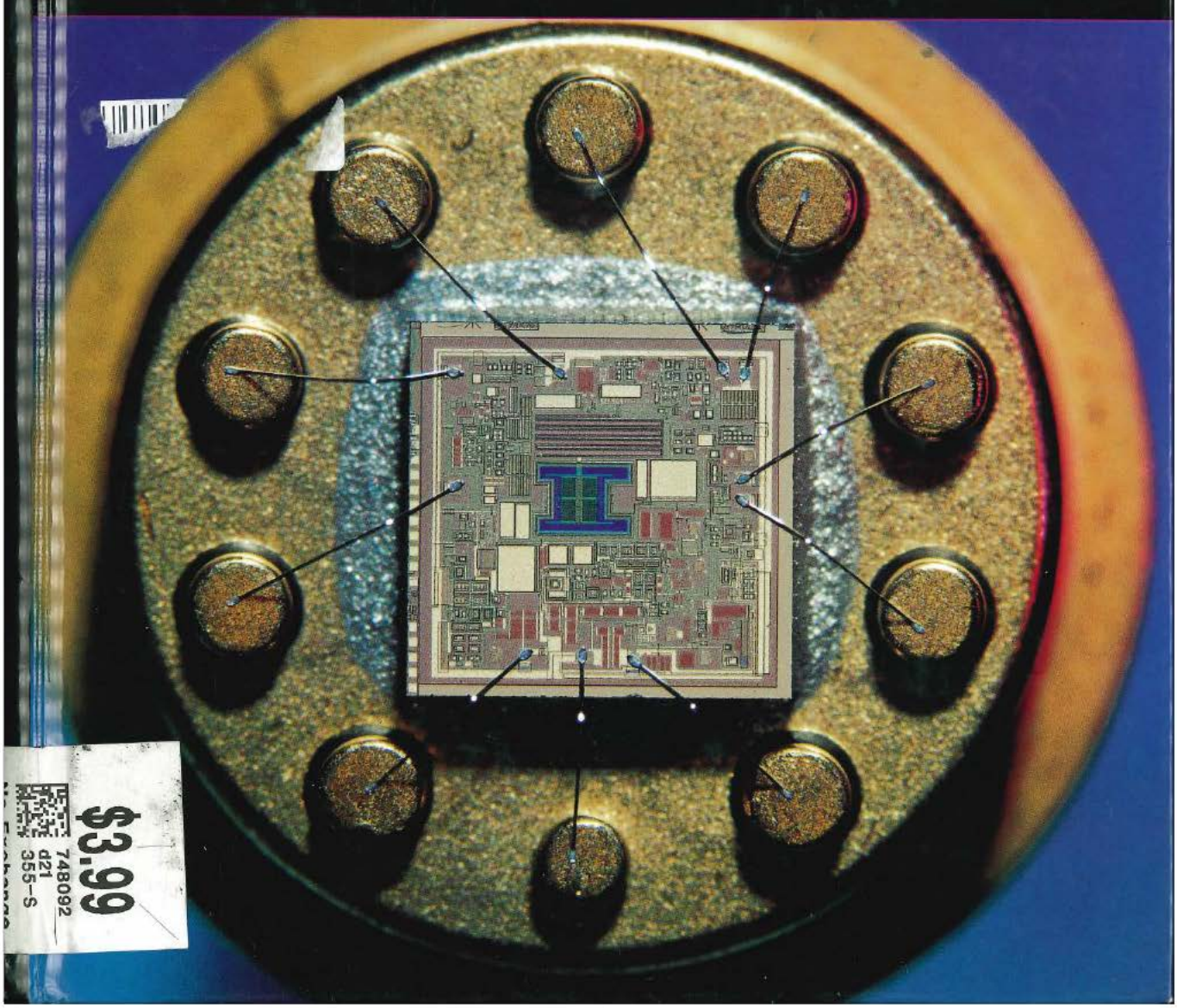
Microelectronic Circuits

FOURTH EDITION

Sedra/Smith

CD-ROM Enclosed



FOURTH EDITION

MICROELECTRONIC CIRCUITS

Adel S. Sedra
University of Toronto

Kenneth C. Smith
University of Toronto and Hong Kong University of Science and Technology

New York Oxford
OXFORD UNIVERSITY PRESS
1998

Oxford University Press

Oxford New York
Athens Auckland Bangkok Bogota Bombay Buenos Aires
Calcutta Cape Town Dar es Salaam Delhi Florence Hong Kong
Istanbul Karachi Kuala Lumpur Madras Madrid Melbourne
Mexico City Nairobi Paris Singapore Taipei Tokyo Toronto Warsaw

and associated companies in

Berlin Ibadan

Published by Oxford University Press, Inc.,
198 Madison Avenue, New York, New York, 10016
http://www.oup-usa.org

Oxford is a registered trademark of Oxford University Press

Staff
Senior Acquiring Editor: *Bill Zobrist*
Editorial/Production/Design Manager: *Elyse Dubin*
Assistant Acquiring Editor: *Krysia Bebick*
Production Coordinator: *John Bauco*
Editorial Assistant: *Jasmine Urmeneta*
Composition: *York Graphic Services*
Production Services: *York Production Services*
Director of Marketing: *Scott Burns*
Assistant Sales Manager: *Marcy Levine*
Advertising Manager: *David Walker*
Copywriter: *Leigh Ann Florek*
Cover Design: *Ed Berg Designs*
Cover Art Direction: *Linda Roppolo*
Manufacturing Director: *Ben Lee*
Manufacturing Controller: *Robin Meredith-Clement*

Library of Congress Cataloging-in-Publication Data

Sedra, Adel S.
Microelectronic circuits / Adel S. Sedra, Kenneth C. Smith. — 4th ed.
p. cm. — (Oxford series in electrical and computer engineering)
Includes bibliographical references and index.
ISBN 0-19-511663-1
1. Electronic circuits. 2. Iutegrated circuits. I. Smith, Kenneth Carless. II. Title. III. Series.
TK7867.S39 1997
621.381—dc21 97-11254
CIP

9 8 7 6 5 4 3

Printed in the United States of America on acid-free paper

Cover Illustration: The chip shown is the ADXL-50 surface-micromachined accelerometer. For the first time, sensor and signal conditioning are combined on a single monolithic chip. In its earliest application, it was a key factor in the improved reliability and reduced cost of modern automotive airbag systems. Photo reprinted with permission of Analog Devices, Inc.

CONDENSED TABLE OF CONTENTS

PREFACE *v*

Chapter 1 INTRODUCTION TO ELECTRONICS *1*

PART I DEVICES AND BASIC CIRCUITS *58*

Chapter 2 OPERATIONAL AMPLIFIERS *60*
Chapter 3 DIODES *122*
Chapter 4 BIPOLAR JUNCTION TRANSISTORS (BJTs) *221*
Chapter 5 FIELD-EFFECT TRANSISTORS (FETs) *353*

PART II ANALOG CIRCUITS *484*

Chapter 6 DIFFERENTIAL AND MULTISTAGE AMPLIFIERS *487*
Chapter 7 FREQUENCY RESPONSE *583*
Chapter 8 FEEDBACK *667*
Chapter 9 OUTPUT STAGES AND POWER AMPLIFIERS *751*
Chapter 10 ANALOG INTEGRATED CIRCUITS *810*
Chapter 11 FILTERS AND TUNED AMPLIFIERS *884*
Chapter 12 SIGNAL GENERATORS AND WAVEFORM-SHAPING CIRCUITS *973*

PART III DIGITAL CIRCUITS *1040*

Chapter 13 MOS DIGITAL CIRCUITS *1042*
Chapter 14 BIPOLAR AND ADVANCED-TECHNOLOGY DIGITAL CIRCUITS *1158*

APPENDIXES

A VLSI FABRICATION TECHNOLOGY *A-1*
B TWO-POST NETWORK PARAMETERS *B-1*
C AN INTRODUCTION TO SPICE *C-1*
D INPUT FILES FOR THE SPICE EXAMPLES *D-1*
E SOME USEFUL NETWORK THEOREMS *E-1*
F SINGLE-TIME-CONSTANT CIRCUITS *F-1*
G DETERMINING THE PARAMETER VALUES OF THE HYBRID-π BJT MODEL *G-1*
H STANDARD RESISTANCE VALUES AND UNIT PREFIXES *H-1*
I ANSWERS TO SELECTED PROBLEMS *I-1*

INDEX *IN-1*

DETAILED TABLE OF CONTENTS

PREFACE *v*

Chapter 1 INTRODUCTION TO ELECTRONICS *1*

Introduction *1*
1.1 Signals *2*
1.2 Frequency Spectrum of Signals *3*
1.3 Analog and Digital Signals *6*
1.4 Amplifiers *9*
1.5 Circuit Models for Amplifiers *19*
1.6 Frequency Response of Amplifiers *28*
1.7 The Digital Logic Inverter *39*
Summary *47*
Bibliography *48*
Problems *48*

PART I DEVICES AND BASIC CIRCUITS *58*

Chapter 2 OPERATIONAL AMPLIFIERS *60*

Introduction *60*
2.1 The Op-Amp Terminals *61*
2.2 The Ideal Op Amp *62*
2.3 Analysis of Circuits Containing Ideal Op Amps—The Inverting Configuration *64*
2.4 Other Applications of the Inverting Configuration *71*
2.4.1 The Inverting Configuration with General Impedances Z_1 and Z_2 *71*
2.4.2 The Inverting Integrator *73*
2.4.3 The Op Amp Differentiator *78*
2.4.4 The Weighted Summer *80*
2.5 The Noninverting Configuration *81*
2.6 Examples of Op-Amp Circuits *85*
2.7 Effect of Finite Open-Loop Gain and Bandwidth on Circuit Performance *92*
2.8 Large-Signal Operation of Op Amps *97*
2.9 DC Imperfections *101*
Summary *108*
Bibliography *109*
Problems *110*

Chapter 3 DIODES *122*

Introduction *122*
3.1 The Ideal Diode *123*
3.2 Terminal Characteristics of Junction Diodes *131*
3.3 Physical Operation of Diodes *137*
3.3.1 Basic Semiconductor Concepts *138*
3.3.2 The *pn* Junction Under Open-Circuit Conditions *143*
3.3.3 The *pn* Junction Under Reverse-Bias Conditions *146*
3.3.4 The *pn* Junction in the Breakdown Region *149*
3.3.5 The *pn* Junction Under Forward-Bias Conditions *151*
3.3.6 Summary *155*
3.4 Analysis of Diode Circuits *155*
3.5 The Small-Signal Model and Its Application *163*
3.6 Operation in the Reverse Breakdown Region—Zener Diodes *172*
3.7 Rectifier Circuits *179*
3.8 Limiting and Clamping Circuits *191*
3.9 Special Diode Types *196*
3.10 The SPICE Diode Model and Simulation Examples *199*
Summary *206*
Bibliography *206*
Problems *207*

Chapter 4 BIPOLAR JUNCTION TRANSISTORS (BJTS) *221*

Introduction *221*
4.1 Physical Structure and Modes of Operation *222*
4.2 Operation of the *npn* Transistor in the Active Mode *223*
4.3 The *pnp* Transistor *232*
4.4 Circuit Symbols and Conventions *234*
4.5 Graphical Representation of Transistor Characteristics *238*
4.6 Analysis of Transistor Circuits at DC *241*
4.7 The Transistor as an Amplifier *253*
4.8 Small-Signal Equivalent Circuit Models *259*
4.9 Graphical Analysis *272*
4.10 Biasing the BJT for Discrete-Circuit Design *276*
4.11 Basic Single-Stage BJT Amplifier Configurations *282*
4.12 The Transistor as a Switch—Cutoff and Saturation *295*
4.13 A General Large-Signal Model for the BJT: The Ebers-Moll (EM) Model *303*
4.14 The Basic BJT Logic Inverter *310*
4.15 Complete Static Characteristics, Internal Capacitances, and Second-Order Effects *315*
4.16 The SPICE BJT Model and Simulation Examples *326*
Summary *331*
Bibliography *332*
Problems *333*

Chapter 5 FIELD-EFFECT TRANSISTORS (FETs) *353*

Introduction *353*
5.1 Structure and Physical Operation of the Enhancement-Type MOSFET *354*
5.2 Current-Voltage Characteristics of the Enhancement MOSFET *366*
5.3 The Deletion-Type MOSFET *376*
5.4 MOSFET Circuits at DC *380*
5.5 The MOSFET as an Amplifier *389*
5.6 Biasing in MOS Amplifier Circuits *400*
5.6.1 Biasing of Discrete MOSFET Amplifiers *400*
5.6.2 Biasing in Integrated-Circuit MOS Amplifiers *402*
5.7 Basic Configurations of Single-Stage IC MOS Amplifiers *408*
5.7.1 The CMOS Common-Source Amplifier *409*
5.7.2 The CMOS Common-Gate Amplifier *413*
5.7.3 The Common-Drain or Source-Follower Configuration *416*
5.7.4 All-NMOS Amplifier Stages *419*
5.7.5 A Final Remark *425*
5.8 The CMOS Digital Logic Inverter *425*
5.9 The MOSFET as an Analog Switch *436*
5.10 The MOSFET Internal Capacitances and High-Frequency Model *441*
5.11 The Junction Field-Effect Transistor (JFET) *447*
5.12 Gallium Arsenide (GaAs) Devices—The MESFET *452*
5.13 The SPICE MOSFET Model and Simulation Examples *458*
Summary *464*
Bibliography *464*
Problems *466*

PART II ANALOG CIRCUITS *484*

Chapter 6 DIFFERENTIAL AND MULTISTAGE AMPLIFIERS *487*

Introduction *487*
6.1 The BJT Differential Pair *487*
6.2 Small-Signal Operation of the BJT Differential Amplifier *492*
6.3 Other Nonideal Characteristics of the Differential Amplifier *504*
6.4 Biasing in BJT Integrated Circuits *508*
6.5 The BJT Differential Amplifier with Active Load *522*
6.6 MOS Differential Amplifiers *527*
6.7 BiCMOS Amplifiers *537*
6.8 GaAs Amplifiers *542*
6.9 Multistage Amplifiers *551*
6.10 SPICE Simulation Example *558*
Summary *563*
Bibliography *564*
Problems *564*

Chapter 7 FREQUENCY RESPONSE *583*

Introduction *583*
7.1 s-Domain Analysis: Poles, Zeros, and Bode Plots *584*
7.2 The Amplifier Transfer Function *590*
7.3 Low-Frequency Response of the Common-Source and Common-Emitter Amplifiers *602*
7.4 High-Frequency Response of the Common-Source and Common-Emitter Amplifiers *610*
7.5 The Common-Base, Common-Gate, and Cascode Configurations *619*
7.6 Frequency Response of the Emitter and Source Followers *626*
7.7 The Common-Collector Common-Emitter Cascade *630*
7.8 Frequency Response of the Differential Amplifier *635*
7.9 SPICE Simulation Examples *645*
Summary *649*
Bibliography *650*
Problems *650*

Chapter 8 FEEDBACK *667*

Introduction *667*
8.1 The General Feedback Structure *668*
8.2 Some Properties of Negative Feedback *670*
8.3 The Four Basic Feedback Topologies *675*
8.4 The Series-Shunt Feedback Amplifier *679*
8.5 The Series-Series Feedback Amplifier *688*
8.6 The Shunt-Shunt and the Shunt-Series Feedback Amplifiers *696*
8.7 Determining the Loop Gain *708*
8.8 The Stability Problem *713*
8.9 Effect of Feedback on the Amplifier Poles *715*
8.10 Stability Study Using Bode Plots *725*
8.11 Frequency Compensation *729*
8.12 SPICE Simulation Examples *735*
Summary *740*
Bibliography *740*
Problems *741*

Chapter 9 OUTPUT STAGES AND POWER AMPLIFIERS *751*

Introduction *751*
9.1 Classification of Output Stages *752*
9.2 Class A Output Stage *753*
9.3 Class B Output Stage *758*

9.4 Class AB Output Stage *764*
9.5 Biasing the Class AB Circuit *767*
9.6 Power BJTs *773*
9.7 Variations on the Class AB Configuration *780*
9.8 IC Power Amplifiers *785*
9.9 MOS Power Transistors *792*
9.10 SPICE Simulation Example *797*
Summary *802*
Bibliography *802*
Problems *803*

