MODULAR SERIES
ON SOLID STATE DEVICES

Robert F. Pierret and Gerold W. Neudeck, Editors

VOLUME I

Semiconductor Fundamentals

ROBERT F. PIERRET
Purdue University

ADDISON-WESLEY PUBLISHING COMPANY
READING, MASSACHUSETTS
MENLO PARK, CALIFORNIA
LONDON • AMSTERDAM
DON MILLS, ONTARIO
SYDNEY

This book is in the
Addison-Wesley Modular Series on Solid State Devices

Library of Congress Cataloging in Publication Data

Pierret, Robert F.
Semiconductor fundamentals.

(Modular series on solid state devices ; v. 1)
Bibliography: p.
Includes index.
1. Semiconductors. I. Title. II. Series: Pierret, Robert F. Modular series on solid state devices ; v. 1.
TK 7871.85.P485 621.3815'2 81-14978
ISBN 0-201-05320-9 AACR2

Reprinted with corrections, July 1983

ISBN 0-201-05320-9
FGHIJ-AL-898765

a system that is beyond the realm of everyday life. Yes, in "reality" the hole is just another subatomic "particle."

2.2.4 Material Classification

Changing the subject somewhat, we would like to conclude this section by citing a very important tie between the band gap of a material, the number of carriers available for transport in a material, and the overall nature of a material. As it turns out, although specifically established for semiconductors, the energy band model of Fig. 2.6 can be applied with only slight modification to all materials. The major difference between materials lies not in the nature of the energy bands, but rather in the magnitude of the energy band gap.

Insulators, as illustrated in Fig. 2.8(a), are characterized by very wide band gaps, with E_G for the insulating materials diamond and SiO_2 being $\simeq$5 eV and $\simeq$8 eV, respectively. In these wide band gap materials the thermal energy available at room temperature excites very few electrons from the valence band into the conduction band; thus very few carriers exist inside the material and the material is therefore a poor conductor of current. The band gap in metals, by way of comparison, is either very small, or no band gap exists at all due to an overlap of the valence and conduction bands (see Fig. 2.8(c)). There is always an abundance of carriers in metals, and hence metals are excellent conductors. Semiconductors present an intermediate case between insulators and metals. At room

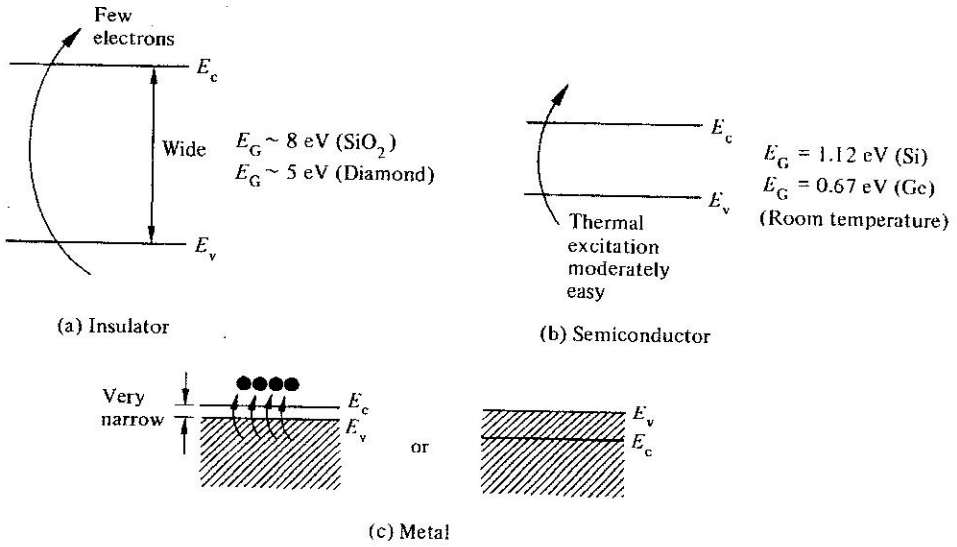


Fig. 2.8 Explanation of the distinction between (a) insulators, (b) semiconductors, and (c) metals using the energy band model.

temperature, $E_G = 1.12$ eV in Si and $E_G = 0.67$ eV in Ge. Thermal energy, by exciting electrons from the valence band into the conduction band, creates a moderate number of carriers in these materials, giving rise in turn to a current-carrying capability intermediate between poor and excellent.

