



# Cutting Efficiency of a Single PDC Cutter on Hard Rock

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## Abstract

Polycrystalline diamond compact (PDC) bits have gained wide popularity in the petroleum industry for drilling soft and moderate-strength formations. However, in hard-formation applications the PDC bit still has limitations even though continuing developments in PDC cutter designs and materials have steadily improved drilling performance. Resolution of the limitations of PDC bits for drilling hard formations will contribute significantly to the price competitiveness of (i) oil and gas recovered from deep, hot, hard-rock formations, and (ii) electricity generated from enhanced geothermal energy reservoirs.

In this paper the cutting efficiency has been analyzed, based on a force model for a single PDC cutter. The cutting efficiency of a single PDC cutter is defined as the ratio of the rock volume removed by a cutter to the force required to remove that volume

of rock. The cutting efficiency is found to be a function of the back-rake angle, depth of cut, and rock properties (e.g., the angle of internal friction).

The highest cutting efficiency is found to occur at specific cutter back-rake angles, which depend on the material properties of the rock. In particular, the cutting efficiency is directly related to the internal angle of friction of the rock being cut.

The results of this analysis can be applied to each PDC cutter on a given bit, then the contributions of the individual cutters can be integrated to model the overall bit performance. Conversely, this analysis can serve as a guideline for developing new PDC bit designs that are optimized for specific rock formations.

## Introduction

PDC bits have gained wide popularity in petroleum and gas drilling due to their long bit life and their ability to maintain a high rate of penetration (ROP). The shearing action induced by fixed drag cutters has proven to be more efficient for penetrating rock than the crushing effect of the teeth or inserts on the rolling cones of a roller bit<sup>1,2,3,4</sup>. However PDC bits have traditionally had limitations when encountering hard formations<sup>5</sup>; hence, they are not yet preferred for hard-rock mining, petroleum/gas, or geothermal energy applications. Enhanced geothermal energy (i.e., geothermal energy recovered from large, +3km, depths) has recently been identified by a MIT-led multidisciplinary expert panel as one of the most promising energy sources in the US<sup>6</sup>. The panel study shows that enhanced geothermal energy has the potential, even for conservative resource estimates, to satisfy the entire US demand for electricity. In order to obtain competitive electricity prices from enhanced geothermal energy, technological advancements are required that will allow cost-effective drilling in hard formations.

The objective of this paper is to develop an analytical model for characterizing the cutting efficiency of PDC bits in hard-rock formations, thereby enabling improvements in the design of future PDC bits.

## PDC Bit

Most PDC bits are composed of a hard matrix body, which is milled out of a solid block of steel or cast from sintered tungsten carbide. The matrix body features blades where the actual PDC cutters are mounted, and open areas, or slots, where the cuttings and mud flow can escape to the annulus. Figure 1 shows a typical 8 1/2" diameter PDC bit from one of the leading PDC bit manufacturers<sup>7</sup>. In Figure 1 the PDC cutters are placed on the gold colored blades. The flow pathways for mud and cuttings are colored blue. Each cutter is fixed on the blade, and rock is removed when the cutters are dragged in a circular path as the bit is rotated at the bottom of the hole (Figure 2). Figure 2 shows a sketch of the circular path for a single cutter while the bit rotates. In Figure 3 a sketch in the vertical plane of a single cutter is shown. The cutter penetrates the rock based on the point load on each cutter produced by the applied weight on bit (WOB). The cutter is tilted with a back-rake angle,  $\phi$ , with respect to the rock. The effectiveness of a PDC bit for removing rock is dependent on several factors. Increased rock hardness reduces ROP. Increased WOB and RPM will increase the ROP if cuttings are removed efficiently<sup>4</sup>. The number of cutters, their back-rake angle, and other design features of the bit will also affect ROP<sup>4</sup>. Lastly, the PDC cutter material and cutter design affect ROP throughout the life of a bit. For instance, cutters fabricated with a fine (10  $\mu$ m) diamond grain size can provide higher ROP and sustain less abrasion damage than cutters with coarse (70  $\mu$ m) diamond grains<sup>7</sup>. Further, rounding the cutter edges and increase in sintering pressures under cutter production make them more thermally stable and gives dramatically improved bit performance when drilling in hard formations<sup>7</sup>.

Of all the different factors mentioned above that can influence PDC bit life and ROP in hard-rock applications, this paper focuses on identifying the effects of cutter orientation and consequently optimizing the orientation to achieve improved performance.

## Cutting Efficiency

To begin the analysis, the cutting efficiency can be related to the volume of rock removed by the cutter and the force exerted to achieve that removal action. The term "specific volume" is introduced here to describe the cutting efficiency; this parameter is defined as:

Specific Volume =

$$\left( \frac{\text{Volume of rock removed in one major chip}}{\text{Maximum force required to remove that volume of rock}} \right)$$

## Volume of Rock Removed By the Cutter in One Chip

The first requirement needed for analyzing the cutting efficiency is the volume of rock removed by the cutter in one major cycle (thus, the major cutting chip). The cutting shape is defined by two curves shown in a two-dimensional view by Figure 4, where

$$\begin{aligned} y_a &= k_a (x - x_0)^2 + y_0 \\ y_b &= k_b x \end{aligned} \quad \dots\dots\dots (1)$$

Here,  $y_a$  describes the failure surface in the rock and  $y_b$  represents the rock surface of the wellbore.

