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54 **System for transmitting television picture information using transform coding of subpictures.**

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(USA); T. OHIRA et al.: "Adaptive orthogonal
transform coding system for NTSC color
television signals"**

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Description

The invention relates to a system for transmitting information of a television picture over a medium, said television picture being organized in television lines and pixels within each line, and being distributed over a plurality of subpictures of uniform dimensions, said system comprising a transformation device for receiving the information of a subpicture for transform into a series of a coefficient bits of respective transformation functions, and comprising an encoder for encoding a subset of k bits of said a coefficient bits into a larger group of n bits by means of a code having at least single error correction capability for said k bits, said k bits containing at least one bit associated to at least one lowest frequency transformation function as most significant bit of the associated coefficient, an output of said transformation device being connected to an input of said medium, said system furthermore comprising a display apparatus which is connected to said medium and which comprises a decoder for receiving said encoded coefficient bits and for therefrom reconstructing the information of the associated subpicture. Such a system is known from the article by T. Ohira et al., Orthogonal Transform Coding Systems for NTSC Colour Television Signals, IEEE Transactions Communications, Vol. COM-26, No. 10, October 1978, pages 1545—63. The transmission medium may be a transport medium, for example, a bundle of telephone lines, or a storage medium, for example, a magnetic tape within a video cassette recorder. The encoding serves to limit the redundancy of the picture information so that the bit rate of the information to be stored or transferred is diminished. The reduction may for example be by a factor of four without lessening the subjective display quality. Suitable transformation function are Hotelling, Fourier, Hadamard or Haar functions, hereinafter only the use of Hadamard functions is treated by way of example. The reduction of the redundancy leads to a higher susceptibility to errors. The citation protects eleven bits by a single error correction code having a minimum Hamming-distance of three.

It has been found that under certain circumstances single bit error correction is insufficient. On the other hand, protection of all k coefficient bits by a code of increased correction capability would raise the number of bits by a too large amount. It is an object of the invention to provide improved error protection for certain coefficient bits while leaving the protection of other coefficient bits at their earlier levels. The object of the invention is realized in that it is characterized in that said code has a minimum Hamming-distance of four to allow for at least single error correction, double error detection within said k bits, in that said code has a second, larger, minimum Hamming-distance of at least five to allow for at least double error correction within a subset of at least two bits of said k bits.

In itself an extended error protection system is disclosed in US—A—3,949,208. This document, however, relates to a memory for general use and therefore the particular relevance of each stored bit is not known beforehand. Therefore, according to this citation all bits have to be equally well protected.

Coefficient bits are to be understood to mean herein the bits which are applied to the encoder of the error correction code in order to be encoded. Code bits are to be understood to mean herein the bits which form the output result of the encoder or the information supplied for the decoder. The bits which result from the analog-to-digital conversion of the picture information before it is applied to the transformation device will not be considered, and the bits which are formed after the implementation of the error correction code in order to adapt the signal to be transmitted to the specific properties of the transmission medium (modulation bits) will not be considered either.

In a particular embodiment of a system of the kind described in the preamble, it is proposed to activate the encoder selectively, under the control of subpicturewise received television picture information, to operate in one of a plurality of transformation modes, each mode having a respective set of coefficient bits applied to the medium plus a mode signalling bit group to indicate the transformation mode thus activated. This is described in the prior unpublished Netherlands Patent Application 8003873. The earlier system has a normal mode in which information is transported only as a small number of bits, for example, as a 0 bit or a 1 bit. In incidental cases this is not sufficient and for one (or some) blocks a change-over is made to such an incident mode. For some transformation functions the number of coefficient bits is then increased at the expense of other transformation functions. It is attractive to include said signalling bit group in the error protection code so that the selection of the incident modes is protected.

In a further particular embodiment of a system of the kind described in the preamble it is proposed, for use with colour television picture information which is sampled in the converter with a sample frequency f_s which equal twice the color subcarrier frequency f_{sc} at instants which coincide with the phase positions

$$\pm \frac{\pi}{4} + M\pi,$$

in which $M=0, 1, 2, \dots$, of the colour information signal $u(t)$ in the line signal, that the transformation functions are Hadamard functions. This is described in the prior unpublished Netherlands Patent Application 8004521. According to the earlier system, the colour difference signal is embodied in only a very small number of coefficients, for example, in only two coefficients. In that case it is attractive to include the most significant coefficient bit of each of the two

transformation functions representing the colour difference information in the error correction code.

Preferably, said extra-protected coefficient bits comprise the most significant coefficient bits of the transformation function which indicate the mean luminance of the relevant subpicture. It has been found that the protection of these most significant bits offers a good subjective result. The number of bit errors to be corrected preferably equals the number of bits forming the third number of coefficient bits.

The invention also relates to a picture converting device for use in a system of the described kind in which the encoder forms a non-systematic code. It has been found that, contrary to many other cases, the non-systematic code then offers an attractive compromise between means and results. Preferably, the number of data bits protected by the error correction code amounts to six and said second number preferably also amounts to six. This results in a simple implementation with usually adequate correction possibilities.

The invention also relates to a display apparatus for use in a system of the described kind in which the decoder forms a feasible series of data bits on the basis of at least one series of a number of code bits which has been found to be incorrect but not correctable. Sometimes the correction is then correctly performed but sometimes also incorrectly. However, it has been found that in many cases the extra-protected coefficient bits can still be correctly recovered.

The invention also relates to a decoder for use in a system or a display apparatus of the described which is constructed as a switching module. It may be a single integrated circuit or, for example, a module consisting of several integrated circuits.

Brief description of the figures.

The invention will be described in detail hereinafter with reference to some Figures.

Figure 1 diagrammatically shows a system in which the invention can be incorporated,

Figure 2 shows, as background information, some possibilities for the coding of a subpicture by means of Hadamard functions,

Figure 3 shows more relevant information for a subpicture consisting of 4×4 pixels,

Figure 4 shows some configurations of subpictures comprising 4×4 pixels,

Figure 5 shows five possibilities for the numbers of coefficient bits used for the elements of the series of transformation functions,

Figure 6 shows the generator matrix of a specimen code,

Figure 7 shows the set of code words formed for this code,

Figure 8 shows an encoder for this code,

Figure 9 shows the parity check matrix for the specimen code,

Figure 10 shows a table with the syndrome words which can be formed,

Figure 11 shows a block diagram of a decoder for the specimen code,

Figure 12 shows a detail of the decoder,

Figure 13 shows a table with syndrome words and associated error indication bits,

Figure 14 shows a detail of the decoder,

Figure 15 shows a table with feasible (n, k) code, and

Figure 16 shows the generator matrix of a $(12, 4)$ code.

Figure 17 shows the relationship between code bits and (guessed) data bits for the Figures 6, 9.

Brief description of the system

Figure 1 diagrammatically shows a system in which the invention can be incorporated. Element 120 is a camera tube. The signal which is scanned line-wise is converted into a digital representation in element 122. Transformation device 146 first of all comprises a Hadamard transformation device 124 and an encoder 126 for an error correction code as will be described hereinafter. Block 128 represents an interleaving device for spreading over a larger time interval faults cause by burst errors. Block 130 represents a modulator for converting the series of code bits received into channel symbols: this series is thus adapted to the properties of the transmission medium. For example, given lower limits are implemented as regards the length of intervals without transitions (run length limitation). Line 144 represents the transmission medium which may be a storage medium or, for example, a communication channel. The elements 120 to 130 may together form part of a picture converting device, for example, a camera. Block 132 represents a demodulator for recovering the code bits from the series of channel symbols regenerated. Element 134 represents the opposite member of the block 128 and serves for de-interleaving. Element 136 represents the decoder for reconstructing or choosing the data bits from the code bits. Element 138 forms the opposite member of the block 124 for cancelling the Hadamard transformation and for recovering a binary coded amplitude signal per pixel. Element 140 represents a digital-to-analog converter. Element 142 represents the display element which is in this case constructed as a cathode ray tube. The elements 132 to 142 may form part of a display apparatus.

