

Fundamentals of

ELECTRIC CIRCUITS

Second Edition

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Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.

Consider the series RLC circuit shown in Fig. 14.21 in the frequency domain. The input impedance is

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C} \quad (14.22)$$

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad (14.23)$$

Resonance results when the imaginary part of the transfer function is zero, or

$$\text{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0 \quad (14.24)$$

The value of ω that satisfies this condition is called the *resonant frequency*. Thus, the resonance condition is

$$\omega_0 L = \frac{1}{\omega_0 C} \quad (14.25)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad (14.26)$$

Since $\omega_0 = 2\pi f_0$,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \quad (14.27)$$

Note that at resonance:

1. The impedance is purely resistive, thus, $\mathbf{Z} = R$. In other words, the LC series combination acts like a short circuit, and the entire voltage is across R.
2. The voltage \mathbf{V}_s and the current \mathbf{I} are in phase, so that the power factor is unity.
3. The magnitude of the transfer function $\mathbf{H}(\omega) = \mathbf{Z}(\omega)$ is minimum.
4. The inductor voltage and capacitor voltage can be much more than the source voltage.

The frequency response of the circuit's current magnitude

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad (14.28)$$

is shown in Fig. 14.22; the plot only shows the symmetry illustrated in this graph when the frequency axis is a logarithm. The average power

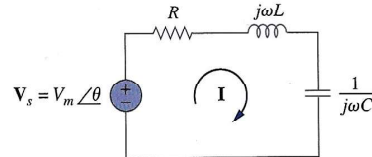


Figure 14.21 The series resonant circuit.

Note No. 4 becomes evident from the fact that

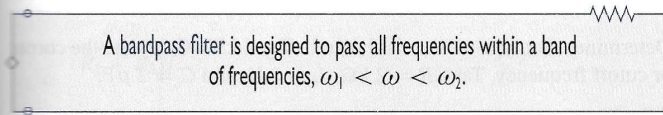
$$|V_L| = \frac{V_m}{R} \omega_0 L = QV_m$$

$$|V_C| = \frac{V_m}{R} \frac{1}{\omega_0 C} = QV_m$$

where Q is the quality factor, defined in Eq. (14.38).

We observe that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 0$. Figure 14.36 shows the plot of $|H(\omega)|$. The bandpass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω_0 , the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14.55)$$



Since the bandpass filter in Fig. 14.35 is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor are determined as in Section 14.5. A bandpass filter can also be formed by cascading the lowpass filter (where $\omega_2 = \omega_c$) in Fig. 14.31 with the highpass filter (where $\omega_1 = \omega_c$) in Fig. 14.33.

14.7.4 Bandstop Filter

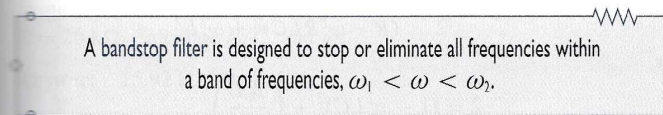
A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a *bandstop*, *bandreject*, or *notch* filter. A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig. 14.37. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)} \quad (14.56)$$

Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Figure 14.38 shows the plot of $|H(\omega)|$. Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14.57)$$

while the half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas in Section 14.5 for a series resonant circuit. Here, ω_0 is called the *frequency of rejection*, while the corresponding bandwidth ($B = \omega_2 - \omega_1$) is known as the *bandwidth of rejection*. Thus,



Notice that adding the transfer functions of the bandpass and the bandstop gives unity at any frequency for the same values of R , L , and C . Of course, this is not true in general but true for the circuits treated here. This is due to the fact that the characteristic of one is the inverse of the other.

In concluding this section, we should note that:

1. From Eqs. (14.50), (14.52), (14.54), and (14.56), the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter as the next section shows.

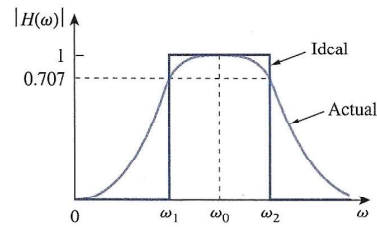


Figure 14.36 Ideal and actual frequency response of a bandpass filter.