From what we have said, the band gap of a material is clearly an important parameter. Actually, it is perhaps *the* most important material parameter; and the value of the silicon band gap at room temperature ($E_G = 1.12$ eV) should definitely be committed to memory.

2.3 CARRIER PROPERTIES

Having formally introduced the electron and the hole, we next seek to learn as much as possible about the nature of these carriers. In this particular section we examine a collage of carrier-related information, information of a general nature including general facts, properties, and terminology.

2.3.1 Charge

Both electrons and holes are charged entities. Electrons are negatively charged, holes are positively charged, and the *magnitude* of the carrier charge, q, is the same for the two types of carriers. In MKS units, $q = 1.6 \times 10^{-19}$ coul. (Please note that, under the convention adopted herein, the electron and hole charges are $-q$ and $+q$, respectively; i.e., the sign of the charge is displayed explicitly.)

2.3.2 Effective Mass

Mass, like charge, is another very basic property of electrons and holes. Unlike charge, however, the carrier mass is not a simple property and cannot be disposed of by simply quoting a number. Indeed, the apparent or effective mass of electrons within a crystal is a function of the semiconductor material (Si, Ge, etc.) and is different from the mass of electrons within a vacuum.

Seeking to obtain insight into the effective mass concept, let us first consider the motion of electrons in a vacuum. If, as illustrated in Fig. 2.9(a), an electron of rest mass m_0 is moving in a vacuum between two parallel plates under the influence of an electric field $\mathscr{E}$, then, according to Newton's second law, the force $\mathbf{F}$ on the electron will be

$$\mathbf{F} = -q\mathscr{E} = m_0 \frac{d\mathbf{v}}{dt} \tag{2.2}$$

where $\mathbf{v}$ is the electron velocity and t is time. Next consider electrons (conduction band electrons) moving between the two parallel end faces of a semiconductor crystal under the influence of an applied electric field, as envisioned in Fig. 2.9(b). Does Eq. (2.2) also describe the overall motion of electrons within the semiconductor crystal? The answer is clearly *no*. Electrons moving inside a semiconductor crystal will collide with semiconductor atoms, thereby causing a periodic deceleration of the carriers. However, should not Eq. (2.2) apply to the portion of the electronic motion occurring *between* collisions?

through a sea of Si atoms and is characterized by an effective mass. Hence, in the donor or pseudo-atom case, the permittivity of free space must be replaced by the permittivity of Si and m_0 must be replaced by m_n^*. We therefore conclude that the binding energy (E_B) of the fifth donor electron is approximately

$$E_B \simeq -\frac{m_n^* q^4}{2(4\pi K_S \varepsilon_0 \hbar)^2} = \frac{m_n^*}{m_0}\frac{1}{K_S^2} E_{H|n=1} \simeq -0.1 \text{ eV} \tag{2.5}$$

where K_S is the Si dielectric constant ($K_S \simeq 12$). Actual donor-site binding energies in Si are listed in Table 2.3. The observed binding energies are seen to be in good agreement with the Eq. (2.5) estimate and, confirming the earlier speculation, are roughly $\simeq 1/20$ the Si band gap energy.

Having established the strength of dopant-site bonding, we are now in a position to tackle the problem of how one visualizes dopant action using the energy band model. Working on the problem, we note first of all that when an electron is released from a donor site it becomes a conduction band electron. If the energy absorbed at the donor site is precisely equal to the electron binding energy, the released electron will moreover have the lowest possible energy in the conduction band — namely, E_c. Adding an energy $|E_B|$ to the bound donor-site electron, in other words, *raises* the electron's energy to E_c. Hence, we are led to conclude that the bound electron occupies an allowed electronic level an energy $|E_B|$ below the conduction band edge, or, as visualized in Fig. 2.12, donor sites can be incorporated into the energy band scheme by adding allowed electronic levels at an energy $E_D = E_c - |E_B|$. Note that the donor-site energy level is represented by a set of dashes, instead of a continuous line, because an electron bound to a donor site is localized in space; i.e., a bound electron does not leave the general Δx vicinity of the donor site. The relative closeness of E_D to E_c of course reflects the fact that $E_c - E_D = |E_B| \simeq (1/20)E_G(\text{Si})$.