Description of the transformation functions

For background information Figure 2 shows some possibilities for the encoding of a subpicture by means of so-called Hadamard functions. The first eight Hadamard functions are shown over a unit time interval at the left of the Figure with bivalent amplitude $(0, 1)$. This row of functions can be indefinitely continued. It can be demonstrated that an arbitrary function can be approximated with any desired accuracy over the unit interval by means of a series of Hadamard functions with adapted amplitude. The same is applicable to a two-dimensional subpicture, four of which are denoted by the reference numerals

20, 22, 24, 26, with bivalent density (white and shaded). Block 20 has an intensity which varies in the horizontal direction according to the function 38 and in the vertical direction according to the function 30: code 3830. The other blocks have the following codes 22: 4034; 24: 3236; 26: 4242. In the approximation of an arbitrary monochrome intensity distribution in the subpicture, the coefficients of the higher order Hadamard functions are almost always relatively small. Such a code can also be used for a polychrome picture (generally trichrome in the case of television pictures). In addition to the luminance signal two colour difference signals are applicable (for red and for blue). The variation of these colour difference signals can also be approximated by means of a series of Hadamard functions with suitable coefficients.

As a continuation Figure 3 shows the sixteen feasible Hadamard functions for approximating a subpicture comprising 4×4 pixels. It is to be noted that this can also be done with subpictures which are not square. However, hereinafter only subpictures comprising 4×4 pixels will be considered. Generally, the number of picture lines and the number of pixels per picture line preferably equals a power of the number two. The number of feasible Hadamard functions for a one-dimensional interval of p pixels (or lines) equals p . Figure 4 shows four feasible dispositions of a subpicture of 4×4 pixels, a shift of one half pixel period being allowed between the pixels of two successive picture lines. Figure 4, example 44, shows an attractive configuration of a subpicture for a PAL system and identical configurations of the subpictures; it has been found that the colour difference signals are then embodied in the functions C6 and C7 of Figure 3 (Figure 3 does not show the shift of the picture lines; furthermore, the subpicture concerns only a single frame). Figure 4, block 46, shows an attractive configuration of a subpicture for an NTSC system and identical configurations of the subpictures; it has been found that the colour difference signals are then embodied in the functions C5 and C7 of Figure 3.

By combinations of the subpictures shown in Figure 4 and possibly of their mirror images, a picture can be divided in many ways; within one picture, subpictures of mutually different configuration may also occur. For further details, reference is made to the cited Netherlands Patent Application 8004521.

The information of each subpicture is transformed by means of the set of Hadamard functions which is symbolized in Figure 3, the coefficients being represented by a bit group. Figure 5 shows five examples for the assignment of a given number of bits to each of these coefficients. The digits in the column 62 correspond to the order of the Hadamard function in Figure 2. The total number of bits in each column (being the sum of the numbers in the column) always equals 38 in Figure 5. This means that the same total dynamics can be realized for the

luminance of a subpicture, because each Hadamard function has the same mean value, that is to say half the luminance "white" in Figure 3. The combination of column 60 of Figure 5 and the subpicture configuration 44 shown in Figure 4 represents an attractive realization for a PAL system in which the colour difference signals are embodied in the coefficients of the Hadamard functions C6 and C7 of Figure 3. The five columns 52 through 60 shown in Figure 5 represent the numbers of transformation modes in which the picture can be represented. Notably the first four columns represent as many selectable cases. Column 52 represents a "normal" situation with an "average" picture, and the other columns represent cases in which a coefficient which is normally small is "large" in a special case. Such a coefficient can then be represented by a larger number of bits so that it is emphasized.

Column 54 emphasizes C5, C9, C13

Column 56 emphasizes C5, C9, C13,

Column 58 emphasizes C5, C6, C13, C9,

the emphasis being at the expense of mainly C0.

It is to be noted that such different transformation modes can also be realized when the combination of the column 60 represents the normal mode. The selection from the four (or possibly another number) transformation modes is signalled by a signalling bit group (of two bits in the case of realization according to the columns 52... 58 of Figure 5). The subpicture of 16 pixels is thus coded by 40 data bits, 38 of which are coefficient bits; this means $2\frac{1}{2}$ bit per pixel. Other adaptive transformation methods are also known per se.

Description of the error protection code.

In the embodiment in accordance with the invention, six of said 40 data bits which represent the relevant subpicture information are protected by a non-systematic $(n, k)=(12, 6)$ code, which means that the number of bits is increased by six; however, within the set of twelve bits all bits are code bits and an original data bit may not be recovered from one associated code bit. As regards the first four columns of Figure 5, these six data bits may represent the four most significant bits of the coefficient C0 plus the two bits of the signalling bit group. Alternatively, in the columns 54, 56 and 58, the three most significant bits of C0 plus the most significant bit of C8 may be selected. In column 60 of Figure 5, the six protected bits may be the four most significant bits of the coefficient of the transformation function C0 and the most significant bits of the functions C6, C7 which represent the colour difference signals.

Figure 6 shows the generator matrix of the specimen code used and Figure 7 shows the table with the 12-bit code words thus formed (column 64) on the basis of the relevant data words (column 66). Column 68 states the weight of the associated code word, i.e. the number of code bits having the value "1". Ignoring the code word which consists entirely of "0" bits, the minimum

weight of a code word equals four. This means that the minimum Hamming distance of the code also equals four, so that at all times one error bit can be corrected in a code word and two error bits can be detected. Figure 8 shows an encoder for implementing the generator matrix of Figure 6. The six data bits arrive in parallel on the inputs 72; the twelve code bits are also outputted in parallel on the outputs 74. The matrix is implemented by means of EXCLUSIVE-OR elements such as the element 70. It is alternatively possible to perform the coding operation by means of a read-only memory which comprises (at least) six inputs and twelve outputs. The interleaving, modulation and parallel/series conversion mechanisms have been omitted in this Figure.

When a code word is received which comprises two incorrect bits, it can be detected as being an incorrect code word. In that case there are a number of code words which are situated from the relevant code at a Hamming distance which equals two. For the decoding of this incorrect code word it is not relevant to determine the correct code word from the feasible choices. However, it is important that the most likely correct data word is selected in the case of an incorrect code word. It appears that the table of Figure 7 can be divided into four subtables. All data words of each subtable have the same value for the two extreme left (most significant) bits. Each code word of a subtable then has a Hamming distance of at least 5 from each code word of each other subtable. For the code word which consists entirely of zeros, this can be verified most easily. Consequently, the two most significant data bits are protected by a code which corrects two bit errors. It also appears that when two bit errors occur in a code word, one in an even position and one in an odd position, correction is also feasible. This appears from the fact that there is no code word having the weight "4" with a "1" in an even bit position as well as in an odd bit position.

Figure 9 shows the parity check matrix [H] of the specimen code which satisfies $[G] \cdot [H] = 0$. Multiplication of a twelve-bit word by the parity check matrix [H] produces a six-bit syndrome word. If the twelve-bit word is an error-free code word, the syndrome word consists exclusively of "0" bits. Figure 10 shows a table with the syndrome words. The first two columns state the digital and the binary representation of the syndrome words. The further column states the order of incorrect code bits (0—11) causing the relevant syndrome word. If there are several possibilities, they are separated by a semicolon. For all feasible $2^{12} = 4k$ different incorrect words, only those comprising the lowest number of incorrect bits are indicated for each syndrome word. For example, the syndrome word "4" is caused by an error in code bit "10", the syndrome word "6" by an error in the code bits 3 and 8, the syndrome word "24" by two feasible combinations of two incorrect code bits, the syndrome word "16" by three of such feasible combina-

tions, and the syndrome word "48" by a large number of feasible combinations of three incorrect code bits. Because the code is linear, the effect of disturbances may be summed. The error pattern (4, 8) and the error pattern (5, 9) may be summed and produce the syndrome word "000011". However, this syndrome word now indicates the error pattern (0, 1), and the error pattern (4, 5, 8, 9) is not considered. The same syndrome word "000011" is also found by bit-wise modulo-2 addition of the syndrome words "111111" (63) and "111100" (60), and also by addition of the syndrome words "111101" (61) and "111110" (62). The table states all twelve error words which contain only one incorrect code bit, and also all $66 = \frac{1}{2} \times 12 \times 11$ incorrect words which contain two incorrect code bits. It appears that the following errors can be corrected:

a) a single incorrect code bit in a code word is always corrected because a unique syndrome word is associated therewith,

b) two incorrect code bits in a code word are also always corrected if they do not both belong to the even code bits of the code word or both to the odd code bits of the code word,

c) when two code bits of the code word are incorrect, both bits having an even or an odd order number, the two most significant data bits are correct or will be properly corrected. In the case of eight different results for the syndrome word (sixteen feasible error patterns in the code word), one data bit must be guessed. In the case of six other different results for the syndrome word (fourteen feasible error patterns in a code word), two data bits must be guessed. In the case of one different result for the syndrome word (several feasible error patterns), the entire data word must be guessed because in that case at least three code bits are incorrect.