Chapter 10 ANALOG INTEGRATED CIRCUITS *810*

Introduction *810*
10.1 The 741 Op-Amp Circuit *811*
10.2 DC Analysis of the 741 *815*
10.3 Small-Signal Analysis of the 741 Input Stage *822*
10.4 Small-Signal Analysis of the 741 Second Stage *828*
10.5 Analysis of the 741 Output Stage *830*
10.6 Gain and Frequency Response of the 741 *835*
10.7 CMOS Op Amps *840*
10.8 Alternative Configurations for CMOS and BiCMOS Op Amps *850*
10.9 Data Converters—An Introduction *856*
10.10 D/A Converter Circuits *860*
10.11 A/D Converter Circuits *864*
10.12 SPICE Simulation Example *870*
Summary *874*
Bibliography *875*
Problems *876*

Chapter 11 FILTERS AND TUNED AMPLIFIERS *884*

Introduction *884*
11.1 Filter Transmission, Types, and Specification *885*
11.2 The Filter Transfer Function *889*
11.3 Butterworth and Chebyshev Filters *892*
11.4 First-Order and Second-Order Filter Functions *900*
11.5 The Second-Order LCR Resonator *909*
11.6 Second-Order Active Filters Based on Inductor Replacement *915*
11.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology *923*
11.8 Single-Amplifier Biquadratic Active Filters *929*
11.9 Sensitivity *938*

11.10 Switched-Capacitor Filters *941*
11.11 Tuned Amplifiers *946*
11.12 SPICE Simulation Examples *959*
Summary *965*
Bibliography *966*
Problems *967*

Chapter 12 SIGNAL GENERATORS AND WAVEFORM-SHAPING CIRCUITS *973*

Introduction *973*
12.1 Basic Principles of Sinusoidal Oscillators *974*
12.2 Op Amp-RC Oscillator Circuits *980*
12.3 LC and Crystal Oscillators *988*
12.4 Bistable Multivibrators *994*
12.5 Generation of Square and Triangular Waveforms Using Astable Multivibrators *1002*
12.6 Generation of a Standardized Pulse—The Monostable Multivibrator *1007*
12.7 Integrated-Circuit Timers *1009*
12.8 Nonlinear Waveform-Shaping Circuits *1014*
12.9 Precision Rectifier Circuits *1018*
12.10 SPICE Simulation Examples *1026*
Summary *1030*
Bibliography *1030*
Problems *1031*

PART III DIGITAL CIRCUITS *1040*

Chapter 13 MOS DIGITAL CIRCUITS *1042*

Introduction *1042*
13.1 Digital Circuit Design: An Overview *1043*
13.1.1 Digital IC Technologies and Logic-Circuit Families *1043*
13.1.2 Logic-Circuit Characterization *1045*
13.1.3 Styles for Digital System Design *1048*
13.1.4 Design Abstraction and Computer Aids *1048*
13.2 Design and Performance Analysis of the CMOS Inverter *1049*
13.3 CMOS Logic-Gate Circuits *1058*
13.4 Pseudo-NMOS Logic Circuits *1070*
13.5 Pass-Transistor Logic Circuits *1080*
13.6 Dynamic Logic Circuits *1090*
13.7 Latches and Flip-Flops *1097*
13.8 Multivibrator Circuits *1106*
13.9 Semiconductor Memories: Types and Architectures *1113*

13.10 Random-Access Memory (RAM) Cells *1116*
13.11 Sense Amplifiers and Address Decoders *1125*
13.11.1 The Sense Amplifier *1125*
13.11.2 The Row-Address Decoder *1131*
13.11.3 The Column-Address Decoder *1133*
13.12 Read-Only Memory (ROM) *1133*
13.13 SPICE Simulation Example *1140*
Summary *1144*
Bibliography *1146*
Problems *1146*

Chapter 14 Bipolar and Advanced-Technology Digital Circuits *1158*

Introduction *1158*
14.1 Dynamic Operation of the BJT Switch *1159*
14.2 Early Forms of BJT Digital Circuits *1163*
14.3 Transistor-Transistor Logic (TTL or T^2L) *1167*
14.4 Characteristics of Standard TTL *1180*
14.5 TTL Families with Improved Performance *1187*
14.6 Emitter-Coupled Logic (ECL) *1195*
14.7 BiCMOS Digital Circuits *1211*
14.8 Gallium-Arsenide Digital Circuits *1216*
14.9 SPICE Simulation Example *1224*
Summary *1230*
Bibliography *1231*
Problems *1232*

APPENDIXES

A VLSI Fabrication Technology *A-1*
B Two-Port Network Parameters *B-1*
C An Introduction to SPICE *C-1*
D Input Files for the SPICE Examples *D-1*
E Some Useful Network Theorems *E-1*
F Single-Time-Constant Circuits *F-1*
G Determining the Parameter Values of the Hybrid-π BJT Model *G-1*
H Standard Resistance Values and Unit Prefixes *H-1*
I Answers to Selected Problems *I-1*

Index *IN-1*

CHAPTER 1

Introduction to Electronics

Introduction
1.1 Signals
1.2 Frequency Spectrum of Signals
1.3 Analog and Digital Signals
1.4 Amplifiers
1.5 Circuit Models for Amplifiers
1.6 Frequency Response of Amplifiers
1.7 The Digital Logic Inverter
Summary
Bibliography
Problems

INTRODUCTION

The subject of this book is modern electronics, a field that has come to be known as **microelectronics. Microelectronics** refers to the integrated-circuit (IC) technology that at the time of this writing is capable of producing circuits that contain millions of components in a small piece of silicon (known as a **silicon chip**) whose area is in the order of 100 mm^2. One such microelectronic circuit, for example, is a complete digital computer, which accordingly is known as a **microcomputer** or, more generally, a **microprocessor.**

In this book we shall study electronic devices that can be used singly (in the design of **discrete circuits**) or as components of an **integrated-circuit (IC)** chip. We shall study the design and analysis of interconnections of these devices, which form discrete and integrated circuits of varying complexity and perform a wide variety of functions. We shall also learn about available IC chips and their application in the design of electronic systems.

The purpose of this first chapter is to introduce some basic concepts and terminology. In particular, we shall learn about signals and about one of the most important signal-processing functions electronic circuits are designed to perform, namely, signal amplification. We shall then look at models for linear amplifiers. These models will be employed in subsequent chapters in the design and analysis of actual amplifier circuits.

Whereas the amplifier is the basic element of analog circuits, the logic inverter plays this role in digital circuits. We shall therefore take a preliminary look at the digital inverter, its circuit function, and important characteristics.

In addition to motivating the study of electronics, this chapter serves as a bridge between the study of linear circuits and that of the subject of this book: the design and analysis of electronic circuits.

1.1 SIGNALS

Signals contain information about a variety of things and activities in our physical world. Examples abound: Information about the weather is contained in signals that represent the air temperature, pressure, wind speed, etc. The voice of a radio announcer reading the news into a microphone provides an acoustic signal that contains information about world affairs. To monitor the status of a nuclear reactor, instruments are used to measure a multitude of relevant parameters, each instrument producing a signal.

To extract required information from a set of signals, the observer (be it a human or a machine) invariably needs to **process** the signals in some predetermined manner. This **signal processing** is usually most conveniently performed by electronic systems. For this to be possible, however, the signal must first be converted into an electric signal, that is, a voltage or a current. This process is accomplished by devices known as **transducers.** A variety of transducers exist, each suitable for one of the various forms of physical signals. For instance, the sound waves generated by a human can be converted into electric signals using a microphone, which is in effect a pressure transducer. It is not our purpose here to study transducers; rather, we shall assume that the signals of interest already exist in the electrical domain and represent them by one of the two equivalent forms shown in Fig. 1.1. In Fig. 1.1(a) the signal is represented by a voltage source $v_s(t)$ having a source resistance R_s. In the alternate representation of Fig. 1.1(b) the signal is represented by a current source $i_s(t)$ having a source resistance R_s. Although the two representations are equivalent, that in Fig. 1.1(a) (known as the Thévenin form) is preferred when R_s is low. The representation of Fig. 1.1(b) (known as the Norton form) is preferred when R_s is high.

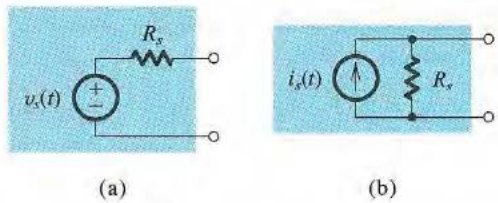


Fig. 1.1 Two alternative representations of a signal source: **(a)** the Thévenin form, and **(b)** the Norton form.

From the discussion above, it should be apparent that a signal is a time-varying quantity that can be represented by a graph such as that shown in Fig. 1.2. In fact, the information content of the signal is represented by the changes in its magnitude as time progresses; that is, the information is contained in the "wiggles" in the signal waveform. In general, such waveforms are difficult to characterize mathematically. In other words, it is not easy to describe succinctly an arbitrary looking waveform such as that of Fig. 1.2. Of course, such a description is of great importance for the purpose of designing appropriate signal-processing circuits that perform desired functions on the given signal.

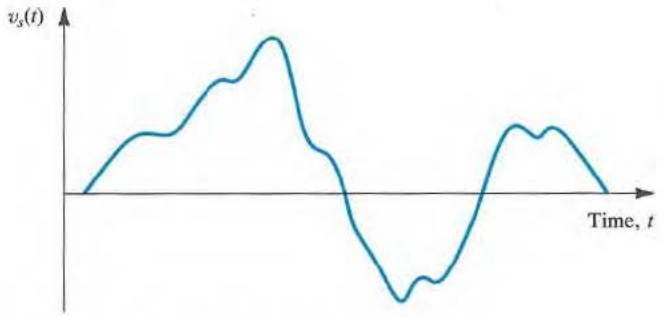


Fig. 1.2 An arbitrary voltage signal $v_s(t)$.

1.2 FREQUENCY SPECTRUM OF SIGNALS

An extremely useful characterization of a signal, and for that matter of any arbitrary function of time, is in terms of its **frequency spectrum.** Such a description of signals is obtained through the mathematical tools of **Fourier series** and **Fourier transform.**[1] We are not interested at this point in the details of these transformations; suffice it to say that they provide the means for representing a voltage signal $v_s(t)$ or a current signal $i_s(t)$ as the sum of sine-wave signals of different frequencies and amplitudes. This makes the sine wave a very important signal in the analysis, design, and testing of electronic circuits. Therefore, we shall briefly review the properties of the sinusoid.

Figure 1.3 shows a sine-wave voltage signal $v_a(t)$,

$$v_a(t) = V_a \sin \omega t \tag{1.1}$$

where V_a denotes the peak value or amplitude in volts and ω denotes the angular frequency in radians per second; that is, $\omega = 2\pi f$ rad/s, where f is the frequency in hertz, $f = 1/T$ Hz, and T is the period in seconds.

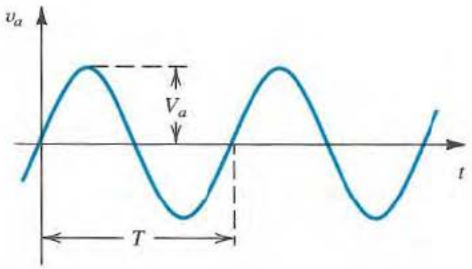


Fig. 1.3 Sine-wave voltage signal of amplitude V_a and frequency $f = 1/T$ Hz. The angular frequency $\omega = 2\pi f$ rad/s.

[1] The reader who has not yet studied these topics should not be alarmed. No detailed application of this material will be made until Chapter 7. Nevertheless, a general understanding of Section 1.2 should be very helpful when studying early parts of this book.

The sine-wave signal is completely characterized by its peak value V_a, its frequency ω, and its phase with respect to an arbitrary reference time. In this case the time origin has been chosen so that the phase angle is 0. It should be mentioned that it is common to express the amplitude of a sine-wave signal in terms of its root-mean-square (rms) value, which is equal to the peak value divided by $\sqrt{2}$. Thus the rms value of the sinusoid $v_a(t)$ of Fig. 1.3 is $V_a/\sqrt{2}$. For instance, when we speak of the wall power supply in our homes as being 120 V, we mean that it has a sine waveform of $120\sqrt{2}$ volts peak value.

Returning now to the representation of signals as the sum of sinusoids, we note that the Fourier series is utilized to accomplish this task for the special case when the signal is a periodic function of time. On the other hand, the Fourier transform is more general and can be used to obtain the frequency spectrum of a signal whose waveform is an arbitrary function of time.

The Fourier series allows us to express a given periodic function of time as the sum of an infinite number of sinusoids whose frequencies are harmonically related. For instance, the symmetrical square-wave signal in Fig. 1.4 can be expressed as

$$v(t) = \frac{4V}{\pi}(\sin \omega_0 t + \tfrac{1}{3} \sin 3\omega_0 t + \tfrac{1}{5} \sin 5\omega_0 t + \cdots) \tag{1.2}$$

where V is the amplitude of the square wave and $\omega_0 = 2\pi/T$ (T is the period of the square wave) is called the **fundamental frequency.** Note that because the amplitudes of the harmonics progressively decrease, the infinite series can be truncated, with the truncated series providing an approximation to the square waveform.

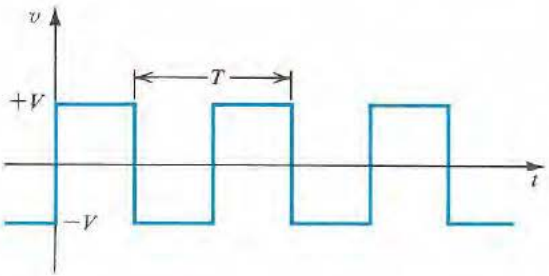


Fig. 1.4 A symmetrical square-wave signal of amplitude V.

The sinusoidal components in the series of Eq. (1.2) constitute the frequency spectrum of the square-wave signal. Such a spectrum can be graphically represented as in Fig. 1.5, where the horizontal axis represents the angular frequency ω in radians per second.

The Fourier transform can be applied to a nonperiodic function of time, such as that depicted in Fig. 1.2, and provides its frequency spectrum as a continuous function of frequency, as indicated in Fig. 1.6. Unlike the case of periodic signals, where the spectrum consists of discrete frequencies (at ω_0 and its harmonics), the spectrum of a nonperiodic signal contains in general all possible frequencies. Nevertheless, the essential parts of the spectra of practical signals are usually confined to relatively short segments of the frequency (ω) axis—an observation that is very useful in the processing of such signals. For instance, the spectrum of audible sounds such as speech and music extends from about 20 Hz to about 20 kHz—a frequency range known as the **audio band.** Here we should note that

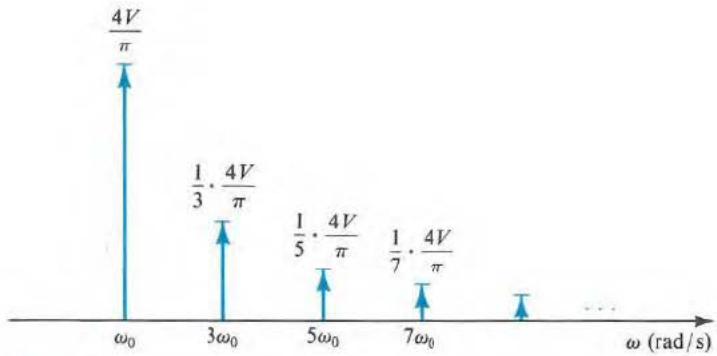


Fig. 1.5 The frequency spectrum (also known as the line spectrum) of the periodic square wave of Fig. 1.4.

although some musical tones have frequencies above 20 kHz, the human ear is incapable of hearing frequencies that are much above 20 kHz.

We conclude this section by noting that a signal can be represented either by the manner in which its waveform varies with time, as for the voltage signal $v_a(t)$ shown in Fig. 1.2, or in terms of its frequency spectrum, as in Fig. 1.6. The two alternative representations are known as the time-domain representation and the frequency-domain representation, respectively. The frequency-domain representation of $v_a(t)$ will be denoted by the symbol $V_a(\omega)$.

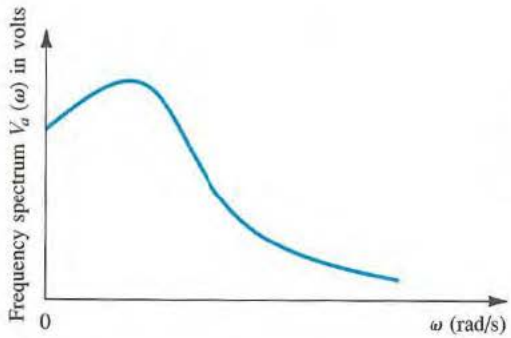


Fig. 1.6 The frequency spectrum of an arbitrary waveform such as that in Fig. 1.2.