The actual visualization of dopant action using the energy band model is pictured in Fig. 2.13. Examining the left-hand side of Fig. 2.13(a), one finds all of the donor sites filled with bound electrons at temperatures $T \to 0°\text{K}$. This is true because very little thermal energy is available to excite electrons from the donor sites into the conduction band at these very low temperatures. The situation changes, of course, as the temperature

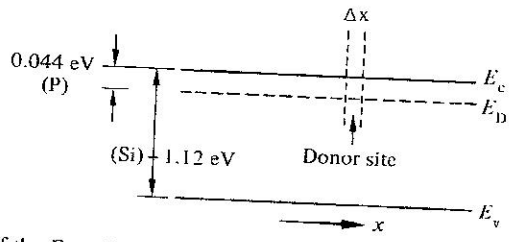


Fig. 2.12 Addition of the $E = E_D$ donor levels to the energy band diagram. Dashes of width Δx emphasize the localized nature of the bound donor-site states.

Table 2.3. Dopant-site Binding Energies.

Donors	$\lvert E_B \rvert$	Acceptors	$\lvert E_B \rvert$
Sb	0.039 eV	B	0.045 eV
P	0.044 eV	Al	0.057 eV
As	0.049 eV	Ga	0.065 eV
		In	0.16 eV

is increased, with more and more of the weakly bound electrons being donated to the conduction band. At room temperature the ionization of the donor sites is all but total, giving rise to the situation pictured at the extreme right of Fig. 2.13(a). Although we have concentrated on donors, the situation for acceptors is completely analogous. As visualized in Fig. 2.13(b), acceptors introduce allowed electronic levels into the forbidden gap at an energy slightly above the valence band edge. At low temperatures, all of these sites will be empty—there is insufficient energy at temperatures $T \to 0°K$ for a valence band electron to make the transition to an acceptor site. Increasing temperature, implying an increased store of thermal energy, facilitates electrons jumping from the valence band onto the acceptor levels. The removal of electrons from the valence band of course creates holes. At room temperature, essentially all of the acceptor sites are filled with an excess electron and an increased hole concentration is effected in the material.

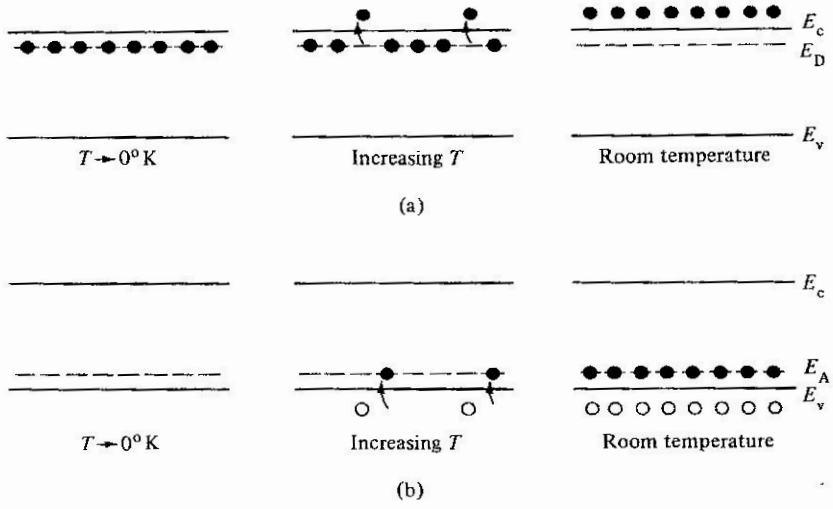


Fig. 2.13 Visualization of (a) donor and (b) acceptor action using the energy band model.

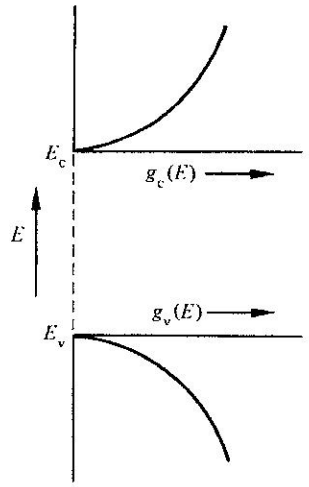


Fig. 2.14 General energy dependence of $g_c(E)$ and $g_v(E)$ near the band edges. $g_c(E)$ and $g_v(E)$ are the density of states in the conduction and valence bands, respectively.