Description of the decoder.

Figure 11 shows a block diagram of a decoder for use with the described code. The correct or incorrect twelve-bit code words $\hat{C}$ arrive on input 76. Element 78 is the syndrome generator which generates a six-bit syndrome word on output 80 by means of the parity check matrix [H], and which conducts the six most significant code bits on output 82. Element 84 is a read-only memory which is addressed by the six syndrome bits; a synchronizing clock system has been omitted for the sake of simplicity. The read-only memory 84 outputs nine error indication bits: six on output 86 and three further error indication bits on output 88. The first six error indication bits $e_0 \dots e_5$ indicate the errors detected in the six most significant code bits; for $(64-8-6-1)=49$ different syndrome words this is the only correction required. In the multiple EXCLUSIVE-OR element 87 these error indication bits are modulo-2 added to the associated, more significant code bits $\hat{C}_0 \dots \hat{C}_5$. In element 90, the six-bit data word m' is reconstructed from the six repaired code bits received. The described circuit elements, exclud-

ing read-only memory 84, are shown in detail in Figure 12. Figure 13 shows a table with syndrome words and the associated groups of error indication bits. The first column of the latter Figure contains a decimal representation of the syndrom words while the second column contains a binary representation. The third column states the six error indication bits appearing on output 86 (Figure 11) which provide the primary correction. The fourth column states the further error indication bits on output 88 which serve to solve ambiguous cases. The decimal representations of the latter three error indication bits are shown in the last column. The meaning of the latter three error indication bits is as follows:

- 0: after the operation by means of the six first error indication bits, no further modification of the data content of the six most significant code bits is required.
- 1: (in this case no correction has been performed in element 87): the entire data word must be guessed because there are more than two incorrect code bits.
- 2: in addition to the correction performed by the first group of error indication bits, the data bits m_2 must be guessed.
- 3: in addition to the correction performed by the first group of error indication bits, the data bit m_3 must be guessed.
- 4: the correction by the first group of error indication bits has not taken place, data bit m_4 must be guessed.
- 5: the correction by the first group of error indication bits need not be performed, data bit m_5 must be guessed.
- 6: the data bits m_2 and m_4 must be guessed.
- 7: the data bits m_3 and m_5 must be guessed.

The connections 76, 80, 82 and 86 of the circuit shown in Figure 12 have already been mentioned. The logic operations are again performed by means of EXCLUSIVE-OR-elements. Connection 92 represents the output of the multiple EXCLUSIVE-OR element 87 of Figure 11. In block 90 of Figure 11, a provisional six-bit data word is reconstructed from the first six code bits thus provisionally corrected, said reconstructed data word being presented on output 94. The latter connection is also shown in Figure 12, together with the internal construction of the element 90. The latter can be constructed in the described manner by means of only four exclusive-OR-elements and by cross-wiring in order to modify the sequence of some code bits. In block element 96, the data bits which are not unambiguously defined are guessed under the control of the three last error indication bits. The decoding of the error indication bits is shown in Figure 14. The circuit shown in Figure 14 comprises two inverters, such as the element 98, a logic OR-gate 101, and five logic AND-gates such as the element 100. On the outputs 102... 110 the signals necessitating the guessing operation successively appear for the data bits m_2 , m_3 , m_4 , m_5 and for the entire

data word. The further guessing in the element 96 can be performed in various ways:

a. guessing implies: assuming to be one: the output signal of Figure 14 is bit-wise combined with the associated data bits in an OR-function.

b. analogously, guessing may imply: assuming to be "zero".

c. guessing implies that the treatment depends on the data content, for example, the value for the preceding subpicture is substituted. Typical properties of the transmission may also be taken into account, for example, the fact that an error from 0 to 1 is more probable than *vice versa*, or the effect of neighbouring bits may be considered. This is not elaborated herein.

Figure 15 shows a table of (N, K) codes with at most twelve code bits and at least 2 data bits, with unequal protection among the data bits. The first column states the length of a code word, i.e. the value of N. The second column states the length of a data word, i.e. the value of K. The third column states the degree of error protection for each of the series data bits as expressed in the minimum Hamming-distance D; it is well known that the minimum Hamming-distance is a convenient way of expressing the error protection capability of a code. A value of D=2 implies that single bit errors may be detected, at D=3 single bit errors may be corrected, and so on. The minimum Hamming-distance as defined for a certain bit implies that the correction or detection capability with respect to reproduction of the correct value of this bit is assured even if the maximum amount of errors covered by this value of D occurs. Therefore, for a (11, 4) code the two most significant bits are protected against two incorrect code bits, while an additional incorrect bit can be detected. The protection for the other two data bits is less. For some cases more possibilities exist, for example, for the (11, 2) code. Sometimes more than two protection levels are feasibly, for example, in the (12, 4) code. Thus, one data bit may be reconstructed even if three errors occur, a second data bit may be reconstructed even if two errors occur, while an additional error is signalled correctly in that case, two further data bits may be reconstructed correctly if one error occurs, while an additional error is signalled correctly. The generator matrix of the latter code is shown in Figure 16.

The code already described with reference to the Figure 6, 9 represents a special case of a class of [N, K]=[4n, 2n] code with $n \geq 3$ and Hamming distances for the relevant data bits of (n+2, n+2, 4, 4, ...) where the differences between the Hamming distances are attractive. A very favourable generator matrix for such a code is given by:

$$[G_{4n}] = \begin{pmatrix} e_n & e_n & 0_n & 1_n \\ 1_n & 0_n & e_n & e_n \\ A_n & A_n & 0_n^{n-1} & 0_n^{n-1} \\ 0_n^{n-1} & 0_n^{n-1} & A_n & A_n \end{pmatrix}$$

Therein, e_n , $\emptyset_n$ and 1_n are vectors having the length of n bits (10000...), (000...) and (111...1), respectively; $\emptyset_{n^{n-1}}$ is an $(n-1) \times n$ matrix consisting exclusively of ones; A_n is a matrix comprising a first column consisting exclusively of zeroes, supplemented at the right-hand side by an $(n-1) \times (n-1)$ unity matrix. The code according to the Figures 6, 9, however, has slightly more attractive properties in the case $n=3$. These $(4n, 2n)$ codes may be decoded in an effective way by

- 1 error in the even positions
- 1 error in the odd positions
- 2 errors in the even positions
- 2 errors in the odd positions

Thus, m_0 is indeed protected against two errors. The same is applicable to the data bit m_1 . Once m_0 and m_1 have been found, they may be used for the guesses for the other, less-protected data bits. These bits m_2 , m_3 , m_4 and m_5 are protected against one bit error, i.e. they can be correctly recovered from the word $\hat{c}$ received if no more than one bit error is present in $\hat{c}$. In the case of two bit errors in $\hat{c}$, m_2 , m_3 , m_4 and m_5 will either be correctly decoded or guessed because of the deadlock.

Claims

1. A system for transmitting information of a television picture over a medium, said television picture being organized in television lines and pixels within each line, and being distributed over a plurality of subpictures of uniform dimensions, said system comprising a transformation device (146) for receiving the information of a subpicture for transform into a series of a coefficient bits of respective transformation functions, and comprising an encoder (126) for encoding a subset of k bits of said a coefficient bits into a larger group of n bits by means of a code having at least single error correction capability for said k bits, said k bits containing at least one bit associated to at least one lowest frequency transformation function as most significant bit of the associated coefficient, an output of said transformation device being connected to an input of said medium, said system furthermore comprising a display apparatus (142) which is connected to said medium and which comprises a decoder for receiving said encoded coefficient bits and for therefrom reconstructing the information of the associated subpicture, characterized in that said code has a minimum Hamming-distance of four to allow for at least single error correction, double error detection within said k bits, in that said code has a second, larger, minimum Hamming-

means of majority decisions. This can be elaborated as follows. Figure 17 first shows the relationship between the code bits c and the data bits m of the Figures 6, 9. After that guesses are given for all data bits (indicated by circlexes). In the case of a voting deadlock for m_2 , m_3 , m_4 , m_5 (2×0 , 2×1), the relevant data bit must be guessed. Assume that there are two bit errors in the word received (the guesses for m_0 are considered):

exactly one of the guesses 1), 2) and 3)

is wrong and exactly one of the guesses 4) and 5); therefore the majority is right.

exactly two of the guesses 1), 2) and 3) are wrong or none of the guesses is wrong; therefore, the majority is right.

one or two of the guesses 4) and 5) are wrong; therefore, the majority is right.

distance of at least five to allow for at least double error correction within a subset of at least two bits of said k bits.