The PDC cutter breaks the rock, and thereby controls the shape of curve  $y_a$ , by developing a crack that separates a fragment, or cutting chip, from the rock matrix. The process of forming a cutting chip can be divided into two phases: crack initiation and crack propagation, as illustrated by Figure 4. The position of the crack initiation is determined primarily by the stress magnitude, which will be infinite on the cutter tip if we assume a homogeneous, linear-elastic rock material and a rigid PDC cutter body. Linear elasticity is an acceptable assumption for hard rocks<sup>8,9</sup>. Since the compressive strength of the rock is orders of magnitude higher than the tensile strength, the crack is initiated by tension. The direction of the initial crack at the cutter tip is  $\psi_0$  (crack initial angle). Once the crack is initiated, it will continue to propagate as a shear fracture until the chip is formed. The crack angle ( $\psi$ ) will change since the failure stress state is changing from tensile to maximum shear failure. The coefficients in Equation 1 are derived in reference<sup>9</sup> and are given as

$$\begin{aligned} x_0 &= \frac{h_c}{2 \tan \alpha} \\ y_0 &= -\frac{h_c}{2} \\ k_a &= \frac{0.25}{h_c} (1 + c_\psi \sin \psi_0) \\ k_b &= \tan \alpha \end{aligned} \quad \dots\dots\dots (2)$$

The shaded area in Figure 4 is

$$ds = (y_b - y_a) dx \quad (3)$$

The whole cross sectional area of the cutter can be obtained by integrating Equation 3 from  $x_0$  to  $x_1$

$$S = \int_{x_0}^{x_1} (y_b - y_a) dx \quad (4)$$

Carrying out the integration, notice that

$$x_1 = 1.25 \frac{e^{2\alpha}}{2 \tan \alpha} (1 - \sin \psi_0) h_c \quad (5)$$

and rearrange to get the area of the cutter shown in Figure 4 as

$$S = c_1 h_c + c_2 h_c^2 \quad (6)$$

where

$$c_1 = 0.0833 \left( 1 + c_\psi \sin \psi_0 \right) \left( \frac{a_0 - b_0 \sin \psi_0}{2 \tan \alpha} \right)^2$$

$$c_2 = -\frac{1}{4 \tan \alpha} \left\{ a_0 - b_0 \sin \psi_0 + [b_0 (1 - \sin \psi_0)]^2 - 1 \right\} \quad (7)$$

and

$$a_0 = 1.25 e^{2\alpha} - 1$$

$$b_0 = 1.25 e^{2\alpha} \quad (8)$$

To calculate the cutting-chip volume, the two-dimensional x, y cutting chip area  $S$  has to be multiplied with the width of the cuttings in the horizontal, z-direction given in Figure 5. The cutting chip volume is then given, approximately, by

$$V = w_e S \quad (9)$$

where  $w_e$  is the equivalent width. The actual integration to obtain the entire 3-dimensional surface over the shear plane is too complex to be performed by any analytical method, so some simplification is needed. Since the thickness of the chip is not large (small depth of cut), the shear surface can be treated as a plane without losing too much precision. The plane is further projected onto the rock surface to get a half ellipse with the axis of  $a_0$  and  $b_0$  as shown on Figure 5. The equivalent width  $w_e$  can then be calculated as the area of the ellipse, which is given as the numerator in Equation 10, divided by the half length (see Figure 5).

$$w_e = \frac{\frac{\pi}{8} a_0 b_0}{a_0 / 2} \quad (10)$$

Since  $w_e$  is then only related to  $b_0$ , and  $b_0$  is directly related to the depth of cut by<sup>3</sup>

$$b_0 = 2 \sqrt{d_c h_c - h_c^2} \quad (11)$$

the equivalent width can then be calculated as

$$w_e = 1.57 \sqrt{d_c h_c - h_c^2} \quad (12)$$

Now the full solution can be implemented to calculate the volume  $V$  based on Equation 9.

## Specific Volume

A parameter, namely specific volume, is introduced to measure the efficiency of cutting. The specific volume,  $V_0$ , is defined as the volume of rock removed by the cutter divided by the resultant force on the cutter

$$V_0 = \frac{V}{F} \quad (13)$$

where  $F$  is the resultant force of the horizontal and vertical forces acting on the cutter

$$F = \sqrt{P_h^2 + P_v^2 + P_s^2} \quad (14)$$

and  $V$  is the volume of rock removed in one major chip and is given by Equation 9.

Equation 13 provides then a measure of the cutting efficiency in terms of specific volume. The highest efficiency occurs at the maximum specific volume.

Figures 6 through 8 show the specific volume as a function of back-rake angle and depth of cut, respectively. As stated previously, the specific volume represents the cutting efficiency to a certain extent. Two very important conclusions can be drawn from these plots. These are:

1) The specific volume reaches its local maximum at  $0^\circ$  and  $25^\circ$  back-rake angle. This indicates that the best settings for back-rake angle are either at  $0^\circ$  or at  $25^\circ$ . The specific volume at  $0^\circ$  back-rake angle is not accurate because the correlation between the crack initial angle and the load angle is not accurate at this point. However, by comparing Figure 6 and Figure 7 (which is plotted using the actual crack initial angle based on maximum shear stresses<sup>9</sup>), it is certain the error at this point is not large. In fact, Figure 6 gives the same conclusions as Figure 7.

Also, the effect of back-rake angle on the specific volume becomes less significant as the depth of cut decreases. However, even when the depth of cut is less than  $0.02 \text{ in}$ , there is still a noticeable effect produced by the back-rake angle, especially at  $0^\circ$ . This implies that the back-rake angle of a PDC cutter becomes more important at larger depth of cut.

2) The specific volume versus depth of cut did not give any local maximum within the range of  $0.01 \sim 0.08 \text{ in}$ , except at  $0.08 \text{ in}$ . This indicates that no optimum depth of cut could be obtained. From the plot, it is clear that the larger the depth of cut, the more efficient the cutting. However, as the depth of cut increases, the incremental increase in specific volume drops. This suggests that a high rate of penetration for a PDC bit is

beneficial only in terms of cutting efficiency. In practice, the depth of cut is limited because of equipment capacity. A depth of cut around 0.04 *in.* is optimum.

