

Evaluation of PDC bit ROP models and the effect of rock strength on model coefficients



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ABSTRACT

Current low oil price conditions have renewed the emphasis on drilling optimization, in order to save time drilling oil and gas wells and reduce operational costs. Rate of penetration (ROP) modeling is a key tool in optimizing drilling parameters, namely bit weight and rotary speed, for faster drilling processes. With a novel, all automated data visualization and ROP modeling tool developed in Excel VBA, ROP Plotter, this work investigates model performance and the impact of rock strength on model coefficients of two different PDC bit ROP models: Hareland and Rampersad (1994) and Motahhari *et al.* (2010). These two PDC bit models are compared against a base case, general ROP relation developed by Bingham (1964) in three different sandstone formations in the vertical section of a Bakken shale horizontal well. For the first time, an attempt has been made to isolate the effect of varying rock strength on ROP model coefficients by investigating lithologies with otherwise similar drilling parameters. Additionally, a comprehensive discussion on the importance of selecting appropriate model coefficients bounds is conducted. Rock strength, accounted for in Hareland's and Motahhari's models but not in Bingham's, results in higher values of constant multiplier model coefficients for the former models, in addition to an increased RPM term exponent for Motahhari's model. Hareland and Rampersad's model is shown to perform best out of the three models with this particular dataset. The effectiveness and applicability of traditional ROP modeling is brought to question, as such models rely on a set of empirical coefficients that incorporate the effect of many drilling factors not accounted for in the model's formulation and are unique to a particular lithology.

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1. Introduction

PDC (Polycrystalline Diamond Compact) bits are the dominant bit type utilized in drilling oil and gas wells today. Bit performance is typically measured by the rate of penetration (ROP), an indication of how fast the well is drilled in terms of length of hole drilled per unit time. Drilling optimization has been at the forefront of energy companies' agendas for decades now, and it gains further importance during the current low oil price environment (Hareland and Rampersad, 1994). The first step in optimizing drilling parameters to produce the best possible ROP is the development of an accurate model relating measurements obtained at the surface to drilling rate.

Several ROP models, including models developed specifically for

a certain bit type, have been published in the literature. These ROP models typically contain a number of empirical coefficients that are lithology dependent and may impair comprehension of the relationship between drilling parameters and rate of penetration. The purpose of this study is to analyze model performance and how model coefficients respond to field data with varying drilling parameters, in particular rock strength, for two PDC bit models (Hareland and Rampersad, 1994; Motahhari *et al.*, 2010). Model coefficients and performance are also compared against a base case ROP model (Bingham, 1964), a simplistic relation that served as the first ROP model widely applied throughout industry and still currently in use. Drilling field data in three sandstone formations with varying rock strengths is investigated, and model coefficients for these three models are computed and compared against one another. It is postulated that coefficients for Hareland's and Motahhari's models in each rock formation will span a wider range than Bingham's model coefficients, as varying rock strength is not accounted for explicitly in the latter formulation. Model

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performance is also evaluated, leading to the choice of the best ROP model for the Bakken shale region in North Dakota.

The ROP models included in this work consist of inflexible equations that relate a few drilling parameters to drilling rate and contain a set of empirical coefficients which combine the influence of hard to model drilling mechanisms, such as hydraulics, cutter rock interaction, bit design, bottom hole assembly characteristics, mud type, and hole cleaning. Although these traditional ROP models generally do not perform well when compared against field data, they provide an important stepping stone to newer modeling techniques. Modern, more powerful, statistics based models with increased flexibility can improve the accuracy of ROP modeling. Gandelman (2012) has reported significant enhancement in ROP modeling by employing artificial neural networks instead of traditional ROP models in oil wells in the pre salt basins offshore Brazil. Artificial neural networks are also successfully utilized for ROP prediction in the works of Bilgesu et al. (1997), Moran et al. (2010) and Esmaeili et al. (2012). However, such improvement in ROP modeling comes at the expense of model interpretability. Therefore, traditional ROP models are still relevant and provide an effective method to analyze how a specific drilling parameter affects rate of penetration.

ROPPlotter, a field data visualization and ROP modeling software developed in Microsoft Excel VBA (Soares, 2015), is employed in calculating model coefficients and comparing model performance.

2. Materials and methods

2.1. ROPPlotter

For the three sandstone formations analyzed in this work, ROPPlotter computes model coefficients that best fit drilling field data. ROPPlotter capabilities include: importing field data with diverse formatting, data filtering by standard deviation or absolute value, data visualization with selectable axis bounds, automated lithology highlighting and ROP modeling. The software facilitates ROP model implementation by guiding the user in providing the necessary parameters for model calculations and utilizing Excel Solver in a loop to fit model coefficients for different lithologies. ROPPlotter also produces tables for comparison between ROP models coefficients and performance with ease.

Drilling data quality and reliability pose major challenges for the oil and gas industry. In order to conduct a more meaningful analysis of ROP models, a data filtering process is initiated to remove data outliers when field data are loaded into ROPPlotter:

The data filtering process is facilitated through the form presented in Fig. 1. Drilling parameter statistics are displayed for the entire original dataset (top left corner), for a selected rock formation (top right corner) and for the modified dataset after data outliers have been excluded (bottom right corner). Data outliers can be detected either by specific threshold values of drilling

Full Dataset: 7414 Data Pts.
Depth Range: 4257ft to 9129.75ft

ROP

Minimum ROP: 1.54ft/hr
Maximum ROP: 2699.43ft/hr
Average ROP: 73.24ft/hr
ROP Std. Dev.: 86.23ft/hr

RPM

Minimum RPM: 2.06
Maximum RPM: 74.45
Average RPM: 41.35
RPM Std. Dev.: 5.92

WOB

Minimum WOB: .36klb
Maximum WOB: 223.05klb
Average WOB: 27.29klb
WOB Std. Dev.: 13.20klb

Flow Rate

Minimum Q: 156.33gal/min
Maximum Q: 397.04gal/min
Average Q: 362.53gal/min
Q Std. Dev.: 19.11gal/min

Flag All Data

Current Formation: Tyler - Sandstone 872 Data Pts.
Depth Range: 7091.25ft to 7513.75ft

ROP

Minimum ROP: 13.89ft/hr
Maximum ROP: 323.04ft/hr
Average ROP: 57.40ft/hr
ROP Std. Dev.: 26.34ft/hr

RPM

Minimum RPM: 39.47
Maximum RPM: 41.25
Average RPM: 40.24
RPM Std. Dev.: .41

WOB

Minimum WOB: 3.84klb
Maximum WOB: 182.88klb
Average WOB: 26.62klb
WOB Std. Dev.: 15.72klb

Flow Rate

Minimum Q: 332.00gal/min
Maximum Q: 337.58gal/min
Average Q: 334.65gal/min
Q Std. Dev.: 1.10gal/min

Flag Formation Data

Formations:

Rierdon - Limestone	286	0
Piper - Limestone	260	0
Spearfish - Sandstone	225	0
Pine Salt - Sandstone	818	0
Broom Creek - Sandstone	405	0
Tyler - Sandstone	874	2
Kibbey Lime - Limestone	111	0
Kibbey Lime - Shale	359	0

Modified Dataset: 7412 Data Pts.
Depth Range: 4257ft to 9129.75ft

ROP

Minimum ROP: 1.54ft/hr
Maximum ROP: 2699.43ft/hr
Average ROP: 73.23ft/hr
ROP Std. Dev.: 86.24ft/hr

RPM

Minimum RPM: 2.06
Maximum RPM: 74.45
Average RPM: 41.35
RPM Std. Dev.: 5.92

WOB

Minimum WOB: .36klb
Maximum WOB: 207.23klb
Average WOB: 27.23klb
WOB Std. Dev.: 12.81klb

Flow Rate

Minimum Q: 156.33gal/min
Maximum Q: 397.04gal/min
Average Q: 362.53gal/min
Q Std. Dev.: 19.10gal/min

Import Filtered Data

Data Flagging Criteria:

☐ Absolute Value ☒ Standard Deviation

☐ ROP ft/hr

☐ WOB klb

☐ RPM rev/min

☐ Q gpm

ROP std. dev. above/below avg.

