000 \text{ lbf} = 899 \times 10^3 \text{ lbf} \qquad \textit{Ans.}$$

The line of action of $F_H$ is below the centroid by an amount

$$y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A_{proj}} = -\frac{\frac{1}{12}(50 \text{ ft})(24 \text{ ft})^3(\sin 90°)}{(12 \text{ ft})(24 \text{ ft})(50 \text{ ft})} = -4 \text{ ft}$$

Thus $F_H$ is 12 + 4 = 16 ft, or two-thirds, down from the free surface or 8 ft from the bottom, as might have been evident by inspection of the triangular pressure distribution.

The vertical component $F_V$ equals the weight of the parabolic portion of fluid above the curved surface. The geometric properties of a parabola are shown in Fig. E2.7*b*. The weight of this amount of water is

$$F_V = \rho g(\tfrac{2}{3} x_0 z_0 b) = (62.4 \text{ lbf/ft}^3)(\tfrac{2}{3})(10 \text{ ft})(24 \text{ ft})(50 \text{ ft})$$
$$= 499{,}000 \text{ lbf} = 499 \times 10^3 \text{ lbf} \qquad \textit{Ans.}$$

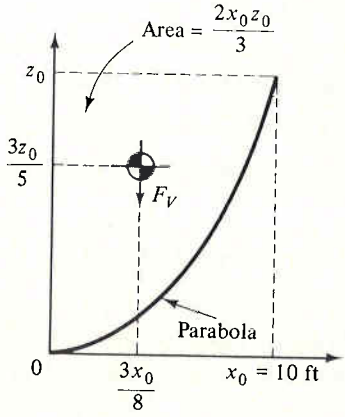


Fig. E2.7b



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This acts downward on the surface at a distance $3x_0/8 = 3.75$ ft over from the origin of coordinates. Note that the vertical distance $3z_0/5$ in Fig. E2.7*b* is irrelevant.

The total resultant force acting on the dam is

$$F = (F_H^2 + F_V^2)^{1/2} = [(499)^2 + (899)^2]^{1/2} = 1028 \times 10^3 \text{ lbf}$$

As seen in Fig. E2.7*c*, this force acts down and to the right at an angle of $29° = \tan^{-1}\frac{499}{899}$. The force $F$ passes through the point $(x, z) = (3.75 \text{ ft}, 8 \text{ ft})$. If we move down along the 29° line until we strike the dam, we find an equivalent center of pressure on the dam at

$$x_{CP} = 5.43 \text{ ft} \qquad z_{CP} = 7.07 \text{ ft} \qquad \textit{Ans.}$$

This definition of CP is rather artificial, but this is an unavoidable complication of dealing with a curved surface.

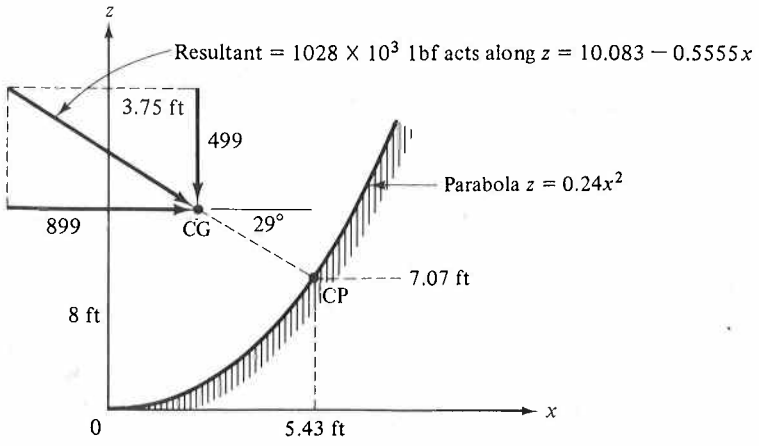


Fig. E2.7c

## 2.7 HYDROSTATIC FORCES IN LAYERED FLUIDS

The formulas for plane and curved surfaces in Secs. 2.5 and 2.6 are valid only for a fluid of uniform density. If the fluid is layered with different densities, as in Fig. 2.15, a single formula cannot resolve the problem because the slope of the linear pressure distribution changes between layers. However, the formulas apply separately to each layer, and thus the appropriate remedy is to compute and sum the separate layer forces and moments.

Consider the slanted plane surface immersed in a two-layer fluid in Fig. 2.15. The slope of the pressure distribution becomes steeper as we move down into the denser second layer. The total force on the plate does *not* equal the pressure at the centroid times the plate area, but the plate portion in each layer does satisfy the formula, so that we can sum forces to find the total

$$F = \sum F_i = \sum p_{CG_i} A_i \tag{2.46}$$

Similarly, the centroid of the plate portion in each layer can be used to locate the center of pressure on that portion

$$y_{CP_i} = -\frac{\rho_i g \sin\theta_i I_{xx_i}}{p_{CG_i} A_i} \qquad x_{CP_i} = -\frac{\rho_i g \sin\theta_i I_{xy_i}}{p_{CG_i} A_i} \tag{2.47}$$



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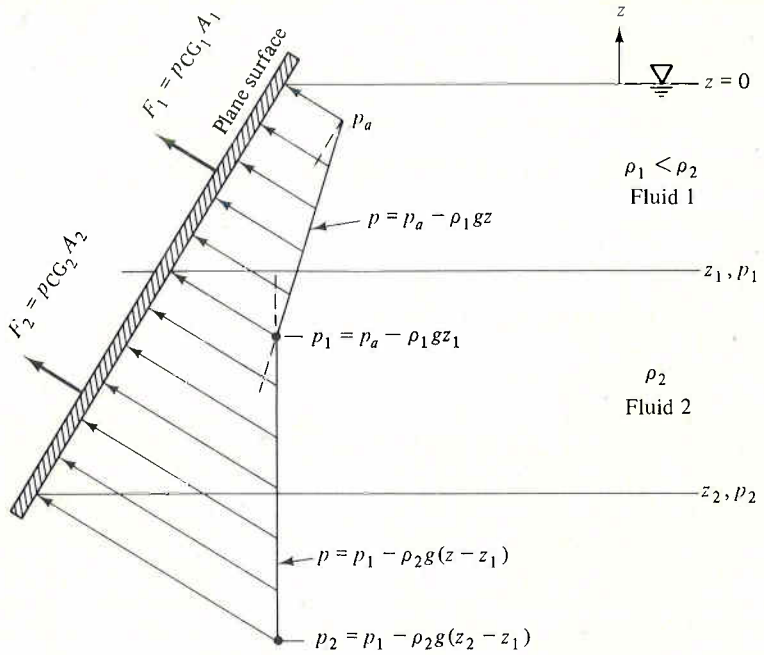


**Fig. 2.15** Hydrostatic forces on a surface immersed in a layered fluid must be summed together in separate pieces.

These formulas locate the center of pressure of that particular $F_i$ with respect to the centroid of that particular portion of plate in the layer, not with respect to the centroid of the entire plate. The center of pressure of the total force $F = \sum F_i$ can then be found by summing moments about some convenient point such as the surface. The following example will illustrate.

**EXAMPLE 2.8** A tank 20 ft deep and 7 ft wide is layered with 8 ft of oil, 6 ft of water, and 4 ft of mercury. Compute (*a*) the total hydrostatic force and (*b*) the resultant center of pressure of the fluid on the right-hand side of the tank.

*Solution*

*Part (a)* Divide the end panel into three parts as sketched in Fig. E2.8 and find the hydrostatic pressure at the centroid of each part, using the relation (2.38) in steps as in Fig. E2.8:

$$p_{CG_1} = (55.0 \text{ lbf/ft}^3)(4 \text{ ft}) = 220 \text{ lbf/ft}^2$$

$$p_{CG_2} = (55.0)(8) + 62.4(3) = 627 \text{ lbf/ft}^2$$

$$p_{CG_3} = (55.0)(8) + 62.4(6) + 846(2) = 2506 \text{ lbf/ft}^2$$

These pressures are then multiplied by the respective panel areas to find the force on each portion:

$$F_1 = p_{CG_1} A_1 = (220 \text{ lbf/ft}^2)(8 \text{ ft})(7 \text{ ft}) = 12{,}300 \text{ lbf}$$

$$F_2 = p_{CG_2} A_2 = 627(6)(7) = 26{,}300 \text{ lbf}$$

$$F_3 = p_{CG_3} A_3 = 2506(4)(7) = 70{,}200 \text{ lbf}$$

$$F = \sum F_i = \overline{108{,}800} \text{ lbf} \qquad \textit{Ans. (a)}$$



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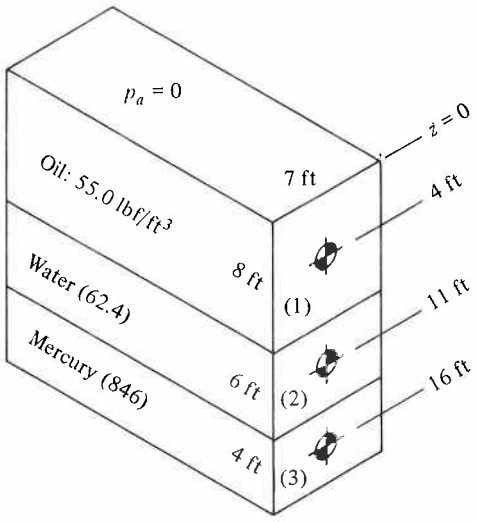


**Fig. E2.8**

*Part (b)* Equations (2.47) can be used to locate the CP of each force $F_i$, noting that $\theta = 90°$ and $\sin\theta = 1$ for all parts. The moments of inertia are $I_{xx_1} = (7\text{ ft})(8\text{ ft})^3/12 = 298.7\text{ ft}^4$, $I_{xx_2} = 7(6)^3/12 = 126.0\text{ ft}^4$, and $I_{xx_3} = 7(4)^3/12 = 37.3\text{ ft}^4$. The centers of pressure are thus at

$$y_{CP_1} = -\frac{\rho_1 g I_{xx_1}}{F_1} = -\frac{(55.0\text{ lbf/ft}^3)(298.7\text{ ft}^4)}{12{,}300\text{ lbf}} = -1{:}33\text{ ft}$$

$$y_{CP_2} = -\frac{62.4(126.0)}{26{,}300} = -0.30\text{ ft} \qquad y_{CP_3} = -\frac{846(37.3)}{70{,}200} = -0.45\text{ ft}$$

This locates $z_{CP_1} = -4 - 1.33 = -5.33\text{ ft}$, $z_{CP_2} = -11 - 0.30 = -11.30\text{ ft}$, and $z_{CP_3} = -16 - 0.45 = -16.45\text{ ft}$. Summing moments about the surface then gives

$$\sum F_i z_{CP_i} = F z_{CP}$$

or
$$12{,}300(-5.33) + 26{,}300(-11.30) + 70{,}200(-16.45) = 108{,}800 z_{CP}$$

or
$$z_{CP} = -\frac{1{,}518{,}000}{108{,}800} = -13.95\text{ ft} \qquad \textit{Ans. (b)}$$

The center of pressure of the total resultant force on the right side of the tank lies 13.95 ft below the surface.

## 2.8 BUOYANCY AND STABILITY

The same principles used to compute hydrostatic forces on surfaces can be applied to the net pressure force on a completely submerged or floating body. The results are the two laws of buoyancy discovered by Archimedes in the third century B.C.:

1. A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.
2. A floating body displaces its own weight in the fluid in which it floats.



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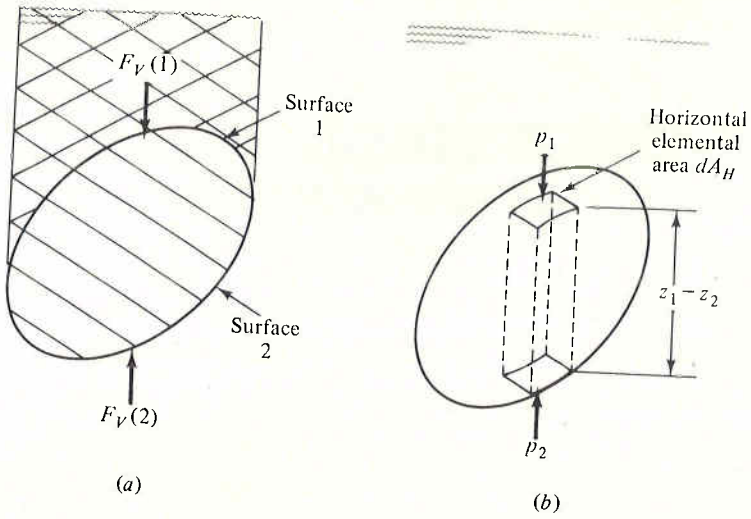


**Fig. 2.16** Two different approaches to the buoyant force on an arbitrary immersed body: (*a*) forces on upper and lower curved surfaces; (*b*) summation of elemental vertical-pressure forces.

These two laws are easily derived by referring to Fig. 2.16. In Fig. 2.16*a*, the body lies between an upper curved surface 1 and a lower curved surface 2. From Eq. (2.45) for vertical force, the body experiences a net upward force

$$\begin{aligned} F_B &= F_V(2) - F_V(1) \\ &= (\text{fluid weight above 2}) - (\text{fluid weight above 1}) \\ &= \text{weight of fluid equivalent to body volume} \end{aligned} \tag{2.48}$$

Alternatively, from Fig. 2.16*b*, we can sum the vertical forces on elemental vertical slices through the immersed body

$$F_B = \int_{\text{body}} (p_2 - p_1)\, dA_H = -\rho g \int (z_2 - z_1)\, dA_H = (\rho g)(\text{body volume}) \tag{2.49}$$

These are identical results and equivalent to law 1 above.

Equation (2.49) assumes that the fluid has uniform specific weight. The line of action of the buoyant force passes through the center of volume of the displaced body; i.e., its center of mass is computed as if it had uniform density. This point through which $F_B$ acts is called the *center of buoyancy*, commonly labeled $B$ or CB on a drawing. Of course the point $B$ may or may not correspond to the actual center of mass of the body's own material, which may have variable density.

Equation (2.49) can be generalized to a layered fluid (LF) by summing the weights of each layer of density $\rho_i$ displaced by the immersed body

$$(F_B)_{\text{LF}} = \sum \rho_i g(\text{displaced volume})_i \tag{2.50}$$

Each displaced layer would have its own center of volume, and one would have to sum moments of the incremental buoyant forces in order to find the center of buoyancy of the immersed body.

Since liquids are relatively heavy, we are conscious of their buoyant forces, but gases also exert buoyancy on any body immersed in them. For example, human beings have an average specific weight of about 60 lbf/ft$^3$. We may record the


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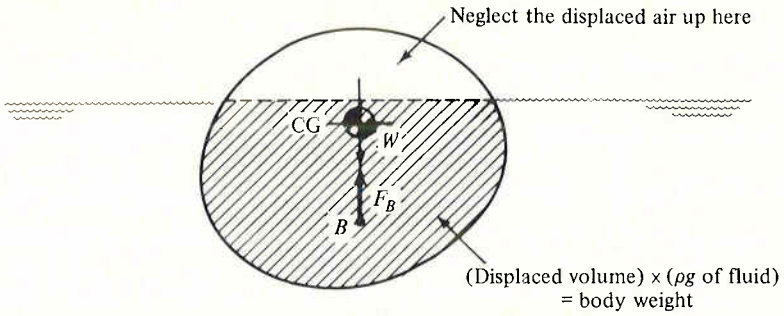


**Fig. 2.17** Static equilibrium of a floating body.

weight of a person as 180 lbf and thus estimate his total volume as 3.0 ft$^3$. However, in so doing we are neglecting the buoyant force of the air surrounding the person. At standard conditions, the specific weight of air is 0.0763 lbf/ft$^3$; hence the buoyant force is approximately 0.23 lbf. If measured in vacuo, the person would weigh about 0.23 lbf more. For balloons and blimps the buoyant force of air instead of being negligible is the controlling factor in the design. Also, many flow phenomena, e.g., natural convection of heat and vertical mixing in the ocean, are strongly dependent upon seemingly small buoyant forces.

Floating bodies are a special case; only a portion of the body is submerged, the remainder poking up out of the free surface. This is illustrated in Fig. 2.17, where the shaded portion is the displaced volume. Equation (2.49) is modified to apply to this smaller volume

$$F_B = (\rho g)(\text{displaced volume}) = \text{floating-body weight} \tag{2.51}$$

Not only does the buoyant force equal the body weight but they are *collinear* since there can be no net moments for static equilibrium. Equation (2.51) is the mathematical equivalent of Archimedes' law 2 previously stated.

**EXAMPLE 2.9** A block of concrete weighs 100 lbf in air and "weighs" only 60 lbf when immersed in fresh water (62.4 lbf/ft$^3$). What is the average specific weight of the block?