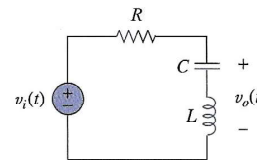


Figure 14.37 A bandstop filter.

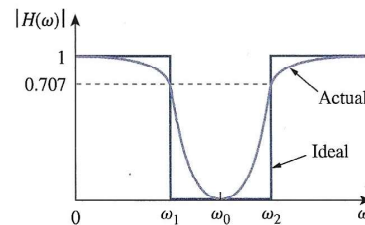


Figure 14.38 Ideal and actual frequency response of a bandstop filter.

In this section, we have used ω_c for the center frequency of the bandpass filter instead of ω_0 as in Chapter 14, to avoid confusing ω_0 with the fundamental frequency of the input signal.

sufficiently large ($\omega_c \gg \omega_0$, e.g., making C small), a large number of the harmonics can be passed. On the other hand, by making ω_c sufficiently small ($\omega_c \ll \omega_0$), we can block out all the ac components and pass only dc, as shown typically in Fig. 17.43(b). (See Fig. 17.2(a) for the Fourier series expansion of the square wave.)

Similarly, the output of a bandpass filter depends on the input signal, the transfer function of the filter $H(\omega)$, its bandwidth B , and its center frequency ω_c . As illustrated in Fig. 17.44(a), the filter passes all the harmonics of the input signal within a band of frequencies ($\omega_1 < \omega < \omega_2$) centered around ω_c . We have assumed that ω_0 , $2\omega_0$, and $3\omega_0$ are within that band. If the filter is made highly selective ($B \ll \omega_0$) and $\omega_c = \omega_0$, where ω_0 is the fundamental frequency of the input signal, the filter passes only the fundamental component ($n = 1$) of the input and blocks out all higher harmonics. As shown in Fig. 17.44(b), with a square wave as input, we obtain a sine wave of the same frequency as the output. (Again, refer to Fig. 17.2(a).)

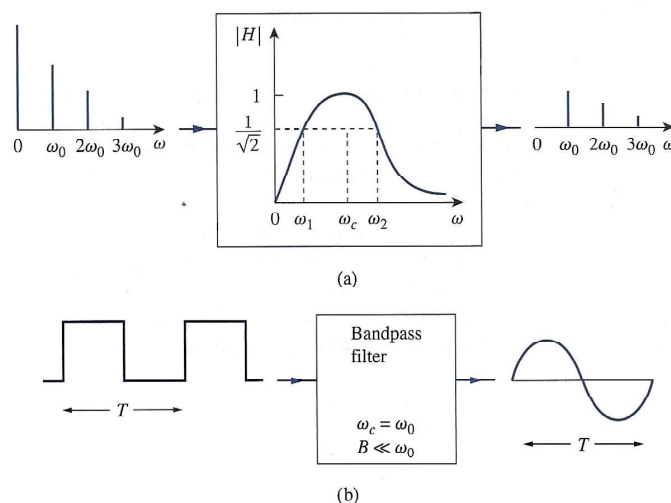


Figure 17.44 (a) Input and output spectra of a bandpass filter, (b) the bandpass filter passes only the fundamental component when $B \ll \omega_0$.

EXAMPLE 17.14

If the sawtooth waveform in Fig. 17.45(a) is applied to an ideal lowpass filter with the transfer function shown in Fig. 17.45(b), determine the output.

Solution:

The input signal in Fig. 17.45(a) is the same as the signal in Fig. 17.9. From Practice Prob. 17.2, we know that the Fourier series expansion is

$$x(t) = \frac{1}{2} - \frac{1}{\pi} \sin \omega_0 t - \frac{1}{2\pi} \sin 2\omega_0 t - \frac{1}{3\pi} \sin 3\omega_0 t - \dots$$

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Answer

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