
MECHANISMS AND MECHANICAL DEVICES SOURCEBOOK

Fourth Edition

**NEIL SCLATER
NICHOLAS P. CHIRONIS**

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PREFACE

This is the fourth edition of *Mechanisms and Mechanical Devices Sourcebook*, a well-illustrated reference book containing a wide range of information on both classical and modern mechanisms and mechanical devices. This edition contains three new chapters: one on basic mechanisms; the second on mobile robots; and the third on new directions in mechanical engineering. The chapter on basic mechanisms provides an overview of the physical principles of mechanics; the chapter on mobile robots examines existing scientific and military mobile robots and the scientific and engineering research in advanced robotics; the chapter on new directions in mechanical engineering reviews the present status and future prospects for microtechnology, highlighting progress in and acceptance of microelectromechanical systems (MEMS). Also included in the chapter are articles on nanotechnology, focused on the role mechanical engineers are playing in this burgeoning science. The field of nanotechnology now involves several branches of engineering as well as the physical, chemical, biological, and medical sciences. A previous section on rapid prototyping has been updated and upgraded as a separate chapter.

This edition contains a large core of archival drawings and text describing and illustrating proven mechanisms and mechanical devices carried over from previous editions. This core has been reorganized to make topics of interest to readers easier to find. Some previously published pages were deleted because their content was deemed to be of little value in future designs, and some figures have been redrawn to make them easier to understand. An extensive and comprehensive index has been provided to make this core a valuable reference resource for engineers, designers, inventors, students, hobbyists, and all enthusiasts for things mechanical. The 11 chapters in this core illustrate practical design solutions that can be recycled into new products.

The first edition of this book, published in 1991, did not mention the influence of electronics and computer science on mechanical engineering and mechanical design. However, since that time a sea change has occurred in the practice of mechanical engineering; today it is difficult to find any contemporary mechanical system or appliance that does not in some way include electronic components or circuits that improve its performance, simplify its operation, or provide for additional safety features. Those components might be as simple as solid-state rectifiers or light-emitting diodes (LEDs) or as complex as microprocessor-based modules that permit the product or system to operate autonomously.

The chapter on basic mechanisms provides the reader with a useful introduction to much of the content of this book; it will also serve as a refresher tutorial for those who have studied mechanical principles in the past and want to get up to speed on the fundamentals again. Topics covered include the inclined plane, screw jack, levers, linkages, gears, cams, and clutches. A previous tutorial chapter on motion control systems that contained illustrations and text describing control schemes and key components has been retained, and a former chapter on industrial robots has been revised and updated with new illustrations and specifications for some of the latest industrial robots. The new chapter on mobile robots extends the book's coverage of robotics and points out their growing economic and technical importance in scientific exploration and research as well as military missions and emergency services.

The new chapter on rapid prototyping discusses the emerging leaders in the field and reports on the trends: increasing popularity of 3-D plastic, paper, and wax models for engineering and design evaluation, and the extrapolation of existing technologies into the fabrication of functional metal and ceramic products. Replacement metal parts for older out-of-production machines are now being made rapidly and cost-effectively by eliminating the high cost and time delay involved in remaking the metal or ceramic dies or casting molds used in mass-production manufacturing.

The earlier articles on MEMS have been revised by reporting on the new developments and significant gains in the complexity of those devices; some MEMS are now being produced in large commercial volumes in established markets. The choices in material alternatives to silicon are discussed, and new microphotographs show more sophisticated multilayer devices.

The impact of electronic controls and communications circuits on mechanical engineering is nowhere more evident than on the latest motor vehicles. Microprocessors and electronics abound: they now control the engines and transmissions in all kinds of motor vehicles, and they have improved vehicle performance and fuel efficiency. Vehicular safety has also been improved by electronically deployed air-bags, antilock braking (ABS), stability or skid control (ESC), traction control (TC), and tire-pressure monitoring. Communication systems summon aid for drivers involved in accidents or breakdowns, and onboard navigation systems now provide map displays of streets to guide drivers.

With the exception of illustrations generously contributed by corporations, and government laboratories (see Acknowledgments), all of the figures in the tutorial Chapters 1 to 4 and 18 and 19 were drawn by this author on a Dell personal computer with software included in the Microsoft Windows XP package. Also, the five illustrations on the front cover of this book were derived from selected figures in those chapters.

Much of the archival core in this edition was first collected from a variety of published sources by Douglas C. Greenwood, then an editor of *Product Engineering* magazine; it first appeared in three volumes published by McGraw-Hill between 1959 and 1964. Nicholas Chironis edited and reorganized much of this content and supplemented it with contemporary technical articles to form the first edition of this book. In subsequent editions this core has been reorganized and new material has been added. References to manufacturers or publications that no longer exist have since been deleted because they are no longer valid sources for further information. The terms *devices* and *mechanisms* used to describe objects in the core pages have been used interchangeably and only some of them have been changed. However, the comprehensive index accounts for these differences in designation. The names of the inventors of these mechanisms and devices have been retained so that readers can research the status of any patents once held by them.

—Neil Sclater

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CHAPTER 1

BASICS OF

MECHANISMS

INTRODUCTION

Complex machines from internal combustion engines to helicopters and machine tools contain many mechanisms. However, it might not be as obvious that mechanisms can be found in consumer goods from toys and cameras to computer drives and printers. In fact, many common hand tools such as scissors, screwdrivers, wrenches, jacks, and hammers are actually true mechanisms. Moreover, the hands and feet, arms, legs, and jaws of humans qualify as functioning mechanisms as do the paws and legs, flippers, wings, and tails of animals.

There is a difference between a *machine* and a *mechanism*: All machines transform energy to do work, but only some mechanisms are capable of performing work. The term *machinery* means an assembly that includes both machines and mechanisms. Figure 1a illustrates a cross section of a machine—an internal combustion engine. The assembly of the piston, connecting rod, and crankshaft is a mechanism, termed a *slider-crank mechanism*. The basic schematic drawing of that mechanism, Fig. 1b, called a *skeleton outline*, shows only its fundamental structure without the technical details explaining how it is constructed.

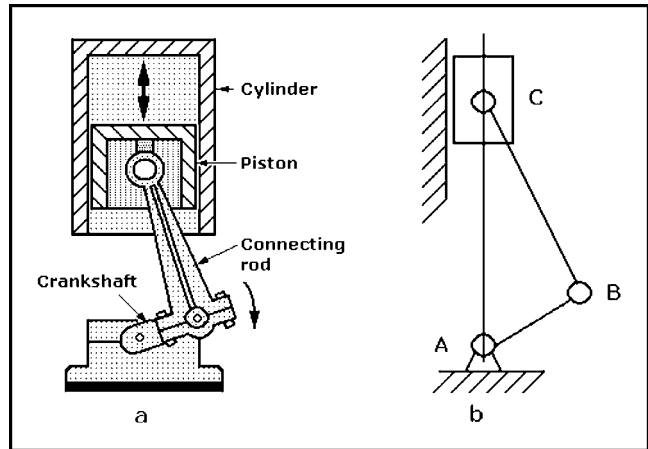


Fig. 1 Cross section of a cylinder of an internal combustion engine showing piston reciprocation (a), and the skeleton outline of the linkage mechanism that moves the piston (b).

PHYSICAL PRINCIPLES

Efficiency of Machines

Simple machines are evaluated on the basis of efficiency and mechanical advantage. While it is possible to obtain a larger force from a machine than the force exerted upon it, this refers only to force and not energy; according to the law of conservation of energy, *more work cannot be obtained from a machine than the energy supplied to it*. Because work = force \times distance, for a machine to exert a larger force than its initiating force or operator, that larger force must be exerted through a correspondingly shorter distance. As a result of friction in all moving machinery, the energy produced by a machine is less than that applied to it. Consequently, by interpreting the law of conservation of energy, it follows that:

$$\text{Input energy} = \text{output energy} + \text{wasted energy}$$

This statement is true over any period of time, so it applies to any unit of time; because power is work or energy per unit of time, the following statement is also true:

$$\text{Input power} = \text{output power} + \text{wasted power}$$

The *efficiency of a machine is the ratio of its output to its input*, if both input and output are expressed in the same units of energy or power. This ratio is always less than unity, and it is usually expressed in percent by multiplying the ratio by 100.

$$\text{Percent efficiency} = \frac{\text{output energy}}{\text{input energy}} \times 100$$

or

$$\text{Percent efficiency} = \frac{\text{output power}}{\text{input power}} \times 100$$

A machine has high efficiency if most of the power supplied to it is passed on to its load and only a fraction of the power is wasted. The efficiency can be as high as 98 percent for a large electrical generator, but it is likely to be less than 50 percent for a screw jack. For example, if the input power supplied to a 20-hp motor with an efficiency of 70 percent is to be calculated, the foregoing equation is transposed.

$$\begin{aligned} \text{Input power} &= \frac{\text{output power}}{\text{percent efficiency}} \times 100 \\ &= \frac{20 \text{ hp}}{70} \times 100 = 28.6 \text{ hp} \end{aligned}$$

Mechanical Advantage

The *mechanical advantage* of a mechanism or system is the ratio of the load or weight W , typically in pounds or kilograms, divided by the effort or force F exerted by the initiating entity or operator, also in pounds or kilograms. If friction has been considered or is known from actual testing, the mechanical advantage, MA, of a machine is:

$$\text{MA} = \frac{\text{load}}{\text{effort}} = \frac{W}{F}$$

However, if it is assumed that the machine operates without friction, the ratio of W divided by F is called the *theoretical mechanical advantage*, TA.

$$TA = \frac{\text{load}}{\text{effort}} = \frac{W}{F}$$

Velocity Ratio

Machines and mechanisms are used to translate a small amount of movement or distance into a larger amount of movement or

distance. This property is known as the *velocity ratio*: it is defined as the ratio of the distance moved by the effort per second divided by the distance moved by the load per second for a machine or mechanism. It is widely used in determining the mechanical advantage of gears or pulleys.

$$VR = \frac{\text{distance moved by effort/second}}{\text{distance moved by load/second}}$$

INCLINED PLANE

The *inclined plane*, shown in Fig. 2, has an incline length l (AB) = 8 ft and a height h (BC) = 3 ft. The inclined plane permits a smaller force to raise a given weight than if it were lifted directly from the ground. For example, if a weight W of 1000 lb is to be

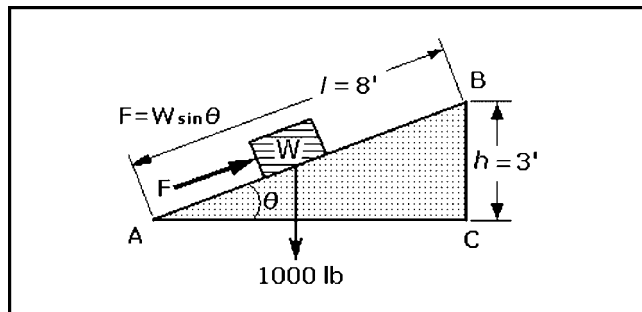


Fig. 2 Diagram for calculating mechanical advantage of an inclined plane.

raised vertically through a height BC of 3 ft without using an inclined plane, a force F of 1000 lb must be exerted over that height. However, with an inclined plane, the weight is moved over the longer distance of 8 ft, but a force F of only $3/8$ of 1000 or 375 lb would be required because the weight is moved through a longer distance. To determine the mechanical advantage of the inclined plane, the following formula is used:

$$F = W \sin \theta \quad \sin \theta = \frac{\text{height } h}{\text{length } l}$$

where height $h = 3$ ft, length $l = 8$ ft, $\sin \theta = 0.375$, and weight $W = 1000$ lb.

$$F = 1000 \times 0.375$$

$$F = 375 \text{ lb}$$

$$\text{Mechanical advantage MA} = \frac{\text{load}}{\text{effort}} = \frac{W}{F} = \frac{1000}{375} = 2.7$$

PULLEY SYSTEMS

A single pulley simply changes the direction of a force so its mechanical advantage is unity. However, considerable mechanical advantage can be gained by using a combination of pulleys. In the typical pulley system, shown in Fig. 3a, each block contains two pulleys or sheaves within a frame or shell. The upper block is fixed and the lower block is attached to the load and moves with it. A cable fastened at the end of the upper block passes around four pulleys before being returned to the operator or other power source.

Figure 3b shows the pulleys separated for clarity. To raise the load through a height h , each of the sections of the cable A, B, C, and D must be moved to a distance equal to h . The operator or other power source must exert a force F through a distance $s = 4h$ so that the velocity ratio of s to h is 4. Therefore, the theoretical mechanical advantage of the system shown is 4, corresponding to the four cables supporting the load W . The theoretical mechanical advantage TA for any pulley system similar to that shown equals the number of parallel cables that support the load.

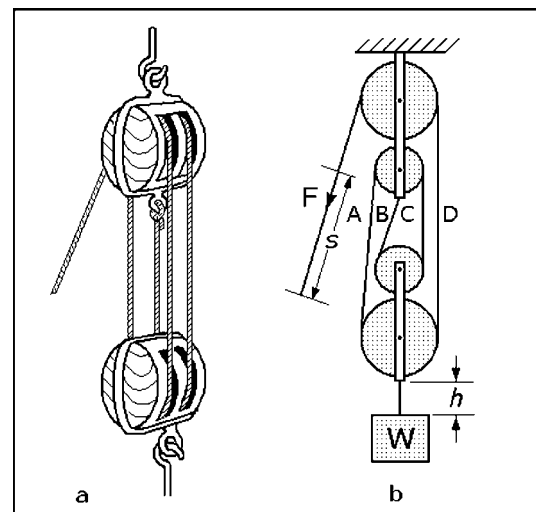


Fig. 3 Four cables supporting the load of this pulley combination give it a mechanical advantage of 4.

SCREW-TYPE JACK

Mechanisms are often required to move a large load with a small effort. For example, a car jack allows an ordinary human to lift a car which may weigh as much as 6000 lb, while the person only exerts a force equivalent to 20 or 30 lb.

The *screw jack*, shown in Fig. 4, is a practical application of the inclined plane because a screw is considered to be an inclined plane wrapped around cylinder. A force F must be exerted at the end of a length of horizontal bar l to turn the screw to raise the load (weight W) of 1000 lb. The 5-ft bar must be moved through a complete turn or a circle of length $s = 2\pi l$ to advance the load a distance h of 1.0 in. or 0.08 ft equal to the pitch p of the screw. The pitch of the screw is the distance advanced in a complete turn. Neglecting friction:

$$F = \frac{W \times h}{s}$$

where $s = 2\pi l = 2 \times 3.14 \times 5$, $h = p = 0.08$, and $W = 1000$ lb

$$F = \frac{1000 \times 0.08}{2 \times 3.14 \times 5} = \frac{80}{31.4} = 2.5 \text{ lb}$$

$$\text{Mechanical advantage MA} = \frac{\text{load}}{\text{effort}} = \frac{2\pi l}{p} = \frac{31.4}{0.08} = 393$$

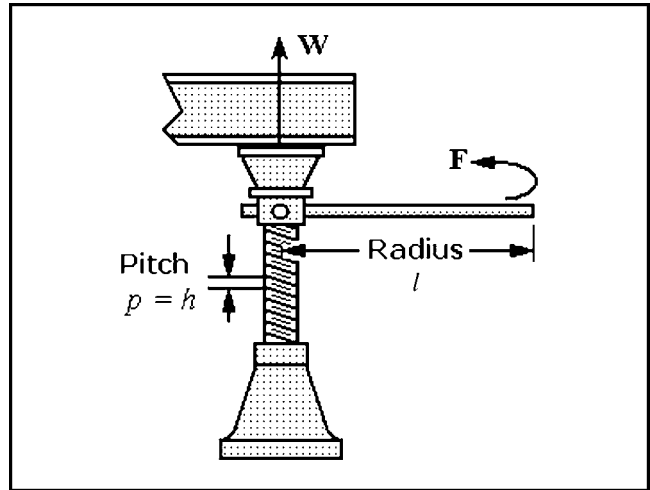


Fig. 4 Diagram for calculating the mechanical advantage of a screw jack.

LEVERS AND MECHANISMS

Levers

Levers are the simplest of mechanisms; there is evidence that Stone Age humans used levers to extend their reach or power; they made them from logs or branches to move heavy loads such as rocks. It has also been reported that primates and certain birds use twigs or sticks to extend their reach and act as tools to assist them in obtaining food.

A lever is a rigid beam that can rotate about a fixed point along its length called the *fulcrum*. Physical effort applied to one end of the beam will move a load at the other end. The act of moving the fulcrum of a long beam nearer to the load permits a large load to be lifted with minimal effort. This is another way to obtain *mechanical advantage*.

The three *classes of lever* are illustrated in Fig. 5. Each is capable of providing a different level of mechanical advantage. These levers are called *Class 1*, *Class 2*, and *Class 3*. The differences in the classes are determined by:

- Position along the length of the lever where the effort is applied
- Position along the length of the lever where the load is applied
- Position along the length of the lever where the fulcrum or pivot point is located

Class 1 lever, the most common, has its fulcrum located at or about the middle with effort exerted at one end and load positioned at the opposite end, both on the same side of the lever. Examples of *Class 1* levers are playground seesaw, crowbar, scissors, claw hammer, and balancing scales.

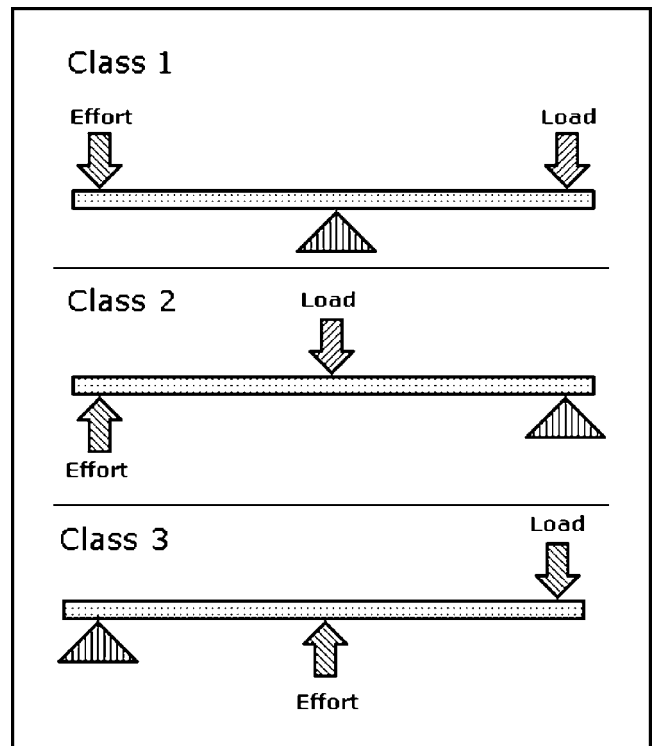


Fig. 5 Three levers classified by the locations of their fulcrums, loads, and efforts.

Class 2 lever has its fulcrum at one end; effort is exerted at the opposite end, and the opposing load is positioned at or near the middle. Examples of Class 2 levers are wheelbarrow, simple bottle openers, nutcracker, and foot pump for inflating air mattresses and inflatable boats.

Class 3 lever also has its fulcrum on one end; load is exerted at the opposite end, and the opposing effort is exerted on or about the middle. Examples of Class 3 levers are shovel and fishing rod where the hand is the fulcrum, tweezers, and human and animal arms and legs.

The application of a Class 1 lever is shown in Fig. 6. The lever is a bar of length AB with its fulcrum at X, dividing the length of the bar into parts: l_1 and l_2 . To raise a load W through a height of h , a force F must be exerted downward through a distance s . The triangles AXC and BXD are similar and proportional; therefore, ignoring friction:

$$\frac{s}{h} = \frac{l_1}{l_2} \text{ and mechanical advantage MA} = \frac{l_1}{l_2}$$

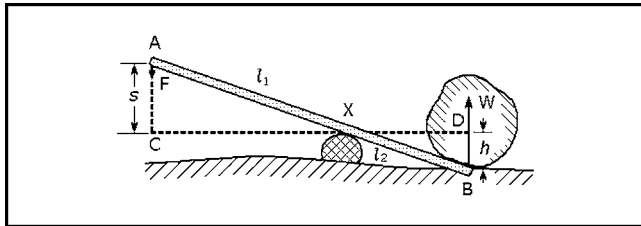


Fig. 6 Diagram for calculating the mechanical advantage of a simple lever for raising a weight.

Winches, Windlasses, and Capstans

Winches, windlasses, and capstans are machines that convert rotary motion into linear motion, usually with some mechanical

advantage. These machines are essentially Class 1 levers: effort is applied to a lever or crank, the fulcrum is the center of the drum, and the load is applied to the rope, chain, or cable.

Manually operated windlasses and capstans, mechanically the same, were originally used on sailing ships to raise and lower anchors. Operated by one or more levers by one or more sailors, both had barrels or drums on which rope or chain was wound. In the past, windlasses were distinguished from capstans; windlasses had horizontal drums and capstans had vertical drums. The modern term *winch* is now the generic name for any manual or power-operated drum for hauling a load with cable, chain, or rope. The manually operated winch, shown in Fig. 7, is widely used today on sailboats for raising and trimming sails, and sometimes for weighing anchors.

Ignoring friction, the mechanical advantage of all of these machines is approximately the *length of the crank* divided by the *diameter of the drum*. In the winch example shown, when the left end of the line is held under tension and the handle or crank is turned clockwise, a force is applied to the line entering on the right; it is attached to the load to perform such useful work as raising or tensioning sails.

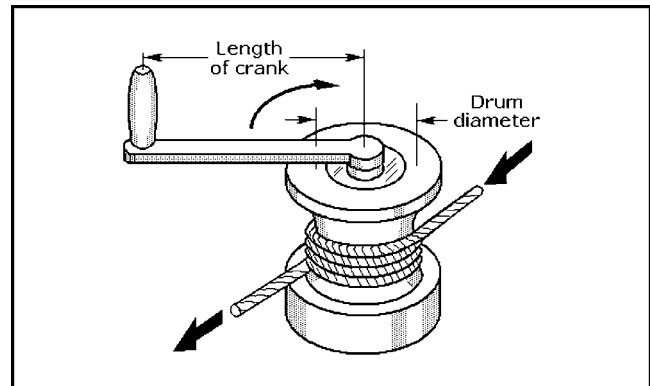


Fig. 7 Diagram for calculating the mechanical advantage of a manually operated winch for raising anchors or sails.

LINKAGES

A *linkage* is a mechanism formed by connecting two or more levers together. Linkages can be designed to change the direction of a force or make two or more objects move at the same time. Many different fasteners are used to connect linkages together yet allow them to move freely such as pins, end-threaded bolts with nuts, and loosely fitted rivets. There are two general classes of linkages: *simple planar linkages* and more complex *specialized linkages*; both are capable of performing tasks such as describing straight lines or curves and executing motions at differing speeds. The names of the linkage mechanisms given here are widely but not universally accepted in all textbooks and references.

Linkages can be classified according to their primary functions:

- *Function generation*: the relative motion between the links connected to the frame
- *Path generation*: the path of a tracer point
- *Motion generation*: the motion of the coupler link

Simple Planar Linkages

Four different simple planar linkages shown in Fig. 8 are identified by function:

- *Reverse-motion linkage*, Fig. 8a, can make objects or force move in opposite directions; this can be done by using the input link as a lever. If the fixed pivot is equidistant from the moving pivots, output link movement will equal input link movement, but it will act in the opposite direction. However, if the fixed pivot is not centered, output link movement will not equal input link movement. By selecting the position of the fixed pivot, the linkage can be designed to produce specific mechanical advantages. This linkage can also be rotated through 360°.
- *Push-pull linkage*, Fig. 8b, can make the objects or force move in the same direction; the output link moves in the same direction as the input link. Technically classed as a four-bar linkage, it can be rotated through 360° without changing its function.

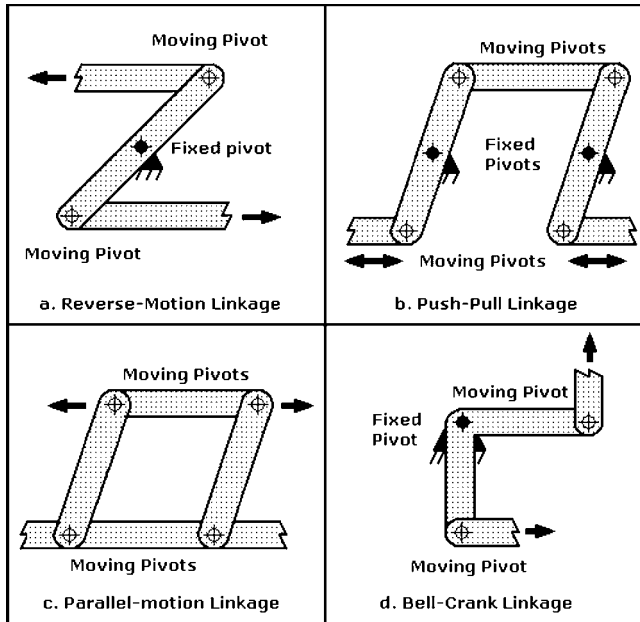


Fig. 8 Functions of four basic planar linkage mechanisms.

- *Parallel-motion linkage*, Fig. 8c, can make objects or forces move in the same direction, but at a set distance apart. The moving and fixed pivots on the opposing links in the parallelogram must be equidistant for this linkage to work correctly. Technically classed as a four-bar linkage, this linkage can also be rotated through 360° without changing its function. Pantographs that obtain power for electric trains from overhead cables are based on parallel-motion linkage. Drawing pantographs that permit original drawings to be manually copied without tracing or photocopying are also adaptations of this linkage; in its simplest form it can also keep tool trays in a horizontal position when the toolbox covers are opened.
- *Bell-crank linkage*, Fig. 8d, can change the direction of objects or force by 90° . This linkage rang doorbells before electric clappers were invented. More recently this mechanism has been adapted for bicycle brakes. This was done by pinning two bell cranks bent 90° in opposite directions together to form tongs. By squeezing the two handlebar levers linked to the input ends of each crank, the output ends will move together. Rubber blocks on the output ends of each crank press against the wheel rim, stopping the bicycle. If the pins which form a fixed pivot are at the midpoints of the cranks, link movement will be equal. However, if those distances vary, mechanical advantage can be gained.

Specialized Linkages

In addition to changing the motions of objects or forces, more complex linkages have been designed to perform many specialized functions: These include drawing or tracing straight lines; moving objects or tools faster in a retraction stroke than in an extension stroke; and converting rotating motion into linear motion and vice versa.

The simplest specialized linkages are four-bar linkages. These linkages have been versatile enough to be applied in many different applications. Four-bar linkages actually have only three moving links but they have one fixed link and four pin joints or pivots. A useful mechanism must have at least four links but closed-loop assemblies of three links are useful elements in structures. Because any linkage with at least one fixed link is a mechanism, both the parallel-motion and push-pull linkages mentioned earlier are technically machines.

Four-bar linkages share common properties: three rigid moving links with two of them hinged to fixed bases which form a *frame*. Link mechanisms are capable of producing rotating, oscillating, or reciprocating motion by the rotation of a crank. Linkages can be used to convert:

- Continuous rotation into another form of continuous rotation, with a constant or variable angular velocity ratio
- Continuous rotation into oscillation or continuous oscillation into rotation, with a constant or variable velocity ratio
- One form of oscillation into another form of oscillation, or one form of reciprocation into another form of reciprocation, with a constant or variable velocity ratio

There are four different ways in which four-bar linkages can perform inversions or complete revolutions about fixed pivot points. One pivoting link is considered to be the *input* or *driver* member and the other is considered to be the *output* or *driven* member. The remaining moving link is commonly called a *connecting link*. The fixed link, hinged by pins or pivots at each end, is called the *foundation link*.