*Solution* A free-body diagram of the submerged block shows a balance between the apparent weight, the buoyant force, and the actual weight

$$\sum F_z = 0 = 60 + F_B - 100$$

or

$$F_B = 40 \text{ lbf} = (62.4)(\text{block volume in ft}^3)$$

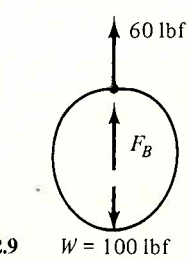


**Fig. E2.9**



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Solving gives the volume of the block as $40/62.4 = 0.641\ \text{ft}^3$. Therefore the specific weight of the block is

$$(\rho g)_{\text{block}} = \frac{100\ \text{lbf}}{0.641\ \text{ft}^3} = 156\ \text{lbf/ft}^3 \qquad \textit{Ans.}$$

Occasionally, a body will have exactly the right weight and volume for its ratio to equal the specific weight of the fluid. If so, the body will be *neutrally buoyant* and will remain at rest at any point where it is immersed in the fluid. Small neutrally buoyant particles are sometimes used in flow visualization, and a neutrally buoyant body called a *Swallow float* [2] is used to track oceanographic currents. A submarine can achieve positive, neutral, or negative buoyancy by pumping water in or out of its ballast tanks.

## Stability

A floating body as in Fig. 2.17 may not approve of the position in which it is floating. If so, it will overturn at the first opportunity and is said to be statically *unstable*, like a pencil balanced upon its point. The least disturbance will cause it to seek another equilibrium position which is stable. Engineers must design to avoid floating instability. The only way to tell for sure whether a floating position is stable is to "disturb" the body a slight amount mathematically and see whether it develops a restoring moment which will return it to its original position. If so, it is stable; if not, unstable. Such calculations for arbitrary floating bodies have been honed to a fine art by naval architects [3], but we can at least outline the basic principle of the static-stability calculation. Figure 2.18 illustrates the computation for the usual case of a symmetric floating body. The steps are as follows:

1. The basic floating position is calculated from Eq. (2.51). The body's center of mass $G$ and center of buoyancy $B$ are computed.

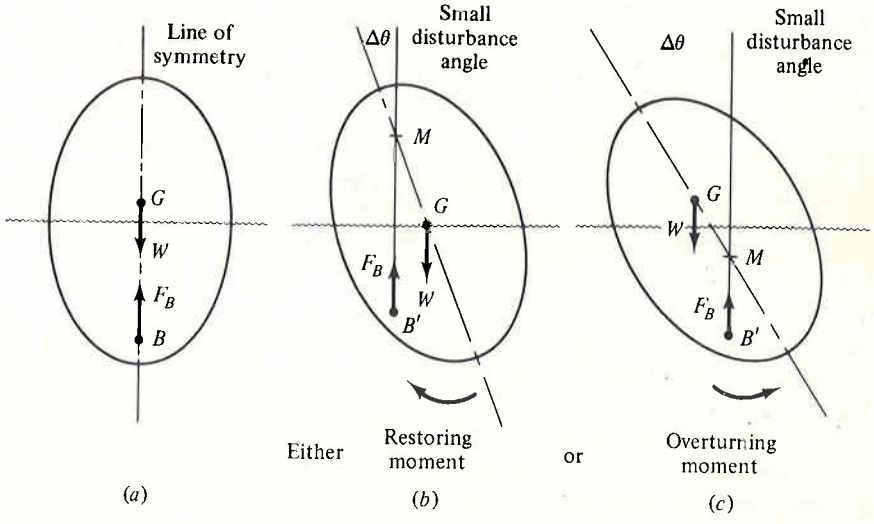


**Fig. 2.18** Calculation of the metacenter $M$ of a floating body to determine its static stability: (*a*) initial floating position; (*b*) $B'$ moves far out (point $M$ above $G$ denotes stability); (*c*) $B'$ moves slightly (point $M$ below $G$ denotes instability).



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2. The body is tilted a small angle $\Delta\theta$ and a new waterline established for the body to float at this angle. The new position $B'$ of the center of buoyancy is calculated. A vertical line drawn upward from $B'$ intersects the line of symmetry at a point $M$, called the *metacenter*, which is independent of $\Delta\theta$ for small angles.
3. If point $M$ is above $G$, that is, if the *metacentric height* $\overline{MG}$ is positive, a restoring moment is present and the original position is stable. If $M$ is below $G$ (negative $\overline{MG}$), the body is unstable and will overturn if disturbed. Stability increases with increasing $\overline{MG}$.

Thus the metacentric height is a property of the cross section for the given weight, and its value gives an indication of the stability of the body. For a body of varying cross section and draft, such as a ship, the computation of the metacenter can be very involved. We shall use a two-dimensional, or uniform, cross section as an example.

**EXAMPLE 2.10** A barge has a uniform rectangular cross section of width $2L$ and vertical draft of height $H$. Determine (*a*) the metacentric height for a small tilt angle and (*b*) the range of ratio $L/H$ for which the barge is statically stable. Assume that the center of mass $G$ of the barge is exactly at the waterline, as shown in Fig. E2.10*a*.

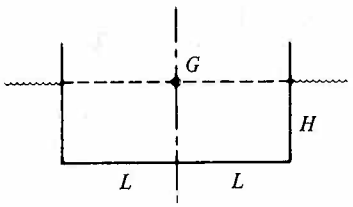


**Fig. E2.10a**

*Solution* The barge in its tilted position $\Delta\theta$ is shown in Fig. E2.10*b*. Because of symmetry the tilted body floats, again with $G$ at the waterline, and triangles $aeG$ and $fbG$ are identical in size. The new center of buoyancy $B'$ can be found by summing moments of the submerged areas about, say, the point $G$ and setting this equal to the total submerged area ($2LH$) times the

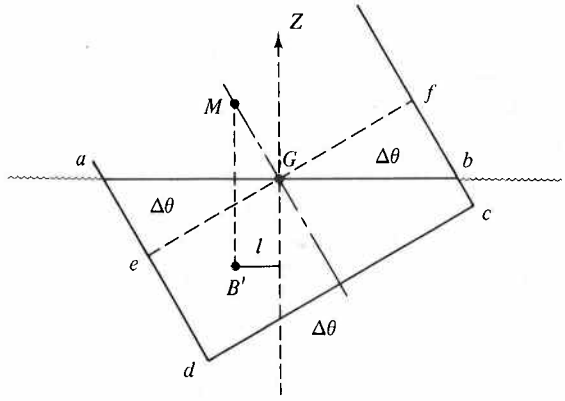


**Fig. E2.10b**



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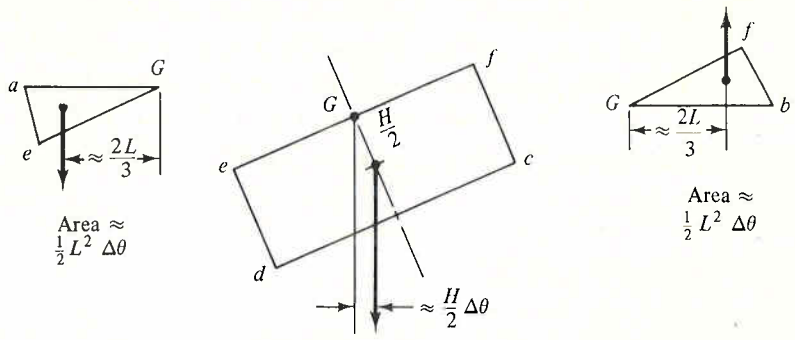


Fig. E2.10c

distance $l$. The submerged area $aGbcde$ is the sum of rectangle $efdc$ plus triangle $aeG$ minus triangle $fbG$, and the centroids of each of these pieces can easily be found for small angles of tilt (Fig. E2.10$c$). Summing moments of area counterclockwise about $G$, we have

$$2(\tfrac{1}{2}L^2\,\Delta\theta)\frac{2L}{3} - 2HL\left(\frac{H}{2}\Delta\theta\right) \equiv 2HLl$$

We can solve this for the distance $l = (L^2/3H - H/2)\,\Delta\theta$. From the tilted position showing $M$ above, the distance $\overline{MG} \approx l/\Delta\theta$, for small angles. Thus we have the approximate result for metacentric height, independent of $\Delta\theta$,

$$\overline{MG} = \frac{L^2}{3H} - \frac{H}{2} \qquad \textit{Ans. (a)}$$

This can be positive or stable only if $L^2 > 3H^2/2$ or

$$L > 1.225H \qquad \textit{Ans. (b)}$$

The wider the barge relative to its draft, the more stable it is.

## 2.9 PRESSURE DISTRIBUTION IN RIGID-BODY MOTION

In rigid-body motion, all particles are in combined translation and rotation and there is no relative motion between particles. With no relative motion, there are no strains or strain rates, so that the viscous term $\mu\nabla^2\mathbf{V}$ in Eq. (2.13) vanishes, leaving a balance between pressure, gravity, and particle acceleration

$$\nabla p = \rho(\mathbf{g} - \mathbf{a}) \qquad (2.52)$$

The pressure gradient acts in the direction $\mathbf{g} - \mathbf{a}$, and lines of constant pressure (including the free surface, if any) are perpendicular to this direction. The general case of combined translation and rotation of a rigid body is shown in Fig. 2.19. If the center of rotation is at point $O$ and the translational velocity is $V_0$ at this point, the velocity of an arbitrary point $P$ on the body is given by[1]

$$\mathbf{V} = \mathbf{V}_0 + \boldsymbol{\Omega} \times \mathbf{r}_0 \qquad (2.53)$$

[1] For a more detailed derivation of rigid-body motion, see Ref. 4, sec. 2.7.



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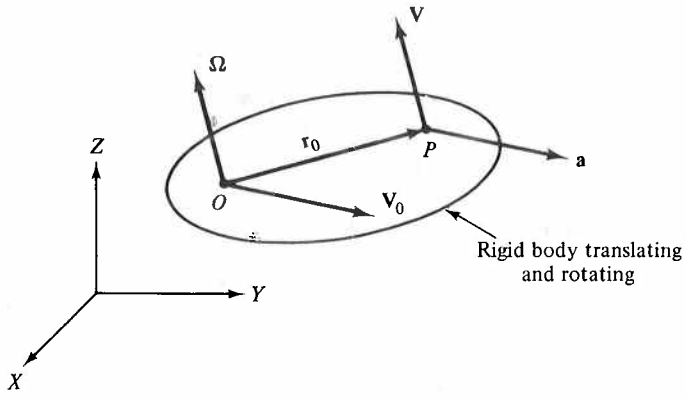


**Fig. 2.19** Kinematics of rigid-body velocity and acceleration.

where $\boldsymbol{\Omega}$ is the angular-velocity vector and $\mathbf{r}_0$ is the position of point $P$. Differentiating, we obtain the most general form of the acceleration of a rigid body

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_0) + \frac{d\boldsymbol{\Omega}}{dt} \times \mathbf{r}_0 \tag{2.54}$$

Looking at the right-hand side, we see that the first term is the translational acceleration, the second term is the *centripetal acceleration*, whose direction is from point $P$ perpendicular toward the axis of rotation, and the third term is the linear acceleration due to changes in the angular velocity. It is rare for all three of these terms to apply to any one fluid flow. In fact, fluids can rarely move in rigid-body motion unless restrained by confining walls for a long time. For example, suppose a tank of water is in a car which starts a constant acceleration. The water in the tank would begin to slosh about, and that sloshing would damp out very slowly until finally the particles of water would be in approximately rigid-body acceleration. This would take so long that the car would have reached hypersonic speeds. Nevertheless, we can at least discuss the pressure distribution in a tank of rigidly accelerating water. The following is an example where the water in the tank will reach uniform acceleration rapidly.

**EXAMPLE 2.11** A tank of water 1 m deep is in free fall under gravity with negligible drag. Compute the pressure at the bottom of the tank if $p_a = 101$ kPa.

*Solution* Being unsupported in this condition, the water particles tend to fall downward as a rigid hunk of fluid. In free fall with no drag, the downward acceleration is $\mathbf{a} = \mathbf{g}$. Thus Eq. (2.52) for this situation gives $\nabla p = \rho(\mathbf{g} - \mathbf{g}) = 0$. The pressure in the water is thus *constant everywhere* and equal to the atmospheric pressure 101 kPa. In other words, the walls are doing no service in sustaining the pressure distribution which would normally exist.

## Uniform Linear Acceleration

In this general case of uniform rigid-body acceleration, Eq. (2.52) applies, $\mathbf{a}$ having the same magnitude and direction for all particles. With reference to Fig. 2.20, the parallelogram sum of $\mathbf{g}$ and $-\mathbf{a}$ gives the direction of the pressure gradient or



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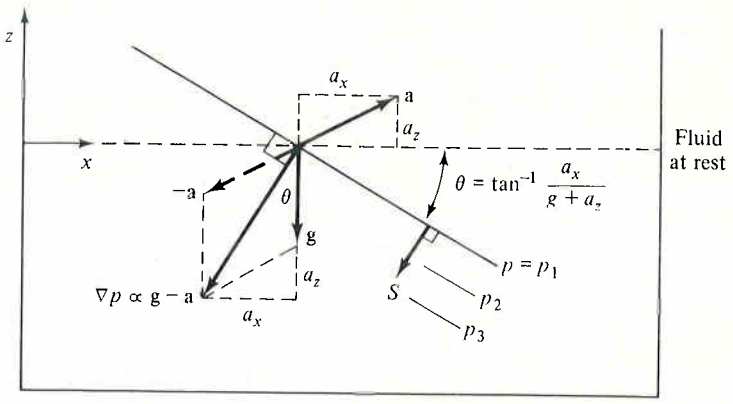


**Fig. 2.20** Tilting of constant-pressure surfaces in a tank of liquid in rigid-body acceleration.

greatest rate of increase of $p$. The surfaces of constant pressure must be perpendicular to this and are thus tilted at an angle $\theta$ such that

$$\theta = \tan^{-1} \frac{a_x}{g + a_z} \tag{2.55}$$

One of these tilted lines is the free surface, which is found by the requirement that the fluid retain its volume unless it spills out. The rate of increase of pressure in the direction $\mathbf{g} - \mathbf{a}$ is greater than in ordinary hydrostatics and is given by

$$\frac{dp}{ds} = \rho G \qquad \text{where } G = [a_x^2 + (g + a_z)^2]^{1/2} \tag{2.56}$$

These results are independent of the size or shape of the container as long as the fluid is continuously connected throughout the container.

**EXAMPLE 2.12** A drag racer rests her coffee mug on a horizontal tray while she accelerates at 7 m/s$^2$. The mug is 10 cm deep and 6 cm in diameter and contains coffee 7 cm deep at rest. (*a*) Assuming rigid-body acceleration of the coffee, determine whether it will spill out of the mug. (*b*) Calculate the gage pressure in the corner at point $A$ if the density of coffee is 1010 kg/m$^3$.

*Solution*

*Part (a)* The free surface tilts at the angle $\theta$ given by Eq. (2.55) regardless of the shape of the mug. With $a_z = 0$ and standard gravity

$$\theta = \tan^{-1} \frac{a_x}{g} = \tan^{-1} \frac{7.0}{9.81} = 35.5^\circ$$

If the mug is symmetric about its central axis, the volume of coffee is conserved if the tilted surface intersects the original rest surface exactly at the centerline, as shown in Fig. E2.12. Thus the deflection at the left side of the mug is

$$z = (3 \text{ cm})(\tan \theta) = 2.14 \text{ cm} \qquad \textit{Ans. (a)}$$

This is less than the 3-cm clearance available, so the coffee will not spill unless it was sloshed during the start-up of acceleration.



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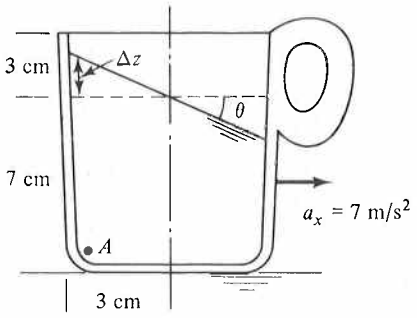


**Fig. E2.12**

*Part (b)* When at rest, the gage pressure at point $A$ is given by Eq. (2.20)

$$p_A = \rho g(z_{\text{surf}} - z_A) = (1010 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.07 \text{ m}) = 694 \text{ N/m}^2 = 694 \text{ Pa}$$

During acceleration, Eq. (2.56) applies, with $G = [(7.0)^2 + (9.81)^2]^{1/2} = 12.05 \text{ m/s}^2$. The distance $\Delta s$ down normal from the tilted surface to point $A$ is

$$\Delta s = (7.0 + 2.14) \cos \theta = 7.44 \text{ cm}$$

Thus the pressure at point $A$ becomes

$$p_A = \rho G \, \Delta s = 1010(12.05)(0.0744) = 906 \text{ Pa} \qquad \textit{Ans. (b)}$$

which is an increase of 31 percent over the pressure when at rest.