2.4.2 The Fermi Function

Whereas the density of states tells one how many states exist at a given energy E, the Fermi function $f(E)$ specifies how many of the existing states at the energy E will be filled with an electron, or equivalently,

$f(E)$ specifies, under equilibrium conditions, the probability that an available state at an energy E will be occupied by an electron.

Mathematically speaking, the Fermi function is simply a probability density function. In mathematical symbols,

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \tag{2.7}$$

where

E_F = Fermi energy or Fermi level
k = Boltzmann constant ($k = 8.62 \times 10^{-5}$eV/°K)
T = temperature in °K

Seeking insight into the nature of the newly introduced function, let us begin by investigating the Fermi function's energy dependence. Consider first temperatures where $T \to 0$°K. As $T \to 0$°K, $(E - E_F)/kT \to -\infty$ for all energies $E < E_F$ and $(E - E_F)/kT \to +\infty$ for all energies $E > E_F$. Hence $f(E < E_F) \to 1/[1 + \exp(-\infty)] = 1$ and $f(E > E_F) \to 1/[1 + \exp(+\infty)] = 0$. This result is plotted in Fig. 2.15(a) and is simply

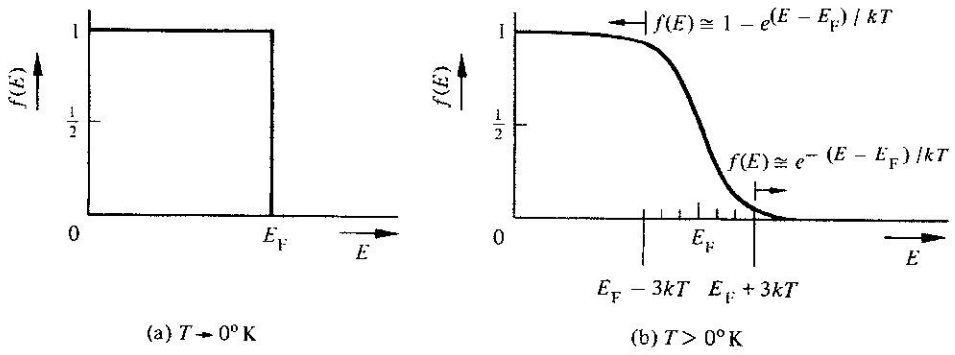


Fig. 2.15 Energy dependence of the Fermi function. (a) $T \to 0°K$; (b) generalized $T > 0°K$ plot with the energy coordinate expressed in kT units.

interpreted as meaning that all states at energies below E_F will be filled and all states at energies above E_F will be empty for temperatures $T \to 0°K$. In other words, there is a sharp cutoff in the filling of allowed energy states at the Fermi energy E_F when the system temperature approaches absolute zero.

Let us next consider temperatures $T > 0°K$. Examining the Fermi function we make the following pertinent observations.

i) If $E = E_F$, $f(E_F) = 1/2$.

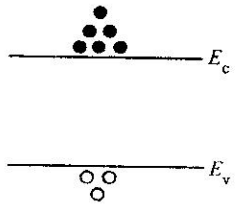
ii) If $E \geq E_F + 3kT$, $\exp[(E-E_F)/kT] \gg 1$ and $f(E) \simeq \exp[-(E-E_F)/kT]$. Consequently, above $E_F + 3kT$ the Fermi function or filled-state probability decays exponentially to zero with increasing energy. Moreover, most states at energies $3kT$ or more above E_F will be empty.

iii) If $E \leq E_F - 3kT$, $\exp[(E-E_F)/kT] \ll 1$ and $f(E) \simeq 1 - \exp[(E-E_F)/kT]$. Below $E_F - 3kT$, therefore, $[1 - f(E)]$, the probability that a given state will be *empty*, decays exponentially to zero with decreasing energy. Most states at energies $3kT$ or more below E_F will be filled.

iv) At room temperature ($T \simeq 300°K$), $kT \simeq 0.026$ eV and $3kT \simeq 0.078$ eV $\ll E_G$(Si). Compared to the Si band gap, the $3kT$ energy interval that appears prominently in the $T > 0°K$ formalism is typically quite small.

