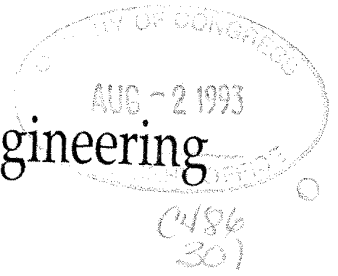


Pocket Book of
Electrical Engineering
Formulas



Richard C. Dorf
Ronald J. Tallarida



CRC Press
Boca Raton Ann Arbor London Tokyo

Library of Congress
Cataloging-in-Publication Data

TK153
.D68
1993

Dorf, Richard C.
Pocket book of electrical engineering formulas / Richard
C. Dorf and Ronald J. Tallarida
p. cm.

Includes bibliographical references and index.

ISBN 0-8493-4473-5

1. Electric engineering—Mathematics—Handbooks,
manuals, etc. 2. Mathematics—Formulae. I. Tallarida,
Ronald J. II. Title.

TK153.D68 1993
621.3'0212—dc20

93-9634
CIP

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International Standard Book Number 0-8493-4473-5

Library of Congress Card Number 93-9634

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

Neutron (rest mass) (m_n)	1.675×10^{-27} kg (939.6 MeV)
Planck constant (h)	6.63×10^{-34} J·s
Proton (rest mass) (m_p)	1.673×10^{-27} kg (938.3 MeV)
Speed of light (c)	3.00×10^8 m·s ⁻¹

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15 Circuits with Energy Storage Elements

1. Capacitors

Capacitance is a measure of the ability of a device to store energy in the form of separated charge or in the form of an electric field:

$$q = Cv$$

where q is the charge, v is the voltage across the element, and C is the capacitance measured in farads (F).

The current through a capacitor is

$$i = C \frac{dv}{dt}$$

The voltage across a capacitor C is

$$v = \frac{1}{C} \int_{t_0}^t i d\tau + v(t_0)$$

where $v(t_0)$ is the voltage at t_0 .

2. Inductors

Inductance is a measure of the ability of a device to store energy in the form of a magnetic field. The voltage across an inductor is

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$$v = L \frac{di}{dt}$$

where i is the current through the inductor and L is the inductance measured in henrys (H).

The current in an inductor is

$$i = \frac{1}{L} \int_{t_0}^t v d\tau + i(t_0)$$

3. Energy Stored in Inductors and Capacitors

$$\omega = \frac{1}{2} Cv^2$$

and

$$\omega = \frac{1}{2} Li^2$$

4. Series and Parallel Inductors

A series connection of N inductors can be represented by one series equivalent inductor L_s as

$$L_s = \sum_{n=1}^N L_n$$

A parallel connection of N inductors can be represented by one equivalent inductor L_p as

$$\frac{1}{L_p} = \sum_{n=1}^N \frac{1}{L_n}$$

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5. Series and Parallel Capacitors

The equivalent capacitance of a set of N parallel capacitors is simply the sum of the individual capacitances:

$$C_p = \sum_{n=1}^N C_n$$

A series connection of N capacitors can be represented by one equivalent capacitance C_s :

$$\frac{1}{C_s} = \sum_{n=1}^N \frac{1}{C_n}$$

6. The Natural Response of an RL or RC Circuit

The *natural response* of a circuit depends only on the internal energy storage of the circuit and not on external sources. The natural response of a series connection of a resistor R and a capacitor C is

$$v = V_0 e^{-t/RC}$$

where $v(0) = V_0$ is the initial voltage on the capacitor and v is the capacitor voltage.

The natural response of a series connection of a resistor R and inductor L is

$$i = I_0 e^{-Rt/L}$$

where $i(0) = I_0$ is the initial current and i is the inductor current.

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7. The Forced Response of an RL or RC Circuit Excited by a Constant Source

The *forced response* of a circuit is the behavior exhibited in reaction to one or more independent signal source. The forced response of an RC circuit is

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/RC}$$

where $v(\infty)$ is the steady-state value at $t = \infty$.

The forced response of an RL circuit is

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

where $\tau = L/R$.

8. The Natural Response of a RLC Circuit

The differential equation for a parallel connection of an R , L , and C is

$$\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = 0$$

where v is the capacitor (see Figure 15.1).

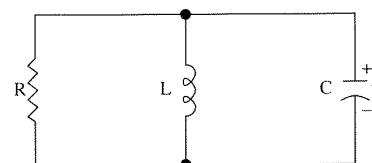


FIGURE 15.1. Parallel RLC circuit.

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The differential equation for the series connection of R , L , and C is

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

where i is the current through the inductor (see Figure 15.2).

The characteristic equation is

$$s^2 + a_1 s + a_0 = 0$$

or

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Then the roots of the characteristic equation are

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

where $\omega_0 = 1/\sqrt{LC}$ is called the resonant frequency.

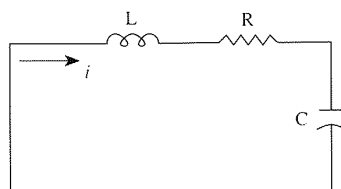


FIGURE 15.2. Series RLC circuit.

The roots of the characteristic equation assume three possible conditions.

1. Two real and distinct roots when $\alpha^2 > \omega_0^2$.
2. Two real equal roots when $\alpha^2 = \omega_0^2$.
3. Two complex roots when $\alpha^2 < \omega_0^2$.

When the two roots are real and distinct, the circuit is said to be *overdamped*. When the roots are both real and equal, the circuit is *critically damped*. When the two roots are complex conjugates, the circuit is said to be *underdamped*.

The overdamped natural response is

$$x = A_1 e^{-s_1 t} + A_2 e^{-s_2 t}$$

where $x = v$ for the parallel RLC circuit and $x = i$ for the series RLC circuit.

When the two roots are equal, the natural response is

$$x = e^{-\alpha t} (A_1 t + A_2)$$

When the circuit is underdamped, we have

$$x = e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, the damped resonant frequency.