## Rock Properties in the Model

There are two basic parameters in the linear-elastic derived force model<sup>9</sup>: the half wedge angle,  $\alpha$ , and the ratio of the normal stress over shear stress on the failure curve,  $k_0$ . These parameters were used in the model as constants. According to the laboratory observations, the half-wedge angle takes the value of 25°.  $k_0$  takes the value of 1.5 based on the stress analysis. However, these two parameters should be functions of rock properties such as internal friction angle and grain size. In fact, the ratio of normal stress over shear stress on the failure curve can be approximated as

$$k_0 = \frac{\sigma_s}{\tau_s} \dots\dots\dots (13)$$

and the wedge angle is a function of internal friction angle also.

These two parameters will change with changes in rock type since the internal friction angle varies according to the rock type. The changes in these two parameters may affect the cutting efficiency. In order to examine the effect of a rock property, such as internal friction angle, on the specific volume, two more calculations were made and the results appear in Figures 9 through 12.

These plots show some very interesting facts, as expected. First, at  $\alpha = 20^\circ$ , there are two maxima of specific volume at back-rake angles of 0° and 20°, with the 20° being significant. This indicates that the optimum back-rake angles are 0° and 20° instead of the 0° and 25° values noted in the case of  $\alpha = 25^\circ$ . At  $\alpha = 30^\circ$ , the optimum back rake changed to 0° and 30°, though the optimum (i.e., local maximum) value is not pronounced for this value of the half wedge angle.

Second, there appears a local maximum value of the specific volume versus depth of cut. At low back-rake angles, this optimum depth of cut is about 0.06 *in.*, whereas at high back-rake angles the optimum depth of cut is lower (about 0.05 *in.*). This means that if the wedge angle is about 40°, the optimum depth of cut should be addressed.

Further investigation is needed to fully understand the relationship between the wedge angle and the angle of internal friction.

## Conclusions

The specific volume, as defined in this paper, reaches its maximum at back-rake angles of 0° and 25°. This suggests that back-rake angles of both 0° and 25° are best settings from the viewpoint of cutting efficiency for the hard rock being analyzed.

The specific volume has no local maximum when plotted against depth of cut. However, increasing specific volume observed for increasing depth of cut indicates that a higher cutting efficiency is obtained at larger depths of cut.

The analysis of the cutting efficiency of a single cutter gives two very important suggestions:

Cutter back rake is, for the specific properties of the tested rock, best at 0° and 25°. Since a 0° back-rake angle is more efficient than 25°, this suggests that a 0° to 5° back-rake angle is best from the standpoint of cutting efficiency.

A larger depth of cut yields a higher cutting efficiency. However, too large a depth of cut will cause problems with the drilling equipment. A range of 0.04 to 0.06 *in.* is suggested.

## Acknowledgement

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## NOMENCLATURE

$P_v$	=	vertical force on a single PDC cutter
$P_h$	=	drag force on a single PDC cutter
$P_s$	=	side force on a single PDC cutter
$d_c$	=	diameter of the PDC cutter
$h_c$	=	depth of cut
$\phi$	=	back rake angle
$\varphi$	=	side rake angle
$\alpha$	=	half wedge angle
$\psi_0$	=	crack initial angle
$\sigma_s$	=	normal stress on the maximum shear stress trajectory
$\tau_s$	=	shear stress on the maximum shear stress trajectory
$F$	=	resultant force on a PDC cutter
$V$	=	volume removed by a PDC cutter in a major chip
$S$	=	cross sectional area of the chip
$V_0$	=	specific volume

## REFERENCES

1. WISE J. L., GROSSMAN, J. W., WRIGHT, E. K., GRONEWALD, P. J., BERTAGNOLLI, K., AND COOLEY, C. H., Latest Results of Parameter Studies on PDC Drag Cutters for Hard-Rock Drilling; *GRC Transactions*, Vol. 29, 2005.
2. SWENSON, D. V., WESENBERG, D. L., AND JONES, A. K., Analytical and Experimental Investigations of Rock Cutting Using Polycrystalline Diamond Compact Drag Cutters; *SPE 10150*, presented at the 56th Annual Fall Technical Conference and Exhibition of the Society of Petroleum Engineers, San Antonio, TX, Oct. 5-7, 1981.
3. WARREN, T. M., AND ARMAGOST, W. K., Laboratory Drilling Performance of PDC Bits; *SPE 15617*, presented at the 61st Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, New Orleans, LA, Oct 5-8, 1986.
4. ZEUCH, D. H., AND FINGER, J. T., Rock Breakage Mechanisms with a PDC Cutter; *SPE 14219*, presented at the 60th Annual Technical Conference and Exhibition of the Society of Petroleum Engineers, Las Vegas, NV, Sept. 22-25, 1985.
5. BRETT, J. F., WARREN, T. M., AND BEHR, S.M., Bit Whirl: A New Theory of PDC Bit Failure; *SPE 19571*, presented at the 64th SPE Annual Technical Conference and Exhibition, San Antonio, TX, Oct. 8-11, 1989.

6. TESTER, W. J., et al., The Future of Geothermal Energy; *an assessment by an MIT-led interdisciplinary panel, MIT, 2006.*
7. WISE, J. L., Geometry and Material Choices Govern Hard-Rock Drilling Performance of PDC Drag Cutters; *ARMA/USRMS 05-883, presented at The 40th U.S. Symposium on Rock Mechanics, Anchorage, AK, June 25-29, 2005.*
8. NYGAARD, R., AND HARELAND, G., Calculating Unconfined Rock Strength from Drilling Data; *1<sup>st</sup> Canada-U.S. Rock Mechanics Symposium, Vancouver, British Columbia, Canada, May 27-31, 2007.*
9. YAN, W., Single PDC Cutter Force Modeling for Hard Rock Cutting; *Ph.D. Dissertation, New Mexico Tech, May 1997.*

## FIGURES AND TABLES



Figure 1. ReedHycalog™ 8-1/2 inch diameter DSX148 drag PDC bit<sup>7</sup>.