## Rigid-Body Rotation

As a second special case, consider rotation of the fluid about the $z$ axis without any translation, as sketched in Fig. 2.21. We assume that the container has been rotating long enough at constant $\Omega$ for the fluid to have attained rigid-body rotation. The fluid acceleration will then be the centripetal term in Eq. (2.54). In the coordinates of Fig. 2.21 the angular-velocity and position vectors are given by

$$\boldsymbol{\Omega} = \mathbf{k}\Omega \qquad \mathbf{r}_0 = \mathbf{i}_r r \tag{2.57}$$

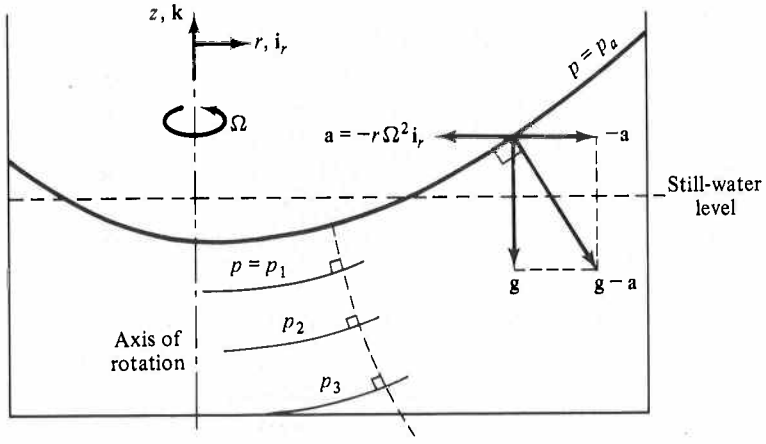


**Fig. 2.21** Development of paraboloid constant-pressure surfaces in a fluid in fluid-body rotation. The dashed line along the direction of maximum pressure increase is an exponential curve.



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Then the acceleration is given by

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_0) = -r\Omega^2 \mathbf{i}_r \tag{2.58}$$

as marked in the figure, and Eq. (2.52) for the force balance becomes

$$\nabla p = \mathbf{i}_r \frac{\partial p}{\partial r} + \mathbf{k} \frac{\partial p}{\partial z} = \rho(\mathbf{g} - \mathbf{a}) = \rho(r\Omega^2 \mathbf{i}_r - g\mathbf{k}) \tag{2.59}$$

Equating like components, we find the pressure field by solving two first-order partial differential equations

$$\frac{\partial p}{\partial r} = \rho r \Omega^2 \qquad \frac{\partial p}{\partial z} = -\rho g \tag{2.60}$$

This is our first specific example of the generalized three-dimensional problem described by Eqs. (2.14) for more than one independent variable. The right-hand sides of (2.60) are known functions of $r$ and $z$. One can proceed as follows: integrate the first equation "partially," i.e., holding $z$ constant, with respect to $r$. The result is

$$p = \tfrac{1}{2}\rho r^2 \Omega^2 + f(z) \tag{2.61}$$

where the "constant" of integration is actually a function $f(z)$.[1] Now differentiate this with respect to $z$ and compare with the second relation of (2.60):

$$\frac{\partial p}{\partial z} = 0 + f'(z) = -\rho g$$

or

$$f(z) = -\rho g z + C \tag{2.62a}$$

where $C$ is a constant. Thus Eq. (2.61) now becomes

$$p = \text{const} - \rho g z + \tfrac{1}{2}\rho r^2 \Omega^2 \tag{2.62b}$$

This is the pressure distribution in the fluid. The value of $C$ is found by specifying the pressure at one point. If $p = p_0$ at $(r, z) = (0, 0)$, then $C = p_0$. The final desired distribution is

$$p = p_0 - \rho g z + \tfrac{1}{2}\rho r^2 \Omega^2 \tag{2.63}$$

The pressure is linear in $z$ and parabolic in $r$. If we wish to plot a constant-pressure surface, say, $p = p_1$, Eq. (2.63) becomes

$$z = \frac{p_0 - p_1}{\rho g} + \frac{r^2 \Omega^2}{2g} = a + br^2 \tag{2.64}$$

Thus the surfaces are paraboloids of revolution, concave upward, with their minimum point on the axis of rotation. Some examples are sketched in Fig. 2.21.

As in the previous example of linear acceleration, the position of the free surface is found by conserving the volume of fluid. For a noncircular container with the axis of rotation off center, as in Fig. 2.21, a lot of laborious mensuration is required, and a single problem will take you all weekend. However, the calculation is easy for

[1] This is because $f(z)$ vanishes when differentiated with respect to $r$. If you don't see this, you should review your calculus.



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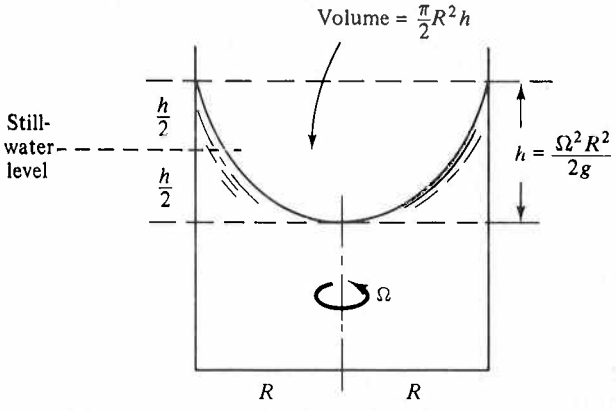


**Fig. 2.22** Determining the free-surface position for rotation of a cylinder of fluid about its central axis.

a cylinder rotating about its central axis, as in Fig. 2.22. Since the volume of a paraboloid is one-half the base area times its height, the still-water level is exactly halfway between the high and low points of the free surface. The center of the fluid drops an amount $h/2 = \Omega^2R^2/4g$, and the edges rise an equal amount.

**EXAMPLE 2.13** The coffee cup in Example 2.12 is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigid-body mode occurs. Find (*a*) the angular velocity which will cause the coffee to just reach the lip of the cup and (*b*) the gage pressure at point *A* for this condition.

*Solution*

*Part (a)* The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance $h/2$ in Fig. 2.22. Thus

$$\frac{h}{2} = 0.03 \text{ m} = \frac{\Omega^2R^2}{4g} = \frac{\Omega^2(0.03 \text{ m})^2}{4(9.81 \text{ m/s}^2)}$$

Solving, we obtain

$$\Omega^2 = 1308 \quad \text{or} \quad \Omega = 36.2 \text{ rad/s} = 345 \text{ r/min} \qquad \textit{Ans. (a)}$$

*Part (b)* To compute the pressure, it is convenient to put the origin of coordinates $r$ and $z$ at the bottom of the free-surface depression, as shown in Fig. E2.13. The gage pressure here is

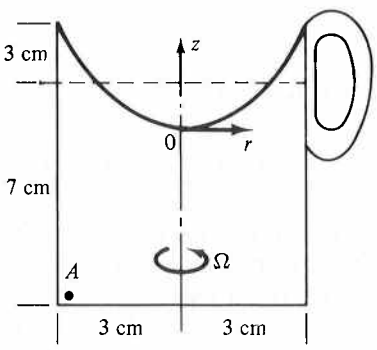


**Fig. E2.13**



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$p_0 = 0$, and point $A$ is at $(r, z) = (3\text{ cm}, -4\text{ cm})$. Equation (2.63) can then be evaluated

$$\begin{aligned} p_A &= 0 - (1010\text{ kg/m}^3)(9.81\text{ m/s}^2)(-0.04\text{ m}) \\ &\quad + \tfrac{1}{2}(1010\text{ kg/m}^3)(0.03\text{ m})^2(1308\text{ rad}^2/\text{s}^2) \\ &= 396\text{ N/m}^2 + 594\text{ N/m}^2 = 990\text{ Pa} \end{aligned}$$ *Ans. (b)*

This is about 43 percent greater than the still-water pressure $p_A = 694$ Pa.

Here, as in the linear-acceleration case, it should be emphasized that the paraboloid pressure distribution (2.63) sets up in *any* fluid under rigid-body rotation, regardless of the shape or size of the container. The container may even be closed and filled with fluid. It is only necessary that the fluid be continuously interconnected throughout the container. The following example will illustrate a peculiar case in which one can visualize an imaginary free surface extending outside the walls of the container.

**EXAMPLE 2.14** A U-tube with a radius of 10 in and containing mercury to a height of 30 in is rotated about its center at 180 r/min until a rigid-body mode is achieved. The diameter of the tubing is negligible. Atmospheric pressure is 2116 lbf/ft$^2$. Find the pressure at point $A$ in the rotating condition.

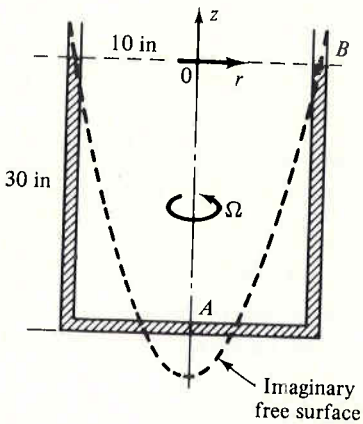


**Fig. E2.14**

*Solution* Convert the angular velocity into radians per second

$$\Omega = (180\text{ r/min})\frac{2\pi}{60} = 18.85\text{ rad/s}$$

From Table 2.1 we find for mercury that $\rho g = 846$ lbf/ft$^3$ and hence $\rho = 846/32.2 = 26.3$ slugs/ft$^3$. At this high rotation rate the free surface will slant upward at a fierce angle [about 84°; check this from Eq. (2.64)], but the tubing is so thin that the free surface will remain at approximately the same 30-in height, point $B$. Placing our origin of coordinates at


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this height, we can calculate the constant $C$ in Eq. (2.62*b*) from the condition $p_B = 2116$ lbf/ft$^2$ at $(r, z) = (10$ in, $0)$:

$$p_B = 2116 \text{ lbf/ft}^2 = C - 0 + \tfrac{1}{2}(26.3 \text{ slugs/ft}^3)(\tfrac{10}{12} \text{ ft})^2(18.85 \text{ rad/s})^2$$

or

$$C = 2116 - 3245 = -1129 \text{ lbf/ft}^2$$

We then obtain $p_A$ by evaluating Eq. (2.63) at $(r, z) = (0, -30$ in)

$$p_A = -1129 - (846 \text{ lbf/ft}^3)(-\tfrac{30}{12} \text{ ft}) = -1129 + 2115 = 986 \text{ lbf/ft}^2 \qquad \textit{Ans.}$$

This is less than atmospheric pressure, and we can see why if we follow the free-surface paraboloid down from point $B$ along the dashed line in the figure. It will cross the horizontal portion of the U-tube (where $p$ will be atmospheric) and fall *below* point $A$. From Fig. 2.22 the actual drop from point $B$ will be

$$h = \frac{\Omega^2 R^2}{2g} = \frac{(18.85)^2(\frac{10}{12})^2}{2(32.2)} = 3.83 \text{ ft} = 46 \text{ in}$$

Thus $p_A$ is about 16 inHg below atmospheric pressure, or about $\frac{16}{12}(846) = 1128$ lbf/ft$^2$ below $p_a = 2116$ lbf/ft$^2$, which checks with the answer above. When the tube is at rest,

$$p_A = 2116 - 846(-\tfrac{30}{12}) = 4231 \text{ lbf/ft}^2$$

Hence rotation has reduced the pressure at point $A$ by 77 percent. Further rotation can reduce $p_A$ to near-zero pressure, and cavitation can occur.

An interesting by-product of this analysis for rigid-body rotation is that the lines everywhere parallel to the pressure gradient form a family of curved surfaces, as sketched in Fig. 2.21. They are everywhere orthogonal to the constant-pressure surfaces, and hence their slope is the negative inverse of the slope computed from Eq. (2.64):

$$\left.\frac{dz}{dr}\right|_{\text{GL}} = -\frac{1}{(dz/dr)_{p=\text{const}}} = -\frac{1}{r\Omega^2/g}$$

where GL stands for gradient line

or

$$\frac{dz}{dr} = -\frac{g}{r\Omega^2} \tag{2.65}$$

Separating the variables and integrating, we find the equation of the pressure-gradient surfaces

$$r = C_1 \exp\left(-\frac{\Omega^2 z}{g}\right) \tag{2.66}$$

Notice that this result and Eq. (2.64) are independent of the density of the fluid. In the absence of friction and Coriolis effects, Eq. (2.66) defines the lines along which the apparent net gravitational field would act on a particle. Depending upon its density, a small particle or bubble would tend to rise or fall in the fluid along these exponential lines, as demonstrated experimentally in Ref. 5. Also, buoyant



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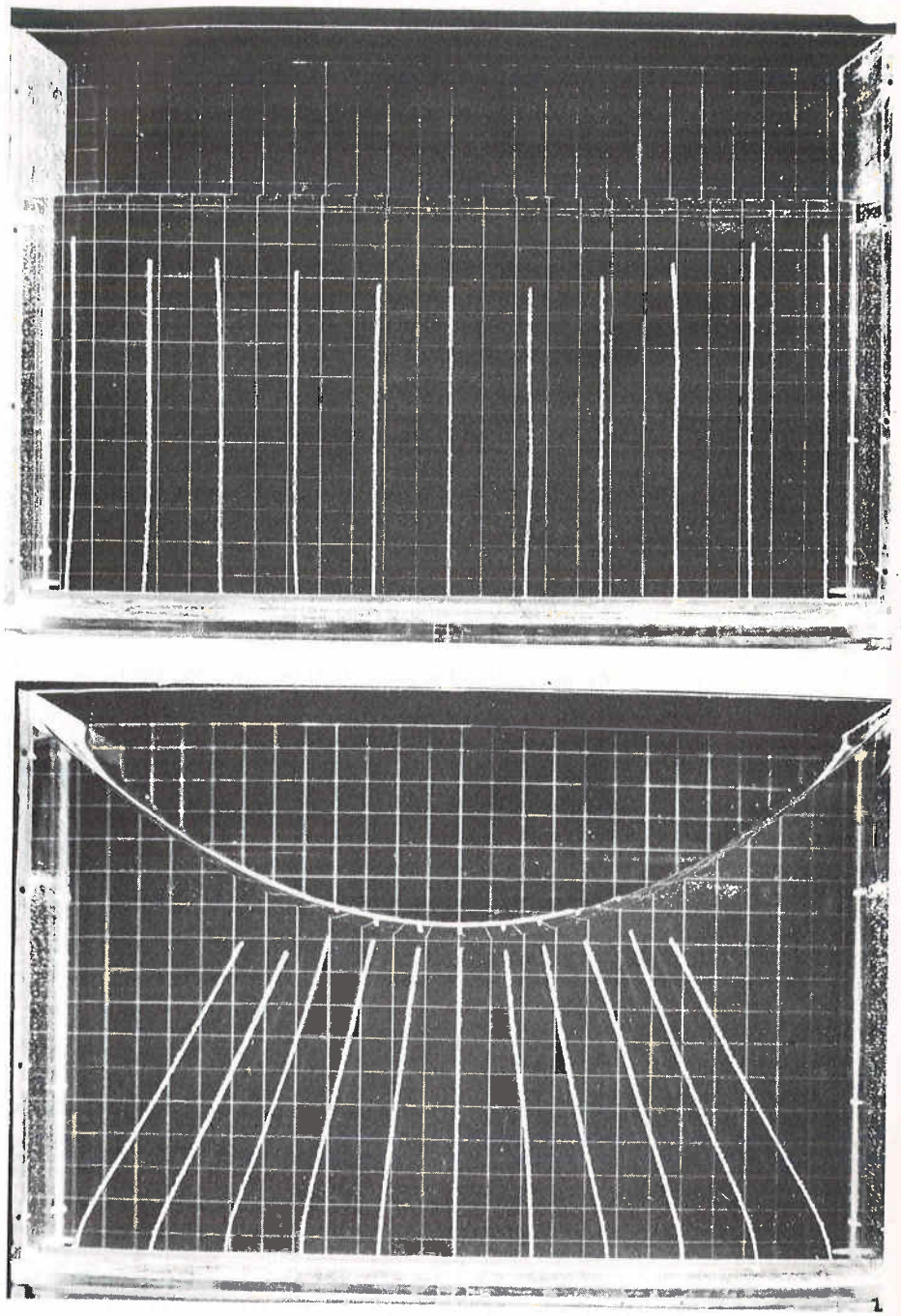

**Fig. 2.23** Experimental demonstration with buoyant streamers of the fluid force field in rigid-body rotation: (*top*) fluid at rest (streamers hang vertically upward); (*bottom*) rigid-body rotation (streamers are aligned with the direction of maximum pressure gradient). (*From Ref. 5, courtesy of R. Ian Fletcher.*)


Page 40 of 64 ABIOMED Ex. 1032

streamers would align themselves with these exponential lines, thus avoiding any stress other than pure tension. Figure 2.23 shows the configuration of such streamers before and during rotation.

