SEMAT AND KATZ

Physics

Physics

Books by Henry Semat

Fundamentals of Physics THIRD EDITION

Introduction to Atomic and Nuclear Physics THIRD EDITION

Physics in the Modern World

With Robert Katz

Physics

Physics

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Preface

This book is intended for students of science and engineering; it aims to develop both an understanding of the important concepts of physics and some analytical skill in the solutions of problems. The mathematical level of the book is such that it may be used by students who are taking a course in calculus concurrently.

The notations and methods of the calculus are introduced early in the text, beginning with the concept of a derivative in the discussion of motion, and are then extended to more complex problems as the student progresses both in physics and in mathematics. Vector algebra is, of course, also used. The vector notation is introduced at the beginning of the text in treating displacements; it is then extended to include the use of the dot product and cross product of two vectors, and the resolution of a vector into components with the aid of unit vectors. These vector methods are used extensively in the sections on Mechanics and Electricity.

The method of exposition and the division of the subject matter into six parts—Mechanics, Heat, Wave Motion and Sound, Electricity and Magnetism, Optics, and Atomics and Nucleonics—follow closely those of the senior author's *Fundamentals of Physics,* now in its third edition. However, the treatment of much of the material is entirely new, as are over two hundred of the figures. The problems at the end of each chapter are graded in difficulty, and many illustrative examples are provided in the text both to clarify concepts and to guide the student in the analytical approach to the solutions of problems. Included among these problems are some involving selected derivations and some requiring the use of calculus. Answers to odd-numbered problems are given in the Appendix, and a booklet containing all the answers is available to the instructor and may be distributed to his students if he so desires.

Three systems of units are developed and used in the sections on Mechanics, Heat, and Sound; these are the British engineering system, and the cgs and mks metric systems. In the section on Electricity and Magnetism, the rationalized mks system is the primary one, but the unrationalized Gaussian system is also developed because of its wide use in all of physics and much of engineering. The Gaussian system is introduced in the early chapters of Electricity; in subsequent

chapters it is used only when new concepts are introduced or when important equations are developed. The forms taken by each important equation in both the mks and the Gaussian systems are presented in a table at the end of the appropriate chapter, while the necessary conversion factors for the units and constants of each system appear in another table. Problems at the end of each chapter are given in both sets of units.

In developing the subject of Electricity and Magnetism we were guided by the recommendations of the Coulomb Law Committee of the American Association of Physics Teachers and we followed these recommendations fairly closely. We take this opportunity of expressing our indebtedness and appreciation to this Committee.

We have used modern concepts and examples throughout the book, and in addition have devoted an entire separate section to Atomics and Nucleonics. The reason for this section is that the traditional two- or three-semester course in physics is actually a terminal course for a great many students; this course will be their only opportunity to learn about atomic and nuclear physics from a physicist's point of view. Wherever a separate course in atomic and nuclear physics is part of the curriculum, this section may be omitted from the general course.

We wish to thank Professor Karl D. Larsen, Head of the Department of Physics at Lafayette College, Professor I. Wallerstein of Purdue University, and Professor Robert A. Becker of the University of Illinois, each of whom read the manuscript and made many valuable criticisms and helpful suggestions.

We are also indebted to several colleagues at our respective institutions who read different parts of the manuscript and made valuable criticisms and helpful suggestions. At Kansas State College Professor R. M. Kerchner, Head of the Electrical Engineering Department, read the section on Electricity and Magnetism; Professor L. D. Ellsworth of the Physics Department read the sections on Electricity and Magnetism and Optics, and Professor Basil Curnutte, Jr., of the Physics Department read the section on Optics. At the City College of New York, Professor M. W. Zemansky, Chairman of the Physics Department, read the section on Heat, Professor Fred C. Rose read the section on Electricity and Magnetism, and Professor Robert I. Wolff read the section on Optics.

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January, 1958

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29

The Magnetic Field

29-1 Natural and Permanent Magnets

Natural magnets, called lodestones, have been known since ancient times. The lodestone, a magnetic oxide of iron called magnetite (Fe_3O_4), was mentioned by Thales of Miletus. By the eleventh century the magnetic compass was known to the Chinese, and in the twelfth century references to

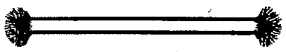

Fig. 29-1 Iron filings cling to the poles of a bar magnet.

the compass were made in Western Europe. The lodestone is capable of attracting pieces of iron and of imparting permanent magnetism to other pieces of iron so that these too could attract iron filings. If an iron bar is magnetized, as the result of being near a piece of lodestone, and is then dipped into iron filings, the filings will cling mostly to the ends of the bar, as shown in Figure 29-1. These ends are called the *poles* of the magnet.

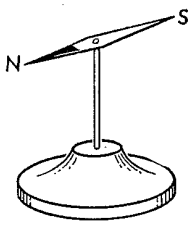

Fig. 29-2 A simple magnetic compass needle.

The use of the magnet as a compass depends upon the fact that if a permanent magnet is suspended so that it can swing freely in a horizontal plane, the magnet will set itself in nearly a north-south direction. The end which points toward the north is called the north-seeking pole, or simply the *north pole*, while the other end is called the *south pole*. A line joining the south to the north pole is called the *axis* of the magnet. A simple magnetic compass is shown in Figure 29-2. When such a compass is mounted upon a sheet of cork and is floated in water, the cork sheet remains at rest, implying that the net force on the compass is zero.

A ship's compass, a somewhat more elaborate instrument, is shown in Figure 29-3. It consists of a compass card fastened to a set of parallel magnets with their north poles facing in the same direction and mounted

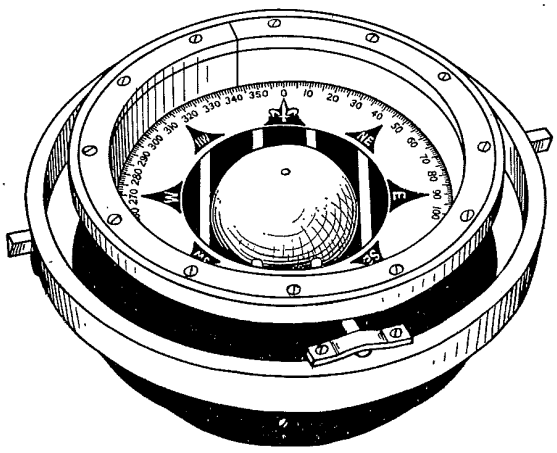

Fig. 29-3 A ship's compass. The compass card is mounted on four parallel magnets.

so that the whole assembly rotates freely in a horizontal plane. The case housing the compass is mounted in a set of bearings so that it can remain horizontal even when the ship rolls and pitches. A marker on the case housing the compass shows the direction in which the ship is headed.

29-2 Coulomb's Law of Force

Suppose that a bar magnet is suspended from its center by a string so that it can swing freely in a horizontal plane. If the north pole of a second bar magnet is brought near the north pole of the first one, there will be a force of repulsion between these poles. The suspended magnet will move so that its north pole goes away from the north pole of the approaching magnet. On the other hand, there will be a force of attraction when the north pole of one magnet approaches the south pole of the other one. In his book, *De Magnete*, William Gilbert (1540–1603) first showed that the earth could be considered as a huge spherical magnet with two poles, and that the compass needle was oriented by the forces exerted by the earth's magnetism.

About two centuries later Charles Augustin de Coulomb (1736–1806) measured the law of force between the poles of two magnets. A long thin magnet was suspended from a fine wire, and the pole of another thin magnet was brought near a like pole of the suspended magnet, as shown in Figure

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-4. Because of the force between these two poles, the suspended magnet perienced a torque which caused it to rotate about the wire as an axis, us twisting the wire. Coulomb udied the force between the poles two magnets as a function of the istance between them; he also udied the force between mag- ets of different strengths, keeping he distance between poles con- tant. He found that the force etween two poles of two magnets aried inversely with the square of he distance between them, and aried directly with the strengths f the poles. The results of Cou- omb's experiments could be ummed up by stating that *the orce between the two magnetic poles is proportional to the product of the strengths of the poles and inversely proportional to the square of the distance between them*, or

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$$F = k_1 \frac{p_1 p_2}{r^2}, \qquad (29\text{-}1)$$

in which p_1 is the pole strength of the first magnet, p_2 is the pole strength of the second magnet, r is the distance between the poles, k_1 is a constant of proportionality, and F is the force between them. The force between the two poles is directed along the line joining the two poles; it is attractive between unlike poles and repulsive between like poles.

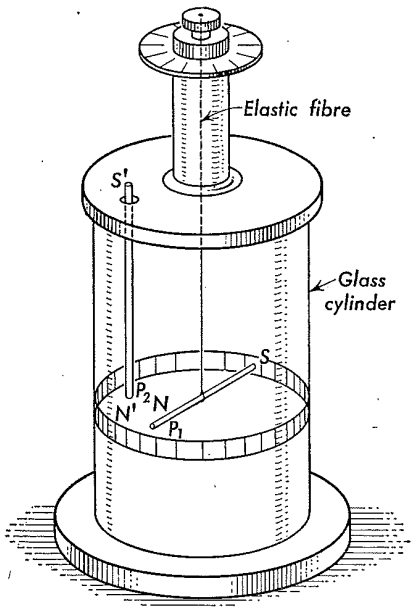

Fig. 29-4 Coulomb type torsion balance. NS, magnet of pole strength p_1, is suspended from elastic fiber. $N'S'$ is another magnet of pole strength p_2.

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Figure

Following the analogy of Coulomb's law for electric charges, we may write this proportionality in the form of an equation, but we must first define either the constant of proportionality k_1 or the magnitude of the unit pole. As before, in the *cgs* system of units, the constant of proportionality k_1 is set equal to 1, when the medium is vacuum, and the subsequent equation is used to define a *unit pole*. The units based upon this equation are then called the *cgs electromagnetic units* (abbreviated emu). In the form of an equation, we have

cgs emu

$$\mathbf{F}_2 = \frac{p_1 p_2}{r^2} \mathbf{1}_r \qquad (29\text{-}1a)$$

for the force in dynes between two poles in vacuum (or in air for which this equation is very nearly correct). In Equation (29-1a) $\mathbf{F}_2$ is the force on the pole p_2 in dynes, r is the distance between the poles in centimeters, and the unit vector $\mathbf{1}_r$ is directed from the pole p_1 to the pole p_2. The pole strength is expressed in unit poles, writing a north pole as positive and a south pole as negative. *A cgs unit pole is* implicitly *defined* in Equation (29-1a) *as one which, placed one centimeter from a like pole in vacuum, will repel it with a force of one dyne.*

In the mks system of units, the constant of proportionality is written as $\dfrac{1}{4\pi\mu_0}$, so that Coulomb's law becomes

mks units

$$\boxed{\mathbf{F}_2 = \frac{1}{4\pi\mu_0}\frac{p_1 p_2}{r^2}\mathbf{1}_r.} \tag{29-1b}$$

The constant μ_0 is assigned the value $4\pi \times 10^{-7}$, for reasons which must be deferred until a subsequent chapter. Equation (29-1b) implicitly defines an *mks unit pole* called a *weber*. In Equation (29-1b) $\mathbf{F}_2$ is the force on pole p_2 expressed in newtons, p_1 and p_2 are expressed in webers, and r is expressed in meters. *An mks unit pole of strength one weber is one which, placed one meter from a like pole in vacuum, will repel it with a force of* $10^7/(4\pi)^2$ *nt.* This is a force of $10^{12}/(4\pi)^2$ dynes. From Equation (29-1a) two cgs unit poles placed 1 m apart repel each other with a force of 10^{-4} dyne. Thus we see that

$$1 \text{ weber} = 10^8/(4\pi) \text{ cgs poles.} \tag{29-2}$$

In a form appropriate to Equation (29-1b) the value of μ_0 may be given as

$$\mu_0 = 4\pi \times 10^{-7}\,\frac{\text{weber}^2}{\text{nt m}^2}\,. \tag{29-3}$$

As the theory of magnetism is developed, other dimensions will be stated for μ_0 which are equivalent to the dimensions given above.

The properties of a magnet and of magnetic poles appear to be similar to the properties of electric charge, and yet there are extremely significant differences. For example, when a long magnetized needle is broken in two, new poles appear at the broken ends, so that each piece of the needle is observed to contain a north pole and a south pole. *An isolated magnetic pole has never been observed in nature.* Poles always occur in pairs. In calculating the forces on a magnet due to the presence of other magnets, we must always take into account the forces due to the north and the south poles of the magnets generating the force.

In a Coulomb type of experiment however, it is possible to use magnets long enough so that when two poles are placed near each other, the other poles are far enough removed so that they do not appreciably affect the result. The pole of a magnet is simply a small region of the magnet which acts as a center of force rather than the location of isolated magnetic charge. In spite of the fact that we shall find it convenient to speak of the magnetic field in terms of magnetic poles, we must always bear in mind that the smallest element of magnetism observed physically is not a magnetic pole but a magnetic dipole.

29-3 Intensity of Magnetic Field

Because of the effect that a magnet produces on magnetic materials placed anywhere in its neighborhood, we can say that a *magnetic field* exists in the neighborhood of a magnet. We can make a quantitative statement of the intensity of the magnetic field at some point in space by placing the north pole of a long, thin, needlelike magnet at this point. If the pole strength of this magnet is p, its north pole will experience a force $\mathbf{F}$. The *magnetic field intensity* $\mathbf{H}$ at this point is defined as the force $\mathbf{F}$ divided by the pole strength p, or, in the form of an equation,

$$\boxed{\mathbf{H} = \frac{\mathbf{F}}{p}.} \qquad (29\text{-}4)$$

The *magnetic field intensity* $\mathbf{H}$ is a vector quantity, whose magnitude is given by the magnitude of the force on a unit north pole, and whose direction is the direction of the force on a north pole. Algebraically, it is conventional to represent a north pole as a positive pole and a south pole as a negative pole. In cgs emu the force is stated in dynes, the pole strength is cgs unit poles, and H is given in *oersteds*. If the intensity of the magnetic field H is known at a particular point in space, the force that would be experienced by each pole of a magnet placed at this point is given by Equation (29-4). For example, if the north pole of a long thin magnet having a pole strength of 2 cgs poles is placed at a point where the intensity is 45 oersteds, the force on it is 90 dynes.

Illustrative Example. A bar magnet 24 cm long has a pole strength of 800 cgs unit poles. Calculate the intensity of the magnetic field at point A 16 cm from the magnet, measured along the line which is the perpendicular bisector of the bar.

Referring to Figure 29-5, we note that the point A is 16 cm from the center C of the bar and 20 cm from each end, N and S. For purposes of calculation, let us imagine a long thin magnet of unit pole strength with its north pole at A,

and apply Coulomb's law to calculate the force exerted on it by each of the poles of the bar magnet. The force of repulsion on the unit north pole at A due to the north pole N is, from Equation (29-1a)

$$F = \frac{800 \times 1}{400} \text{ dynes} = 2 \text{ dynes}$$

directed away from N along NA. The force of attraction on the unit north pole at A due to the south pole S is also 2 dynes but is directed toward S along AS. Adding these two forces vectorially, we get 2.4 dynes acting parallel to the axis of the magnet as the force on a unit north pole at A. The intensity of the magnetic field at A is therefore

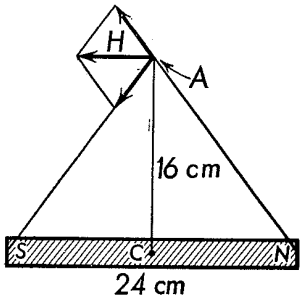


Fig. 29-5

$$H = \frac{2.4 \text{ dynes}}{\text{cgs pole}} = 2.4 \text{ oersteds.}$$

If the north pole is taken away and if a south pole is placed at A, it will experience an equal force in the opposite direction. A magnet so small that it can be considered as though both poles are practically at A will line up parallel to the direction of the field at A, with the north pole pointing in the direction of the field. A magnetic field may therefore be mapped by small compass needles, which will orient themselves parallel to the direction of the field.

When iron filings are introduced into a magnetic field, the filings become magnetized. The magnetized filings orient themselves in the direction of the field. Figure 29-6 shows a typical arrangement of iron filings in the neighborhood of a bar magnet as seen in one plane.

As we shall see in a subsequent chapter, a magnetic field may also be generated by a wire carrying current, and, in fact, this is the most common method for the generation of a magnetic field. This has been recognized in the choice of the units of magnetic field intensity H in the mks system where H is generally stated in units of *amperes per meter*. Until we have developed the concepts appropriate to the generation of the magnetic field by current in a wire, we shall represent the magnetic field in units of *newtons per weber* in the mks system of units, a form appropriate to the definition of H in Equation (29-4). The relationship between the mks and cgs units of the magnetic field intensity H may be stated as

$$1 \frac{\text{nt}}{\text{weber}} = 4\pi \times 10^{-3} \text{ oersted.} \qquad (29\text{-}5)$$

This conversion factor may be verified by recalling that

$$1 \text{ nt} = 10^5 \text{ dynes,}$$

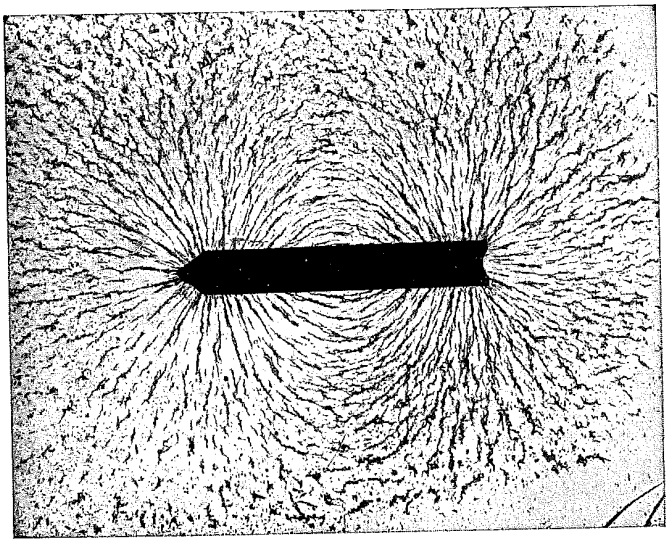


Fig. 29-6 Pattern formed by iron filings in the neighborhood of a bar magnet.

and $$1 \text{ weber} = \frac{10^8}{4\pi} \text{ cgs poles.}$$

Thus $$1 \frac{\text{nt}}{\text{weber}} = 1 \frac{\text{nt}}{\text{weber}} \times \frac{10^5 \text{ dynes}}{1 \text{ nt}} \times \frac{1 \text{ weber}}{10^8/4\pi \text{ cgs pole}}$$

$$= 4\pi \times 10^{-3} \frac{\text{dyne}}{\text{cgs pole}}$$

$$= 4\pi \times 10^{-3} \text{ oersted.}$$

29-4 Magnetic Moment

If a bar magnet of length s and pole strength p is placed in a uniform magnetic field of intensity H, its north pole will experience a force Hp in the direction of the field, while its south pole will experience an equal force in the opposite direction, as shown in Figure 29-7(a). If the magnet is not parallel to the field, the magnet will experience a torque tending to rotate it so that its alignment is parallel to the field. The magnet will experience two forces equal in magnitude but opposite in direction, so that the net force on the magnet is zero.