The properties cited above are reflected and summarized in the $T > 0°K$ Fermi function plot displayed in Fig. 2.15(b).

Before concluding the discussion here, it is perhaps worthwhile to reemphasize that the Fermi function applies only under equilibrium conditions. The Fermi function, however, is universal in the sense it applies with equal validity to all materials —insulators, semiconductors, and metals. Although introduced in relationship to semiconductors, the Fermi function is not dependent in any way on the special nature of semiconductors, but is simply a statistical function associated with electrons in general.



ic representation of carrier energy distributions.

signifies that the semiconductor is p-type. Note finally that the dashed E_i line also typically appears on the energy band diagrams characterizing extrinsic semiconductors. The E_i line in such cases represents the expected positioning of the Fermi level *if* the material were intrinsic, and serves as a reference energy level dividing the upper and lower halves of the band gap.

2.5 EQUILIBRIUM CARRIER CONCENTRATIONS

We have arrived at a somewhat critical point in the carrier modeling process. For the most part, this section simply embodies the culmination of our modeling efforts, with working relationships for the equilibrium carrier concentrations being established to reinforce the qualitative carrier information presented in previous sections. Unfortunately, the emphasis on the development of mathematical relationships makes the final assault on the carrier modeling summit the most difficult part of the journey—the reader cannot relax. A comment is also in order concerning the presentation herein of alternative forms for the carrier relationships. The alternative forms can be likened to the different kinds of wrenches used, for example, in home and automobile repairs. The open-end wrench, the box wrench, and the ratchet wrench all serve the same general purpose. In some applications one can use any of the wrenches. In other applications, however, a special situation restricts the type of wrench employed or favors the use of one wrench over another. The same is true of the alternative carrier relationships. Finally, expressions that are widely quoted or find widespread usage have been enclosed in brackets. A single set of brackets signifies a moderately important result; a double set of brackets, a very important result.

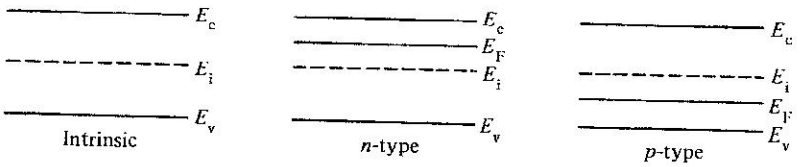


Fig. 2.18 "At a glance" representation of intrinsic (left), n-type (middle), and p-type (right) semiconductor materials using the energy band diagram.

at or near room temperature,

$$p = N_A \tag{2.34a}$$

Thus, making use of Eq. (2.19b),

$$n_i e^{(E_i - E_F)/kT} = N_A \tag{2.42}$$

and

$$\left[E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right)\right] \qquad p\text{-type material} \tag{2.43}$$

From Eqs. (2.41) and (2.43) it is obvious that the Fermi level moves systematically upward in energy from E_i with increasing donor doping and systematically downward in energy from E_i with increasing acceptor doping. The exact Fermi level positioning in Si at room temperature, nicely reinforcing the foregoing statement, is displayed in Fig. 2.21. Also note that for a given semiconductor material and ambient temperature there exist maximum nondegenerate donor and acceptor concentrations, doping concentrations above which the material becomes degenerate. In Si at room temperature the maximum nondegenerate doping concentrations are $N_D \simeq 1.6 \times 10^{18}/\text{cm}^3$ and $N_A \simeq 7.7 \times 10^{17}/\text{cm}^3$. The large Si doping values required for degeneracy, we should interject, have led to the common usage of "highly doped" (or n^+-material/p^+-material) and "degenerate" as interchangeable terms. Finally, the question may arise, "What procedure should be followed in computing E_F when one is not sure whether a material