2. A system as claimed in Claim 1, wherein $n=2k$ and k is an even number equal to or greater than six.

3. A system as claimed in Claim 1 or 2, in which the encoder can be selectively activated to operate in one of a plurality of transformation modes under the control of subpicture-wise received television picture information, a unique set of numbers of coefficient bits being used for the relevant elements of the second series of transformation functions in each transformation mode, a signalling bit group also being applied to the transmission medium in order to indicate the transformation mode thus activated, characterized in that said signalling bit group is included in the error-protected bit group.

4. A system as claimed in Claim 1 or 2, for use with colour television picture information which is sampled in the picture converting device with a sample frequency f_s which equals twice the colour subcarrier frequency f_{sc} at instants which coincide with the phase positions

$$\pm \frac{\pi}{4} + M\pi,$$

in which $M=0, 1, 2, \dots$, of the colour information signal $u(t)$ in the line signal, the transformation functions being Hadamard functions, characterized in that the most significant coefficient bit of each of the two transformation functions representing the colour difference information is included in the error correction code, together with said number of coefficient bits.

5. A system as claimed in Claim 1, 2, 3 or 4, characterized in that said group of protected bits comprise the most significant coefficient bits of

the transformation function which indicates the mean luminance of the relevant subpicture.

6. A picture converting device for use in a system as claimed in any of the Claims 1 to 5, characterized in that the encoder forms a non-systematic code, in that more than $(n-k)$ code bits are formed each from a logic combination of more than one data bit.

7. A picture converting device as claimed in Claim 6, characterized in that the number of data bits protected by the error correction code amounts to six, said second number also amounting to six.

8. A display apparatus for use in a system as claimed in any of the Claims 1 to 5, characterized in that the decoder upon reception of at least one series of code bits found incorrect but not correctable regenerates an estimated series of data bits therefor.

9. A decoder for use in a system as claimed in any of the Claims 1 to 5, or in a display apparatus as claimed in Claim 8, characterized in that it is constructed as one or more integrated circuit modules.

Patentansprüche

1. System zum Übertragen von Information eines Fernsehbildes über ein Medium, wobei das Fernsehbild aus Fernsehzeilen und Bildelementen in jeder Zeile aufgebaut und auf eine Anzahl von Teilbildern mit einheitlichen Abmessungen verteilt ist, wobei das System einen Umsetzer (146) zum Empfangen der Information eines Teilbildes zum Umsetzen in eine Reihe von a Koeffizientenbits jeweiliger Umsetzfunktionen und einen Coder (126) zum Codieren eines Teilsatzes von k Bits dieser a Koeffizientenbits in eine grössere Gruppe von n Bits mittels eines Codes mit zumindest Einzelfehlerkorrekturfähigkeit für die k Bits, wobei die k Bits zumindest ein Bit enthält, das zumindest eine Niedrigstfrequenzumsetzungsfunktion als Bit mit höchster Wertigkeit des zugeordneten Koeffizienten zugeordnet ist, wobei ein Ausgang des Umsetzers mit einem Eingang des Mediums verbunden ist, wobei das System weiter ein Anzeigegerät (142) enthält, das mit dem Medium verbunden ist und einen Decoder zum Empfang der codierten Koeffizientenbits enthält und daraus die Information des zugeordneten Teilbildes rekonstruiert, dadurch gekennzeichnet, dass der Code einen Mindest-Hamming-Abstand von vier zum Erlauben zumindest von Einzelfehlerkorrektur und Doppelfehlerdetektierung innerhalb dieser k Bits hat, dass dieser Code einen zweiten grösseren Hamming-Abstand von zumindest fünf hat, die zumindest Doppelfehlerkorrektur in einem Teilsatz von zumindest zwei Bits der k Bits ermöglicht.

2. System nach Anspruch 1, dadurch gekennzeichnet, dass $n=2k$ und k eine gerade Anzahl gleich oder grösser als sechs ist.

3. System nach Anspruch 1 oder 2, in dem der Coder zum Betreiben in einer Anzahl von

Umsetzungsbetriebsarten unter der Steuerung der teilbildweise empfangenen Fernsehbilddinformation selektiv aktivierbar ist, wobei ein eindeutiger Anzahlsatz von Koeffizientenbits für die betreffenden Elemente der zweiten Reihe von Umsetzungsfunktionen in jeder Umsetzungsbetriebsart verwendet wird, wobei eine Signalisierungsbitgruppe auch dem Übertragungsmedium zugeführt wird, um die auf diese Weise aktivierte Umsetzungsbetriebsart anzugeben, dadurch gekennzeichnet, dass die Signalisierungsbitgruppe in die fehlergeschützte Bitgruppe aufgenommen ist.

4. System nach Anspruch 1 oder 2 zur Verwendung mit Farbfernsehbilddinformation, die in der Bildumwandlungsanordnung mit einer Abtastfrequenz f_s abgetastet ist, die gleich dem zweifachen Farbhilfsträgerfrequenz f_{sc} zu Zeitpunkten ist, die mit den Phasenpositionen

$$\pm \frac{\pi}{4} + M\pi,$$

zusammenfallen, worin $M=0, 1, 2, \dots$ des Farbinformationssignals $u(t)$ im Zeilensignal ist, wobei die Umsetzungsfunktionen Hadamard-Funktionen sind, dadurch gekennzeichnet, dass das bedeutsamste Koeffizientenbit einer jeden der zwei die Farbdifferenzinformation darstellenden Umsetzungsfunktionen in den Fehlerkorrekturcode zusammen mit der erwähnten Anzahl von Koeffizientenbits aufgenommen ist.

5. System nach Anspruch 1, 2, 3 oder 4, dadurch gekennzeichnet, dass die Gruppe geschützter Bits die Koeffizientenbits der Umsetzungsfunktion mit höchster Wertigkeit enthält, wobei diese Umsetzungsfunktion die mittlere Leuchtdichte des betreffenden Teilbildes angibt.

6. Bildumwandlungsanordnung zur Verwendung in einem System nach einem oder mehreren der Ansprüche 1 bis 5, dadurch gekennzeichnet, dass der Coder einen nicht systematischen Code bildet, in dem mehr als $(n-k)$ Codebits aus je einer logischen Kombination von mehr als einem Datenbit gebildet werden.

7. Bildumwandlungsanordnung nach Anspruch 6, dadurch gekennzeichnet, dass die Anzahl der vom Fehlerkorrekturcode geschützter Datenbits sechs beträgt, wobei die zweite Anzahl ebenfalls sechs beträgt.

8. Anzeigegerät zur Verwendung in einem System nach einem oder mehreren der Ansprüche 1 bis 5, dadurch gekennzeichnet, dass der Decoder beim Empfang zumindest einer Reihe von fehlerhaft, aber nicht korrigierbar befundene Codebits dafür eine geschätzte Reihe von Datenbits regeneriert.

9. Decoder zur Verwendung in einem System nach einem oder mehreren der Ansprüche 1 bis 5, oder in einem Anzeigegerät nach Anspruch 8, dadurch gekennzeichnet, dass er als einen oder als mehrere integrierte Schaltungsmoduln aufgebaut ist.