Following the terminology developed in electrostatics where we called an assemblage of two equal and opposite charges an electric dipole, we call a magnet with its two equal and opposite poles a *magnetic dipole*. The *magnetic dipole moment* $\mathbf{m}$ is a vector quantity directed from the south pole

of the magnet to its north pole and is of magnitude given by the product of the pole strength p by the separation of the poles s. In the form of an equation the magnitude of the dipole moment is given by

$$m = ps. \tag{29-6}$$

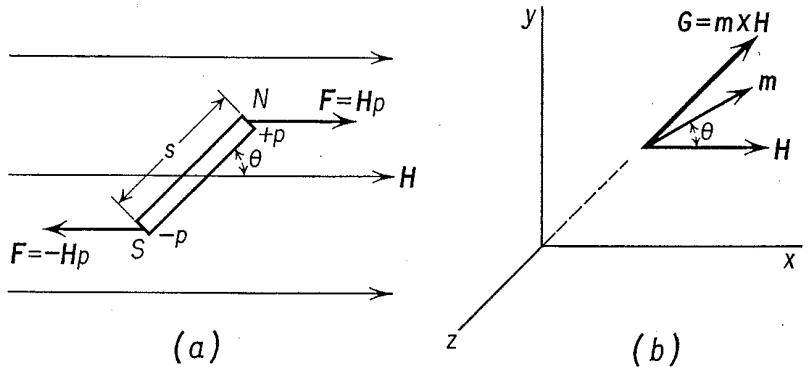


Fig. 29-7 (a) Forces on a bar magnet in a uniform magnetic field. The plane of the paper is the x-y plane. (b) When the magnetic moment vector **m** and the magnetic field **H** lie in the x-y plane as shown, the torque **G** on the magnet is parallel to the z axis.

If **s** is the displacement vector directed from the south pole of the magnet to its north pole, the magnetic dipole moment vector **m** is given by the equation

$$\boxed{\mathbf{m} = p\mathbf{s},} \tag{29-6a}$$

which may be compared to the corresponding definitions in electricity given in Equation (25-11). The cgs electromagnetic units of magnetic dipole moment are *cgs pole centimeters*, while the mks units of dipole moment may be expressed as *weber meters*.

Let us calculate the magnitude of the torque G about the center of the magnet due to the effect of the magnetic field **H** upon the magnetic dipole **m**, as shown in Figure 29-7(b). If the dipole-moment vector makes an angle θ with **H**, the torque is

$$G = Hps \sin \theta,$$

or, in terms of the magnetic moment m as defined in Equations (29-6), the torque is given by

$$G = Hm \sin \theta. \tag{29-7}$$

As shown in the figure, the torque tends to rotate the magnet in the clockwise direction. Following the right-hand rule, the torque vector is in the

$-z$ direction. Using the notation of the vector product developed in Section 7-4, we may write

$$\boxed{\mathbf{G} = \mathbf{m} \times \mathbf{H}.} \tag{29-7a}$$

We may apply this result to the measurement of the magnetic field. If a small bar magnet of moment of inertia I is suspended in a magnetic field H, the torque on the magnet will be given by Equation (29-7), and if the angle made by the magnet with the field direction is small, we may replace sin θ by θ. Observing that the direction of the torque vector is opposite to the direction of the angular displacement, we obtain

$$G = -Hm\theta \tag{29-8}$$

By comparing Equation (29-8) with Equation (12-2), we see that the magnet is subjected to an elastic restoring torque which is proportional to the angular displacement θ and opposite in direction. The magnet will therefore vibrate in simple harmonic motion. The period of the motion T will be given by Equation (12-17) as

$$T = 2\pi\sqrt{\frac{I}{mH}}. \tag{29-9}$$

Thus the magnetic field intensity H may be determined by measuring the period of vibration, T of a magnet of known magnetic moment M and moment of inertia I.

Illustrative Example. A bar magnet 8 cm long has a mass of 20 gm and a pole strength of 60 cgs unit poles. Determine (a) its magnetic moment and (b) its period when placed in a field whose intensity is 100 oersteds.

(a) The magnetic moment of the bar magnet is, from Equation (29-6),

$$m = 60 \text{ poles} \times 8 \text{ cm} = 480 \text{ pole cm}.$$

To determine the period it is necessary to find the moment of inertia. The moment of inertia of a uniform bar about an axis through its center of gravity is given by the expression

$$I = \tfrac{1}{12}ML^2,$$

where M is the mass of the bar and L is its length. Therefore

$$I = \tfrac{1}{12} \times 20 \text{ gm} \times 64 \text{ cm}^2 = 106.7 \text{ gm cm}^2.$$

(b) Substituting these values in Equation (29-9), we find

$$T = 2\pi\sqrt{\frac{106.7}{480 \times 100}} \text{ sec} = 0.296 \text{ sec}.$$

29-5 Atomic Magnetic Moments

When a small magnet is placed in a uniform magnetic field, it experiences a torque. When the magnet is placed in a *nonuniform* field, the field acting at the south pole of the magnet may be given by $\mathbf{H}_s$, while that acting at the north pole of the magnet may be given by $\mathbf{H}_n = \mathbf{H}_s + \Delta\mathbf{H}$, as shown in Figure 29-8, where $\Delta\mathbf{H}$ is the vector difference between the fields at these two poles. The net force on the magnet is due to the change in the field $p\,\Delta\mathbf{H}$, for if p is the pole strength of the magnet, the force on the magnet is

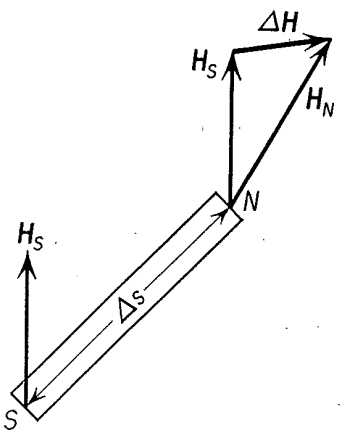


Fig. 29-8

$$\mathbf{F} = p\mathbf{H}_n - p\mathbf{H}_s = p\,\Delta\mathbf{H}.$$

Let us suppose that the magnet is physically small, of length Δs. We multiply and divide the right-hand side of the equation above by the quantity Δs to find

$$\mathbf{F} = p\,\Delta s\,\frac{\Delta\mathbf{H}}{\Delta s}.$$

The product $p\,\Delta s$ is the magnetic moment m of the magnet, while the quantity $\Delta\mathbf{H}/\Delta s$ is the rate of change of the magnetic field directed from the south to the north pole. Thus we have, in the limit of extremely small magnets,

$$\mathbf{F} = m\frac{\Delta\mathbf{H}}{\Delta s} = m\frac{d\mathbf{H}}{ds}, \tag{29-10}$$

where the direction in which we take the derivative of the magnetic field intensity is the direction of the magnetic moment.

A method of measuring atomic magnetic moments devised by Stern and Gerlach (1921) was to generate a beam of atoms, to send them through a nonuniform or inhomogeneous magnetic field of known inhomogeneity, and to observe the deflection of the beam. In the Stern-Gerlach experiment a beam of silver atoms was obtained by heating silver to a high temperature in an oven in vacuum. A narrow beam of silver atoms coming from an oven O, Figure 29-9, after passing through defining slits S_1 and S_2, was allowed to pass through an inhomogeneous magnetic field and strike a plate P. The inhomogeneous field was produced by an electromagnet with specially designed pole pieces. One pole piece was in the form of a knife-edge, while the other pole piece had a channel cut in it parallel to the

knife-edge. From the pattern on the photographic plate, shown in Figure 29-10, and a knowledge of the speed of the particles, the magnetic moment

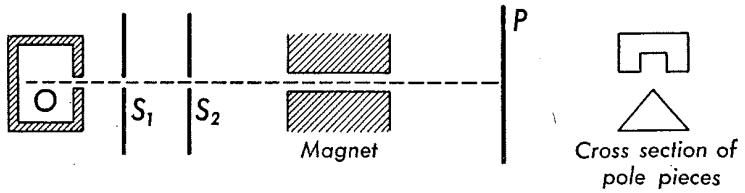


Fig. 29-9 Arrangement of apparatus in Stern-Gerlach experiment.

of silver could be calculated. The magnetic dipole moment of silver, m_{Ag}, is now known to be

$$m_{Ag} = 0.93 \times 10^{-20} \text{ emu}.$$

It may be remarked parenthetically that the magnetic moment of the silver atom, as measured in the Stern-Gerlach experiment, is actually the magnetic moment of the valence electron of silver. A glance at Table 5 of Appendix A shows that silver, in the normal state, has a single electron,

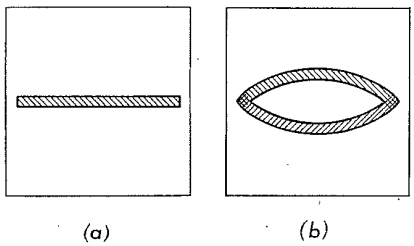


Fig. 29-10 Type of pattern made by a beam of silver atoms (a) without magnetic field on; (b) with magnetic field on.

an s electron outside of a closed, stable configuration of the other 46 electrons. The sum of the magnetic moments of these 46 electrons is zero. The magnetic moment of an electron is ascribed to its intrinsic spin. One interesting result of the above experiment was that the silver atom was deflected in only one of two possible directions, as indicated by the splitting of the line into two lines. The interpretation of this is that a spinning electron, when in a magnetic field, cannot have any arbitrary direction in space. The spin of an electron may be considered as a vector quantity, since there is an angular momentum associated with it directed along the axis of spin. The magnetic moment is also a vector quantity directed along this axis. When an electron enters a magnetic field, its axis can take only one of two possible orientations, either parallel to the direction of the

magnetic field or antiparallel, that is, oppositely directed, depending upon the direction of its spin. This limitation of the possible orientations of the axis of spin of an electron is called *space quantization*.

Measurements of the magnetic moment, by the methods of Stern and Gerlach and by other methods developed subsequently, have been made of atoms, molecules, and subatomic particles such as electrons, protons, neutrons, and many nuclei. These measurements are an important part of our knowledge of the properties of matter.

29-6 Terrestrial Magnetism

We have already shown that a bar magnet, when suspended so that it can swing freely about a vertical axis, will ultimately come to rest in an approximately north-south position. The use of a compass is based upon this

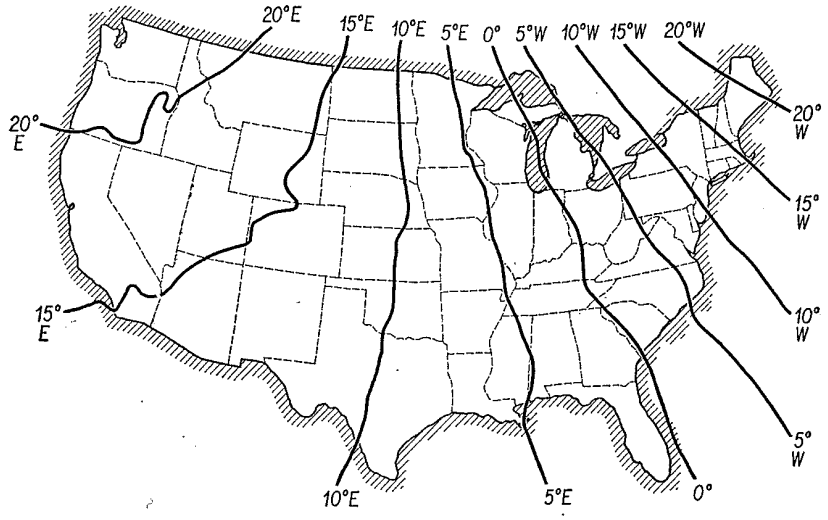


Fig. 29-11 Chart showing isogonic lines for the United States, 1955. From the 1955 edition of chart 3077, of the U.S. Coast and Geodetic Survey, showing distribution of magnetic declination throughout the United States. (Courtesy of Coast and Geodetic Survey.)

observation. It is important, however, to know how much the magnetic north-south direction differs from the geographic or true north-south direction. Various surveys have been made to determine this difference for various points of the earth's surface. This difference is termed the *declination* of the magnetic from the geographic north-south direction. It differs from point to point on the earth's surface and also is found to vary in a

complex manner with time. Charts are plotted for any region of the earth's surface with lines showing equal declinations, or *isogonic* lines, marked on them. One such chart is shown in Figure 29-11. In New York the declination is about 11°W, in Detroit it happens to be 3°W, while in San Francisco it is about 18°E.

The declination gives the direction of the horizontal component only of the earth's magnetic field at any one point. The fact that there is a vertical component to the earth's magnetic field can be shown by the following simple experiment. A long unmagnetized steel needle is mounted so that it can pivot about a horizontal axis through its center of gravity. It is balanced in the horizontal position and pointed in the direction in which a compass needle would point. The needle is now magnetized by means of a strong magnet and then allowed to swing freely. It will be found that the magnetized needle no longer balances in the horizontal position but dips with its north pole downward. The angle of dip varies from place to place over the earth. At New York the angle of dip is 72° with the horizontal, near the equator it is almost 0°, and at the magnetic pole in the Northern Hemisphere it is 90°. In this manner the earth's magnetic poles are located.

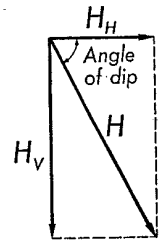


Fig. 29-12 Angle of dip.

When the dip needle is properly used as described above, its direction is that of the earth's magnetic field at the given place. The horizontal component H_H of the intensity of the earth's magnetic field can easily be determined by means of an oscillating horizontal magnet, as described above. By means of a simple vector diagram, illustrated in Figure 29-12, the total magnetic field intensity H and its vertical component H_V can be computed.

The earth behaves as though it contained a very large magnet with its south pole in the Northern Hemisphere located at and below Boothia Peninsula in northern Canada, and the other pole in the Southern Hemisphere near Ross Sea in Antarctica. The origin of the earth's magnetic field is at present unknown. The hypothesis that seems most reasonable is that the earth's magnetic field is produced by electric currents within the earth, probably within its core. If the above hypothesis is accepted, one must explain the source of these electric currents.

29-7 A Molecular Theory of Magnets

An insight into the origin of the magnetic properties of magnetic substances can be obtained from the following simple experiment. First magnetize a long steel needle or steel strip; then break the magnet in half. By dipping

these parts in iron filings, it will be found, as sketched in Figure 29-13, that each half is itself a magnet. By testing the polarity of each part, it will be found that opposite poles appear on the faces where the break was made. If one of these parts is again broken in half, we will again have two magnets. This process can be repeated until the magnets become too small to handle. Each part broken from the original magnet will always be found to be magnetized. This suggests that a magnet is made up of very tiny magnets, perhaps of molecular size.

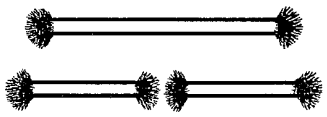


Fig. 29-13 A bar magnet broken in half forms two bar magnets.

On this theory of magnetism, magnetic substances are assumed to contain little magnets of approximately molecular size. When the bar of steel is unmagnetized, these molecular magnets are oriented at random, as sketched in Figure 29-14(a), so that the bar shows no polarity and has no net magnetic moment. But when the bar of steel is brought into a magnetic field, its molecular magnets rotate until they line up with the field, as shown in Figure 29-14(b); the bar thus acquires magnetization, magnetic poles, and a magnetic moment. When the steel bar is removed from the magnetic field, most of these little magnets retain their orientation so that the steel bar remains permanently magnetized, although its magnetic moment may not be as big as it was when in the magnetic field. Other substances, such as soft iron, lose most of their magnetism when taken out of the magnetic field.

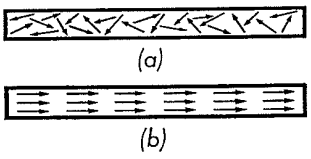


Fig. 29-14 (a) Molecular magnets in an unmagnetized steel bar have random orientations. (b) When the steel bar is magnetized, the molecular magnets line up parallel to the external field, producing poles at the ends of the bar.

Other evidence in support of this molecular theory of magnetism may be cited. If a magnetized steel bar is dropped, hammered, or otherwise jarred, it loses some of its magnetism. If it is heated to a temperature beyond about 800°C, it loses its magnetic properties, probably because of the fact that the increased vibratory motion of the molecules destroys the orientation of the molecular magnets. This molecular theory is not, of course, the complete explanation. One would like to know the origin of these elementary magnets and why some substances are magnetic while others apparently are not magnetic. We shall consider this topic again after we have shown the relationship between electricity and magnetism.

29-8 A Theory of Magnetism

From Coulomb's law it is possible to develop a theory of magnetism based upon the magnetic dipole as the basic element of magnetism, and upon the auxiliary concept of the magnetic pole. Such a theory of magnetism is quite analogous to the theory of electrostatics developed in earlier chapters. In this theory the *magnetic field intensity* **H** is analogous to the electric field intensity **E**. The contribution to the magnetic field intensity at a point due to a magnetic pole of strength p can be written in an equation analogous to Equation (23-2) as

$$\mathbf{H} = k_1 \frac{p}{r^2} \mathbf{1}_r, \tag{29-11}$$

where, in the cgs system of units,

cgs units $$k_1 = 1$$

in vacuum, and **H** is expressed in oersteds when p is expressed in cgs unit poles and r is expressed in centimeters. In the mks system of units,

mks units $$k_1 = \frac{1}{4\pi\mu_0} \tag{29-12}$$

in vacuum, and **H** is expressed in newtons per weber when p is expressed in webers, r is expressed in meters, and

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{weber}^2}{\text{nt m}^2}.$$

In many problems associated with the magnetic fields due to permanent magnets, it is helpful to define a magnetic potential similar to the electric potential defined in electrostatics. One essential difference between electrostatics and magnetostatics lies in the fact that there are no *conductors* of magnetism. Another difference which we have already emphasized is that the isolated magnetic pole is a useful mental construct rather than an experimentally observed entity.

To carry the analogy further, we utilize the concept of *magnetic lines of force*, developed as in Section 23-5 for the electric field. There are 4π lines of force emanating from a cgs unit north pole, and $1/\mu_0$ lines of force emanating from an mks unit north pole of pole strength 1 weber, as in Equations (23-5). Since the source of the magnetic field is the magnetic dipole, every volume element must contain as much positive magnetism as negative magnetism, so that the net number of magnetic lines of force emerging from any Gaussian surface is zero.

In dealing with the magnetic field in the material medium, we define the *magnetization* **M** as the *magnetic moment per unit volume*, just as, in a dielectric, we spoke of the polarization **P** as the electric moment per unit volume. The magnetic medium differs from the dielectric medium in that a permanent state of magnetization is commonly found in magnetic materials, while the polarization of most dielectric materials is only found in the presence of an electric field.

It is possible to produce a state of permanent electric polarization in some waxes by cooling them slowly in an electric field. These permanently polarized waxes are called *electrets*, and generate an electric field similar to the magnetic field generated by magnets. When electrets are left alone for a time, electric charges, in the form of atmospheric ions, collect on their surfaces in such a way as to neutralize the electric field generated by the electret. The electret may be revived by scraping away its surface, removing the collected ions. The fact that permanent magnets are not diminished in strength by exposure to the atmosphere may be cited as evidence that free magnetic poles do not exist in the atmosphere.

In addition to the permanent magnetization, we describe the induced magnetization in a magnetic material by the magnetic susceptibility χ_m (chi sub m) so that the induced magnetization could be written as

$$\mathbf{M} = \chi_m \mathbf{H}, \tag{29-13}$$

analogous to Equation (25-12).

Just as it was convenient to describe the electric field in the material medium by means of the electric displacement **D**, so it is convenient to describe the magnetic field in terms of the magnetic induction **B**, defined by equations analogous to Equation (25-24) as

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}. \tag{29-14a}$$

In the mks system of units, the magnetization **M**, the magnetic moment per unit volume, is expressed in webers per square meter, and the product $\mu_0 \mathbf{H}$ is similarly expressed in webers per square meter. Thus the units of magnetic induction **B** are expressed in webers per square meter. We may describe the relationship between **B** and **H** in a linear medium through the equation

$$\mathbf{B} = \mu \mathbf{H}, \tag{29-15a}$$

where

$$\mu = \kappa_m \mu_0. \tag{29-16}$$

In these equations μ is called the *permeability* of the medium, and κ_m is called the relative permeability of the medium. The value of κ_m for vacuum is unity.