Three inversions or linkage rotations of a four-bar chain are shown in Figs. 9, 10, and 11. They are made up of links AB, BC, CD, and AD. The forms of the three inversions are defined by the position of the shortest link with respect to the link selected as the foundation link. The ability of the driver or driven links to make complete rotations about their pivots determines their functions.

Drag-link mechanism, Fig. 9, demonstrates the first inversion. The shortest link AD between the two fixed pivots is the foundation link, and both driver link AB and driven link CD can make full revolutions.

Crank-rocker mechanism, Fig. 10, demonstrates the second inversion. The shortest link AB is adjacent to AD, the foundation link. Link AB can make a full 360° revolution while the opposite link CD can only oscillate and describe an arc.

Double-rocker mechanism, Fig. 11, demonstrates the third inversion. Link AD is the foundation link, and it is opposite the shortest link BC. Although link BC can make a full 360° revolution, both pivoting links AB and CD can only oscillate and describe arcs.

The fourth inversion is another *crank-rocker mechanism* that behaves in a manner similar to the mechanism shown in Fig. 10,

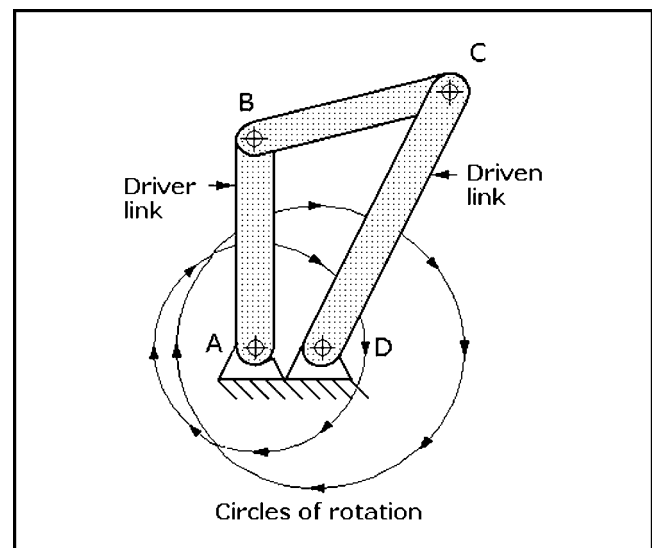


Fig. 9 Four-bar drag-link mechanism: Both the driver link AB and driven link CD can rotate through 360° . Link AD is the foundation link.

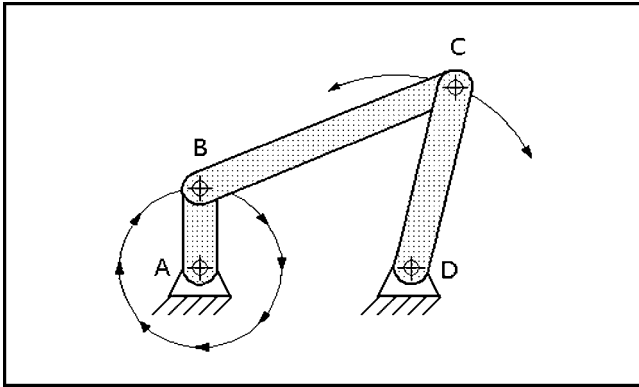


Fig. 10 Crank-rocker mechanism: Link AB can make a 360° revolution while link CD oscillates with C describing an arc. Link AD is the foundation link.

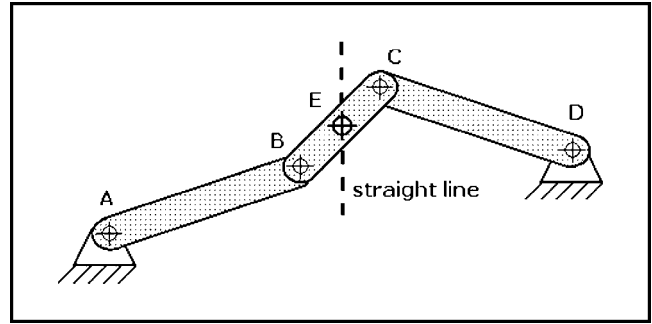


Fig. 12 Watt's straight-line generator: The center point E of link BC describes a straight line when driven by either links AB or CD.

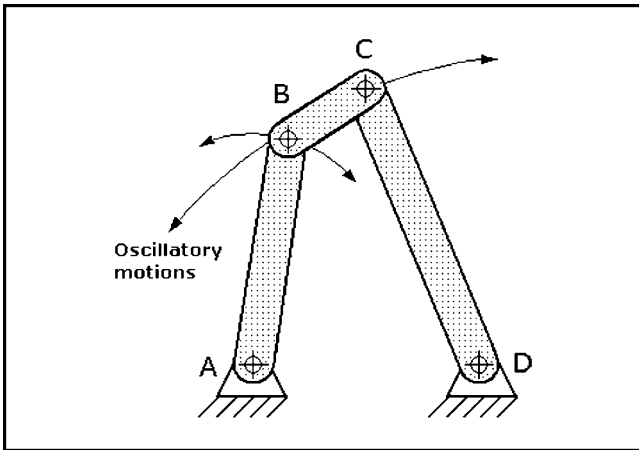


Fig. 11 Double-rocker mechanism: Short link BC can make a 360° revolution, but pivoting links AB and CD can only oscillate, describing arcs.

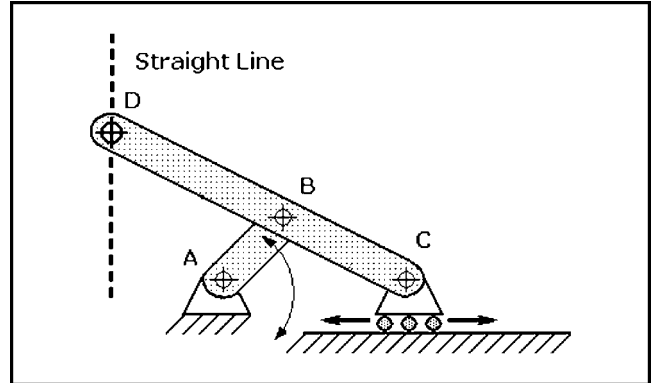


Fig. 13 Scott Russell straight-line generator: Point D of link DC describes a straight line as driver link AB oscillates, causing the slider at C to reciprocate left and right.

but the longest link, CD, is the foundation link. Because of this similarity between these two mechanisms, the fourth inversion is not illustrated here. A drag-link mechanism can produce either a nonuniform output from a uniform input rotation rate or a uniform output from a nonuniform input rotation rate.

Straight-Line Generators

Figures 12 to 15 illustrate examples of classical linkages capable of describing straight lines, a function useful in many different kinds of machines, particularly machine tools. The dimensions of the rigid links are important for the proper functioning of these mechanisms.

Watt's straight-line generator, illustrated in Fig. 12, can describe a short vertical straight line. Equal length links AB and CD are hinged at A and D, respectively. The midpoint E of connecting link BC traces a figure eight pattern over the full mechanism excursion, but a straight line is traced in part of the excursion because point E diverges to the left at the top of the stroke and to the right at the bottom of the stroke. This linkage was used by Scottish instrument maker, James Watt, in a steam-driven beam pump in about 1769, and it was a prominent mechanism in early steam-powered machines.

Scott Russell straight-line generator, shown in Fig. 13, can also describe a straight line. Link AB is hinged at point A and pinned to link CD at point B. Link CD is hinged at point C which restricts it to horizontal oscillating movement.

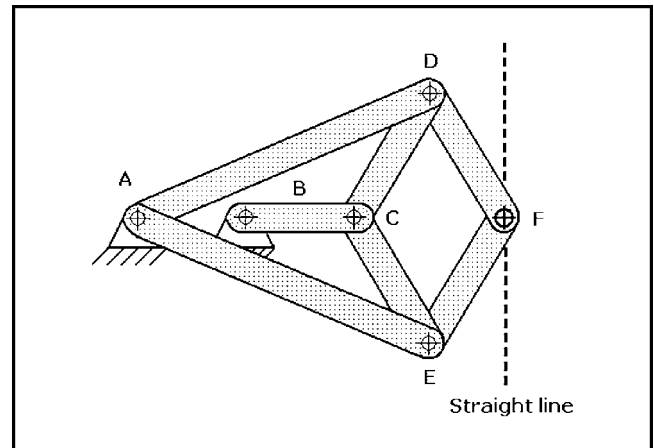


Fig. 14 Peaucellier's straight-line generator: Point F describes a straight line when either link AD or AE acts as the driver.

point F can describe arcs of any radius. However, the linkage can be restricted to tracing straight lines (infinite radiuses) by selecting link lengths for AD and AE. In this figure they are 45 units long. This linkage was invented in 1873 by the French engineer, Captain Charles-Nicolas Peaucellier.

Tchebicheff's straight-line generator, shown in Fig. 15, can also describe a horizontal line. Link CB with E as its midpoint traces a straight horizontal line for most of its transit as links AB and DC are moved to the left and right of center. To describe this straight line, the length of the foundation link AD must be twice the length of link CB. To make this mechanism work as a straight-line generator, CB is 10 units long, AD is 20 units long, and both AB and DC are 25 units long. With these dimensions, link CB will assume a vertical position when it is at the right and left extremes of its travel excursion. This linkage was invented by nineteenth-century Russian mathematician, Pafnuty Tchebicheff or Chebyshev.

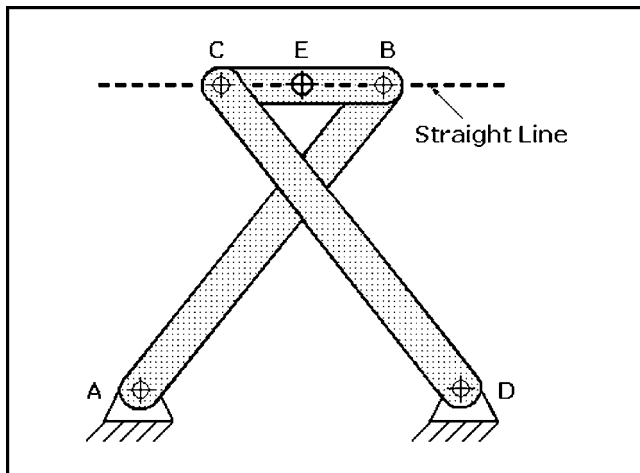


Fig. 15 Tchebicheff's straight-line generator: Point E of link CB describes a straight line when driven by either link AB or DC. Link CB moves into a vertical position at both extremes of its travel.

Rotary/Linear Linkages

Slider-crank mechanism (or a simple crank), shown as Fig. 16, converts rotary to linear motion and vice versa, depending on its application. Link AB is free to rotate 360° around the hinge while link BC oscillates back and forth because point C is hinged to a roller which restricts it to linear motion. Either the slider or the rotating link AB can be the driver.

This mechanism is more familiar as the piston, connecting rod, and crankshaft of an internal combustion engine, as illustrated in Fig. 1. The piston is the slider at C, the connecting rod is link BC, and the crankshaft is link AB. In a four-stroke engine, the piston is pulled down the cylinder by the crankshaft, admitting

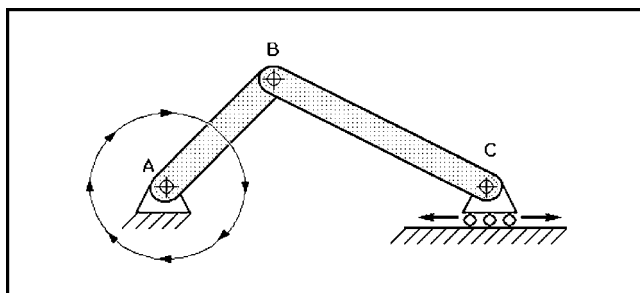


Fig. 16 Slider-crank mechanism: This simple crank converts the 360° rotation of driver link AB into linear motion of link BC, causing the slider at C to reciprocate.

the air-fuel mixture; in the compression stroke the piston is driven back up the cylinder by the crankshaft to compress the air-fuel mixture. However, the roles change in the combustion stroke when the piston drives the crankshaft. Finally, in the exhaust stroke the roles change again as the crankshaft drives the piston back to expel the exhaust fumes.

Scotch-yoke mechanism, pictured in Fig. 17, functions in a manner similar to that of the simple crank mechanism except that its linear output motion is sinusoidal. As wheel A, the driver, rotates, the pin or roller bearing at its periphery exerts torque within the closed yoke B; this causes the attached sliding bar to reciprocate, tracing a sinusoidal waveform. Part a shows the sliding bar when the roller is at 270°, and part b shows the sliding bar when the roller is at 0°.

Rotary-to-linear mechanism, drawn in Fig. 18, converts a uniform rotary motion into an intermittent reciprocating motion. The three teeth of the input rotor contact the steps in the frame or yoke, exerting torque 3 times per revolution, moving the yoke with attached bar. Full linear travel of the yoke is accomplished in 30° of rotor rotation followed by a 30° delay before returning the yoke. The reciprocating cycle is completed 3 times per revolution of the input. The output is that of a step function.

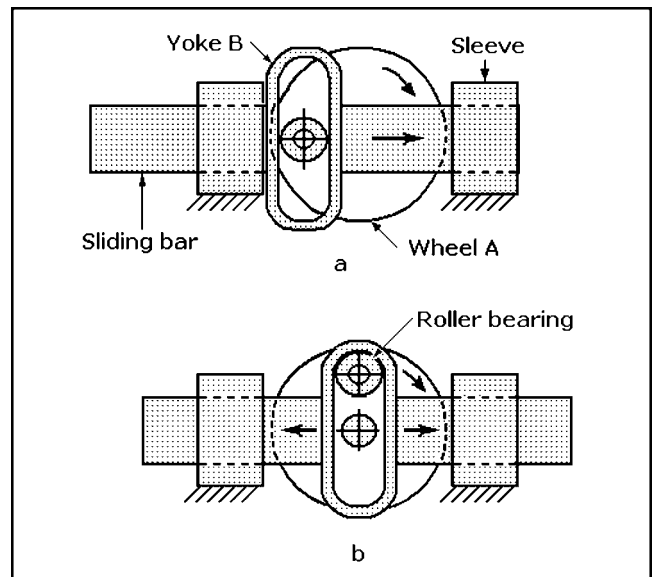


Fig. 17 Scotch-yoke mechanism translates the rotary motion of the wheel with a peripheral roller into reciprocating motion of the yoke with supporting bars as the roller exerts torque within the yoke. The yoke is shown in its left (270°) position in (a) and in its center (0°) position in (b).

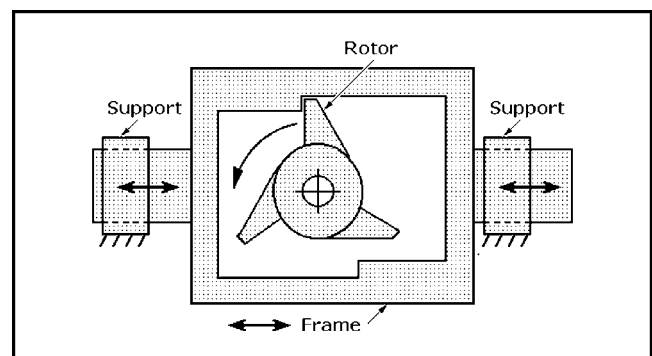


Fig. 18 Rotary-to-linear mechanism converts the uniform rotation of the 3-tooth rotor into a reciprocating motion of the frame and supporting bars. The reciprocating cycle is completed 3 times per rotor revolution.

SPECIALIZED MECHANISMS

Geneva wheel mechanism, illustrated in Fig. 19, is an example of intermittent gearing that converts continuous rotary motion into intermittent rotary motion. Geneva wheel C makes a quarter turn for every turn of lever AB attached to driving wheel A. When pin B on lever AB turns clockwise, it enters one of the four slots of geneva wheel C; the pin moves downward in the slot, applying enough torque to the geneva wheel to turn it counterclockwise $\frac{1}{4}$ revolution before it leaves the slot. As wheel A continues to rotate clockwise, it engages the next three slots in a sequence to complete one geneva wheel rotation. If one of the slots is obstructed, the pin can only move through part of the revolution, in either direction, before it strikes the closed slot, stopping the rotation of the geneva wheel. This mechanism has been used in mechanical windup watches, clocks, and music boxes to prevent overwinding.

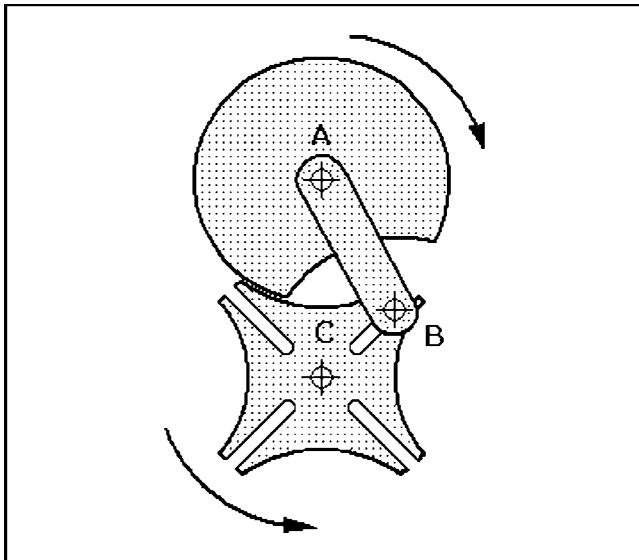


Fig. 19 Geneva wheel escapement mechanism: Pin B at the end of lever AB (attached to wheel A) engages a slot in geneva wheel C as wheel A rotates clockwise. Pin B moves down the slot, providing torque to drive the geneva wheel counterclockwise $\frac{1}{4}$ revolution before it exits the first slot; it then engages the next three slots to drive the geneva wheel through one complete counterclockwise revolution.

Swing-arm quick-return mechanism, drawn as Fig. 20, converts rotary motion into nonuniform reciprocating motion. As drive link AB rotates 360° around pin A, it causes the slider at B to reciprocate up and down along link CD. This, in turn, causes CD to oscillate left and right, describing an arc. Link DE, pinned to D with a rolling slider pinned at E, moves slowly to the right before being returned rapidly to the left.

Whitworth quick-return mechanism, shown as Fig. 21, converts rotary motion to nonuniform reciprocating motion. Drive link AB rotates 360° about pin A causing the slider at B to reciprocate back and forth along link CD; this, in turn, causes link CD to rotate 360° around point C. Link DE is pinned to link CD at D

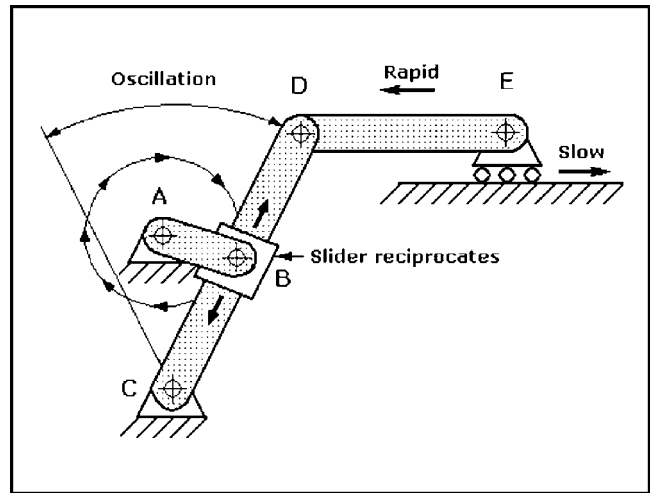


Fig. 20 Swing-arm quick-return mechanism: As drive link AB rotates 360° around A, it causes the slider at B to reciprocate up and down along link CD, causing CD to oscillate through an arc. This motion drives link DE in a reciprocating motion that moves the rolling slider at E slowly to the right before returning it rapidly to the left.

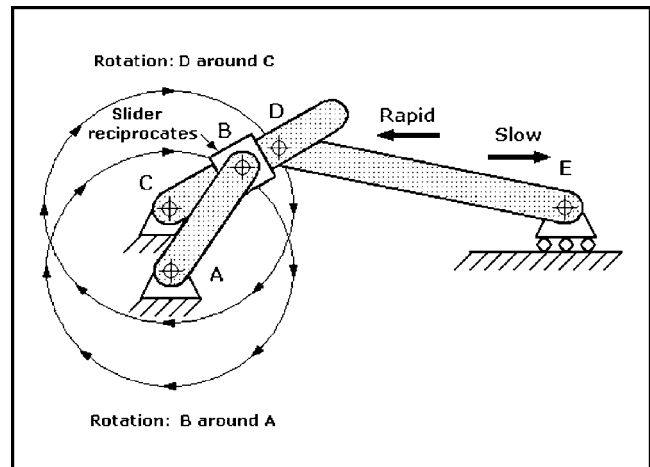


Fig. 21 Whitworth's quick-return mechanism: As drive link AB rotates 360° around A, it causes the slider at B to reciprocate back and forth along link CD, which, in turn causes CD to rotate 360° around C. This motion causes link DE to reciprocate, first moving rolling slider at E slowly to the right before returning it rapidly to the left.

and a rolling slider at E. The slider at E is moved slowly to the right before being returned rapidly to the left. This mechanism, invented in the nineteenth century by English engineer, Joseph Whitworth, has been adapted for shapers, machine tools with moving arms that cut metal from stationary workpieces. A hardened cutting tool attached at the end of the arm (equivalent to point E) advances slowly on the cutting stroke but retracts

rapidly on the backstroke. This response saves time and improves productivity in shaping metal.

Simple ratchet mechanism, drawn as Fig. 22, can only be turned in a counterclockwise direction. The ratchet wheel has many wedge-shaped teeth that can be moved incrementally to turn an oscillating drive lever. As driving lever AB first moves clockwise to initiate counterclockwise movement of the wheel, it drags pawl C pinned at B over one or more teeth while pawl D prevents the wheel from turning clockwise. Then, as lever AB reverses to drive the ratchet wheel counterclockwise, pawl D is released, allowing the wheel to turn in that direction. The amount of backward incremental motion of lever AB is directly proportional to pitch of the teeth: smaller teeth will reduce the degree of rotation while larger teeth will increase them. The contact surfaces of the teeth on the wheel are typically inclined, as shown, so they will not be disengaged if the mechanism is subjected to vibration or shock under load. Some ratchet mechanisms include a spring to hold pawl D against the teeth to assure no clockwise wheel rotation as lever AB is reset.

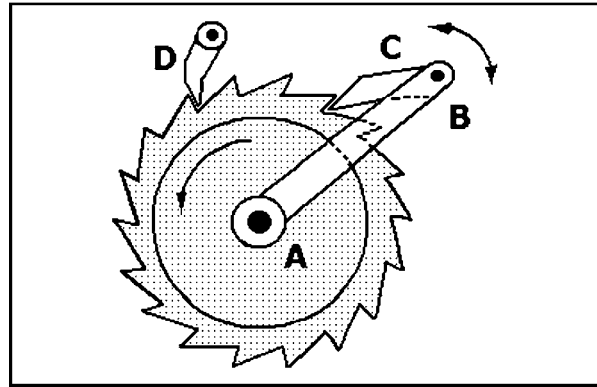


Fig. 22 This ratchet wheel can be turned only in a counterclockwise direction. As driving lever AB moves clockwise, it drags pawl C, pinned at B over one or more teeth while pawl D prevents the wheel from turning clockwise. Then as lever AB reverses to drive the ratchet wheel counterclockwise, pawl D is released allowing the wheel to turn in that direction.

GEARS AND GEARING

A *gear* is a wheel with evenly sized and spaced teeth machined or formed around its perimeter. Gears are used in rotating machinery not only to transmit motion from one point to another, but also for the mechanical advantage they offer. Two or more gears transmitting motion from one shaft to another is called a *gear train*, and *gearing* is a system of wheels or cylinders with meshing teeth. Gearing is chiefly used to transmit rotating motion but can also be adapted to translate reciprocating motion into rotating motion and vice versa.

Gears are versatile mechanical components capable of performing many different kinds of power transmission or motion control. Examples of these are

- Changing rotational speed
- Changing rotational direction
- Changing the angular orientation of rotational motion
- Multiplication or division of torque or magnitude of rotation
- Converting rotational to linear motion, and its reverse
- Offsetting or changing the location of rotating motion

The teeth of a gear can be considered as levers when they mesh with the teeth of an adjoining gear. However, gears can be rotated continuously instead of rocking back and forth through short distances as is typical of levers. A gear is defined by the number of its teeth and its diameter. The gear that is connected to the source of power is called the *driver*, and the one that receives power from the driver is the *driven gear*. It always rotates in a direction opposing that of the driving gear; if both gears have the same number of teeth, they will rotate at the same speed. However, if the number of teeth differs, the gear with the smaller r number of teeth will rotate faster. The size and shape of all gear teeth that are to mesh properly for working contact must be equal.

Figure 23 shows two gears, one with 15 teeth connected at the end of shaft A, and the other with 30 teeth connected at the end of shaft B. The 15 teeth of smaller driving gear A will mesh with 15 teeth of the larger gear B, but while gear A makes one revolution gear B will make only $\frac{1}{2}$ revolution.

The number of teeth on a gear determines its diameter. When two gears with different diameters and numbers of teeth are meshed

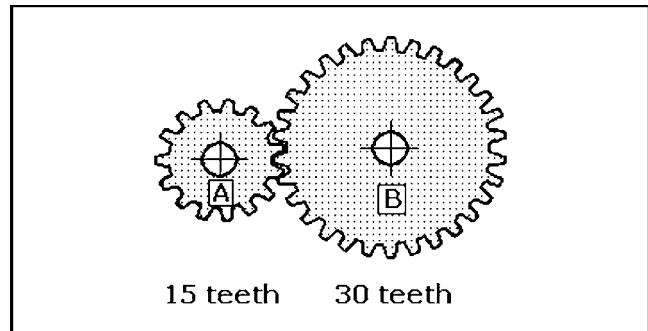


Fig. 23 Gear B has twice as many teeth as gear A, and it turns at half the speed of gear A because gear speed is inversely proportional to the number of teeth on each gear wheel.

together, the number of teeth on each gear determines gear ratio, velocity ratio, distance ratio, and mechanical advantage. In Fig. 23, gear A with 15 teeth is the driving gear and gear B with 30 teeth is the driven gear. The gear ratio GR is determined as:

$$\begin{aligned} \text{GR} &= \frac{\text{number of teeth on driven gear B}}{\text{number of teeth on driving gear A}} \\ &= \frac{30}{15} = \frac{2}{1} \text{ (also written as 2:1)} \end{aligned}$$

The number of teeth in both gears determines the rotary distance traveled by each gear and their angular speed or velocity ratio. The angular speeds of gears are inversely proportional to the numbers of their teeth. Because the smaller driving gear A in Fig. 23 will revolve twice as fast as the larger driven gear B, velocity ratio VR is:

$$\text{VR} = \frac{\text{velocity of driving gear A}}{\text{velocity of driven gear B}} = \frac{2}{1} \text{ (also written as 2:1)}$$

In this example load is represented by driven gear B with 30 teeth and the effort is represented by driving gear A with 15 teeth. The distance moved by the load is twice that of the effort. Using the general formula for mechanical advantage MA:

$$MA = \frac{\text{load}}{\text{effort}} = \frac{30}{15} = 2$$

Simple Gear Trains

A gear train made up of multiple gears can have several drivers and several driven gears. If the train contains an odd number of gears, the output gear will rotate in the same direction as the input gear, but if the train contains an even number of gears, the output gear will rotate opposite that of the input gear. The number of teeth on the intermediate gears does not affect the overall velocity ratio, which is governed purely by the number of teeth on the first and last gear.