Revendications

1. Système pour transmettre de l'information d'une image de télévision par un moyen de transmission, l'image de télévision étant organisée en lignes de télévision et en pixels dans chaque ligne, et étant répartie sur plusieurs sous-images de dimensions uniformes, le système comprenant un dispositif de transformation (146) destiné à recevoir l'information d'une sous-image en vue de la transformer en une série de a bits de coefficients de fonctions de transformation respectives, et comprenant un codeur (126) pour coder un sous-jeu de k bits des a bits de coefficients en un plus grand groupe de n bits au moyen d'un code ayant au moins une capacité de correction d'erreurs simples pour k bits, les k bits contenant au moins un bit associé à au moins une fonction de transformation à fréquence la plus basse en tant que bit le plus significatif du coefficient associé, une sortie du dispositif de transformation étant connectée à une entrée au moyen de transmission le système comprenant, en outre, un appareil de visualisation (142) qui est connecté au moyen de transmission et qui comprend un décodeur destiné à recevoir les bits de coefficients codés et pour reconstruire à partir de ces bits l'information de la sous-image associée, caractérisé en ce que le code présente une distance de Hamming minimale de quatre, afin de permettre au moins la correction d'erreurs simples, la détection d'erreurs doubles dans les k bits, le code présentant une seconde distance de Hamming minimale, plus grande, d'au moins cinq, afin de permettre au moins la correction d'erreurs doubles dans un sous-jeu d'au moins deux bits des k bits.

2. Système suivant la revendication 1, dans lequel $n=2k$ et k est un nombre pair égal ou supérieur à six.

3. Système suivant la revendication 1 ou 2, dans lequel le codeur peut être activé sélectivement pour fonctionner selon un parmi plusieurs modes de transformation sous la commande d'une information d'image de télévision reçue par sous-image, un jeu unique de nombres de bits de coefficients étant utilisé pour les éléments en question de la seconde série de fonctions de transformation dans chaque mode de transformation, un groupe de bits de signalisation étant également appliqué au moyen de transmission, afin d'indiquer le mode de transformation ainsi activé, caractérisé en ce que le groupe

de bits de signalisation est inclus dans le groupe de bits protégés contre les erreurs.

4. Système suivant la revendication 1 ou 2 à utiliser avec de l'information d'image de télévision couleur qui est échantillonnée dans le dispositif convertisseur d'image à une fréquence d'échantillonnage f_s égale au double de la fréquence de la sous-porteuse couleur f_{sc} à des instants qui coïncident avec les positions de phase

$$\pm \frac{\pi}{4} + M\pi,$$

où $M=0, 1, 2, \dots$, du signal d'information couleur $u(t)$ dans le signal de ligne, les fonctions de transformation étant des fonctions de Hadamard, caractérisé en ce que le bit de coefficient le plus significatif de chacune des deux fonctions de transformation représentant l'information de différence de couleur est inclus dans le code de correction d'erreurs, en même temps que le nombre de bits de coefficients.

5. Système suivant la revendication 1, 2, 3 ou 5, caractérisé en ce que le groupe de bits protégés comprend les bits de coefficients les plus significatifs de la fonction de transformation qui indique la luminance moyenne de la sous-image en question.

6. Dispositif convertisseur d'image à utiliser dans un système suivant l'une quelconque des revendications 1 à 5, caractérisé en ce que le codeur forme un code non systématique, en ce que plus de $(n-k)$ bits de code sont formés, chacun à partir d'une combinaison logique de plus d'un bit de données.

7. Dispositif convertisseur d'image suivant la revendication 6, caractérisé en ce que le nombre de bits de données protégés par le code de correction d'erreurs s'élève à six, le second nombre s'élevant également à six.

8. Appareil de visualisation à utiliser dans un système suivant l'une quelconque des revendications 1 à 5, caractérisé en ce que le décodeur, à la réception d'au moins une série de bits de code s'avérant incorrects, mais impossibles à corriger, régénère une série estimée de bits de données pour cette série.

9. Décodeur à utiliser dans un système suivant l'une quelconque des revendications 1 à 5 ou dans un appareil de visualisation suivant la revendication 8, caractérisé en ce qu'il est construit sous la forme d'un ou de plusieurs modules en circuit intégré.