In the cgs system of units Equation (29-14a) may be re-expressed as

cgs units

$$\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}. \tag{29-14b}$$

The units of magnetic induction **B** are expressed in *gausses*. The terminology for this system of units is not so well worked out as in the case of mks units, so that each quantity in Equation (29-14b) is expressed in different units. Thus **H** is expressed in oersteds and **M** is expressed in cgs poles per square centimeter. The linear relationship between **B** and **H**, for a linear medium, is then expressed as

cgs units $$\mathbf{B} = \kappa_m \mathbf{H}. \tag{29-15b}$$

This neat analogy between the electric field and the magnetic field is very useful, but it neglects the most important of all magnetic effects, the generation of a magnetic field by an electric current. It was discovered by Hans Christian Oersted in 1820 that an electric current in a wire caused a compass needle to be deflected. Subsequent investigations have shown that the magnetic field exerts a force on a moving charge, and that a changing magnetic field induces a current in a wire. The theory outlined above must be supplemented to include these effects.

In contrast to the case of the electric field in which the electric displacement **D** plays a minor role, important in simplifying calculations but not at the foundation of electrostatics, the magnetic induction **B** is of great significance, for, as we shall see, **B** describes the effects of the field on a moving charge, while **H** describes the effects of the field upon a pole. From a fundamental physical viewpoint the pole is a property of bulk matter and is not a fundamentally significant magnetic parameter. Since isolated poles do not exist in nature, we are forced to relegate the magnetic field intensity **H** to the role of an auxiliary quantity and to think of the magnetic induction **B** as the fundamental magnetic-field vector. The sources of the magnetic field are today understood to be electric currents and the magnetic dipole moments of the fundamental physical particles, while the probe by means of which we are enabled to examine the magnetic field is either a moving charged particle or an elementary magnetic dipole.

Problems

29-1. A bar magnet having a pole strength of 400 cgs poles and a length of 5 cm is placed so that its south pole is at the origin and its north pole is located at the point whose coordinates are (5 cm, 0). Find the magnitude and direction of the force exerted by this magnet on the north pole of a long, slender, needlelike magnet whose pole strength is 20 cgs poles, when that pole is placed at a point whose coordinates are (a) (0, 10 cm), (b) (−10 cm, 0), and (c) (5 cm, 3 cm).

29-2. A bar magnet having a pole strength of 10^{-5} weber is placed along the y axis so that its south pole is at the origin and its north pole is located at $y =$ 0.1 m. Find the magnitude and direction of the force exerted by this magnet on the north pole of a needlelike magnet of pole strength 10^{-6} weber when that north pole is located at a point whose coordinates are (a) (0, −0.2 m), (b) (0.24 m, 0), and (c) (0.03 m, 0.05 m).

29-3. Two identical bar magnets are placed side by side with their axes parallel and separated by a distance of 10 cm. Find the magnitude and direction of the force between them when like poles face each other. The pole strength of each magnet is 300 cgs poles, and the length of each magnet is 20 cm.

29-4. Repeat the calculation of Problem 29-3 in the case that unlike poles face each other and the pole strength of the magnets is 10^{-5} weber. Perform your calculations in mks units.

29-5. What is the intensity of the magnetic field at a point 20 cm from the north pole of a bar magnet, measured along the axis of the magnet, if the magnet is 30 cm long and has a pole strength of 400 cgs poles?

29-6. What is the magnetic field intensity generated by the bar magnet at each of the points indicated in Problem 29-1?

29-7. What is the magnetic field intensity generated by the bar magnet at each of the points indicated in Problem 29-2?

29-8. A bar magnet 4 cm long, having a pole strength of 12 cgs poles, is placed in a uniform magnetic field with the axis of the magnet perpendicular to the direction of the magnetic field. The intensity of the magnetic field is 45 oersteds. Determine the torque acting on this magnet.

29-9. A small compass needle oscillates with a period of 2.5 sec when placed at a point where the horizontal component of the earth's magnetic field is 0.2 oersted. Determine the magnetic moment of the compass needle if its moment of inertia about the axis of rotation is 24 gm cm^2.

29-10. A bar magnet 6 cm long is suspended by a wire passing through its center. The magnet is placed in a uniform field with the axis of the magnet perpendicular to the direction of the field. The torque, as measured by the twist of the wire, is 1,500 dynes cm. Determine (a) the magnetic moment of the magnet and (b) the pole strength. The intensity of the field is 50 oersteds.

29-11. A small bar magnet has a period of oscillation of 2 sec when pivoted in a uniform magnetic field of 36 nt/weber. When it is used to measure the intensity of a second magnetic field, the period is found to be 0.40 sec. Determine the intensity of the second magnetic field.

29-12. A uniformly magnetized rod has a pole strength of 10 webers. The magnet has a cross-sectional area of 1 cm^2 and a length of 10 cm. (a) What is the magnetic moment of the magnet? (b) What is the magnetization of the magnet?

29-13. By analogy with the electric field in a material medium (Sections 25-6 to 25-8), write a formula for the magnetic field intensity H due to a magnetic pole of pole strength p when that pole is immersed in a medium of relative permeability κ_m (a) in cgs units and (b) in mks units.

29-14. From the result of Problem 29-13, state Coulomb's law of force between two poles immersed in a medium of relative permeability κ_m (a) in mks units and (b) in cgs units.

30

Magnetic Fields of Currents

30-1 Magnetic Field around an Electric Current

The first evidence for the existence of a magnetic field around an electric current was observed in 1820 by Hans Christian Oersted (1777–1851). He found that a wire carrying current caused a freely pivoted compass needle

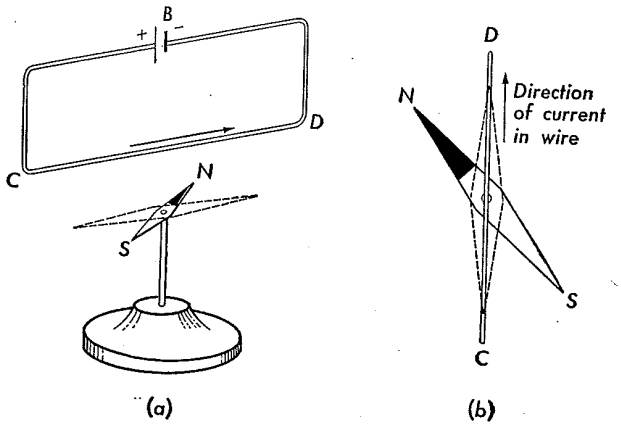


Fig. 30-1 Oersted's experiment. Compass needle is deflected toward the west when the wire CD carrying current is placed above it and the direction of the current is toward the north, from C to D.

in its vicinity to be deflected. If the current in a long straight wire is directed from C to D, as shown in Figure 30-1, a compass needle below it, whose initial orientation is shown in dotted lines, will have its north pole deflected to the left and its south pole deflected to the right. If the current in the wire is reversed and directed from D to C, then the north pole will be deflected to the right, as seen from above. In terms of the forces acting on the poles, these forces are clearly perpendicular to the direction of the current and to the line from the nearest portion of the wire to the pole itself.

The magnetic field in the neighborhood of a wire carrying current can be investigated either by exploring the region with a small compass or by using iron filings. When a wire carrying current is passed perpendicularly through a plane board and iron filings are sprinkled on the board, the

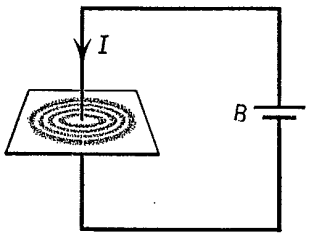


Fig. 30-2 Pattern formed by iron filings showing the circular magnetic field around a wire carrying current.

filings form a circular pattern, as shown in Figure 30-2. Thus the magnetic field generated by the wire carrying current is circular in a plane at right angles to the current. The circles are concentric, with their common center at the position of the wire. The direction of the magnetic field can be

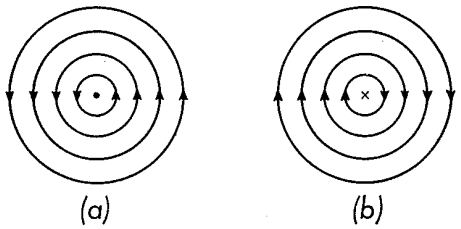


Fig. 30-3 Direction of the magnetic field around a wire (a) when the current is out of the paper; (b) when the current is into the paper. The dot represents a head-on view of an arrow while the cross represents a rear view.

determined with the aid of a small compass. If we look along the wire so that the current is coming toward us, the magnetic field is counterclockwise. If we draw a dot to represent a head-on view of an arrow and a cross to represent a rear view of an arrow, we may show the direction of the magnetic field, associated with current in a wire perpendicular to the plane of the paper when the current is coming toward the reader in Figure 30-3(a), and when the current is away from the reader in Figure 30-3(b). A small compass placed anywhere in the field will orient itself tangent to one of these circles with its north pole in the direction of the arrow.

When the wire carrying current is bent in the form of a circular loop, as shown in Figure 30-4, the direction of the magnetic field around each small portion of the wire may be determined from the above observations.

If the current in the loop is counterclockwise when viewed from the positive x axis, the direction of the magnetic field at the center of the loop is perpendicular to the plane of the loop and in the positive x direction.

A simple way of determining the direction of the magnetic field relative to the direction of the current is given by the *right-hand rule.* If the current

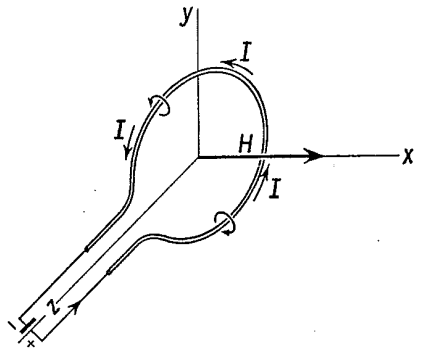


Fig. 30-4 Magnetic field produced by a current in a circular loop of wire. The magnetic field at the center is at right angles to the plane of the loop.

is in a straight portion of the wire, then, if we imagine the thumb of the right hand to be placed along the wire and pointing in the direction of the current, the curled fingers of the right hand will point in the direction of the magnetic field. If the current flows in a circular path, then, if we imagine the fingers of the right hand curving in the direction of the current, the thumb will point in the direction of the magnetic field inside the coil, as in Figure 30-4.

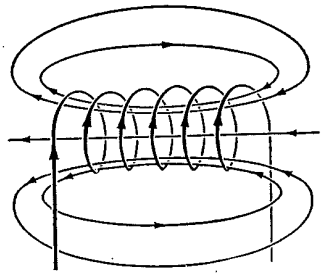


Fig. 30-5 Magnetic field due to a current in a long cylindrical coil or helix.

When, as in Figure 30-5, the wire is wound in the form of a helix which is very long in comparison with its diameter, the magnetic field outside the coil is very similar to that of a bar magnet, while the field inside the coil is uniform and parallel to the axis of the coil. At large distances the field produced by a circular loop of wire carrying current is indistinguishable from the field generated by a small magnet. According to a theory first presented by Ampère, it is possible to imagine that all magnetic effects are due to circulating currents, and to attribute the magnetic fields generated by permanent magnets to circulating currents within molecules. These currents are associated with orbital motions of electrons about some attractive center within the molecule.

30-2 Magnetic Field Generated by a Wire Carrying Current

As soon as Oersted's discovery was announced, scientists in Europe began repeating and extending these experiments. Interest in Oersted's results was aroused because this was the first time a force had been observed which was not directed along the line joining the two bodies but was perpendicular to this line. Since Newton had developed the concept of universal gravitation, both electrostatic and magnetostatic forces had been studied. All these forces obeyed an inverse square law and were directed along the line joining the bodies responsible for the forces. Biot and Savart (1820) determined the magnetic field produced by the current in a long straight wire. The experiments of Biot and Savart showed that the magnetic field intensity H at a distance a from a long straight wire carrying current I was directly proportional to the magnitude of the current and inversely proportional to the distance from the wire; that is,

$$H = k_2 \frac{2I}{a}, \tag{30-1}$$

where k_2 is a constant of proportionality. The direction of the magnetic field intensity was always normal to the wire and to the perpendicular from

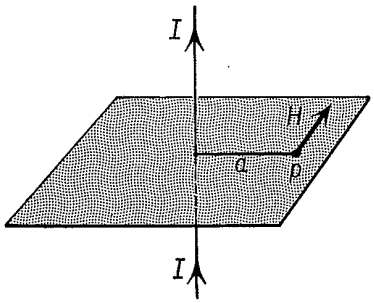


Fig. 30-6 The field at a point P due to a long straight wire is perpendicular to the plane formed by the wire and the normal a dropped from P to the wire. In this figure H is directed into the plane of the paper, as given by the right-hand rule.

the point where the field was being evaluated to the wire. The direction of the magnetic field was given by the right-hand rule, as shown in Figure 30-6.

The value of the constant k_2 depends upon the system of units used to measure H, I, and a. In the mks system of units, the current I is expressed in amperes, the distance a is expressed in meters, and the magnetic field intensity H is expressed in mks units, stated in Section 29-3 as newtons per weber. In this system of units the constant k_2 is $\frac{1}{4\pi}$, and Equation (30-1) becomes

$$H = \frac{I}{2\pi a}. \tag{30-1a}$$

In Section 29-3 we mentioned that the units of H in the mks system were commonly expressed as amperes per meter. The justification for expressing H in these terms is provided by Equation (30-1a). The units of H may be stated equivalently as newtons per weber or as amperes per meter, and the choice of units will depend upon which form is more convenient for the problem in hand.

In one form of cgs units called the *Gaussian* system of units, toward which we have been building our cgs unit system, the units of electrical quantities such as charge, current, electric field intensity, electric displacement, potential, and resistance are based upon the statcoulomb as the unit of charge, while units of magnetic quantities, the oersted and the gauss, are based upon the cgs unit pole. In this system the constant k_2 is $1/c$, where c is the velocity of light in centimeters/per second, given approximately as $c = 3 \times 10^{10}$ cm/sec. To simplify the presentation of the material of this chapter, all equations will be presented in the mks system of units.

Traditionally, Equation (30-1) has been used to define a new unit of current called the *electromagnetic unit of current*, by setting k_2 equal to 1. This unit is called the emu of current, or the *abampere*. Electromagnetic units of current will rarely be used in this book.

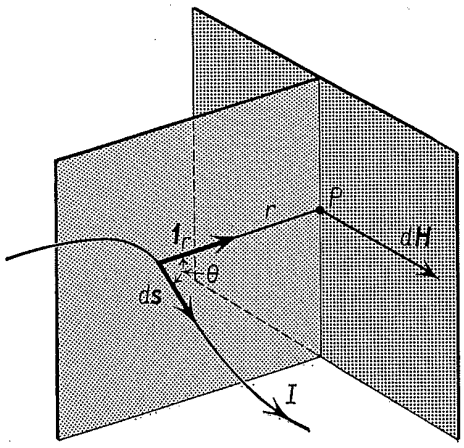


Fig. 30-7 The magnetic field intensity $d\mathbf{H}$ at any point P a distance r from an element $d\mathbf{s}$ of wire carrying current I is perpendicular to the plane of $d\mathbf{s}$ and $\mathbf{1}_r$, where $\mathbf{1}_r$ is the unit vector directed from $d\mathbf{s}$ to P. The element $d\mathbf{H}$ is shown coming out of the plane of the paper when $d\mathbf{s}$ and $\mathbf{1}_r$ lie in the plane of the paper.

30-3 Magnetic Field Generated by Current Element

The results of the experiments of Biot and Savart on the magnetic field around a wire carrying current form the basis of a fundamental physical law for the determination of magnetic fields produced by currents. If we consider any small element of wire of length ds in which there is a current I (see Figure 30-7), its contribution $d\mathbf{H}$ to the magnetic field intensity at a

point P located a distance r from it, is given by

$$d\mathbf{H} = \frac{I\, d\mathbf{s} \times \mathbf{1}_r}{4\pi r^2}. \tag{30-2}$$

The unit vector $\mathbf{1}_r$ is directed from the current element to the field point P. The vector $d\mathbf{s}$ is tangent to the direction of the current. The direction of $d\mathbf{H}$ is perpendicular to both $d\mathbf{s}$ and $\mathbf{1}_r$. Equation (30-2) is sometimes called the law of Biot and Savart, and also *Ampère's law.*

The total magnetic field intensity $\mathbf{H}$ at a point P due to a given current distribution is found by summing vectorially (that is, integrating) the contributions $d\mathbf{H}$ from all the elements $d\mathbf{s}$ of the wire. For most geometrical arrangements of the conductor, this summation is beyond the reach of the calculus and must be carried out by numerical methods of integration or by use of an electronic computer. We shall consider only two cases, the flat circular coil and the long straight wire.

30-4 Magnetic Field of a Flat Circular Coil

Let us use Ampère's law to calculate the magnetic field intensity at the center of a single circular loop of wire carrying a current I. As shown in

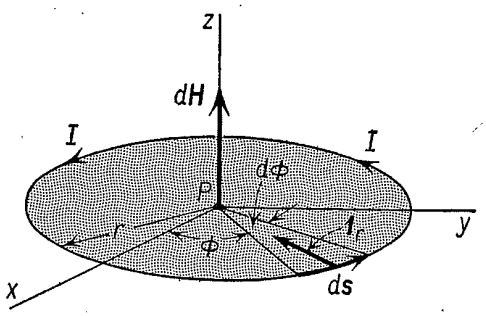


Fig. 30-8

Figure 30-8, each element of the loop of wire makes a right angle with the line drawn from that element to the field point P at the center of the circle. The direction of the magnetic field $d\mathbf{H}$ due to each element of the loop $d\mathbf{s}$ is in the positive z direction when the current is in the counterclockwise direction, as shown in the figure. Since the contribution to the magnetic field from every element of current is in the same direction, the problem of obtaining the vector sum of the contributions to the total magnetic field intensity is reduced to a scalar sum, and we may set up the problem in the form of an integral.

From Equation (30-2) we have

$$d\mathbf{H} = \frac{I\, d\mathbf{s} \times \mathbf{1}_r}{4\pi r^2}.$$

The factor $d\mathbf{s} \times \mathbf{1}_r$ is equal in magnitude to the product of the magnitude ds by the magnitude of $\mathbf{1}_r$ (which is unity), by the sine of the angle between them. Since the angle between the two vectors is 90°, the magnitude of this vector product is simply the magnitude ds. The direction of this vector product is in the $+z$ direction, so that we may represent it as

$$d\mathbf{s} \times \mathbf{1}_r = ds\, \mathbf{1}_z,$$

where $\mathbf{1}_z$ is a unit vector in the positive z direction. Thus we have

$$d\mathbf{H} = \frac{I\, ds}{4\pi r^2} \mathbf{1}_z,$$

where the magnitude of $d\mathbf{H}$ is the factor multiplying $\mathbf{1}_z$.

In order to carry out the integration, we must represent the variables of the problem in terms of some simple parameter. Let us measure the angular position of an element of current by the angle ϕ, from the x axis, positive in the counterclockwise direction. The angle subtended by the element of length ds is $d\phi$ such that

$$ds = r\, d\phi.$$

Thus we have

$$dH = \frac{Ir\, d\phi}{4\pi r^2},$$

and, integrating around the circumference of the circle by letting ϕ go from 0 to 2π, we find

$$H = \int_0^{2\pi} \frac{I\, d\phi}{4\pi r}$$

$$= \frac{I}{2r}.$$

The complete representation of the magnetic field intensity at the center of a single circular loop of wire of radius r carrying a current I in the counterclockwise direction is given by

$$\mathbf{H} = \frac{I}{2r} \mathbf{1}_z. \tag{30-3}$$

In Equation (30-3) the appropriate choice of units to represent H is amperes per meter when I is in amperes and r is expressed in meters.