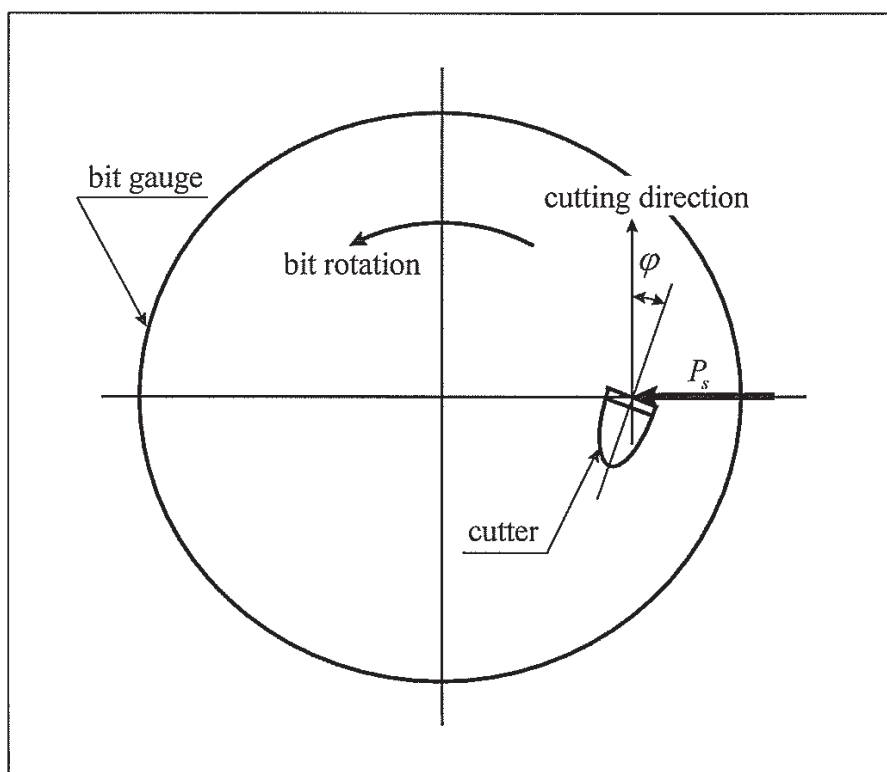


Figure 2. Sketch showing the cutting path, cutting direction, and side force on the cutter when bit rotates.

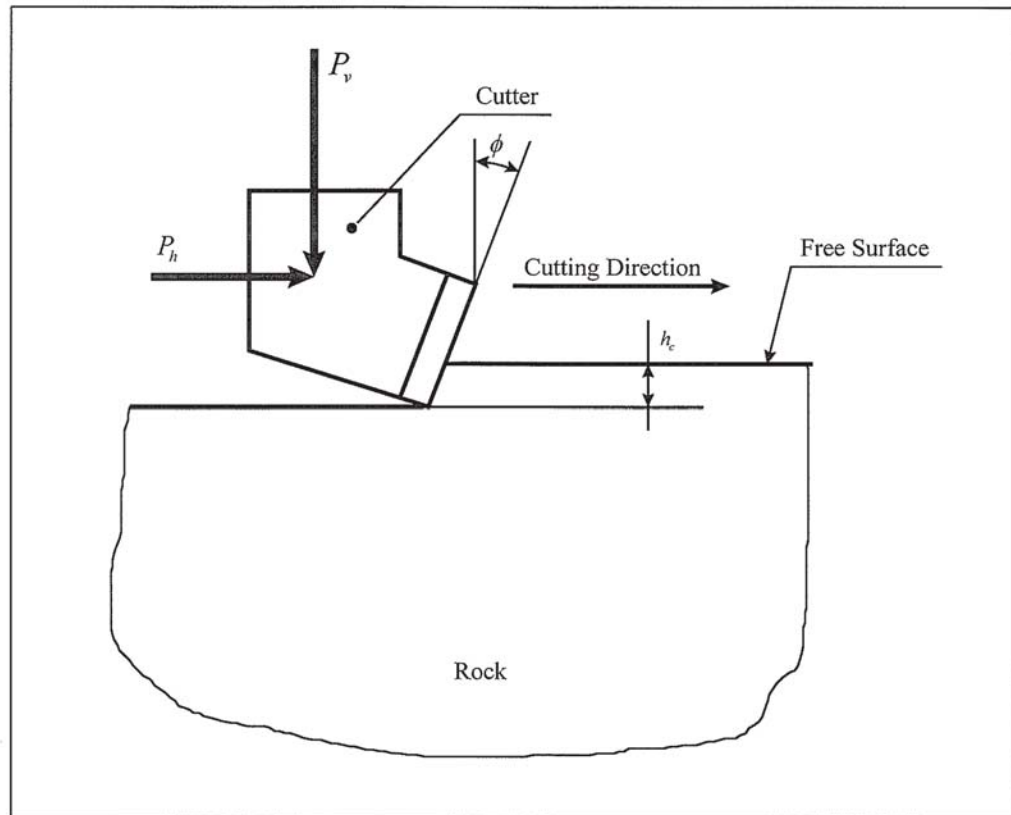


Figure 3. Sketch showing the PDC cutter and associated vertical and horizontal force components.