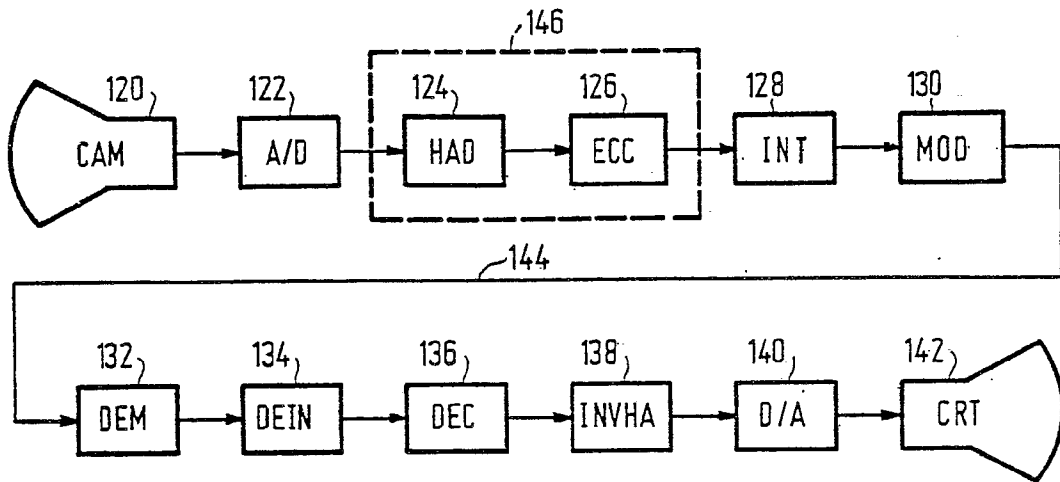


FIG.1

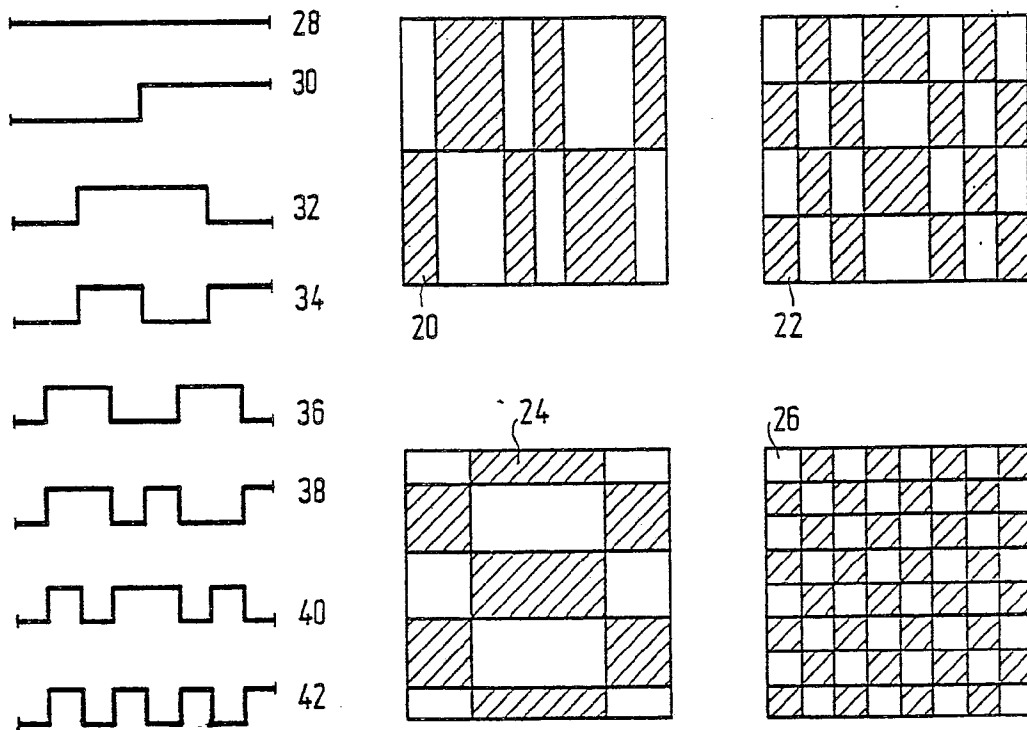
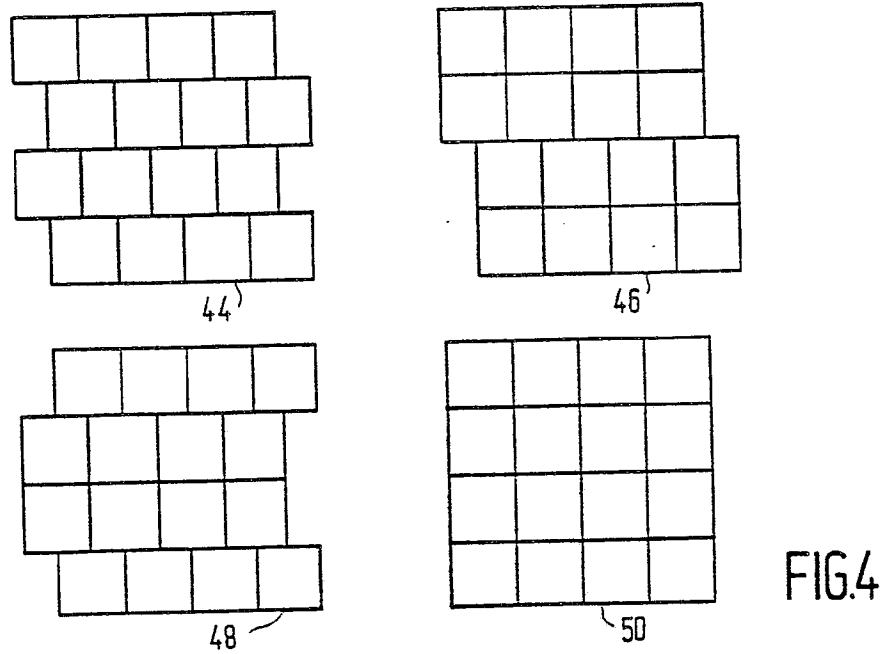
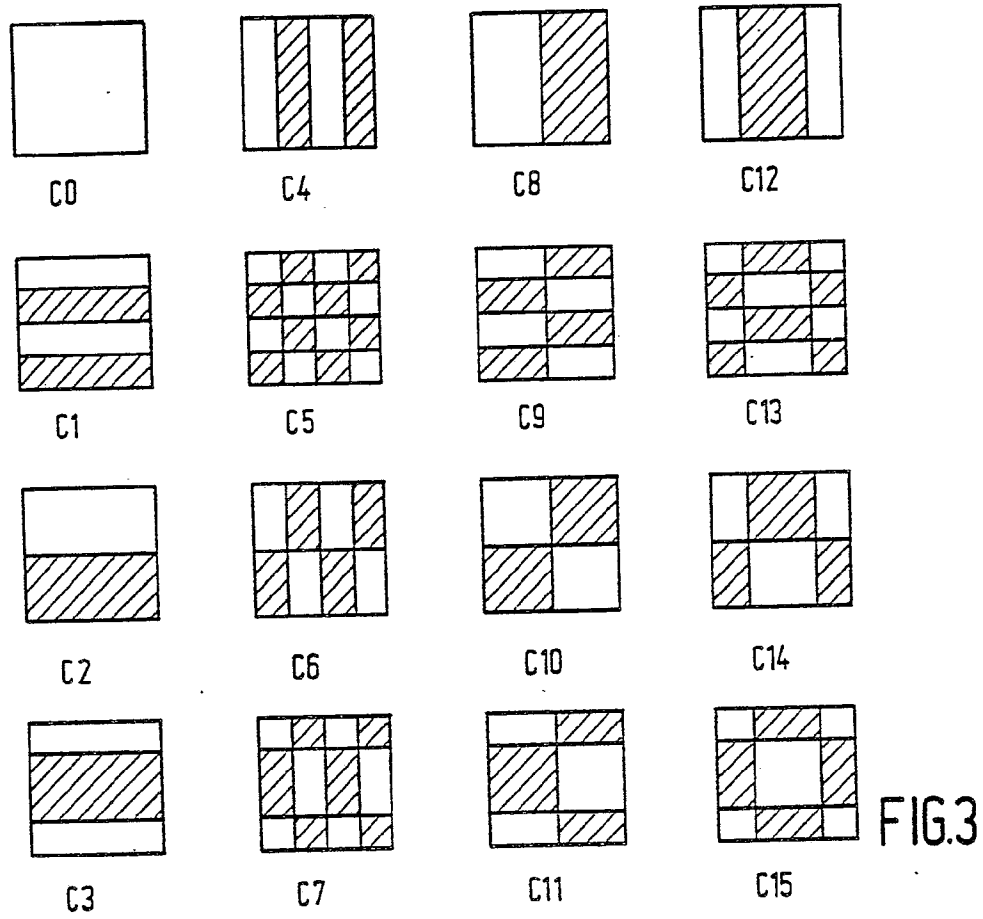


FIG.2



9	6	6	5	9	0
3	2	3	3	4	1
4	4	4	3	5	2
4	3	3	3	4	3
3	3	3	3	4	4
0	2	2	2	2	5
0	0	0	2	5	6
0	0	0	0	5	7
4	4	4	4	5	8
0	2	2	2	1	9
3	2	2	2	3	10
2	2	2	2	1	11
4	4	3	3	4	12
0	2	2	2	1	13
2	2	2	2	1	14
0	0	0	0	0	15
52	54	56	58	60	62

FIG.5

$$[G] = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

FIG.6

<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>	<u>2</u>	<u>3</u>
000001	000001010101	4	100101	011100100010	5
000010	000010101010	4	100110	011111011101	9
000011	000011111111	8	100111	011110001000	5
000100	010101010000	4	101000	100010000111	5
000101	010100000101	4	101001	100011010010	5
000110	010111111010	8	101010	100000101101	5
000111	010110101111	8	101011	100001111000	5
001000	101010100000	4	101100	110111010111	9
001001	101011110101	8	101101	110110000010	5
001010	101000001010	4	101110	110101111101	9
001011	101001011111	8	101111	110100101000	5
001100	111111110000	8	110000	001100111100	6
001101	111110100101	8	110001	001101101001	6
001110	111101011010	8	110010	001110010110	6
001111	111100001111	8	110011	001111000011	6
010000	000100011011	5	110100	011001101100	6
010001	000101001110	5	110101	011000111001	6
010010	000110110001	5	110110	011011000110	6
010011	000111100100	5	110111	011010010011	6
010100	010001001011	5	111000	100110011100	6
010101	010000011110	5	111001	100111001001	6
010110	010011100001	5	111010	100100110110	6
010111	010010110100	5	111011	100101100011	6
011000	101110111011	9	111100	110011001100	6
011001	101111101110	9	111101	110010011001	6
011010	101100010001	5	111110	110001100110	6
011011	101101000100	5	111111	110000110011	6
011100	111011101011	9	000000	000000000000	0
011101	111010111110	9			
011110	111001000001	5			
011111	111000010100	5			
100000	001000100111	5			
100001	001001110010	5			
100010	001010001101	5			
100011	001011011000	5			
100100	011101110111	9			