In the event that we have a coil of N turns of wire wound as a flat circular coil rather than a single circular loop, each turn of the coil contributes a magnetic field at the center of the coil, as given by Equation (30-3). Since these field contributions are all in the same direction, the total magnetic field is N times that given by Equation (30-3). A circular coil of wire of N turns carrying current I in the counterclockwise direction, when viewed from the positive z axis, as in Figure 30-8, has at its center a magnetic field $\mathbf{H}$ given by

$$\mathbf{H} = \frac{NI}{2r}\mathbf{1}_z. \tag{30-4}$$

One of the early forms of current-measuring instruments was the *tangent galvanometer*, which consisted of a single circular loop of wire with a short compass needle placed at its center, as shown in Figure 30-9. When a current I was sent through the loop, it set up a magnetic field of intensity

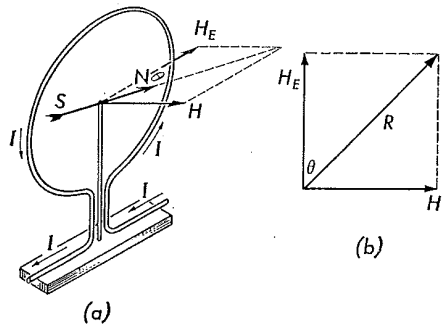


Fig. 30-9 (a) Tangent galvanometer. The plane of the circular coil is parallel to the earth's magnetic field. (b) The two magnetic fields which act on the N-pole of the small compass needle. H_E is the horizontal component of the earth's magnetic field; H is the intensity of the magnetic field produced by the current. R is the resultant of these two field intensities. $H = H_E \tan \theta$.

H at its center, producing a force Hp on each pole of the magnet. In the figure the force on the north pole is shown directed to the right, while that on the south pole is directed to the left. These two forces produced a torque causing the compass needle to rotate out of the plane of the coil. The horizontal component of the earth's magnetic field produced an opposite torque on the compass needle. The compass needle came to rest at some angle θ with the plane of the loop, at which angle the two opposing torques were equal in magnitude. A simple calculation shows that the current in the loop is proportional to the tangent of the angle θ, hence the name tangent galvanometer. This was the instrument refined by Lord Kelvin as the detector for use in the first successful operation of the Atlantic cable.

Illustrative Example. A circular coil of 30 closely wound turns of 15 cm average radius carries a current of 2 amp. Determine the magnitude and direction of the magnetic field at the center of this coil.

Assuming that the turns are so closely wound that the current in each one contributes the same amount to the intensity of the magnetic field at the center of the coil, the magnetic field intensity is of magnitude

$$H = \frac{NI}{2r}.$$

Substituting numerical values, we have

$$H = \frac{30 \text{ turns} \times 2 \text{ amp}}{2 \times 0.15 \text{ m}} = 200 \frac{\text{amp}}{\text{m}}.$$

Note that the *turn* is a dimensionless quantity. The direction of the magnetic field is perpendicular to the plane of the coil, as given by the right-hand rule.

30-5 Magnetic Field of a Long Straight Wire

The experimental result of Biot and Savart given in Equation (30-1) may be calculated from Equation (30-2) by integrating the contributions to the field from each element of the wire. The field point P is located a distance a from the wire, as shown in Figure 30-10. Let us measure the distance s along the wire from the foot of the perpendicular from P to the wire.

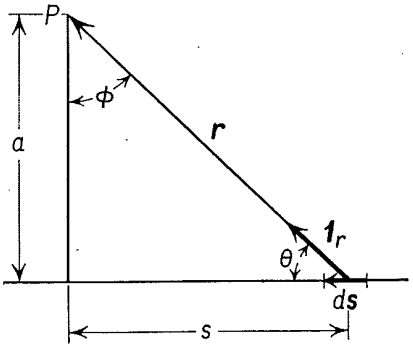


Fig. 30-10

In terms of the angle θ between the element ds and the unit vector $\mathbf{1}_r$ from the element ds to the field point P, the distance r may be represented as

$$r = \frac{a}{\sin \theta} = a \csc \theta.$$

To express the length of the element ds in terms of the distance a and the angle θ, we first note that the length s is given by

$$s = a \cot \theta,$$

so that, taking differentials to obtain the magnitude of ds,

$$ds = -a \csc^2 \theta \, d\theta.$$

We note that contribution to the magnetic field from each element of wire is in the same direction at P; that is, it is directed out of the plane of the

paper, toward the reader. If we call this direction the direction of z, then

$$d\mathbf{H} = + \frac{I\, ds \sin\theta}{4\pi r^2} \mathbf{1}_z.$$

Substituting appropriate expressions into the above equation, we find

$$dH = - \frac{Ia \csc^2\theta\, d\theta \sin\theta}{4\pi a^2 \csc^2\theta}$$

$$= - \frac{I \sin\theta\, d\theta}{4\pi a}.$$

Since all contributions to the magnetic field at P are in the same direction, we may find the magnetic field intensity at P by integrating between the limits of $\theta = \pi$ to $\theta = 0$, corresponding to limits on s of $s = -\infty$ to $s = +\infty$. Thus we have

$$H = \int_{\pi}^{0} - \frac{I}{4\pi a} \sin\theta\, d\theta,$$

$$H = \frac{I}{2\pi a},$$

as previously stated in Equation (30-1a).

30-6 Field of a Solenoid and a Toroid

A solenoid is made by winding wire on a cylindrical form; the length s of the cylinder is generally much larger than its radius r. The adjacent turns of the wire are generally close together, as shown in Figure 30-11. The field inside the solenoid may be calculated from Ampère's law. The field is uniform over the entire region of the solenoid except near the ends, as

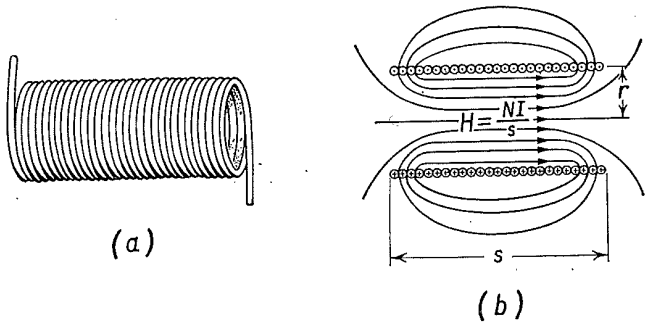


Fig. 30-11 (a) Windings of a solenoid. (b) Magnetic field intensity inside a solenoid. Current is coming out of the wires on top and going into the wires on bottom.

shown in the figure. The field intensity within a solenoid will be shown in Section 33-6 to be

$$H = \frac{NI}{s}, \tag{30-5}$$

where N is the number of turns of the solenoid, s is its length, and I is the current in it.

A toroid is made by winding wire on a ring or doughnut-shaped form called a *torus*. The field of the toroid is entirely confined to the space within the toroid. A toroid may be generated by bending a long straight solenoid

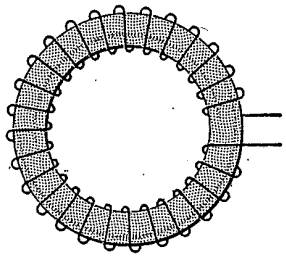


Fig. 30-12 Toroid.

in the form of a ring. The magnetic field intensity within the toroid is given by Equation (30-5), where s now represents the mean circumference of the ring, shown in Figure 30-12.

The solenoid and the toroid are frequently used as standard means of achieving known, uniform magnetic fields.

30-7 Equivalence of a Moving Charge and a Current

Our original definition of current was based on the flow of charges through a surface in a given time interval; it remains to be shown that a set of moving charges will produce the same magnetic effect as a current in a wire. This was first shown experimentally by H. A. Rowland in 1876. He used an ebonite disk having metallic sectors distributed near the rim of the disk. The metallic sectors were charged electrically, and the disk was set into rapid rotation. A magnetic needle suspended near the disk was deflected by the magnetic field set up by the moving charges. The direction of the deflection was the same as that which would have been produced by the current in a circular loop of wire coinciding with the rim of the disk. When the direction of rotation was reversed, the deflection of the magnetic needle was also reversed. More recently (1929) R. C. Tolman set a charged cylinder oscillating about its axis and observed that this produced an alternating magnetic field, the same as that produced in the neighborhood

of an alternating current. These experiments show that, as far as the magnetic effect is concerned, a moving charge and a current in a wire are equivalent.

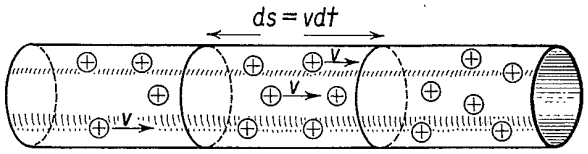


Fig. 30-13

Figure 30-13 shows a tube in which a current I consists of the motion of positive charges with uniform velocity v. From the definition of current

$$I = \frac{dq}{dt},$$

where dq is the quantity of charge which passes through a complete cross section of the tube in a time dt. The charge will traverse a length ds of the tube, where

$$ds = v\,dt.$$

Eliminating dt from these two equations yields

$$I\,ds = v\,dq.$$

In other words, a charge dq moving with velocity v may be considered to be equivalent to a current element of length ds carrying a current I.

We may find the magnetic field H generated by a moving charge q by substituting the above results into Equation (30-2). We find

$$\boxed{\mathbf{H} = \frac{q\mathbf{v} \times \mathbf{1}_r}{4\pi r^2}.} \qquad (30\text{-}6)$$

30-8 Coulomb's Law and Ampère's Law

Let us consider the magnetic field generated by a charged particle mounted on an airplane. If we calculate the magnetic field generated by the moving charge from Equation (30-6), we note that, to an observer on the ground, the charge is moving with the velocity v of the airplane, but, to an observer on the airplane, the charge is moving with zero velocity. Thus an observer on the surface of the earth is able to detect a magnetic field due to the motion of the charged particle on the airplane, but an observer on the airplane cannot detect a magnetic field due to the charge

on the airplane, for that charge has no motion with respect to him. By definition, the magnetic field intensity H in a particular reference frame is the force on a unit north pole at rest in that frame. If our formulation is to be consistent, we must expect that the force exerted by a charge upon a magnetic pole is the same when these are in relative motion, regardless

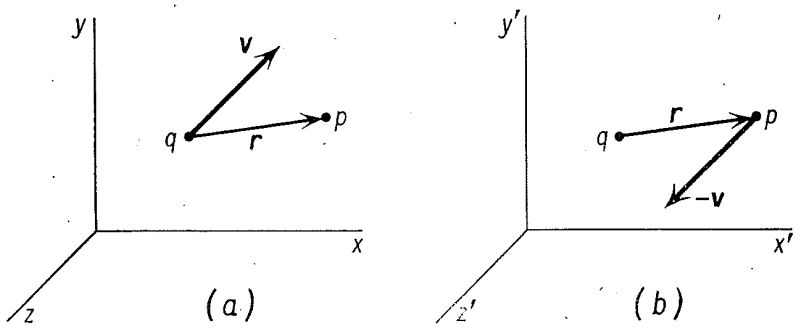


Fig. 30-14 (a) System of axes in which the magnetic pole is at rest. (b) System of axes in which the charge q is at rest.

of whether the pole is considered to be at rest or the charge is considered to be at rest.

In the coordinate frame in which the pole of strength p is at rest, as shown in Figure 30-14(a), the charge q is moving with velocity $\mathbf{v}$, and the magnetic field $\mathbf{H}$ at the position of the pole, due to the moving charge, is given by Equation (30-6) as

$$\mathbf{H} = q\frac{\mathbf{v}\times\mathbf{1}_r}{4\pi r^2}.$$

The force on the magnetic pole at this point is therefore

$$\mathbf{F} = \mathbf{H}p = qp\frac{\mathbf{v}\times\mathbf{1}_r}{4\pi r^2}.$$

If we consider the frame of reference to be fixed with respect to the charge q, as shown in Figure 30-14(b), we must consider the charge to be at rest and the pole to be moving with a velocity $\mathbf{v}' = -\mathbf{v}$. Substituting this into the above equation, and rearranging the position of q in the equation we find

$$\mathbf{F} = p(-\mathbf{v}')\times\frac{q\,\mathbf{1}_r}{4\pi r^2}$$

for the force exerted on the moving pole by the fixed charge. From the illustrative example of Section 25-8, we note that the terms grouped to the

right of the cross in the above equation represent precisely the electric displacement **D** generated by the charge q at the position of the pole p, for

$$\mathbf{D} = \frac{q\,\mathbf{1}_r}{4\pi r^2}.$$

Thus the force on a pole p, moving with velocity **v** (dropping the prime) in a coordinate frame in which the electric displacement is **D**, is given by

$$\mathbf{F} = -p(\mathbf{v}\times\mathbf{D}).$$

Thus the force experienced by a moving magnetic pole consists of two parts. One part is due to the magnetic field **H**; this force on the pole is independent of its state of rest or motion. The second part depends upon the motion of the poles; this force is due to the velocity of the pole and to the electric displacement **D**. In the form of an equation, the force on a magnetic pole, moving with velocity **v** in a coordinate frame in which the magnetic field intensity is **H** and the electric displacement is **D**, is given by

$$\mathbf{F} = p(\mathbf{H} - \mathbf{v}\times\mathbf{D}). \tag{30-7}$$

Another interpretation of Equation (30-7) is that the force exerted on a magnetic pole by a charged particle may be calculated in one of two ways: (a) for the case in which the pole is considered to be at rest and the charge in motion, we can use Ampère's law; (b) for the case in which the charge is considered to be at rest and the pole is in motion, we can use Coulomb's law, utilizing the concept of electric displacement. In Rowland's experiment a moving charge was seen to deflect a compass needle. From Equation (30-7) we see that we must expect a compass needle to be deflected when the needle is moving in a region of space in which there is an electric

TABLE 30-1 PRINCIPAL EQUATIONS IN MKS AND GAUSSIAN UNITS

Equation	MKS	Gaussian	
(30-2)	$d\mathbf{H} = \frac{I\,d\mathbf{s}\times\mathbf{1}_r}{4\pi r^2}$	$d\mathbf{H} = \frac{I\,d\mathbf{s}\times\mathbf{1}_r}{r^2 c}$	Current element
(30-1a)	$H = \frac{I}{2\pi a}$	$H = \frac{2I}{ac}$	Straight wire
(30-5)	$H = \frac{NI}{s}$	$H = \frac{4\pi NI}{sc}$	Solenoid or toroid
(30-6)	$\mathbf{H} = \frac{q\mathbf{v}\times\mathbf{1}_r}{4\pi r^2}$	$\mathbf{H} = \frac{q\mathbf{v}\times\mathbf{1}_r}{r^2 c}$	Moving charge
(30-7)	$\mathbf{F} = p(\mathbf{H} - \mathbf{v}\times\mathbf{D})$	$\mathbf{F} = p\left(\mathbf{H} - \frac{\mathbf{v}}{c}\times\mathbf{D}\right)$	Pole

field, for example, if a compass needle moves between the plates of a charged capacitor.

TABLE 30-2 CONVERSION FACTORS RELATING MKS AND GAUSSIAN UNITS

Quantity	Symbol	MKS Unit	Gaussian Unit	
Pole	p	1 weber	$= \frac{10^8}{4\pi}$ unit pole	(emu)
Magnetic field intensity	H	$1\frac{\text{nt}}{\text{weber}} = 1\frac{\text{amp}}{\text{m}}$	$= 4\pi \times 10^{-3}$ oersted	(emu)
Charge	q	1 coul	$= 3 \times 10^9$ stcoul	(esu)
Current	I	1 amp	$= 3 \times 10^9$ statamperes	(esu)
Displacement	D	1 coul/m^2	$= 3 \times 10^5$ stcoul/cm^2	(esu)

Problems

30-1. Determine (a) in mks units and (b) in Gaussian units the intensity of the magnetic field at the center of a circular coil of 6 cm radius when it carries a current of 15 amp.

30-2. Determine (a) in mks units and (b) in Gaussian units the magnetic field intensity at the center of a closely wound circular coil of 75 turns, whose average radius is 9 cm, when the coil carries a current of 4 amp.

30-3. A circular coil of 20 cm radius consists of a single turn of wire and has a small compass needle suspended at its center so that it can swing freely about a vertical axis. The plane of the coil is set parallel to the earth's magnetic field. If the horizontal component of the earth's magnetic field is 0.2 oersted, determine the angle through which the compass needle is deflected when a current of 3 amp is sent through the coil.

30-4. Determine (a) in mks units and (b) in Gaussian units the magnetic field intensity at a distance of 0.25 m from a long straight wire in which there is a current of 32 amp.

30-5. A small compass needle, whose magnetic moment is 40 cgs poles cm and whose moment of inertia is 6.0 gm cm^2, is placed at a point 15 cm from a long straight wire. When a current is sent through the wire, the needle oscillates with a period of 7.2 sec. Determine (a) the magnetic field intensity at the position of the compass and (b) the current in the wire.

30-6. A solenoid 0.75 m long and 0.08 m in diameter is wound with 400 turns of wire. Determine the magnetic field intensity inside this solenoid when it carries a current of 6.5 amp.

30-7. A long straight wire carries a current of 25 amp. The north pole of a long bar magnet is placed 5 cm from this wire. If the pole strength of this magnet is 40 cgs poles, determine the direction and magnitude of the force on this north pole.

30-8. A long straight wire carries a current of 20 amp. The wire is parallel

to the z axis, and the current flows in the positive z direction. In addition to the magnetic field generated by the current in the wire, there is an external magnetic field in the positive z direction of intensity 5 amp/m. Find the resultant magnetic field (a) at a point in the x-y plane whose coordinates are (5 m, 0) and (b) at a point in the x-y plane whose coordinates are (0, 10 m).

30-9. Derive an equation, in Gaussian units, for the magnetic field intensity at a point in the neighborhood of a long straight wire carrying current.

30-10. Derive an equation, in Gaussian units, for the magnetic field intensity at the center of a circular loop of wire.

30-11. Find the magnetic field intensity within a toroid of mean radius 10 cm wound with 100 turns of wire when the current in the toroid is 4 amp. The circular cross section of the toroid has a diameter of 1 cm.

30-12. Two long straight wires are 18 cm apart and carry currents of 36 amp each. Determine the magnetic field intensity at a point midway between them (a) when these currents are in the same direction and (b) when the currents are in opposite directions. Express your answers in both Gaussian and mks units.

30-13. In the Oersted experiment a long straight wire placed 4 cm above a compass needle carries a current of 28 amp directed to the north. Determine the torque on the compass needle if its magnetic moment is 75 cgs pole cm.

30-14. Find the force on a magnetic pole of 10^{-4} weber moving in the $+x$ direction with a speed of 10^3 m/sec between the plates of a parallel-plate capacitor in vacuum. The plates of the capacitor are 1 cm apart and are charged to a potential difference of 500 volts. The electric field between the plates of the capacitor is in the $+y$ direction.

31

Forces on Moving Charges and Currents

31-1 Force on a Charge Moving in a Magnetic Field

Ampère was the first to show that wires carrying current experienced forces when placed in magnetic fields. Later it was shown that charged particles moving in magnetic fields also experience forces. Common applications of these phenomena are the electric motor, the galvanometer, and the cathode-ray tube. Since a moving electric charge is equivalent to a current, we shall first consider the force acting on a charge q moving with velocity $\mathbf{v}$ in a magnetic field of induction $\mathbf{B}$. Experiment shows that the force $\mathbf{F}$ acting on the charge q is at right angles to the directions of both $\mathbf{v}$ and $\mathbf{B}$. This provides us with a basic distinction between the two magnetic field vectors $\mathbf{B}$ and $\mathbf{H}$.