## 2.10 PRESSURE MEASUREMENT

There are many devices available for measuring pressure, both in a static and in a moving stream. All take advantage of the fact that pressure applied to a finite area of material causes a force and stress and displacement in the material. These mechanical effects can then be quantified in various ways:

1. A force balance
2. The height of a liquid column (manometer)
3. Direct displacement measurement
4. Indirect (electrical) measurement of displacement

Force balances are commonly used to *calibrate* pressure instruments rather than as devices for routine measurement. The manometer, analyzed in Sec. 2.4, is a simple and inexpensive device with no moving parts except the liquid column itself. Manometers can be constructed with extreme accuracy by using fluids of small density differences, tilted liquid columns, and a micrometer-mounted pointer or eyepiece to locate the meniscus accurately. Examples are given in Fig. 2.24.

When measuring pressure in a moving fluid (the so-called *static pressure*), care must be taken not to disturb the flow. The best way to do this is to take the measurement through a *static hole* in the wall of the flow, as illustrated for the two instruments in Fig. 2.24. The hole should be normal to the wall, and burrs should be avoided. If the hole is small enough (typically 1 mm diameter), there will be no flow into the measuring tube once the pressure has adjusted to a steady value. Thus the flow is almost undisturbed. An oscillating flow pressure, however, can cause a large error due to possible dynamic response of the tubing. Other devices of smaller

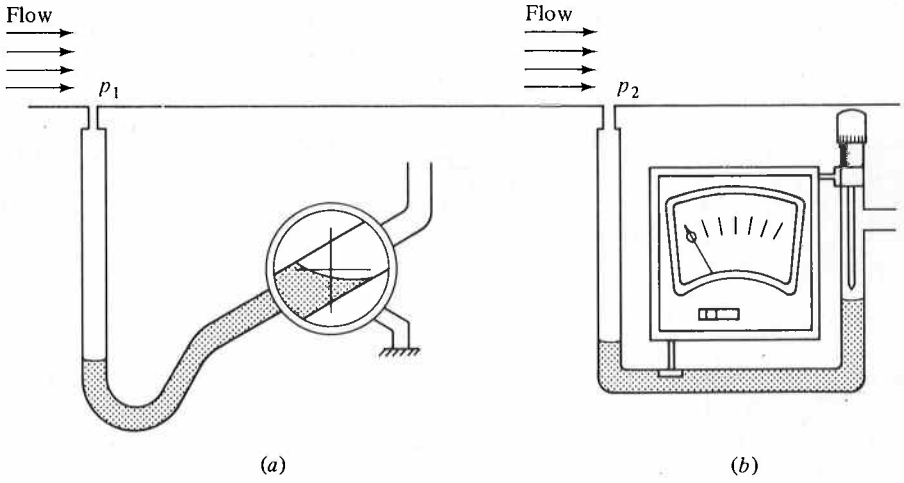


**Fig. 2.24** Two types of accurate manometers for precise measurements: (*a*) tilted tube with eyepiece; (*b*) micrometer pointer with ammeter detector.



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dimensions are used for dynamic-pressure measurements (see Fig. 2.26*d*). Note that the manometers in Fig. 2.24 are arranged to measure the absolute pressures $p_1$ and $p_2$. If the pressure difference $p_1 - p_2$ is desired, a significant error is incurred by subtracting two independent measurements, and it would be far better to connect both ends of one instrument to the two static holes $p_1$ and $p_2$ so that one manometer reads the difference directly.

The third type of pressure gage is a direct displacement measurement. An inexpensive and reliable device is the *bourdon tube*, sketched in Fig. 2.25. A curved tube with a flattened cross section will deflect outward when pressurized internally. The deflection can be measured by a linkage attached to a calibrated dial-gage pointer. Extreme accuracy can be obtained through proper design, and commercial bourdon gages are available with an accuracy of $\pm 0.1$ percent of full scale.

The fourth type of pressure gage measures the displacement of the gage sensing element by electrical means. The common techniques are (1) resistive, (2) capacitive, (3) inductive, (4) reluctive, (5) piezoelectric, and (6) eddy currents. Some examples are sketched in Fig. 2.26.

The reluctive device in Fig. 2.26*a* is called a *linear variable differential transformer* (LVDT). Note that it uses a bourdon tube for convenience as a source of mechanical displacement from the applied pressure. The strain gages in Fig. 2.26*b* are attached to a membrane which deflects under pressure, while the potentiometer gage (Fig. 2.26*c*) has a small bellows which deflects the pickup on the variable resistor. The piezoelectric transducer (Fig. 2.26*d*) uses the discovery of Pierre and Jacques Curie in 1880 that certain quartz crystals generate an electric charge when stressed. The piezoelectric gage has no internal fluid volume; i.e., the diaphragm is installed flush with the wall and reacts almost instantaneously to a change in local pressure. Thus the dynamic response is excellent, and piezoelectric sensors are in wide use for rapid transient-pressure measurements such as blast waves or turbulent-pressure fluctuations.

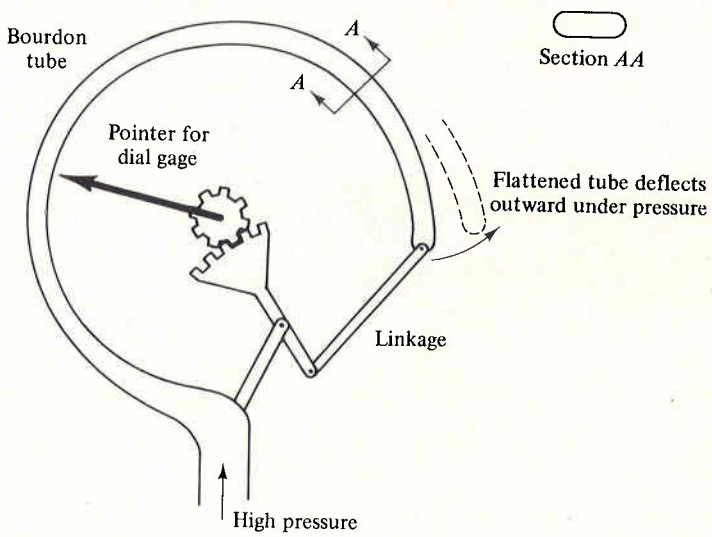


**Fig. 2.25** Schematic of a bourdon-tube device for mechanical measurement of high pressures.



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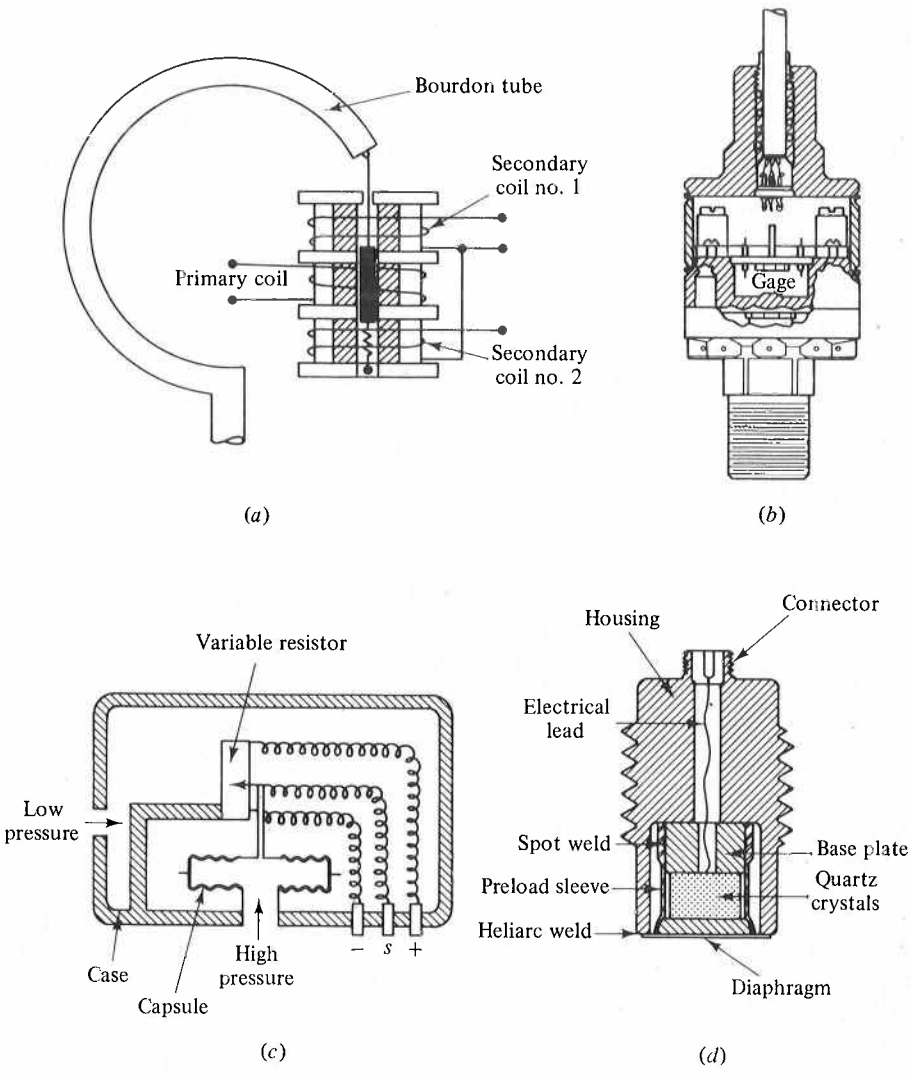


**Fig. 2.26** Four types of pressure transducers with electric output: (*a*) differential transformer (core motion causes a change in transformer reluctance); (*b*) strain-gage transducer (membrane deflection causes gage strain); (*c*) potentiometer transducer (capsule deflection causes variable-resistance change); (*d*) piezoelectric transducer (diaphragm deflection causes induced charge in crystals).

All the pressure devices with electric sensors have the great advantage that their output can be recorded on a paper chart or magnetic tape or other such permanent storage. This advantage serves to offset the relatively high cost ($100 to $2000) of electromechanical transducers.

Further discussion of pressure-measuring devices can be found in Refs. 7 to 9.

## SUMMARY

This chapter has been devoted entirely to the computation of pressure distributions and the resulting forces and moments in a static fluid or a fluid with a known velocity field. All hydrostatic (Secs. 2.3 to 2.8) and rigid-body (Sec. 2.9) problems



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are solved in this manner and are classic cases which every student should understand. In many irrotational and almost all viscous flows, both pressure and velocity are unknowns and are solved together as a system of equations in the chapters which follow.

## REFERENCES


1. "U.S. Standard Atmosphere, 1962," U.S. Government Printing Office, Washington, 1962.
2. G. Neumann and W. J. Pierson, Jr., "Principles of Physical Oceanography," Prentice-Hall, Englewood Cliffs, N.J., 1966.
3. T. C. Gillmer, "Modern Ship Design," chap. 5, United States Naval Institute, Annapolis, Md., 1970.
4. D. T. Greenwood, "Principles of Dynamics," Prentice-Hall, Englewood Cliffs, N.J., 1965.
5. R. I. Fletcher, The Apparent Field of Gravity in a Rotating Fluid System, *Am. J. Phys.* vol. 40, pp. 959–965, July 1972.
6. National Committee for Fluid Mechanics Films, "Illustrated Experiments in Fluid Mechanics," M.I.T. Press, Cambridge, Mass., 1972.
7. J. P. Holman, "Experimental Methods for Engineers," 4th ed., McGraw-Hill, New York, 1983.
8. R. P. Benedict, "Fundamentals of Temperature, Pressure, and Flow Measurement," 3d ed., Wiley, New York, 1984.
9. T. G. Beckwith, N. L. Buck, and R. D. Marangoni, "Mechanical Measurements," 3d ed., Addison-Wesley, Reading, Mass., 1982.


## RECOMMENDED FILMS

Films numbered 5, 17, and 24 in Appendix C.

# Problems

**PROBLEM DISTRIBUTION**

| Section | Topic | Problems |
|---|---|---|
| 2.1 | Pressure and pressure gradient | 2.1–2.5 |
| 2.2 | Static equilibrium, gage pressure | 2.6–2.8 |
| 2.3 | Hydrostatic pressure; barometers | 2.9–2.24 |
| 2.3 | The atmosphere | 2.25–2.33 |
| 2.4 | Manometers | 2.34–2.47 |
| 2.5 | Force on plane surfaces | 2.48–2.81 |
| 2.6 | Force on curved surfaces | 2.82–2.101 |
| 2.7 | Forces in layered fluids | 2.102–2.103 |
| 2.8 | Buoyancy, Archimedes' principle | 2.104–2.126 |
| 2.8 | Stability of floating bodies | 2.127–2.136 |
| 2.9 | Uniform acceleration | 2.137–2.151 |
| 2.9 | Rigid-body rotation | 2.152–2.160 |
| 2.10 | Pressure measurements | None |

**2.1** Figure 2.1 and Eqs. (2.3) and (2.4) show that the pressure is the same in all directions at a point in a fluid at rest. How does this analysis differ from the discussion in Sec. 1.2 and Fig. 1.1*c* and *d*, which showed that Mohr's circle shrinks to a point in a hydrostatic fluid?

**2.2** For small changes $\Delta z$ the hydrostatic relation Eq. (2.20) becomes $\Delta p = \rho g \Delta z$, yet in Eq. (2.3) we would appear to get the relation $\Delta p = \frac{1}{2}\rho g \Delta z$. Explain the factor of $\frac{1}{2}$ in this second relation.



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**2.3** For the two-dimensional stress field shown in Fig. P2.3 it is found that

$$\sigma_{xx} = 2000 \text{ lbf/ft}^2 \qquad \sigma_{yy} = 1800 \text{ lbf/ft}^2$$

$$\sigma_{xy} = \sigma_{yx} = 100 \text{ lbf/ft}^2$$

Find the shear and normal stress in pounds per square foot on the plane $AA$ cutting through at a 30° angle as shown.

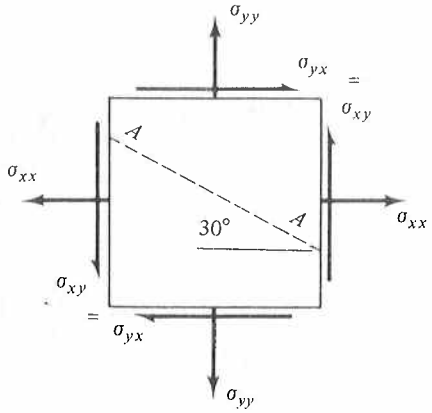


Fig. P2.3

**2.4** For the stress field of Fig. P2.3 we are given

$$\sigma_{xx} = 2000 \text{ lbf/ft}^2 \qquad \sigma_{yy} = 1800 \text{ lbf/ft}^2$$

$$\sigma_n(AA) = 2100 \text{ lbf/ft}^2$$

Compute the shear stress on plane $AA$ and the shear stress $\sigma_{xy}$.

**2.5** Equation (2.13) is a vector equation which has three components. Write out its $y$ component in full.

**2.6** Atlanta, Georgia has an average altitude of 1100 ft. On a day when atmospheric pressure equals the U.S. standard value, gage $A$ in a laboratory experiment reads 99 kPa and gage $B$ reads 97 kPa. Express these readings in gage pressure or vacuum pressure as the case may be.

**2.7** Any pressure reading can be expressed as a head or length $h = p/\rho g$. What is standard sea-level pressure expressed in (*a*) meters of water, (*b*) feet of water, (*c*) inches of mercury, and (*d*) millimeters of mercury?

**2.8** On a day when the barometer reads 29.5 inHg a pressure gage reads 1.3 lbf/in$^2$ vacuum. Express the absolute pressure in the gage in (*a*) inches of mercury and (*b*) pascals.

**2.9** Express the absolute pressure at point $C$ in Fig. 2.4 in pascals if it lies 85 cm below the surface and the barometer reads 29.7 inHg.

**2.10** Express a pressure of 2 kPa in terms of a column height of 20°C fluid for (*a*) benzene, (*b*) methanol, and (*c*) seawater.

**2.11** What depth change in feet in (*a*) water, (*b*) seawater, and (*c*) mercury will cause a pressure increase of 1 atm?

**2.12** The deepest known point in the ocean is 11,034 m in the Mariana Trench in the Pacific. Assuming seawater to have a constant specific weight of 10,050 N/m$^3$, what is the pressure at this point in atmospheres?

**2.13** Repeat Prob. 2.12 by integrating the more exact nonlinear pressure-density relation for water, Eq. (1.38), from the surface to the sea bottom. What is the percent change in the computed bottom pressure?

**2.14** The closed tank in Fig. P2.14 is at 20°C. If the pressure at point $A$ is 90 kPa absolute, what is the absolute pressure at point $B$ in kilopascals? What percent error do you make by neglecting the specific weight of the air?