FIG. 7

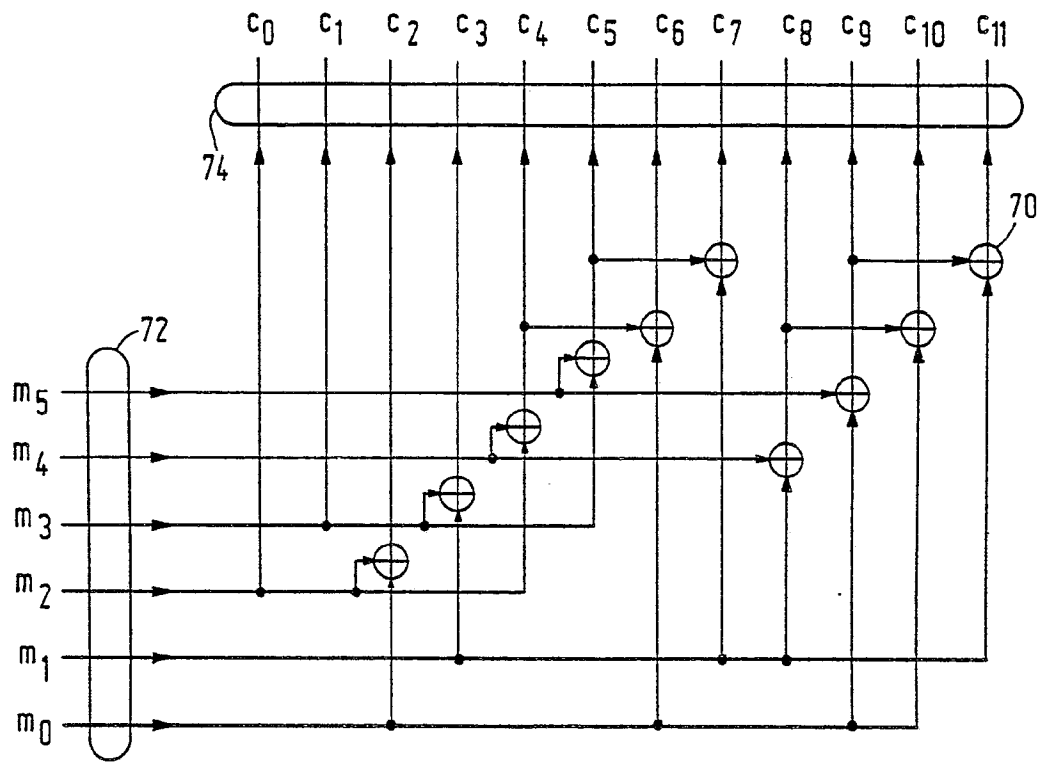


FIG. 8

$$[H] = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

FIG. 9

<u>1</u>	<u>2</u>	<u>3</u>		<u>1</u>	<u>2</u>	<u>3</u>
0	000000	no errors		32	100000	1, 3 ; 5, 7 ; 9, 11 ;
1	000001	4, 8 ; 6, 10 ;		33	100001	2 ;
2	000010	5, 9 ; 7, 11 ;		34	100010	5, 11 ; 7, 9 ;
3	000011	0, 1 ;		35	100011	0, 3 ;
4	000100	10 ;		36	100100	0, 4 ; 2, 6 ;
5	000101	6 ;		37	100101	0, 8 ; 2, 10 ;
6	000110	3, 8 ;		38	100110	1, 8 ;
7	000111	3, 4 ;		39	100111	1, 4 ;
8	001000	11 ;		40	101000	9 ;
9	001001	2, 9 ;		41	101001	2, 11 ;
10	001010	7 ;		42	101010	5 ;
11	001011	2, 5 ;		43	101011	2, 7 ;
12	001100	10, 11 ;		44	101100	9, 10 ;
13	001101	6, 11 ;		45	101101	6, 9 ;
14	001110	7, 10 ;		46	101110	5, 10 ;
15	001111	6, 7 ;		47	101111	5, 6 ;
16	010000	0, 2 ; 4, 6 ; 8, 10 ;		48	110000	0, 4, 8 ; 1, 5, 9 ; etc.
17	010001	4, 10 ; 6, 8 ;		49	110001	0 ;
18	010010	3 ;		50	110010	1 ;
19	010011	1, 2 ;		51	110011	2, 3 ;
20	010100	8 ;		52	110100	0, 6 ; 2, 4 ;
21	010101	4 ;		53	110101	0, 10 ; 2, 8 ;
22	010110	3, 10 ;		54	110110	1, 10 ;
23	010111	3, 6 ;		55	110111	1, 6 ;
24	011000	1, 5 ; 3, 7 ;		56	111000	1, 7 ; 3, 5 ;
25	011001	0, 9 ;		57	111001	0, 11 ;
26	011010	1, 9 ; 3, 11 ;		58	111010	1, 11 ; 3, 9 ;
27	011011	0, 5 ;		59	111011	0, 7 ;
28	011100	8, 11 ;		60	111100	8, 9 ;
29	011101	4, 11 ;		61	111101	4, 9 ;
30	011110	7, 8 ;		62	111110	5, 8 ;
31	011111	4, 7 ;		63	111111	4, 5 ;

FIG. 10

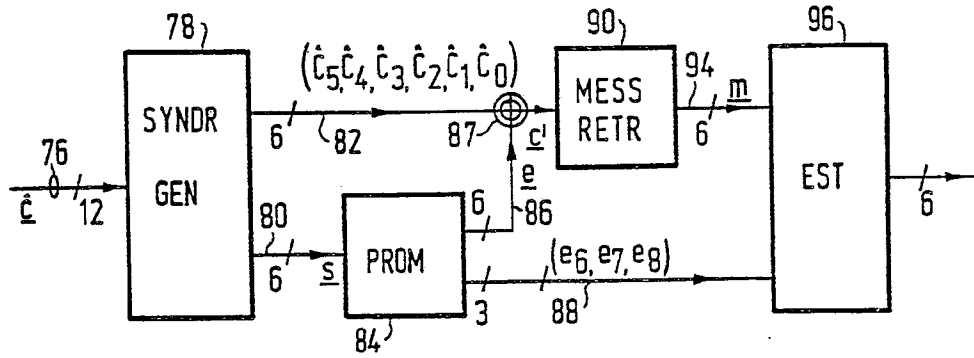


FIG.11

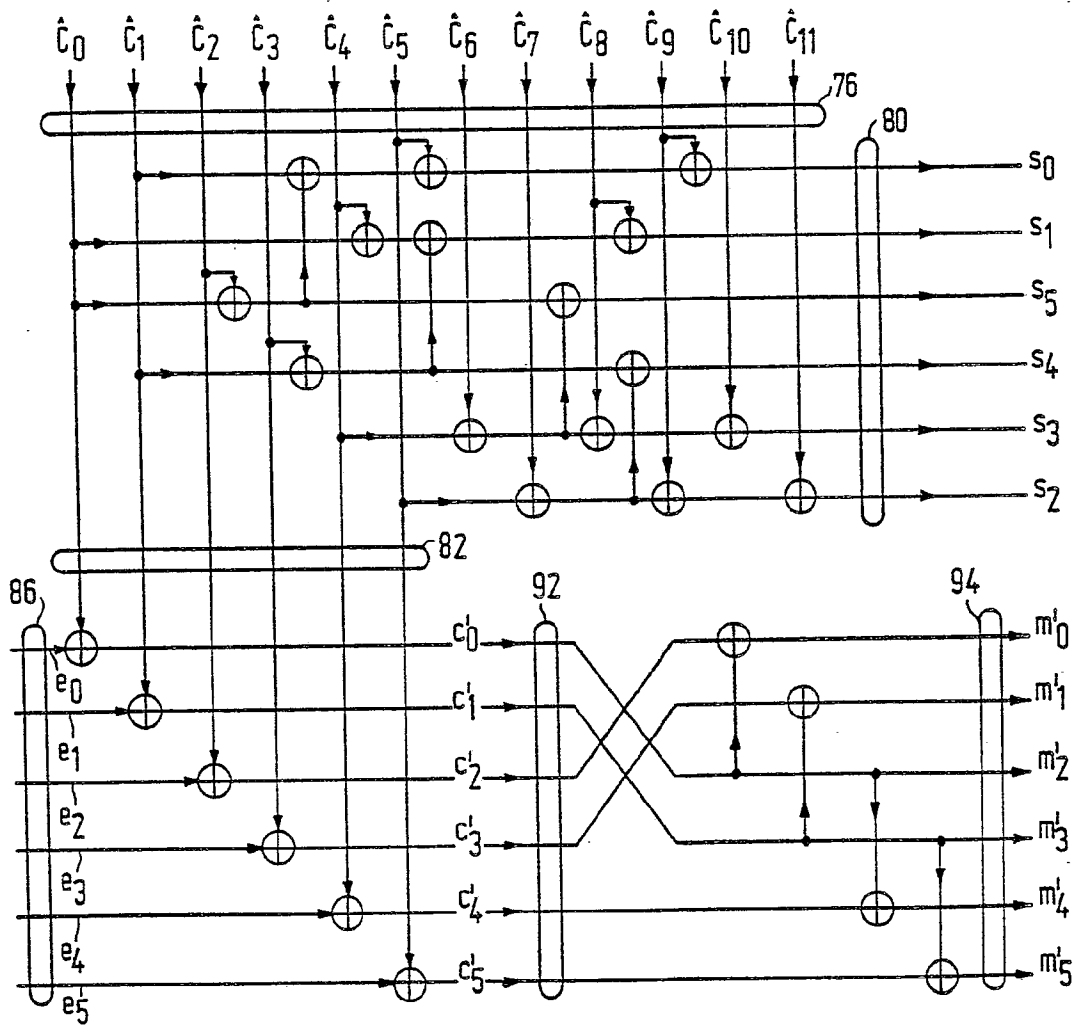
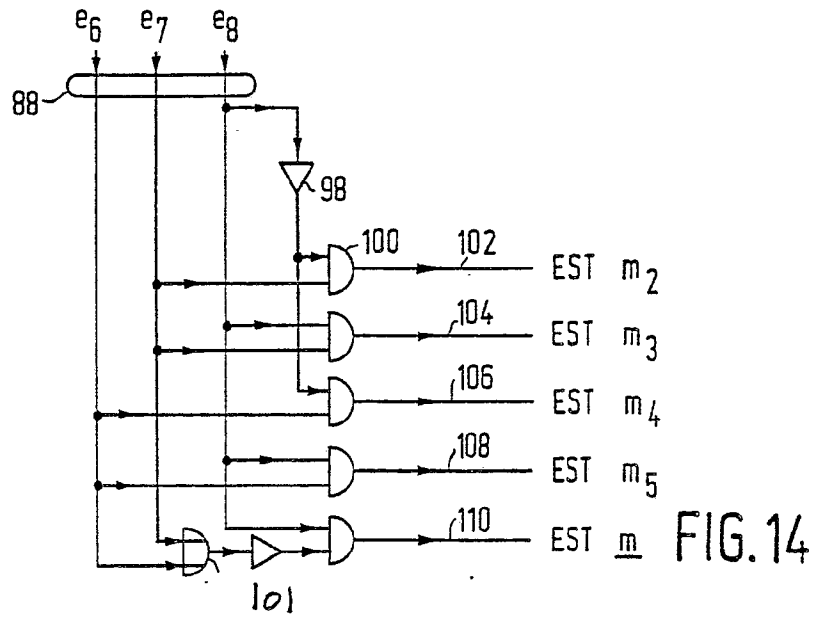