WOB std. dev. above/below avg.

RPM std. dev. above/below avg.

Q std. dev. above/below avg.

Flagged Data Points:

Depth (ft)	ROP (ft/hr)	WOB (klbs)	RPM	Q (gpm)	Data Row
7142.75	40.57	133.10	40.03	335.08	2503
7143.00	44.91	147.77	40.00	335.00	2504
7236.75	56.17	112.26	40.66	335.60	2655
7237.25	59.48	176.55	41.25	334.55	2656
7241.00	323.04	63.97	39.57	337.58	2657
7241.50	297.39	35.97	39.62	337.56	2658
7242.00	235.49	20.78	40.00	337.00	2659
7242.50	141.92	20.48	40.65	335.91	2660

Delete Flagged Data Point Delete All Flagged Data Points

Fig. 1. Data filtering form in ROPPlotter.

parameters or by a designated number of standard deviations away from the mean. The flagged points are displayed in a table in the bottom left corner, where the user can choose which data points are indeed outliers to be removed. Once data outliers have been excluded from the dataset, the main controlling form of ROPPlotter is displayed to facilitate the implementation of several capabilities:

The form above is accompanied by a ROP vs. depth plot fully governed by user interaction with the command buttons in Fig. 2. Examples of plots generated by ROPPlotter can be seen in Figs. 6–8 and Figs. 9–11 later in this manuscript. On the top right hand corner, plot bounds can be adjusted by either scrolling with the provided arrows or typing numbers in the boxes for each axis. The left side of the form offers many distinct tools to enhance data visualization. ROP modeling capabilities are positioned on right side, including comparison of model coefficients and performance with tables generated spontaneously. Currently, six different ROP models are included in ROPPlotter's library: Bingham (1964), Bourgoyne and Young (1974), Winters et al. (1987), Hareland and Rampersad (1994), Hareland et al. (2010), and Motahhari et al.

(2010). The ROP models developed by Bourgoyne and Young (1974), Winters et al. (1987), and Hareland et al. (2010) were not included in this study since they are not PDC bit specific models (Bingham's model was included as a base case). By selecting an ROP model and clicking the “Add Model to Plot” in Fig. 2, the user is guided by model specific forms to provide additional input parameters necessary for the model computations:

Fig. 3 displays the form specific to Hareland and Rampersad (1994) drag bit model. There are many bit design and geometry factors required for this model's calculation, which are shown on the left side of Fig. 3. Note that all model specific forms allow for user specified model coefficient bounds and initial guesses, a fundamental necessity for proper ROP model computations as later discussed in Section 3.2.

2.2. Bakken Shale case study

The depth based dataset utilized in this investigation spans twenty different rock formations in a 4871 ft (1485 m) long (4257 ft

The screenshot shows the ROPPlotter software interface. At the top, there are buttons for "Import New Data" and "Save Plot". Below "Import New Data", it says "Make sure data is on first sheet of Workbook." and displays "Imported Field Data" with a "Depth Range: 4257ft to 9128ft", "ROP Range: 2ft/hr to 2699ft/hr", and "Data Points: 7234". To the right, "Depth Bounds (ft):" are set to "Top: 7041" and "Bottom: 7564", while "ROP Bounds (ft/hr):" are set to "Low: 9" and "High: 155". There are "Reset Bounds" and "Zoom In on Formation" buttons. Below this, the "Formation Boundary:" is set to "Tyler - Sandstone", with a "Formation Depth Range: 7091ft to 7514ft" and "Formation ROP Range: 14ft/hr to 108ft/hr". A "Formation Data Pts.: 854" is also shown. On the left, a vertical stack of buttons includes "Hide Formation Boundary", "Show Formation Boundary", "Formation Color Code On", "Lithology Color Code On", "Data Filter Color Code", and "Color Code Off". On the right, "ROP Models:" is set to "Motahhari PDC Bit" and "Solver Method:" is set to "Min form. error". Below these are buttons for "Add Model to Plot", "Delete Model from Plot", "Delete All Models from Plot", "Evaluate Models from Plot", "Coefficient Analysis", and "Delete All Models Calcs.". At the bottom, "Overbalance:" controls include "Show OB on Plot" and "Hide OB on Plot" buttons, "Overbalance Range:" set to "0psi to 0psi", "Max OB at Depth:" set to "0ft", and "Overbalance Bounds (psi):" with "Low:" and "High:" input fields and arrows.

Fig. 2. Main controlling form of ROPPlotter.

Bit Properties:

Bit Diameter, Db = in

Cutter Diameter, dc = in

Number of Cutters, Nc =

Backrake Angle, theta = deg

Siderake Angle, alpha = deg

Bit Wear, Wf =

Model Coefficients Bounds:

Min. Bound	Coefficient	Max. Bound	Initial Guess
<input type="text" value="0.001"/>	< a <	<input type="text" value="10000"/>	<input type="text" value="5"/>
<input type="text" value="0.5"/>	< b <	<input type="text" value="1"/>	<input type="text" value="0.5"/>
<input type="text" value="0.5"/>	< c <	<input type="text" value="1.5"/>	<input type="text" value="0.5"/>

Add Model to Plot

Fig. 3. Hareland and Rampersad (1994) drag bit model form in ROPPlotter.

[1298 m] true vertical depth [TVD] to 9128 ft [2782 m] TVD) vertical section of a Bakken shale horizontal well in North Dakota. Despite the fact that time based data were not available, depth based data are preferred for ROP modeling analysis. It provides a straightforward method to correlate the intervals drilled with formation tops provided by geologists, facilitating the fitting of a distinct set of model coefficients for each lithology. A simplified stratigraphic column for this segment is shown in Fig. 4:

This entire interval was drilled by Marathon with the same Smith 616 PDC bit. Three rock formations with comparable average ROP, weight on bit (WOB) and rotary speed (rotations per minute, RPM) were selected for evaluation of the three ROP models. Even though all three lithologies chosen for this analysis were sandstones, rock type was not part of the selection criteria. In fact, these three sandstone formations have varying rock strength and the methodology applied in this study could be extended to any other lithologies, as long as they present similar drilling parameters (ROP, WOB, and RPM). The Broom Creek sandstone is a fine to medium grained sandstone of late Pennsylvanian and early Permian age. Interbedded layers of shale and dolomite are present in this rock. Average porosity is 14% and the maximum value reaches 20% (Sorensen et al., 2009). The Tyler formation is of early Pennsylvanian age and has been widely studied by the petroleum industry due its prolific oil production since 1954. It is divided into two sub systems, with interbedded shales in the lower system and

interbedded limestones in the Upper Tyler. Porosity ranges from 10% to 20% and permeability is greater than 100 md (Nesheim and Nordeng, 2014). Both the Broom Creek and Tyler Sandstones belong to the Minnelusa group. The Ratcliffe sandstone is a thin sub unit of the Madison group and of Mississippian age. It contains interbedded limestones and anhydrites (Blue et al., 1986).