We have already shown that the force on a magnetic pole at rest is determined by the magnetic intensity $\mathbf{H}$. We have seen that moving charges and currents produce $\mathbf{H}$. The relation between the magnetic induction $\mathbf{B}$ and the magnetic intensity $\mathbf{H}$ has been given in Equation (29-15a) as

$$\mathbf{B} = \mu\mathbf{H},$$

where μ is the permeability of the medium.

The force $\mathbf{F}$ on a charge q moving with velocity $\mathbf{v}$ in a field of induction $\mathbf{B}$ is given by

$$\mathbf{F} = k_3 q\mathbf{v}\times\mathbf{B}. \tag{31-1}$$

As shown in Figure 31-1 the force is perpendicular to the plane formed by the vectors $\mathbf{v}$ and $\mathbf{B}$ and is directed in accordance with the right-hand rule for the vector product. The constant k_3 is a constant of proportionality which is assigned the value unity in the mks system of units. Thus if the force $\mathbf{F}$ is expressed in newtons, the charge q in coulombs, the velocity $\mathbf{v}$ in meters per second, and the magnetic induction $\mathbf{B}$ in webers per square meter, Equation (31-1) may be rewritten as

$$\boxed{\mathbf{F} = q\mathbf{v}\times\mathbf{B}.} \tag{31-2}$$

We may consider Equation (31-2) as a definition of **B**. From the above equation we see that if **v** is parallel to **B**, the force on the moving charge is zero. This suggests a method of determining the direction of **B** by use of cathode-ray tubes. If a charge of 1 coul moving perpendicular to the field with a velocity of 1 m/sec experiences a force of 1 nt, then the field has an induction of 1 weber/m^2.

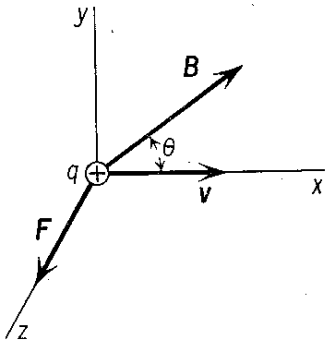


Fig. 31-1 The force **F** exerted on a charge q moving with velocity **v** in a magnetic field of induction **B**. **B** and **v** are in the x-y plane; **F** is along the z axis.

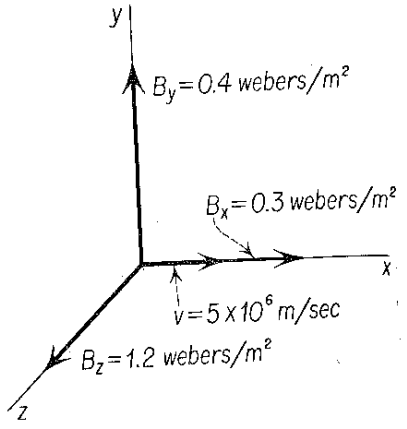


Fig. 31-2

Illustrative Example. A proton whose charge is 1.60×10^{-19} coul is moving in the x direction with a velocity of 5×10^6 m/sec in a magnetic field whose components are $B_x = 0.3$ weber/m^2, $B_y = 0.4$ weber/m^2, $B_z = 1.2$ weber/m^2. See Figure 31-2. Find the x, y, and z components of the force on the proton.

The x component of the field is parallel to the velocity; thus the force on the moving charge due to B_x is zero.

From Equation (31-2) the direction of the force on the particle due to the y component of the field is in the z direction, and since the angle between **v** and $\mathbf{B}_y$ is 90° we have

$$F_z = qvB_y = 1.60 \times 10^{-19} \text{ coul} \times 5 \times 10^6 \text{ m/sec} \times 0.4 \text{ weber/m}^2,$$

so that

$$F_z = 3.2 \times 10^{-13} \text{ nt}.$$

Applying the right-hand rule for the vector product, we see that the force on the particle due to the z component of the field is in the $-y$ direction. The angle between **v** and $\mathbf{B}_z$ is 90°. The y component of the force on the particle is therefore

$$F_y = -1.60 \times 10^{-19} \text{ coul} \times 5 \times 10^6 \text{ m/sec} \times 1.2 \text{ weber/m}^2,$$

$$F_y = -9.6 \times 10^{-13} \text{ nt}.$$

The resultant force F is of magnitude

$$F = (F_y^2 + F_z^2)^{1/2} = 9.74 \times 10^{-13} \text{ nt}.$$

In the Gaussian system of units the constant k_3 has the value $1/c$, and we must express the units of B in gausses and of H in oersteds. As before, we must express q in statcoulombs, I in statamperes, v in centimeters per second, and F in dynes. The relationship between B and H is

$$\mathbf{B} = \kappa_m \mathbf{H}. \tag{31-3}$$

While κ_m is a pure number, we may think of it as expressed in units of gausses per oersted for convenience in calculation.

The relationship between gausses and webers per square meter may be found by calculating, from Equation (31-1), the magnetic induction in gausses required to exert a force of 1 nt on a charge of 1 coul moving perpendicularly to the magnetic field with a velocity of 1 m/sec. By definition, this field is 1 weber/m^2. The known numerical values are

$$F = 1 \text{ nt} = 10^5 \text{ dynes},$$

$$q = 1 \text{ coul} = 3 \times 10^9 \text{ stcoul},$$

$$v = 1 \text{ m/sec} = 10^2 \text{ cm/sec},$$

$$c = 3 \times 10^{10} \text{ cm/sec},$$

$$\theta = 90°.$$

From Equation (31-1) expressed in cgs units, we have

$$F = q \frac{v}{c} B \sin \theta,$$

$$10^5 \text{ dynes} = \frac{3 \times 10^9 \text{ stcoul} \times 10^2 \text{ cm/sec}}{3 \times 10^{10} \text{ cm/sec}} \times B,$$

$$B = 10^4 \text{ gausses}.$$

Thus a magnetic field whose induction is 10^4 gausses will produce the same effect on a moving charge as a field whose induction is 1 weber/m^2. In the form of an equation we may write

$$1 \text{ weber/m}^2 = 10^4 \text{ gausses}. \tag{31-4}$$

31-2 Path of a Charged Particle in a Magnetic Field

When a charged particle is projected into a uniform magnetic field, the component of the field parallel to the velocity exerts no force on the particle. It is convenient to resolve the velocity of the particle into components parallel and perpendicular to the field, and to consider separately the motion of the particle perpendicular to the field.

Let us consider a charged particle that is moving with velocity **v** perpendicular to a magnetic field of induction **B**. The particle will experience a force Bqv constant in magnitude and directed perpendicularly to the velocity of the particle. Hence the particle will move in a circular path with uniform speed v. The force exerted by the magnetic field on the charged particle is the centripetal force which produces an acceleration $a = v^2/r$, where r is the radius of the circular path. Applying Newton's second law to this case, we get

$$F = \frac{mv^2}{r} = Bqv,$$

from which

$$r = \frac{mv}{Bq}. \tag{31-5}$$

If the motion of the charged particle is not transverse to the field, the symbol v in Equation (31-5) must be interpreted as the component of the velocity which is perpendicular to the magnetic field. The motion of the particle will then consist of a motion of translation parallel to the field and, at the same time, a motion in a circular orbit in a plane perpendicular to the field. The effect of these two simultaneous movements is to produce a corkscrewlike motion, or a helical motion whose axis is parallel to the magnetic field.

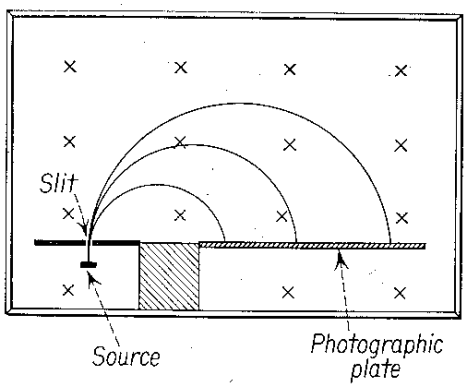


Fig. 31-3 Beta-ray spectrometer.

Note that the force on the particle moving in the magnetic field is always perpendicular to its velocity. The magnetic field therefore *does no work* on the charged particle, but only deflects it. This property of a uniform magnetic field may be utilized to measure the velocities of charged particles. In a *beta-ray spectrometer*, used to measure the velocities of electrons emitted in radioactive disintegrations, electrons or beta rays are emitted into a uniform magnetic field of known magnetic induction B in a

direction perpendicular to the field. The particles are deflected into circular orbits and strike a photographic film after completing a semicircle, as shown in Figure 31-3. The diameter of the orbit can be measured from the photographic film; the momentum mv of the electrons can then be determined, and the energy of the emitted particles may also be calculated.

The deflection of charged particles in magnetic fields is today widely utilized in scientific and technical apparatus such as beta-ray spectrometers,

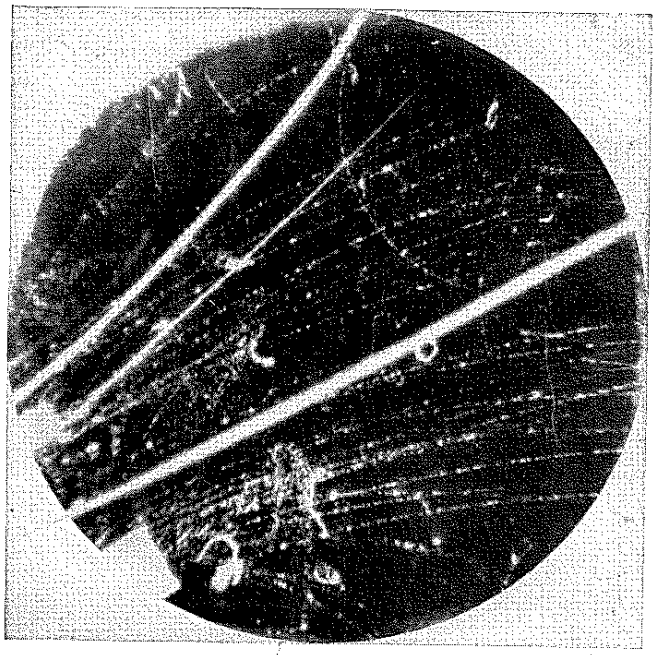


Fig. 31-4 Cloud chamber photograph showing curved paths of particles moving at right angles to a magnetic field. The magnetic field is directed into the paper and the particles originate from a source at the left. The three heavy tracks are those of protons; the numerous light tracks curved in the opposite direction are those of electrons. (Photograph by H. R. Crane.)

mass spectrometers, cathode-ray tubes (as used in television), the cyclotron (accelerates charged particles for nuclear research), the magnetron (a vacuum tube for the production of microwaves used in radar), and many others. The deflection of charged particles in a magnetic field may be seen in the accompanying photograph of tracks in a Wilson cloud chamber, shown in Figure 31-4. In this photograph the magnetic field is perpendicular to the paper and is directed into the plane of the paper.

31-3 Force on a Charged Particle in an Electromagnetic Field

We have already seen that a charged particle in an electric field of intensity **E** experiences a force $q\mathbf{E}$. This force does not depend upon the velocity of the particle. When a charged particle moves with a velocity **v**, it will experience a force given by the equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}). \tag{31-6}$$

Equation (31-6) is simply a combination of results already stated in Equations (23-1a) and (31-2). We may think of Equation (31-6) as the defining equation for the electric field intensity **E** and the magnetic induction **B**. To determine these fields in a region of space, we may imagine that a probe charge q is directed through the region. The force on the particle is determined by observing the deflection of the particle, and the electric and magnetic fields are determined by calculation from Equation (31-6). In a similar way we may imagine that Equation (30-7) is the defining equation for the magnetic intensity **H** and the electric displacement **D**, through the force on a moving pole.

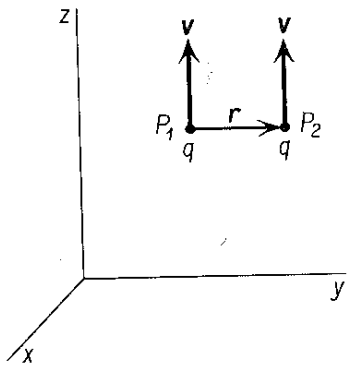


Fig. 31-5

In our initial discussion of Coulomb's law of force between magnetic poles, in Section 29-2, we stated that the constant μ_0 was to be assigned to a value of $4\pi \times 10^{-7}$ for reasons which had to be deferred to a later chapter. At this point we wish to justify the choice of a numerical value for μ_0. To do this we will calculate the ratio of the magnetic force to the electric force between two moving charged particles.

Let us consider the force between two equal charges q moving with the same velocity **v** in a direction parallel to the z axis, as shown in Figure 31-5. The instantaneous positions of the charges are P_1 and P_2, as shown in the figure. The particles are fixed in position relative to each other. The electric force on the charge at P_2 due to the presence of an equal charge at P_1 is $\mathbf{F}_e$, whose magnitude is

$$F_e = \frac{q^2}{4\pi\epsilon_0 r^2}.$$

The magnetic force on the charge at P_2 may be obtained by first computing the magnetic field intensity H generated by the motion of the charge at P_1

with respect to the coordinate frame from Equation (30-6),

$$\mathbf{H} = \frac{q\mathbf{v} \times \mathbf{1}_r}{4\pi r^2}.$$

The relationship between $\mathbf{B}$ and $\mathbf{H}$ in vacuum is given by $\mathbf{B} = \mu_0\mathbf{H}$, so that

$$\mathbf{B} = \frac{\mu_0 q\mathbf{v} \times \mathbf{1}_r}{4\pi r^2}.$$

Since the velocity $\mathbf{v}$ is in the z direction and the unit vector $\mathbf{1}_r$ is in the y direction, from P_1 to P_2, the magnetic induction $\mathbf{B}$ is in the $-x$ direction. The magnitude of the magnetic force F_m on the charged particle at P_2 may be obtained from

$$\mathbf{F}_m = q\mathbf{v} \times \mathbf{B},$$

$$F_m = qvB \sin\theta.$$

At P_2 the magnetic induction $\mathbf{B}$ is in the $-x$ direction, and the velocity of the particle is in the z direction. Since the two vectors $\mathbf{v}$ and $\mathbf{B}$ are perpendicular to each other, the factor $\sin\theta$ in the vector product is equal to 1, and the magnitude of the magnetic force is equal to

$$F_m = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}.$$

The ratio of the magnetic force to the electric force must be a pure number, without dimensions. We obtain

mks units
$$\frac{F_m}{F_e} = \mu_0\epsilon_0 v^2. \tag{31-7a}$$

Thus the quantity $\mu_0\epsilon_0$ must have the dimensions of $1/v^2$; that is, its dimensions must be $(\text{sec/m})^2$.

When the same calculation is carried out in Gaussian units, we find that

Gaussian units
$$\frac{F_m}{F_e} = \frac{v^2}{c^2}. \tag{31-7b}$$

The ratio of the magnetic force to the electric force must be the same in whatever system of units the calculation is carried out. In Gaussian units both the velocity of the particle and the velocity of light are expressed in centimeters per second, while in mks units the velocity of the particle is expressed in meters per second. In order to equate the results of the two calculations, let us now express all velocities in meters per second. This will not alter the ratio in Equation (31-7b). We obtain

$$\mu_0\epsilon_0 v^2 = \frac{v^2}{c^2},$$

or

mks units $$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \tag{31-8}$$

where c is the velocity of light in meters per second.

Our procedure in developing the mks system of electrical units was first to define the size of the coulomb from the statcoulomb. This determined the choice of the constant ϵ_0. Once a choice of ϵ_0 had been made, there could be no freedom of choice of the constant μ_0, for the magnitude of that constant was determined by the connection between electric and magnetic forces, as developed in Equation (31-8).

31-4 Force on a Wire Carrying Current in a Magnetic Field

We have already seen in Section 30-7 that a charge dq moving with a velocity $\mathbf{v}$ could be considered equivalent to a current I in an element of wire $\mathbf{ds}$ through the relationship

$$\mathbf{v}\, dq = I\, \mathbf{ds}. \tag{31-9}$$

From Equation (31-2) the force on a moving charge element dq is given by

$$d\mathbf{F} = dq\, \mathbf{v} \times \mathbf{B},$$

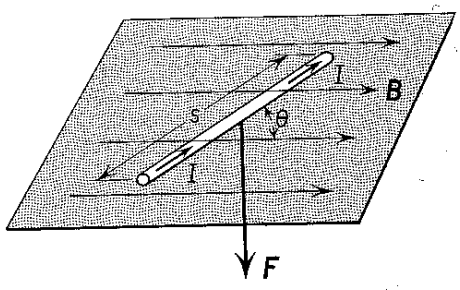


Fig. 31-6

and, substituting from Equation (31-9), we find the force $d\mathbf{F}$ on a current element $I\, \mathbf{ds}$ to be

$$d\mathbf{F} = I\, \mathbf{ds} \times \mathbf{B}. \tag{31-10}$$

This force is perpendicular to the plane formed by the current element and the direction of the field. For the special case of a long straight conductor of length s in a uniform field of magnetic induction B, the magnitude of the force F on the wire is given by

$$F = BIs \sin \theta, \tag{31-11}$$

where θ is the angle between the direction of the current in the wire and the direction of B, as shown in Figure 31-6. The force is perpendicular to the

plane formed by s and B, and its direction may be found by the right-hand rule for the vector product of Equation (31-10). The force on a wire carrying current in a magnetic field is the basis for the operation of electric motors and galvanometers.

Illustrative Example. A wire 5 m long lies along the x axis. Find the components of the force on the wire when it carries a current of 2 amp in the positive x direction if it is placed in a uniform magnetic field for which

$$B_x = 0.3 \text{ weber/m}^2,$$

$$B_y = 0.4 \text{ weber/m}^2,$$

$$B_z = 1.2 \text{ webers/m}^2.$$

Since the wire lies along the x axis, the force on the wire due to B_x is zero. The angle made by the wire with the y axis and with the z axis is 90°. The force exerted on the wire due to the y component of the field is in the $+z$ direction. Calling this force F_z, we have

$$\begin{aligned} F_z &= B_y I s \\ &= 0.4 \text{ weber/m}^2 \times 2 \text{ amp} \times 5 \text{ m} \\ &= 4.0 \text{ nt}. \end{aligned}$$

The force exerted on the wire due to the z component of the field is in the $-y$ direction. Calling this force F_y, we have

$$\begin{aligned} F_y &= -B_z I s \\ &= -1.2 \text{ weber/m}^2 \times 2 \text{ amp} \times 5 \text{ m} \\ &= -12 \text{ nt}. \end{aligned}$$

31-5 Torque on a Coil Carrying Current

Let us assume that a rectangular coil of sides a and b is carrying current I in a region of space in which there is a uniform field of magnetic induction B. If the plane of the coil is perpendicular to the direction of B, as shown in Figure 31-7 where B is directed into the paper, symmetrical elements of the coil at g and h at the top and bottom of the coil experience equal and opposite forces. Since the forces on symmetrical elements of the wire are directed along the same straight line, the resultant force on the coil and the resultant torque on the coil due to these elements are zero. Similarly, the elements at c and d contribute a net force of zero and a net torque of zero.

When the orientation of the coil is changed so that the normal to the plane of the coil makes an angle ϕ with the direction of B, as shown in Figure 31-8, the symmetrical elements g and h at the top and bottom of the coil still experience collinear forces in the $-z$ and $+z$ direction, respectively, which are equal and opposite, and therefore contribute zero net force and

torque. The forces on elements at c and d are still equal and opposite, directed parallel to the x axis in the figure, but they are no longer in the same straight line; they are now separated by a distance $a \sin \phi$ [Figure 31-8(a)].