FIG.12

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
0	000000	000000	000	0	0	32	100000	000000	111	0	7
1	000001	000000	100	0	4	33	100001	001000	000	8	0
2	000010	000000	101	0	5	34	100010	000000	101	0	5
3	000011	110000	000	48	0	35	100011	100100	000	36	0
4	000100	000000	000	0	0	36	100100	001000	010	8	2
5	000101	000000	000	0	0	37	100101	100000	110	32	6
6	000110	000100	000	4	0	38	100110	010000	000	16	0
7	000111	000110	000	6	0	39	100111	010010	000	18	0
8	001000	000000	000	0	0	40	101000	000000	000	0	0
9	001001	001000	000	8	0	41	101001	001000	000	8	0
10	001010	000000	000	0	0	42	101010	000001	000	1	0
11	001011	001001	000	9	0	43	101011	001000	000	8	0
12	001100	000000	000	0	0	44	101100	000000	000	0	0
13	001101	000000	000	0	0	45	101101	000000	000	0	0
14	001110	000000	000	0	0	46	101110	000001	000	1	0
15	001111	000000	000	0	0	47	101111	000001	000	1	0
16	010000	000000	110	0	6	48	110000	000000	001	0	1
17	010001	000000	100	0	4	49	110001	100000	000	32	0
18	010010	000100	000	4	0	50	110010	010000	000	16	0
19	010011	011000	000	24	0	51	110011	001100	000	12	0
20	010100	000000	000	0	0	52	110100	100000	010	32	2
21	010101	000010	000	2	0	53	110101	100000	110	32	6
22	010110	000100	000	4	0	54	110110	010000	000	16	0
23	010111	000100	000	4	0	55	110111	010000	000	16	0
24	011000	000100	011	4	3	56	111000	010000	011	16	3
25	011001	100000	000	32	0	57	111001	100000	000	32	0
26	011010	010000	111	16	7	58	111010	010000	111	16	7
27	011011	100001	000	33	0	59	111011	100000	000	32	0
28	011100	000000	000	0	0	60	111100	000000	000	0	0
29	011101	000010	000	2	0	61	111101	000010	000	2	0
30	011110	000000	000	0	0	62	111110	000001	000	1	0
31	011111	000010	000	2	0	63	111111	000011	000	3	0

FIG. 13



$$[G] = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

FIG.16

7	2	54
8	2	64
8	3	544
9	2	74
9	3	554
9	4	5444
10	2	76
10	3	664
10	4	5544
10	5	54444
11	2	86,94
11	3	764
11	4	6644
11	5	55444
12	2	96
12	3	766
12	4	7644
12	5	55554
12	6	554444

FIG.15

$$\begin{array}{lll}
 C_0 = m_2 & C_4 = m_4 + m_2 & C_8 = m_4 + m_1 \\
 C_1 = m_3 & C_5 = m_5 + m_3 & C_9 = m_5 + m_0 \\
 C_2 = m_2 + m_0 & C_6 = m_4 + m_2 + m_0 & C_{10} = m_4 + m_1 + m_0 \\
 C_3 = m_3 + m_1 & C_7 = m_5 + m_3 + m_1 & C_{11} = m_5 + m_1 + m_0
 \end{array}$$

$$\begin{array}{l}
 m_0: \quad 1) \hat{C}_0 + \hat{C}_2 \\
 \quad \quad 2) \hat{C}_4 + \hat{C}_6 \\
 \quad \quad 3) \hat{C}_8 + \hat{C}_{10} \\
 \quad \quad 4) \hat{C}_1 + \hat{C}_5 + \hat{C}_9 \\
 \quad \quad 5) \text{ if } \hat{C}_1 + \hat{C}_5 + \hat{C}_9 = \hat{C}_1 + \hat{C}_7 + \hat{C}_{11} = \hat{C}_3 + \hat{C}_5 + \hat{C}_{11}, \\
 \quad \quad \quad \text{then } \hat{C}_1 + \hat{C}_5 + \hat{C}_9 \text{ else } 1 + \hat{C}_1 + \hat{C}_5 + \hat{C}_9.
 \end{array}$$

$$\begin{array}{l}
 m_1: \quad 1) \hat{C}_1 + \hat{C}_3 \\
 \quad \quad 2) \hat{C}_5 + \hat{C}_7 \\
 \quad \quad 3) \hat{C}_9 + \hat{C}_{11} \\
 \quad \quad 4) \hat{C}_0 + \hat{C}_4 + C_8 \\
 \quad \quad 5) \text{ if } \hat{C}_0 + \hat{C}_4 + \hat{C}_8 = \hat{C}_0 + \hat{C}_6 + \hat{C}_{10} = \hat{C}_2 + \hat{C}_4 + \hat{C}_{10}, \\
 \quad \quad \quad \text{then } \hat{C}_0 + \hat{C}_4 + \hat{C}_8 \text{ else } 1 + \hat{C}_0 + \hat{C}_4 + \hat{C}_8.
 \end{array}$$

$$\begin{array}{ll}
 m_2: \quad 1) \hat{C}_0 & m_3: \quad 1) \hat{C}_1 \\
 \quad \quad 2) \hat{C}_2 + m_0 & \quad \quad 2) \hat{C}_3 + m_1 \\
 \quad \quad 3) \hat{C}_4 + \hat{C}_8 + m_1 & \quad \quad 3) \hat{C}_5 + \hat{C}_9 + m_0 \\
 \quad \quad 4) \hat{C}_6 + \hat{C}_{10} + m_1 & \quad \quad 4) \hat{C}_7 + \hat{C}_{11} + m_0
 \end{array}$$

$$\begin{array}{ll}
 m_4: \quad 1) \hat{C}_0 + \hat{C}_4 & m_5: \quad 1) \hat{C}_1 + \hat{C}_5 \\
 \quad \quad 2) \hat{C}_2 + \hat{C}_6 & \quad \quad 2) \hat{C}_3 + \hat{C}_7 \\
 \quad \quad 3) \hat{C}_8 + m_1 & \quad \quad 3) \hat{C}_9 + m_0 \\
 \quad \quad 4) \hat{C}_{10} + m_1 + m_0 & \quad \quad 4) \hat{C}_{11} + m_0 + m_1
 \end{array}$$

FIG.17