Rock strength, quantified in this dataset as unconfined compressive strength (UCS), generally increases with depth, as expected due to rock compaction. Data filtering capabilities in ROPPlotter are employed in removing data point outliers to improve data reliability, yielding the following dataset:

In Table 1, average values of ROP, WOB, and RPM are similar across the three sandstone formations. Hence, variations in ROP model coefficients due to changing UCS can be analyzed. RPM is the drilling parameter with lowest standard deviations throughout the lithologies, while UCS has the highest standard deviation (more than 50% of the average value in the Tyler Sandstone).

The second set of properties necessary for ROP model calculations are bit properties, assumed to be constant in this study. This vertical section was drilled with a Smith PDC bit, model 616 (6 blades, 16 mm cutters). Based on vendor information (SHARC, 2015) and common industry practice (Rajabov et al., 2012), the following bit properties were determined for this investigation:

Bit diameter and type (Smith 616 PDC bit) were provided in the

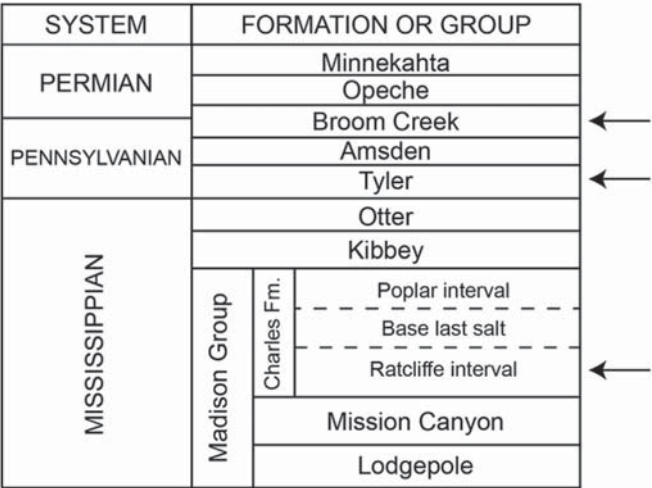


Fig. 4. Generalized stratigraphic column for the Williston Basin, North Dakota, based on Theley (2014). Formations included in this study are marked with arrows.

Table 1
Bakken Shale dataset filtered by removing data points with ROP or WOB values two standard deviations above or below the average.

Formation	Broom Creek Sandstone	Tyler Sandstone	Ratcliffe Sandstone
Original Data Points	405	874	134
Filtered Data Points	389	854	131
Depth (ft)	6897 to 7090	7091 to 7514	8006 to 8073
ROP Min. (ft/hr)	15.10	13.89	32.68
ROP Max. (ft/hr)	103.28	108.07	88.33
ROP Std. Dev. (ft/hr)	22.01	22.54	14.07
ROP Avg. (ft/hr)	57.58	56.63	59.96
WOB Min. (klb)	4.64	3.84	9.73
WOB Max. (klb)	41.11	60.90	36.36
WOB Std. Dev. (klb)	5.18	4.77	8.55
WOB Avg. (klb)	22.89	24.73	23.22
RPM Min. (rev/min)	6.80	39.58	39.68
RPM Max. (rev/min)	43.17	41.22	42.00
RPM Std. Dev. (rev/min)	3.67	0.40	0.50
RPM Avg. (rev/min)	39.47	40.24	40.68
UCS Min. (psi)	2229	2020	8540
UCS Max. (psi)	23,268	23,462	14,576
UCS Std. Dev. (psi)	3398	5696	1451
UCS Avg. (psi)	8118	9656	11,131

BHA report by the operator. The bit manufacturer website (SHARC, 2015) indicates the number of cutters and cutter diameter on the bit description. Bit wear is a difficult, if not impossible, parameter to measure in real time as the formations are being drilled. However, from Table 1, it is clear that the three sandstone formations selected for analysis are located towards the end of the bit run (4257 ft [1298 m] TVD to 9128 ft [2782 m] TVD). Hence, the bit wear value in Table 2 conveys a conservative estimate that the bit is at 50% operating capacity, producing 50% of the total ROP that an unworn bit would generate in the same circumstances. Although the PDC cutters' siderake and backrake angle are not provided by the bit manufacturer, they can be inferred from common industry practice. According to Rajabov et al. (2012), a cutter backrake angle of 10° minimizes the mechanical specific energy (MSE) necessary to drill through different lithologies. Additionally, a siderake angle of 30° represents the breaking point after which increasing siderake angle drastically increases MSE (Rajabov et al., 2012). A diagram of a single PDC bit cutter displaying backrake and siderake angles is presented below (See Fig. 5):

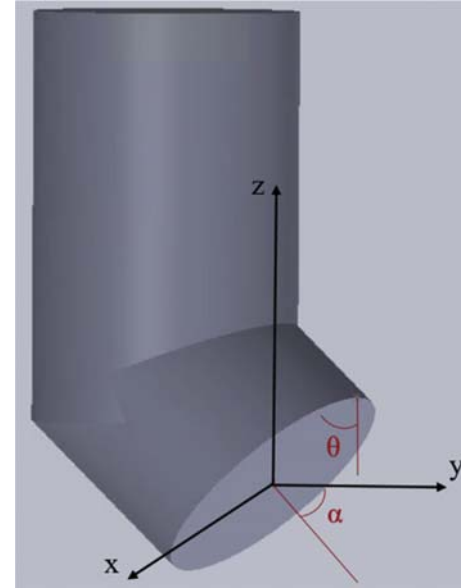


Fig. 5. Representation of a single PDC cutter with siderake angle α and backrake angle θ .

3. Theory

3.1. ROP models – governing equations

In this section, governing equations for the three ROP models are analyzed, revealing what drilling parameters each model takes into account. Variable nomenclature and units for some models have been modified from the original publications for consistency in comparison between the ROP models employed in this study.

Before presenting ROP models specific to PDC bits, Bingham's (1964) base case ROP model, an empirical relation applicable to all bit types, is introduced:

$$ROP = a \text{ RPM} \left(\frac{WOB}{D_B} \right)^b \quad (1)$$

where ROP is the rate of penetration (ft/hr), WOB is the weight on bit (klb), RPM is the rotary speed (revolutions/sec), D_B is the bit diameter (in), and a and b are the dimensionless constants for each rock formation. The two fitting parameters in this model (a and b) represent a measure of the drillability of each lithology (Bingham, 1964). Establishing minimum and maximum coefficient bounds is essential for proper computation of these two model coefficients to fit the model to field data, as discussed in the next section.