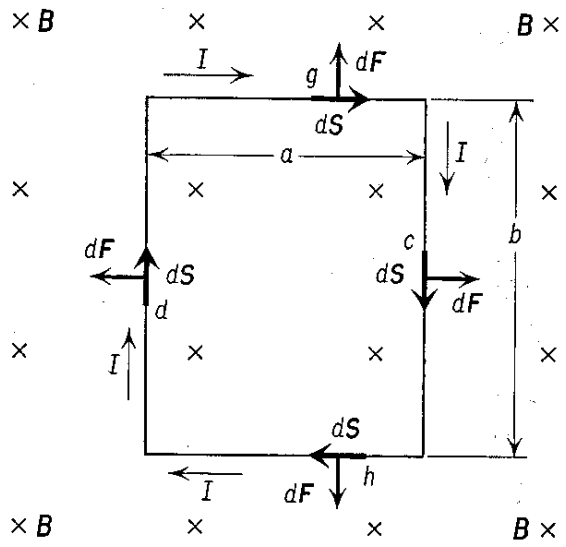


Fig. 31-7

The force on each side b of the coil is given by Equation (31-11) as

$$F = Bib$$

for the angle θ between the vertical wires, and the direction of B is 90°. The resultant force on the coil is zero, for the forces on each side of the coil are in opposite directions, but the torque G on the coil is now

$$G = BIba \sin \phi.$$

Since the area A of the coil is given by

$$A = ba,$$

we may express the torque on the coil as

$$G = BIA \sin \phi. \tag{31-12}$$

If the coil is closely wound, having N turns, a torque equal to that given by Equation (31-12) is contributed by each turn so that we have

$$G = NBIA \sin \phi. \tag{31-13}$$

Although Equation (31-13) was derived for rectangular loops or coils

of wire, it may be seen that coils of any shape, whose conductors lie in a single plane, may be approximated by a collection of rectangular coils laid side by side, as shown in Figure 31-9. Thus Equation (31-13) gives the torque on a coil of any shape when placed in a uniform magnetic field.

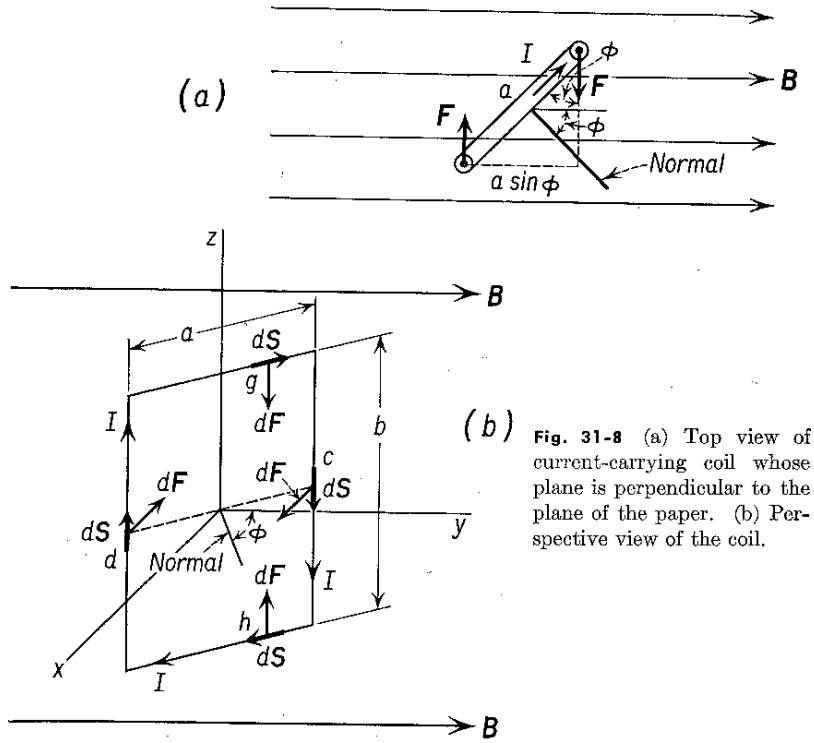


Fig. 31-8 (a) Top view of current-carrying coil whose plane is perpendicular to the plane of the paper. (b) Perspective view of the coil.

The operation of a moving-coil galvanometer, or *D'Arsonval galvanometer*, is based upon the torque on a coil in a magnetic field. The essentials of such a galvanometer are a strong permanent magnet shaped so that the two poles face each other across a gap in which the magnetic induction is fairly uniform, a rectangular coil of many turns of fine wire, and a fine spring attached to the coil. This spring may be in the form of a spirallike fine watch spring, in which case the coil is mounted on jeweled bearings, or the spring may be in the form of a long wire, one end of which is rigidly attached at the top of the frame housing the coil and the other end is attached to the coil and is part of the conductor leading current into the coil. As seen from Equation (31-13), the torque on the coil is proportional to the current through the coil. At a particular value of the current, the coil is deflected until the restoring torque of the spring is equal to the torque

generated by the magnetic field. The deflection may be indicated by a pointer moving over a scale or by means of a beam of light reflected from

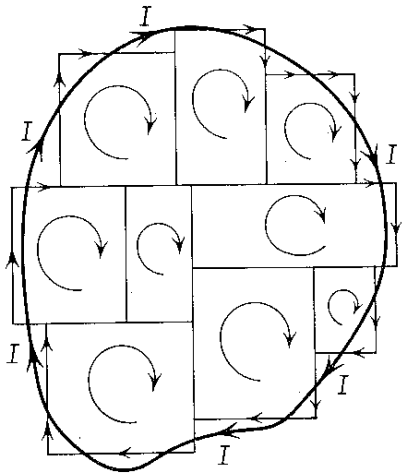


Fig. 31-9 A plane coil of arbitrary shape may be approximated by a set of rectangular coils. Note that the currents on all interior wires cancel out.

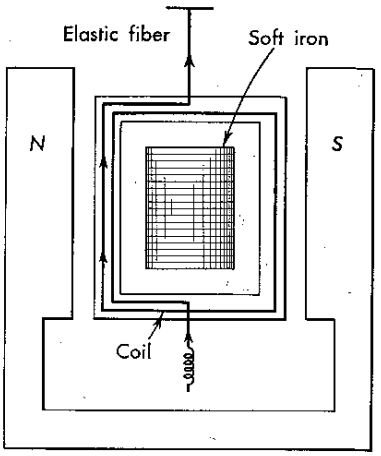


Fig. 31-10 Essentials of a D'Arsonval type of galvanometer. The soft iron core is introduced to shape the magnetic field so that it is radial, thus obtaining a galvanometer in which the deflection is proportional to the current through the coil.

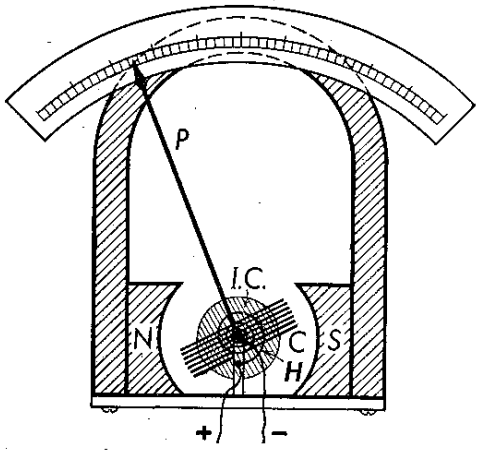


Fig. 31-11 Essentials of a moving coil galvanometer. C is the moving coil, N and S are the poles of a permanent magnet, $I.C.$ is the soft iron core, P is the pointer, and H is the hairspring.

a mirror which turns with the coil. Schematic diagrams of the essentials of a D'Arsonval galvanometer, and a typical panel-type moving-coil galvanometer are shown in Figures 31-10 and 31-11. In practice the pole

pieces are curved so that the magnetic field is nearly radial. This produces a galvanometer in which the deflection is nearly proportional to the angular displacement.

31-6 Force between Two Parallel Conductors

When two parallel conductors carry current, there is a force between them which is repulsive when the currents are in opposite directions and attractive when the currents are in the same direction. The force is due to the interaction between the current in one wire and the magnetic field at that wire due to current in the other wire. We may calculate the force between the two conductors by utilizing the results of the preceding chapter. If the current in the wire shown as 1 in Figure 31-12 is I_1, the magnetic field intensity at the position of wire 2, due to the current I_1 in wire 1, is

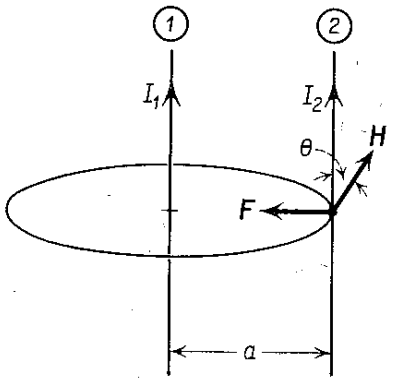


Fig. 31-12

$$H = \frac{I_1}{2\pi a}, \qquad (30\text{-}1a)$$

where a is the distance between the wires. The magnetic induction B is related to the magnetic field intensity H in vacuum or in air through the equation

$$B = \mu_0 H;$$

thus the magnetic induction due to the current in wire 1 at the position of wire 2 is

$$B = \frac{\mu_0 I_1}{2\pi a}.$$

The direction of **B** is parallel to the direction of **H** and is shown in the figure as directed into the plane of the paper, perpendicular to the wire 2. We may find the force on a length s of wire 2 by application of Equation (31-11). We note that the angle θ between the direction of the current in wire 2 and the direction of **B** at wire 2 is 90°. Thus we have

$$F = BsI_2$$

$$= \frac{\mu_0 I_1 I_2 s}{2\pi a}.$$

The force per unit length of wire is given by

$$\frac{F}{s} = \frac{\mu_0 I_1 I_2}{2\pi a}. \tag{31-14}$$

From application of the rule for the vector product, as given in Equation (31-10) for the direction of the force, the force on wire 2 may be seen to be toward wire 1. An equal but opposite force is exerted on wire 2, as may be seen from an application of the same procedure to wire 2. When the currents are in opposite directions, the force is of the same magnitude but is repulsive rather than attractive.

From Equation (31-14) we note that the units of μ_0 may be expressed as nt/amp^2 as well as weber/nt m^2 or weber/amp m. The units in which μ_0 is expressed is a matter of convenience. Thus we may write

$$\mu_0 = 4\pi \times 10^{-7} \text{ nt/amp}^2. \tag{31-15}$$

It is possible through Equation (31-14) to establish a definition of the ampere in terms of the mechanical quantities of force and distance and the constant μ_0. Thus an ampere may be defined as that current flowing in two infinitely long parallel wires separated by a distance of 1 m in air which causes a force of 2×10^{-7} nt per meter of length between them. In a device called the current balance, the force between two coils of wire connected in series and separated by a distance of a few centimeters may be used to make an absolute determination of current.

When there is an alternating current in a coil, the current is first in one direction, then in the other, following a sine or cosine function of time. The currents in adjacent turns are in the same direction, so that there is an attractive force between adjacent turns which varies from some maximum value to zero and back again. This periodic force may cause the wires of the coil to vibrate and produce an audible hum; such a noise is sometimes heard in a transformer of a radio set.

Illustrative Example. Find the force between two parallel conductors in a power distributing station. The conductors are 10 m long, carry a current of 150 amp each, and are separated by a distance of 1 cm.

From Equation (31-14)

$$F = \frac{\mu_0 I_1 I_2 s}{2a}.$$

Substituting numerical values, we have

$$F = \frac{4\pi \times 10^{-7} \text{ nt/amp}^2 \times 150 \text{ amp} \times 150 \text{ amp} \times 10 \text{ m}}{2 \times 0.01 \text{ m}},$$

$$F = 4.5 \text{ nt}.$$

TABLE 31-1 PRINCIPAL EQUATIONS IN MKS AND GAUSSIAN UNITS

Equation	MKS	Gaussian	
(31-2)	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$	$\mathbf{F} = q\dfrac{\mathbf{v} \times \mathbf{B}}{c}$	Moving charge
(31-3)	$\mathbf{B} = \kappa_m \mu_0 \mathbf{H}$	$\mathbf{B} = \kappa_m \mathbf{H}$	
(31-5)	$r = \dfrac{mv}{Bq}$	$r = \dfrac{mvc}{Bq}$	Circular path
(31-6)	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\mathbf{F} = q\left(\mathbf{E} + \dfrac{\mathbf{v} \times \mathbf{B}}{c}\right)$	Charge
(31-10)	$d\mathbf{F} = I\, d\mathbf{s} \times \mathbf{B}$	$d\mathbf{F} = \dfrac{I}{c}\, d\mathbf{s} \times \mathbf{B}$	Element of wire carrying current
(31-13)	$G = NBIA \sin \phi$	$G = \dfrac{NBIA \sin \phi}{c}$	Torque on coil
(31-14)	$\dfrac{F}{s} = \dfrac{\mu_0 I_1 I_2}{2\pi a}$	$\dfrac{F}{s} = \dfrac{2I_1 I_2}{ac^2}$	Parallel wires

TABLE 31-2 CONVERSION FACTORS RELATING MKS AND GAUSSIAN UNITS

Quantity	Symbol	MKS Unit	Gaussian Unit	
Magnetic intensity	H	1 amp/m = 1 nt/weber	$= 4\pi \times 10^{-3}$ oersted	(emu)
Magnetic induction	B	1 weber/m^2	$= 10^4$ gausses	(emu)
Electric intensity	E	1 nt/coul = 1 volt/m	$= \dfrac{10^{-4}}{3}\dfrac{\text{statvolt}}{\text{cm}}$	(esu)
			$= \dfrac{10^{-4}}{3}\dfrac{\text{dyne}}{\text{stcoul}}$	(esu)
Electric displacement	D	1 coul/m^2	$= 3 \times 10^5$ stcoul/cm^2	(esu)
Charge	q	1 coul	$= 3 \times 10^9$ stcoul	(esu)
Current	I	1 amp	$= 3 \times 10^9$ statamperes	(esu)

Problems

31-1. A proton of charge $+4.8 \times 10^{-10}$ stcoul moves with a speed of 10^9 cm/sec at right angles to the direction of a magnetic field of 300 gausses. Find the force on the proton due to its motion with respect to the magnetic field.

31-2. The mass of a proton is 1.67×10^{-24} gm. What will be the radius of the circle in which the proton of Problem 31-1 moves?

31-3. In a beta-ray spectrometer an electron moving perpendicularly to a magnetic field of magnetic induction 150 gausses moves in a circle of 15 cm diameter. Find the momentum of the electron.

31-4. A proton moves with a velocity of 10^4 m/sec in the $+x$ direction in a magnetic field whose components are $B_x = 0.03$ weber/m^2, $B_y = 0.04$ weber/m^2, $B_z = 14$ webers/m^2. Find the components of the force on the proton.

31-5. A particle of charge 10 μcoul has a velocity whose components are $v_x = 3$ m/sec, $v_y = 4$ m/sec. The particle moves in a magnetic field whose components are $B_x = 0.5$ weber/m^2, $B_y = 1.2$ webers/m^2, and $B_z = 1.5$ webers/m^2. Find the components of the force on the particle.

31-6. The magnetic induction between the poles of a large electromagnet is 1,000 gausses in a direction from east to west. The field extends vertically for a distance of 5 cm. A wire is suspended vertically in the field, and a current of 15 amp is sent through it. Determine the magnitude and direction of the force on the wire.

31-7. A wire parallel to the y axis carries a current of 10 amp. The wire is 0.5 m long. The magnetic induction in the vicinity of the wire has an x component of 0.3 weber/m^2, a y component of -1.2 webers/m^2, and a z component of 0.5 weber/m^2. Find the components of the force on the wire.

31-8. A current of 8 amp is directed downward in a vertical wire 80 cm long which is suspended in a place where the intensity of the earth's magnetic field has a horizontal component of 0.25 oersted and a vertical component of 0.75 oersted. Determine the magnitude and the direction of the force on the wire.

31-9. A long uniform solenoid is wound with 1,000 turns/m of length and carries a current of 10 amp. An electron, whose velocity component parallel to the axis of the solenoid is 10^5 m/sec and whose velocity perpendicular to the axis of the solenoid is 10^6 m/sec, is ejected from a radioactive source at the center of the solenoid. The path described by the electron is a helix. (a) Find the radius of the cross section of this helix and (b) find the pitch (number of turns per meter) of the helix. The charge of an electron is -4.8×10^{-10} stcoul, and the mass of an electron is 9.10×10^{-28} gm.

31-10. A rectangular coil of 150 turns of fine wire is suspended in a magnetic field with its plane parallel to the direction of the field. The dimensions of the coil are 2 cm by 4 cm. When a current of 0.5 amp is sent through the coil, a torque of 18,000 dynes cm acts on it. Determine the magnetic induction of this field.

31-11. A straight wire 12 cm long is placed in a magnetic field of 350 gausses directed at right angles to the length of the wire. A delicate spring balance attached to the wire measures a force of 160 dynes when a current is sent through the wire. Determine the current in the wire.

31-12. A proton whose mass is 1.65×10^{-27} kg travels in a circular path of 0.35 m radius in a magnetic field of 1.2 webers/m^2. Determine (a) the velocity of the proton and (b) its period of revolution.

31-13. A rectangular galvanometer coil has 500 turns wound on a frame 2 cm by 4 cm. It hangs between the poles of a magnet where the flux density is 0.080 weber/m^2. Determine the torque which acts on this coil when the current in it is 3×10^{-8} amp.

31-14. Derive Equation (31-7b).

31-15. Two electrons are moving down the axis of a television tube with a velocity of 10^7 m/sec. The electrons are separated by a distance of 0.01 cm. (a) What is the electric force between them? (b) What is the magnetic force between them? (c) What is the resultant force between them?

31-16. A circular coil of wire of radius 10 cm and containing 50 turns is placed in a magnetic field in which B is 1.5 webers/m^2. The current in the coil is 3 amp. (a) What is the torque on the coil when its plane is parallel to the field? (b) When the normal to the plane of the coil makes an angle of 30° with the field?

31-17. One end of a flexible copper wire is fastened to a post while the other goes over a fixed pulley and has a weight W hanging from it. The portion of the wire from A to the pulley is horizontal and is in a uniform magnetic field of flux density B in the vertical direction. A current I is sent through the wire; determine the radius of the circular arc into which the wire is bent.

32

Electromagnetic Induction

32-1 Motion of a Wire in a Magnetic Field

When a wire moves through a uniform magnetic field of induction B, in a direction at right angles to the field and to the wire itself, the electric charges within the conductor experience forces due to their motion through this magnetic field. The positive charges are held in place in the conductor by the action of interatomic forces, but the free electrons, usually one or two per atom, are caused to drift to one side of the conductor, thus setting up an electric field E within the conductor which opposes the further drift of electrons. The magnitude of this electric field E may be calculated by equating the force it exerts on a charge q, to the force on this charge due to its motion through the magnetic field of induction B; thus

$$Eq = Bqv,$$

from which

$$E = Bv.$$

If, as a result of the motion of the wire through the magnetic field, a charge q is moved a distance s along the wire against the internal electric field E, a quantity of work $\mathscr{W}$ is done by the agency moving the wire, given by the expression

$$\mathscr{W} = Eqs = Bvqs.$$

Thus an electromotive force is generated within the wire as a result of its motion through the magnetic field. The electromotive force across the ends of the wire is the work per unit charge done by the agency moving the wire. The emf $\mathscr{E}$ is thus

$$\mathscr{E} = \frac{\mathscr{W}}{q} = \frac{Bqvs}{q},$$

so that

$$\boxed{\mathscr{E} = Bsv.} \qquad (32\text{-}1)$$

As shown in Figure 32-1, the direction of the emf is the direction in which positive charges are made to move by the action of the magnetic field, and therefore is opposite to the direction of the induced electric field E within the wire.