In simple gear trains, high or low gear ratios can only be obtained by combining large and small gears. In the simplest basic gearing involving two gears, the driven shaft and gear revolves in a direction opposite that of the driving shaft and gear. If it is desired that the two gears and shafts rotate in the same direction, a third *idler gear* must be inserted between the driving gear and the driven gear. The idler revolves in a direction opposite that of the driving gear.

A simple gear train containing an idler is shown in Fig. 24. Driven idler gear B with 20 teeth will revolve 4 times as fast counterclockwise as driving gear A with 80 teeth turning clockwise. However, gear C, also with 80 teeth, will only revolve one turn clockwise for every four revolutions of idler gear B, making the velocities of both gears A and C equal except that gear C turns in the same direction as gear A. In general, the velocity ratio of the first and last gears in a train of simple gears is not changed by the number of gears inserted between them.

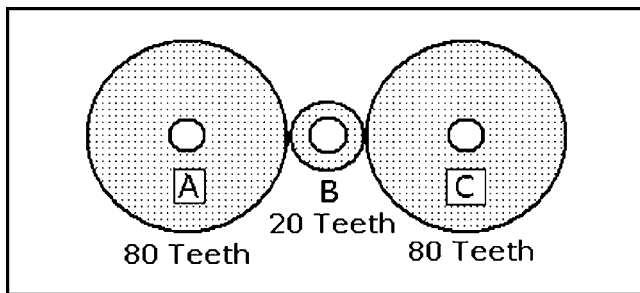


Fig. 24 Gear train: When gear A turns once clockwise, gear B turns four times counter clockwise, and gear wheel C turns once clockwise. Gear B reverses the direction of gear C so that both gears A and C turn in the same direction with no change in the speed of gear C.

Compound Gear Trains

More complex compound gear trains can achieve high and low gear ratios in a restricted space by coupling large and small gears on the same axle. In this way gear ratios of adjacent gears can be multiplied through the gear train. Figure 25 shows a set of compound gears with the two gears B and D mounted on the middle shaft. Both rotate at the same speed because they are fastened together. If gear A (80 teeth) rotates at 100 rpm clockwise, gear B (20 teeth) turns at 400 rpm counterclockwise because of its velocity ratio of 1 to 4. Because gear D (60 teeth) also turns at 400 rpm and its velocity ratio is 1 to 3 with respect to gear C

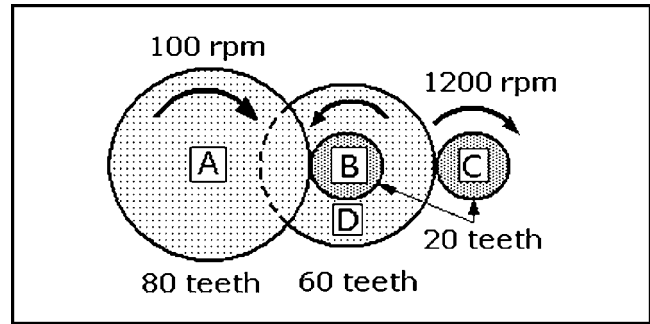


Fig. 25 Compound gears: Two gears B and D are mounted on a central shaft and they turn at the same speed. If gear A rotates at 100 rpm clockwise, gears B and D turn counterclockwise at 400 rpm, and gear C, driven by gear D, turns clockwise at 1200 rpm.

(20 teeth), gear C will turn at 1200 rpm clockwise. The velocity ratio of a compound gear train can be calculated by multiplying the velocity ratios for all pairs of meshing gears. For example, if the driving gear has 45 teeth and the driven gear has 15 teeth, the velocity ratio is $^{15}/_{45} = 1/3$.

Gear Classification

All gears can be classified as either external gears or internal or annual gears:

- *External gears* have teeth on the outside surface of the disk or wheel.
- *Internal or annual gears* have teeth on the inside surface of a ring or cylinder.

Spur gears are cylindrical external gears with teeth that are cut straight across the edge of the disk or wheel parallel to the axis of rotation. The spur gears shown in Fig. 26a are the simplest gears. They normally translate rotating motion between two parallel shafts. An *internal* or *annual* gear, as shown in Fig. 26b, is a variation of the spur gear except that its teeth are cut on the inside of a ring or flanged wheel rather than on the outside. Internal gears usually drive or are driven by a pinion. The disadvantage of a simple spur gear is its tendency to produce thrust that can misalign other meshing gears along their respective shafts, thus reducing the face widths of the meshing gears and reducing their mating surfaces.

Rack gears, as the one shown in Fig. 26c, have teeth that lie in the same plane rather than being distributed around a wheel. This gear configuration provides straight-line rather than rotary motion. A rack gear functions like a gear with an infinite radius.

Pinions are small gears with a relatively small number of teeth which can be mated with rack gears.

Rack and pinion gears, shown in Fig. 26c, convert rotary motion to linear motion; when mated together they can transform the rotation of a pinion into reciprocating motion, or vice versa. In some systems, the pinion rotates in a fixed position and engages the rack which is free to move; the combination is found in the steering mechanisms of vehicles. Alternatively, the rack is fixed while the pinion rotates as it moves up and down the rack: Funicular railways are based on this drive mechanism; the driving pinion on the rail car engages the rack positioned between the two rails and propels the car up the incline.

Bevel gears, as shown in Fig. 26d, have straight teeth cut into conical circumferences which mate on axes that intersect, typically at right angles between the input and output shafts. This class of gears includes the most common straight and spiral bevel gears as well as miter and hypoid gears.

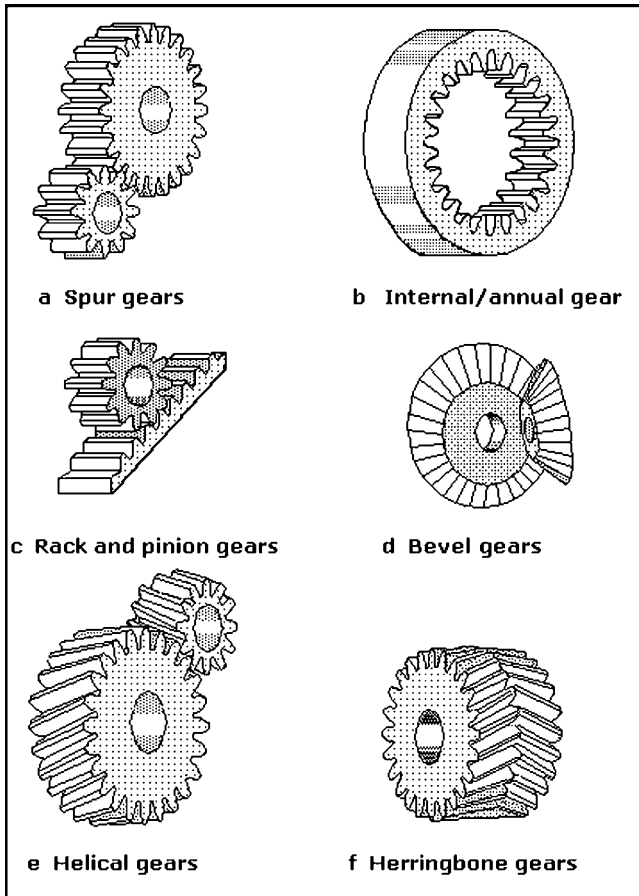


Fig. 26 Gear types: Eight common types of gears and gear pairs are shown here.

Straight bevel gears are the simplest bevel gears. Their straight teeth produce instantaneous line contact when they mate. These gears provide moderate torque transmission, but they are not as smooth running or quiet as spiral bevel gears because the straight teeth engage with full-line contact. They permit medium load capacity.

Spiral bevel gears have curved oblique teeth. The spiral angle of curvature with respect to the gear axis permits substantial tooth overlap. Consequently, the teeth engage gradually and at least two teeth are in contact at the same time. These gears have lower tooth loading than straight bevel gears and they can turn up to 8 times faster. They permit high load capacity.

Miter gears are mating bevel gears with equal numbers of teeth used between rotating input and output shafts with axes that are 90° apart.

Hypoid gears are helical bevel gears used when the axes of the two shafts are perpendicular but do not intersect. They are commonly used to connect driveshafts to rear axles of automobiles, and are often incorrectly called *spiral gearing*.

Helical gears are external cylindrical gears with their teeth cut at an angle rather than parallel to the axis. A simple helical gear, as shown in Fig. 26e, has teeth that are offset by an angle with respect to the axis of the shaft so that they spiral around the shaft in a helical manner. Their offset teeth make them capable of smoother and quieter action than spur gears, and they are capable of driving heavy loads because the teeth mesh at an acute angle rather than at 90°. When helical gear axes are parallel they are called parallel helical gears, and when they are at right angles they are called helical gears. Herringbone and worm gears are based on helical gear geometry.

Herringbone or double helical gears, as shown in Fig. 26f, are helical gears with V-shaped right-hand and left-hand helix angles side by side across the face of the gear. This geometry neutralizes axial thrust from helical teeth.

Worm gears, also called *screw gears*, are other variations of helical gearing. A worm gear has a long, thin cylindrical form with one or more continuous helical teeth that mesh with a helical gear. The teeth of the worm gear slide across the teeth of the driven gear rather than exerting a direct rolling pressure as do the teeth of helical gears. Worm gears are widely used to transmit rotation, at significantly lower speeds, from one shaft to another at a 90° angle.

Face gears have straight tooth surfaces, but their axes lie in planes perpendicular to shaft axes. They are designed to mate with instantaneous point contact. These gears are used in right-angle drives, but they have low load capacities.

Practical Gear Configurations

Isometric drawing Fig. 27 shows a *special planetary gear configuration*. The external driver spur gear (lower right) drives the outer ring spur gear (center) which, in turn, drives three internal planet spur gears; they transfer torque to the driven gear (lower left). Simultaneously, the central planet spur gear produces a summing motion in the pinion gear (upper right) which engages a rack with a roller follower contacting a radial disk cam (middle right).

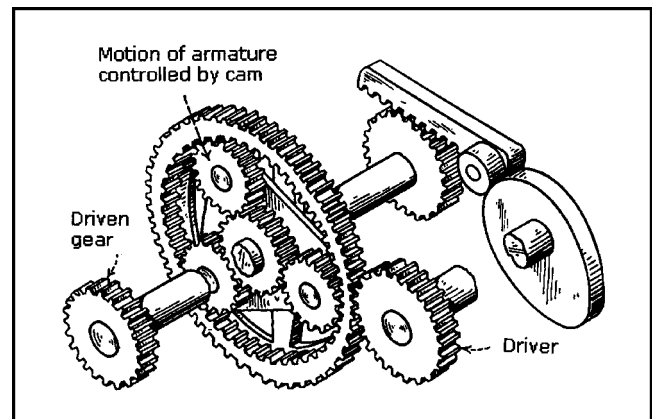


Fig. 27 A special planetary-gear mechanism: The principal of relative motion of mating gears illustrated here can be applied to spur gears in a planetary system. The motion of the central planet gear produces the motion of a summing gear.

Isometric drawing Fig. 28 shows a *unidirectional drive*. The output shaft B rotates in the same direction at all times, regardless of the rotation of the input shaft A. The angular velocity of output shaft B is directly proportional to the angular velocity of input shaft A. The spur gear C on shaft A has a face width that is twice as wide as the faces on spur gears F and D, which are mounted on output shaft B. Spur gear C meshes with idler E and with spur gear D. Idler E meshes with the spur gears C and F. Output shaft B carries two free-wheel disks, G and H, which are oriented unidirectionally.

When input shaft A rotates clockwise (bold arrow), spur gear D rotates counterclockwise and it idles around free-wheel disk H. Simultaneously, idler E, which is also rotating counterclockwise, causes spur gear F to turn clockwise and engage the rollers on free-wheel disk G. Thus, shaft B is made to rotate clockwise. On the other hand, if the input shaft A turns counterclockwise

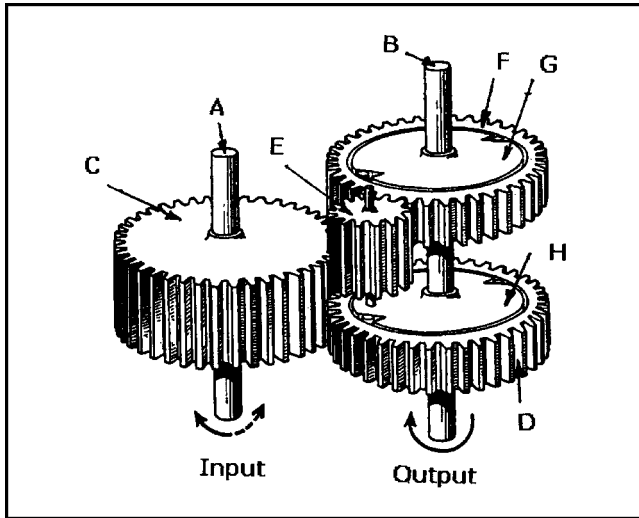


Fig. 28 The output shaft of this unidirectional drive always rotates in the same direction regardless of the direction of rotation of the input shaft.

(dotted arrow), spur gear F will idle while spur gear D engages free-wheel disk H, which drives shaft B so that it continues to rotate clockwise.

Gear Tooth Geometry

The geometry of gear teeth, as shown in Fig. 29, is determined by pitch, depth, and pressure angle.

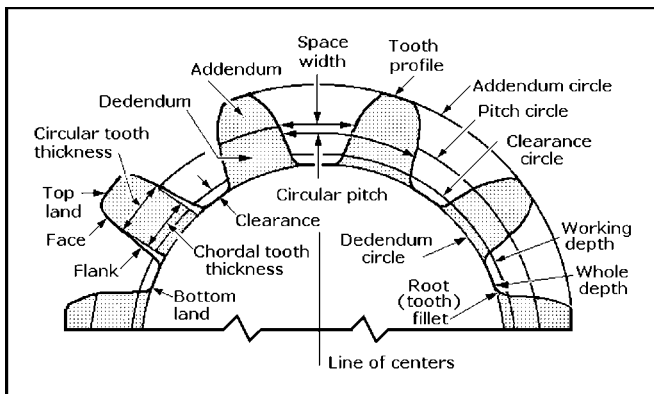


Fig. 29 Gear-tooth geometry.

Gear Terminology

addendum: The radial distance between the *top land* and the *pitch circle*. This distance is measured in inches or millimeters.

addendum circle: The circle defining the outer diameter of the gear.

circular pitch: The distance along the pitch circle from a point on one tooth to a corresponding point on an adjacent tooth. It is also the sum of the *tooth thickness* and the *space width*. This distance is measured in inches or millimeters.

clearance: The radial distance between the *bottom land* and the *clearance circle*. This distance is measured in inches or millimeters.

contact ratio: The ratio of the number of teeth in contact to the number of teeth not in contact.

dedendum: The radial distance between the *pitch circle* and the *dedendum circle*. This distance is measured in inches or millimeters.

dedendum circle: The theoretical circle through the *bottom lands* of a gear.

depth: A number standardized in terms of pitch. Full-depth teeth have a *working depth* of $2/P$. If the teeth have equal *addenda* (as in standard interchangeable gears), the addendum is $1/P$. Full-depth gear teeth have a larger contact ratio than stub teeth, and their working depth is about 20 percent more than stub gear teeth. Gears with a small number of teeth might require *undercutting* to prevent one interfering with another during engagement.

diametral pitch (P): The ratio of the number of teeth to the *pitch diameter*. A measure of the coarseness of a gear, it is the index of tooth size when U.S. units are used, expressed as teeth per inch.

pitch: A standard pitch is typically a whole number when measured as a *diametral pitch (P)*. *Coarse pitch gears* have teeth larger than a diametral pitch of 20 (typically 0.5 to 19.99). *Fine-pitch gears* usually have teeth of diametral pitch greater than 20. The usual maximum fineness is 120 diametral pitch, but involute-tooth gears can be made with diametral pitches as fine as 200, and cycloidal tooth gears can be made with diametral pitches to 350.

pitch circle: A theoretical circle upon which all calculations are based.

pitch diameter: The diameter of the *pitch circle*, the imaginary circle that rolls without slipping with the pitch circle of the mating gear, measured in inches or millimeters.

pressure angle: The angle between the *tooth profile* and a line perpendicular to the *pitch circle*, usually at the point where the pitch circle and the tooth profile intersect. Standard angles are 20° and 25° . It affects the force that tends to separate mating gears. A high pressure angle decreases the *contact ratio*, but it permits the teeth to have higher capacity and it allows gears to have fewer teeth without *undercutting*.

Gear Dynamics Terminology

backlash: The amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle. It is the shortest distance between the noncontacting surfaces of adjacent teeth.

gear efficiency: The ratio of output power to input power taking into consideration power losses in the gears and bearings and from windage and the churning of the gear lubricant.

gear power: A gear's load and speed capacity. It is determined by gear dimensions and type. Helical and helical-type gears have capacities to approximately 30,000 hp, spiral bevel gears to about 5000 hp, and worm gears to about 750 hp.

gear ratio: The number of teeth in the larger gear of a pair divided by the number of teeth in the *pinion* gear (the smaller gear of a pair). It is also the ratio of the speed of the pinion to the speed of the gear. In reduction gears, the ratio of input speed to output speed.

gear speed: A value determined by a specific pitchline velocity. It can be increased by improving the accuracy of the gear teeth and the balance of all rotating parts.

undercutting: The recessing in the bases of gear tooth flanks to improve clearance.

PULLEYS AND BELTS

Pulleys and belts transfer rotating motion from one shaft to another. Essentially, pulleys are gears without teeth that depend on the frictional forces of connecting belts, chains, ropes, or cables to transfer torque. If both pulleys have the same diameter, they will rotate at the same speed. However, if one pulley is larger than the other, mechanical advantage and velocity ratio are gained. As with gears, the velocities of pulleys are inversely proportional to their diameters. A large drive pulley driving a smaller driven pulley by means of a belt or chain is shown in Fig. 30. The smaller pulley rotates faster than the larger pulley in the same direction as shown in Fig. 30a. If the belt is crossed, as shown in Fig. 30b, the smaller pulley also rotates faster than the larger pulley, but its rotation is in the opposite direction.

A familiar example of belt and pulley drive can be seen in automotive cooling fan drives. A smooth pulley connected to the engine crankshaft transfers torque to a second smooth pulley coupled to the cooling fan with a reinforced rubber endless belt. Before reliable direct-drive industrial electric motors were developed, a wide variety of industrial machines equipped with smooth pulleys of various diameters were driven by endless leather belts from an overhead driveshaft. Speed changes were achieved by switching the belt to pulleys of different diameters on the same

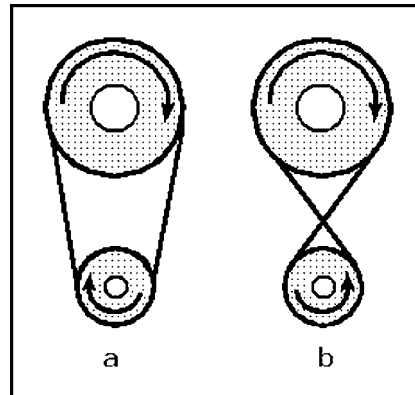


Fig. 30 Belts on pulleys: With a continuous belt both pulleys rotate in the same direction (a), but with a crossed belt both pulleys rotate in opposite directions (b).

machine. The machines included lathes and milling machines, circular saws in sawmills, looms in textile plants, and grinding wheels in grain mills. The source of power could have been a water wheel, windmill, or a steam engine.

SPROCKETS AND CHAINS

Sprockets and chains offer another method for transferring rotating motion from one shaft to another where the friction of a drive belt would be insufficient to transfer power. The speed relationships between sprockets of different diameters coupled by chains are the same as those between pulleys of different diameters coupled by belts, as shown in Fig. 30. Therefore, if the chains are crossed, the sprockets will rotate in different directions. Bicycles

have sprocket and chain drives. The teeth on the sprockets mesh with the links on the chains. Powered winches on large ships act as sprockets because they have teeth that mate with the links of heavy chain for raising anchors. Another example can be seen in tracked equipment including bulldozers, cranes, and military tanks. The flexible treads have teeth that mate with teeth on driving sprockets that propel these machines.

CAM MECHANISMS

A *cam* is a mechanical component capable of transmitting motion to a follower by direct contact. In a cam mechanism, the cam is the driver and the driven member is called the *follower*. The follower can remain stationary, translate, oscillate, or rotate. The general form of a plane cam mechanism is illustrated in the kinematic diagram Fig. 31. It consists of two shaped members A and B with smooth, round, or elongated contact surfaces connected to a third body C. Either body A or body B can be the driver, while the other body is the follower. These shaped bodies can be replaced by an equivalent mechanism. Points 1 and 2 are pin-jointed at the centers of curvature of the contacting surfaces. If any change is made in the relative positions of bodies A and B, points 1 and 2 are shifted, and the links of the equivalent mechanisms have different lengths.

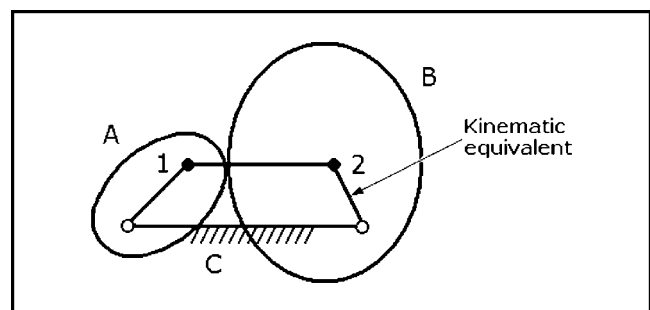


Fig. 31 Basic cam mechanism and its kinematic equivalent. Points 1 and 2 are centers of curvature of the contact point.

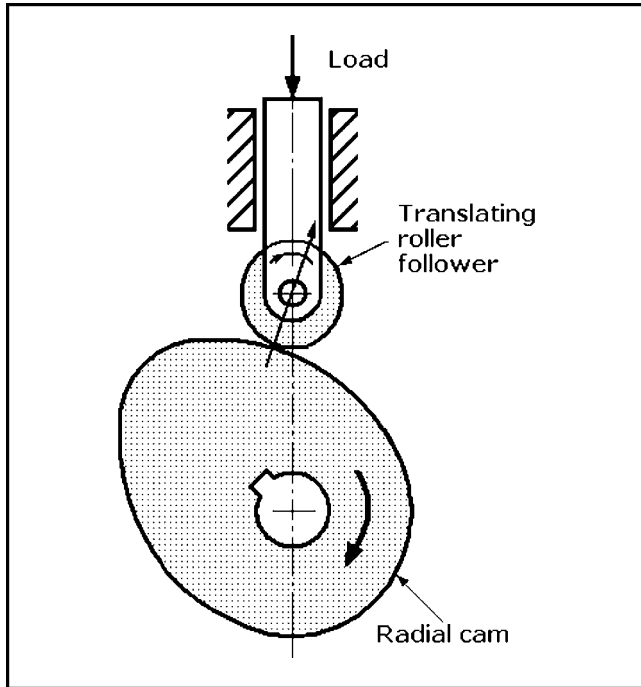


Fig. 32 Radial open cam with a translating roller follower. The roller is kept in contact with the cam by the mass of the load.

A widely used open radial-cam mechanism is shown in Fig. 32. The roller follower is the most common follower used in these mechanisms because it can transfer power efficiently between the cam and follower by reducing friction and minimizing wear between them. The arrangement shown here is called a *gravity constraint cam*; it is simple and effective and can be used with rotating disk or end cams if the weight of the follower system is enough to keep it in constant contact with the cam profile. However, in most practical cam mechanisms, the cam and follower are constrained at all operating speeds by preloaded compression springs. Cams can be designed by three methods:

- Shaping the cam body to some known curve, such as a spiral, parabola, or circular arc
- Designing the cam mathematically to determine follower motion and then plotting the tabulated data to form the cam
- Drawing the cam profile freehand using various drafting curves

The third method is acceptable only if the cam motion is intended for low speeds that will permit the use of a smooth, “bumpless” curve. In situations where higher loads, mass, speed, or elasticity of the members are encountered, a detailed study must be made of both the dynamic aspects of the cam curve and the accuracy of cam fabrication.

Many different kinds of machines include cams, particularly those that operate automatically such as printing presses, textile looms, gear-cutters, and screw machines. Cams open and close the valves in internal combustion engines, index cutting tools on machine tools, and operate switches and relays in electrical control equipment. Cams can be made in an infinite variety of shapes from metal or hard plastic. Some of the most important cams will be considered here. The possible applications of mechanical cams are still unlimited despite the introduction of *electronic cams* that mimic mechanical cam functions with appropriate computer software.

Classification of Cam Mechanisms

Cam mechanisms can be classified by their input/output motions, the configuration and arrangement of the follower, and the shape of the cam. Cams can also be classified by the kinds of motions made by the follower and the characteristics of the cam profile. The possible kinds of input/output motions of cam mechanisms with the most common disk cams are shown in Figs. 33a to e; they are examples of rotating disk cams with translating followers. By contrast, Fig. 33f shows a follower arm with a roller that swings or oscillates in a circular arc with respect to the follower hinge as the cam rotates. The follower configurations in Figs. 33a to d are named according to their characteristics: a *knife-edge*; b, e, and f *roller*; c *flat-faced*; and d *spherical-faced*. The face of the flat follower can also be oblique with respect to the cam. The follower is an element that moves either up and down or side to side as it follows the contour of the cam.

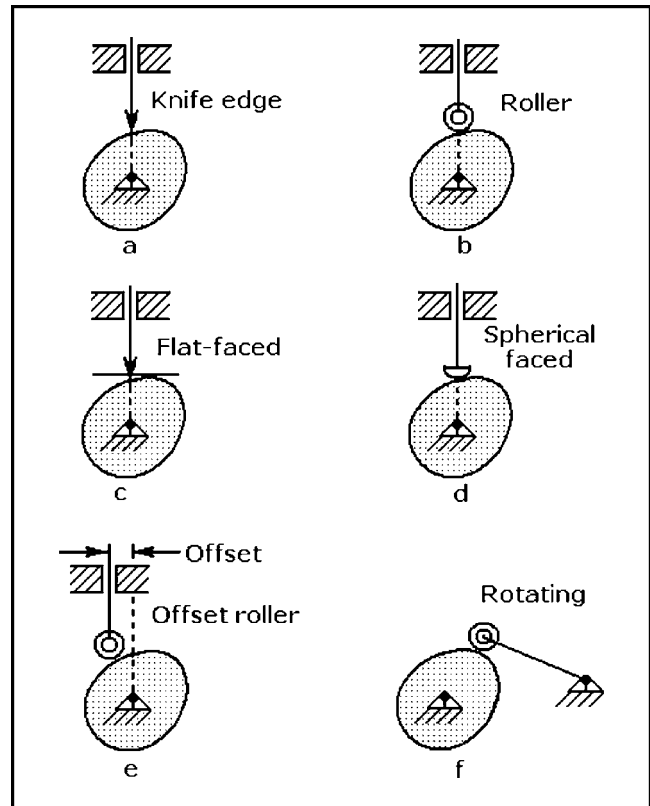


Fig. 33 Cam configurations: Six different configurations of radial open cams and their followers.