Hareland and Rampersad (1994) proposed a general drag bit model based on cutter rock interaction:

$$ROP = \frac{14.14 N_c \text{ RPM } A_v}{D_B} \quad (2)$$

where N_c is the number of cutters and A_v is the area of rock compressed ahead of a cutter (in^2). A_v can be modified for different drag bit types, and for PDC bits it is formulated as:

$$A_v = \cos \alpha \sin \theta \left[\left(\frac{d_c}{2} \right)^2 \cos^{-1} \left(1 - \frac{4 \text{ WOB}}{\cos \theta \pi N_c d_c^2 \sigma_c} \right) \left(\frac{2 \text{ WOB}}{\cos \theta \pi N_c \sigma_c} - \frac{4 \text{ WOB}^2}{(\cos \theta \pi N_c d_c \sigma_c)^2} \right)^{0.5} \left(\frac{\text{WOB}}{\cos \theta \pi N_c \sigma_c} \right) \right] \quad (3)$$

where α is the cutter side rake angle, θ is the cutter back rake angle, d_c is the cutter diameter (in), and σ_c is the uniaxial (or unconfined) compressive rock strength (psi). The compressed rock area in Eq. (3) can also be adapted to account for bit wear, but model application becomes quite difficult, involving additional fitting parameters. Therefore, for simplicity, the bit wear will be represented by a constant multiplying the right hand side of Eq. (2), illustrating the operating capacity of the bit ($W_{f,max} = 1$). Hareland's model coefficients are introduced in a single term:

$$COR = \frac{a}{(\text{RPM}^b \text{ WOB}^c)} \quad (4)$$

where a , b , and c are cutter geometry correction factors. These three model coefficients are lumped into COR – a lithology coefficient that multiplies the right hand side of Eq. (2). It is important to note that b and c , exponents on the RPM and WOB terms, respectively, are in the denominator of Eq. (4). Looking at Eq. (2) and Eq. (3), ROP has a linear relationship with RPM and a quadratic relation with WOB. Hence, coefficient b reduces the linear influence of RPM on ROP and coefficient c reduces the quadratic influence of WOB on ROP. The final form of Hareland's PDC bit model becomes:

$$ROP = W_f \frac{a}{(\text{RPM}^b \text{ WOB}^c)} \frac{14.14 N_c \text{ RPM}}{D_B} \cos \alpha \sin \theta \left[\left(\frac{d_c}{2} \right)^2 \cos^{-1} \left(1 - \frac{4 \text{ WOB}}{\cos \theta \pi N_c d_c^2 \sigma_c} \right) \left(\frac{2 \text{ WOB}}{\cos \theta \pi N_c \sigma_c} - \frac{4 \text{ WOB}^2}{(\cos \theta \pi N_c d_c \sigma_c)^2} \right)^{0.5} \left(\frac{\text{WOB}}{\cos \theta \pi N_c \sigma_c} \right) \right] \quad (5)$$

Table 2

Constant bit properties for Bakken Shale Case Study.

Bit property	Value	Units
Bit Diameter, D_b	8.75	in
Number of Cutters, N_c	50	
Cutter Diameter, d_c	0.63	in
Cutter Siderake Angle, α	30	deg
Cutter Backrake Angle, θ	10	deg
Bit Wear, W_f	0.50	

Motahhari et al. (2010) developed a PDC bit specific ROP model, incorporating a wear function and rock strength:

$$ROP = W_f \left(\frac{G RPM^\gamma WOB^\alpha}{D_B S} \right) \quad (6)$$

where S is the confined rock strength (psi), G is a model coefficient related to bit rock interactions and bit geometry, and α and γ are ROP model exponents. Note that the confined compressive strength (CCS) in Motahhari's PDC bit ROP model (S in Eq. (6)) is the rock strength measured at in situ confining stress conditions, and not the same as the unconfined compressive strength (UCS) in Hareland's drag bit model (σ_c in Eq. (5)). In the Bakken Shale dataset, only UCS measurements were reported (Table 1). Even though CCS can be expressed in terms of UCS or vice versa by assuming a Mohr Coulomb (linear) or Griffith Theory (parabolic) failure envelope, the same UCS values will be utilized in both Hareland's and Motahhari's models in order to compare coefficients consistently. The wear function in Eq. (6), W_f , is difficult to implement and introduces additional fitting parameters. Similar to the procedure for Hareland's model, W_f is simplified as a constant with maximum value of 1.

Table 3 compares drilling parameters and bit properties accounted for in each of the three ROP models described above:

Table 3 is divided into two categories of parameters: variable and constant properties. Variable properties represent drilling parameters continuously measured during drilling, which can change at every data point (see Table 1). On the other hand, while parameters in the "Constant (Bit) Properties" section can obviously change throughout the drilling of an entire well (especially bit wear), they are assumed to be constant for a specific lithology (see Table 2).

Looking at Table 3, it is clear that WOB, RPM and bit diameter are the most important factors in modeling ROP according to the selected models, as they are present in all three models' formulations. Both Hareland's and Motahhari's models include a rock strength term (assumed to be the same in this study) and a bit wear term. Additionally, Hareland's model introduces many bit geometry factors not accounted for in the other two models. None of the three models consider the influence of drilling fluid flow rate on ROP, as other models published in literature do (Bourgoyne and Young, 1974; Winters et al., 1987). The model coefficients, or fitting parameters, for all three models are discussed in the next section.

3.2. ROP models – coefficient bounds

Model coefficients in the three ROP models presented in the

previous section can be classified into three distinct categories: a constant that multiplies the entire ROP formulation, a weight on bit term exponent, and a RPM term exponent. According to the theory, ROP should vary linearly with RPM and WOB up until an inflection point. Increasing rotary speed or bit weight after such point will actually result in a reduction of drilling rate, due to inadequate hole cleaning, excessive pipe vibrations or pipe buckling (Gandelman, 2012). In this study, model coefficient lower bounds will force at least a square root relation between RPM/WOB and ROP, deemed appropriate as discussed at the end of this section. Table 4 illustrates coefficient categorization and provides model coefficient bounds for calculation of all fitting parameters in this investigation:

Model coefficients, or fitting parameters, are computed by minimizing the average error between the ROP model fit and ROP field data for a specific rock formation. Soares (2015) has reported on the importance of selecting appropriate coefficient bounds in order to achieve good model fits to field data. Looking at the coefficient bounds table above, the constant multiplier category of coefficients displays a wide range between lower and upper bounds. The upper bound is set to the same order of magnitude as UCS values seen on the field, allowing Hareland's and Motahhari's models to balance the division by rock strength (not formulated in Bingham's model).