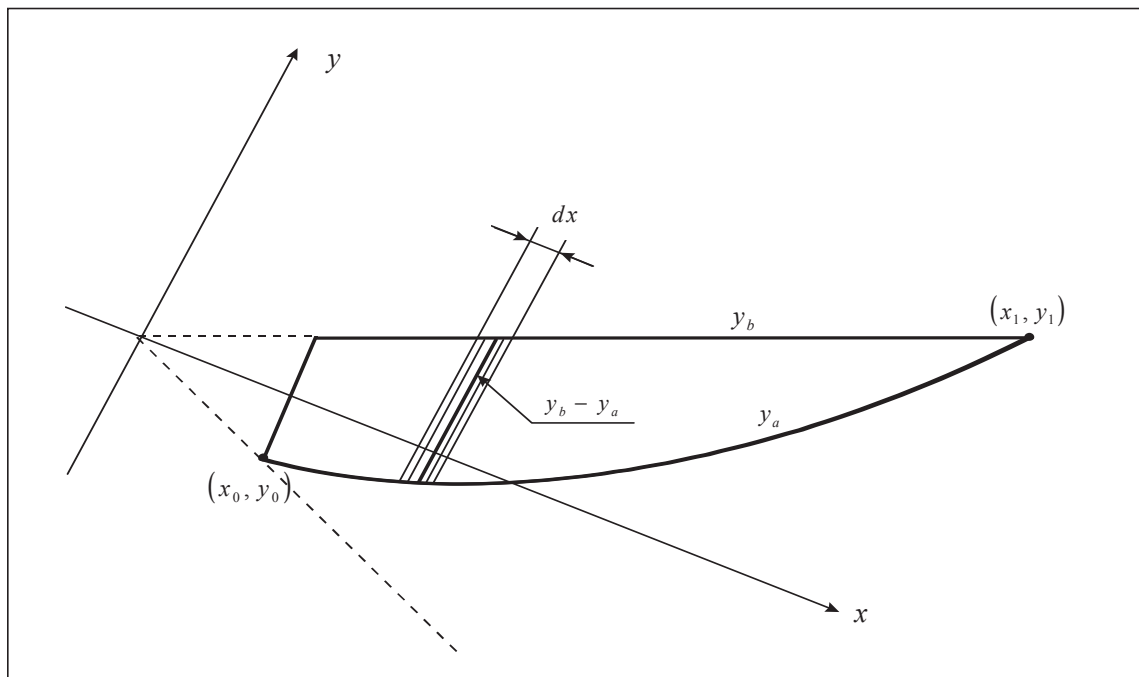


Figure 4. Sketch showing the chip dimension.

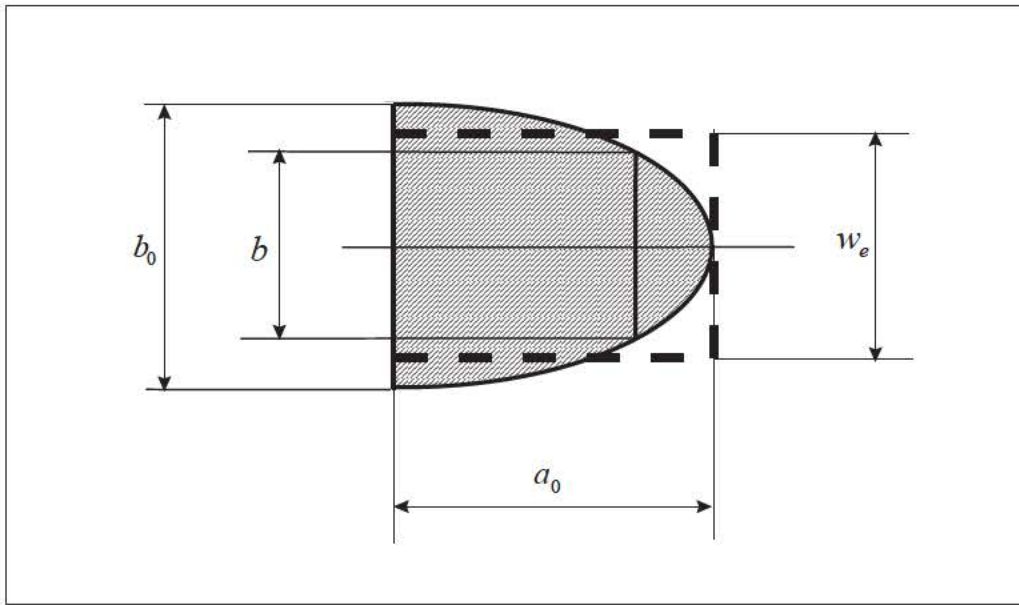


Figure 5. Sketch showing the simplified shear plane for calculating  $w_e$ .