Exercises

1.1 Find the frequencies f and ω of a sine-wave signal with a period of 1 ms.

Ans. $f = 1000$ Hz; $\omega = 2\pi \times 10^3$ rad/s

1.2 What is the period T of sine waveforms characterized by frequencies of (a) $f = 60$ Hz? (b) $f = 10^{-3}$ Hz? (c) $f = 1$ MHz?

Ans. 16.7 ms; 1000 s; 1 μs

1.3 When the square-wave signal of Fig. 1.4, whose Fourier series is given in Eq. (1.2), is applied to a resistor, the total power dissipated may be calculated directly using the relationship $P = 1/T \int_0^T (v^2/R)\, dt$, or indirectly by summing the contribution of each of the harmonic components, that is, $P = P_1 + P_3 + P_5 + \cdots$, which may be found directly from rms values. Verify that the two approaches are equivalent. What fraction of the energy of a square wave is in its fundamental? In its first five harmonics? In its first seven? First nine? In what number of harmonics is 90% of the energy? (Note that in counting harmonics, the fundamental at ω_0 is the first, the one at $2\omega_0$ is the second, etc.)

Ans. 0.81; 0.93; 0.95; 0.96; 3

1.3 ANALOG AND DIGITAL SIGNALS

The voltage signal depicted in Fig. 1.2 is called an **analog signal.** The name derives from the fact that such a signal is analogous to the physical signal that it represents. The magnitude of an analog signal can take on any value; that is, the amplitude of an analog signal exhibits a continuous variation over its range of activity. The vast majority of signals in the world around us are analog. Electronic circuits that process such signals are known as **analog circuits.** A variety of analog circuits will be studied in this book.

An alternative form of signal representation is that of a sequence of numbers, each number representing the signal magnitude at an instant of time. The resulting signal is called a **digital signal.** To see how a signal can be represented in this form—that is, how signals can be converted from analog to digital form—consider Fig. 1.7(a). Here the curve represents a voltage signal, identical to that in Fig. 1.2. At equal intervals along the time axis we have marked the time instants t_0, t_1, t_2, and so on. At each of these time instants the magnitude of the signal is measured, a process known as **sampling.** Figure 1.7(b) shows a representation of the signal of Fig. 1.7(a) in terms of its samples. The signal of Fig. 1.7(b) is defined only at the sampling instants; it no longer is a continuous function of time, but rather, it is a **discrete-time signal.** However, since the magnitude of each sample can take any value in a continuous range, the signal in Fig. 1.7(b) is still an analog signal.

Now if we represent the magnitude of each of the signal samples in Fig. 1.7(b) by a number having a finite number of digits, then the signal amplitude will no longer be continuous; rather, it is said to be **quantized, discretized,** or **digitized.** The resulting digital signal then is simply a sequence of numbers that represent the magnitudes of the successive signal samples.

The choice of number system to represent the signal samples affects the type of digital signal produced and has a profound effect on the complexity of the digital circuits required to process the signals. It turns out that the **binary** number system results in the simplest possible digital signals and circuits. In a binary system, each digit in the number takes on one of only two possible values, denoted 0 and 1. Correspondingly, the digital signals in

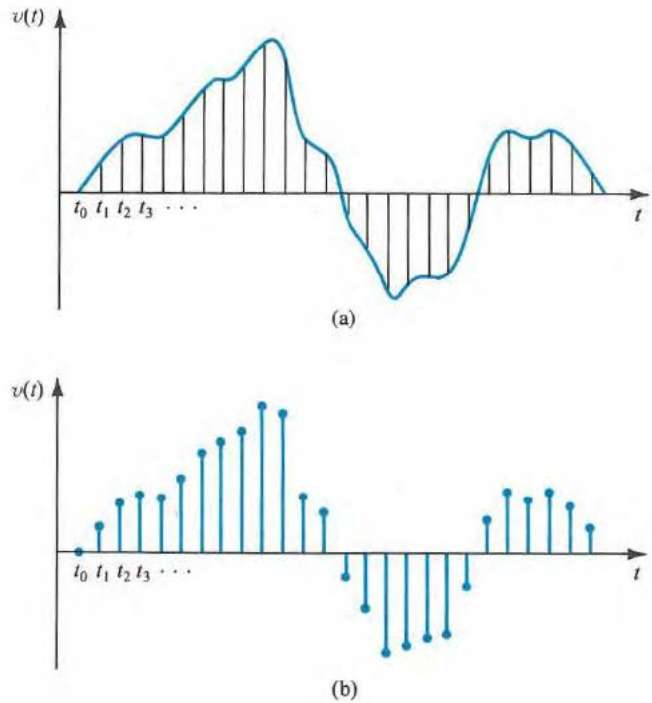


Fig. 1.7 Sampling the continuous-time analog signal in **(a)** results in the discrete-time signal in **(b).**

binary systems need have only two voltage levels, which can be labeled low and high. As an example, in some of the digital circuits studied in this book, the levels are 0 V and +5 V. Figure 1.8 shows the time variation of such a digital signal. Observe that the waveform is a pulse train with 0 V representing a 0 signal, or logic 0, and +5 V representing logic 1.

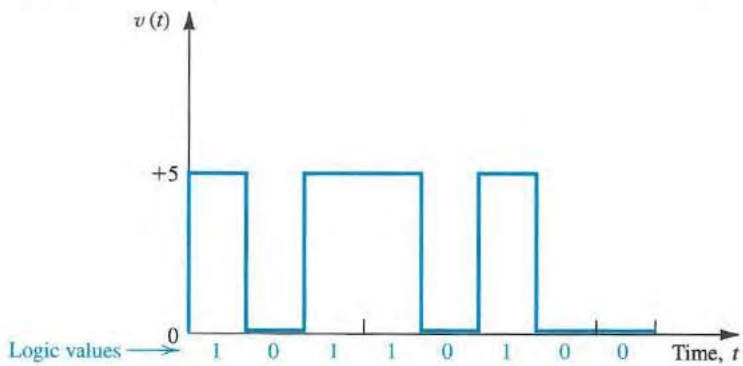


Fig. 1.8 Variation of a particular binary digital signal with time.

If we use N *binary digits* (bits) to represent each sample of the analog signal, then the digitized sample value can be expressed as

$$D = b_0 2^0 + b_1 2^1 + b_2 2^2 + \cdots + b_{N-1} 2^{N-1} \tag{1.3}$$

where $b_0, b_1, \ldots, b_{N-1}$, denote the N bits and have values of 0 or 1. Here bit b_0 is the **least significant bit** (LSB), and bit b_{N-1} is the **most significant bit** (MSB). Conventionally, this binary number is written as $b_{N-1}b_{N-2} \ldots b_0$. We observe that such a representation quantizes the analog sample into one of 2^N levels. Obviously the greater the number of bits (i.e., the larger the N), the closer the digital word D approximates the magnitude of the analog sample. That is, increasing the number of bits reduces the *quantization error* and increases the resolution of the analog-to-digital conversion. This improvement is, however, usually obtained at the expense of more complex and hence more costly circuit implementations. It is not our purpose here to delve into this topic any deeper; we merely want the reader to appreciate the nature of analog and digital signals. Nevertheless, it is an opportune time to introduce a very important circuit building block of modern electronic systems: the **analog-to-digital converter** (A/D or ADC) shown in block form in Fig. 1.9. The ADC accepts at its input the samples of an analog signal and provides for each input sample the corresponding N-bit digital representation (according to Eq. 1.3) at its N output terminals. Thus although the voltage at the input might be, say, 6.51 V, at each of the output terminals (say, at the ith terminal), the voltage will be either low (0 V) or high (5 V) if b_i is supposed to be 0 or 1, respectively. We shall study the ADC and its dual circuit the **digital-to-analog converter** (D/A or DAC) in Chapter 10.

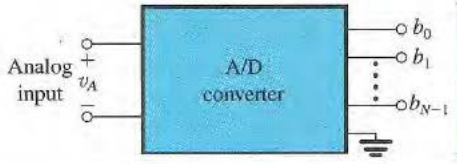


Fig. 1.9 Block-diagram representation of the analog-to-digital converter (ADC).

Once the signal is in digital form, it can be processed using digital circuits. Of course digital circuits can deal also with signals that do not have an analog origin such as the signals that represent the various instructions of a digital computer.

Since digital circuits deal exclusively with binary signals, their design is simpler than that of analog circuits. Furthermore, digital systems can be designed using a relatively few different kinds of digital circuit blocks. However, a large number (e.g., hundreds of thousands or even millions) of each of these blocks are usually needed. Thus the design of digital circuits poses its own set of challenges to the designer, but provides reliable and economic implementations of a great variety of signal processing functions, some of which are not possible with analog circuits. At the present time, more and more of the signal processing functions are being performed digitally. Examples around us abound: from the digital watch and the calculator to digital audio systems and, very soon, digital television. Moreover, some longstanding analog systems such as the telephone communication system are now almost entirely digital. And we should not forget the most important of all digital systems, the digital computer.

The basic building blocks of digital systems are logic circuits and memory circuits. We shall study both in this book, beginning in Section 1.7 with the most fundamental digital circuit, the digital logic inverter.

One final remark: Although the digital processing of signals is at present all-pervasive, there remain many signal processing functions that are best performed by analog circuits. Indeed, many electronic systems include both analog and digital parts. It follows that a good electronics engineer must be proficient in the design of both analog and digital circuits. Such is the aim of this book.

Exercise

1.4 Consider a 4-bit digital word $D = b_3b_2b_1b_0$ (see Eq. 1.3) used to represent an analog signal v_A that varies between 0 V and +15 V.

(a) Give D corresponding to v_A = 0 V, 1 V, 2 V, and 15 V.

(b) What change in v_A causes a change from 0 to 1 in: (i) b_0, (ii) b_1, (iii) b_2, and (iv) b_3?

(c) If v_A = 5.2 V, what do you expect D to be? What is the resulting error in representation?

Ans. (a) 0000, 0001, 0010, 1111; (b) +1 V, +2 V, +4 V, +8 V; (c) 0101, −4%

1.4 AMPLIFIERS

In this section, we shall introduce a fundamental signal-processing function that is employed in some form in almost every electronic system, namely, signal amplification.

Signal Amplification

From a conceptual point of view the simplest signal-processing task is that of **signal amplification.** The need for amplification arises because transducers provide signals that are said to be "weak," that is, in the microvolt (μV) or millivolt (mV) range and possessing little energy. Such signals are too small for reliable processing, and processing is much easier if the signal magnitude is made larger. The functional block that accomplishes this task is the **signal amplifier.**

It is appropriate at this point to discuss the need for **linearity** in amplifiers. When amplifying a signal, care must be exercised so that the information contained in the signal is not changed and no new information is introduced. Thus when feeding the signal shown in Fig. 1.2 to an amplifier, we want the output signal of the amplifier to be an exact replica of that at the input, except of course for having larger magnitude. In other words, the "wiggles" in the output waveform must be identical to those in the input waveform. Any change in waveform is considered to be **distortion** and is obviously undesirable.

An amplifier that preserves the details of the signal waveform is characterized by the relationship

$$v_o(t) = Av_i(t) \tag{1.4}$$

CHAPTER 11

Filters and Tuned Amplifiers

Introduction
11.1 Filter Transmission, Types, and Specification
11.2 The Filter Transfer Function
11.3 Butterworth and Chebyshev Filters
11.4 First-Order and Second-Order Filter Functions
11.5 The Second-Order LCR Resonator
11.6 Second-Order Active Filters Based on Inductor Replacement
11.7 Second-Order Active Filters Based on the Two-Integrator-Loop Topology
11.8 Single-Amplifier Biquadratic Active Filters
11.9 Sensitivity
11.10 Switched-Capacitor Filters
11.11 Tuned Amplifiers
11.12 SPICE Simulation Examples
Summary
Bibliography
Problems

INTRODUCTION

In this chapter, we study the design of an important building block of communications and instrumentation systems, the electronic filter. Filter design is one of the very few areas of engineering for which a complete design theory exists, starting from specification and ending with a circuit realization. A detailed study of filter design requires an entire book, and indeed such textbooks exist. In the limited space available here, we shall concentrate on a selection of topics that provide an introduction to the subject as well as a useful arsenal of filter circuits and design methods.

The oldest technology for realizing filters makes use of inductors and capacitors, and the resulting circuits are called **passive LC filters.** Such filters work well at high frequencies; however, in low-frequency applications (dc to 100 kHz) the required inductors are

large and physically bulky, and their characteristics are quite nonideal. Furthermore, such inductors are impossible to fabricate in monolithic form and are incompatible with any of the modern techniques for assembling electronic systems. Therefore, there has been considerable interest in finding filter realizations that do not require inductors. Of the various possible types of **inductorless filters,** we shall study **active-RC filters** and **switched-capacitor filters.**

Active-RC filters utilize op amps together with resistors and capacitors and are fabricated using discrete, hybrid thick-film, or hybrid thin-film technology. However, for large-volume production, such technologies do not yield the economies achieved by monolithic fabrication. At the present time, the most viable approach for realizing fully integrated monolithic filters is the switched-capacitor technique.

The last topic studied in this chapter is the tuned amplifier commonly employed in the design of radio and TV receivers. Although tuned amplifiers are in effect bandpass filters, they are studied separately because their design is based on somewhat different techniques.

11.1 FILTER TRANSMISSION, TYPES, AND SPECIFICATION

The filters we are about to study are linear circuits that can be represented by the general two-port network shown in Fig. 11.1. The filter **transfer function** $T(s)$ is the ratio of the output voltage $V_o(s)$ to the input voltage $V_i(s)$,

$$T(s) \equiv \frac{V_o(s)}{V_i(s)} \tag{11.1}$$

The filter **transmission** is found by evaluating $T(s)$ for physical frequencies, $s = j\omega$, and can be expressed in terms of its magnitude and phase as

$$T(j\omega) = |T(j\omega)|\, e^{j\phi(\omega)} \tag{11.2}$$

The magnitude of transmission is often expressed in decibels in terms of the **gain function**

$$G(\omega) \equiv 20\, log|T(j\omega)|, \text{ dB} \tag{11.3}$$

or, alternatively, in terms of the **attenuation function**

$$A(\omega) \equiv -20\, log|T(j\omega)|, \text{ dB} \tag{11.4}$$

A filter shapes the frequency spectrum of the input signal, $|V_i(j\omega)|$, according to the magnitude of the transfer function $|T(j\omega)|$, thus providing an output $V_o(j\omega)$ with a spectrum

$$|V_o(j\omega)| = |T(j\omega)|\, |V_i(j\omega)| \tag{11.5}$$

Also, the phase characteristics of the signal are modified as it passes through the filter according to the filter phase function $\phi(\omega)$.

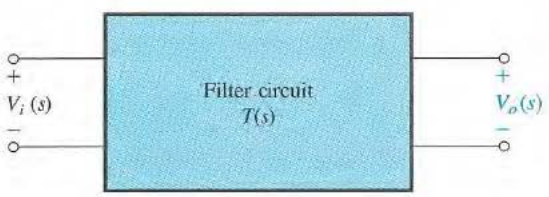


Fig. 11.1 The filters studied in this chapter are linear circuits represented by the general two-port network shown. The filter transfer function $T(s) \equiv V_o(s)/V_i(s)$.

We are specifically interested here in filters that perform a **frequency-selection** function: **passing** signals whose frequency spectrum lies within a specified range, and **stopping** signals whose frequency spectrum falls outside this range. Such a filter has ideally a frequency band (or bands) over which the magnitude of transmission is unity (the filter **passband**) and a frequency band (or bands) over which the transmission is zero (the filter **stopband**). Figure 11.2 depicts the ideal transmission characteristics of the four major filter types: **low-pass** (LP) in Fig. 11.2(a), **high-pass** (HP) in Fig. 11.2(b), **bandpass** (BP) in Fig. 11.2(c), and **bandstop** (BS) or **band-reject** in Fig. 11.2(d). These idealized characteristics, by virtue of their vertical edges, are known as **brick-wall** type responses.