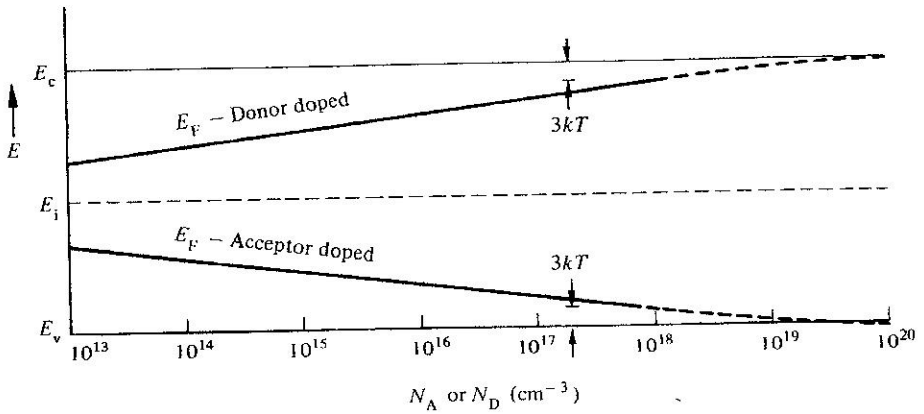


Fig. 2.21 Fermi level positioning in Si at room temperature as a function of the doping concentration. The solid E_F lines were established using Eq. (2.41) for donor doped material and Eq. (2.43) for acceptor doped material ($kT = 0.026$ eV, and $n_i = 10^{10}/\text{cm}^3$).

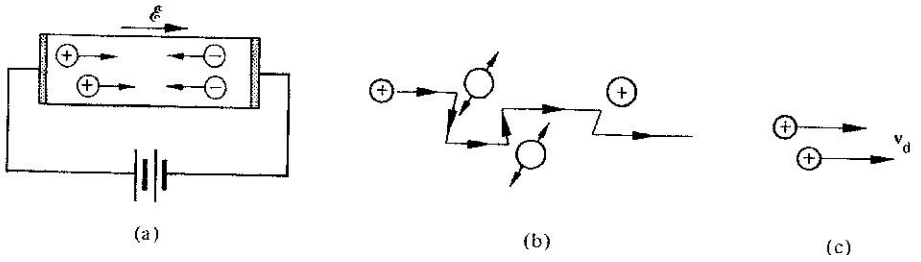


Fig. 3.1 Visualization of carrier drift: (a) motion of carriers within a biased semiconductor bar; (b) drifting hole on a microscopic or atomic scale; (c) carrier drift on a macroscopic scale.

frequently interrupted (the carriers are said to be scattered). The net result, pictured in Fig. 3.1(b), is carrier motion generally along the direction of the electric field, but in a disjointed fashion involving repeated periods of acceleration and subsequent decelerating collisions.

The microscopic drifting motion of a single carrier is obviously complex and quite difficult to analyze quantitatively in any detail. Fortunately, measurable quantities are *macroscopic* observables which reflect the average or overall motion of the carriers. Averaging over all electrons or holes at any given time, we find that the resultant motion of each carrier type can be described in terms of a constant drift velocity, $\mathbf{v}_d$. In other words, on a macroscopic scale, drift may be visualized (see Fig. 3.1(c)) as nothing more than all carriers of a given type moving along at a constant velocity in a direction parallel or antiparallel to the applied electric field.

By way of clarification, it is important to point out that the drifting motion of the carriers arising in response to an applied electric field is actually superimposed upon the always-present thermal motion of the carriers. Electrons in the conduction band and holes in the valence band gain and lose energy via collisions with the semiconductor lattice and are nowhere near stationary even under equilibrium conditions. In fact, under equilibrium conditions the thermally related carrier velocities average $\sim 1/1000$ the speed of light at room temperature! As pictured in Fig. 3.2, however, the thermal motion of the carriers is completely random. Thermal motion therefore averages out to zero on a macroscopic scale, does not contribute to current transport, and can be conceptually neglected.

3.1.2 Drift Current

Let us next turn to the task of determining the current flowing within a semiconductor as a result of carrier drift. By definition

I (current) = the charge per unit time crossing an arbitrarily chosen plane of observation oriented normal to the direction of current flow.