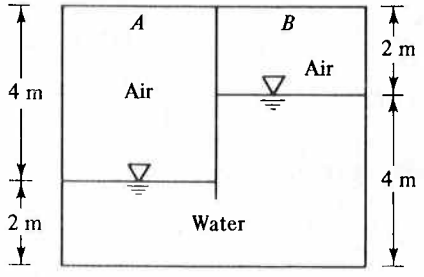


Fig. P2.14

**2.15** The answer to Prob. 1.44 is that mercury best fits $B = 41{,}350$ and $n = 6$. Consider this a reward if you read ahead this far. Now use this formula to determine the pressure in the Mariana Trench (Prob. 2.12) if the ocean were filled with mercury instead of seawater.

**2.16** The air-oil-water system in Fig. P2.16 is at 20°C. Knowing that gage $A$ reads 15 lbf/in$^2$ absolute and gage $B$ reads 1.25 lbf/in$^2$ less than gage $C$, compute (*a*) the specific weight of the oil in pounds force per cubic foot and (*b*) the actual reading of gage $C$ in pounds force per square inch absolute.



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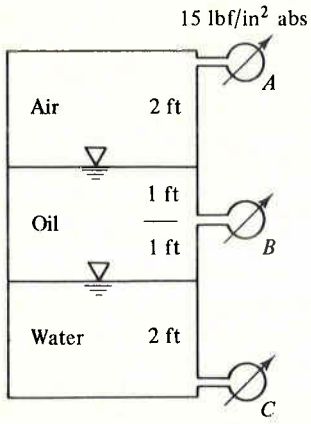


Fig. P2.16

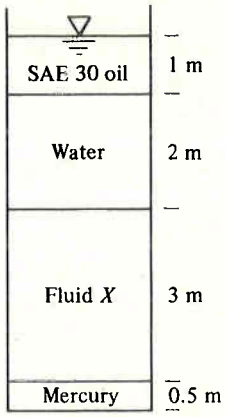


Fig. P2.18

**2.17** The system in Fig. P2.17 is at 20°C. If the pressure at point $A$ is 2000 lbf/ft$^2$, determine the pressures at points $B$, $C$, and $D$ in pounds force per square foot.

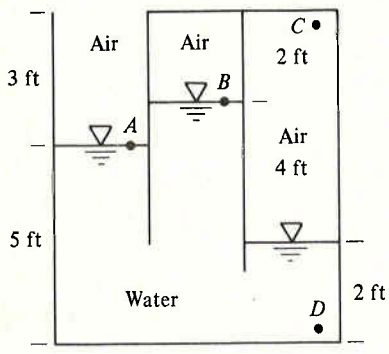


Fig. P2.17

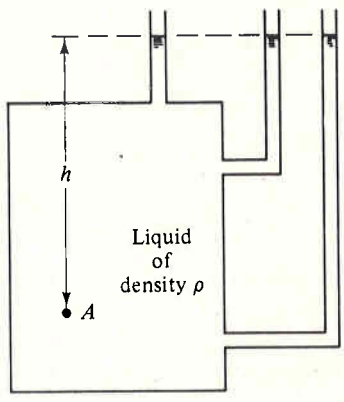


Fig. P2.19

**2.18** The system in Fig. P2.18 is at 20°C. If atmospheric pressure is 101.33 kPa and the pressure at the bottom of the tank is 237 kPa, what is the specific gravity of fluid $X$?

**2.19** When open tubes called *piezometers* are connected to a tank of liquid under pressure, the liquid rises to the *piezometer head*, or *pressure head*, of the liquid. If $p_A$ is the pressure at point $A$, show that all three piezometers rise to the same level $h = p_A/\rho g$.

**2.20** The hydraulic jack in Fig. P2.20 is filled with oil at 56 lbf/ft$^3$. Neglecting the weight of the two pistons, what force $F$ on the handle is required to support the 2000-lbf weight for this design?

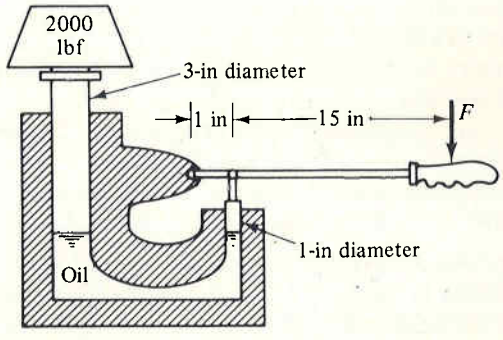


Fig. P2.20



ABIOMED Ex. 1032

**2.21** The Red Sea, being in a region of high evaporation, is very salty. The average pressure at a depth of 20 m in the Red Sea is 202 kPa above atmospheric pressure. What is the density of Red Sea water in kilograms per cubic meter? What is its percentage salt content by weight?

**2.22** The fuel gage for a gasoline tank in a car reads proportional to the bottom gage pressure as in Fig. P2.22. If the tank is 30 cm deep and accidentally contains 2 cm of water plus gasoline, how many centimeters of air remain at the top when the gage erroneously reads "full"?

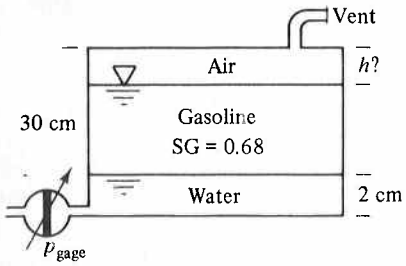


**Fig. P2.22**

**2.23** At 20°C gage $A$ reads 300 kPa absolute. What is the height $h$ of the water in centimeters? What should gage $B$ read in kilopascals absolute?

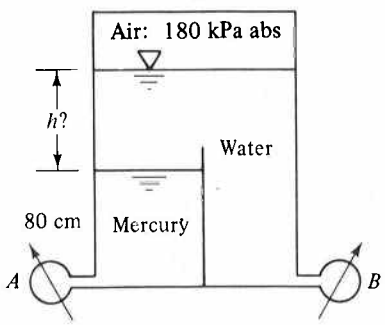


**Fig. P2.23**

**2.24** The U-tube in Fig. P2.24 has a 1-cm ID and contains mercury as shown. If 10 $cm^3$ of water is poured into the right-hand leg, what will the free-surface height in each leg be after the sloshing has died down?

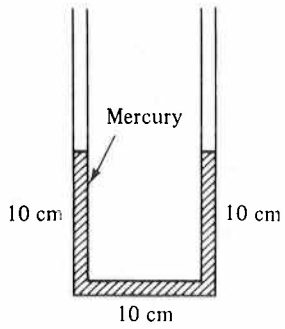


**Fig. P2.24**

**2.25** If air were incompressible at its nominal specific weight of 11.8 $N/m^3$, how high in meters would the atmosphere have to be to cause sea-level pressure to be its nominal value of 2116 $lbf/ft^2$?

**2.26** The atmosphere of Venus is 90 percent $CO_2$, but assume it to be 100 percent. Its surface pressure is about 2 MPa, and its average surface temperature is 190°C. For an isothermal atmosphere, estimate the pressure at an altitude of 20 km. Take $g = 28.5\ ft/s^2$.

**2.27** From the standard atmosphere in Fig. 2.7, estimate the mass of the entire atmosphere in kilograms, assuming the radius of the earth to be 6400 km.

**2.28** Investigate the effect of a doubling of the lapse rate on atmospheric pressure. If $T_0$ is 20°C and $p_a$ = 99 kPa, compute the pressure at 3500 m for a lapse rate of (*a*) 0.0005 K/m and (*b*) 0.001 K/m. What do you conclude?

**2.29** Sea-level temperature and pressure and the atmospheric lapse rate vary from day to day. At a certain time sea-level temperature is 50°F and sea-level pressure is 29 inHg. An airplane overhead registers a temperature of 20°F and a pressure of 11 $lbf/in^2$ absolute. Estimate the altitude of the plane in feet.

**2.30** What percentage error do you make in Prob. 2.29 if you simply assume a standard lapse rate of 0.003566 °R/ft?

**2.31** From Fig. 2.7 standard pressure at $z = 30$ km is 1.20 kPa. From 30 to 40 km the temperature rises approximately linearly from −45 to −20°C. Estimate the pressure at $z = 40$ km in pascals.

**2.32** On a cold day at the North Pole sea-level temperature is −50°C and remains approximately isothermal up to 3000 m. Estimate the atmospheric pressure there at $z = 3000$ m if sea-level pressure is 100 kPa.



ABIOMED Ex. 1032

**2.33** On a summer day in Narragansett, Rhode Island, sea-level temperature is 80°F, sea-level pressure is 29.5 inHg, and the lapse rate is 0.0036 °R/ft. Estimate the specific weight of the local atmosphere in pounds force per cubic foot at an altitude of 5000 ft.

**2.34** In Fig. P2.34 fluid 2 is carbon tetrachloride and fluid 1 is glycerin. If $p_a = 101$ kPa, determine the absolute pressure at point $A$.

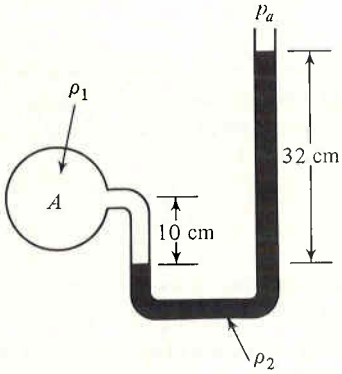


**Fig. P2.34**

**2.35** In Fig. P2.35 all fluids are at 20°C. Determine the pressure difference between points $A$ and $B$.

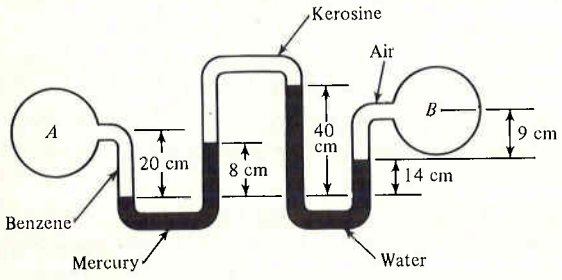


**Fig. P2.35**

**2.36** In Fig. P2.36 all fluids are at 20°C. If $p_B - p_A = 99$ kPa, what must the height $H$ be in centimeters?

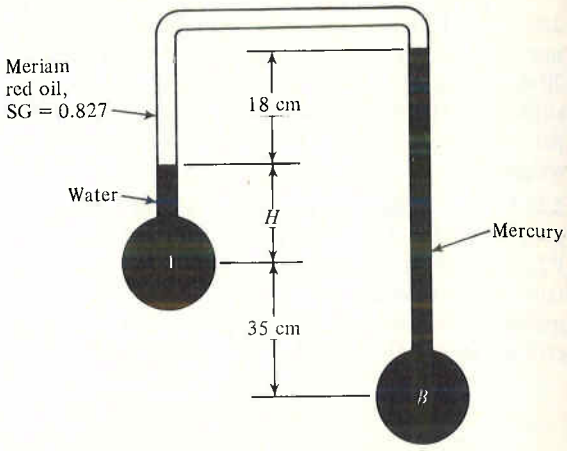


**Fig. P2.36**

**2.37** For Fig. 2.9 if fluid 1 is water and fluid 2 is mercury, and $z_A = 0$, $z_1 = -10$ cm, what is the level $z_2$ for which $p_A = p_{atm}$?

**2.38** For Fig. 2.9 fluid 1 is air and fluid 2 is mercury. The right leg is fitted with a scale graduated to read $p_A$ in kilopascals gage. What should the difference between scale graduations in millimeters be?

**2.39** The inclined manometer in Fig. P2.39 contains Meriam red manometer oil, SG = 0.827. Assume that the reservoir is very large. If the inclined arm is fitted with graduations 1 in apart, what should the angle $\theta$ be if each graduation corresponds to 1 lbf/ft$^2$ gage pressure for $p_A$?

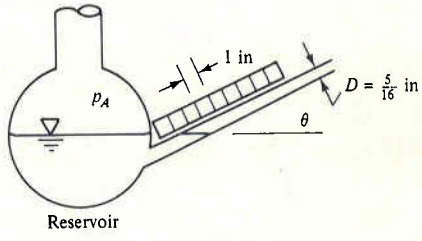


**Fig. P2.39**

**2.40** Repeat Prob. 2.39 for a finite reservoir approximating a half-full hemisphere with a diameter of 6 in.



ABIOMED Ex. 1032

**2.41** The system in Fig. P2.41 is at 20°C. Compute the pressure at point $A$ in pounds force per square foot absolute.

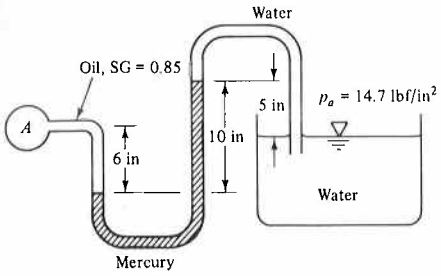


**Fig. P2.41**

**2.42** Very small pressure differences $p_A - p_B$ can be measured accurately by the two-fluid differential manometer in Fig. P2.42. Density $\rho_2$ is only slightly larger than the upper fluid $\rho_1$. Derive an expression for the proportionality between $h$ and $p_A - p_B$ if the reservoirs are very large.

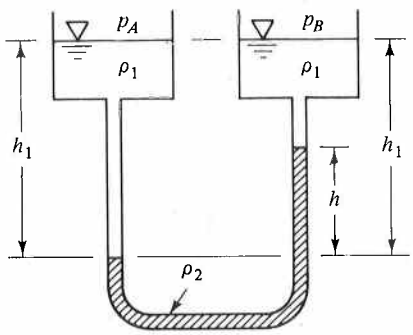


**Fig. P2.42**

**2.43** To illustrate the effectiveness of the system in Fig. P2.42, consider that a height difference of 1 cm in a fluid with SG = 0.82 corresponds to a pressure difference of 80.3 Pa. In Fig. P2.42, what difference $p_A - p_B$ corresponds to $h = 1$ cm when $SG_1 = 0.81$ and $SG_2 = 0.83$? What is the increase in accuracy?

**2.44** Water flows downward in a pipe at 45°, as shown in Fig. P2.44. The pressure drop $p_1 - p_2$ is partly due to gravity and partly due to friction. The mercury manometer reads a 6-in height difference. What is the total pressure drop $p_1 - p_2$ in pounds force per square inch? What is the pressure drop due to friction only between 1 and 2 in pounds force per square inch? Does the manometer reading correspond only to friction drop? Why?

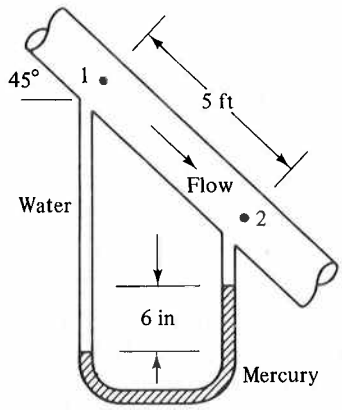


**Fig. P2.44**

**2.45** Determine the gage pressure at point $A$ in pascals. Is it higher or lower than atmospheric?

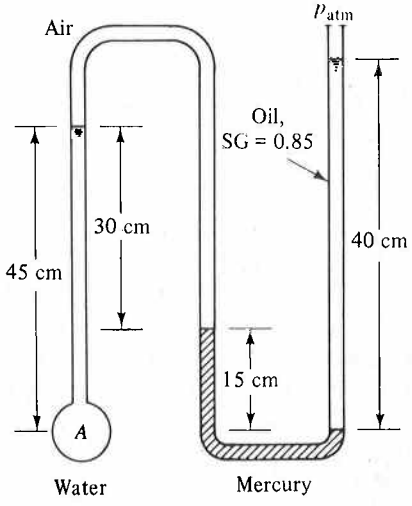


**Fig. P2.45**

**2.46** In Fig. P2.46, what will the level $h$ of the oil in the right-hand tube be in centimeters? Both tubes are open to the atmosphere.



ABIOMED Ex. 1032

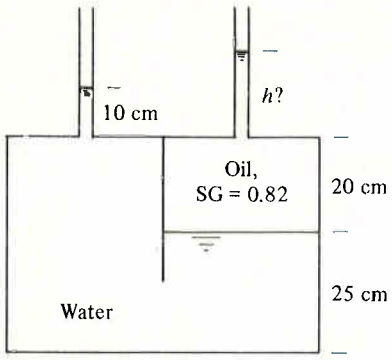


Fig. P2.46

**2.47** The cylindrical tank in Fig. P2.47 is being filled with water at 20°C by a pump which produces an absolute pressure of 150 kPa at the inlet. At the instant shown the air pressure is 100 kPa and $H = 25$ cm. If the air compresses isothermally, what will $H$ be when the flow ceases because the tank becomes hydrostatic?