Filter Specification

The filter design process begins with the filter user specifying the transmission characteristics required of the filter. Such a specification cannot be of the form shown in Fig. 11.2 because physical circuits cannot realize these idealized characteristics. Figure 11.3 shows realistic specifications for the transmission characteristics of a low-pass filter. Observe that since a physical circuit cannot provide constant transmission at all passband frequencies, the specifications allow for deviation of the passband transmission from the ideal 0 dB, but places an upper bound, A_{max} (dB), on this deviation. Depending on the application, A_{max}

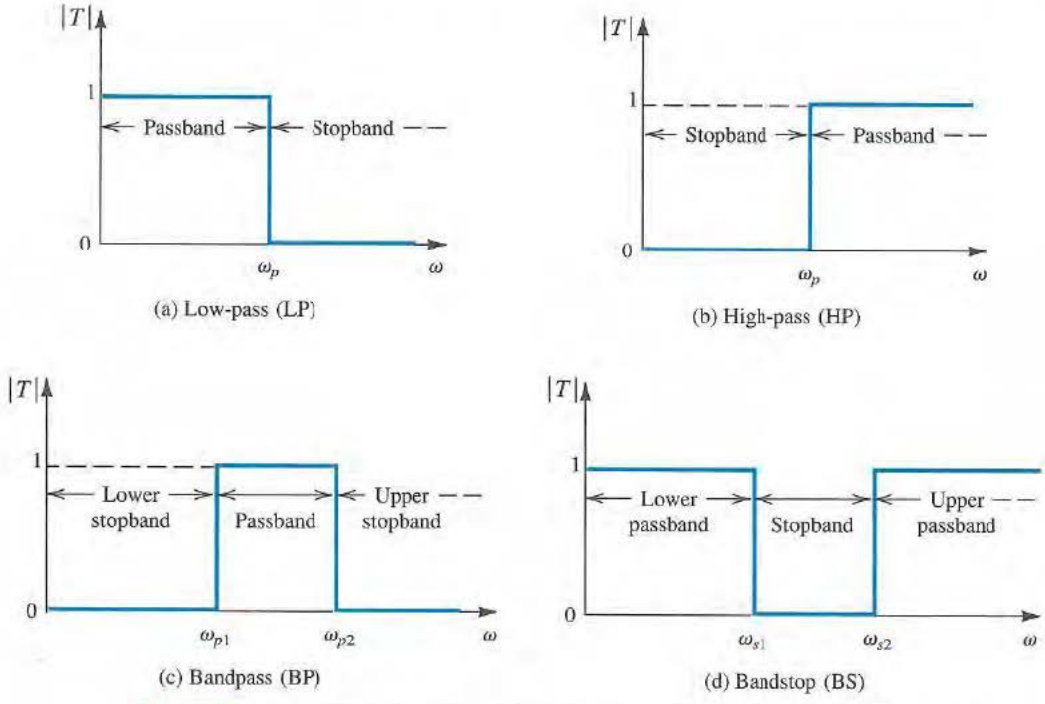


Fig. 11.2 Ideal transmission characteristics of the four major filter types: **(a)** low-pass (LP), **(b)** high-pass (HP), **(c)** bandpass (BP), and **(d)** bandstop (BS).

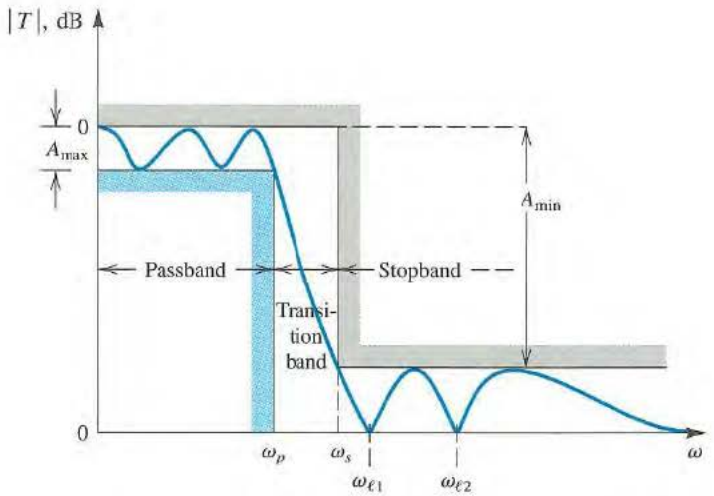


Fig. 11.3 Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

typically ranges from 0.05 to 3 dB. Also, since a physical circuit cannot provide zero transmission at all stopband frequencies, the specifications in Fig. 11.3 allow for some transmission over the stopband. However, the specifications require the stopband signals to be attenuated by at least $A_{\min}$ (dB) relative to the passband signals. Depending on the filter application, $A_{\min}$ can range from 20 to 100 dB.

Since the transmission of a physical circuit cannot change abruptly at the edge of the passband, the specifications of Fig. 11.3 provide for a band of frequencies over which the attenuation increases from near 0 dB to $A_{\min}$. This **transition band** extends from the passband edge ω_p to the stopband edge ω_s. The ratio ω_s/ω_p is usually used as a measure of the sharpness of the low-pass filter response and is called the **selectivity factor.** Finally, observe that for convenience the passband transmission is specified to be 0 dB. The final filter, however, can be given a passband gain, if desired, without changing its selectivity characteristics.

To summarize, the transmission of a low-pass filter is specified by four parameters:

1. the passband edge, ω_p;
2. the maximum allowed variation in passband transmission, $A_{\max}$;
3. the stopband edge, ω_s; and
4. the minimum required stopband attenuation, $A_{\min}$.

The more tightly one specifies a filter—that is, lower $A_{\max}$, higher $A_{\min}$, and/or a selectivity ratio ω_s/ω_p closer to unity—the closer the response of the resulting filter to the ideal. However, the resulting filter circuit must be of higher order and thus more complex and expensive.

In addition to specifying the magnitude of transmission, there are applications in which the phase response of the filter is also of interest. The filter design problem, however, is considerably complicated when both magnitude and phase are specified.

Once the filter specifications have been decided upon, the next step in the design is to find a transfer function whose magnitude meets the specification. To meet specification, the magnitude response curve must lie in the unshaded area in Fig. 11.3. The curve shown in the figure is for a filter that *just* meets specifications. Observe that for this particular filter, the magnitude response *ripples* throughout the passband with the ripple peaks being all equal. Since the peak ripple is equal to A_{max} it is usual to refer to A_{max} as the **passband ripple** and to ω_p as the **ripple bandwidth.** The particular filter response shown ripples also in the stopband, again with the ripple peaks all equal and of such a value that the minimum stopband attenuation achieved is equal to the specified value, A_{min}. Thus this particular response is said to be **equiripple** in both the passband and the stopband.

The process of obtaining a transfer function that meets given specifications is known as **filter approximation.** Filter approximation is usually performed using computer programs (Snelgrove, 1982; Ouslis and Sedra, 1995) or filter design tables (Zverev, 1967). In simpler cases, filter approximation can be performed using closed-form expressions, as will be seen in Section 11.3.

Finally, Fig. 11.4 shows transmission specifications for a bandpass filter and the response of a filter that meets these specifications. For this example we have chosen an approximation function that does not ripple in the passband; rather, the transmission decreases monotonically on both sides of the center frequency, attaining the maximum allowable deviation at the two edges of the passband.

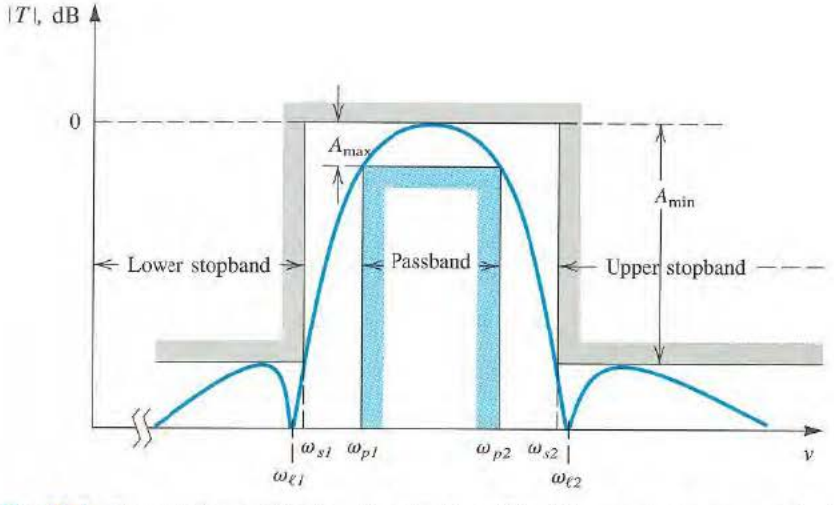


Fig. 11.4 Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

Exercises

11.1 Find approximate values of attenuation corresponding to filter transmissions of: 1, 0.99, 0.9, 0.8, 0.7, 0.5, 0.1, 0.

Ans. 0, 0.1, 1, 2, 3, 6, 20, ∞ (dB).

11.2 If the magnitude of passband transmission is to remain constant to within $\pm 5\%$, and if the stopband transmission is to be no greater than 1% of the passband transmission, find $A_{\max}$ and $A_{\min}$.

Ans. 0.9 dB; 40 dB.

11.2 THE FILTER TRANSFER FUNCTION

The filter transfer function $T(s)$ can be written as the ratio of two polynomials as

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0} \tag{11.6}$$

The degree of the denominator, N, is the **filter order.** For the filter circuit to be stable, the degree of the numerator must be less than or equal to that of the denominator; $M \leq N$. The numerator and denominator coefficients, $a_0, a_1, \ldots, a_M$ and $b_0, b_1, \ldots, b_{N-1}$, are real numbers. The polynomials in the numerator and denominator can be factored, and $T(s)$ can be expressed in the form

$$T(s) = \frac{a_M (s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)} \tag{11.7}$$

The numerator roots, $z_1, z_2, \ldots, z_M$, are the **transfer-function zeros,** or **transmission zeros;** and the denominator roots, $p_1, p_2, \ldots, p_N$, are the **transfer-function poles,** or the **natural modes**[1]**.** Each transmission zero or pole can be either a real or a complex number. Complex zeros and poles, however, must occur in conjugate pairs. Thus, if $-1 + j2$ happens to be a zero, then $-1 - j2$ also must be a zero.

Since in the filter stopband the transmission is required to be zero or small, the filter transmission zeros are usually placed on the $j\omega$ axis at stopband frequencies. This indeed is the case for the filter whose transmission function is sketched in Fig. 11.3. This particular filter can be seen to have infinite attenuation (zero transmission) at two stopband frequencies: ω_{t1} and ω_{t2}. The filter then must have transmission zeros at $s = +j\omega_{t1}$ and $s = +j\omega_{t2}$. However, since complex zeros occur in conjugate pairs, there must also be transmission zeros at $s = -j\omega_{t1}$ and $s = -j\omega_{t2}$. Thus the numerator polynomial of this filter will have the factors $(s + j\omega_{t1})(s - j\omega_{t1})(s + j\omega_{t2})(s - j\omega_{t2})$, which can be written as $(s^2 + \omega_{t1}^2)(s^2 + \omega_{t2}^2)$. For $s = j\omega$ (physical frequencies) the numerator becomes $(-\omega^2 + \omega_{t1}^2)(-\omega^2 + \omega_{t2}^2)$, which indeed is zero at $\omega = \omega_{t1}$ and $\omega = \omega_{t2}$.

[1] Throughout this chapter we use the names *poles* and *natural modes* interchangeably.

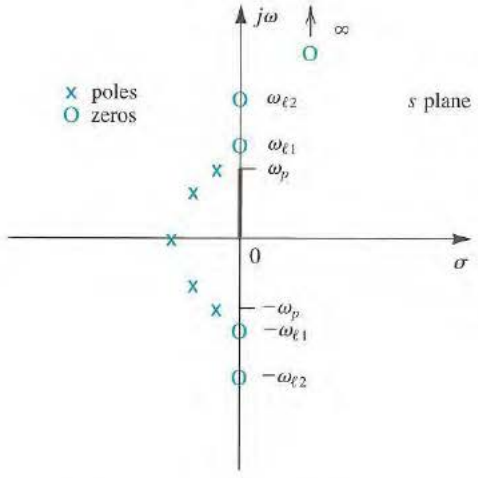


Fig. 11.5 Pole–zero pattern for the low-pass filter whose transmission is sketched in Fig. 11.3. This filter is of fifth order ($N = 5$).

Continuing with the example in Fig. 11.3, we observe that the transmission decreases toward $-\infty$ as ω approaches ∞. Thus the filter must have one or more transmission zeros at $s = \infty$. In general, the number of transmission zeros at $s = \infty$ is the difference between the degree of the numerator polynomial, M, and the degree of the denominator polynomial, N, of the transfer function in Eq. (11.6). This is because as s approaches ∞, $T(s)$ approaches a_M/s^{N-M} and thus is said to have $N - M$ zeros at $s = \infty$.

For a filter circuit to be stable, all its poles must lie in the left half of the s-plane, and

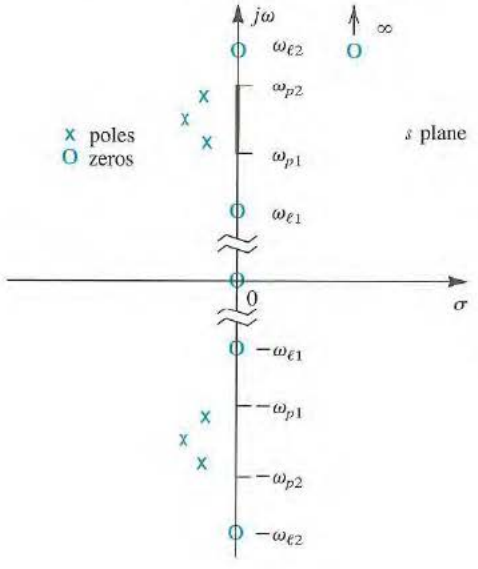


Fig. 11.6 Pole–zero pattern for the bandpass filter whose transmission function is shown in Fig. 11.4. This filter is of sixth order ($N = 6$).

thus $p_1, p_2, \ldots, p_N$ must all have negative real parts. Figure 11.5 shows typical pole and zero locations for the low-pass filter whose transmission function is depicted in Fig. 11.3. We have assumed that this filter is of fifth order ($N = 5$). It has two pairs of complex conjugate poles and one real-axis pole, for a total of five poles. All the poles lie in the vicinity of the passband, which is what gives the filter its high transmission at passband frequencies. The five transmission zeros are at $s = \pm j\omega_{l1}$, $s = \pm j\omega_{l2}$, and $s = \infty$. Thus the transfer function for this filter is of the form

$$T(s) = \frac{a_4 (s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0} \tag{11.8}$$

As another example, consider the bandpass filter whose magnitude response is shown in Fig. 11.4. This filter has transmission zeros at $s = \pm j\omega_{l1}$ and $s = \pm j\omega_{l2}$. It also has one or more zeros at $s = 0$ and one or more zeros at $s = \infty$ (because the attenuation decreases toward $-\infty$ as ω approaches 0 and ∞). Assuming that only one zero exists at each of $s = 0$ and $s = \infty$, the filter must be of sixth order, and its transfer function takes the form

$$T(s) = \frac{a_5 s (s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^6 + b_5 s^5 + \cdots + b_0} \tag{11.9}$$

A typical pole–zero plot for such a filter is shown in Fig. 11.6.

As a third and final example, consider the low-pass filter whose transmission function is depicted in Fig. 11.7(a). We observe that in this case there are no finite values of ω at

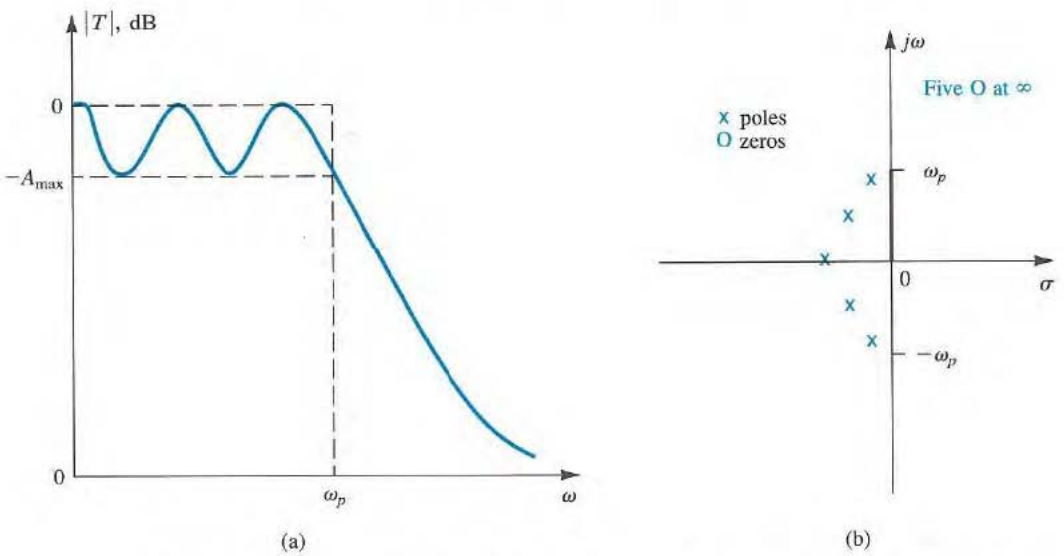


Fig. 11.7 (a) Transmission characteristics of a fifth-order low-pass filter having all transmission zeros at infinity. (b) Pole–zero pattern for the filter in (a).

which the attenuation is infinite (zero transmission). Thus it is possible that all the transmission zeros of this filter are at $s = \infty$. If this is the case, the filter transfer function takes the form

$$T(s) = \frac{a_M}{s^N + b_{N-1}s^{N-1} + \cdots + b_0} \tag{11.10}$$

Such a filter is known as an **all-pole filter.** Typical pole–zero locations for a fifth-order all-pole low-pass filter are shown in Fig. 11.7(b).