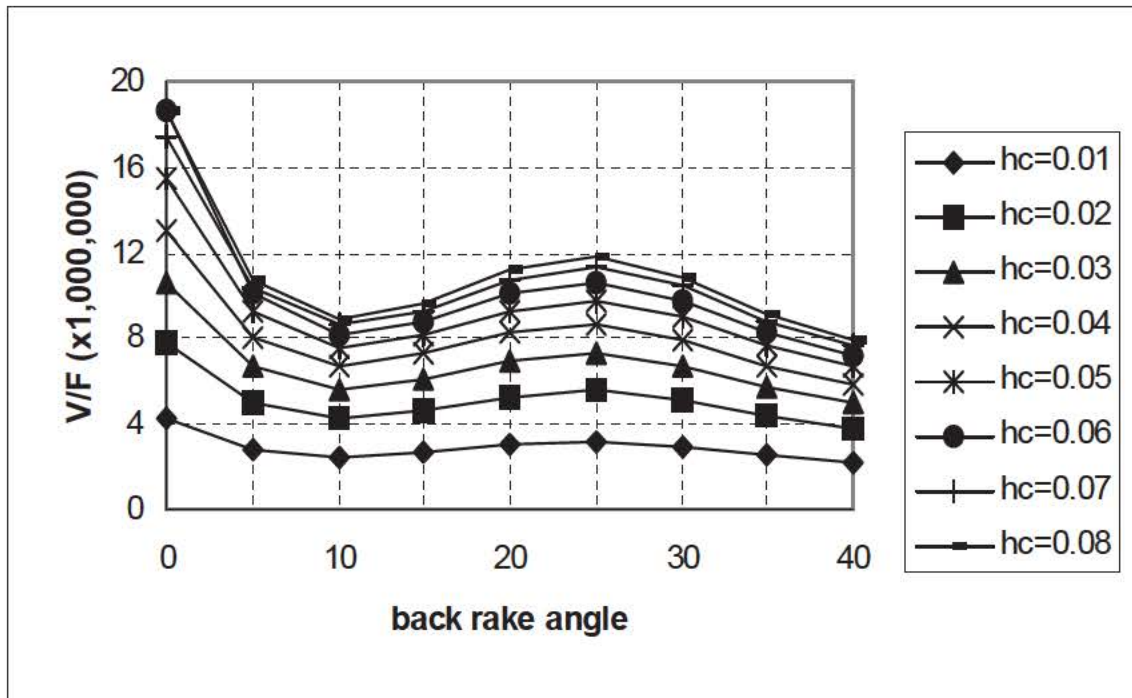


Figure 6. Specific volume versus back-rake angle,  $\psi_0$  calculated based on Equation 11.  $H_c$  is rock cuttings thickness.

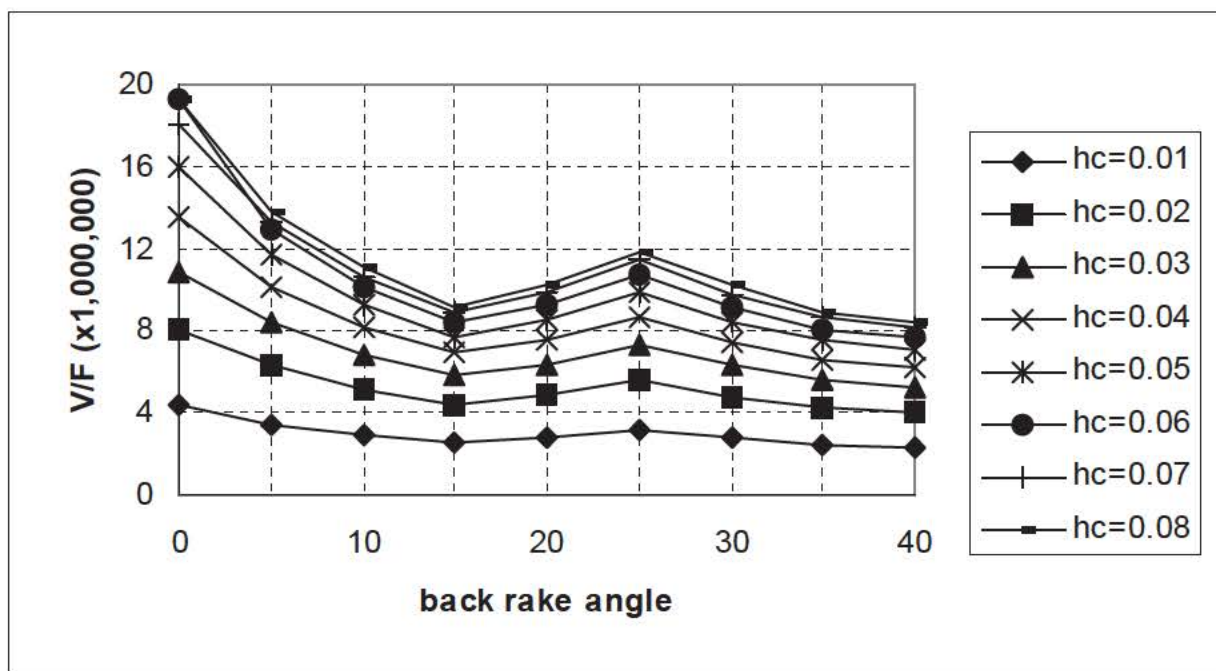


Figure 7. Specific volume versus back-rake angle.  $\psi_0$  calculated from orientation of maximum shear stresses<sup>9</sup>.

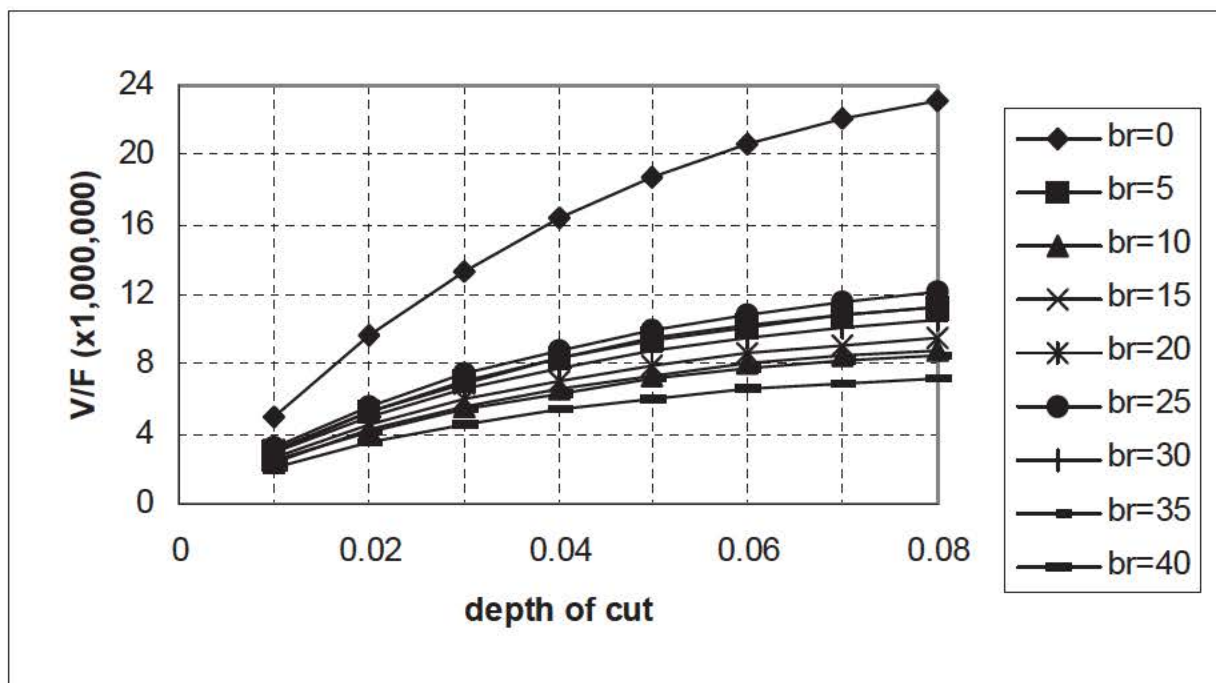


Figure 8. Specific volume versus depth of cut.