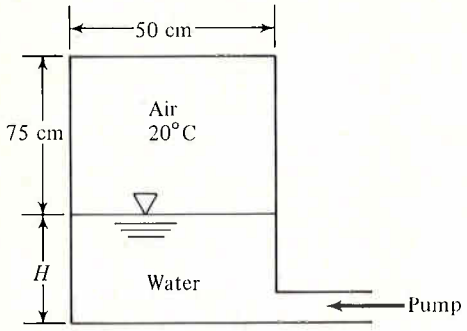


Fig. P2.47

**2.48** The tank in Prob. 2.46 is 30 cm wide into the paper. Compute the force in newtons on the left-hand side panel of the tank and its center-of-pressure position.

**2.49** Repeat Prob. 2.48 but instead compute the force and center of pressure on the right-hand side panel.

**2.50** Repeat Example 2.5 but instead let the hinge be at point $A$ and let point $B$ rest against a smooth bottom.

**2.51** Gate $AB$ in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric pressure, compute the force $F$ on the gate and its center-of-pressure position $X$.

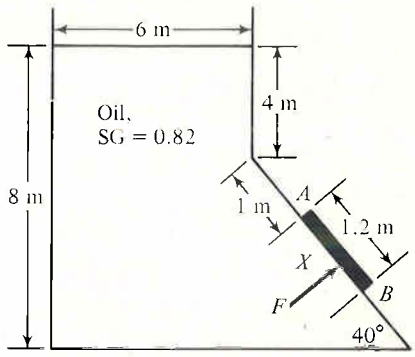


Fig. P2.51

**2.52** A vertical submerged gate 8 ft wide and 10 ft high is hinged at the top and held closed by water whose level is 5 ft above the gate top. What horizontal force applied at the bottom of the gate is required to open it?

**2.53** A vat filled with oil (SG = 0.82) is 7 m long and 3 m deep and has a trapezoidal cross section 4 m wide at the bottom and 6 m wide at the top. Compute (*a*) the weight of oil in the vat, (*b*) the force on the vat bottom, and (*c*) the force on the trapezoidal end panel.

**2.54** The gate $AB$ in Fig. P2.54 is 6 ft wide into the paper, hinged at point $A$, and restrained by a stop at point $B$. Compute the force on the stop and the reactions at $A$ if the water depth $h$ is 10 ft.

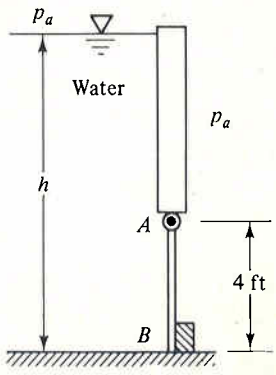


Fig. P2.54

**2.55** In Fig. P2.54 the stop $B$ will break if the force on it equals 10,000 lbf. For what water depth $h$ is this condition reached?

**2.56** In Fig. P2.54 the hinge $A$ will break if its horizontal reaction equals 9000 lbf. For what water depth $h$ is this condition reached?



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**2.57** Neglecting $p_a$, find the hydrostatic force on the triangular gate $ABC$ in Fig. P2.57 and the position of its center of pressure.

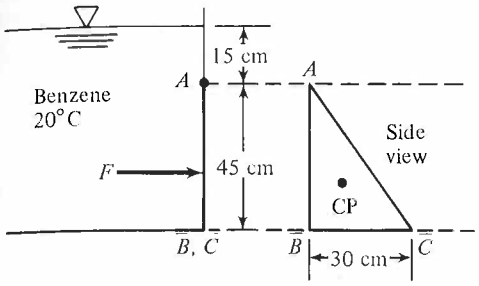


Fig. P2.57

**2.58** In Fig. P2.58 the cover gate $AB$ closes a circular opening 80 cm in diameter. The gate is held closed by a 200-kg mass as shown. Assume standard gravity at 20°C. At what water level $h$ will the gate be dislodged? Neglect the weight of the gate.

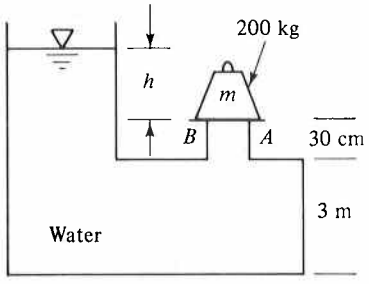


Fig. P2.58

**2.59** When freshly poured between wooden forms, concrete approximates a fluid with SG = 2.40. Figure P2.59 shows an 8- by 10-ft by 6-in slab poured between wooden forms which are connected by four corner bolts $A$, $B$, $C$, $D$ as shown. Neglecting end effects, compute the forces in all four bolts.

**2.60** Find the net hydrostatic force per unit width on the rectangular gate $AB$ in Fig. P2.60 and its line of action.

**2.61** A cylindrical wooden-stave barrel is 4 ft in diameter and 6 ft high. It is held together by steel hoops at the top and bottom, each with a cross section of 0.35 in$^2$. If the barrel is filled with apple juice (SG = 1.02), compute the tension stress in each hoop in pounds force per square inch absolute.

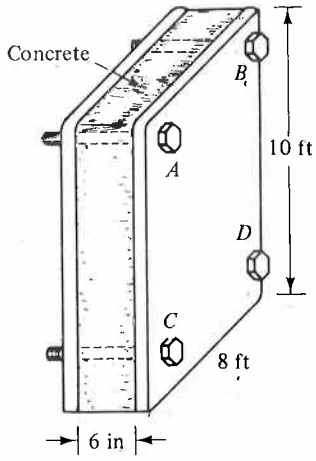


Fig. P2.59

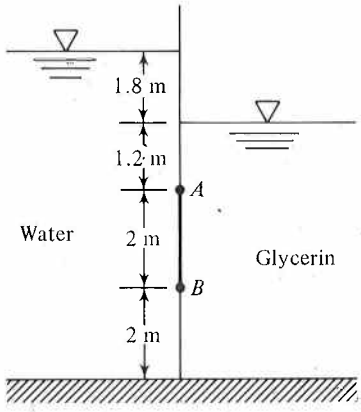


Fig. P2.60

**2.62** The gate $AB$ in Fig. P2.62 is 15 ft long and 10 ft wide into the paper and hinged at $B$ with a stop at $A$. Neglecting the weight of the gate, compute the water level $h$ for which the gate will start to fall.

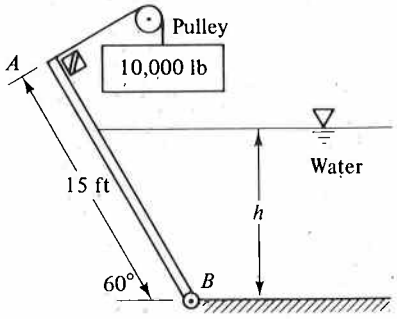


Fig. P2.62



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**2.63** Repeat Prob. 2.62 by including the weight of the gate, which is 1-in-thick steel, SG = 7.85.

**2.64** The turbine inlet duct coming from a large water dam is 3 m in diameter. It is closed by a vertical circular gate, whose center point is 50 m below the dam water level. Compute the force on the gate and its center of pressure.

**2.65** Gate $AB$ in Fig. P2.65 is semicircular, hinged at $B$, and held by a horizontal force $P$ at $A$. What force $P$ is required for equilibrium?

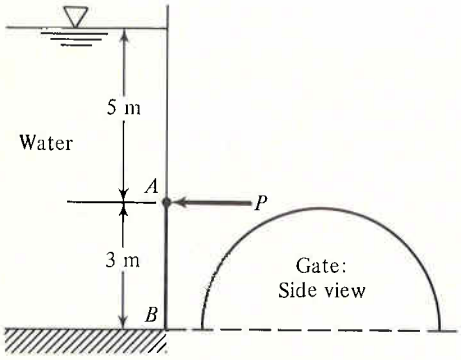


Fig. P2.65

**2.66** Dam $ABC$ in Fig. P2.66 is 40 m wide into the paper and made of concrete weighing 22 kN/m$^3$. Find the hydrostatic force on surface $AB$ and its moment about point $C$. Could this force tip the dam over?

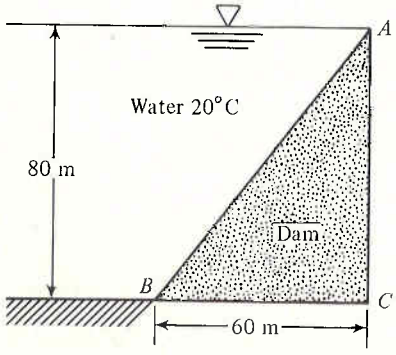


Fig. P2.66

**2.67** Isosceles triangle gate $AB$ in Fig. P2.67 is hinged at $A$ and weightless. What horizontal force $P$ is required at point $B$ for equilibrium?

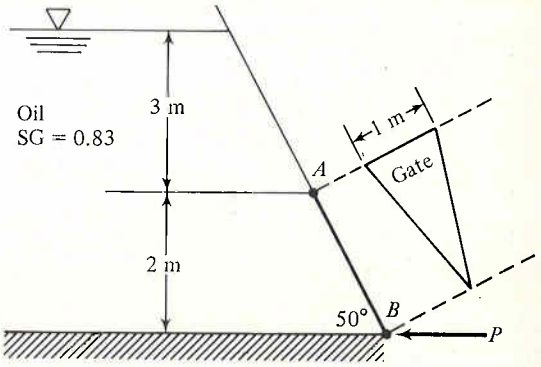


Fig. P2.67

**2.68** The tank in Fig. P2.68 is 50 cm wide into the paper. Compute the hydrostatic force on (*a*) the lower panel $BC$ and (*b*) the upper panel $AD$. Neglect atmospheric pressure.

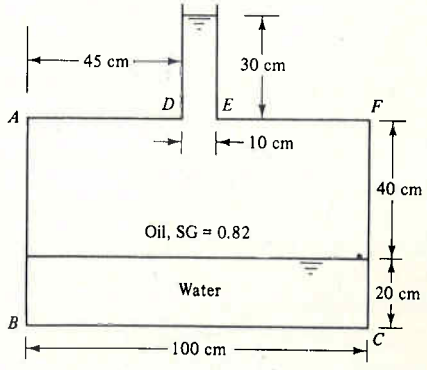


Fig. P2.68

**2.69** The water tank in Fig. P2.69 is pressurized, as shown by the mercury-manometer reading. Determine the hydrostatic force per unit depth on gate $AB$.



ABIOMED Ex. 1032

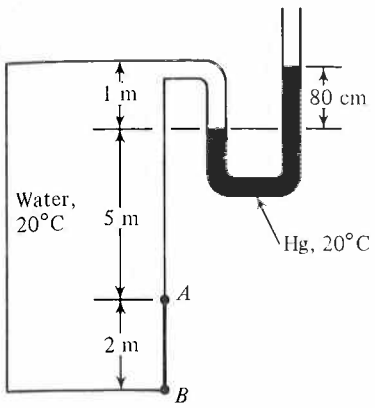


**Fig. P2.69**

**2.70** Calculate the force and center of pressure on one side of the vertical triangular panel $ABC$ in Fig. P2.70. Neglect $p_{atm}$.

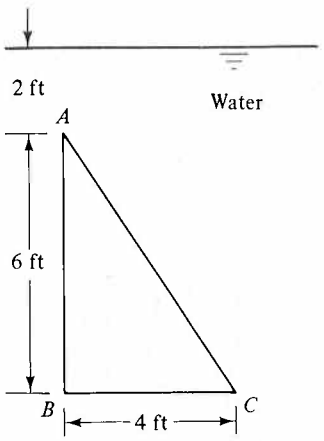


**Fig. P2.70**

**2.71** In Fig. P2.71 gate $AB$ is 3 m wide into the paper and is connected by a rod and pulley to a concrete sphere (SG = 2.40). What diameter of the sphere is just sufficient to keep the gate closed?

**2.72** The V-shaped container in Fig. P2.72 is hinged at $A$ and held together by cable $BC$ at the top. If cable spacing is 1 m into the paper, what is the cable tension?

**2.73** Gate $AB$ is 5 ft wide into the paper and opens to let fresh water out when the ocean tide is dropping. The hinge at $A$ is 2 ft above the freshwater level. At what ocean level $h$ will the gate first open? Neglect the gate weight.

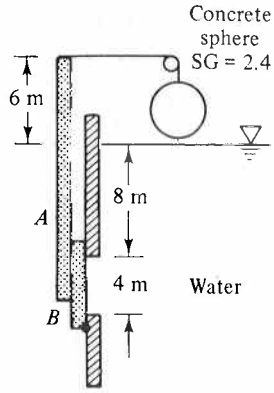


**Fig. P2.71**

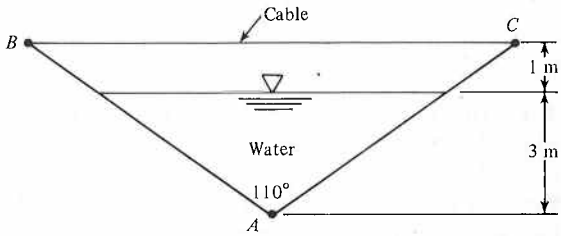


**Fig. P2.72**

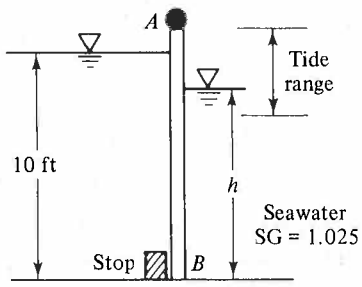


**Fig. P2.73**

**2.74** In Prob. 2.73 investigate analytically whether the required height $h$ is independent of the gate width $b$ into the paper.



ABIOMED Ex. 1032

**2.75** Compute the force on one side of the parabolic panel $ABC$ in Fig. P2.75, neglecting atmospheric pressure. Compute the vertical distance down to the center of pressure knowing that $I_{xx} = 2bh^3/7$ for a parabola about a horizontal axis through $A$.

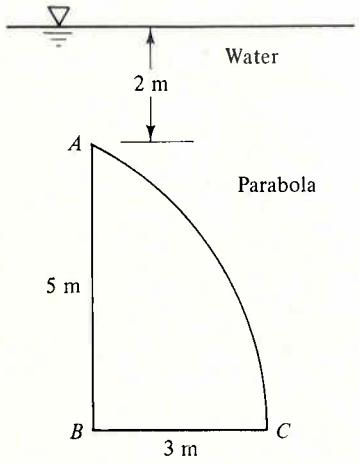


**Fig. P2.75**

**2.76** The huge conical beaker in Fig. P2.76 is filled with kerosine (SG = 0.62). An evacuated hemisphere $ABC$ of diameter 2 m rests at the bottom as shown. Compute the total force on the circular base $AB$ of the hemisphere. Don't ask what this problem really means.

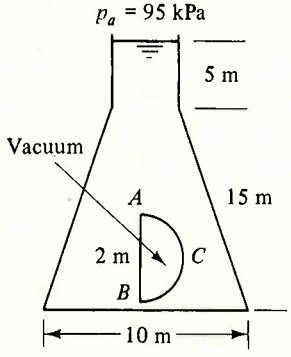


**Fig. P2.76**

**2.77** The circular gate $ABC$ in Fig. P2.77 has a 1-m radius and is hinged at $B$. Compute the force $P$ just sufficient to keep the gate from opening when $h = 10$ m. Neglect atmospheric pressure.

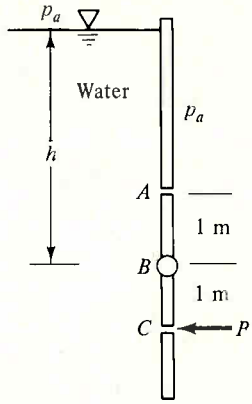


**Fig. P2.77**

**2.78** Repeat Prob. 2.77 to derive an analytic expression for $P$ as a function of $h$. Is there anything unusual about your solution?

**2.79** Gate $ABC$ in Fig. P2.79 is 1 m square and is hinged at $B$. It will open automatically when the water level $h$ becomes high enough. Determine the lowest height for which the gate will open. Neglect atmospheric pressure.

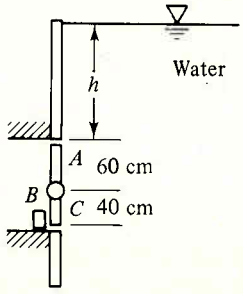


**Fig. P2.79**

**2.80** Repeat Prob. 2.79 to determine $h$ analytically for fluid of arbitrary density $\rho$. Is your solution unusual?

**2.81** Gate $AB$ is 7 ft into the paper and weighs 3000 lbf when submerged. It is hinged at $B$ and rests against a smooth wall at $A$. Determine the water level $h$ at the left which will just cause the gate to open.



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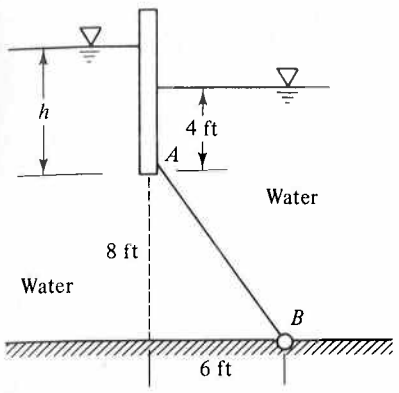


Fig. P2.81

**2.82** The dam in Fig. P2.82 is a quarter-circle 50 m wide into the paper. Determine the horizontal and vertical components of the hydrostatic force against the dam and the point CP where the resultant strikes the dam.