There are two basic types of follower: *in-line* and *offset*. The centerline of the in-line follower passes through the centerline of the camshaft. Figures 33a to d show five followers that move in a plane perpendicular to the axis of rotation of the camshaft. By contrast, the centerline of the offset follower, as illustrated in Fig. 33e, does not pass through the centerline of the camshaft. The amount of offset is the horizontal distance between the two centerlines. Follower offset reduces the side thrust introduced by the roller follower. Figure 33f illustrates a translating or swing-arm rotating follower that must be constrained to maintain contact with the cam profile.

The most common rotating disk or plate cams can be made in a variety of shapes including offset round, egg-shaped, oval, and cardioid or heart-shaped. Most cams are mounted on a rotating shaft. The cam and follower must be constrained at all operating

speeds to keep them in close contact throughout its cycle if a cam mechanism is to function correctly. Followers are typically spring-loaded to maintain constant contact with the shaped surface of the cam, but gravity constraint is still an option.

If it is anticipated that a cam mechanism will be subjected to severe shock and vibration, a *grooved disk cam*, as shown in Fig. 34, can be used. The cam contour is milled into the face of a disk so that the roller of the cam follower will be confined and continuously constrained within the side walls of the groove throughout the cam cycle. The groove confines the follower roller during the entire cam rotation. Alternatively, the groove can be milled on the outer circumference of a cylinder or barrel to form a *cylindrical or barrel cam*, as shown in Fig. 35. The follower of this cam can translate or oscillate. A similar groove can also be milled around the conical exterior surface of a *grooved conical cam*.

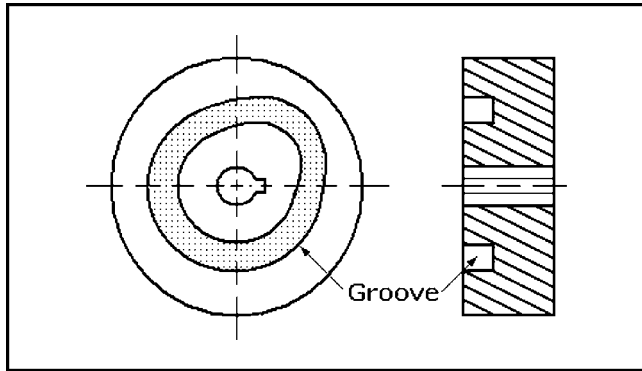


Fig. 34 Grooved cam made by milling a contoured cam groove into a metal or plastic disk. A roller follower is held within the grooved contour by its depth, eliminating the need for spring-loading.

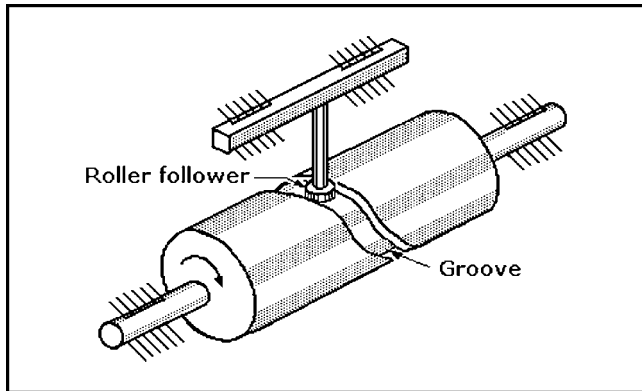


Fig. 35 Cylindrical or barrel cam: A roller follower tracks the groove precisely because of the deep contoured groove milled around the circumference of the rotating cylinder.

By contrast, the barrel-shaped *end cam*, shown in Fig. 36, has a contour milled on one end. This cam is usually rotated, and its follower can also either translate or oscillate, but the follower system must be carefully controlled to exercise the required constraint because the follower roller is not confined by a groove. Another distinct form of cam is the *translating cam*, as shown in Fig. 37. It is typically mounted on a bed or carrier that moves back and forth in a linear reciprocal motion under a stationary vertical translating follower, usually with a roller. However, the cam can also be mounted so that it remains stationary while a follower system moves in a linear reciprocal motion over the limited range of the cam.

The unusual dual-rotary cam configuration shown in Fig. 38 is a *constant-diameter cam*; it consists of two identical disk cams

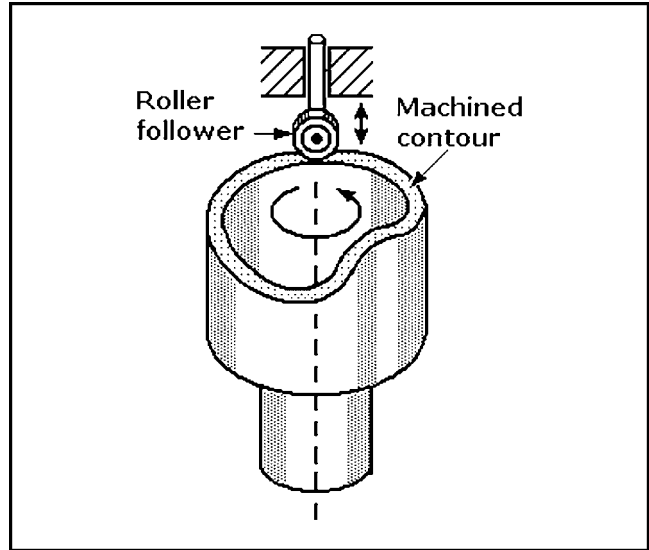


Fig. 36 End cam: A roller follower tracks a cam contour machined at the end of this rotating cylindrical cam.

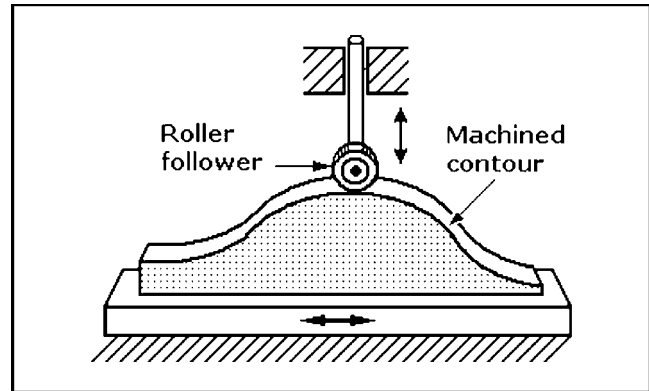


Fig. 37 Translating cam: A roller follower either tracks the reciprocating motion of the cam profile or is driven back and forth over a stationary cam profile.

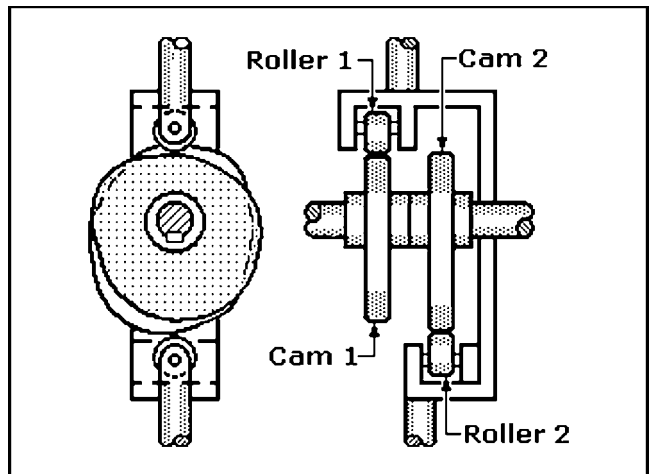


Fig. 38 Constant-diameter cam: Two identical cams, 1 and 2, are separated on the same shaft and offset at an angle that provides a virtual constant diameter. Cam 1 with roller follower 1 is the functioning cam, and cam 2 with roller follower 2 constrains cam 1 to smooth its motion.

with followers mounted a fixed distance apart on a common shaft, but the cams are offset so that if superimposed their contours form a virtual circle of constant diameter. Cam 1 is the functional cam while cam 2 acts as a constraint, effectively canceling out the irregular motion that occurs with a single rotary cam and follower.

The motions of the followers of all of these cam mechanisms can be altered to obtain a different sequence by changing the contour of the cam profile. The timing of the sequence of disk and cylinder cams can be changed by altering the rotational speed of their camshafts. The timing of the sequence of the translation cam can be changed by altering the rate of reciprocal motion of the bed on which it is mounted on its follower system. The rotation of the follower roller does not influence the motion of any of the cam mechanisms.

Cam Terminology

Figure 39 illustrates the nomenclature for a radial open disk cam with a *roller follower* on a plate cam.

base circle: The circle with the shortest radius from the cam center to any part of the cam profile.

cam profile: The outer surface of a disk cam as it was machined.

follower travel: For a *roller follower* of a disk cam it is the vertical distance of follower travel measured at the center point of the roller as it travels from the *base circle* to the *cam profile*.

motion events: When a cam rotates through one cycle, the follower goes through rises, dwells, and returns. A *rise* is the motion of the follower away from the cam center; a *dwell* occurs when the follower is resting; and a *return* is the motion of the follower toward the cam center.

pitch curve: For a *roller follower* of a disk cam it is the path generated by the center point of the roller as the follower is rotated around a stationary plate cam.

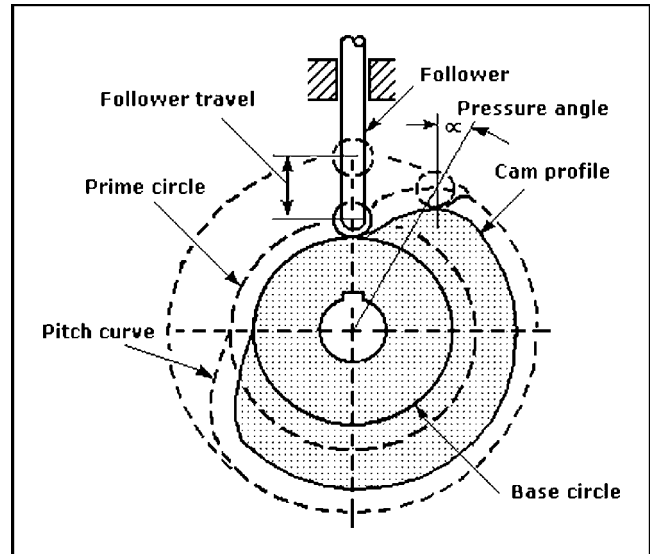


Fig. 39 Cam nomenclature: This diagram identifies the industry-accepted technical terms for cam features.

pressure angle: For a *roller follower* of a disk cam it is the angle at any point between the normal to the pitch curve and the instantaneous direction of follower motion. This angle is important in cam design because it indicates the steepness of the cam profile.

prime circle (reference circle): For a *roller follower* of a disk cam it is the circle with the shortest radius from the cam center to the pitch curve.

stroke or throw: The longest distance or widest angle through which the follower moves or rotates.

working curve: The working surface of a cam that contacts the follower. For a *roller follower* of a plate cam it is the path traced by the center of the roller around the cam profile.

CLUTCH MECHANISMS

A *clutch* is defined as a coupling that connects and disconnects the driving and driven parts of a machine; an example is an engine and a transmission. Clutches typically contain a driving shaft and a driven shaft, and they are classed as either externally or internally controlled. *Externally controlled clutches* can be controlled either by friction surfaces or components that engage or mesh positively. *Internally controlled clutches* are controlled by internal mechanisms or devices; they are further classified as *overload*, *overriding*, and *centrifugal*. There are many different schemes for a driving shaft to engage a driven shaft.

Externally Controlled Friction Clutches

Friction-Plate Clutch. This clutch, shown in Fig. 40, has a control arm, which when actuated, advances a sliding plate on the driving shaft to engage a mating rotating friction plate on the same shaft; this motion engages associated gearing that drives the driven shaft. When reversed, the control arm disengages the sliding plate. The friction surface can be on either plate, but is typically only on one.

Cone Clutch. A clutch operating on the same principle as the friction-plate clutch except that the control arm advances a cone on the driving shaft to engage a mating rotating friction cone on the same shaft; this motion also engages any associated gearing that drives the driven shaft. The friction surface can be on either cone but is typically only on the sliding cone.

Expanding Shoe Clutch. This clutch is similar to the friction-plate clutch except that the control arm engages linkage that forces several friction shoes radially outward so they engage the inner surface of a drum on or geared to the driven shaft.

Externally Controlled Positive Clutches

Jaw Clutch. This clutch is similar to the plate clutch except that the control arm advances a sliding jaw on the driving shaft to make positive engagement with a mating jaw on the driven shaft.

Other examples of externally controlled positive clutches are the *planetary transmission clutch* consisting essentially of a sun gear keyed to a driveshaft, two planet gears, and an outer driven

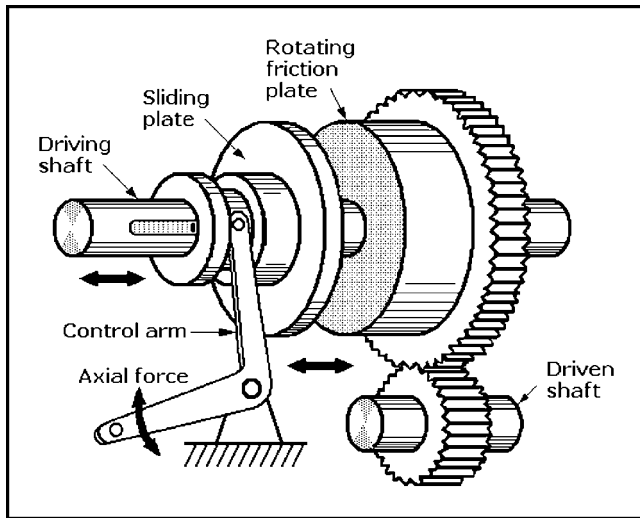


Fig. 40 Friction plate clutch: When the left sliding plate on the driving shaft is clamped by the control arm against the right friction plate idling on the driving shaft, friction transfers the power of the driving shaft to the friction plate. Gear teeth on the friction plate mesh with a gear mounted on the driven shaft to complete the transfer of power to the driven mechanism. Clutch torque depends on the axial force exerted by the control arm.

ring gear. The *pawl and ratchet clutch* consists essentially of a pawl-controlled driving ratchet keyed to a driven gear.

Internally Controlled Clutches

Internally controlled clutches can be controlled by springs, torque, or centrifugal force. The *spring and ball radial-detent clutch*, for example, disengages when torque becomes excessive, allowing the driving gear to continue rotating while the driveshaft stops rotating. The *wrapped-spring clutch* consists of two separate rotating hubs joined by a coil spring. When driven in the right direction, the spring tightens around the hubs increasing the friction grip. However, if driven in the opposite direction the spring relaxes, allowing the clutch to slip.

The *expanding-shoe centrifugal clutch* is similar to the externally controlled *expanding shoe clutch* except that the friction shoes are pulled in by springs until the driving shaft attains a pre-set speed. At that speed centrifugal force drives the shoes radially outward so that they contact the drum. As the driveshaft rotates faster, pressure between the shoes and drum increases, thus increasing clutch torque.

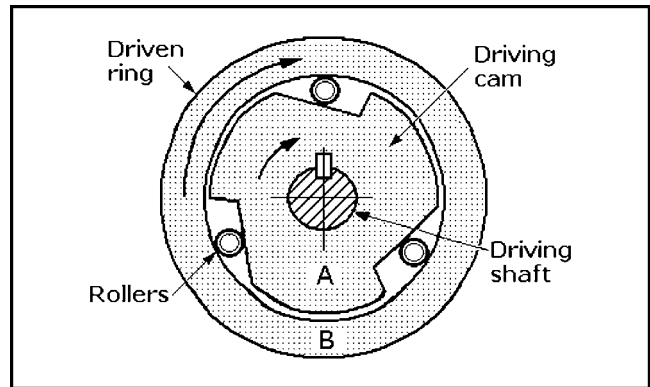


Fig. 41 Overrunning clutch: As driving cam A revolves clockwise, the rollers in the wedge-shaped gaps between cam A and outer ring B are forced by friction into those wedges and are held there; this locks ring B to cam A and drives it clockwise. However, if ring B is turned counterclockwise, or is made to revolve clockwise faster than cam A, the rollers are freed by friction, the clutch slips, and no torque is transmitted.

The *overrunning or overriding clutch*, as shown in Fig. 41, is a specialized form of a cam mechanism, also called a *cam and roller clutch*. The inner driving cam A has wedge-shaped notches on its outer rim that hold rollers between the outer surface of A and the inner cylindrical surfaces of outer driven ring B. When driving cam A is turning clockwise, frictional forces wedge the rollers tightly into the notches to lock outer driven ring B in position so it also turns in a clockwise direction. However, if driven ring B is reversed or runs faster clockwise than driving cam A (when it is either moving or immobile) the rollers are set free, the clutch will slip and no torque is transmitted. Some versions of this clutch include springs between the cam faces and the rollers to ensure faster clutching action if driven ring B attempts to drive driving cam A by overcoming residual friction. A version of this clutch is the basic free-wheel mechanism that drives the rear axle of a bicycle.

Some low-cost, light-duty overrunning clutches for one-direction-only torque transmission intersperse cardioid-shaped pellets called *sprags* with cylindrical rollers. This design permits cylindrical internal drivers to replace cammed drivers. The sprags bind in the concentric space between the inner driver and the outer driven ring if the ring attempts to drive the driver. The torque rating of the clutch depends on the number of sprags installed. For acceptable performance a minimum of three sprags, equally spaced around the circumference of the races, is usually necessary.

GLOSSARY OF COMMON MECHANICAL TERMS

acceleration: The time rate of change of velocity of a body. It is always produced by force acting on a body. Acceleration is measured as feet per second per second (ft/s^2) or meters per second per second (m/s^2).

component forces: The individual forces that are the equivalent of the resultant.

concurrent forces: Forces whose lines of action or directions pass through a common point or meet at a common point.

crank: A side link that revolves relative to the frame.

crank-rocker mechanism: A four-bar linkage characterized by the ability of the shorter side link to revolve through 360° while the opposing link rocks or oscillates.

couple: Two equal and opposite parallel forces that act at diametrically opposite points on a body to cause it to rotate around a point or an axis through its center.

displacement: Distance measured from a fixed reference point in a specified direction; it is a vector quantity; units are measured in inches, feet, miles, centimeters, meters, and kilometers.

double-crank mechanism: A four-bar linkage characterized by the ability of both of its side links to oscillate while the shortest link (opposite the foundation link) can revolve through 360°.

dynamics: The study of the forces that act on bodies not in equilibrium, both balanced and unbalanced; it accounts for the masses and accelerations of the parts as well as the external forces acting on the mechanisms: It is a combination of *kinetics* and *kinematics*.

efficiency of machines: The ratio of a machine's output divided by its input is typically expressed as a percent. There are energy or power losses in all moving machinery caused primarily by friction. This causes inefficiency, so a machine's output is always less than its input; both output and input must be expressed in the same units of power or energy. This ratio, always a fraction, is multiplied by 100 to obtain a percent. It can also be determined by dividing the machine's mechanical advantage by its velocity ratio and multiplying that ratio by 100 to get a percent.

energy: A physical quantity present in three-dimensional space in which forces can act on a body or particle to bring about physical change; it is the capacity for doing work. Energy can take many forms, including mechanical, electrical, electromagnetic, chemical, thermal, solar, and nuclear. Energy and work are related and measured in the same units: foot-pounds, ergs, or joules; it cannot be destroyed, but it can be wasted.

- *Kinetic energy* is the kind of energy a body has when it is in motion. Examples are a rolling soccer ball, a speeding automobile, or a flying airplane.
- *Potential energy* is the kind of energy that a body has because of its position or state. Examples are a concrete block poised at the edge of a building, a shipping container suspended above ground by a crane, or a roadside bomb.

equilibrium: In mechanics, a condition of balance or static equilibrium between opposing forces. An example is when there are equal forces at both ends of a seesaw resting on a *fulcrum*.

force: Strength or energy acting on a body to push or pull it; it is required to produce acceleration. Except for gravitation, one body cannot exert a force on another body unless the two are in contact. The Earth exerts a force of attraction on bodies, whether they are in contact or not. Force is measured in poundals (lb-ft/s²) or newtons (kg-m/s²).

fulcrum: A pivot point or edge about which objects are free to rotate.

kinematic chain: A combination of links and pairs without a fixed link.

kinematics: The study of the motions of bodies without considering how the variables of force and mass influence the motion. It is described as the geometry of motion.

kinetics: The study of the effects of external forces including gravity upon the motions of physical bodies.

linear motion: Motion in a straight line. An example is when a car is driving on a straight road.

lever: A simple machine that uses opposing torque around a fulcrum to perform work.

link: A rigid body with pins or fasteners at its ends to connect it to other rigid bodies so it can transmit a force or motion. All machines contain at least one link, either in a fixed position relative to the Earth or capable of moving the machine and the link during the motion; this link is the *frame* or *fixed link* of the machine.

linkages: Mechanical assemblies consisting of two or more levers connected to produce a desired motion. They can also be mechanisms consisting of rigid bodies and lower *pairs*.

machine: An assembly of mechanisms or parts or mechanisms capable of transmitting force, motion, and energy from a power source; the objective of a machine is to overcome some form of resistance to accomplish a desired result. There are two functions of machines: (1) the transmission of relative motion and (2) the transmission of force; both require that the machine be strong and rigid. While both machines and mechanisms are combinations of rigid bodies capable of definite relative motions, machines transform energy, but mechanisms do not. A *simple machine* is an elementary mechanism. Examples are the lever, wheel and axle, pulley, inclined plane, wedge, and screw.

machinery: A term generally meaning various combinations of machines and mechanisms.

mass: The quantity of matter in a body indicating its inertia. Mass also initiates gravitational attraction. It is measured in ounces, pounds, tons, grams, and kilograms.

mechanical advantage: The ratio of the load (or force W) divided by the effort (or force F) exerted by an operator. If friction is considered in determining mechanical advantage, or it has been determined by the actual testing, the ratio W/F is the mechanical advantage MA. However, if the machine is assumed to operate without friction, the ratio W/F is the theoretical mechanical advantage TA. Mechanical advantage and velocity ratio are related.

mechanics: A branch of physics concerned with the motions of objects and their response to forces. Descriptions of mechanics begin with definitions of such quantities as acceleration, displacement, force, mass, time, and velocity.

mechanism: In mechanics, it refers to two or more rigid or resistant bodies connected together by joints so they exhibit definite relative motions with respect to one another. Mechanisms are divided into two classes:

- *Planar:* Two-dimensional mechanisms whose relative motions are in one plane or parallel planes.
- *Spatial:* Three-dimensional mechanisms whose relative motions are not all in the same or parallel planes.

moment of force or torque: The product of the force acting to produce a turning effect and the perpendicular distance of its line of action from the point or axis of rotation. The perpendicular distance is called the *moment arm* or the *lever arm torque*. It is measured in pound-inches (lb-in.), pound-feet (lb-ft), or newton-meters (N-m).

moment of inertia: A physical quantity giving a measure of the *rotational inertia* of a body about a specified axis of rotation; it depends on the mass, size, and shape of the body.

nonconcurrent forces: Forces whose lines of action do not meet at a common point.

noncoplanar forces: Forces that do not act in the same plane.

oscillating motion: Repetitive forward and backward circular motion such as that of a clock pendulum.

pair: A joint between the surfaces of two rigid bodies that keeps them in contact and relatively movable. It might be as simple as a pin, bolt, or hinge between two links or as complex as a universal joint between two links. There are two kinds of pairs in mechanisms classified by the type of contact between the two bodies of the pair: *lower pairs* and *higher pairs*.

- Lower pairs are *surface-contact pairs* classed either as *revolute* or *prismatic*. Examples: a hinged door is a revolute pair and a sash window is a prismatic pair.
- Higher pairs include *point, line, or curve pairs*. Examples: paired rollers, cams and followers, and meshing gear teeth.

power: The time rate of doing work. It is measured in foot-pounds per second (ft-lb/s), foot-pounds per minute (ft-lb/min), horsepower, watts, kilowatts, newton-meters/s, ergs/s, and joules/s.

reciprocating motion: Repetitive back and forth linear motion as that of a piston in an internal combustion engine.

rotary motion: Circular motion as in the turning of a bicycle wheel.

resultant: In a system of forces, it is the single force equivalent of the entire system. When the resultant of a system of forces is zero, the system is in equilibrium.

skeleton outline: A simplified geometrical line drawing showing the fundamentals of a simple machine devoid of the actual details of its construction. It gives all of the geometrical information needed for determining the relative motions of the main links. The relative motions of these links might be complete circles, semicircles, or arcs, or even straight lines.

statics: The study of bodies in equilibrium, either at rest or in uniform motion.

torque: An alternative name for *moment of force*.

velocity: The time rate of change with respect to distance. It is measured in feet per second (ft/s), feet per minute (ft/min), meters per second (m/s), or meters per minute (m/min).

velocity ratio: A ratio of the distance movement of the effort divided by the distance of movement of the load per second for a machine. This ratio has no units.

weight: The force on a body due to the gravitational attraction of the Earth; weight $W = \text{mass } n \times \text{acceleration } g$ due to the Earth's gravity; mass of a body is constant but g , and therefore W vary slightly over the Earth's surface.

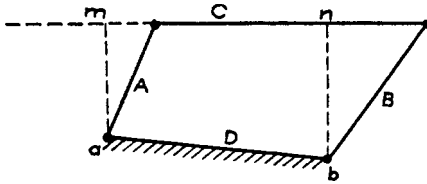
work: The product of force and distance: the distance an object moves in the direction of force. Work is not done if the force exerted on a body fails to move that body. Work, like energy, is measured in units of ergs, joules, or foot-pounds.

CHAPTER 5

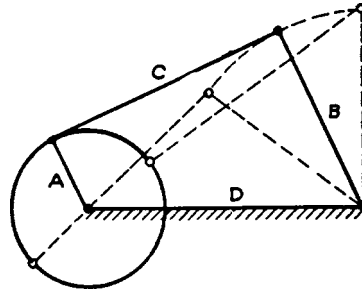
LINKAGES: DRIVES AND MECHANISMS

FOUR-BAR LINKAGES AND TYPICAL INDUSTRIAL APPLICATIONS

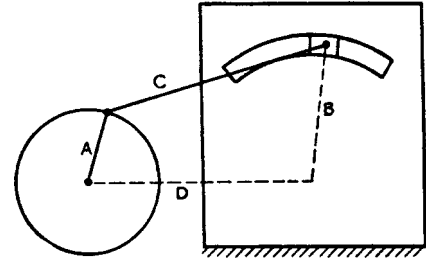
All mechanisms can be broken down into equivalent four-bar linkages. They can be considered to be the basic mechanism and are useful in many mechanical operations.



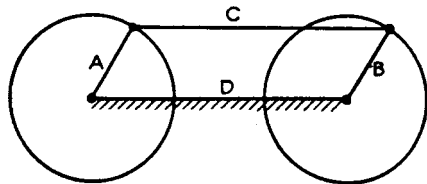
FOUR-BAR LINKAGES—Two cranks, a connecting rod and a line between the fixed centers of the cranks make up the basic four-bar linkage. Cranks can rotate if A is smaller than B or C or D . Link motion can be predicted.



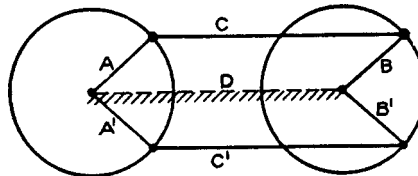
CRANK AND ROCKER—the following relations must hold for its operation:
 $A + B + C > D$; $A + D + B > C$;
 $A + C - B < D$, and $C - A + B > D$.