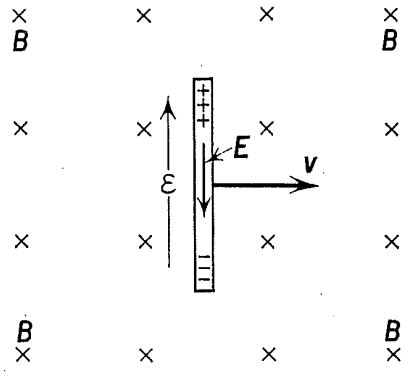


Fig. 32-1

To gain further insight into the effect of moving a wire through a magnetic field, let us suppose that the wire of length s slides over a fixed conductor a consisting of two parallel tracks which are electrically connected at one end, as shown in Figure 32-2. As the wire moves to the right with velocity v, the induced emf in the wire produces a current I in the closed circuit, in the direction of the emf $\mathcal{E}$ in the moving wire. We have seen in Section 31-4 that a wire carrying current in a direction perpendicular to the magnetic field experiences a force given by

$$F = BIs.$$

In the figure this force is directed to the left. In order to satisfy the principle of conservation of energy, the agency moving the wire to the right

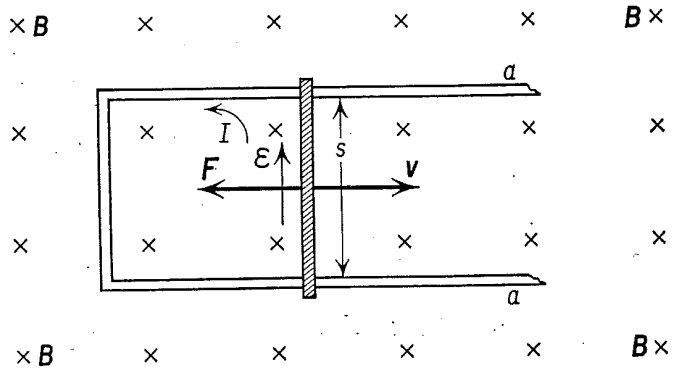


Fig. 32-2

must exert a force equal and opposite to the force F above, and expend mechanical power $\mathcal{P}$ such that

$$\mathcal{P} = Fv = BIsv.$$

At the same time the electrical power generated is

$$\mathcal{P} = \mathcal{E}I.$$

Equating the mechanical power expended and the electrical power generated, we find

$$\mathcal{E} = Bsv,$$

which is Equation (32-1).

In calculating the emf developed by a wire moving through a magnetic field, we have used two different points of view. The first calculation was made from essentially a microscopic point of view in which our attention was directed to the forces on isolated charges within the wire. The second calculation was made from a macroscopic viewpoint, in which our attention was directed to the force on the wire and to the emf. The same result was obtained in each case.

Many practical devices, such as electric generators and motors, are designed so that conductors move across magnetic fields. In using Equation (32-1) to discuss the operation of these devices, it must be remembered that B, s, and v were all considered to be perpendicular to one another. If they are not mutually perpendicular in a particular case, then only the components of the three quantities which are mutually perpendicular are to be considered. As the wire moves through the magnetic field, it is often described as "cutting" the lines of magnetic induction. Equation (32-1) then shows that the emf induced in a wire depends upon the number of lines of magnetic induction cut per unit time.

32-2 Magnetic Flux and Flux Density

It is convenient to represent the magnetic induction B by lines of magnetic induction, sometimes called lines of magnetic *flux*. The direction of the magnetic induction is tangent to the flux lines, and the magnitude of the magnetic induction is given in the usual way by the number of lines per unit area passing through a surface perpendicular to the flux lines. The total number of lines passing perpendicularly through an element of area is then called the *magnetic flux* Φ (capital phi) through that area. If the area of an element perpendicular to B is ΔA, then the flux $\Delta\Phi$ through that element is given by

$$\Delta\Phi = B\,\Delta A. \tag{32-2}$$

In the mks system of units, the magnetic induction is stated in units of webers per square meter, and the area is stated in units of square meters. The flux is expressed in units of webers. For this reason the magnetic induction B is often referred to as the *flux density*.

In the Gaussian system of units, the magnetic induction B is expressed in gausses, the area in square centimeters, and the flux in *maxwells*. We have already seen that

$$1 \text{ weber/m}^2 = 10^4 \text{ gausses},$$

so that

$$1 \text{ weber} = 10^4 \text{ gausses} \times 10^4 \text{ cm}^2,$$

or

$$1 \text{ weber} = 10^8 \text{ maxwells}.$$

Following the procedure we have used throughout the development of electricity and magnetism, unless otherwise indicated all equations are expressed in the mks system of units, and the principal ones will be restated in the Gaussian system of units in a table at the end of this chapter.

The magnetic flux Φ is a scalar quantity, but the magnetic induction $\mathbf{B}$ is a vector quantity; the area $\Delta\mathbf{A}$ may be considered as a vector quantity. In dealing with closed surfaces, as in Gauss's theorem in electrostatics, we considered the direction of an area as that of an outward drawn normal. Although in the present instance the area is not part of a closed surface, it may be thought of as a *film* or a *cap* bounded by a closed conducting boundary. In choosing the direction of the area vector, we must associate a positive direction of circulation around the boundary of the area in accordance with a *right-hand rule*. Thus if in Figure 32-2 the direction of the area vector is chosen as pointing toward the reader, the positive direction of the current in the wire may be found by directing the thumb of the right hand in the direction of the area vector. The curled fingers of the right hand indicate the positive direction of the current or the emf as in the counterclockwise direction. If the area vector is pointing into the paper, the positive direction of the current is clockwise. Following this convention, we may rewrite Equation (32-2) in vector form as the scalar product of $\mathbf{B}$ and $\Delta\mathbf{A}$ as

$$\Delta\Phi = \mathbf{B}\cdot\Delta\mathbf{A}. \qquad \textbf{(32-2a)}$$

32-3 Faraday's Law of Electromagnetic Induction

The phenomenon of electromagnetic induction was discovered in 1831 by Michael Faraday (1791–1867) in England and independently by Joseph Henry (1797–1878) in the United States. One example of electromagnetic induction is the emf generated in a wire moving through a magnetic field, as discussed in Section 32-1. In Faraday's original experiment the apparatus consisted essentially of two neighboring circuits, shown in Figure 32-3; one circuit, which we shall call the primary circuit, contained a battery B, a coil P, and a key K, for opening and closing the circuit; the second circuit, or secondary circuit, consisted of a coil S and a galvanometer G. Faraday observed that when the key was closed, the galvanometer

in the S circuit gave a momentary deflection and then returned to its zero position and remained there as long as the key was closed. When the key was opened, there was another momentary deflection of the galvanometer, opposite in direction to the previous deflection, and then the galvanometer needle returned to its zero position.

Analyzing this simple experiment, we find that when the key in the primary circuit was closed, a current started flowing through the primary coil P. This current produced a magnetic field in the neighborhood of P and also around the coil S; that is, a change was produced in the magnetic field around the coil S in the secondary circuit. The fact that the galvanometer showed only a momentary deflection can be interpreted by saying that a current was *induced* in the secondary circuit momentarily and that this induced current was due to the *change* in the magnetic field around the secondary circuit. As long as the current in the primary circuit remained constant, the magnetic field around both P and S remained constant, but the galvanometer read zero during this time. But when the magnetic field was again changed, say by opening the key, a current was again induced in the secondary circuit, this time in a direction opposite to that produced when the key was closed.

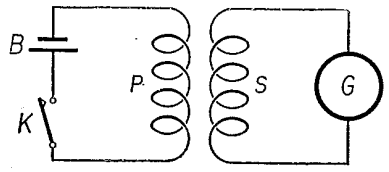


Fig. 32-3 Faraday's experiment on electromagnetic induction.

The results of the above experiment on electromagnetic induction can be explained qualitatively by stating that the change in the magnetic field around the secondary coil *induced an electromotive force* in the coil, and, since the coil is part of a closed circuit, this induced emf produced a current in the secondary circuit.

There are many ways in which the magnetic field around the coil S may be varied. Suppose, for example, that the key of the primary circuit is kept closed so that a steady current flows through the coil P. As long as the magnetic field around the secondary coil S remains constant, there will be no emf induced in it, and the galvanometer will read zero. But if we move S away from P so as to decrease the magnetic field around S, the galvanometer will show a deflection; similarly, if we move S toward P, the galvanometer will show a deflection but now in the opposite direction.

If we put a variable resistor in the primary circuit so that the current in it may be varied, and keep the distance between S and P constant, there will be an induced emf and hence an induced current in the secondary circuit whenever the current in the primary circuit is changed. When the

current in the primary coil is increased, the induced current in the secondary coil will be in one direction; when the current in the primary coil is decreased, the induced current in the secondary coil will be in the opposite direction.

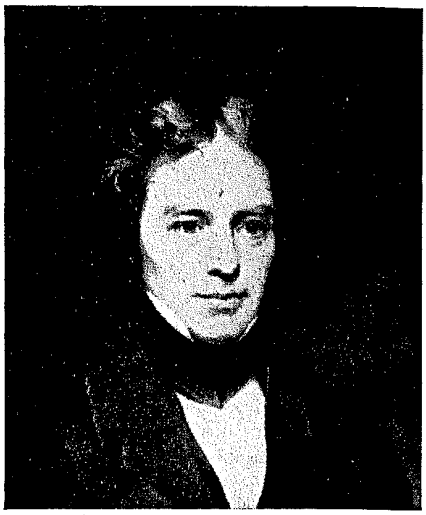


Fig. 32-4 Michael Faraday (1791–1867). Chemist and physicist. Discovered the laws of electrolysis and electromagnetic induction. Introduced the concept of lines of force to help understand the phenomena associated with electric and magnetic fields. (Courtesy of *Scripta Mathematica.*)

The results of many experiments on electromagnetic induction can be stated in the form of a law known as *Faraday's law of electromagnetic induction* as follows:

The electromotive force induced in each turn of wire in any circuit is equal to the time rate of decrease of the magnetic flux through it, or, in the form of an equation:

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}.} \tag{32-3}$$

Faraday's law may be considered as one of the fundamental empirical laws of electromagnetism, or it may be derived by applying Ampère's law and the principle of conservation of energy to typical cases. To show this, let us reconsider the case of a wire moving perpendicularly with constant velocity through a magnetic field of flux density **B**, as shown in Figure 32-2. If the wire moves a distance Δx to the right, the change in area is

$$\Delta A = s\,\Delta x.$$

In accordance with the sign convention, the positive direction of the normal to ΔA is upward toward the reader. The flux density **B** is directed down-

ward, so that $\mathbf{B} \cdot \Delta\mathbf{A}$, the change in flux $\Delta\Phi$ through this area, is

$$\Delta\Phi = -Bs\,\Delta x.$$

If this change in flux takes place in a short time interval Δt, then

$$\frac{\Delta\Phi}{\Delta t} = -Bs\frac{\Delta x}{\Delta t},$$

and, in the limit of short time intervals, we find

$$\frac{d\Phi}{dt} = -Bs\frac{dx}{dt} = -Bsv.$$

Substituting for the quantity on the right-hand side of the above equation from Equation (32-1), we find

$$\mathcal{E} = -\frac{d\Phi}{dt}. \tag{32-3}$$

Although we have derived the result for a moving wire, Equation (32-3) has been found to be true for the emf induced in any closed circuit when the flux through that circuit changes with time.

Whenever the magnetic flux passing through a single turn of wire is changing, the instantaneous emf developed in the loop is given by Equation (32-3), regardless of the reason for the change in the flux. If the coil consists of N turns of wire, an emf will be induced in each turn by the changing magnetic flux. The turns of a coil may be considered as connected in series, so that the emf in the coil will be the sum of the emf's induced in the individual turns. If the rate of change of magnetic flux is the same through each turn, then the total emf induced in the coil will be given by

$$\boxed{\mathcal{E} = -N\frac{d\Phi}{dt}.} \tag{32-4}$$

Illustrative Example. A coil containing 750 turns of wire is wound on a rectangular frame 20 cm by 30 cm. The magnetic induction normal to the area of the coil is 0.3 weber/m^2. The magnetic field is reduced to zero at a uniform rate in 0.25 sec. Determine the magnitude of the emf induced in the coil.

The total flux through the coil is

$$\Phi = BA$$

$$= 0.3\,\frac{\text{weber}}{\text{m}^2} \times 0.06\ \text{m}^2$$

$$= 0.018\ \text{weber}$$

The rate of change of flux is given by

$$\frac{d\Phi}{dt} = \frac{0.018 \text{ weber}}{0.25 \text{ sec}}$$

$$= 0.072 \text{ weber/sec.}$$

Since $N = 750$ turns, we have, from Equation (32-4), neglecting the sign of the emf,

$$\mathcal{E} = 750 \times 0.072 \text{ volts},$$

$$\mathcal{E} = 54 \text{ volts}.$$

32-4 Lenz's Law

The method for determining the direction of the current in a coil produced by an induced emf was first clearly stated by H. F. E. Lenz (1804-1864) in 1834; it is based upon the application of the principle of conservation of energy to the process of electromagnetic induction. Lenz's law states that *the induced current is in such a direction as to oppose, by its magnetic action, whatever change produces the current.*

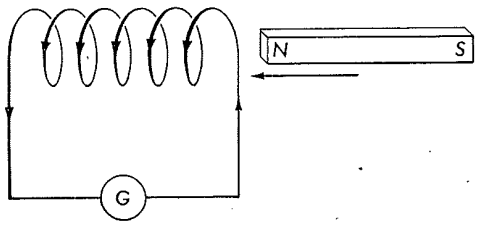


Fig. 32-5 An emf is induced in the coil by the motion of a bar magnet. Direction of the induced current is shown by the arrows on the wire.

Another way of stating Lenz's law is that the direction of the induced current is such as to oppose the change in the magnetic flux in the circuit. If for any reason whatever there is an increase in the magnetic flux through the circuit, the induced current will be in such a direction as to set up a magnetic field to oppose the increase in the magnetic flux through it. Similarly, if there is a decrease in the magnetic flux through the circuit, the induced current will be in such a direction that it will set up a magnetic field which will oppose the decrease in the flux through it. In the case of the moving wire of Figure 32-2, the motion of the wire to the right tended to increase the magnetic flux enclosed within the closed circuit. The increase in magnetic flux was directed into the plane of the paper. According to Lenz's law the induced current in this circuit had to be in the counterclockwise direction, as shown in the figure. The magnetic field generated by the induced current was directed out of the plane of the paper, so as to oppose the change in the flux through the circuit.

Let us suppose that a bar magnet is brought near a coil whose terminals are connected to a galvanometer, as shown in Figure 32-5. When the north pole of the magnet approaches the coil, the galvanometer registers a current in one direction. When the magnet is removed from the coil, the galvanometer registers a current in the opposite direction. We may find the direction of the induced currents by the application of Lenz's law. When the north pole of the bar magnet is brought near one end of the coil, the induced current in the coil will be in such a direction as to set up a magnetic field which will oppose the motion of the north pole toward it; that is, the magnetic field caused by the induced current will repel the north pole of the bar magnet. If the north pole of the magnet is moved away from one end of the coil, the induced current will set up a magnetic field so as to attract the north pole of the bar magnet and oppose its motion away from the coil. Thus work must be done in moving the bar magnet with respect to the coil because of the force which is generated when the magnet is moved. This work is transformed into electric energy, as evidenced by the existence of an induced current in the coil, and the conversion of this energy into heat.

32-5 Electric Dynamo or Generator

The moving wire of Figure 32-2 is a simple form of electric generator. In this illustrative dynamo, mechanical work is done on the wire, and this is converted into electrical energy. More practical forms of generators are usually constructed so that the motion of the conductor is rotational rather than translational. In the simplest case a coil of N turns is rotated about an axis in the plane of the coil, in a magnetic field which is perpendicular to the axis of rotation, as shown in Figure 32-6. The two ends of the coil are connected to two insulated conducting rings called *slip rings*, mounted on the axis and rotating with the coil. Two blocks of carbon, called *brushes*, press against these rings as they rotate and provide electrical contact with the external circuit.

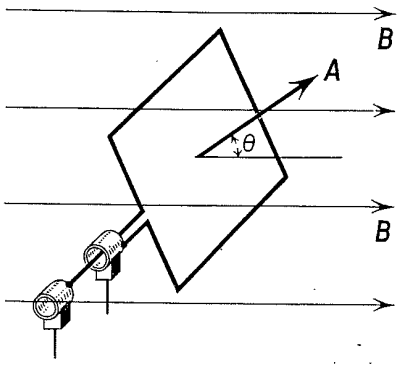


Fig. 32-6

Let us suppose that the coil of area A is rotating in a field of uniform flux density B. If the coil is oriented so that the normal to the plane of the coil makes an angle θ with the lines of flux, the component of the magnetic

field which is normal to the plane of the coil is $B \cos \theta$, so that the flux passing perpendicularly through the coil is

$$\Phi = BA \cos \theta.$$

The emf $\mathcal{E}$ generated in a coil of N turns, as a result of the changing flux through the coil as it rotates, may be obtained from Equation (32-4) as

$$\mathcal{E} = -N \frac{d\Phi}{dt}$$

$$= -NBA \frac{d}{dt} (\cos \theta)$$

$$= NBA \sin \theta \frac{d\theta}{dt}.$$

Let us assume that the coil rotates with uniform angular velocity ω, and that at time $t = 0$, the angle $\theta = 0°$. We have

$$\theta = \omega t,$$

and

$$\frac{d\theta}{dt} = \omega,$$

so that

$$\mathcal{E} = NBA\omega \sin \omega t. \tag{32-5}$$

The emf is zero at time $t = 0$ and varies as a sine function of the time. The emf changes direction after each half revolution of the coil. The emf is said to be an *alternating* emf. The maximum value of the emf occurs when $\theta = \omega t = 90°$; that is, when the plane of the coil is parallel to the magnetic field. The emf is zero when the plane of the coil is perpendicular to the magnetic field ($\theta = 0°$). The variation in the emf induced in the coil is one revolution, as shown in Figure 32-7. The rate of change of the magnetic flux through the coil is greatest when it is passing through the position where it is parallel to the field, and is zero when the coil is perpendicular to the field.

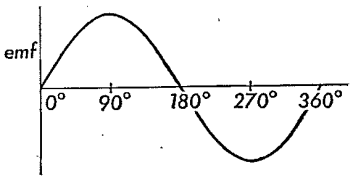


Fig. 32-7 Graph of the alternating emf induced in the coil of the generator during one revolution.

In some small generators, such as those operated by hand and used to supply current to ring bells in rural telephones, the magnetic field may be supplied by a permanent magnet; these generators are called *magnetos*.

In most generators the magnetic field is produced by current in *field coils*. This current may be supplied by a battery or it may be supplied by the generator itself. Instead of a single rotating coil of wire, there are usually several coils, each consisting of many turns of wire, wound on an iron core, rotating in the magnetic field. The whole assembly is called an *armature*. Most large generators have a complicated field structure, with two or more pairs of poles. The induced emf in the coils of an armature is always an alternating emf, and the current in these coils is always an alternating current.

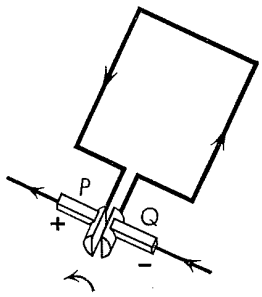


Fig. 32-8 Split-ring commutator. P and Q are brushes in contact with the segments of the commutator.