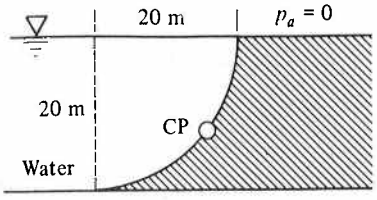


Fig. P2.82

**2.83** Gate $AB$ is a quarter circle 10 ft wide into the paper and hinged at $B$. Find the force $F$ just sufficient to keep the gate from opening. Neglect the weight of the gate.

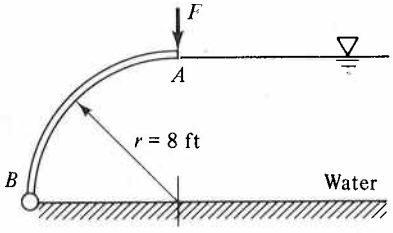


Fig. P2.83

**2.84** Repeat Prob. 2.83 by including the 4000-lbf weight of the gate, which is a steel plate of uniform thickness.

**2.85** Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. P2.85.

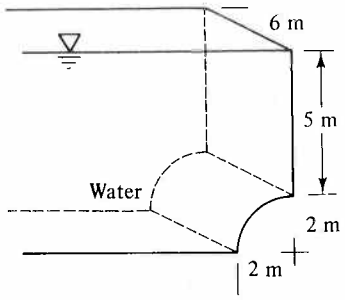


Fig. P2.85

**2.86** Compute the horizontal and vertical components of the hydrostatic force on the hemispherical bulge at the bottom of the tank in Fig. P2.86.

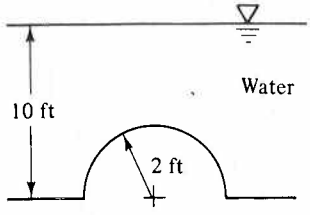


Fig. P2.86

**2.87** The bottle of champagne (SG = 0.96) in Fig. P2.87 is under pressure as shown by the mercury-manometer reading. Compute the net force on the 2-in-radius hemispherical end cap at the bottom of the bottle.

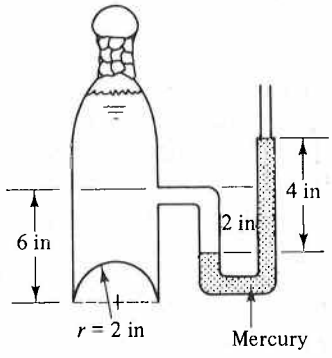


Fig. P2.87



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**2.88** The half-cylinder $ABC$ in Fig. P2.88 is 10 ft wide into the paper. Calculate the net moment of the hydrostatic oil forces on the cylinder about point $C$.

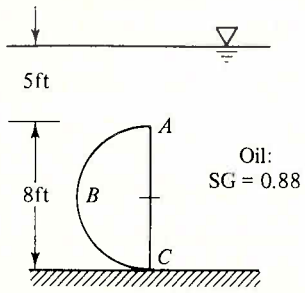


Fig. P2.88

**2.89** Compute the hydrostatic force and its line of action on the semicylindrical bulge $ABC$ in Fig. P2.89.

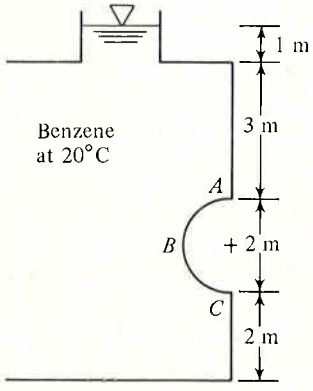


Fig. P2.89

**2.90** A 1-ft-diameter hole in the bottom of the tank in Fig. P2.90 is closed by a conical 45° plug. Neglecting the weight of the plug, compute the force $F$ required to keep the plug in the hole.

**2.91** The hemispherical dome in Fig. P2.91 weighs 30 kN and is filled with water and attached to the floor by six equally spaced bolts. What is the force in each bolt required to hold the dome down?

**2.92** A 4-m-diameter water tank consists of two half-cylinders, each weighing 4.5 kN/m, bolted together as shown in Fig. P2.92. If the support of the end caps is neglected, determine the force induced in each bolt.

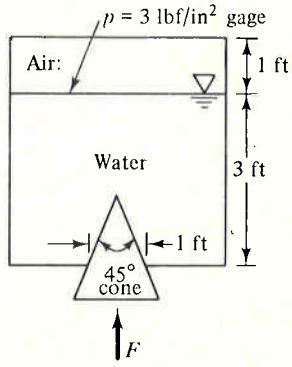


Fig. P2.90

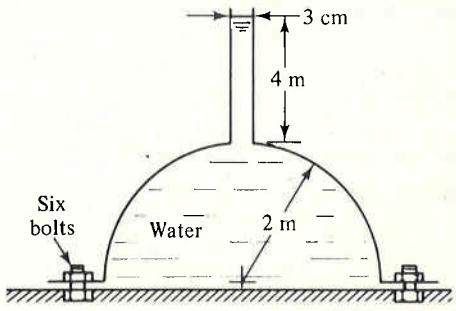


Fig. P2.91

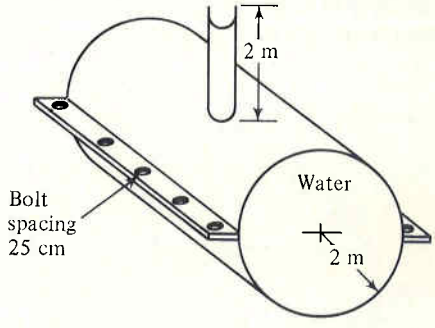


Fig. P2.92

**2.93** The 2-ft-radius cylinder in Fig. P2.93 is 8 ft wide into the paper. Compute the horizontal and vertical components of the water against the cylinder.



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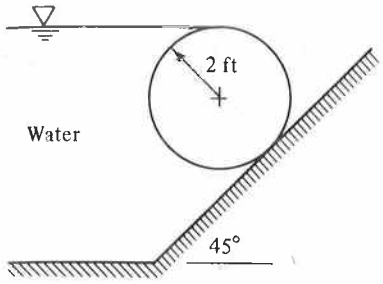


Fig. P2.93

**2.94** The 4-ft-diameter log (SG = 0.80) in Fig. P2.94 is 10 ft long into the paper and dams water as shown. Compute the net vertical and horizontal reactions at point *C*.

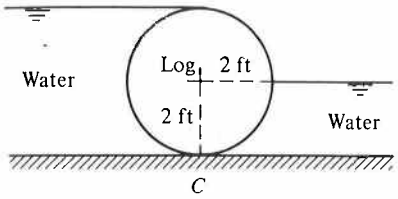


Fig. P2.94

**2.95** The 2-m-diameter cylinder in Fig. P2.95 is 5 m long into the paper and rests in static equilibrium against the smooth wall at point *B*. Compute (*a*) the weight and (*b*) the specific gravity of the cylinder. Assume zero wall friction at point *B*.

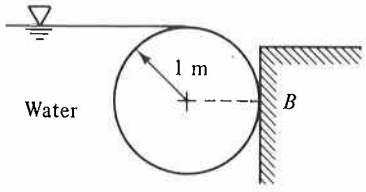


Fig. P2.95

**2.96** The tank in Fig. P2.96 is 2 m wide into the paper. Neglecting atmospheric pressure, compute the hydrostatic (*a*) horizontal force, (*b*) vertical force, and (*c*) resultant force on quarter-circle panel *BC*.

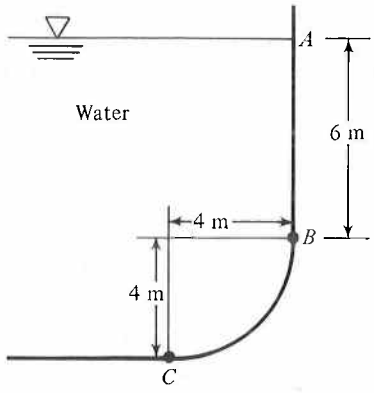


Fig. P2.96

**2.97** Gate *AB* is a quarter-circle 8 ft wide, hinged at *B* and resting against a smooth wall at *A*. Compute the reaction forces at point *A* and at point *B*.

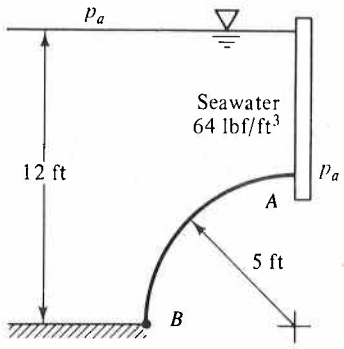


Fig. P2.97

**2.98** Gate *ABC* in Fig. P2.98 is a quarter-circle 10 ft wide into the paper. Compute the horizontal and vertical hydrostatic forces on the gate and the line of action of the resultant force.

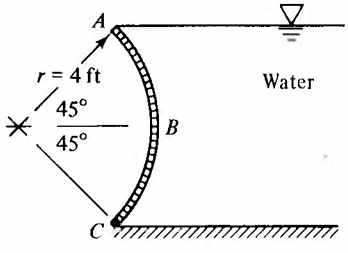


Fig. P2.98



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**2.99** A 2-ft-diameter sphere weighing 500 lbf closes a 1-ft-diameter hole in the bottom of the tank in Fig. P2.99. Compute the force $F$ required to dislodge the sphere from the hole.

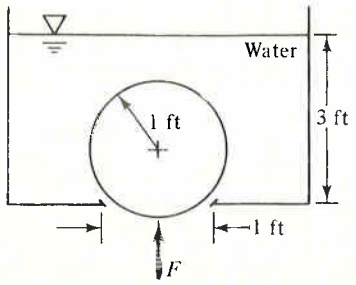


**Fig. P2.99**

**2.100** Pressurized water fills the tank in Fig. P2.100. Compute the net hydrostatic force on the conical surface $ABC$.

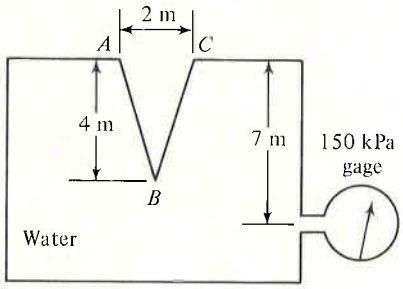


**Fig. P2.100**

**2.101** Gate $AB$ in Fig. P2.101 is 10 m wide into the paper, parabolic in shape, and hinged at $B$. Compute the force $F$ required to hold the gate in equilibrium. Neglect atmospheric pressure.

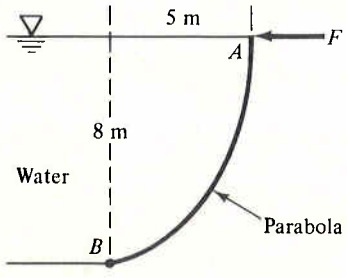


**Fig. P2.101**

**2.102** A horizontal cylindrical tank 10 ft long and 6 ft in diameter is half-filled with water, and the remainder is filled with oil (SG = 0.85). Compute the force and center of pressure on the end circular panel of the tank.

**2.103** The cylindrical tank in Fig. P2.89 has a hemispherical end cap $ABC$ and contains oil and water as shown. Compute the resultant force and line of action of the fluids on the end cap $ABC$.

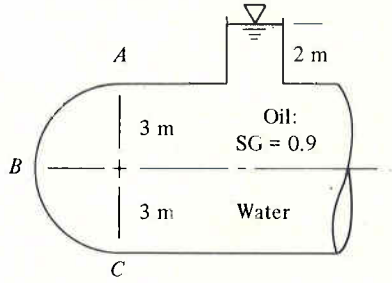


**Fig. P2.103**

**2.104** The can in Fig. P2.104 floats in the position shown. What is its weight in newtons?

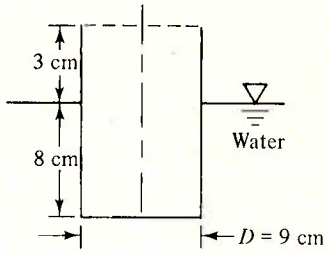


**Fig. P2.104**

**2.105** It is said that Archimedes discovered the buoyancy laws when asked by King Hiero of Syracuse to determine whether or not his new crown was gold (SG = 19.3). Archimedes found the weight of the crown in air to be 13.0 N and its weight in water to be 11.8 N. Was it gold or not?

**2.106** Repeat Prob. 2.105 by assuming that the crown is an alloy of gold (SG = 19.3) and silver (SG = 10.5). For the same measured weights, compute the percentage silver in the crown.



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**2.107** A sphere of buoyant solid molded foam is immersed in seawater (64 lbf/ft$^3$) and moored at the bottom. The sphere radius is 14 in. The mooring line has a tension of 150 lbf. What is the specific weight of the sphere?

**2.108** Repeat Prob. 2.62 assuming that the 10,000-lbf weight is concrete (SG = 2.40) and is hanging submerged in the water.

**2.109** A *hydrometer* floats at a level which is a measure of the specific gravity of the liquid. The stem is of constant diameter $D$, and a weight in the bottom bulb stabilizes the body, as shown in Fig. P2.109. If the total hydrometer weight is 0.03 lbf and the stem diameter is 0.3 in, compute the height $h$ where it will float when the liquid has SG = 1.3.

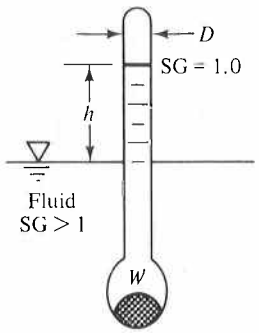


**Fig. P2.109**

**2.110** A hydrometer has a weight of 0.16 N and a stem diameter of 1 cm. What is the distance between scale markings for SG = 1.0 and SG = 1.1? Between SG = 1.1 and SG = 1.2?

**2.111** For the hydrometer of Fig. P2.109 derive an analytic formula for the float position $h$ as a function of SG, $W$, $D$, and the specific weight $\rho_0 g$ of pure water. Are the scale markings linear or nonlinear as a function of SG?

**2.112** A wooden pole (SG = 0.65), 9 cm by 9 cm by 5 m long, hangs vertically from a string in such a way that 3 m is submerged in water and 2 m is above the surface. What is the tension in the string?

**2.113** A spar buoy is a buoyant rod weighted at the bottom so that it floats upright and can be used for measurements or markers. The spar in Fig. P2.113 is wood (SG = 0.6), 2 by 2 in by 12 ft and floats in seawater (SG = 1.025). How many pounds of steel (SG = 7.85) should be added to the bottom so that exactly $h$ = 2 ft of the spar is exposed?

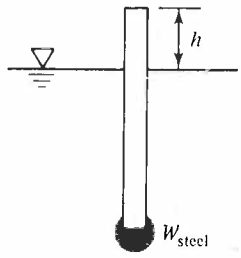


**Fig. P2.113**

**2.114** A right circular cone is 8 cm in diameter and 15 cm high and weighs 1 N in air. How much force is required to push this cone vertex-downward into benzene so that its base is exactly at the surface? How much additional force will push the base 10 cm below the surface?

**2.115** The 2- by 2-in by 12-ft spar buoy of Fig. P2.113 has 6 lb of steel weight attached and has gone aground on a rock 8 ft deep (Fig. P2.115). Compute the angle $\theta$ at which the buoy will lean, assuming that the rock exerts no moment on the buoy.

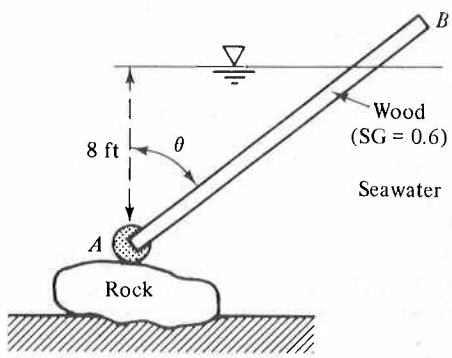


**Fig. P2.115**

**2.116** The solid cube 12 cm on a side in Fig. P2.116 is balanced by a 2-kg mass on the beam scale when the cube is immersed in water (SG = 1.0). What is the specific weight of the cube material?



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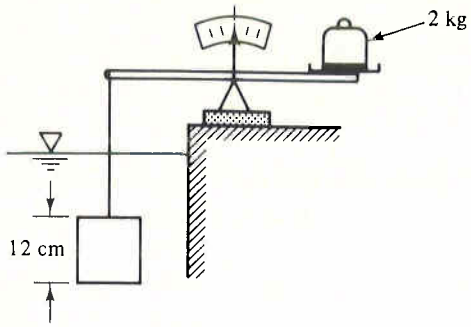


Fig. P2.116

**2.117** The balloon in Fig. P2.117 is filled with helium (molecular weight = 4.00) and pressurized to 110 kPa. The atmosphere conditions are shown. Compute the tension in the mooring line.