Almost all of the filters studied in this chapter have all of their transmission zeros on the $j\omega$-axis, in the filter stopband(s), including[2] $\omega = 0$ and $\omega = \infty$. Also, to obtain high selectivity, all the natural modes will be complex conjugate (except for the case of odd-order filters where one natural mode must be on the real axis). Finally we note that the more selective the required filter response is, the higher its order must be, and the closer its natural modes are to the $j\omega$ axis.

Exercises

11.3 A second-order filter has its poles at $s = -(1/2) \pm j(\sqrt{3}/2)$. The transmission is zero at $\omega = 2$ rad/s and is unity at dc ($\omega = 0$). Find the transfer function.

Ans. $T(s) = \frac{1}{4}\frac{s^2 + 4}{s^2 + s + 1}$

11.4 A fourth-order filter has zero transmission at $\omega = 0$, $\omega = 2$ rad/s, and $\omega = \infty$. The natural modes are $-0.1 \pm j0.8$ and $-0.1 \pm j1.2$. Find $T(s)$.

Ans. $T(s) = \frac{a_3\, s(s^2 + 4)}{(s^2 + 0.2s + 0.65)(s^2 + 0.2s + 1.45)}$

11.5 Find the transfer function $T(s)$ of a third-order all-pole low-pass filter whose poles are at a radial distance of 1 rad/s from the origin and whose complex poles are at 30° angles from the $j\omega$-axis. The dc gain is unity. Show that $|T(j\omega)| = 1/\sqrt{1 + \omega^6}$. Find ω_{3dB} and the attenuation at $\omega = 3$ rad/s.

Ans. $T(s) = 1/(s + 1)(s^2 + s + 1)$; 1 rad/s; 28.6 dB.

11.3 BUTTERWORTH AND CHEBYSHEV FILTERS

In this section, we present two functions that are frequently used in approximating the transmission characteristics of low-pass filters. These functions have the advantage that closed-form expressions are available for their parameters. Thus one can use them in filter design without the need for computers or filter design tables. Their utility, however, is limited to relatively simple applications.

[2] Obviously, a low-pass filter should *not* have a transmission zero at $\omega = 0$, and, similarly, a high-pass filter should not have a transmission zero at $\omega = \infty$.

Although in this section we discuss the design of low-pass filters only, the approximation functions presented can be applied to the design of other filter types through the use of frequency transformations (see Sedra and Brackett, 1978).

The Butterworth Filter

Figure 11.8 shows a sketch of the magnitude response of a Butterworth[3] filter. This filter exhibits a monotonically decreasing transmission with all the transmission zeros at $\omega = \infty$, making it an all-pole filter. The magnitude function for an Nth-order Butterworth filter with a passband edge ω_p is given by

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}}} \tag{11.11}$$

At $\omega = \omega_p$,

$$|T(j\omega_p)| = \frac{1}{\sqrt{1 + \epsilon^2}} \tag{11.12}$$

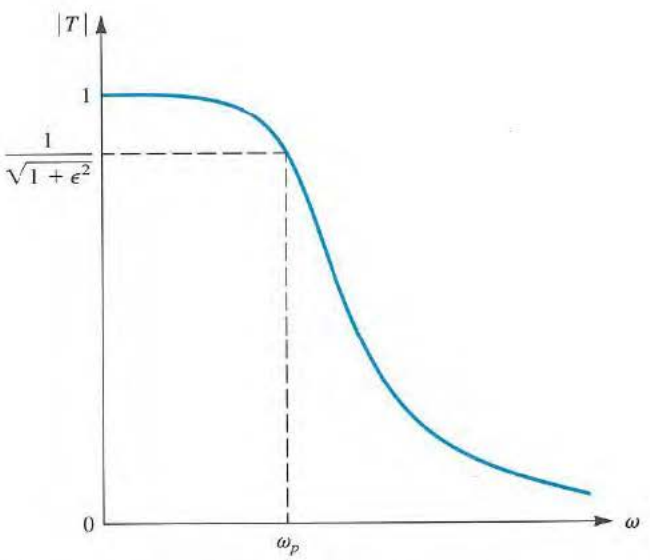


Fig. 11.8 The magnitude response of a Butterworth filter.

[3] The Butterworth filter approximation is named after S. Butterworth, a British engineer who in 1930 was among the first to employ it.

Thus the parameter ϵ determines the maximum variation in passband transmission, A_{max}, according to

$$A_{max} = 20 \log\sqrt{1 + \epsilon^2} \tag{11.13}$$

Conversely, given A_{max}, the value of ϵ can be determined from

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} \tag{11.14}$$

Observe that in the Butterworth response the maximum deviation in passband transmission (from the ideal value of unity) occurs at the passband edge only. It can be shown that the first $2N - 1$ derivatives of $|T|$ relative to ω are zero at $\omega = 0$ (see Van Valkenburg, 1980). This property makes the Butterworth response very flat near $\omega = 0$ and gives the response the name **maximally flat** response. The degree of passband flatness increases as the order N is increased, as can be seen from Fig. 11.9. This figure indicates also that, as should be expected, as the order N is increased the filter response approaches the ideal brick-wall type response.

At the edge of the stopband, $\omega = \omega_s$, the attenuation of the Butterworth filter is given by

$$\begin{aligned} A(\omega_s) &= -20 \log [1/\sqrt{1 + \epsilon^2 (\omega_s/\omega_p)^{2N}}] \\ &= 10 \log[1 + \epsilon^2 (\omega_s/\omega_p)^{2N}] \end{aligned} \tag{11.15}$$

This equation can be used to determine the filter order required, which is the lowest integer value of N that yields $A(\omega_s) \geq A_{min}$.

The natural modes of an Nth-order Butterworth filter can be determined from the graphical construction shown in Fig. 11.10(a). Observe that the natural modes lie on a circle of

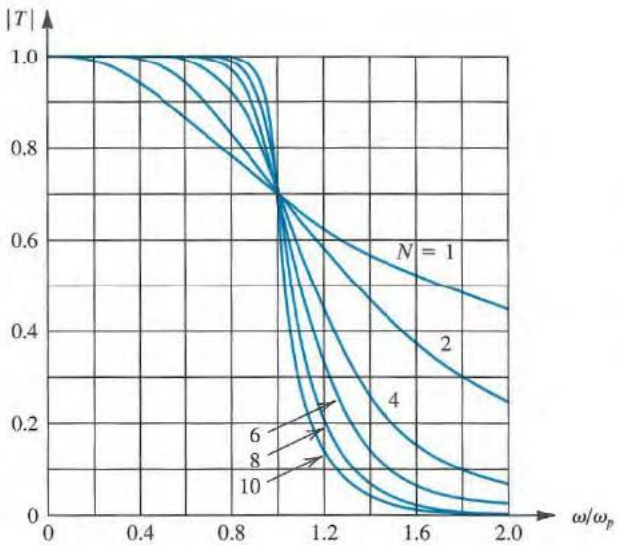


Fig. 11.9 Magnitude response for Butterworth filters of various order with $\epsilon = 1$. Note that as the order increases, the response approaches the ideal brick-wall type transmission.

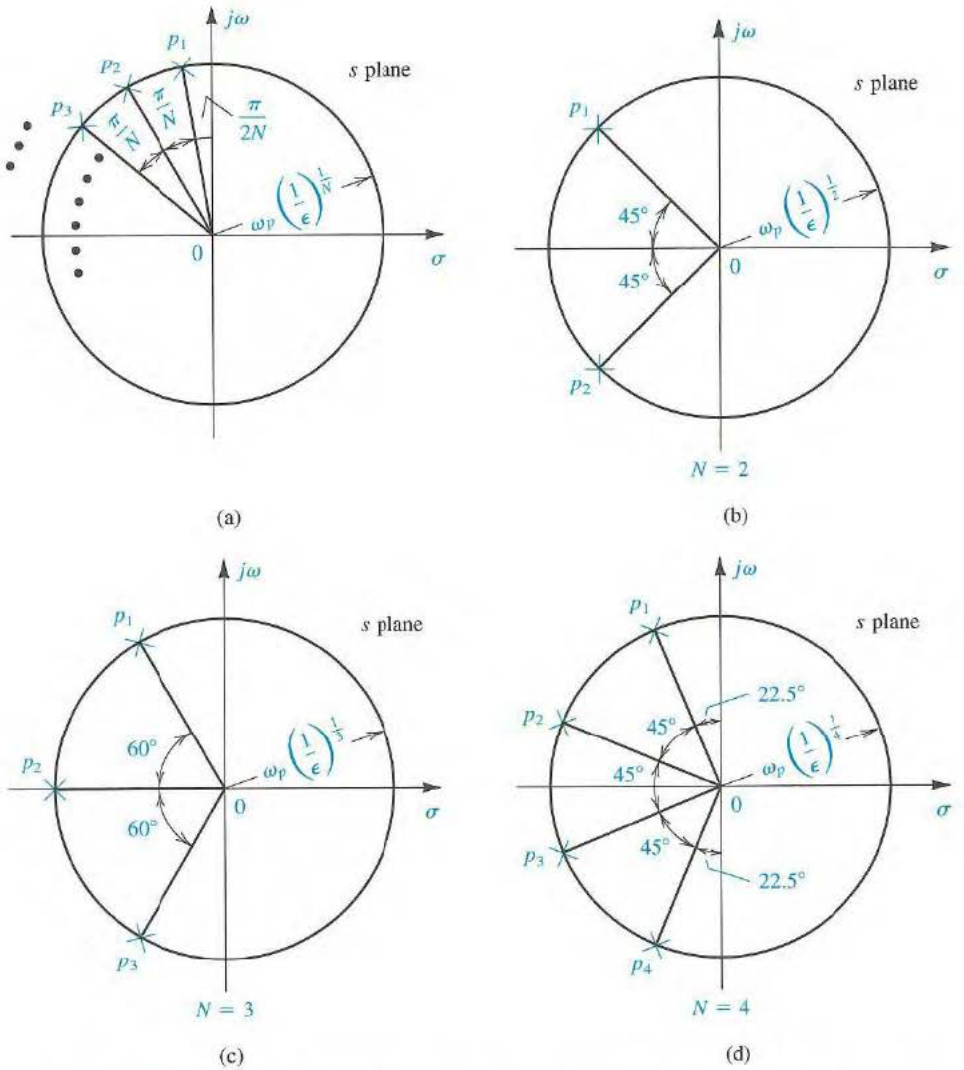


Fig. 11.10 Graphical construction for determining the poles of a Butterworth filter of order N. All the poles lie in the left half of the s-plane on a circle of radius $\omega_0 = \omega_p(1/\epsilon)^{1/N}$, where ϵ is the passband deviation parameter ($\epsilon = \sqrt{10^{A_{max}/10} - 1}$): **(a)** the general case, **(b)** $N = 2$, **(c)** $N = 3$, **(d)** $N = 4$.

radius $\omega_p (1/\epsilon)^{1/N}$ and are spaced by equal angles of π/N, with the first mode at an angle $\pi/2N$ from the $+j\omega$-axis. Since the natural modes all have equal radial distance from the origin they all have the same frequency $\omega_0 = \omega_p (1/\epsilon)^{1/N}$. Figs. 11.10(b), (c), and (d) show the natural modes of Butterworth filters of order $N = 2$, 3, and 4, respectively. Once the N natural modes $p_1, p_2, \ldots, p_N$ are found the transfer function can be written as

$$T(s) = \frac{K\,\omega_0^N}{(s - p_1)(s - p_2)\cdots(s - p_N)} \tag{11.16}$$

where K is a constant equal to the required dc gain of the filter.

To summarize, to find a Butterworth transfer function that meets transmission specifications of the form in Fig. 11.3 we perform the following procedure:

1. Determine ϵ from Eq. (11.14).
2. Use Eq. (11.15) to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A_{\min}$.
3. Use Fig. 11.10(a) to determine the N natural modes.
4. Use Eq. (11.16) to determine $T(s)$.

EXAMPLE 11.1

Find the Butterworth transfer function that meets the following low-pass filter specifications: $f_p = 10$ kHz, $A_{\max} = 1$ dB, $f_s = 15$ kHz, $A_{\min} = 25$ dB, dc gain $= 1$.

SOLUTION

Substituting $A_{\max} = 1$ dB into Eq. (11.14) yields $\epsilon = 0.5088$. Equation (11.15) is then used to determine the filter order by trying various values for N. We find that $N = 8$ yields $A(\omega_s) = 22.3$ dB and $N = 9$ gives 25.8 dB. We thus select $N = 9$.

Figure 11.11 shows the graphical construction for determining the poles. The poles all have the same frequency $\omega_0 = \omega_p (1/\epsilon)^{1/N} = 2\pi \times 10 \times 10^3 (1/0.5088)^{1/9} = 6.773 \times 10^4$ rad/s. The first pole p_1 is given by

$$p_1 = \omega_0 (-\cos 80^\circ + j \sin 80^\circ) = \omega_0 (-0.1736 + j0.9848)$$

Combining p_1 with its complex conjugate p_9 yields the factor $(s^2 + s\,0.3472\,\omega_0 + \omega_0^2)$ in the denominator of the transfer function. The same can be done for the other complex poles, and the complete transfer function is obtained using Eq. (11.16),

$$T(s) = \frac{\omega_0^9}{(s + \omega_0)(s^2 + s\,1.8794\,\omega_0 + \omega_0^2)(s^2 + s\,1.5321\,\omega_0 + \omega_0^2)} \times \frac{1}{(s^2 + s\,\omega_0 + \omega_0^2)(s^2 + s\,0.3472\,\omega_0 + \omega_0^2)} \tag{11.17}$$

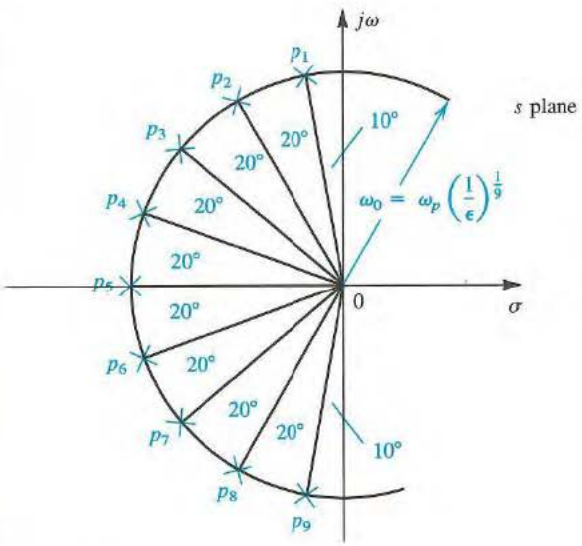


Fig. 11.11 Poles of the ninth-order Butterworth filter of Example 11.1.

The Chebyshev Filter

Figure 11.12 shows representative transmission functions for Chebyshev[4] filters of even and odd order. The Chebyshev filter exhibits an equiripple response in the passband and a monotonically decreasing transmission in the stopband. While the odd-order filter has $|T(0)| = 1$, the even-order filter exhibits its maximum magnitude deviation at $\omega = 0$. In both cases the total number of passband maxima and minima equals the order of the filter, N. All the transmission zeros of the Chebyshev filter are at $\omega = \infty$, making it an all-pole filter.

The magnitude of the transfer function of an Nth-order Chebyshev filter with a passband edge (ripple bandwidth) ω_p is given by

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cos^2[N \cos^{-1}(\omega/\omega_p)]}} \qquad \text{for } \omega \leq \omega_p \tag{11.18}$$

and

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cosh^2[N \cosh^{-1}(\omega/\omega_p)]}} \qquad \text{for } \omega \geq \omega_p \tag{11.19}$$

At the passband edge, $\omega = \omega_p$, the magnitude function is given by

$$|T(j\omega_p)| = \frac{1}{\sqrt{1 + \epsilon^2}}$$

[4] Named after the Russian mathematician P. L. Chebyshev, who in 1899 used these functions in studying the construction of steam engines.