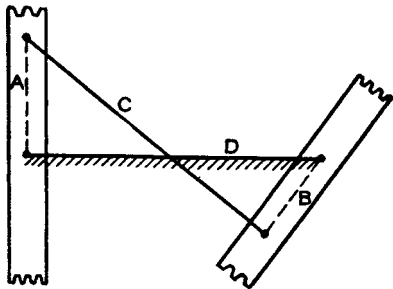
FOUR-BAR LINK WITH SLIDING MEMBER—One crank is replaced by a circular slot with an effective crank distance of B .



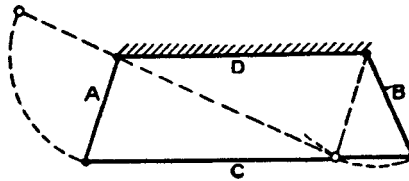
PARALLEL CRANK FOUR-BAR—Both cranks of the parallel crank four-bar linkage always turn at the same angular speed, but they have two positions where the crank can-not be effective.



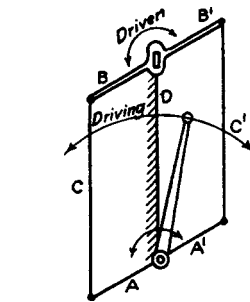
DOUBLE PARALLEL CRANK—This mechanism avoids a dead center position by having two sets of cranks at 90° advancement. The connecting rods are always parallel.



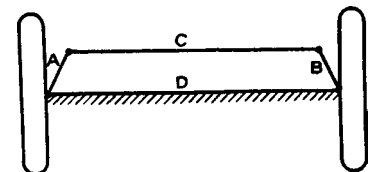
NON-PARALLEL EQUAL CRANK—The centrodes are formed as gears for passing dead center and they can replace ellipticals.



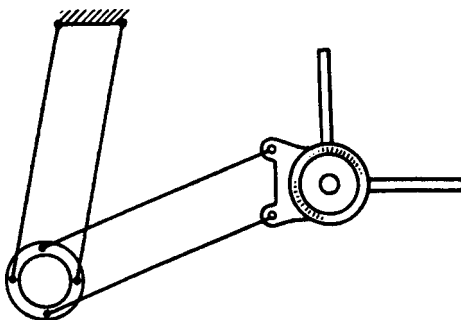
SLOW MOTION LINK—As crank A is rotated upward it imparts motion to crank B . When A reaches its dead center position, the angular velocity of crank B decreases to zero.



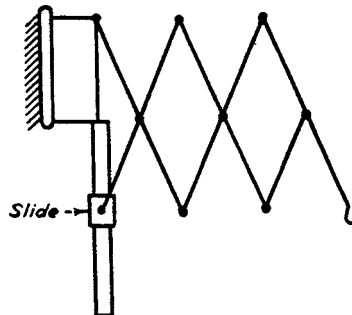
PARALLEL CRANK—Steam control linkage assures equal valve openings.



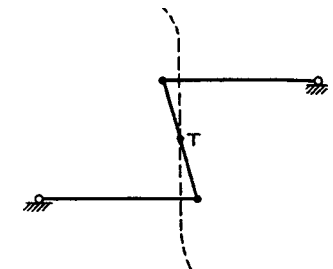
TRAPAZOIDAL LINKAGE—This linkage is not used for complete rotation but can be used for special control. The inside moves through a larger angle than the outside with normals intersecting on the extension of a rear axle in a car.



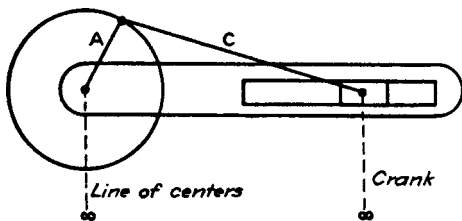
DOUBLE PARALLEL CRANK MECHANISM—This mechanism forms the basis for the universal drafting machine.



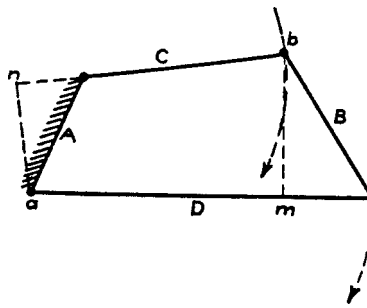
ISOSCELES DRAG LINKS—This "lazy-tong" device is made of several isosceles links; it is used as a movable lamp support.



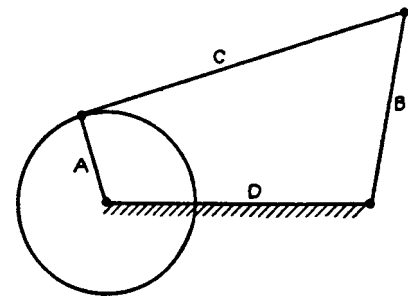
WATT'S STRAIGHT-LINE MECHANISM—Point T describes a line perpendicular to the parallel position of the cranks.



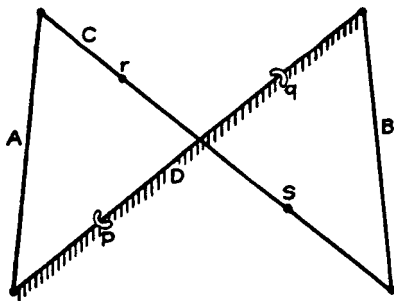
STRAIGHT SLIDING LINK—This is the form in which a slide is usually used to replace a link. The line of centers and the crank *B* are both of infinite length.



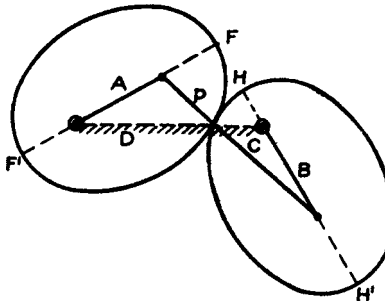
DRAG LINK—This linkage is used as the drive for slotter machines. For complete rotation: $B > A + D - C$ and $B < D + C - A$.



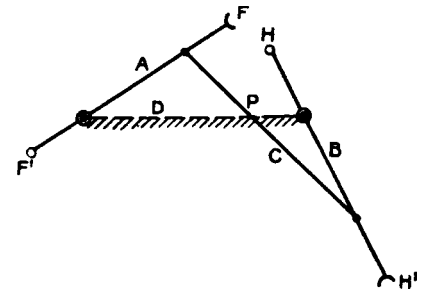
ROTATING CRANK MECHANISM—This linkage is frequently used to change a rotary motion to a swinging movement.



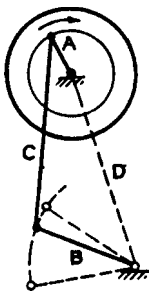
NON-PARALLEL EQUAL CRANK—If crank *A* has a uniform angular speed, *B* will vary.



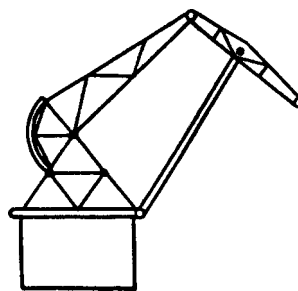
ELLIPTICAL GEARS—They produce the same motion as non-parallel equal cranks.



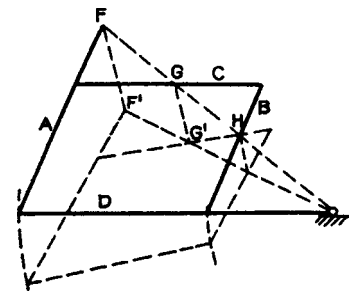
NON-PARALLEL EQUAL CRANK—It is the same as the first example given but with crossover points on its link ends.



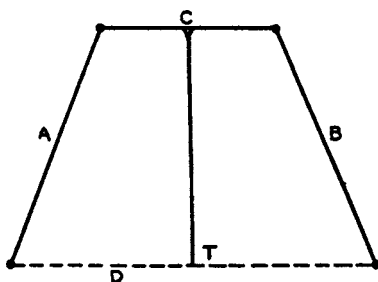
TREADLE DRIVE—This four-bar linkage is used in driving grinding wheels and sewing machines.



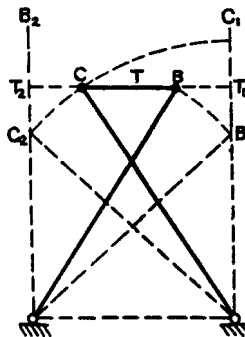
DOUBLE LEVER MECHANISM—This slewing crane can move a load in a horizontal direction by using the D-shaped portion of the top curve.



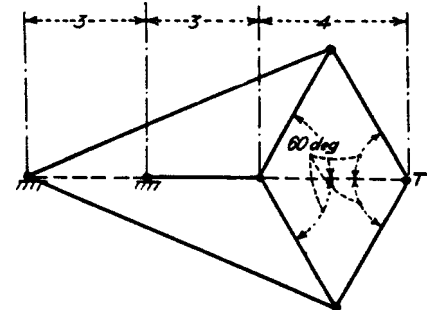
PANTOGRAPH—The pantograph is a parallelogram in which lines through *F*, *G* and *H* must always intersect at a common point.



ROBERT'S STRAIGHT-LINE MECHANISM—The lengths of cranks *A* and *B* should not be less than $0.6 D$; *C* is one half of *D*.



TCHEBICHEFF'S—Links are made in proportion: $AB = CD = 20$, $AD = 16$, $BC = 8$.



PEAUCELLIER'S CELL—When proportioned as shown, the tracing point *T* forms a straight line perpendicular to the axis.

SEVEN LINKAGES FOR TRANSPORT MECHANISMS

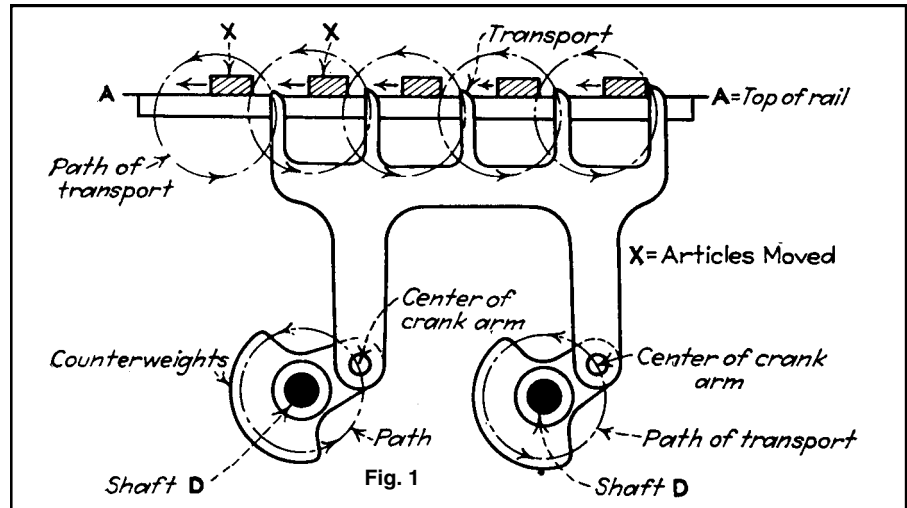


Fig. 1 In this design a rotary action is used. The shafts *D* rotate in unison and also support the main moving member. The shafts are carried in the frame of the machine and can be connected by either a link, a chain and sprocket, or by an intermediate idler gear between two equal gears keyed on the shafts. The rail *A-A* is fixed rigidly on the machine. A pressure or friction plate can hold the material against the top of the rail and prevent any movement during the period of rest.

Transport mechanisms generally move material. The motion, although unidirectional, gives an intermittent advancement to the material being conveyed. The essential characteristic of such a motion is that all points in the main moving members follow similar and equal paths. This is necessary so that the members can be subdivided into sections with projecting parts. The purpose of the projections is to push the articles during the forward motion of the material being transported. The transport returns by a different path from the one it follow in its advancement, and the material is left undisturbed until the next cycle begins. During this period of rest, while the transport is returning to its starting position, various operations can be performed sequentially. The selection of the

particular transport mechanism best suited to any situation depends, to some degree, on the arrangement that can be obtained for driving the materials and the path desired. A slight amount of overtravel is always required so that the projection on the transport can clear the material when it is going into position for the advancing stroke.

The designs illustrated here have been selected from many sources and are typical of the simplest solutions of such problems. The paths, as indicated in these illustrations, can be varied by changes in the cams, levers, and associated parts. Nevertheless, the customary cut-and-try method might still lead to the best solution.

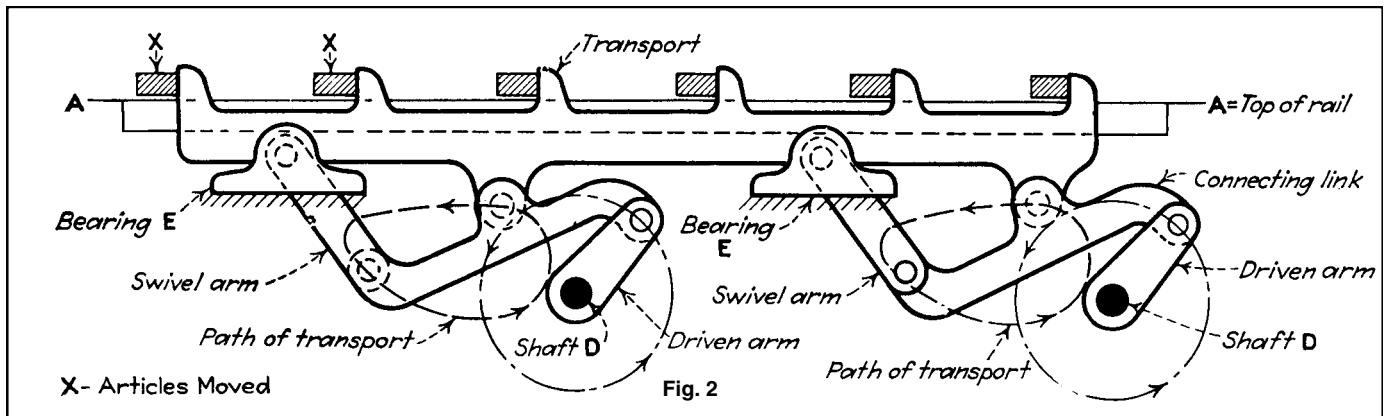


Fig. 2 Here is a simple form of linkage that imparts a somewhat "egg-shaped" motion to the transport. The forward stroke is almost a straight line. The transport is carried on the connecting links. As in the design of Fig. 1, the shafts *D* are driven in unison and are supported in the frame of the machine. Bearings *E* are also supported by the frame of the machine and the rail *A-A* is fixed.

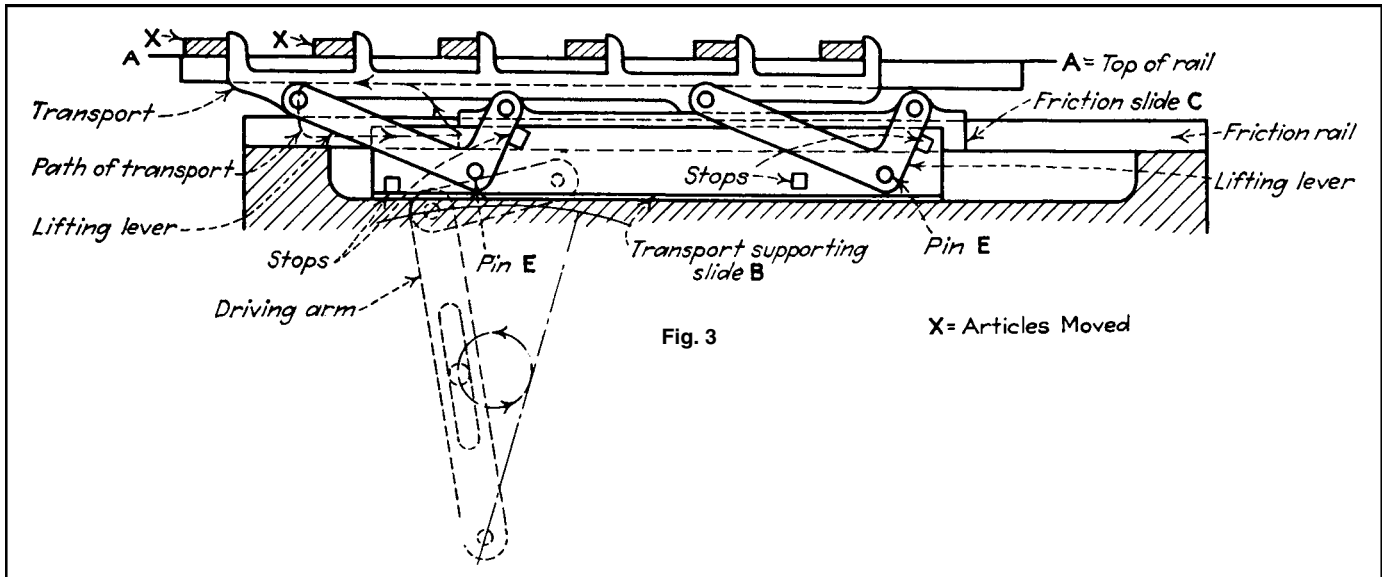


Fig. 3 In another type of action, the forward and return strokes are accomplished by a suitable mechanism, while the raising and lowering is imparted by a friction slide. Thus it can be seen that as the transport supporting slide *B* starts to move to the left, the friction slide *C*, which rests on the friction rail, tends to remain at rest. As a result, the lifting lever starts to turn in a clockwise direction. This motion raises the transport which remains in its raised position against stops until the return stroke starts. At that time the reverse action begins. An adjustment should be provided to compensate for the friction between the slide and its rail. It can readily be seen that this motion imparts a long straight path to the transport.

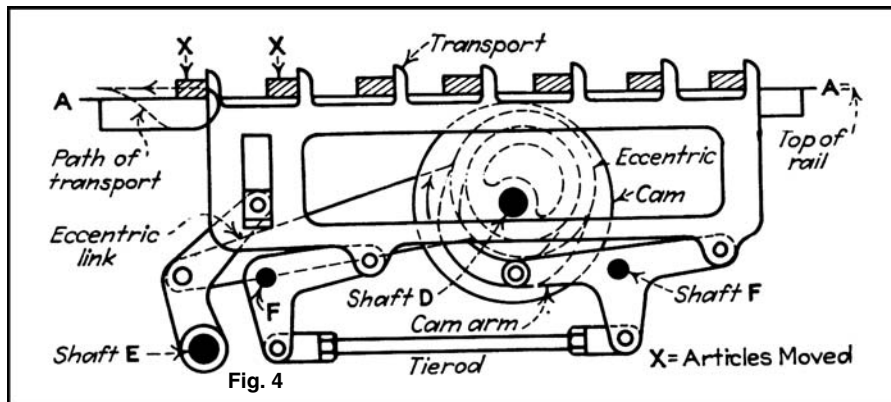


Fig. 4 This drawing illustrates an action in which the forward motion is imparted by an eccentric while the raising and lowering of the transport is accomplished by a cam. The shafts, *F*, *E*, and *D* are positioned by the frame of the machine. Special bell cranks support the transport and are interconnected by a tierod.

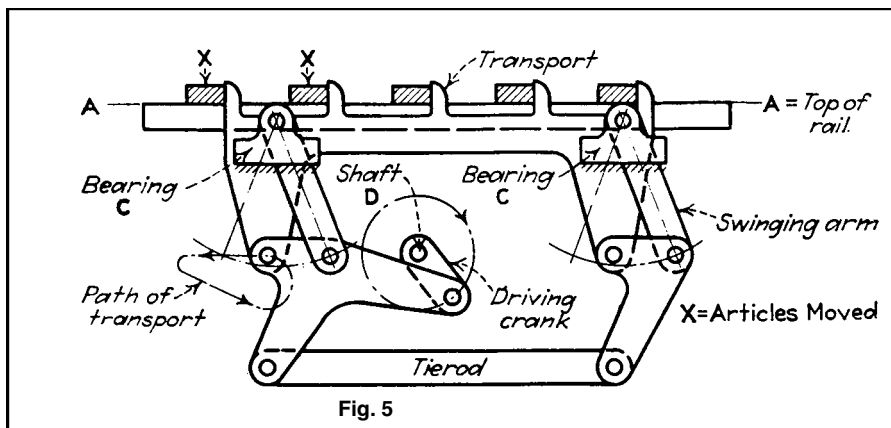


Fig. 5 This is another form of transport mechanism based on a link motion. The bearings *C* are supported by the frame as is the driving shaft *D*.

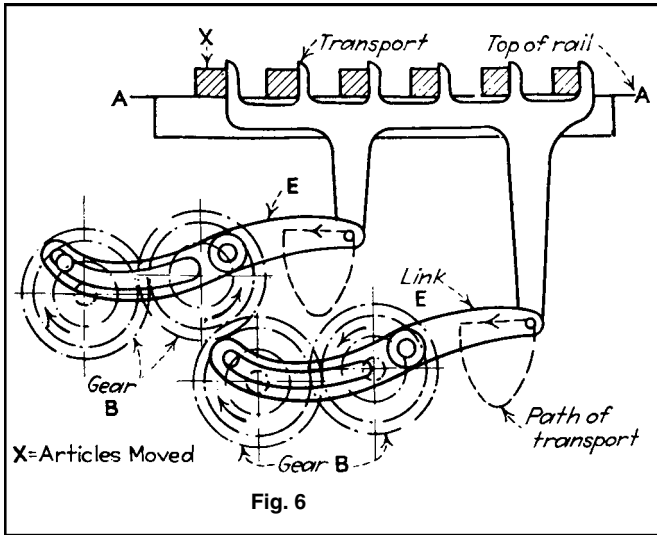


Fig. 6 An arrangement of interconnected gears with equal diameters that will impart a transport motion to a mechanism. The gear and link mechanism imparts both the forward motion and the raising and lowering motions. The gear shafts are supported in the frame of the machine.

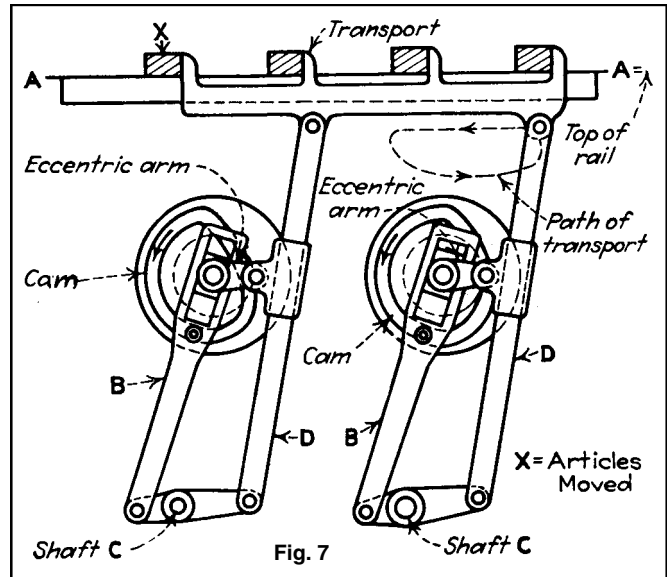


Fig. 7 In this transport mechanism, the forward and return strokes are accomplished by the eccentric arms, while the vertical motion is performed by the cams.

FIVE LINKAGES FOR STRAIGHT-LINE MOTION

These linkages convert rotary to straight-line motion without the need for guides.

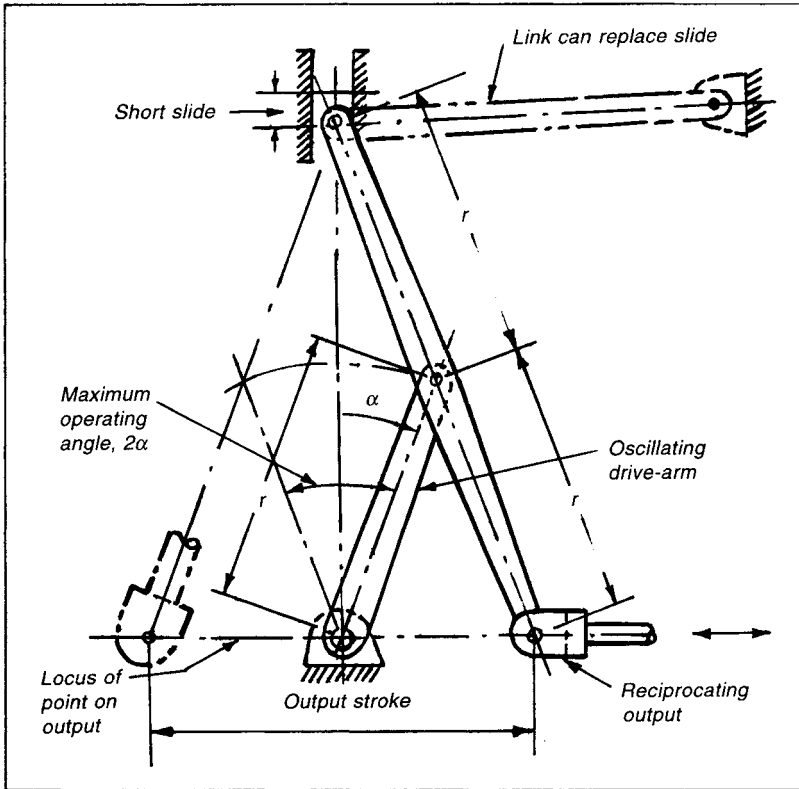


Fig. 1 An Evans' linkage has an oscillating drive-arm that should have a maximum operating angle of about 40° . For a relatively short guideway, the reciprocating output stroke is large. Output motion is on a true straight line in true harmonic motion. If an exact straight-line motion is not required, a link can replace the slide. The longer this link, the closer the output motion approaches that of a true straight line. If the link-length equals the output stroke, deviation from straight-line motion is only 0.03% of the output stroke.

Fig. 2 A simplified Watt's linkage generates an approximate straight-line motion. If the two arms are of equal length, the tracing point describes a symmetrical figure 8 with an almost straight line throughout the stroke length. The straightest and longest stroke occurs when the connecting-link length is about two-thirds of the stroke, and arm length is 1.5 times the stroke length. Offset should equal half the connecting-link length. If the arms are unequal, one branch of the figure-8 curve is straighter than the other. It is straightest when a/b equals $(\text{arm } 2)/(\text{arm } 1)$.

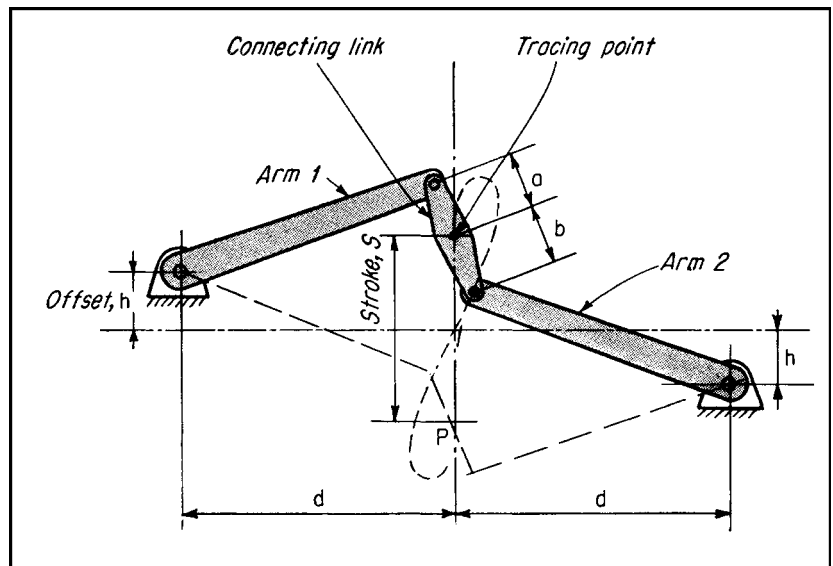


Fig. 3 Four-bar linkage produces an approximately straight-line motion. This arrangement provides motion for the stylus on self-registering measuring instruments. A comparatively small drive displacement results in a long, almost-straight line.