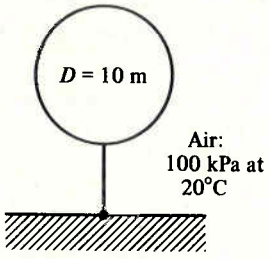


Fig. P2.117

**2.118** A 1-ft-diameter hollow sphere is made of steel (SG = 7.85) with 0.16-in wall thickness. How high will this sphere float in water (SG = 1.0)? How much weight must be added inside to make the sphere neutrally buoyant?

**2.119** When a 5-lb weight is placed on the end of a floating 4- by 4-in by 9-ft wooden beam, the beam tilts at 1.5° with its right upper corner at the surface, as in Fig. P2.119. What is the specific weight of the wood?

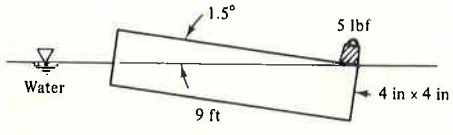


Fig. P2.119

**2.120** A wooden beam (SG = 0.6) is 15 by 15 cm by 4 m and is hinged at $A$. At what angle $\theta$ will the beam float in the water?

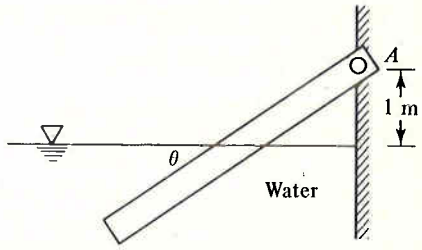


Fig. P2.120

**2.121** A barge weighs 50 tons empty and is 20 ft wide, 50 ft long, and 8 ft high. What will be its *draft*, i.e., its depth below the waterline, when loaded with 130 tons of gravel and floating in seawater (SG = 1.025)? One ton = 2000 lbf.

**2.122** A block of steel (SG = 7.85) will "float" at a mercury-water interface as in Fig. P2.122. What will be the ratio of the distances $a$ and $b$ for this condition?

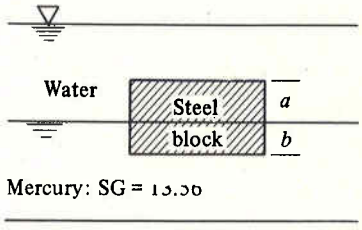


Fig. P2.122

**2.123** In an estuary where fresh water meets and mixes with seawater there often occurs a stratified salinity condition with fresh water on top and salt water on the bottom, as in Fig. P2.123. The interface is called a *halocline*. An idealization of this would be constant density on each side of the halocline as shown. A 35-cm-diameter sphere weighing 50 lbf would "float" near such a halocline. Compute the sphere position for the idealization in Fig. P2.123.



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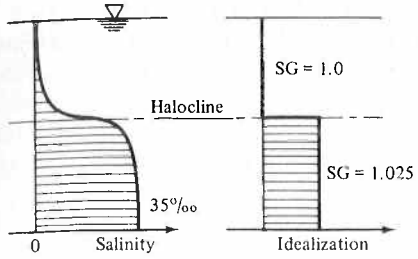


Fig. P2.123

**2.124** A balloon weighing 3 lbf is 5 ft in diameter. It is filled with hydrogen (molecular weight 2.02) at 15 lbf/in$^2$ absolute and 60°F and released. At what altitude in the U.S. Standard Atmosphere will this balloon be neutrally buoyant?

**2.125** A rectangular barge 20 ft wide by 50 ft long by 8 ft deep floats empty with a draft of 3 ft in a canal lock 30 ft wide by 60 ft long and water depth 6 ft when the empty barge is present. If 200,000 lbf of steel is loaded onto the barge, what are the new draft of the barge and the new water depth in the lock?

**2.126** An open can 15 cm in diameter and 30 cm long (Fig. P2.126) is eased downward so that the air within is trapped and compressed by the water. The can weighs 4 N and is made of sheet steel (SG = 7.85). Assuming isothermal compression of the air in the can, compute the height of water intrusion $h$ and the height $L$ of the can above the water when the can is floating. Assume that the position is stable.

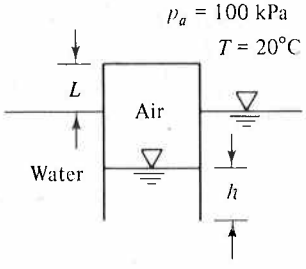


Fig. P2.126

**2.127** A spar buoy 2 by 2 in by 10 ft is made of wood (40 lbf/ft$^3$) and weighted with 4 lbf of steel at its bottom end. For this assumed vertical floating position compute the metacentric height $\overline{MG}$. Is the buoy stable?

**2.128** A cube of side length $L$ and specific gravity $S$ = 0.8 floats in water (SG = 1.0) as shown in Fig. P2.128. Compute the metacentric height $\overline{MG}$. Is the cube stable?

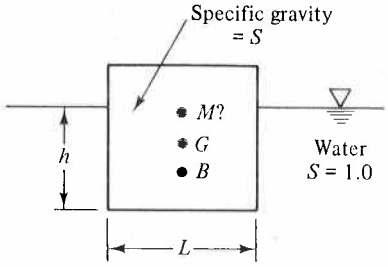


Fig. P2.128

**2.129** Make a complete analytic study of Prob. 2.128 and determine the range of values of $S$ between 0 and 1.0 for which the cube is stable.

**2.130** Investigate the stability of the wooden pole in Prob. 2.112. Is it stable when hanging vertically?

**2.131** A rectangular barge is 6 by 15 by 30 ft long and is piled so high with gravel that its center of gravity is 2 ft above the waterline, as in Fig. P2.131. Is the barge stable for this configuration?

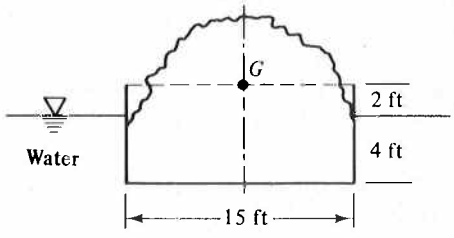


Fig. P2.131

**2.132** A solid right circular cone has SG = 0.99 and floats vertically as in Fig. P2.132. Is this a stable position for the cone?

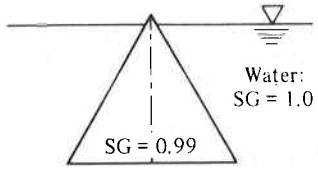


Fig. P2.132



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**2.133** Suppose that the barge in Fig. P2.131 has no gravel but instead is filled with oil (SG = 0.83) to a depth of 5 ft. Is the barge stable? What complicates the analysis?

**2.134** When floating in water (SG = 1.0), an equilateral triangular body (SG = 0.9) might take one of the two positions shown in Fig. P2.134. Which is the more stable position? Assume large width into the paper.

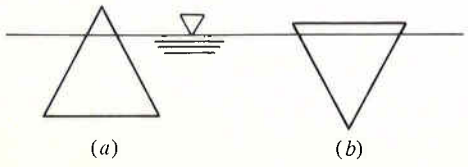


**Fig. P2.134**

**2.135** A solid cylinder is 1 m in diameter and 5 m long, with SG = 0.5. When floating in water (SG = 1.0), what is its stable position? What is its metacentric height?

**2.136** Investigate the stability of the floating can in Prob. 2.126. Make simplifying assumptions if necessary.

**2.137** An open tank of water 3 m deep is being accelerated upward at 4 m/s$^2$. Compute the gage pressure at the bottom of the tank in pascals.

**2.138** The liquid fuel (SG = 0.72) in a rocket is 8 ft deep and vented to the atmosphere. The rocket is accelerating upward at 3 $g$'s. Compute the pressure at the bottom of the fuel tank in pounds force per square inch gage.

**2.139** The tank of water in Fig. P2.139 accelerates to the right with the fluid in rigid-body motion. Compute $a_x$ in meters per second squared. Does the solution change if the fluid is mercury?

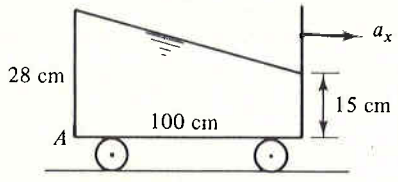


**Fig. P2.139**

**2.140** Compute the gage pressure at point $A$ in Fig. P2.139 if the fluid is mercury (SG = 13.56).

**2.141** The same water tank from Prob. 2.139 is now moving in rigid-body motion up a 30° inclined plane with constant acceleration. Compute $a$ in meters per second squared. Is the acceleration up or down? Compute gage $p_A$.

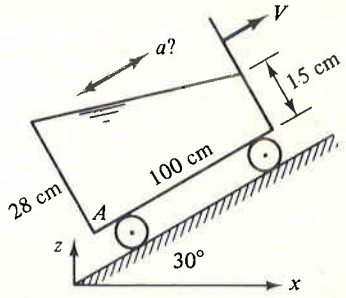


**Fig. P2.141**

**2.142** The tank of water in Fig. P2.142 is 10 cm wide into the paper. If it is accelerated uniformly to the right at 5.0 m/s$^2$, what will the water depth be on side $AB$? What will the pressure force be on panel $AB$?

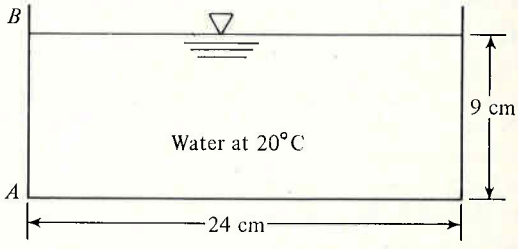


**Fig. P2.142**

**2.143** The tank of water in Fig. P2.143 is full and open to the atmosphere at point $A$. For what acceleration $a_x$ in feet per second squared will the pressure at point $B$ be atmospheric?

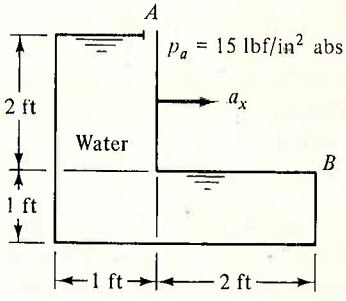


**Fig. P2.143**



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**2.144** For the water tank of Fig. P2.143, what acceleration $a_x$ in feet per second squared will cause the pressure at point $B$ to be zero absolute? Neglect cavitation.

**2.145** A full can of water is 30 cm in diameter and 60 cm high. What gradually applied rigid-body acceleration will cause one-quarter of the water to spill out?

**2.146** A bucket 1 ft deep contains 6 in of water and is being twirled in a vertical circle on a 3 ft rope at 1 r/s. Estimate the gage pressure at the bottom of the bucket when it is at (*a*) its highest and (*b*) its lowest position.

**2.147** The tank of water in Fig. P2.147 accelerates uniformly by freely rolling down a 30° incline. If the wheels are frictionless, what is the angle $\theta$? Can you explain this interesting result?

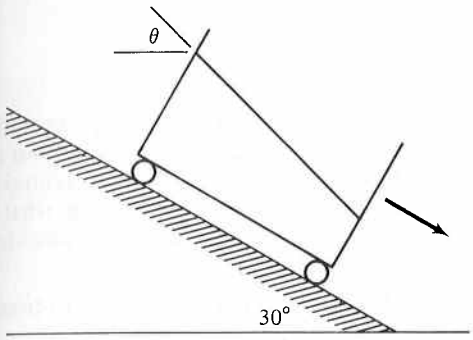


Fig. P2.147

**2.148** A fish tank 12 in deep by 15 by 30 in is to be carried in a station wagon which may experience accelerations of $\frac{1}{2}g$. What is the maximum water depth which will avoid spilling in rigid-body motion? What is the best alignment of the tank with respect to car motion? What is a proper engineering safety factor on water depth to avoid sloshing spills?

**2.149** The 6-ft-radius waterwheel in Fig. P2.149 is being used to lift water with its 1-ft-diameter half-cylinder blades. If the wheel rotates at 10 r/min and rigid-body motion is assumed, what is the water surface angle $\theta$ at position $A$?

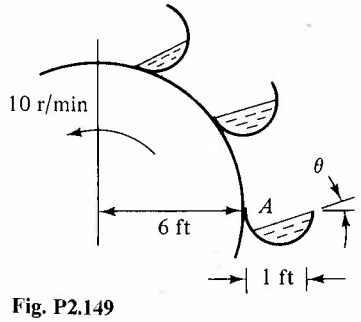


Fig. P2.149

**2.150** A cheap accelerometer, probably worth the price, can be made from a U-tube as in Fig. P2.150. The water-level change $h$ is a measure of $a_x$. If $L = 20$ cm and $D = 1$ cm, what will $h$ be if $a_x = 5$ m/s$^2$? Can the scale markings on the tube be linear multiples of $a_x$? What are the disadvantages of this design?

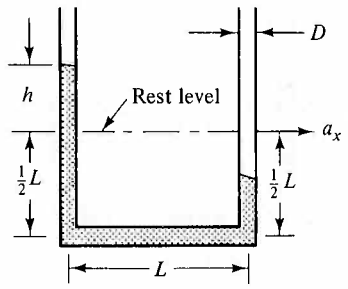


Fig. P2.150

**2.151** The U-tube in Fig. P2.151 is open at $A$ and closed at $D$. If accelerated to the right at uniform $a_x$, what acceleration will cause the pressure at point $C$ to be atmospheric? The fluid is water (SG = 1.0).

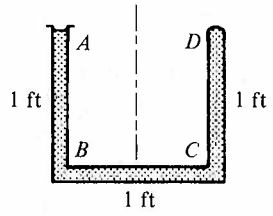


Fig. P2.151

**2.152** A 6-in-diameter cylinder 1 ft high is full of water. If it is rotated about its central axis in rigid-body motion, compute the rate in revolutions per minute for which one-third of the water will spill out.

**2.153** For the cylinder of Prob. 2.152 compute the rotation rate in revolutions per minute for which the bottom will be barely exposed.

**2.154** If the U-tube in Fig. P2.150 is rotated about the right leg at 80 r/min, what will the level $h$ in the left leg be if $L = 20$ cm and $D = 1$ cm?



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**2.155** For what uniform rotation rate in revolutions per minute about axis $C$ will the U-tube in Fig. P2.155 take the configuration shown? The fluid is mercury at 20°C.

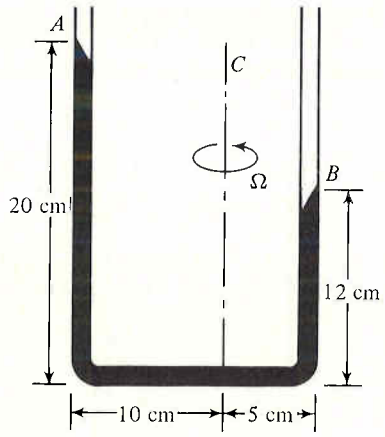


**Fig. P2.155**

**2.156** Suppose that the U-tube of Fig. P2.151 is rotated about axis $DC$. If the fluid is water at 122°F and atmospheric pressure is 2116 lbf/ft$^2$ absolute at what rotation rate will the fluid within the tube begin to vaporize? At what point will this occur?

**2.157** The 45° V-tube in Fig. P2.157 contains water and is open at $A$ and closed at $C$. What uniform rotation rate in revolutions per minute about axis $AB$ will cause the pressure to be equal at points $B$ and $C$? For this condition, at what point in leg $BC$ will the pressure be a minimum?

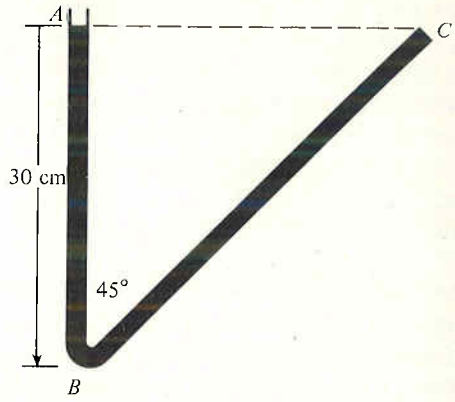


**Fig. P2.157**

**2.158** A 9-in-high, 6-in-diameter can is filled with SAE 30 oil at 20°C and the lid is sealed at standard sea-level pressure. The can is then rotated about its central axis at 3000 r/min. Assuming rigid-body motion, what will the maximum oil pressure in the can be in pounds per square inch absolute?

**2.159** For Prob. 2.158, compute the total hydrostatic force on the bottom of the can in pounds force.

**2.160** A very deep 8-in-diameter can contains 6 in of water overlaid with 6 in of oil (SG = 0.86). If the can is rotated about its central axis at 300 r/min and rigid-body motion ensues, what will the shape of the air-oil and oil-water interfaces be? What will the maximum fluid pressure in the can be in pounds force per square inch gage?



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