Setting lower bounds for WOB and RPM exponents at 0.5 ensures that the ROP models account for these drilling parameters. Due to high variance in the WOB field data in this study (Table 1), ROP models will drive the WOB term exponent coefficients to zero during model fitting if no lower bound is specified, in an effort to reduce overall error. Conversely, model error is increased if the WOB exponent lower bound is increased. Appendix A contains model results for WOB exponent lower bounds equal to 0 (Table 7) and 1 (Table 8), exemplifying this point. It is worth observing that the upper bound for the WOB and RPM term exponents in Hareland's model is lower than for the other two ROP models, as b and c appear in the denominator of Eq. (5). As observed with results shown in Appendix A (Table 7), Hareland's model will drive the WOB term exponent to a value of zero if allowed to, even if that results in higher model error by maintaining the quadratic relationship between WOB and ROP. Similar results are observed with respect to rotary speed if the RPM exponent lower bound is set to zero, showing that the variance in field data induces the Hareland model to remove any influence of those drilling parameters on ROP controlled by model coefficients, although the ensuing error is higher. Hence, setting lower bounds of 0.5 for Hareland's RPM and WOB term exponents yields lower model errors, even though it limits the RPM ROP relation to at most a square root correlation.

4. Results

The ROPPlotter software is utilized to calculate model

Table 3
Comparison between parameters in each ROP model formulation.

Model	Bingham (1964)	Hareland and Rampersad (1994)	Motahhari et al. (2010)
Variable Properties			
Weight-on-Bit, WOB	✓	✓	✓
Rotary Speed, RPM	✓	✓	✓
Unconfined Rock Strength, σ_c		✓	
Confined Rock Strength, S			✓
Constant (Bit) Properties			
Bit Diameter, D_b	✓	✓	✓
Number of Cutters, N_c		✓	
Cutter Diameter, d_c		✓	
Cutter Siderake Angle, α		✓	
Cutter Backrake Angle, θ		✓	
Bit Wear, W_f		✓	✓

Table 4
Model coefficients and their computation bounds in this study.

Model	Bingham (1964)	Hareland and Rampersad (1994)	Motahhari et al. (2010)
Constant multiplier			
Lower Bound	0.001	0.001	0.001
Upper Bound	10,000	10,000	10,000
WOB exponent			
Lower Bound	0.5	0.5	0.5
Upper Bound	10	1.5	10
RPM exponent			
Lower Bound	N/A	0.5	0.5
Upper Bound	N/A	1	10

coefficients that best fit the Bakken Shale dataset with bit properties and coefficient bounds previously prescribed. A graphical representation of model fits to field data is presented in Figs. 6–8:

ROP model performance and coefficients are displayed for each rock formation in Table 5:

The table above serves as an indicator of how each model operates in different lithologies with varying rock strength. Recall from Table 1 that average values for ROP, WOB, and RPM are similar for all three sandstones. UCS values are highest for Racliffe Sandstone (deepest formation), followed by Tyler Sandstone and Broom Creek Sandstone (shallowest formation).

5. Discussion

From Table 5, it is clear that Hareland's model has the best performance in all three lithologies. Additionally, there is a trend of decreasing model error with deeper formations for Bingham's model and Hareland's model. With Motahhari's model, this is not the case, as it performs best in the shallowest lithology. When analyzing model performance in Table 5, it is important to look back at standard deviation values for drilling parameters presented in Table 1 and summarized here for convenience (See Table 6):

Model error is lowest for Bingham and Hareland models on the

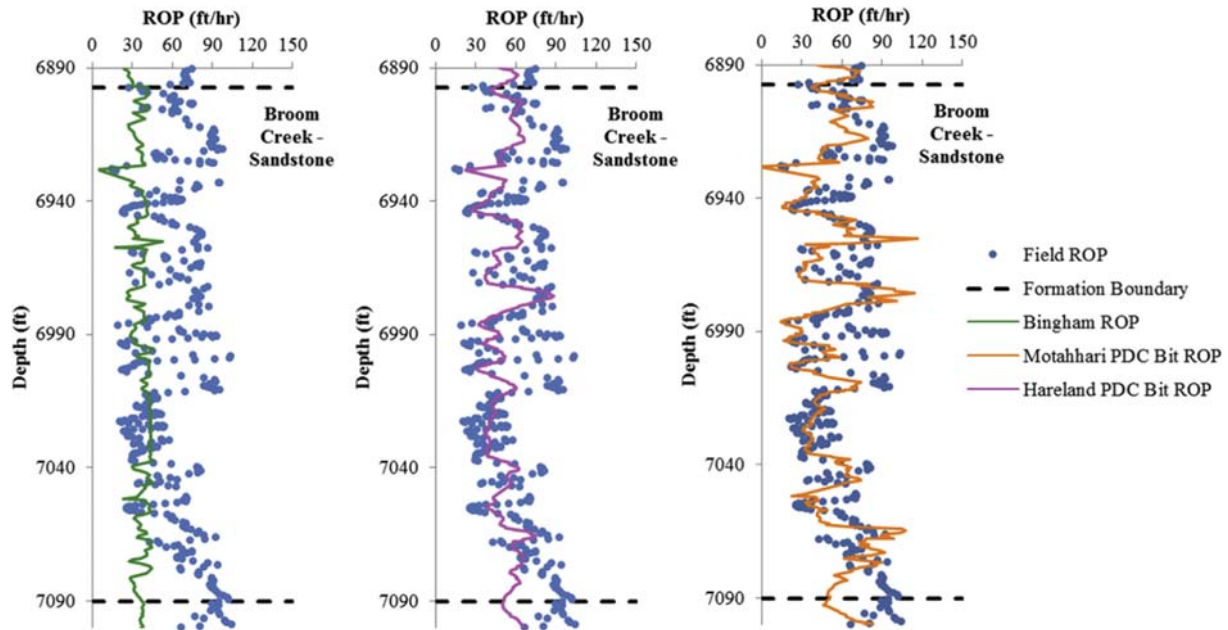


Fig. 6. Broom Creek Sandstone ROP models fit and field data.

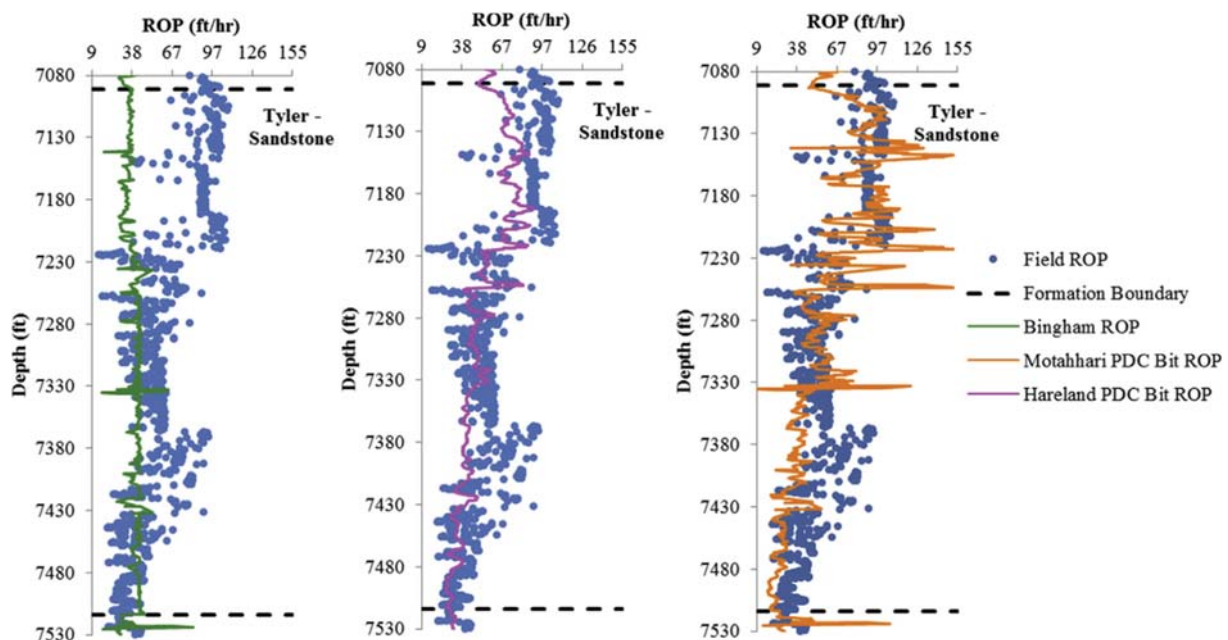


Fig. 7. Tyler Sandstone ROP models fit and field data.