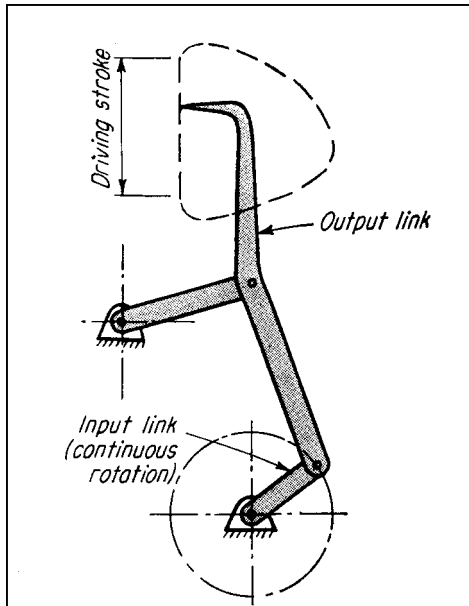
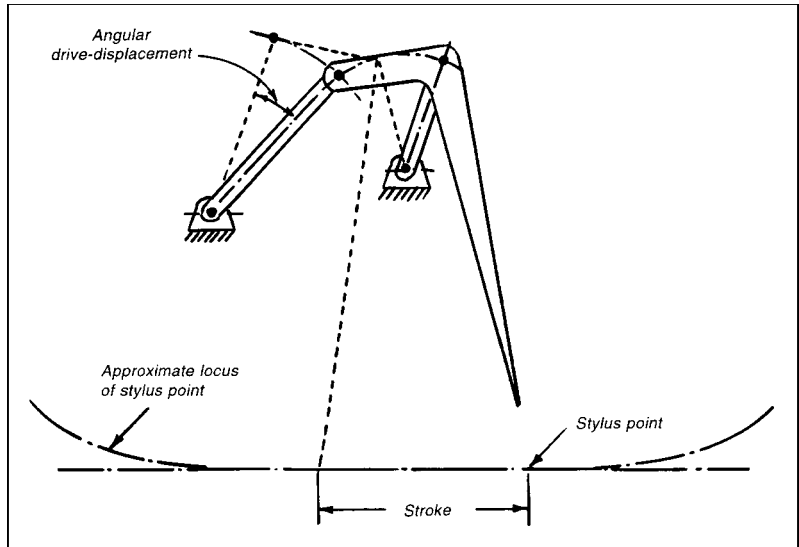


Fig. 4 A D-drive is the result when linkage arms are arranged as shown here. The output-link point describes a path that resembles the letter *D*, so there is a straight part of its cycle. This motion is ideal for quick engagement and disengagement before and after a straight driving stroke.

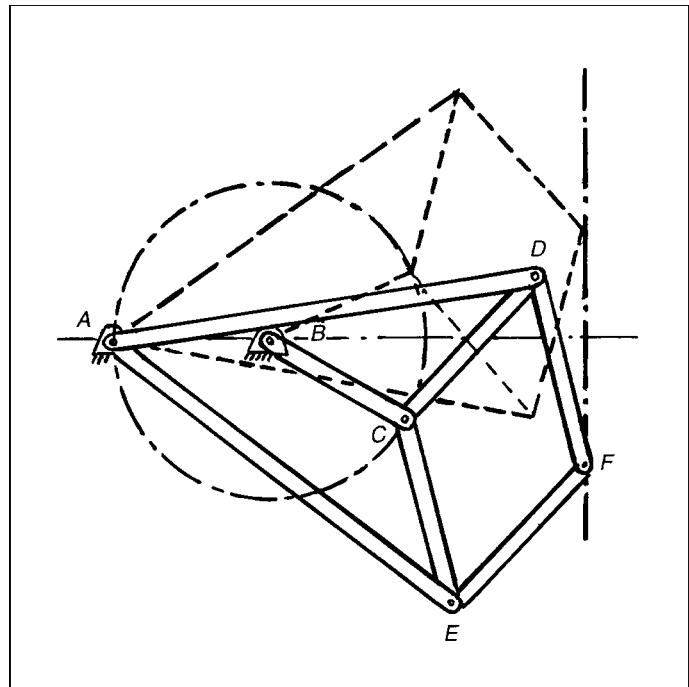


Fig. 5 The “Peaucellier cell” was the first solution to the classical problem of generating a straight line with a linkage. Within the physical limits of the motion, $AC \times AF$ remains constant. The curves described by *C* and *F* are, therefore, inverse; if *C* describes a circle that goes through *A*, then *F* will describe a circle of infinite radius—a straight line, perpendicular to *AB*. The only requirements are that: $AB = BC$; $AD = AE$; and CD, DF, FE, EC be equal. The linkage can be used to generate circular arcs of large radius by locating *A* outside the circular path of *C*.

SIX EXPANDING AND CONTRACTING LINKAGES

Parallel bars, telescoping slides, and other devices that can spark answers to many design problems.

Figs. 1 and 2 Expanding grilles are often put to work as a safety feature. A single parallelogram (fig. 1) requires slotted bars; a double parallelogram (fig. 2) requires none—but the middle grille-bar must be held parallel by some other method.

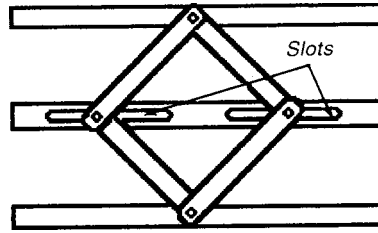


Fig. 1

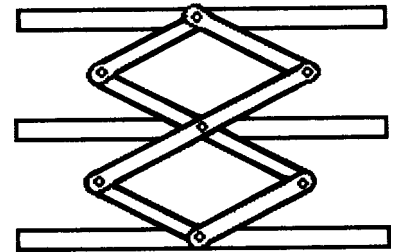


Fig. 2

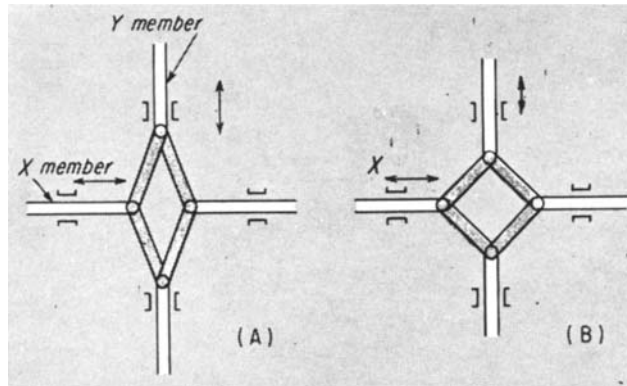


Fig. 3 Variable motion can be produced with this arrangement. In (A) position, the Y member is moving faster than the X member. In (B), speeds of both members are instantaneously equal. If the motion is continued in the same direction, the speed of X will become greater.

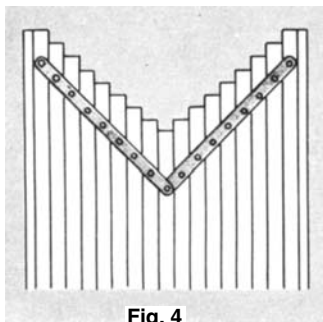


Fig. 4

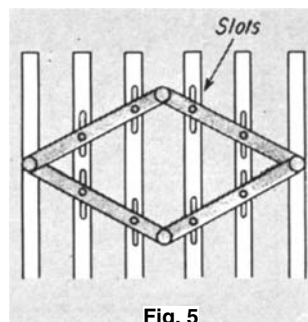


Fig. 5

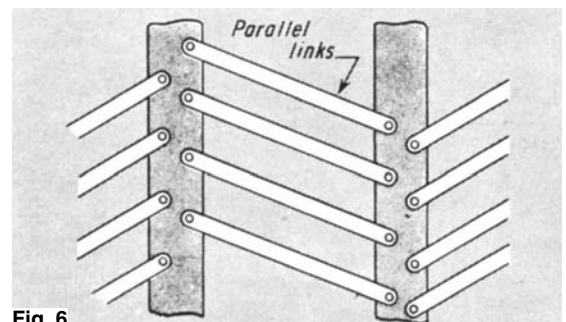


Fig. 6

Figs. 4, 5, and 6 Multibar barriers such as shutters and gates (fig. 4) can take various forms. Slots (fig. 5) allow for vertical adjustment. The space between bars can be made adjustable (fig. 6) by connecting the vertical bars with parallel links.

FOUR LINKAGES FOR DIFFERENT MOTIONS

Fig. 1 No linkages or guides are included in this modified hypocycloid drive which is relatively small in relation to the length of its stroke. The sun gear of pitch diameter D is stationary. The drive shaft, which turns the T-shaped arm, is concentric with this gear. The idler and planet gears, with pitch diameters of $D/2$, rotate freely on pivots in the arm extensions. The pitch diameter of the idler has no geometrical significance, although this gear does have an important mechanical function. It reverses the rotation of the planet gear, thus producing true hypocycloid motion with ordinary spur gears only. Such an arrangement occupies only about half as much space as does an equivalent mechanism containing an internal gear. The center distance R is the sum of $D/2$, $D/4$, and an arbitrary distance d , determined by specific applications. Points A and B on the driven link, which is fixed to the planet, describe straight-line paths through a stroke of $4R$. All points between A and B trace ellipses, while the line AB envelopes an astroid.

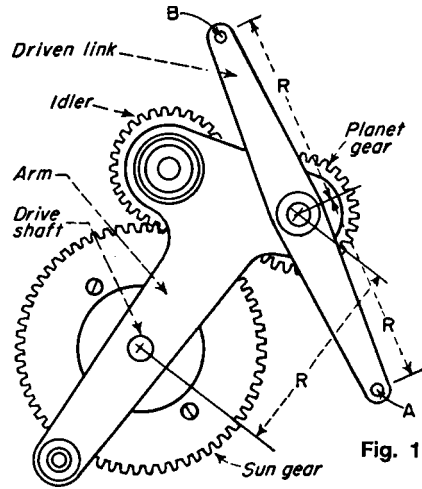


Fig. 1

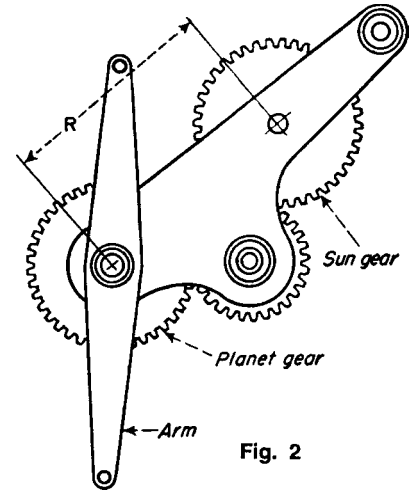


Fig. 2

Fig. 2 A slight modification of the mechanism in Fig. 1 will produce another type of useful motion. If the planet gear has the same diameter as that of the sun gear, the arm will remain parallel to itself throughout the complete cycle. All points on the arm will thereby describe circles of radius R . Here again, the position and diameter of the idler gear have no geometrical importance. This mechanism can be used, for example, to cross-perforate a uniformly moving paper web. The value for R is chosen so that $2\pi R$, or the circumference of the circle described by the needle carrier, equals the desired distance between successive lines of perforations. If the center distance R is made adjustable, the spacing of perforated lines can be varied as desired.

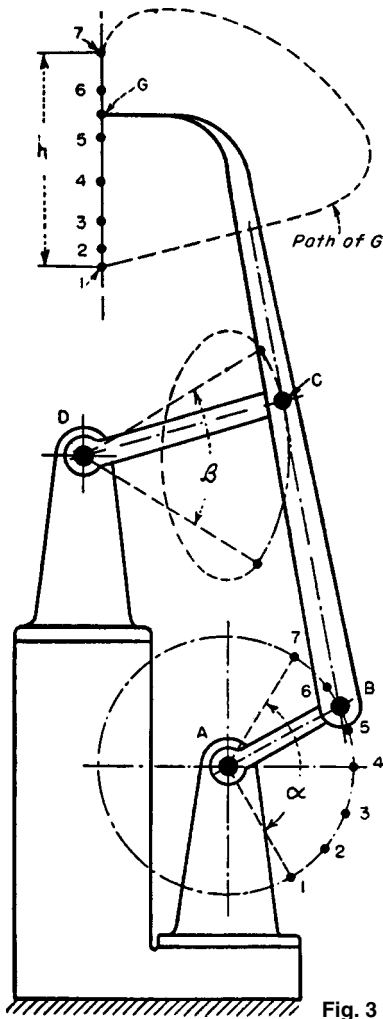


Fig. 3

Fig. 3 To describe a "D" curve, begin at the straight part of path G, and replace the oval arc of C with a circular arc that will set the length of link DC.

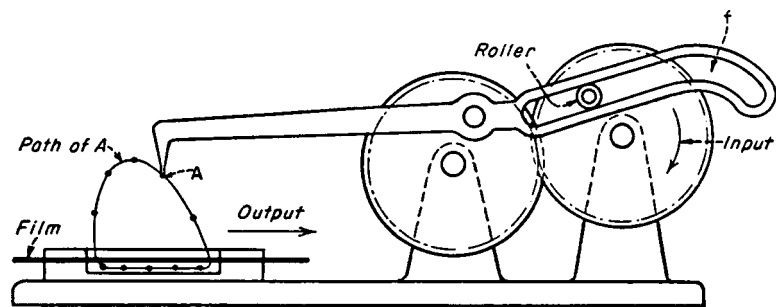


Fig. 4

Fig. 4 This mechanism can act as a film-strip hook that will describe a nearly straight line. It will engage and disengage the film perforation in a direction approximately normal to the film. Slight changes in the shape of the guiding slot f permit the shape of the output curve and the velocity diagram to be varied.

NINE LINKAGES FOR ACCELERATING AND DECELERATING LINEAR MOTIONS

When ordinary rotary cams cannot be conveniently applied, the mechanisms presented here, or adaptations of them, offer a variety of interesting possibilities for obtaining either acceleration or deceleration, or both.

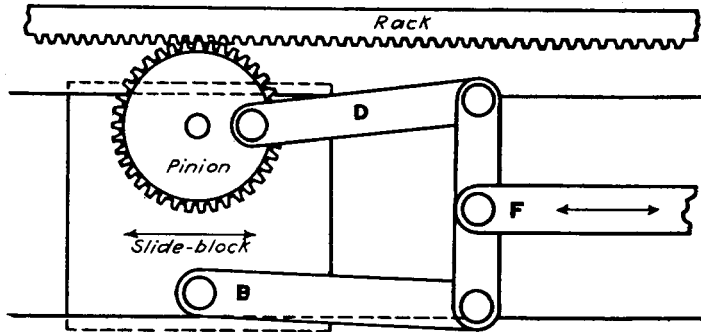


Fig. 1

Fig. 1 A slide block with a pinion and shaft and a pin for link B reciprocates at a constant rate. The pinion has a crankpin for mounting link D, and it also engages a stationary rack. The pinion can make one complete revolution at each forward stroke of the slide block and another as the slide block returns in the opposite direction. However, if the slide block is not moved through its normal travel range, the pinion turns only a fraction of a revolution. The mechanism can be made variable by making the connection link for F adjustable along the length of the element that connects links B and D. Alternatively, the crankpin for link D can be made adjustable along the radius of the pinion, or both the connection link and the crankpin can be made adjustable.

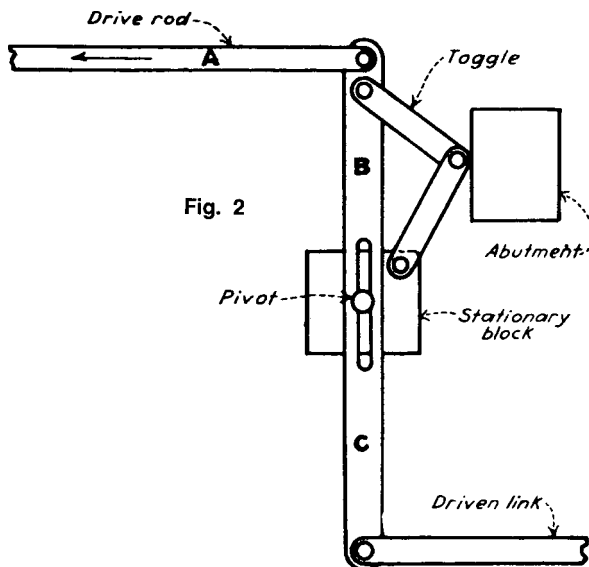


Fig. 2

Fig. 2 A drive rod, reciprocating at a constant rate, rocks link BC about a pivot on a stationary block. A toggle between arm B and the stationary block contacts an abutment. Motion of the drive rod through the toggle causes deceleration of driven link B. As the drive rod moves toward the right, the toggle is actuated by encountering the abutment. The slotted link BC slides on its pivot while turning. This lengthens arm B and shortens arm C of link BC. The result is deceleration of the driven link. The toggle is returned by a spring (not shown) on the return stroke, and its effect is to accelerate the driven link on its return stroke.

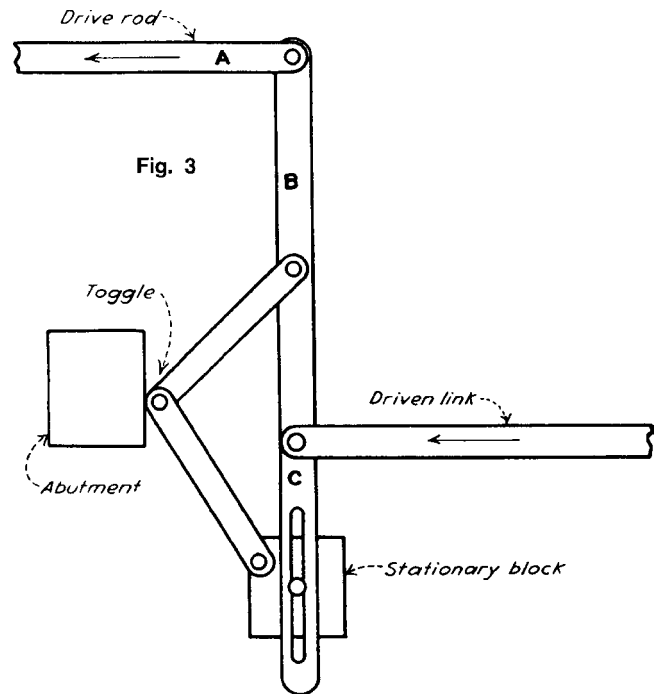


Fig. 3

Fig. 3 The same direction of travel for both the drive rod and the driven link is provided by the variation of the Fig. 2 mechanism. Here, acceleration is in the direction of the arrows, and deceleration occurs on the return stroke. The effect of acceleration decreases as the toggle flattens.

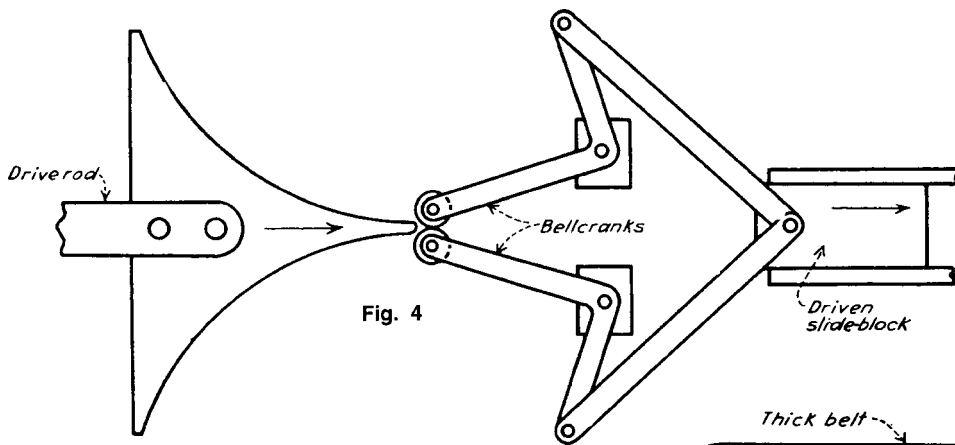


Fig. 4

Fig. 4 A bellcrank motion is accelerated as the rollers are spread apart by a curved member on the end of the drive rod, thereby accelerating the motion of the slide block. The driven elements must be returned by spring to close the system.

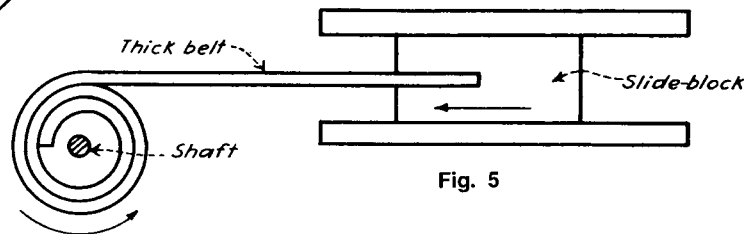


Fig. 5

Fig. 5 A constant-speed shaft winds up a thick belt or similar flexible connecting member, and its effective increase in radius causes the slide block to accelerate. It must be returned by a spring or weight on its reversal.

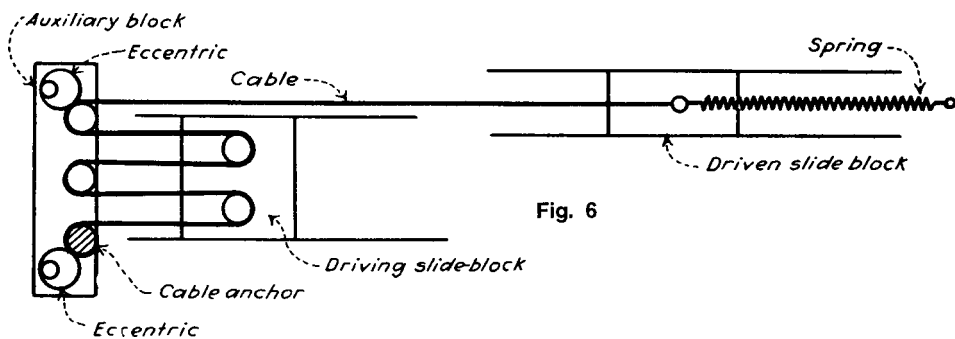


Fig. 6

Fig. 6 An auxiliary block that carries sheaves for a cable which runs between the driving and driven slide block is mounted on two synchronized eccentrics. The motion of the driven block is equal to the length of the cable paid out over the sheaves, resulting from the additive motions of the driving and auxiliary blocks.

Fig. 7 A curved flange on the driving slide block is straddled by rollers that are pivotally mounted in a member connected to the driven slide block. The flange can be curved to give the desired acceleration or deceleration, and the mechanism returns by itself.

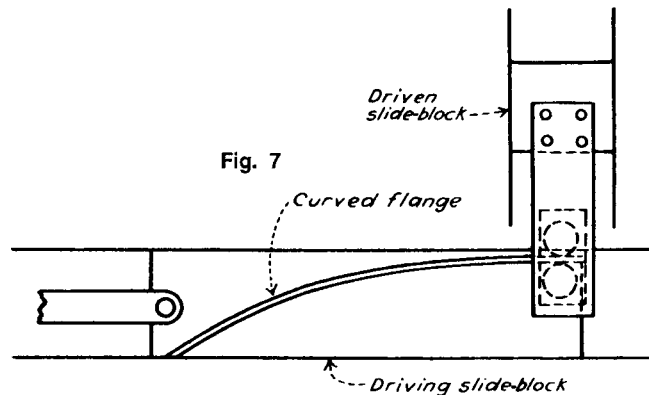


Fig. 7

Fig. 8 The stepped acceleration of the driven block is accomplished as each of the three reciprocating sheaves progressively engages the cable. When the third acceleration step is reached, the driven slide block moves six times faster than the drive rod.

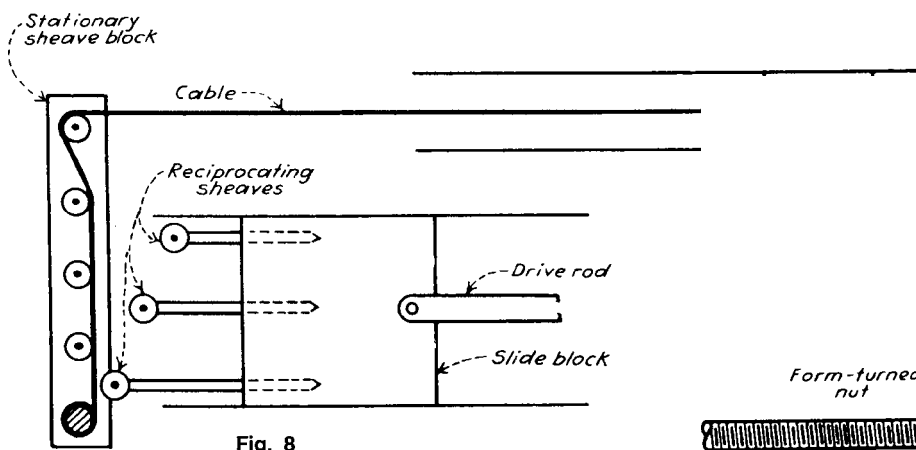


Fig. 8

Fig. 9 A form-turned nut, slotted to travel on a rider, is propelled by reversing its screw shaft, thus moving the concave roller up and down to accelerate or decelerate the slide block.

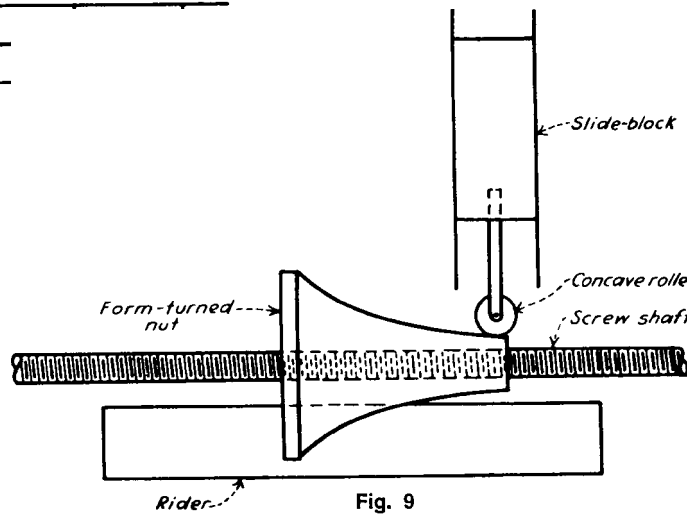


Fig. 9

TWELVE LINKAGES FOR MULTIPLYING SHORT MOTIONS

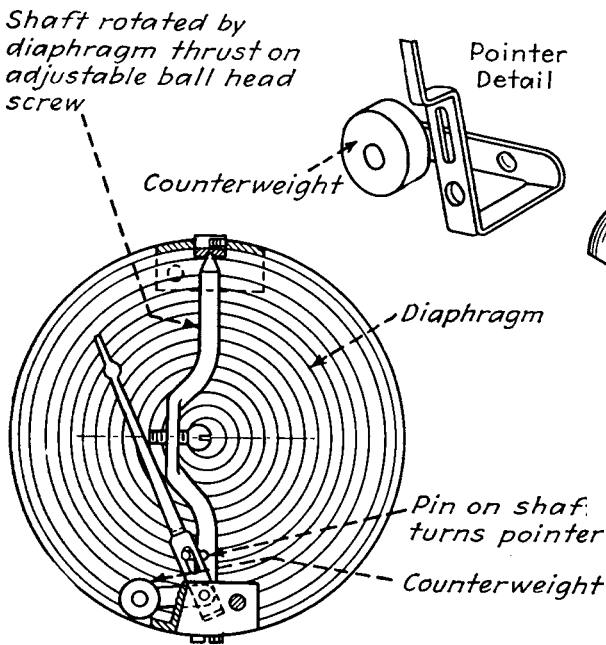


Fig. 1 A lever-type transmission in a pressure gage.