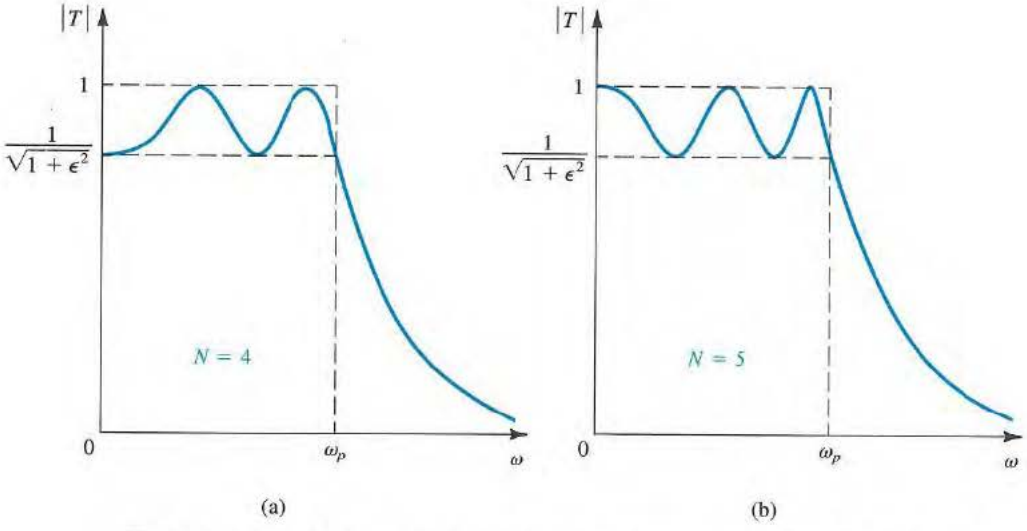


Fig. 11.12 Sketches of the transmission characteristics of representative even- and odd-order Chebyshev filters.

Thus, the parameter ϵ determines the passband ripple according to

$$A_{\max} = 10 \log(1 + \epsilon^2) \tag{11.20}$$

Conversely, given $A_{\max}$, the value of ϵ is determined from

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1} \tag{11.21}$$

The attenuation achieved by the Chebyshev filter at the stopband edge ($\omega = \omega_s$) is found using Eq. (11.19) as

$$A(\omega_s) = 10 \log[1 + \epsilon^2 \cosh^2(N \cosh^{-1}(\omega_s/\omega_p))] \tag{11.22}$$

With the aid of a calculator this equation can be used to determine the order N required to obtain a specified $A_{\min}$ by finding the lowest integer value of N that yields $A(\omega_s) \geq A_{\min}$. As in the case of the Butterworth filter, increasing the order N of the Chebyshev filter causes its magnitude function to approach the ideal brick-wall low-pass response.

The poles of the Chebyshev filter are given by

$$\begin{aligned} p_k = &-\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right) \sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) \\ &+ j\omega_p \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right) \cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right) \qquad k = 1, 2, \ldots, N \end{aligned} \tag{11.23}$$

Finally, the transfer function of the Chebyshev filter can be written as

$$T(s) = \frac{K\,\omega_p^N}{\epsilon\,2^{N-1}(s - p_1)(s - p_2)\cdots(s - p_N)} \tag{11.24}$$

where K is the dc gain that the filter is required to have.

To summarize, given low-pass transmission specifications of the type shown in Fig. 11.3, the transfer function of a Chebyshev filter that meets these specifications can be found as follows:

1. Determine ϵ from Eq. (11.21).
2. Use Eq. (11.22) to determine the order required.
3. Determine the poles using Eq. (11.23).
4. Determine the transfer function using Eq. (11.24).

The Chebyshev filter provides a more efficient approximation than the Butterworth filter. Thus, for the same order and the same $A_{\max}$, the Chebyshev filter provides greater stopband attenuation than the Butterworth filter. Alternatively, to meet identical specifications, one requires a lower order for the Chebyshev than for the Butterworth filter. This point will be illustrated by the following example.

EXAMPLE 11.2

Find the Chebyshev transfer function that meets the same low-pass filter specifications given in Example 11.1; namely, $f_p = 10$ kHz, $A_{\max} = 1$ dB, $f_s = 15$ kHz, $A_{\min} = 25$ dB, dc gain = 1.

SOLUTION

Substituting $A_{\max} = 1$ dB into Eq. (11.21) yields $\epsilon = 0.5088$. By trying various values for N in Eq. (11.22) we find that $N = 4$ yields $A(\omega_s) = 21.6$ dB and $N = 5$ provides 29.9 dB. We thus select $N = 5$. Recall that we required a ninth-order Butterworth filter to meet the same specifications in Example 11.1.

The poles are obtained by substituting in Eq. (11.23) as

$$\begin{aligned} p_1, p_5 &= \omega_p(-0.0895 \pm j0.9901) \\ p_2, p_4 &= \omega_p(-0.2342 \pm j0.6119) \\ p_5 &= \omega_p(-0.2895) \end{aligned}$$

The transfer function is obtained by substituting these values in Eq. (11.24) as

$$\begin{aligned} T(s) = {} & \frac{\omega_p^5}{8.1408\,(s + 0.2895\,\omega_p)(s^2 + s\,0.4684\,\omega_p + 0.4293\,\omega_p^2)} \\ & \times \frac{1}{s^2 + s\,0.1789\,\omega_p + 0.9883\,\omega_p^2} \end{aligned} \tag{11.25}$$

where $\omega_p = 2\pi \times 10^4$ rad/s.

Exercises

D11.6 Determine the order N of a Butterworth filter for which $A_{max} = 1$ dB, $\omega_s/\omega_p = 1.5$, and $A_{min} = 30$ dB. What is the actual value of minimum stopband attenuation realized? If A_{min} is to be exactly 30 dB, to what value can A_{max} be reduced?

Ans. $N = 11$; $A_{min} = 32.87$ dB; 0.54 dB

11.7 Find the natural modes and the transfer function of a Butterworth filter with $\omega_p = 1$ rad/s, $A_{max} = 3$ dB ($\epsilon \simeq 1$), and $N = 3$.

Ans. $-0.5 \pm j\sqrt{3}/2$ and -1; $T(s) = 1/(s + 1)(s^2 + s + 1)$

11.8 Observe that Eq. (11.18) can be used to find the frequencies in the passband at which $|T|$ is at its peaks and at its valleys. (The peaks are reached when the $\cos^2[\ ]$ term is zero, and the valleys correspond to the $\cos^2[\ ]$ term equal to unity.) Find these frequencies for a fifth-order filter.

Ans. Peaks at $\omega = 0$, 0.59 ω_p, and 0.95 ω_p; the valleys at $\omega = 0.31\ \omega_p$ and 0.81 ω_p

D11.9 Find the attenuation provided at $\omega = 2\omega_p$ by a seventh-order Chebyshev filter with a 0.5-dB passband ripple. If the passband ripple is allowed to increase to 1 dB, by how much does the stopband attenuation increase?

Ans. 64.9 dB; 3.3 dB

D11.10 It is required to design a low-pass filter having $f_p = 1$ kHz, $A_{max} = 1$ dB, $f_s = 1.5$ kHz, $A_{min} = 50$ dB. (a) Find the required order of Chebyshev filter. What is the excess stopband attenuation obtained? (b) Repeat for a Butterworth filter.

Ans. (a) $N = 8$, 5 dB. (b) $N = 16$, 0.5 dB

11.4 FIRST-ORDER AND SECOND-ORDER FILTER FUNCTIONS

In this section, we shall study the simplest filter transfer functions, those of first and second order. These functions are useful in their own right in the design of simple filters. First- and second-order filters can also be cascaded to realize a high-order filter. Cascade design is in fact one of the most popular methods for the design of active filters (those utilizing op amps and RC circuits). Because the filter poles occur in complex-conjugate pairs, a high-order transfer function $T(s)$ is factored into the product of second-order functions. If $T(s)$ is odd, there will also be a first-order function in the factorization. Each of the second-order functions (and the first-order function in the case $T(s)$ is odd) is then realized using one of the op amp–RC circuits that will be studied in this chapter, and the resulting blocks are placed in cascade. If the output of each block is taken at the output terminal of an op amp where the impedance level is low (ideally zero), cascading does not change the transfer functions of the individual blocks. Thus the overall transfer function of the cascade is simply the product of the transfer functions of the individual blocks, which is the original $T(s)$.

First-Order Filters

The general first-order transfer function is given by

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0} \tag{11.26}$$

This **bilinear transfer function** characterizes a first-order filter with a natural mode at $s = -\omega_0$, a transmission zero at $s = -a_0/a_1$, and a high-frequency gain that approaches a_1. The numerator coefficients, a_0 and a_1, determine the type of filter (e.g., low-pass, high-pass, etc.). Some special cases together with passive (RC) and active (op amp–RC) realizations are shown in Fig. 11.13. Note that the active realizations provide considerably more versatility over their passive counterparts; in many cases the gain can be set to a desired value, and some transfer-function parameters can be adjusted without affecting others. The output impedance of the active circuit is also very low, making cascading possible. The op amp, however, limits the high-frequency operation of the active circuits.

Another important special case of the first-order filter function is the **all-pass filter** shown in Fig. 11.14. Here the transmission zero and the natural mode are symmetrically located relative to the $j\omega$-axis. (They are said to display mirror-image symmetry with respect to the $j\omega$-axis.) Observe that although the transmission of the all-pass filter is (ideally) constant at all frequencies, its phase shows frequency selectivity. All-pass filters are used as phase shifters and in systems that require phase shaping (e.g., in the design of circuits called *delay equalizers*, which cause the overall time delay of a transmission system to be constant with frequency).

Exercises

D11.11 Using $R_1 = 10\,\text{k}\Omega$, design the op amp–RC circuit of Fig. 11.13(b) to realize a high-pass filter with a corner frequency of 10^4 rad/s and a high-frequency gain of 10.

Ans. $R_2 = 100\,\text{k}\Omega$ and $C = 0.01\,\mu\text{F}$

D11.12 Design the op amp–RC circuit of Fig. 11.14 to realize an all-pass filter with a 90° phase shift at 10^3 rad/s. Select suitable component values.

Ans. Possible choice: $R = R_1 = R_2 = 10\,\text{k}\Omega$, $C = 0.1\,\mu\text{F}$.

Second-Order Filter Functions

The general second-order or **biquadratic** filter transfer function is usually expressed in the standard form

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)\, s + \omega_0^2} \tag{11.27}$$

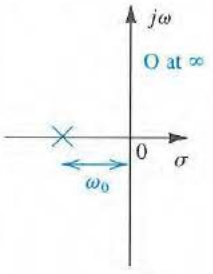
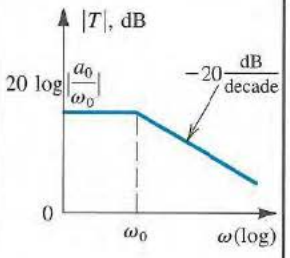
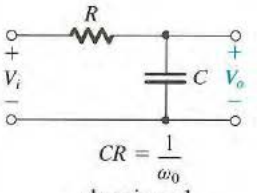
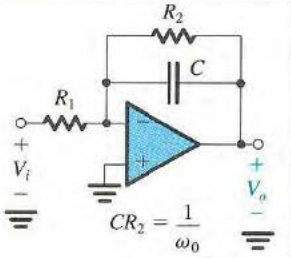
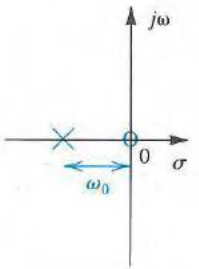
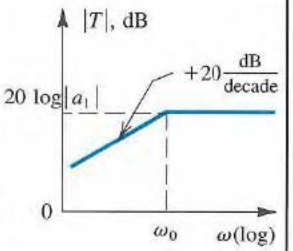
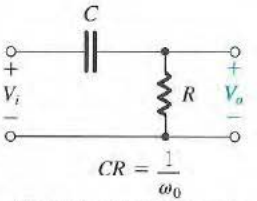
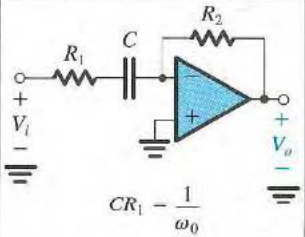
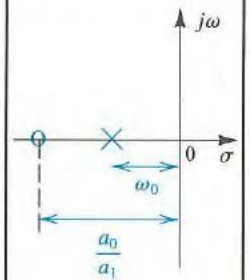
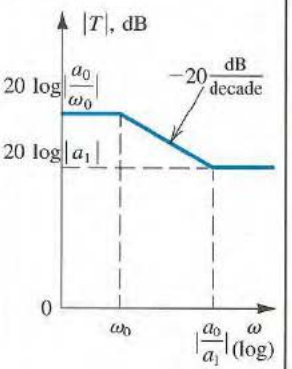
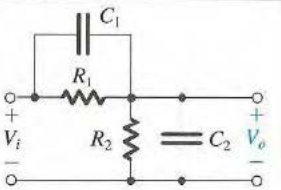
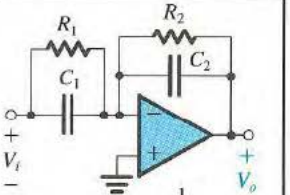
Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T\rvert$	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$			 $CR = \frac{1}{\omega_0}$ dc gain = 1	 $CR_2 = \frac{1}{\omega_0}$ dc gain $= -\frac{R_2}{R_1}$
(b) High-Pass (HF) $T(s) = \frac{a_1 s}{s + \omega_0}$			 $CR = \frac{1}{\omega_0}$ High-frequency gain = 1	 $CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_0}{a_1}$ dc gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$	 $C_2R_2 = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ dc gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$

Fig. 11.13 First-order filters.

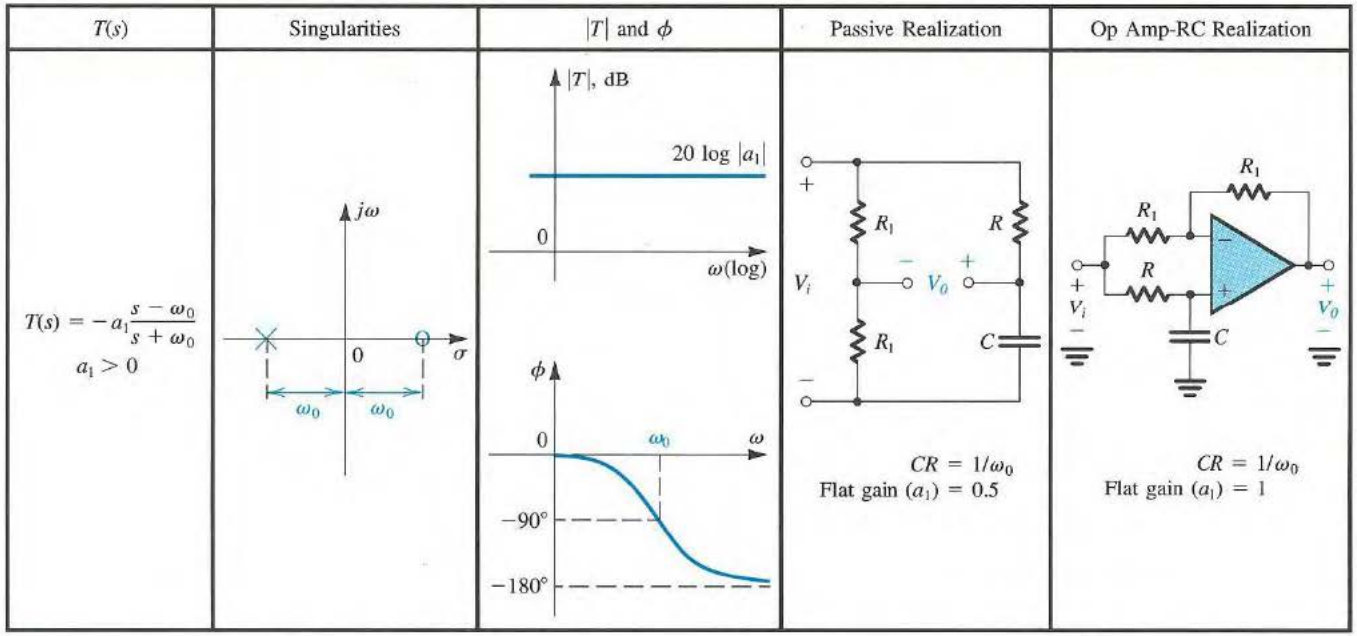


Fig. 11.14 First-order all-pass filter.

where ω_0 and Q determine the natural modes (poles) according to

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - (1/4Q^2)} \tag{11.28}$$

We are usually interested in the case of complex-conjugate natural modes, obtained for $Q > 0.5$. Figure 11.15 shows the location of the pair of complex-conjugate poles in the s-plane. Observe that the radial distance of the natural modes (from the origin) is equal to ω_0, which is known as the **pole frequency.** The parameter Q determines the distance of the poles from the $j\omega$-axis; the higher the value of Q, the closer the poles are to the $j\omega$-axis, and the more selective the filter response becomes. An infinite value for Q locates the poles on the $j\omega$-axis and can yield sustained oscillations in the circuit realization. A negative value of Q implies that the poles are in the right half of the s-plane, which certainly produces oscillations. The parameter Q is called the **pole quality factor**, or simply, **pole-Q.**

The transmission zeros of the second-order filter are determined by the numerator coefficients, a_0, a_1, and a_2. It follows that the numerator coefficients determine the type of second-order filter function (i.e., LP, HP, etc.). Seven special cases of interest are illustrated in Fig. 11.16. For each case we give the transfer function, the s-plane locations of the transfer function singularities, and the magnitude response. Circuit realizations for the various second-order filter functions will be given in subsequent sections.