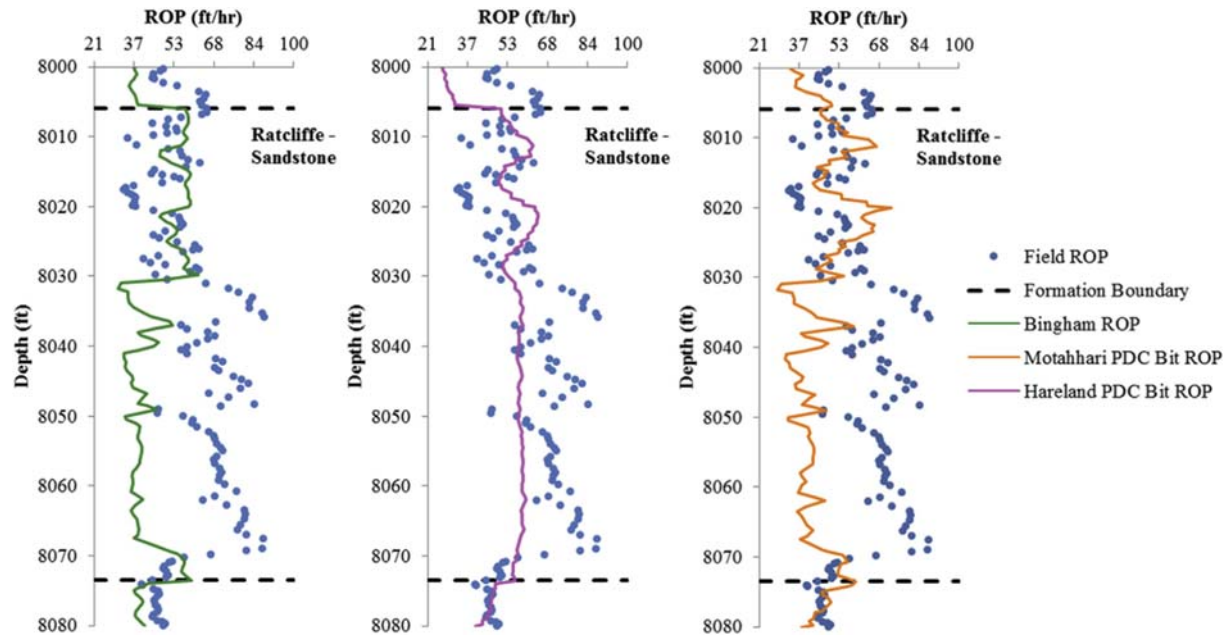


Fig. 8. Ratcliffe Sandstone ROP models fit and field data. Note that in all three formations, Bingham's model acts more as a moving average, while the other two models oscillate more and are able to capture spikes from field data.

Table 5
Model performance and coefficients for the three sandstone formations.

Formation	Broom creek sandstone	Tyler sandstone	Ratcliffe sandstone
Bingham error	40.26%	36.17%	33.05%
Hareland error	25.08%	24.89%	20.20%
Motahhari error	29.32%	37.83%	32.24%
Constant multiplier			
Bingham a	0.60	0.61	0.73
Hareland a	3152	2979	4279
Motahhari G	128.0	172.0	5.93
WOB exponent			
Bingham b	0.50	0.50	0.50
Hareland c	0.50	0.50	0.50
Motahhari α	0.50	0.91	0.50
RPM exponent			
Hareland b	0.51	0.50	0.50
Motahhari γ	2.50	2.06	3.42

Ratcliffe Sandstone formation, which also has lowest ROP and UCS standard deviations and a very low RPM standard deviation (even though it has the highest WOB deviation). Poorer model performance in the other lithologies is most likely explained by a high RPM standard deviation in the Broom Creek Sandstone, a high UCS standard deviation in the Tyler Sandstone, and a higher ROP standard deviation for both formations in comparison to the Ratcliffe Sandstone.

With respect to model coefficients, Bingham's model provides an important base case analysis. Not influenced by rock strength

Table 6
Standard deviations for drilling parameters in the Bakken Shale dataset.

Formation	Broom creek sandstone	Tyler sandstone	Ratcliffe sandstone
ROP Std. Dev. (ft/hr)	22.01	22.54	14.07
WOB Std. Dev. (klb)	5.18	4.77	8.55
RPM Std. Dev. (rev/min)	3.67	0.40	0.50
UCS Std. Dev. (psi)	3398	5696	1451

field data, Bingham's a increases slightly with increasing depth. Constant multipliers for the remaining models were expected to increase with depth (and increasing UCS), which is not the case with Hareland's a in the Tyler Sandstone formation and Motahhari's G in the Ratcliffe Sandstone formation. Hareland's a is 4 orders of magnitude greater than Bingham's a and Motahhari's G is 3 orders of magnitude greater than Bingham's a (except in Ratcliffe Sandstone formation), and those multipliers vary at a much higher rate than Bingham's a . Due to high variance encountered with field data, it is hard to remove the effect of other parameters and analyze only the influence of rock strength on model coefficients. However, in general, Hareland's a and Motahhari's G can be thought of as a means to balance the increasing rock strength with depth (not explicitly accounted for in the base case Bingham model).

All three ROP models try to remove influence of WOB on ROP by reducing the WOB term exponent to the lower bound of 0.5, as expected due to high WOB standard deviation. Note that in Hareland's model, the WOB term exponent is in the denominator but is still driven to the lower bound value. The WOB term exponent of Motahhari's model has a value of 0.91 in the Tyler Sandstone, the only instance in Table 5 where a WOB exponent differs from the lower bound value. This occurrence also represents the highest error seen for the Motahhari model, as well as very high oscillation of the model fit (Fig. 7). The Tyler Sandstone formation contains the largest UCS standard deviation out of the three sandstones, which may be responsible for such singularity.

RPM term exponents are driven to the lower bound for Hareland's model, meaning that the model always utilizes the maximum possible square root relation between RPM and ROP. On the other hand, Motahhari's model utilizes a changing RPM exponent as an additional way to offset rock strength, as γ increases significantly from the Broom Creek Sandstone formation to the Ratcliffe Sandstone formation (and is much greater than Hareland's RPM exponents). Motahhari's RPM term exponent decreases in the Tyler Sandstone formation, since the WOB exponent is significantly greater than the lower bound. Motahhari's model poor performance in the Ratcliffe Sandstone can be explained by a curious choice to lean more on the RPM exponent than on the constant

multiplier to balance increasing rock strength.