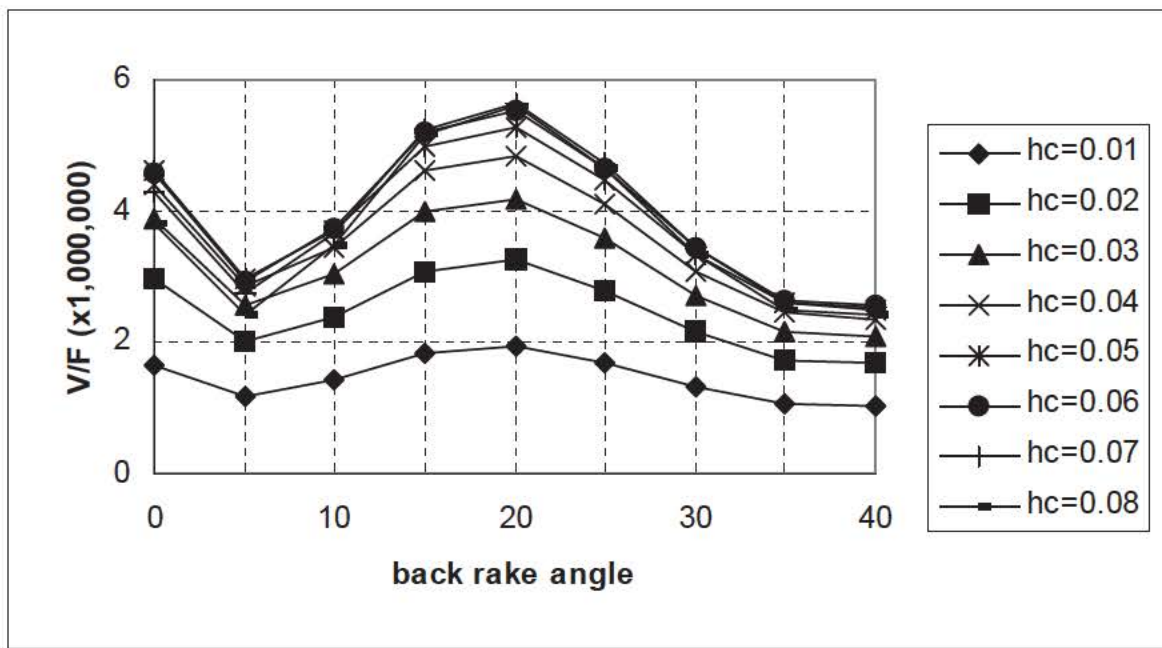


Figure 9. Specific volume versus back-rake angle ( $\alpha = 20^\circ$ ,  $k_0 = 2.14$ ).

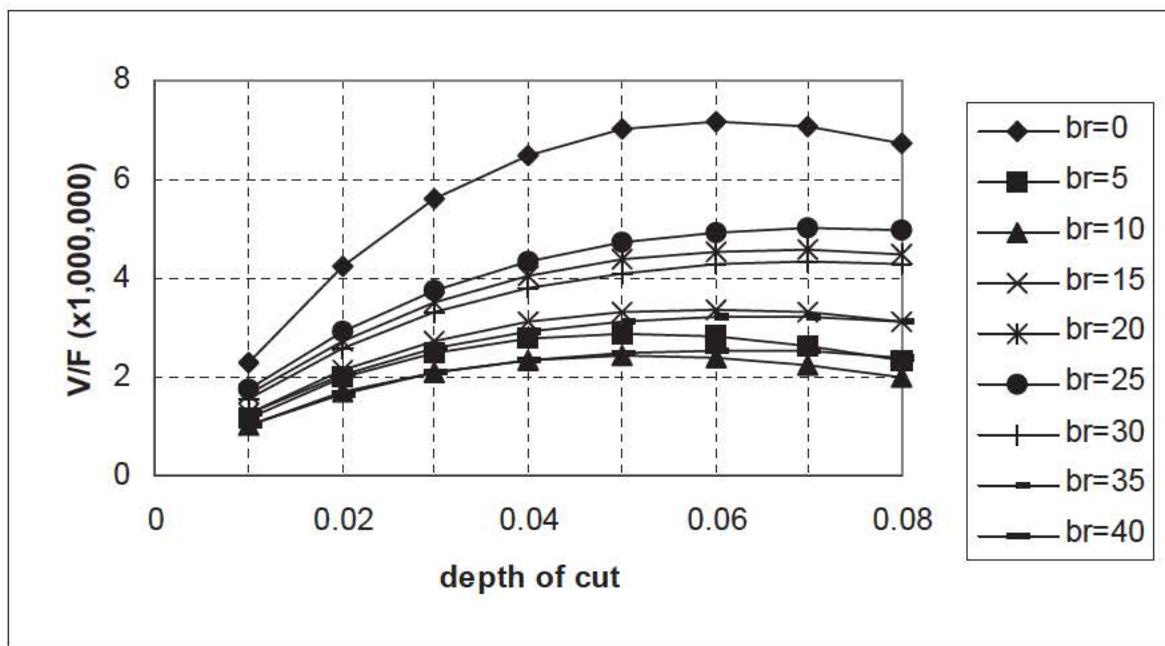


Figure 10. Specific volume versus depth of cut ( $\alpha = 20^\circ$ ,  $k_0 = 2.14$ ).