All seven special second-order filters have a pair of complex-conjugate natural modes characterized by a frequency ω_0 and a Q factor, Q.

In the low-pass (LP) case, shown in Fig. 11.16(a), the two transmission zeros are at $s = \infty$. The magnitude response can exhibit a peak with the details indicated. It can be shown that the peak occurs only for $Q > 1/\sqrt{2}$. The response obtained for $Q = 1/\sqrt{2}$ is the Butterworth, or maximally flat, response.

The high-pass (HP) function shown in Fig. 11.16(b) has both of its transmission zeros at $s = 0$ (dc). The magnitude response shows a peak for $Q > 1/\sqrt{2}$, with the details of the response as indicated. Observe the duality between the LP and HP responses.

Next consider the bandpass filter function shown in Fig. 11.16(c). Here, one transmission zero is at $s = 0$ (dc), and the other is at $s = \infty$. The magnitude response peaks at $\omega = \omega_0$. Thus the **center frequency** of the bandpass filter is equal to the pole frequency ω_0. The selectivity of the second-order bandpass filter is usually measured by its 3-dB

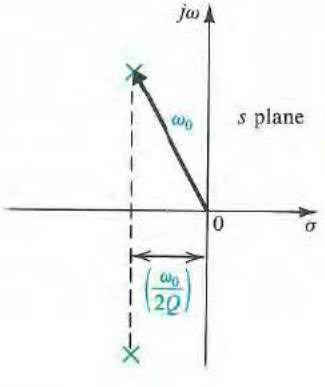


Fig. 11.15 Definition of the parameters ω_0 and Q of a pair of complex conjugate poles.

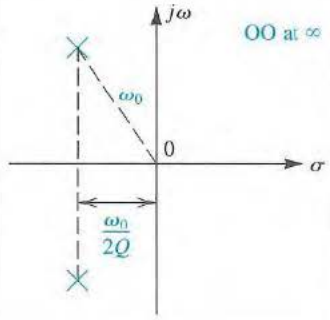
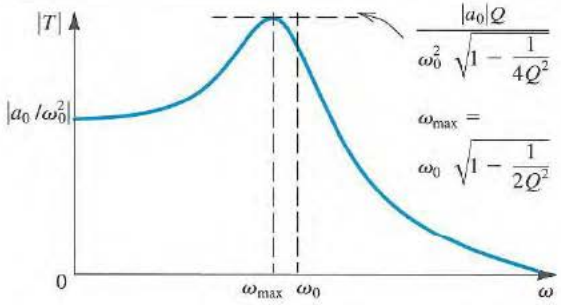
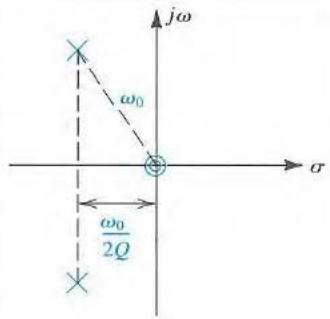
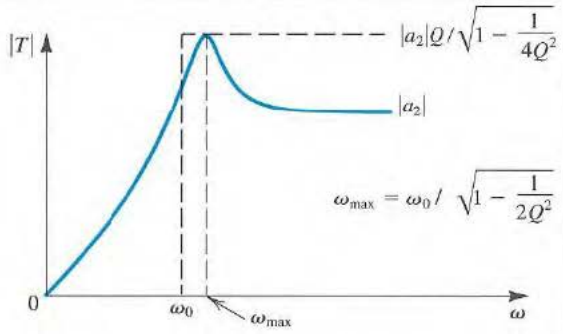
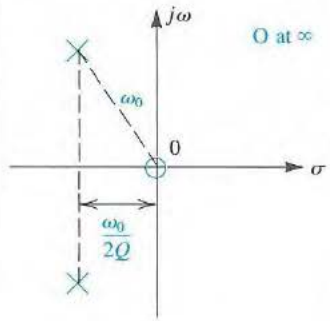
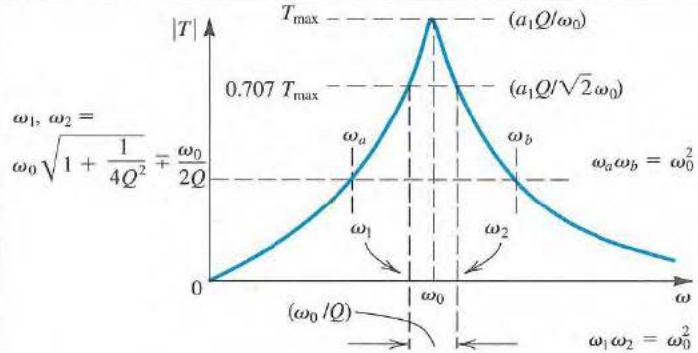
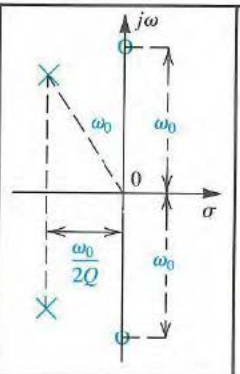
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ dc gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

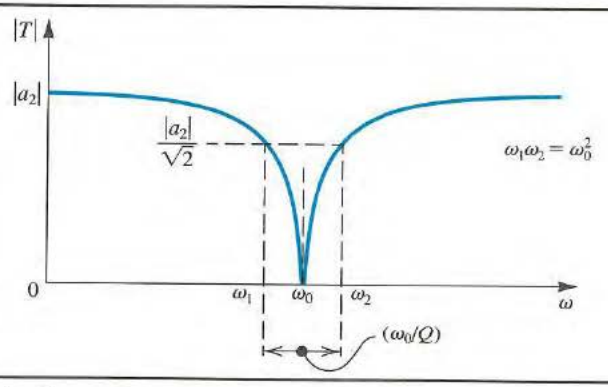
Fig. 11.16 Second-order filtering functions.

(d) Notch

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

dc gain =
high-frequency gain = a_2





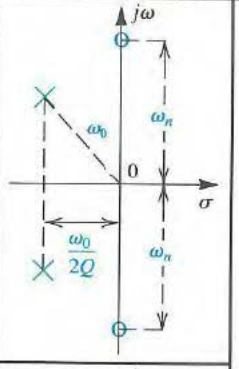
(e) Low-Pass Notch (LPN)

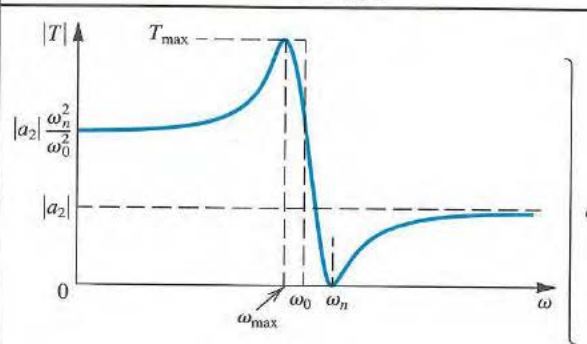
$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_n \geq \omega_0$

dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$

high-frequency gain = a_2





$$\omega_{\max} = \omega_0 \sqrt{\frac{\left(\frac{\omega_n^2}{\omega_0^2}\right)\left(1 - \frac{1}{2Q^2}\right) - 1}{\frac{\omega_n^2}{\omega_0^2} + \frac{1}{2Q^2} - 1}}$$

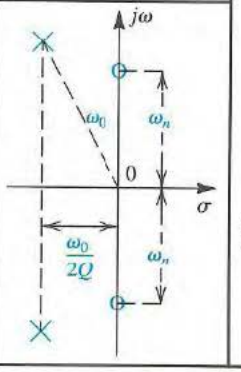
(f) High-Pass Notch (HPN)

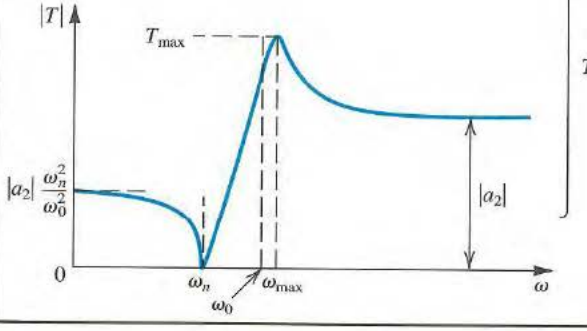
$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_n \leq \omega_0$

dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$

high-frequency gain = a_2





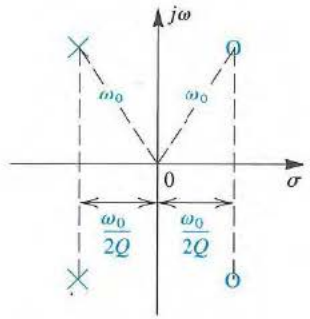
$$T_{\max} = \frac{|a_2| \quad |\omega_n^2 - \omega_{\max}^2|}{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2 \omega_{\max}^2}}$$

Fig. 11.16 (continued)

(g) All-Pass
(AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$



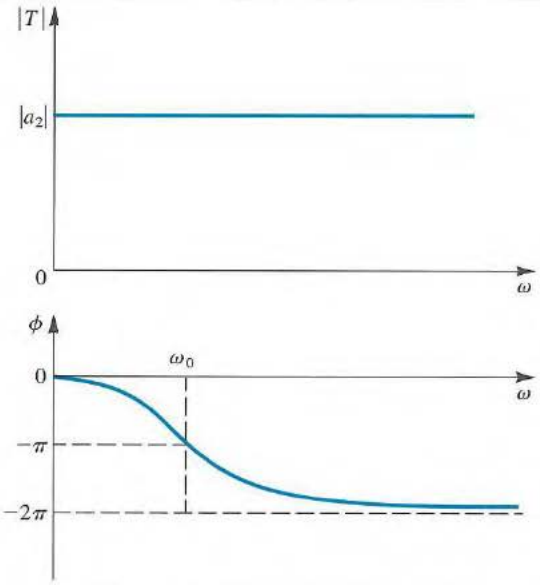


Fig. 11.16 (continued)

bandwidth. This is the difference between the two frequencies ω_1 and ω_2 at which the magnitude response is 3 dB below its maximum value (at ω_0). It can be shown that

$$\omega_1, \omega_2 = \omega_0\sqrt{1 + (1/4Q^2)} \pm \frac{\omega_0}{2Q} \tag{11.29}$$

Thus

$$BW \equiv \omega_2 - \omega_1 = \omega_0/Q \tag{11.30}$$

Observe that as Q increases, the bandwidth decreases and the bandpass filter becomes more selective.

If the transmission zeros are located on the $j\omega$-axis, at the complex conjugate locations $\pm j\omega_n$, then the magnitude response exhibits zero transmission at $\omega = \omega_n$. Thus a **notch** in the magnitude response occurs at $\omega = \omega_n$, and ω_n is known as the **notch frequency.** Three cases of the second-order notch filter are possible: the regular notch, obtained when $\omega_n = \omega_0$ (Fig. 11.16d); the low-pass notch, obtained when $\omega_n > \omega_0$ (Fig. 11.16e); and the high-pass notch, obtained when $\omega_n < \omega_0$ (Fig. 11.16f). The reader is urged to verify the response details given in these figures (a rather tedious task, though!). Observe that in all notch cases, the transmission at dc and at $s = \infty$ is finite. This is so because there are no transmission zeros at either $s = 0$ or $s = \infty$.

The last special case of interest is the all-pass filter whose characteristics are illustrated in Fig. 11.16(g). Here the two transmission zeros are in the right half of the s-plane, at the mirror-image locations of the poles. (This is the case for all-pass functions of any order.) The magnitude response of the all-pass function is constant over all frequencies; the **flat gain,** as it is called, is in our case equal to $|a_2|$. The frequency selectivity of the all-pass function is in its phase response.

Exercises

11.13 For a maximally flat second-order low-pass filter ($Q = 1/\sqrt{2}$) show that at $\omega = \omega_0$ the magnitude response is 3 dB below the value at dc.

11.14 Give the transfer function of a second-order bandpass filter with a center frequency of 10^5 rad/s, a center-frequency gain of 10, and a 3-dB bandwidth of 10^3 rad/s.

Ans. $T(s) = \dfrac{10^4 s}{s^2 + 10^3 s + 10^{10}}$

11.15 (a) For the second-order notch function with $\omega_n = \omega_0$, show that in order for the attenuation to be greater than A dB over a frequency band BW_a, the value of Q is given by

$$Q \leq \frac{\omega_0}{BW_a\sqrt{10^{A/10} - 1}}$$

(*Hint:* First, show that any two frequencies, ω_1 and ω_2, at which $|T|$ is the same, are related by $\omega_1\omega_2 = \omega_0^2$.) (b) Use the result of (a) to show that the 3-dB bandwidth is ω_0/Q, as indicated in Fig. 11.16(d).

11.16 Consider a low-pass notch with $\omega_0 = 1$ rad/s, $Q = 10$, $\omega_n = 1.2$ rad/s, and a dc gain of unity. Find the frequency and magnitude of the transmission peak. Also find the high-frequency transmission.

Ans. 0.986 rad/s; 3.17; 0.69

11.5 THE SECOND-ORDER LCR RESONATOR

In this section we shall study the second-order LCR resonator shown in Fig. 11.17(a). The use of this resonator to derive circuit realizations for the various second-order filter functions will be demonstrated. Also, it will be shown in the next section that replacing the inductor L by a simulated inductance obtained using an op amp–RC circuit results in an op amp–RC resonator. The latter forms the basis of an important class of active-RC filters to be studied in the next section.

The Resonator Natural Modes

The natural modes of the parallel resonance circuit of Fig. 11.17(a) can be determined by applying *an excitation that does not change the natural structure of the circuit*. Two possible ways of exciting the circuit are shown in Figs. 11.17(b) and (c). In Fig. 11.17(b) the resonator is excited with a current source I connected in parallel. Since as far as the natural

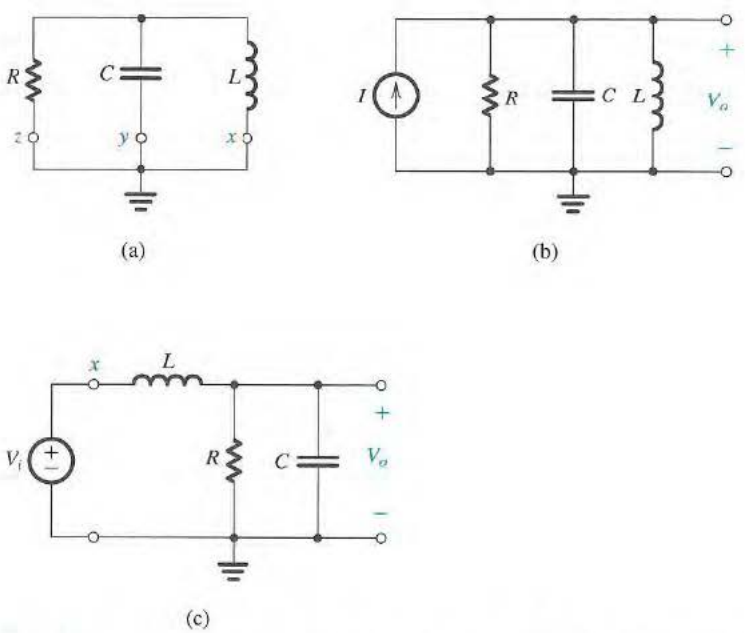


Fig. 11.17 **(a)** The second-order parallel LCR resonator. **(b)** and **(c)** Two ways for exciting the resonator of (a) without changing its *natural structure*. The resonator poles are the poles of V_o/I and V_o/V_i.

response of a circuit is concerned, an independent ideal current source is equivalent to an open circuit, the excitation of Fig. 11.17(b) does not alter the natural structure of the resonator. Thus the circuit in Fig. 11.17(b) can be used to determine the natural modes of the resonator by simply finding the poles of any response function. We can for instance take the voltage V_o across the resonator as the response and thus obtain the response function $V_o/I = Z$, where Z is the impedance of the parallel resonance circuit. It is obviously more convenient, however, to work in terms of the admittance Y; thus

$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{(1/sL) + sC + (1/R)} = \frac{s/C}{s^2 + s(1/CR) + (1/LC)} \tag{11.31}$$

Equating the denominator to the standard form $s^2 + s(\omega_0/Q) + \omega_0^2$ leads to

$$\omega_0^2 = 1/LC \tag{11.32}$$

and

$$\omega_0/Q = 1/CR \tag{11.33}$$

Thus,

$$\omega_0 = 1/\sqrt{LC} \tag{11.34}$$

$$Q = \omega_0 CR \tag{11.35}$$

These expressions should be familiar to the reader from earlier studies of parallel resonance circuits in introductory courses on circuit theory.