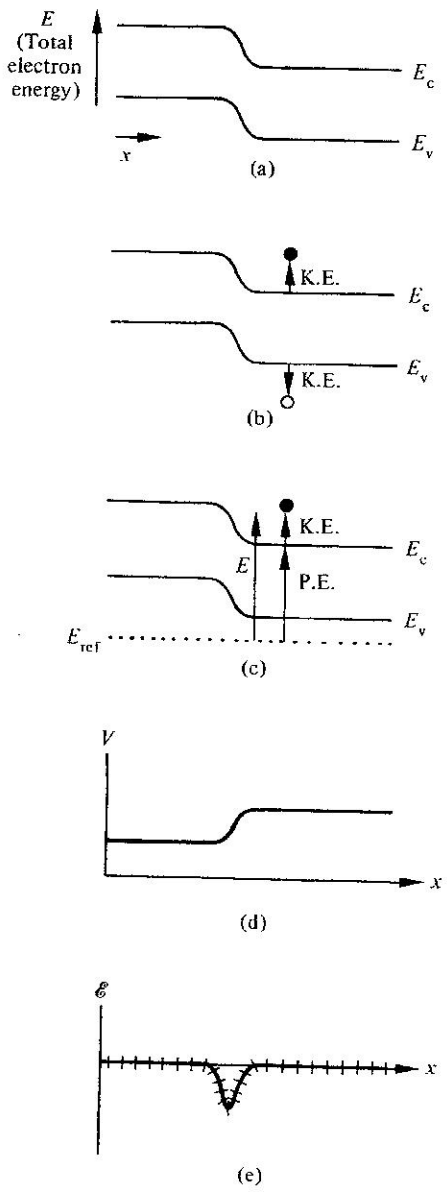


Fig. 3.9 Relationship between band bending and the electric field within a semiconductor: (a) sample energy band diagram; (b) identification of the carrier kinetic energies; (c) specification of the electron potential energy; (d) electrostatic potential and (e) electric field versus position dependence deduced from and associated with the part (a) energy band diagram.

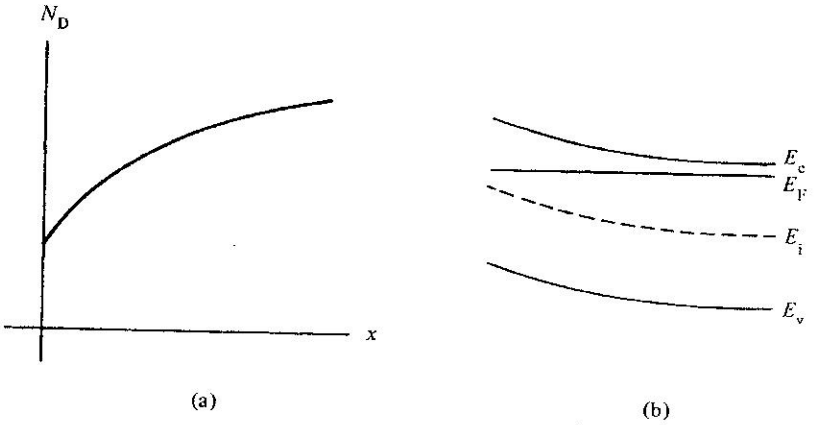


Fig. 3.13 Nonuniformly doped semiconductor: (a) assumed doping variation with position; (b) corresponding equilibrium energy band diagram.

fied, more complex, forms of Eqs. (3.23) result when the argument is extended to degenerate materials. We should also comment that the intermediate results concerning the positional invariance of the equilibrium Fermi level, and the existence of a nonzero electric field inside a nonuniformly doped semiconductor under equilibrium conditions, are important in themselves. Relative to numerical values, it is worthwhile noting that kT/q is a voltage, and at room temperature is approximately equal to 0.026 V.* Hence, for a $N_D = 10^{14}/\text{cm}^3$ Si sample maintained at room temperature, $D_N = (kT/q)\mu_n = (0.026\text{ V})(1360\text{ cm}^2/\text{V-sec}) \cong 35\text{ cm}^2/\text{sec}$. Finally, the Einstein relationship is one of the easiest equations to remember because it rhymes internally—Dee over mu equals kTee over q. The rhyme holds even if the equation is inverted—mu over Dee equals q over kTee.

3.3 RECOMBINATION–GENERATION

3.3.1 Definition–Visualization

Recombination–generation is perhaps the most interesting of the three primary types of carrier action and at the same time the most challenging to totally master. Formally, recombination–generation can be defined as follows:

Generation is a process whereby electrons and holes (carriers) are created.

Recombination is a process whereby electrons and holes are annihilated or destroyed.

*At room temperature $kT \cong 0.026\text{ eV} = (1.6 \times 10^{-19})(0.026)$ joule. Thus $kT/q = (1.6 \times 10^{-19})(0.026)/(1.6 \times 10^{-19})$ joule/coul = 0.026 V.