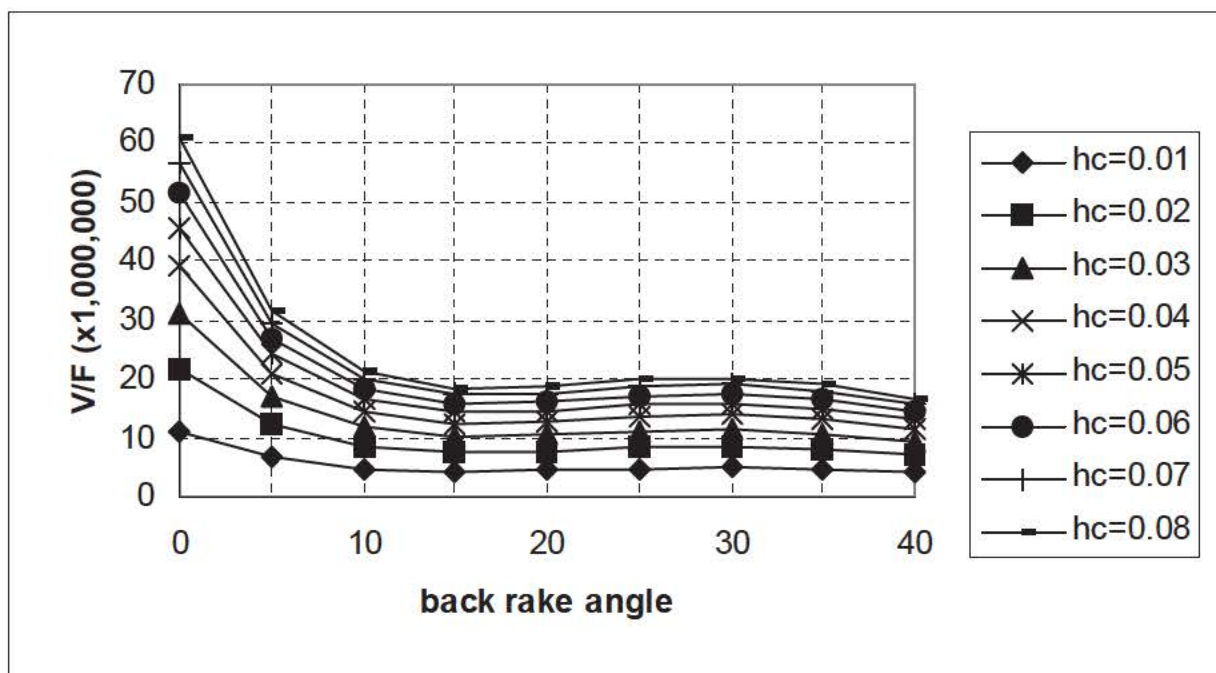


Figure 11. Specific volume versus back-rake angle ( $\alpha = 30^\circ$ ,  $k_0 = 3.73$ ).

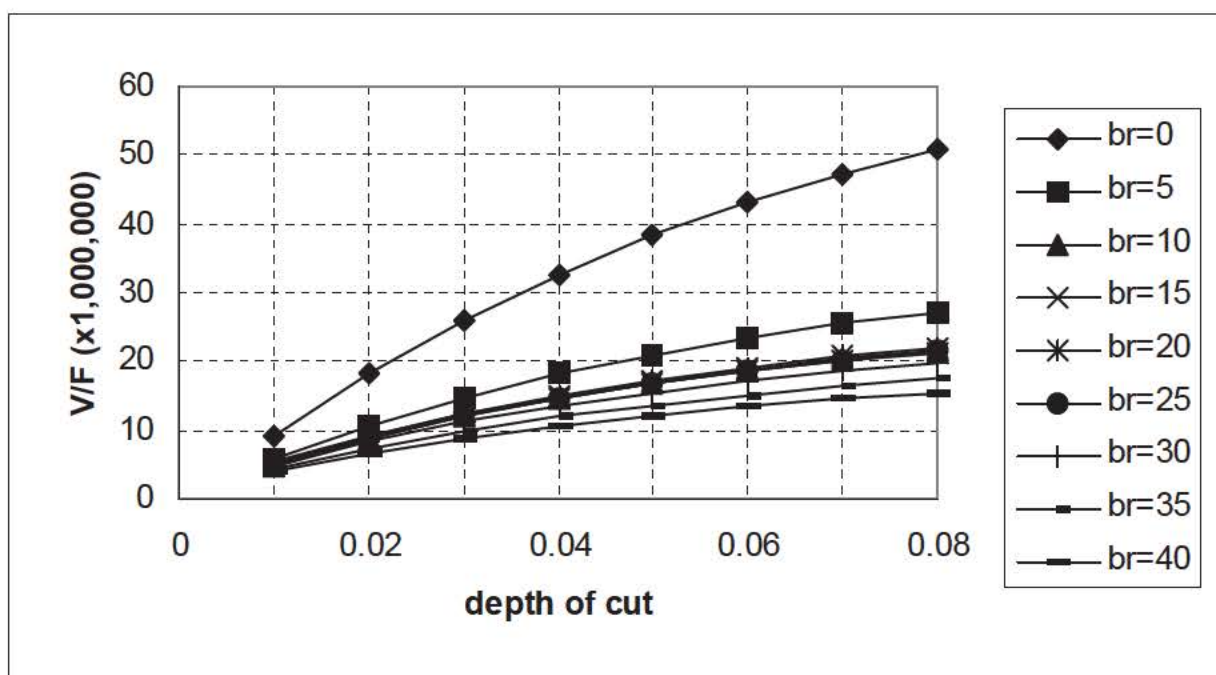


Figure 12. Specific volume versus depth of cut ( $\alpha = 30^\circ$ ,  $k_0 = 3.73$ ).