For some purposes it is desired to have a current which does not reverse its direction, as in electroplating baths. For such cases it is necessary to change the alternating current developed in the armature to current which is always in the same direction in the outside circuit. In a simple form of d-c generator, this is accomplished by a *split-ring commutator*, shown in Figure 32-8. The two ends of the armature are connected to the two insulated halves of the split ring. As the coil rotates, a given brush is always connected to that part of the coil moving in a particular direction through the field, so that one of the brushes is always the positive terminal of the generator and the other brush is always the negative terminal. The current from an armature with a single turn is not constant but is pulsating, as shown in Figure 32-9. The wave form is essentially a sine wave with the negative half cycles reversed.

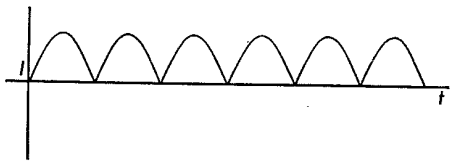


Fig. 32-9 Direct current from a single coil.

In modern d-c generators the armature consists of many coils connected in series, and the commutator contains many segments. Figure 32-10 shows the current from a generator containing two coils. The small variations in the current are referred to as commutator ripple.

A simple generator coil rotated in an unknown magnetic field may be used to measure the magnetic induction, using Equation (32-5), if the dimensions of the coil and the speed of rotation are known, and the emf generated by the rotation of the coil is measured.

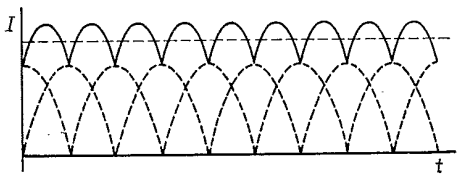


Fig. 32-10 Direct current from a d-c generator having a commutator with many segments. The dotted lines show the current from the two pairs of segments, while the solid line shows the current from such a generator. Note that the current is not constant but fluctuates about some average value shown as a dashed horizontal line.

32-6 Self-Inductance

The law of electromagnetic induction states that an emf is induced in any circuit in which the magnetic flux is changing. The manner in which the change in the magnetic flux is produced does not matter; the changes may be induced by external currents or magnets, or by changes in the circuit itself. The coil of a motor is caused to rotate by current passing through the armature, and as a result of the rotation of the armature, the flux through the coil is changed and a *back emf* is developed. When current is sent through a coil, a magnetic field is established through it, and any changes in the current generate changes in the magnetic flux through the coil. These changes in flux induce an emf in the coil, which, according to Lenz's law, must be in such a direction as to oppose the change in current. The emf induced in the coil is proportional to the rate of change of current in it, or, in the form of an equation,

$$\mathcal{E} = -L\frac{di}{dt}. \tag{32-6}$$

The constant of proportionality L represents a property of the coil which depends upon its dimensions and its geometrical shape; L is called the *self-inductance* of the coil. The minus sign is used to express the fact that the emf induced in a coil by a *change* in current is opposite to the direction of the *change*. In the mks system the unit of inductance is the *henry*, after Joseph Henry. Thus a coil has an inductance of 1 henry if an emf of 1 volt is induced in the coil when the current through it is changing at the rate of 1 amp/sec. A device having inductance is called an *inductor* and is represented schematically in Figure 32-11.

Fig. 32-11 Schematic representation of an inductor.

Another interpretation of the self-inductance L of a circuit may be obtained by comparing Equations (32-4) and (32-6), yielding

$$L\frac{di}{dt} = N\frac{d\Phi}{dt},$$

from which

$$Li = N\Phi,$$

so that

$$L = \frac{N\Phi}{i}. \tag{32-7}$$

The quantity $N\Phi$ is called the *flux linkage* of the circuit, hence the self-inductance L is the flux linkage per unit current of a circuit.

Let us determine the self-inductance of a uniformly wound toroid of N turns, mean length s, and cross-sectional area A. The magnetic field intensity within the toroid is uniform and given by

$$H = \frac{Ni}{s}. \tag{30-5}$$

When a toroid is in vacuum, the magnetic induction within the toroid is given by the equation

$$B = \frac{\mu_0 Ni}{s},$$

and the flux of magnetic induction within the toroid is

$$\Phi = \frac{\mu_0 NiA}{s}.$$

Since

$$L = \frac{N\Phi}{i}, \tag{32-7}$$

we get

$$L = \frac{\mu_0 N^2 A}{s}. \tag{32-8}$$

Thus the self-inductance of a toroid in air is a property of the geometry of the toroid, just as the capacitance of a capacitor is a property of its geometry. Any conducting element in an electrical circuit has the property of inductance. The conductors which connect the various parts of an electric circuit also generate a magnetic field when current passes through them. The inductance associated with the leads is often called *stray* inductance or distributed inductance.

From Equation (32-8) we see that μ_0 may be expressed in terms of the unit of inductance as

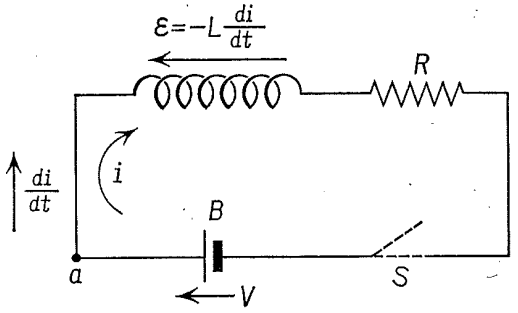
$$\mu_0 = 4\pi \times 10^{-7}\,\frac{\text{henry}}{\text{meter}}, \tag{32-9}$$

and, indeed, these are the units in which μ_0 is most commonly expressed.

32-7 Inductive Transients

Let us consider the simple circuit of Figure 32-12 in which an inductor L and a resistor R are connected in series to a battery B. Initially the switch S is open. We may analyze the behavior of the circuit by applying Kirchhoff's laws (Section 27-6) to the circuit at any instant after the switch is closed. Let us suppose that at a particular time t the current in the circuit

Fig. 32-12



is in the clockwise direction, as shown in the figure, and that the current is increasing so that the direction of the rate of change of current is in the same direction as the current itself. Starting at the point a and applying Kirchhoff's laws by moving a probe charge around the circuit in the direction of the current, we find

$$-L\frac{di}{dt} - iR + V = 0, \tag{32-10}$$

where V is the emf of the battery, and $-L\ (di/dt)$ is the back emf in the inductor.

Equation (32-10) is a differential equation whose solution is given by

$$i = \frac{V}{R}(1 - e^{-\frac{R}{L}t}). \tag{32-11}$$

We may verify this solution by differentiation of Equation (32-11) and substitution into Equation (32-10).

$$\frac{di}{dt} = \frac{V}{R}\frac{R}{L}e^{-\frac{R}{L}t}. \tag{32-12}$$

Substituting for i and for di/dt from Equations (32-11) and (32-12) into Equation (32-10), we find

$$-L\frac{V}{L}e^{(-R/L)t} - V + Ve^{(-R/L)t} + V = 0,$$

establishing the correctness of the solution.

The current in the circuit is zero at the instant the switch is closed, and gradually increases to a maximum value which is determined only by the magnitude of the resistance and the emf of the cell, as shown in Figure 32-13. Initially, the rate of change of current is very large, and the emf induced in the inductor limits the flow of current. At the instant the switch is closed, the emf of the inductor is equal and opposite to the emf of the

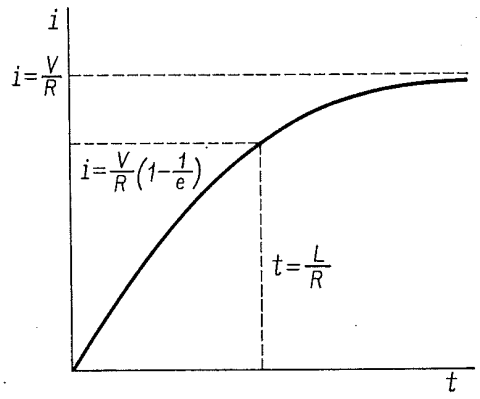


Fig. 32-13 Growth of current in a circuit containing inductance and resistance.

battery, so that the current is zero. When the current reaches a steady value and is no longer changing, there is no induced emf, and the current is determined by Ohm's law. At a time $t = L/R$, called the *time constant* of the circuit, the current has reached to within $1/e$ of its maximum value.

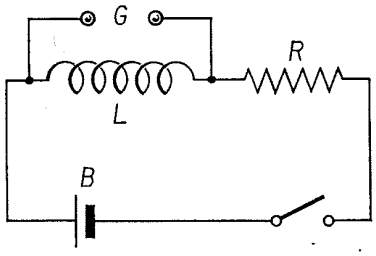


Fig. 32-14 A spark-gap G placed across the terminals of an inductor.

The curve describing the current as a function of time is called a *transient*, for it describes the current shortly after the switch is closed rather than the steady-state current that is established after a long interval of time.

In the process of establishing a current in the circuit, a magnetic field is established in the inductor. If the switch is suddenly opened after the current has reached a steady value, an emf will be induced in the inductor

whose value will depend upon the rate of change of current. The more quickly the current is interrupted, the greater will be the induced emf. If a spark gap is placed across the terminals of the inductor, as shown in Figure 32-14, a spark may pass between the terminals of the spark gap when the switch is opened, because of the large induced emf in the inductor. The energy of the magnetic field is then dissipated into the heat, sound, and radiant energy generated at the spark gap. The emf generated upon opening a circuit containing inductance is often called a switching transient, and is responsible for the large arcs which are often observed when electrical switches are opened.

32-8 Energy Stored in an Inductor

Let us calculate the energy of an inductor when there is a steady-state current I in it. During the transient interval when the current is changing from zero to the maximum value I, the emf of the inductor is given by

$$\mathcal{E} = -L\frac{di}{dt}. \tag{32-6}$$

An amount of power $\mathcal{P}$ is expended by an external source of electrical energy to establish this current. The applied potential difference is opposite to the direction of the induced emf so that

$$\mathcal{P} = -\mathcal{E}i = Li\frac{di}{dt}.$$

The work $d\mathcal{W}$ done by an outside agency in driving current through the inductor against the induced emf in a time dt is

$$d\mathcal{W} = \mathcal{P}\,dt = Li\frac{di}{dt}\,dt,$$

or

$$d\mathcal{W} = Li\,di.$$

Integrating between the limits of $i = 0$ and $i = I$, the final current through the inductor, we have,

$$\mathcal{W} = \int_0^I Li\,di,$$

yielding

$$\mathcal{W} = \tfrac{1}{2}LI^2 \tag{32-13}$$

for the energy of an inductor. When L is in henrys and I is in amperes, $\mathcal{W}$ is in joules.

When the current in a circuit builds up from zero to a value I, energy is supplied to the magnetic field, its value being $\frac{1}{2}LI^2$. As long as the

current remains constant, no additional energy is supplied to the magnetic field. In the circuit of Figure 32-12, all of the energy supplied during the steady state is transformed into heat. When the current decreases from I to zero, the magnetic field also decreases to zero; the energy that was stored in the magnetic field is returned to the circuit.

We can use the equation for the energy of an inductor to determine the energy per unit volume $\mathscr{W}_V$ in a magnetic field by considering an inductor in the form of a toroid. Its magnetic field is confined entirely to the volume within the toroid. The inductance of a toroid is given by

$$L = \frac{\mu_0 N^2 A}{s}, \tag{32-8}$$

hence the energy stored in the magnetic field of the toroid is, from Equation (32-13),

$$\mathscr{W} = \frac{1}{2}\frac{\mu_0 N^2 A}{s} I^2.$$

The magnetic field intensity within the toroid is given by

$$H = \frac{NI}{s}. \tag{30-5}$$

Substituting for N from Equation (30-5) into the above equation for the energy of the toroid, we have

$$\mathscr{W} = \tfrac{1}{2}\mu_0 H^2 A s.$$

The volume within the toroid is given by the product of its cross-sectional area A by its mean circumference s. Thus the energy per unit volume $\mathscr{W}_V$ of the magnetic field is given by

$$\mathscr{W}_V = \tfrac{1}{2}\mu_0 H^2 = \tfrac{1}{2}BH. \tag{32-14}$$

Notice that our procedure for finding the energy stored in the magnetic field has been very similar to the calculation by which we found the energy per unit volume stored in the electric field. In the case of the magnetic field, we utilized the energy in the field of a toroid, while in the electric field we utilized the energy in the field of a capacitor. Recalling that result from Equation (25-5), we may write the energy per unit volume $\mathscr{W}_V$ in the electromagnetic field as

$$\mathscr{W}_V = \tfrac{1}{2}(\epsilon_0 E^2 + \mu_0 H^2). \tag{32-15a}$$

This formula may be rewritten as

$$\mathscr{W}_V = \tfrac{1}{2}(DE + BH). \tag{32-15b}$$

In the mks system of units, the energy density $\mathscr{W}_V$ in the electromagnetic field is in joules per cubic meter when the electric field is expressed in volts per meter and the magnetic field intensity is expressed in amperes per meter.

TABLE 32-1 PRINCIPAL EQUATIONS IN MKS AND GAUSSIAN UNITS

Equation	MKS	Gaussian	
(32-2)	$\Delta\Phi = B\,\Delta A$	Same form as mks	Flux change
(32-3)	$\mathcal{E} = -\dfrac{d\Phi}{dt}$	$\mathcal{E} = -\dfrac{1}{c}\dfrac{d\Phi}{dt}$	Electromagnetic induction
(32-4)	$\mathcal{E} = -N\dfrac{d\Phi}{dt}$	$\mathcal{E} = -\dfrac{N}{c}\dfrac{d\Phi}{dt}$	N-turn coil
(32-6)	$\mathcal{E} = -L\dfrac{di}{dt}$	Same form as mks	Inductance
(32-8)	$L = \dfrac{\mu_0 N^2 A}{s}$	$L = \dfrac{4\pi N^2 A}{sc^2}$	Toroid or long solenoid
(32-13)	$\mathcal{W} = \frac{1}{2}LI^2$	Same form as mks	Energy
(32-14)	$\mathcal{W}_V = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}BH$	$\mathcal{W}_V = \dfrac{H^2}{8\pi} = \dfrac{BH}{8\pi}$	Energy density in vacuum
(32-15b)	$\mathcal{W}_V = \frac{1}{2}(DE + BH)$	$\mathcal{W}_V = \dfrac{1}{8\pi}(DE + BH)$	Energy density

TABLE 32-2 CONVERSION FACTORS RELATING MKS AND GAUSSIAN UNITS

Quantity	Symbol	MKS Unit	Gaussian Unit	
Flux	Φ	1 weber	$= 10^8$ maxwells	(emu)
Inductance	L	1 henry	$= \dfrac{1}{9\times10^{11}}$ stathenry	(esu)
Pole	p	1 weber	$= \dfrac{10^8}{4\pi}$ unit pole	(emu)
Magnetic intensity	H	$1\dfrac{\text{nt}}{\text{weber}} = 1\dfrac{\text{amp}}{\text{m}}$	$= 4\pi\times10^{-3}$ oersted	(emu)
Magnetic induction	B	1 weber/m^2	$= 10^4$ gausses	(emu)
Electric intensity	E	$1\dfrac{\text{nt}}{\text{coul}} = 1\dfrac{\text{volt}}{\text{m}}$	$= \dfrac{10^{-4}}{3}\dfrac{\text{statvolt}}{\text{cm}} = \dfrac{10^{-4}}{3}\dfrac{\text{dyne}}{\text{stcoul}}$	(esu)
Electric displacement	D	1 coul/m^2	$= 3\times10^5$ statcoul/cm^2	(esu)

Problems

32-1. The magnetic flux in a coil having 40 turns changes steadily from zero to 20,000 maxwells in 2 sec. Find the induced emf in the coil.

32-2. The magnetic flux through a coil of 125 turns is changed at a constant rate from zero to 0.40 weber in 2.5 sec. Determine the emf induced in this coil.

32-3. A circular coil of 50 turns and radius 15 cm lies in the x-y plane. The magnetic induction is changed in 0.001 sec at a constant rate from zero to a value whose x component is 0.3 weber/m^2, whose y component is 0.4 weber/m^2, and whose z component is 0.5 weber/m^2. Find the emf induced in the coil.

32-4. A wire 50 cm long is at rest along the x axis. A large magnet generating a uniform field directed along the $+y$ direction of 0.2 weber/m^2 is moved in the $+z$ direction with a speed of 30 m/sec. Find the magnitude and the direction of the emf induced in the wire.

32-5. A wire 75 cm long is moved in the y direction at a speed of 25 m/sec, so that the wire is always parallel to the x axis. The magnetic field has components $B_x = 0.2$ weber/m^2, $B_y = -0.3$ weber/m^2, and $B_z = 0.4$ weber/m^2. Find the emf induced in the wire.

32-6. A short solenoid, connected to a galvanometer, stands on one end upon a table. The north pole of a long bar magnet is brought down from above into the solenoid. Apply Lenz's law to find if the direction of the current induced in the solenoid is clockwise or counterclockwise, as viewed from above.

32-7. A rectangular coil of wire having 10 turns with dimensions of 20 cm by 30 cm is rotating at constant speed of 600 rpm in a magnetic field in which the magnetic induction is 600 gausses. The axis of rotation is perpendicular to the field. Find the maximum value of the emf produced.

32-8. Part of a closed circuit consists of a straight wire 1.5 m long moving at a speed of 2 m/sec perpendicular to a magnetic field of 10,000 gausses. (a) What is the emf induced in the circuit? (b) What is the force on the wire when the induced current is 5 amp?

32-9. A coil of 300 concentrated turns and an area of 800 cm^2 is lying flat on a horizontal table. When the coil is turned over through 180° in 0.10 sec, the average induced emf is 0.024 volt. What is the vertical component of the magnetic intensity of the earth's magnetic field?

32-10. A rectangular coil 12 cm by 25 cm and containing 15 turns is rotating at a constant speed of 1,800 rpm in a magnetic field in which the magnetic induction is 0.15 weber/m^2. The axis of rotation is perpendicular to the field. (a) Determine the maximum emf induced in this coil. (b) If the zero of time is taken at the point where the coil is parallel to the magnetic induction, find the emf in the coil when it has rotated by 53°.

32-11. When the current in a coil is changed at a constant rate of 5 amp/sec, the emf induced in the coil is 0.25 volt. Determine the self-inductance of the coil.

32-12. Derive a formula for the self-inductance of a long solenoid. Assume that the field is uniform everywhere within the solenoid.

32-13. What is the self-inductance of a solenoid 50 cm long and 5 cm in diameter, wound with 400 turns of wire?

32-14. What is the self-inductance of a toroid of mean circumference 25 cm, wound with 500 turns of wire, if the cross-sectional area of each turn is 2 cm^2?

32-15. An inductor has an inductance of 0.01 henry and an internal resistance of 5 ohms. The inductor is connected to the terminals of a battery having an emf of 12 volts. What will be the current in the circuit (a) in 0.001 sec? (b) In 0.01 sec? (c) In 0.1 sec? (d) Determine the time constant of this circuit.

32-16. An inductor of inductance 0.1 henry is connected in series with a 50-ohm resistor. This series combination is connected across the terminals of a 100-volt battery. What will be the energy stored in the magnetic field of the inductor when the current reaches a steady value?

32-17. Solve Equation (32-10) by separating the variables so that it becomes

$$\frac{di}{i - \frac{V}{R}} = -\frac{R}{L}dt,$$

and integrating to obtain Equation (32-11). Evaluate the constant of integration, letting $i = 0$ when $t = 0$.

32-18. Suppose that a switching arrangement is used in the circuit of Figure 32-12 so that the battery is removed from the circuit and a connecting wire is substituted in its place. (a) Show that Equation (32-10) becomes

$$L\frac{di}{dt} + iR = 0.$$

(b) Solve this equation for the current as a function of the time. Evaluate the constant of integration letting $i = I$ when $t = 0$. (c) Plot a graph of this equation and compare it with the graph of Figure 32-13.

32-19. Referring to the toroid of Problem 32-14, calculate (a) the energy in the magnetic field when the current is 10 amp and (b) the energy per unit volume of this field.