It is also important to note that all three models predict an ever increasing ROP with increasing WOB and RPM. As previously discussed, there is an inflection point after which increasing WOB will lead to hole cleaning and pipe buckling issues and increasing RPM will lead to excessive drillstring vibrations, reducing ROP in both cases (Gandelman, 2012).

6. Conclusions

The ROPPlotter software was an important component of this study, automating the computation and comparison of model coefficients and producing easily understandable plots and tables. ROPPlotter can be implemented in numerous future case studies to speed up and enhance ROP modeling analysis.

For the Williston Basin dataset presented in this study, Hareland's model works best and should be used in predicting ROP for future wells in the region. It is proposed that Hareland performs better because it selects the smallest possible WOB and RPM term exponents, emphasizing more on the constant multiplier and avoiding the high variability in field data.

All three models drive the WOB exponent down to the lower bound in order to reduce error, as the WOB field data has relatively high standard deviation. As shown in Appendix A, selection of proper model coefficient bounds is a key factor in the effectiveness of ROP modeling. ROP models, if allowed to, will try to eliminate the influence of WOB on ROP during parameter optimization, completely neglecting one of the most important drilling parameters affecting drilling rate.

By selecting three lithologies with similar average ROP, WOB and RPM, it is possible to better understand how changing rock strength affects ROP model coefficients. Hareland and Motahhari models employ the combined contribution of constant multiplier and RPM term exponent to offset increasing rock strength when compared to Bingham's model. Motahhari's model has raw dependence on the inverse of rock strength, and utilizes both the constant multiplier G and the RPM exponent γ to balance increasing UCS. Hareland's model rock strength component shows up in a complicated term (Eq. (3)), and only the multiplier a changes with increasing rock strength. While Hareland's a value is greater than Motahhari's G , Motahhari's γ (RPM exponent) is always at least quadratic. The combined influences of Motahhari's G and γ on ROP are greater than Hareland's a and b , meaning that bigger model coefficient values are necessary for the Motahhari model to overcome the inverse dependence on rock strength.

After decades of traditional ROP modeling done by prescribing an equation and fitting empirical coefficients to field data for a singular, localized application, the after the fact nature and operational limitations and deficiencies of such modeling are being addressed with real time, data driven modeling and optimization of drilling parameters used to quantify drilling performance and efficiencies. Since ROP is a highly nonlinear, complicated function depending on many parameters specific to that localized application, defining a set equation relating ROP to a few drilling parameters is not sufficient. Indeed, evaluating past drilling performances are important in case studies, seeking ways to improve drilling operations. However such after the fact analyses with traditional models have no predictive capabilities. More powerful advances may be made by coupling real time rig floor measurements of surface and downhole parameters with Wider Windows Statistical Learning Models (WWSLM), which have demonstrated highly accurate and powerful predictive capabilities while simultaneously preserving and expanding the utility of a posteriori assessments (Hegde et al., 2015). Even though the intricacies of WWSLM are beyond the scope of this study, model results with the same dataset

presented in Section 2.2 are displayed in Appendix B for comparison purposes.

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Acronyms

CCS	Confined compressive strength
Marathon	Marathon Oil Corporation
MSE	Mechanical Specific Energy
PDC	Polycrystalline diamond compact
ROP	Rate of penetration
RPM	Rotations per minute
UCS	Unconfined compressive strength
VBA	Visual Basic for Applications
WOB	Weight on bit
WWSLM	Wider Windows Statistical Learning Models

Appendix A

Following the discussion on model coefficient bounds in Section 3.2, model fit results with different WOB term exponent lower bounds are presented in Tables 7 and 8:

By comparing the tables above with Table 5 (WOB exponent lower bound of 0.5), the following observations are made:

- With a WOB exponent lower bound equal to 0 (Table 7), most WOB exponents were driven to the lower bound. Model errors are lower for Bingham and Motahhari models (except Tyler Sandstone), since WOB variance is ignored. The error actually increased for the Hareland model, as the original quadratic influence of WOB on ROP is maintained with the WOB term in the denominator. Bingham's constant multiplier increases at all well depths to make up for the loss of the WOB term. Hareland's constant multiplier decreased in all three sandstones, since the quadratic WOB ROP relation is preserved. Hareland's RPM exponent remained around the lower bound of 0.5. Motahhari's constant multiplier actually decreased in the first two

Table 7
Model performance and coefficients with a WOB exponent lower bound of zero.

Formation	Broom creek sandstone	Tyler sandstone	Ratcliffe sandstone
Bingham error	35.22%	32.49%	20.72%
Hareland error	30.88%	26.66%	31.00%
Motahhari error	28.36%	38.29%	21.83%
Constant multiplier			
Bingham a	1.10	1.08	1.35
Hareland a	635	633	863
Motahhari G	2.73	1.99	56.10
WOB exponent			
Bingham b	0.00	0.00	0.00
Hareland c	0.01	0.00	0.02
Motahhari α	0.00	0.92	0.00
RPM exponent			
Hareland b	0.51	0.50	0.52
Motahhari γ	3.97	3.25	3.28

Table 8
Model performance and coefficients with a WOB exponent lower bound of one.

Formation	Broom creek sandstone	Tyler sandstone	Ratcliffe sandstone
Bingham error	44.33%	39.41%	42.52%
Hareland error	35.54%	41.22%	50.30%
Motahhari error	32.46%	38.33%	40.54%
Constant multiplier			
Bingham <i>a</i>	0.34	0.33	0.36
Hareland <i>a</i>	10,000	10,000	10,000
Motahhari <i>G</i>	10,000	2.27	2.00
WOB exponent			
Bingham <i>b</i>	1.00	1.00	1.00
Hareland <i>c</i>	1.00	1.00	1.00
Motahhari α	1.00	1.00	1.00
RPM exponent			
Hareland <i>b</i>	0.50	0.50	0.50
Motahhari γ	0.88	3.15	3.24

formations, due to an accentuated increase in the RPM exponent (2.50–3.97 and 2.06 to 3.25 – compared to Table 5). The constant multiplier increased in the last formation with a decrease in the RPM exponent. The behavior of Motahhari’s model coefficients are erratic and hard to predict.

- With a WOB exponent lower bound equal to 1 (Table 8), model error increased for all 3 models in all 3 sandstones. All WOB exponents are driven to the new lower bound of 1. Bingham’s constant multiplier decreased in comparison to the base case (Table 5), since more weight is placed on the WOB term. Hareland’s constant multiplier increased to the upper bound, as the WOB ROP quadratic relation is reduced to linear and the

constant needs to make up for the lower WOB term contribution. The RPM exponent stayed at the lower bound. Motahhari’s model behavior is erratic once again, sometimes relying more on the constant multiplier and at other times relying more on the RPM term exponent to offset rock strength.