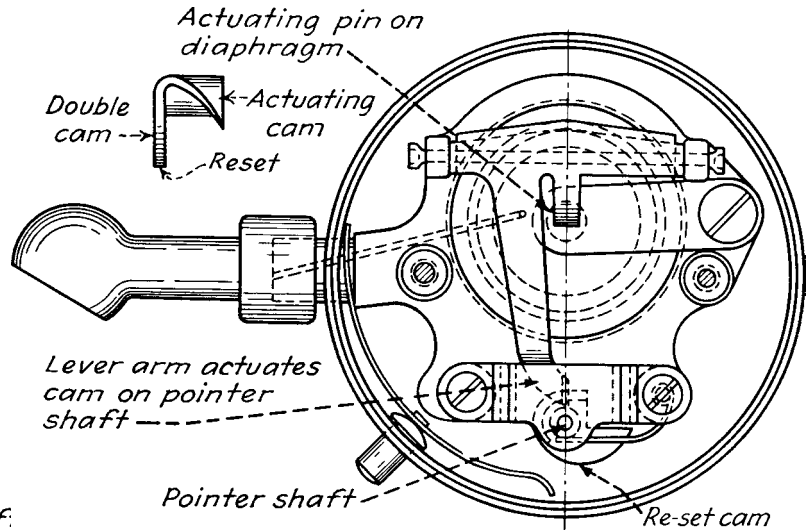


Fig. 2 A lever and cam drive for a tire gage.

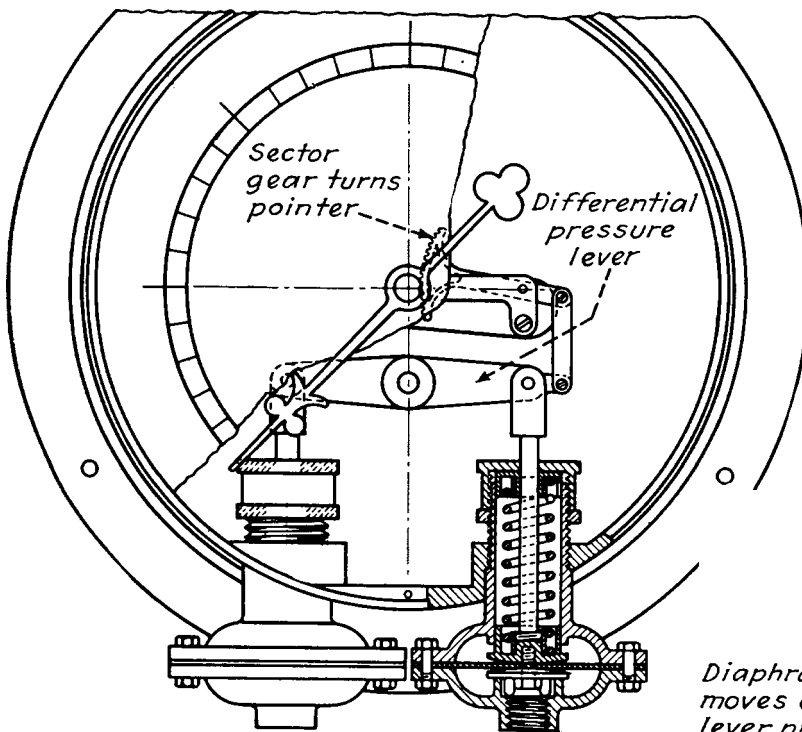


Fig. 3 A lever and sector gear in a differential pressure gage.

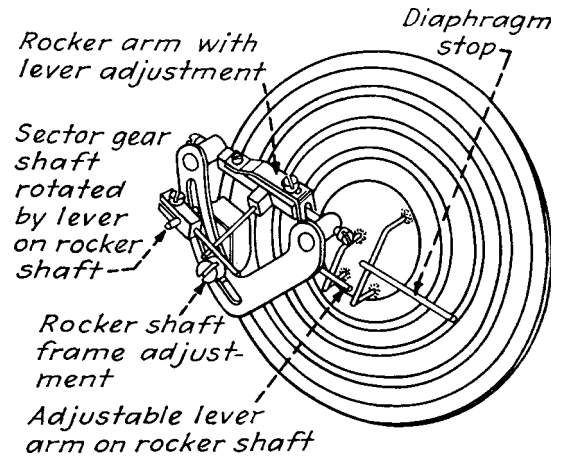


Fig. 4 A sector gear drive for an aircraft airspeed indicator.

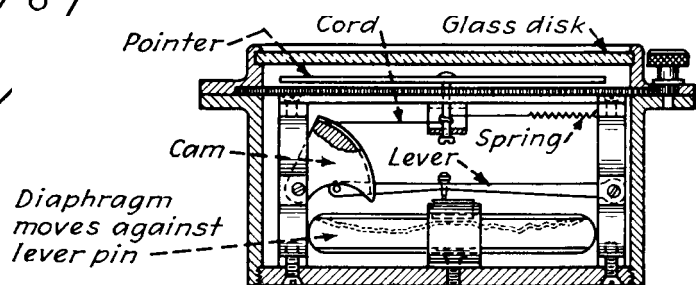


Fig. 5 A lever, cam, and cord transmission in a barometer.

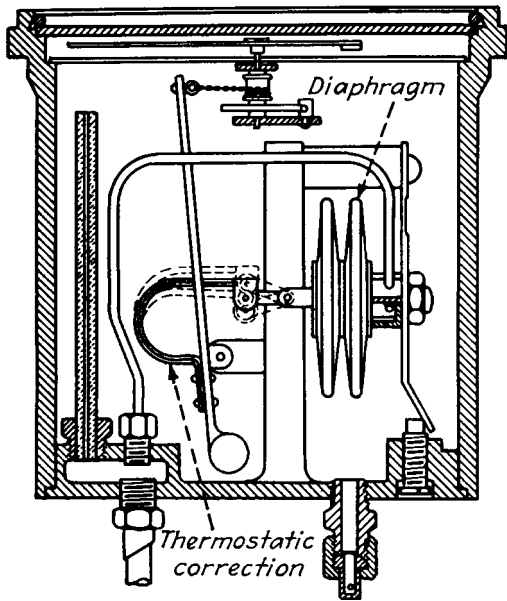


Fig. 6 A link and chain transmission for an aircraft rate of climb instrument.

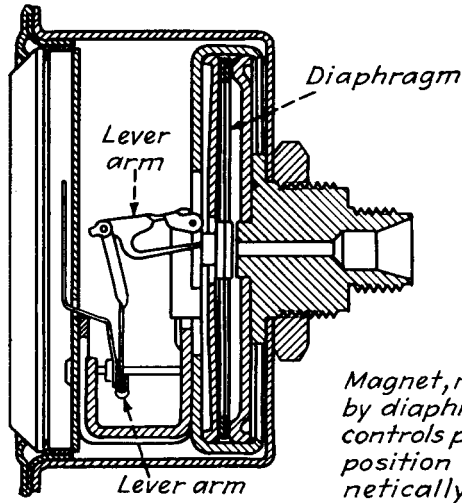


Fig. 7 A lever system in an automobile gasoline tank.

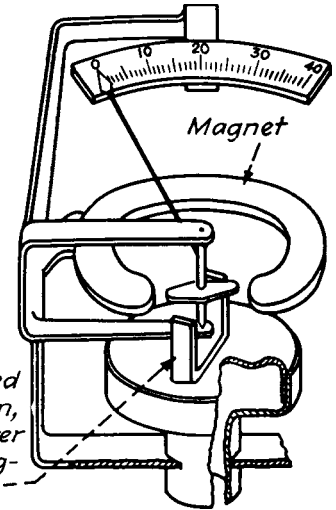


Fig. 8 Interfering magnetic fields for fluid pressure measurement.

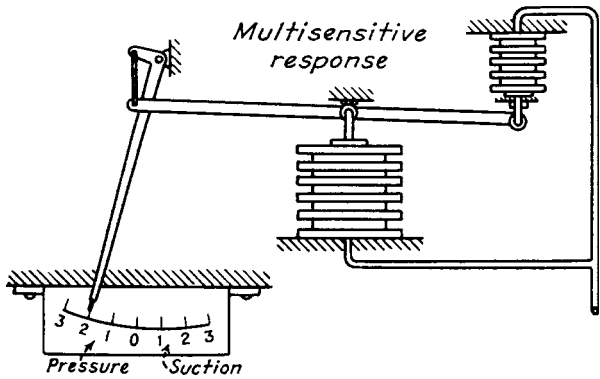


Fig. 9 A lever system for measuring atmospheric pressure variations.

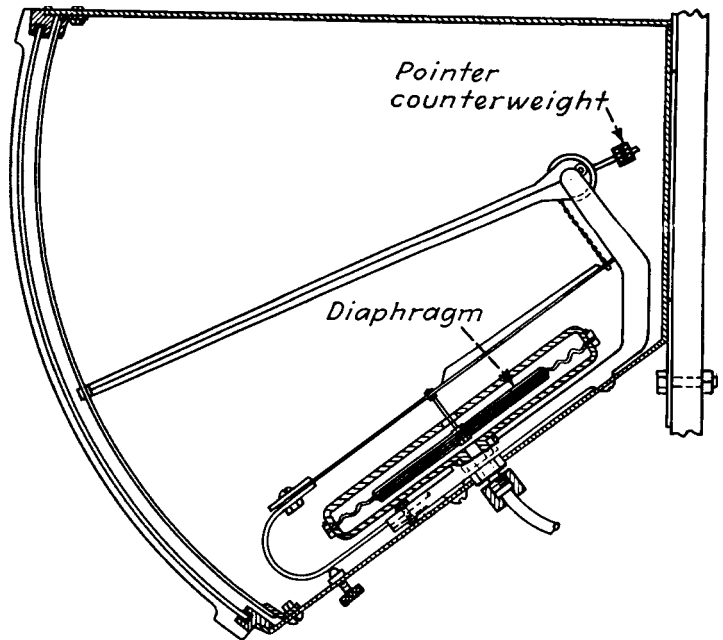


Fig. 10 A lever and chain transmission for a draft gage.

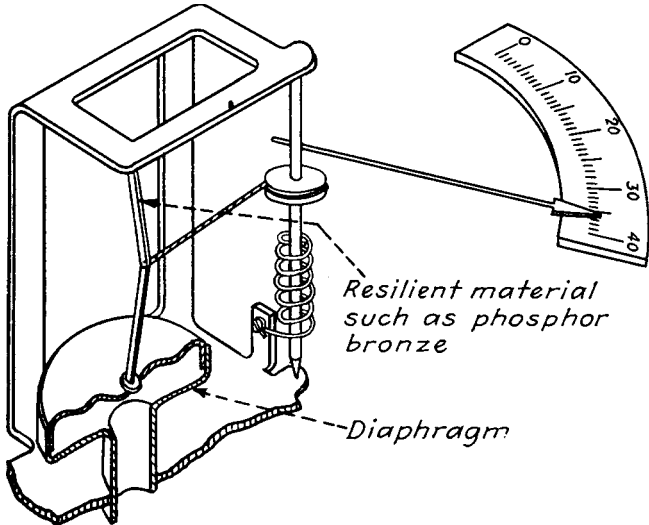


Fig. 11 A toggle and cord drive for a fluid pressure measuring instrument.

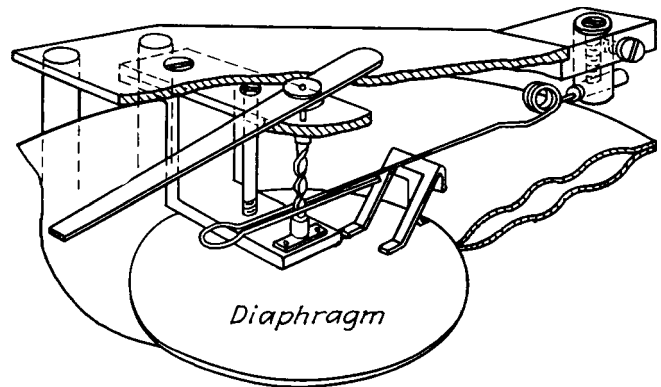
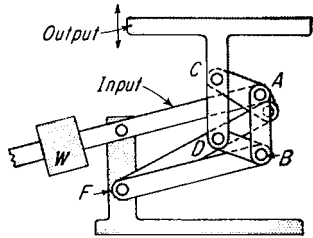


Fig. 12 A spiral feed transmission for a general purpose analog instrument.

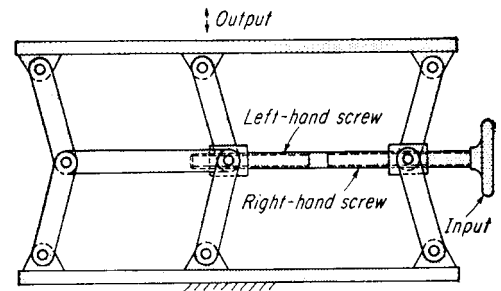
FOUR PARALLEL-LINK MECHANISMS

Eight-bar linkage



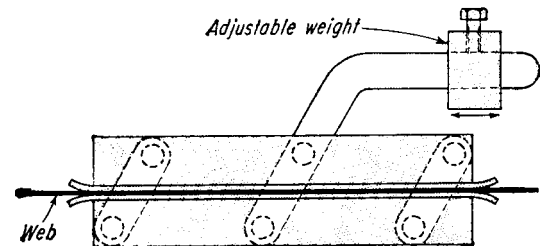
Link *AB* in this arrangement will always be parallel to *EF*, and link *CD* will always be parallel to *AB*. Hence *CD* will always be parallel to *EF*. Also, the linkages are so proportioned that point *C* moves in an approximately straight line. The final result is that the output plate will remain horizontal while moving almost straight up and down. The weight permitted this device to function as a disappearing platform in a theater stage.

Double-handed screw mechanism



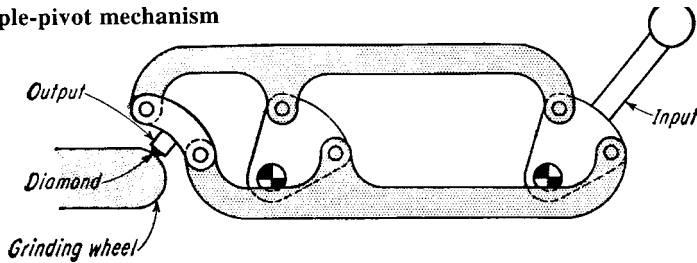
Turning the adjusting screw spreads or contracts the linkage pairs to raise or lower the table. Six parallel links are shown, but the mechanism can be built with four, eight, or more links.

Tensioning mechanism



A simple parallel-link mechanism that produces tension in webs, wires, tapes, and strip steels. Adjusting the weight varies the drag on the material.

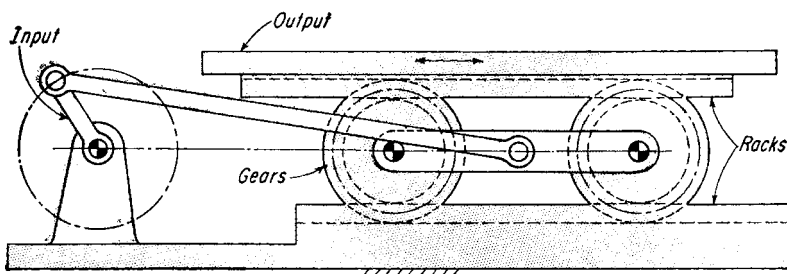
Triple-pivot mechanism



Two triangular plates pivot around fixed points on a machine frame. The output point describes a circular-arc curve. It can round out the cutting surfaces of grinding wheels.

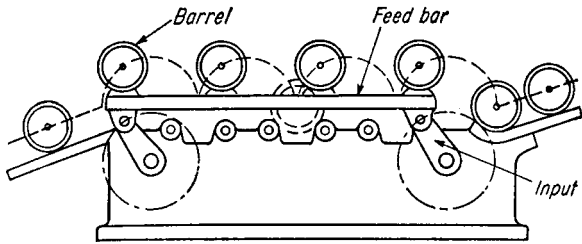
SEVEN STROKE MULTIPLIER LINKAGES

Reciprocating-table drive



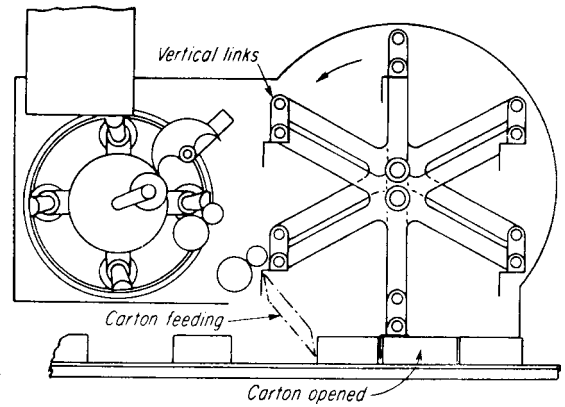
Two gears rolling on a stationary bottom rack drive the movable top rack, which is attached to a printing table. When the input crank rotates, the table will move out to a distance of four times the crank length.

Parallel-link feeder



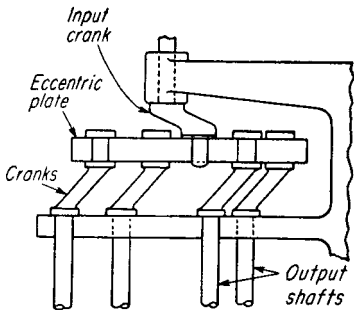
One of the cranks is the input, and the other follows to keep the feeding bar horizontal. The feeder can move barrels from station to station.

Parallelogram linkage



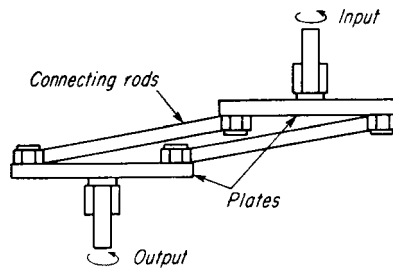
All seven short links are kept in a vertical position while rotating. The center link is the driver. This particular machine feeds and opens cartons, but the mechanism will work in many other applications.

Parallel-link driller



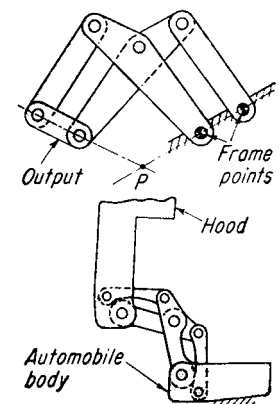
This parallel-link driller powers a group of shafts. The input crank drives the eccentric plate. This, in turn, rotates the output cranks that have the same length at the same speed. Gears would occupy more room between the shafts.

Parallel-plate driver



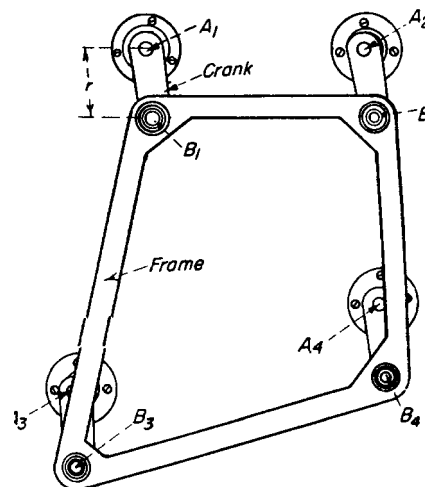
The input and output shafts of this parallel-plate driver rotate with the same angular relationship. The positions of the shafts, however, can vary to suit other requirements without affecting the input-output relationship between the shafts.

Curve-scribing mechanism



The output link rotates so that it appears to revolve around a point moving in space (*P*). This avoids the need for hinges at distant or inaccessible spots. The mechanism is suitable for hinging the hoods of automobiles.

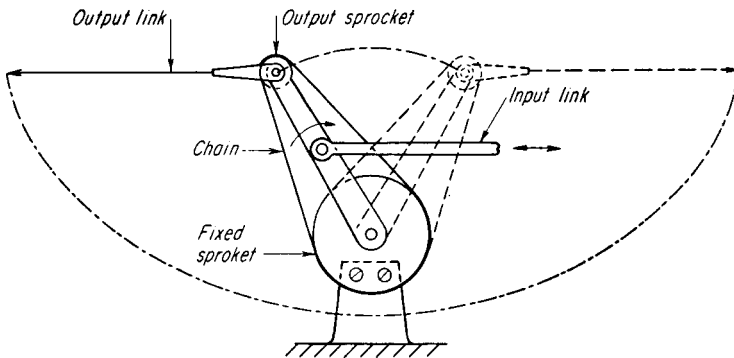
Parallel-link coupling



The absence of backlash makes this parallel-link coupling a precision, low-cost replacement for gear or chain drives that can also rotate parallel shafts. Any number of shafts greater than two can be driven from any one of the shafts, provided two conditions are fulfilled: (1) All cranks must have the same length r ; and (2) the two polygons formed by the shafts *A* and frame pivot centers *B* must be identical. The main disadvantage of this mechanism is its dynamic unbalance, which limits the speed of rotation. To lessen the effect of the vibrations produced, the frame should be made as light as is consistent with strength requirements for the intended application.

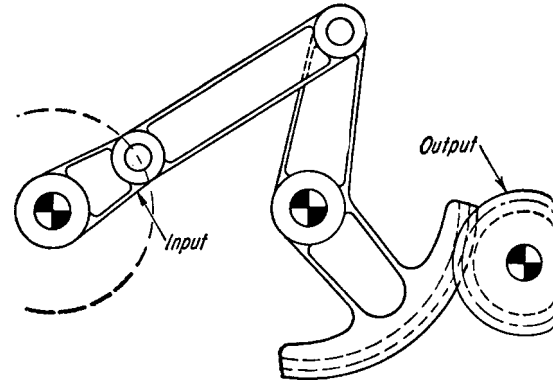
NINE FORCE AND STROKE MULTIPLIER LINKAGES

Wide-angle oscillator



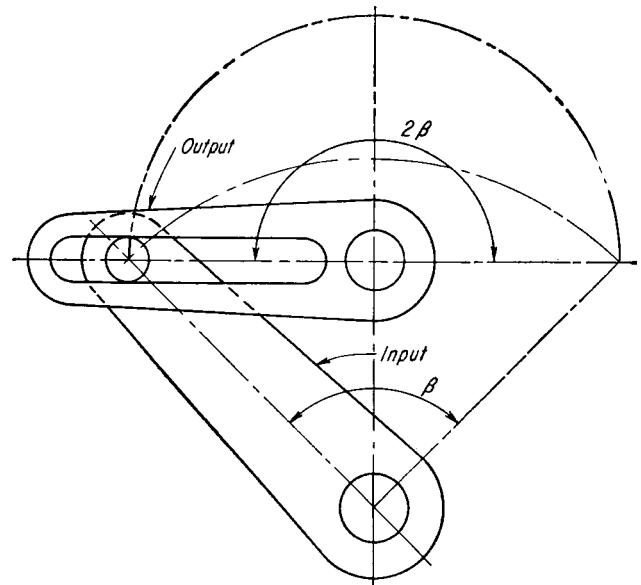
The motion of the input linkage in the diagram is converted into a wide-angle oscillation by the two sprockets and chain. An oscillation of 60° is converted into 180° oscillation.

Gear-sector drive



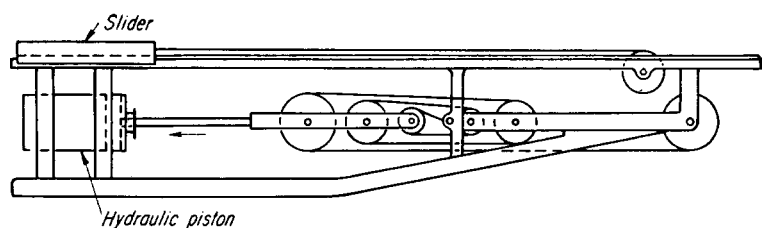
This is actually a four-bar linkage combined with a set of gears. A four-bar linkage usually obtains no more than about 120° of maximum oscillation. The gear segments multiply the oscillation in inverse proportion to the radii of the gears. For the proportions shown, the oscillation is boosted two and one-half times.

Angle-doubling drive



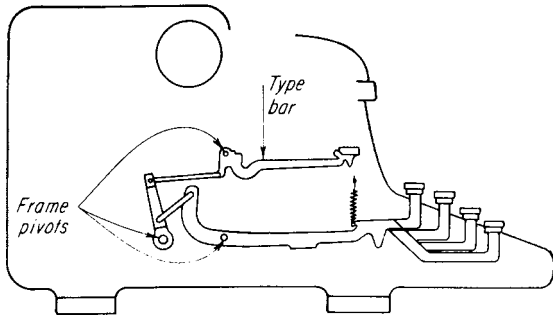
This angle-doubling drive will enlarge the oscillating motion β of one machine member into an output oscillation of 2β . If gears are employed, the direction of rotation cannot be the same unless an idler gear is installed. In that case, the centers of the input and output shafts cannot be too close. Rotating the input link clockwise causes the output to follow in a clockwise direction. For any set of link proportions, the distance between the shafts determines the gain in angle multiplication.

Pulley drive



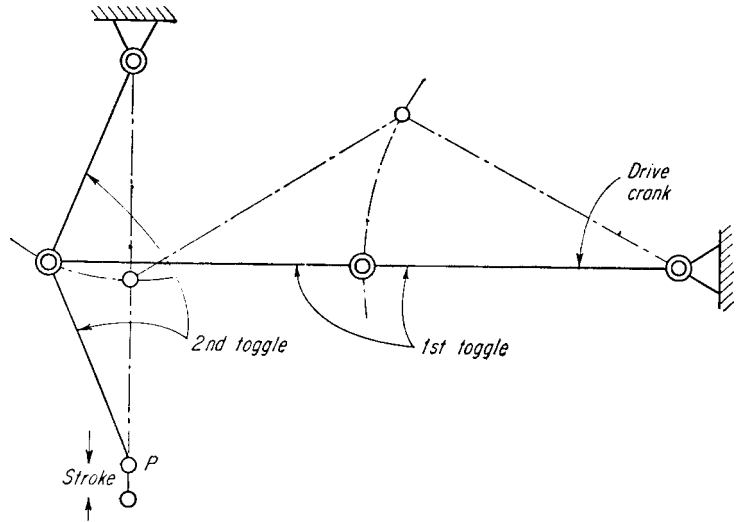
This pulley drive multiplies the stroke of a hydraulic piston, causing the slider to move rapidly to the right for catapulting objects.