An alternative way of exciting the parallel LCR resonator for the purpose of determining its natural modes is shown in Fig. 11.17(c). Here, node x of inductor L has been disconnected from ground and connected to an ideal voltage source V_i. Now, since as far as the natural response of a circuit is concerned, an ideal independent voltage source is equivalent to a short circuit, the excitation of Fig. 11.17(c) does not alter the natural structure of the resonator. Thus we can use the circuit in Fig. 11.17(c) to determine the natural modes of the resonator. These are the poles of any response function. For instance, we can select V_o as the response variable and find the transfer function V_o/V_i. The reader can easily verify that this will lead to the natural modes determined above.

In a design problem, we will be given ω_0 and Q and will be asked to determine L, C, and R. Equations (11.34) and (11.35) are two equations in the three unknowns. The one available degree of freedom can be utilized to set the impedance level of the circuit to a value that results in practical component values.

Realization of Transmission Zeros

Having selected the component values of the LCR resonator so as to realize a given pair of complex-conjugate natural modes, we now consider the use of the resonator to realize a desired filter type (e.g., LP, HP, etc.). Specifically, we wish to find out where to inject the input voltage signal V_i so that the transfer function V_o/V_i is the desired one. Toward that end, note that in the resonator circuit in Fig. 11.17(a) any of the nodes labeled x, y, or z can be disconnected from ground and connected to V_i without altering the circuit's natural

modes. When this is done the circuit takes the form of a voltage divider, as shown in Fig. 11.18(a). Thus the transfer function realized is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \tag{11.36}$$

We observe that *the transmission zeros are the values of s at which $Z_2(s)$ is zero, provided that $Z_1(s)$ is not simultaneously zero, and the values of s at which $Z_1(s)$ is infinite, provided that $Z_2(s)$ is not simultaneously infinite.* This statement makes physical sense: The output will be zero either when $Z_2(s)$ behaves as a short circuit or when $Z_1(s)$ behaves as an open circuit. If there is a value of s at which both Z_1 and Z_2 are zero, then V_o/V_i will be finite and no transmission zero is obtained. Similarly, if there is a value of s at which both Z_1 and Z_2 are infinite, then V_o/V_i will be finite and no transmission zero is realized.

Realization of the Low-Pass Function

Using the scheme outlined above we see that to realize a low-pass function, node x is disconnected from ground and connected to V_i, as shown in Fig. 11.18(b). The transmission zeros of this circuit will be at the value of s for which the series impedance becomes infinite (sL becomes infinite at $s = \infty$) and the value of s at which the shunt impedance becomes zero ($1/[sC + (1/R)]$ becomes zero at $s = \infty$). Thus this circuit has two transmission zeros at $s = \infty$, as an LP is supposed to. The transfer function can be written either by inspection or by using the voltage-divider rule. Following the latter approach, we obtain

$$\begin{aligned} T(s) \equiv \frac{V_o}{V_i} &= \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)} \\ &= \frac{1/LC}{s^2 + s\,(1/CR) + (1/LC)} \end{aligned} \tag{11.37}$$

Realization of the High-Pass Function

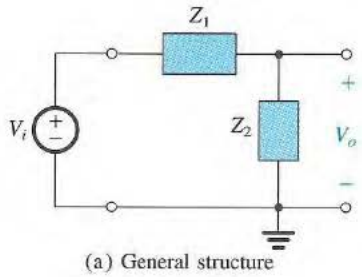
To realize the second-order high-pass function, node y is disconnected from ground and connected to V_i, as shown in Fig. 11.18(c). Here the series capacitor introduces a transmission zero at $s = 0$ (dc), and the shunt inductor introduces another transmission zero at $s = 0$ (dc). Thus, by inspection, the transfer function may be written as

$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2 + s\,(\omega_0/Q) + \omega_0^2} \tag{11.38}$$

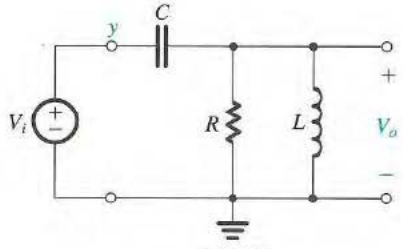
where ω_0 and Q are the natural mode parameters given by Eqs. (11.34) and (11.35) and a_2 is the high-frequency transmission. The value of a_2 can be determined from the circuit by observing that as s approaches ∞, the capacitor approaches a short circuit and V_o approaches V_i, resulting in $a_2 = 1$.

Realization of the Bandpass Function

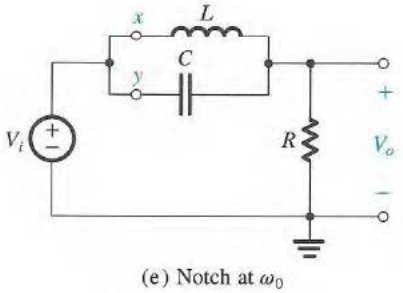
The bandpass function is realized by disconnecting node z from ground and connecting it to V_i, as shown in Fig. 11.18(d). Here the series impedance is resistive, and thus does not



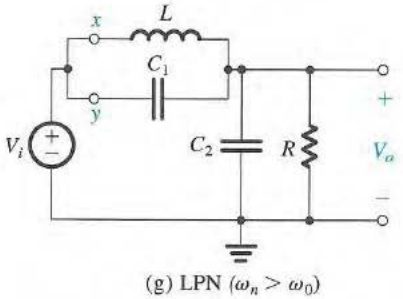
(a) General structure



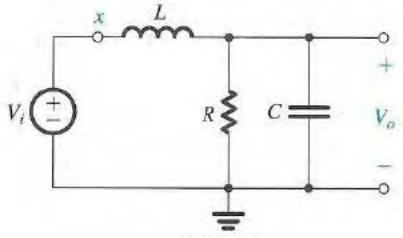
(c) HP



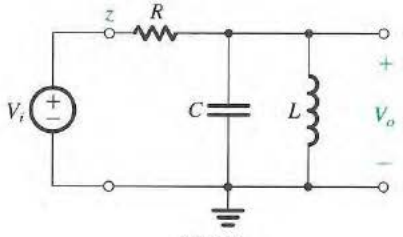
(e) Notch at ω_0



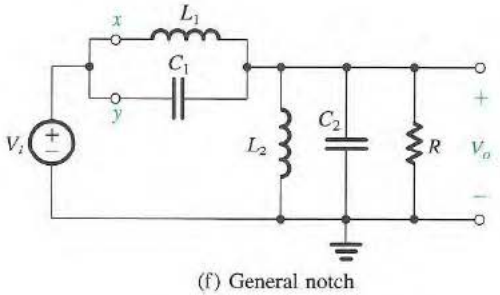
(g) LPN ($\omega_n > \omega_0$)



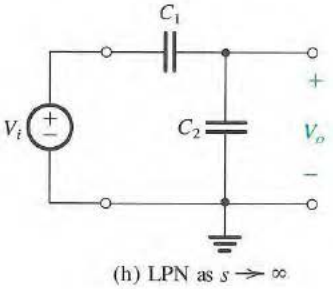
(b) LP



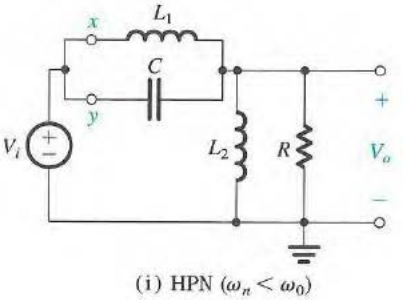
(d) BP



(f) General notch



(h) LPN as $s \to \infty$



(i) HPN ($\omega_n < \omega_0$)

Fig. 11.18 Realization of various second-order filter functions using the LCR resonator of Fig. 11.17(b): **(a)** general structure, **(b)** LP, **(c)** HP, **(d)** BP, **(e)** notch at ω_0, **(f)** general notch, **(g)** LPN ($\omega_n \geq \omega_0$), **(h)** LPN as $s \to \infty$, **(i)** HPN ($\omega_n < \omega_0$).

introduce any transmission zeros. These are obtained as follows: One zero at $s = 0$ is realized by the shunt inductor, and one zero at $s = \infty$ is realized by the shunt capacitor. At the center-frequency ω_0, the parallel LC tuned circuit exhibits an infinite impedance, and thus no current flows in the circuit. It follows that at $\omega = \omega_0$, $V_o = V_i$. In other words, the center-frequency gain of the bandpass filter is unity. Its transfer function can be obtained as follows:

$$\begin{aligned} T(s) &= \frac{Y_R}{Y_R + Y_L + Y_C} = \frac{1/R}{(1/R) + (1/sL) + sC} \\ &= \frac{s\,(1/CR)}{s^2 + s\,(1/CR) + (1/LC)} \end{aligned} \tag{11.39}$$

Realization of the Notch Functions

To obtain a pair of transmission zeros on the $j\omega$-axis we use a parallel resonance circuit in the series arm, as shown in Fig. 11.18(e). Observe that this circuit is obtained by disconnecting both nodes x and y from ground and connecting them together to V_i. The impedance of the LC circuit becomes infinite at $\omega = \omega_0 = 1/\sqrt{LC}$, thus causing zero transmission at this frequency. The shunt impedance is resistive and thus does not introduce transmission zeros. It follows that the circuit in Fig. 11.18(e) will realize the notch transfer function

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\,(\omega_0/Q) + \omega_0^2} \tag{11.40}$$

The value of the high-frequency gain a_2 can be found from the circuit to be unity.

To obtain a notch filter realization in which the notch frequency ω_n is arbitrarily placed relative to ω_0, we adopt a variation on the above scheme. We still use a parallel LC circuit in the series branch, as shown in Fig. 11.18(f) where L_1 and C_1 are selected so that

$$L_1 C_1 = 1/\omega_n^2 \tag{11.41}$$

Thus the L_1C_1 tank circuit will introduce a pair of transmission zeros at $\pm j\omega_n$, provided that the L_2C_2 tank is not resonant at ω_n. Apart from this restriction, the values of L_2 and C_2 must be selected so as to ensure that the natural modes have not been altered; thus

$$C_1 + C_2 = C \tag{11.42}$$

$$L_1 /\!/ L_2 = L \tag{11.43}$$

In other words, when V_i is replaced by a short circuit, the circuit should reduce to the original LCR resonator. Another way of thinking about the circuit of Fig. 11.18(f) is that it is obtained from the original LCR resonator by lifting part of L and part of C off ground and connecting them to V_i.

It should be noted that in the circuit of Fig. 11.18(f), L_2 does *not* introduce a zero at $s = 0$ because at $s = 0$, the L_1C_1 circuit also has a zero. In fact, at $s = 0$ the circuit reduces to an inductive voltage divider with the dc transmission being $L_2/(L_1 + L_2)$. Similar comments can be made about C_2 and the fact that it does *not* introduce a zero at $s = \infty$.

The LPN and HPN filter realizations are special cases of the general notch circuit of Fig. 11.18(f). Specifically, for the LPN,

$$\omega_n > \omega_0,$$

thus

$$L_1C_1 < (L_1//L_2)(C_1 + C_2)$$

This condition can be satisfied with L_2 eliminated (i.e., $L_2 = \infty$ and $L_1 = L$), resulting in the LPN circuit in Fig. 11.18(g). The transfer function can be written by inspection as

$$T(s) \equiv \frac{V_o}{V_i} = a_2 \frac{s^2 + \omega_n^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{11.44}$$

where $\omega_n^2 = 1/LC_1$, $\omega_0^2 = 1/L(C_1 + C_2)$, $\omega_0/Q = 1/CR$, and a_2 is the high-frequency gain. From the circuit we see that as $s \to \infty$, the circuit reduces to that in Fig. 11.18(h), for which

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

Thus

$$a_2 = \frac{C_1}{C_1 + C_2} \tag{11.45}$$

To obtain an HPN realization we start with the circuit of Fig. 11.18(f) and use the fact that $\omega_n < \omega_0$ to obtain

$$L_1C_1 > (L_1//L_2)(C_1 + C_2)$$

which can be satisfied while selecting $C_2 = 0$ (i.e., $C_1 = C$). Thus we obtain the reduced circuit shown in Fig. 11.18(i). Observe that as $s \to \infty$, V_o approaches V_i and thus the high-frequency gain is unity. Thus, the transfer function can be expressed as

$$T(s) \equiv \frac{V_o}{V_i} = \frac{s^2 + (1/L_1C)}{s^2 + s(1/CR) + [1/(L_1//L_2)\,C]} \tag{11.46}$$

Realization of the All-Pass Function

The all-pass transfer function

$$T(s) = \frac{s^2 - s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{11.47}$$

can be written as

$$T(s) = 1 - \frac{s\,2(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2} \tag{11.48}$$

The second term on the right-hand side is a bandpass function with a center-frequency gain of 2. We already have a bandpass circuit (Fig. 11.18d) but with a center-frequency gain of unity. We shall therefore attempt an all-pass realization with a flat gain of 0.5, that is,

$$T(s) = 0.5 - \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

This function can be realized using a voltage divider with a transmission ratio of 0.5 together with the bandpass circuit of Fig. 11.18(d). To effect the subtraction, the output of the all-

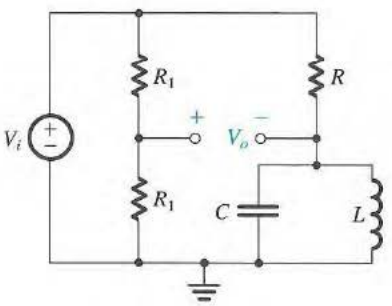


Fig. 11.19 Realization of the second-order all-pass transfer function using a voltage divider and an LCR resonator.

pass circuit is taken between the output terminal of the voltage divider and that of the bandpass filter, as shown in Fig. 11.19. Unfortunately this circuit has the disadvantage of lacking a common ground terminal between the input and the output. An op amp–RC realization of the all-pass function will be presented in the next section.

Exercises

11.17 Use the circuit of Fig. 11.18(b) to realize a second-order low-pass function of the maximally flat type with a 3-dB frequency of 100 kHz.

Ans. Selecting $R = 1\ \text{k}\Omega$, we obtain $C = 1125$ pF and $L = 2.25$ mH.

11.18 Use the circuit of Fig. 11.18(e) to design a notch filter to eliminate a bothersome power-supply hum at a 60-Hz frequency. The filter is to have a 3-dB bandwidth of 10 Hz (i.e., the attenuation is greater than 3 dB over a 10-Hz band around the 60-Hz center frequency; see Exercise 11.15 and Fig. 11.16d). Use $R = 10\ \text{k}\Omega$.

Ans. $C = 1.6\ \mu$F and $L = 4.42$ H (Note the large inductor required. This is the reason passive filters are not practical in low-frequency applications.)

11.6 SECOND-ORDER ACTIVE FILTERS BASED ON INDUCTOR REPLACEMENT

In this section, we study a family of op amp–RC circuits that realize the various second-order filter functions. The circuits are based on an op amp–RC resonator obtained by replacing the inductor L in the LCR resonator with an op amp–RC circuit that has an inductive input impedance.

The Antoniou Inductance-Simulation Circuit

Over the years, many op amp–RC circuits have been proposed for simulating the operation of an inductor. Of these, one circuit invented by A. Antoniou (see Antoniou, 1969) has proved to be the "best." By "best" we mean that the operation of the circuit is very tolerant to the nonideal properties of the op amps, in particular their finite gain and bandwidth. Figure 11.20(a) shows the Antoniou inductance simulation circuit. If the circuit is fed at its