Appendix B

Results from modeling ROP with the WWSLM for this Williston Basin dataset (Section 2.2) are compared with traditional ROP modeling performance in Table 9:

Table 9
Comparison of model performance between traditional ROP models and WWSLM.

Formation	Broom creek sandstone	Tyler sandstone	Ratcliffe sandstone
Bingham error	40.26%	36.17%	33.05%
Hareland error	25.08%	24.89%	20.20%
Motahhari error	29.32%	37.83%	32.24%
SLM error	17.42%	14.32%	9.61%

Table 9 undoubtedly shows the superiority of statistical learning algorithms to predict ROP when compared to traditional ROP modeling. By graphically comparing WWSLM efficacy with the best performing traditional model (Hareland and Rampersad, 1994), it is clear that the SLM is able to capture ROP field data trends much better than Hareland’s model:

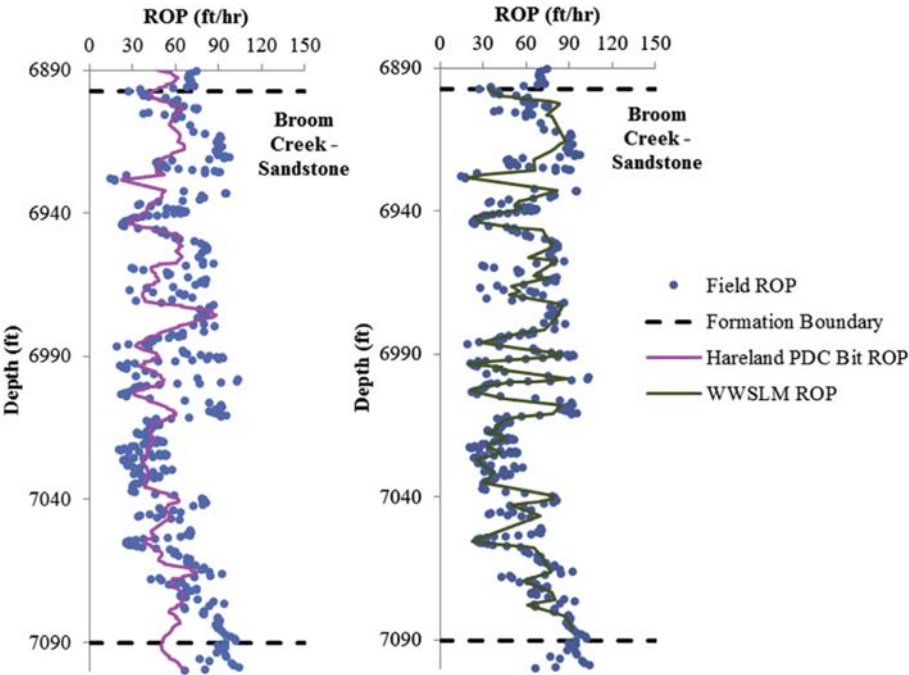


Fig. 9. Broom Creek Sandstone WWSLM and Hareland models fit and field data.

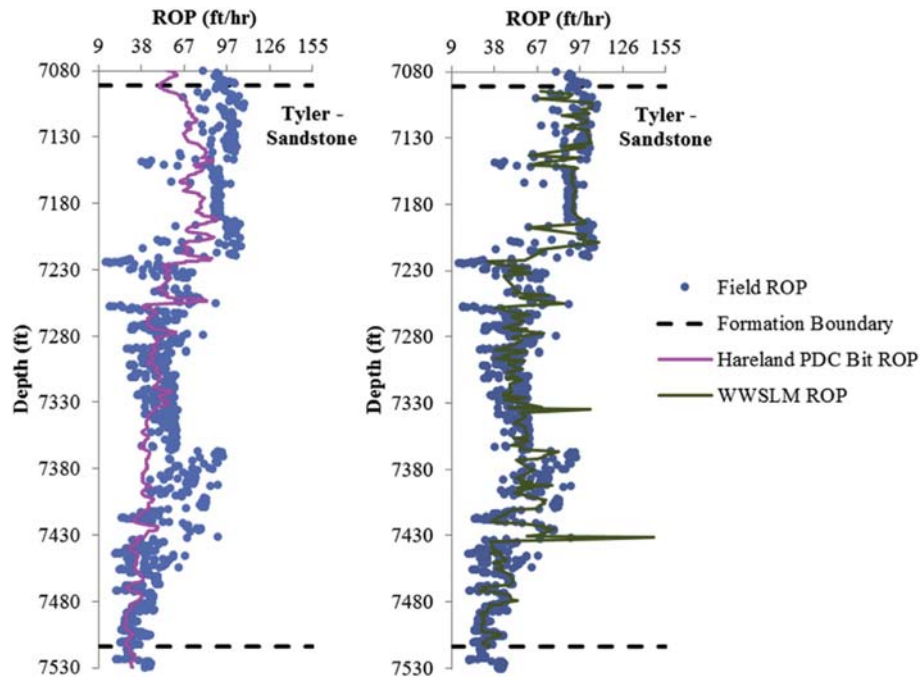


Fig. 10. Tyler Sandstone WWSLM and Hareland models fit and field data.

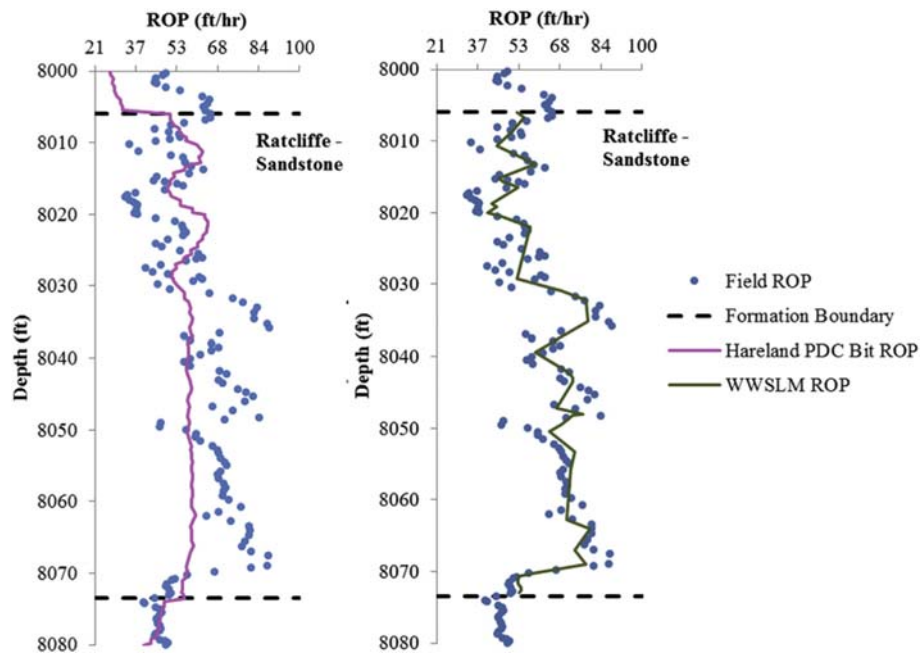


Fig. 11. Ratcliffe Sandstone WWSLM and Hareland models fit and field data.

For more information on the specifics of WWSLM, please refer to Hegde et al. (2015).

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