Typewriter drive



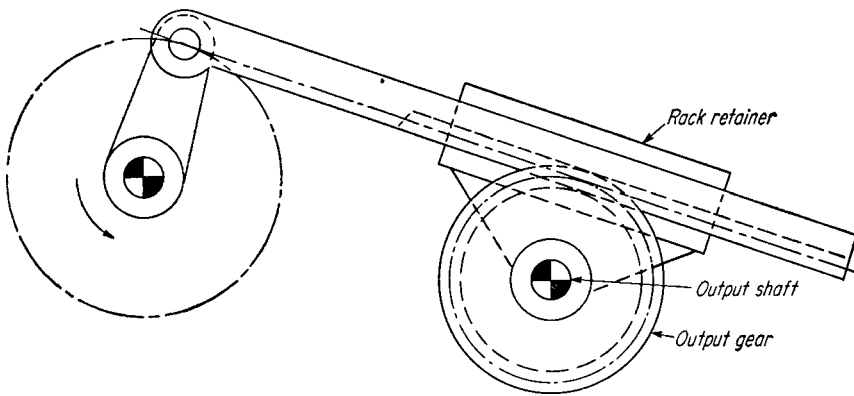
This drive multiplies the finger force of a typewriter, producing a strong hammer action at the roller from a light touch. There are three pivot points attached to the frame. The links are arranged so that the type bar can move in free flight after a key has been struck. The mechanism illustrated is actually two four-bar linkages in series. Some typewriters have as many as four four-bar linkages in a series.

Double-toggle puncher



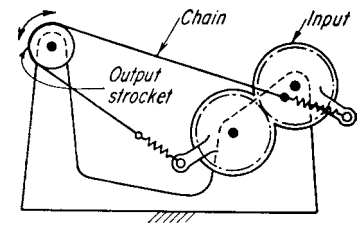
The first toggle of this puncher keeps point *P* in the raised position although its weight can exert a strong downward force (as in a heavy punch weight). When the drive crank rotates clockwise (e.g., driven by a reciprocating mechanism), the second toggle begins to straighten so as to create a strong punching force.

Gear-rack drive



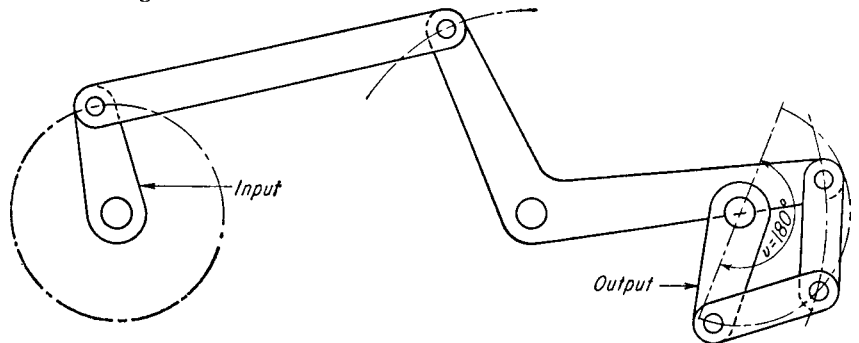
This drive mechanism converts the motion of an input crank into a much larger rotation of the output (from 30° to 360°). The crank drives the slider and gear rack, which in turn rotates the output gear.

Chain drive



Springs and chains are attached to geared cranks of this drive to operate a sprocket output. Depending on the gear ratio, the output will produce a desired oscillation, e.g., two revolutions of output in each direction for each 360° of input.

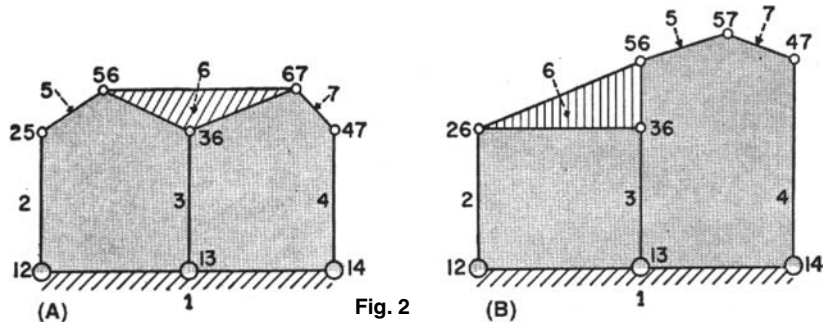
Linkage-train drive



Arranging linkages in series on this drive can increase its angle of oscillation. In the version illustrated, the oscillating motion of the L-shaped rocker is the input for the second linkage. The final oscillation is 180°.

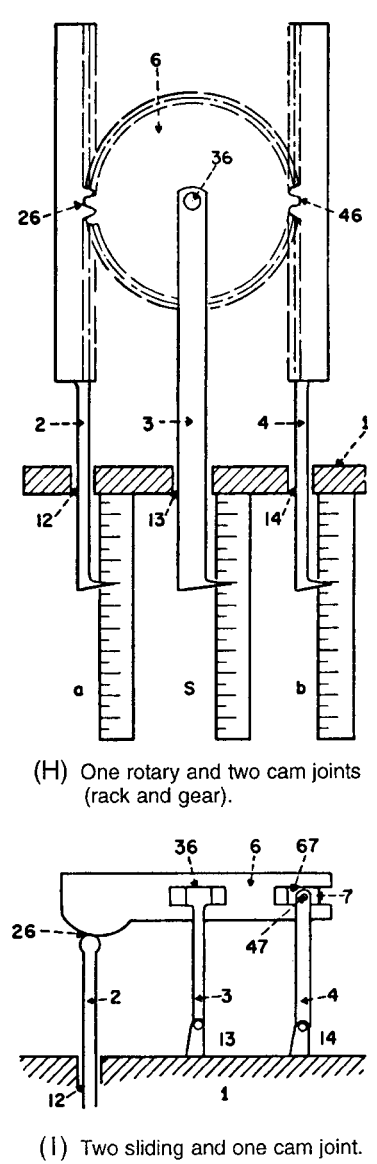
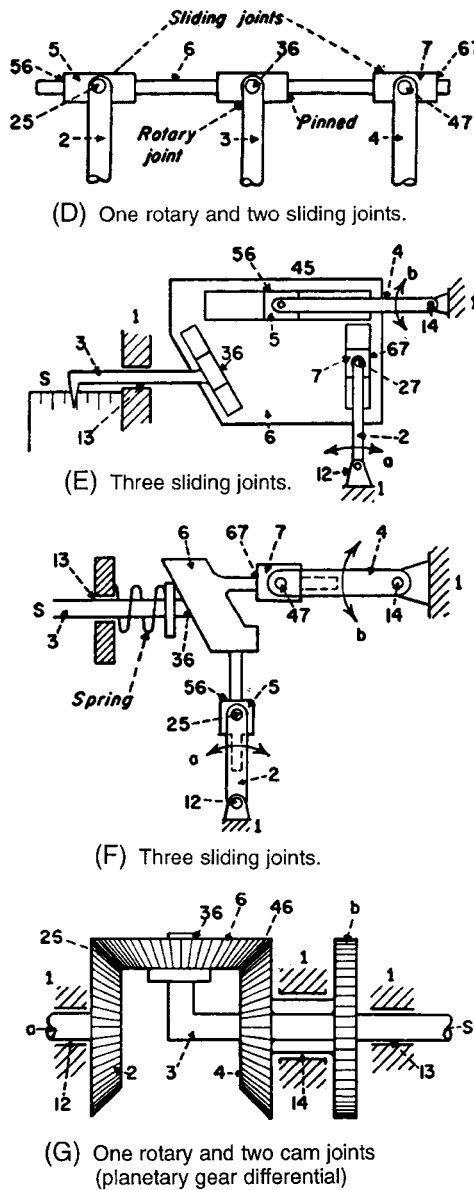
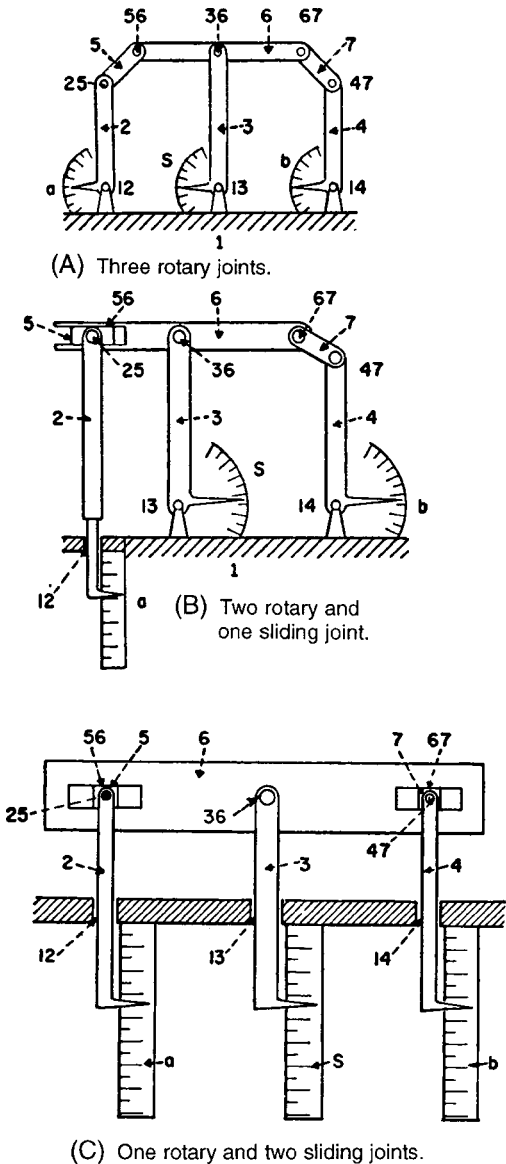
EIGHTEEN VARIATIONS OF DIFFERENTIAL LINKAGE

Figure 1 shows the modifications of the differential linkage shown in Fig. 2(A). These are based on the variations in the triple-jointed intermediate link 6. The links are designated as follows: Frame links: links 2, 3 and 4; two-jointed intermediate links: links 5 and 7; three jointed intermediate links: link 6.

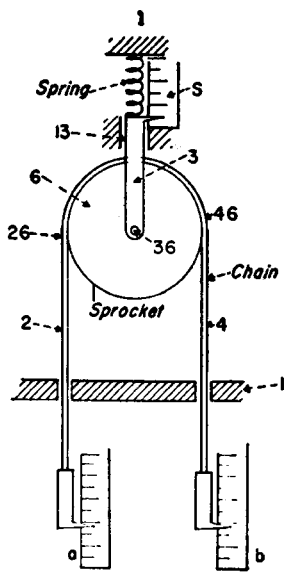


The input motions to be added are a and b ; their sum s is equal to $c_1 a + c_2 b$, where c_1 and c_2 are scale factors. The links are numbered in the same way as those in Fig. 2(A).

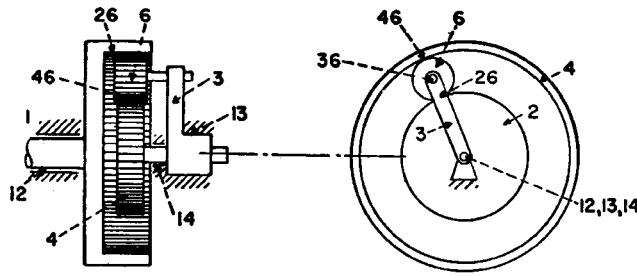
Fig. 1



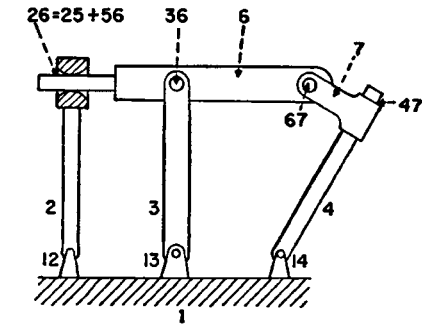
Variations of Differential Linkage (continued)



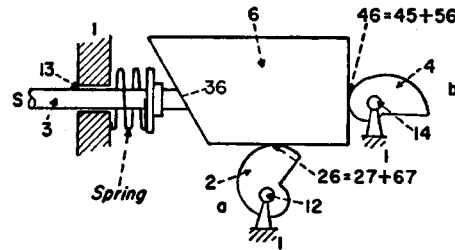
(J) One rotary and two cam joints (chain and sprocket).



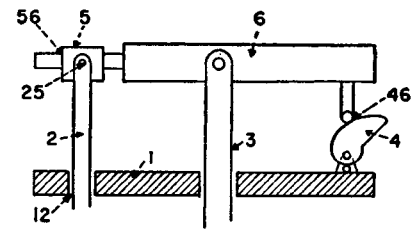
(M) One rotary and two cam joints (planetary gears differential).



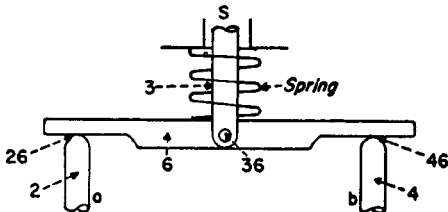
(O) Two rotary and one cam joint.



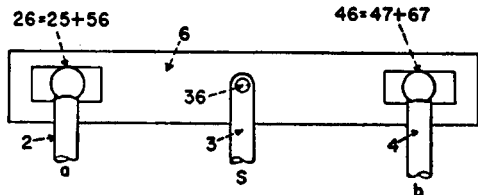
(N) One sliding and two cam joints.



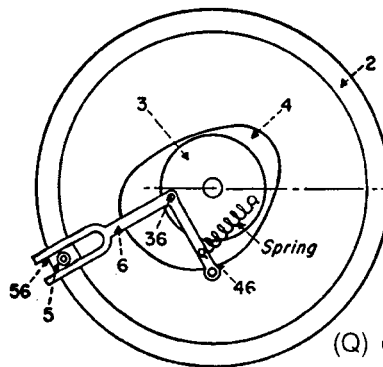
(P) One rotary, one sliding, and one cam joint.



(K) One rotary and two cam joints.



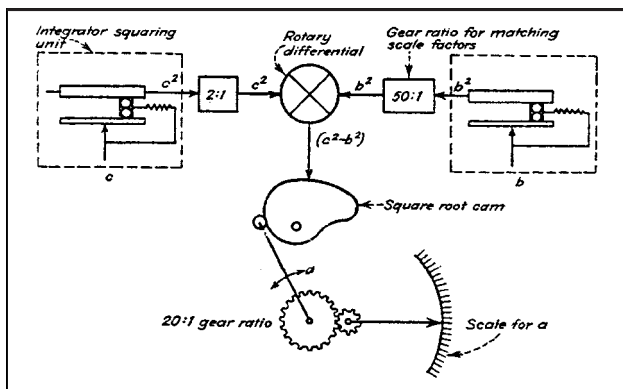
(L) One rotary and two cam joints.



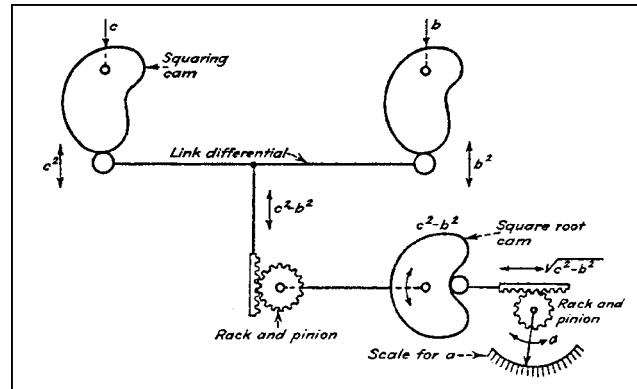
(Q) One rotary, one sliding and one cam joint.

Front view

End view



The integrator method of mechanizing the equation $a = \sqrt{c^2 - b^2}$ is shown in the schematic form. It requires an excessive number of parts.



The cam method of mechanizing $a = \sqrt{c^2 - b^2}$ uses function generators for squaring and a link differential for subtraction. Note the reduction in parts from the integrator method.

FOUR-BAR SPACE MECHANISMS

There are potentially hundreds of them, but only a few have been discovered so far. Here are the best of one class—the four-bar space mechanisms.

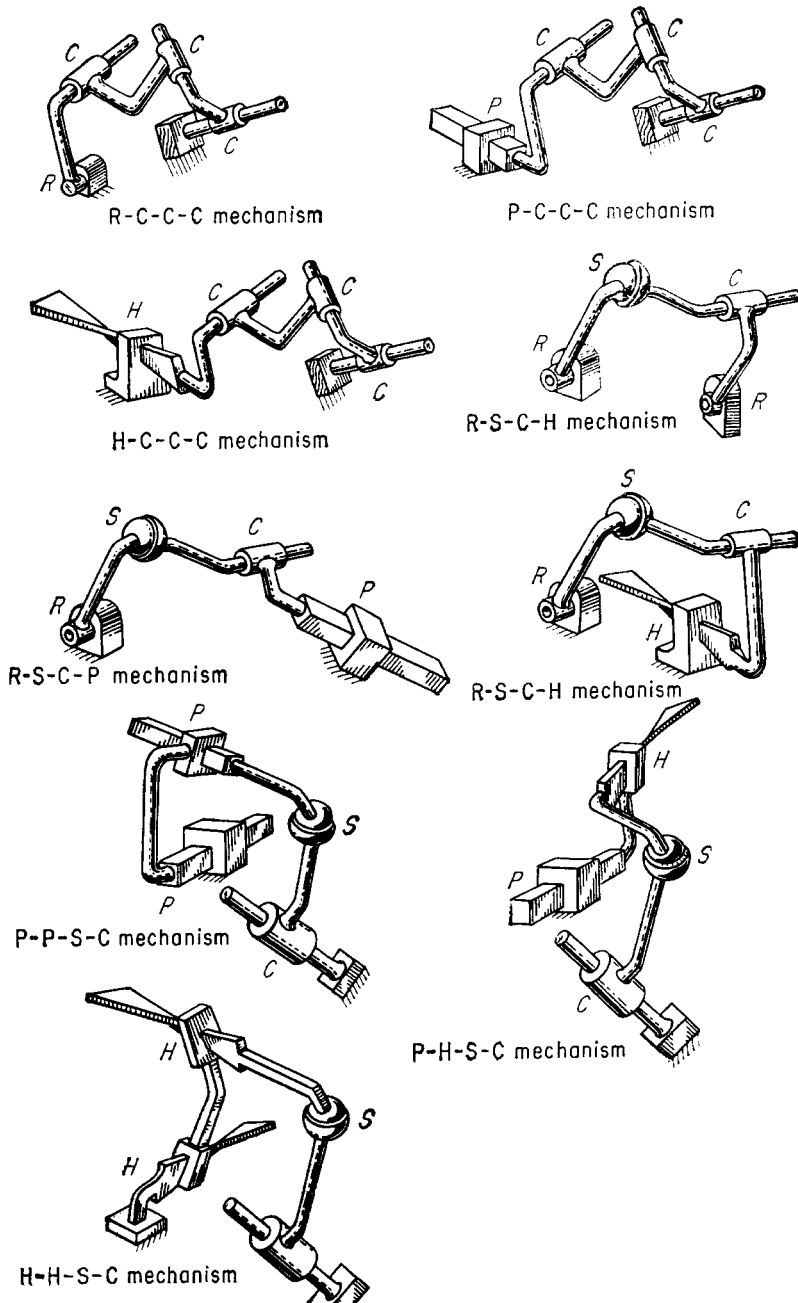


Fig. 1 The nine chosen mechanisms.

A virtually unexplored area of mechanism research is the vast domain of three-dimensional linkage, frequently called space mechanism. Only a comparatively few kinds have been investigated or described, and little has been done to classify those that are known. As a result, many engineers do not know much about them, and applications of space mechanisms have not been as widespread as they could be.

Because a space mechanism can exist with a wide variety of connecting joints or “pair” combinations, it can be identified by the type and sequence of its joints. A listing of all of the physically realizable kinematic pairs has been established, based on the number of degrees-of-freedom of a joint. These pairs are all the known ways of connecting two bodies together for every possible freedom of relative motion between them.

The Practical Nine

The next step was to find the combination of pairs and links that would produce practical mechanisms. Based on the “Kutzbach criterion” (the only known mobility criterion)—determines the degree of freedom of a mechanism due to the constraints imposed by the pairs), 417 different kinds of space mechanisms have been identified. Detailed examination showed many of these to be mechanically complex and of limited adaptability. But the four-link mechanisms had particular appeal because of their mechanical simplicity. A total of 138 different kinds of four-bar mechanisms have been found. Of these, nine have particular merit (Fig. 1).

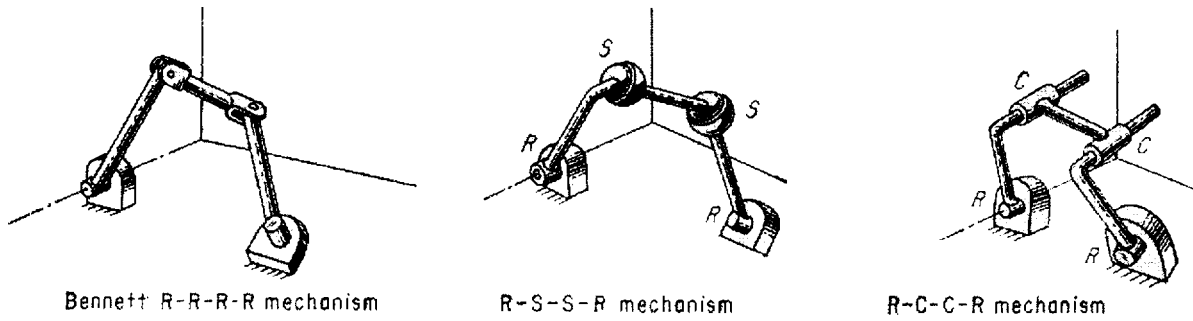


Fig. 2 The three mavericks.

These nine four-link mechanisms are the easiest to build because they contain only those joints that have area contact and are self-connecting. In the table, these joints are the five closed, lower pair types:

- R = Revolute joint, which permits rotation only
- P = Prism joint, which permits sliding motion only
- H = Helix or screw type of joint
- C = Cylinder joint, which permits both rotation and sliding (hence has two degrees of freedom)
- S = Sphere joint, which is the common ball joint permitting rotation in any direction (three degrees of freedom)

All these mechanisms can produce rotary or sliding output motion from a rotary input—the most common mechanical requirements for which linkage mechanisms are designed.

The type letters of the kinematic pairs in the table identify the mechanism by ordering the letter symbols consecutively around the closed kinematic chain. The first letter identifies the pair connecting the input link and the fixed link; the last letter identifies the output link, or last link, with the fixed link. Thus, a mechanism labeled R-S-C-R is a double-crank mechanism with a spherical pair between the input crank and the coupler, and a cylindrical pair between the coupler and the output crank.

The Mavericks

The Kutzbach criterion is inadequate for the job because it cannot predict the existence of such mechanisms as the Bennett R-R-R-R mechanism, the double-ball joint R-S-S-R mechanism, and the R-C-C-R mechanism (Fig. 2). These “special” mechanisms require special geometric conditions to have a single degree of freedom. The R-R-R-R mechanism requires a particular orientation of the revolute axes and a particular ratio of link lengths to function as a single degree of freedom space mechanism. The R-S-S-R configuration, when functioning as a single degree-of-freedom mechanism, will have a passive degree of freedom of its coupler link. When properly constructed, the configuration R-C-C-R will also have a passive degree-of-freedom of its coupler, and it will function as a single-degree space mechanism.

Of these three special four-link mechanisms, the R-S-S-R mechanism is seen as the outstanding choice. It is the most versatile and practical configuration for meeting double-crank motion requirements.

Classification of kinematic pairs

Degree of freedom	Type number*	Type of joint	
		Symbol	Name
1	100	R	Revolute
	010	P	Prism
	001	H	Helix
2	200	T	Torus
	110	C	Cylinder
	101	T _H	Torus-helix
	020
	011
	300	S	Sphere
3	210	S _S	Sphere-slotted cylinder
	201	S _{SH}	Sphere-slotted helix
	120	P _L	Plane
	021
	111
	310	S _G	Sphere-groove
	301	S _{GH}	Sphere-grooved helix
4	220	C _P	Cylinder-plane
	121
	211
5	320	S _P	Sphere-plane
	221
	311

* Number of freedoms, give in the order of N_R, N_T, N_H.

SEVEN THREE-DIMENSIONAL LINKAGE DRIVES

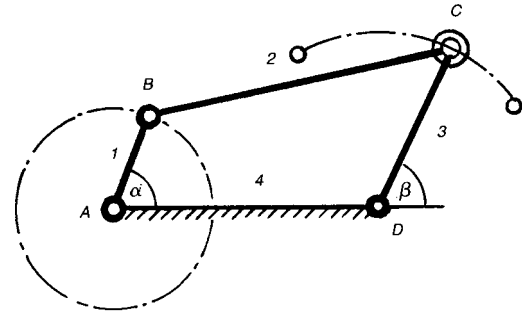
The main advantage of three-dimensional drives is their ability to transmit motion between nonparallel shafts. They can also generate other types of helpful motion. This roundup includes descriptions of seven industrial applications for the drives.

Spherical Crank Drive

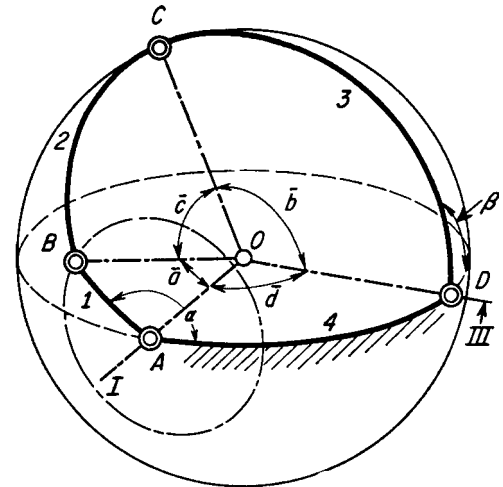
This type of drive is the basis for most three-dimensional linkages, much as the common four-bar linkage is the basis for the two-dimensional field. Both mechanisms operate on similar principles. (In the accompanying sketches, a is the input angle, and β the output angle. This notation has been used throughout this section.)

In the four-bar linkage, the rotary motion of driving crank 1 is transformed into an oscillating motion of output link 3. If the fixed link is made the shortest of all, then it is a double-crank mechanism; both the driving and driven members make full rotations.

The spherical crank drive, link 1 is the input, link 3 the output. The axes of rotation intersect at point O ; the lines connecting AB , BC , CD , and DA can be considered to be parts of great circles of a sphere. The length of the link is best represented by angles a , b , c , and d .



Four-bar linkage



The Spherical Crank

Spherical-Slide Oscillator Drive

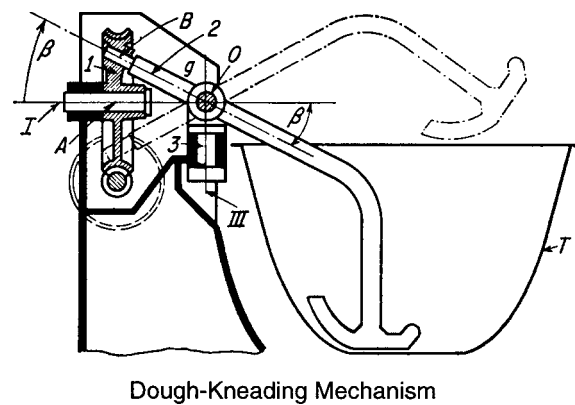
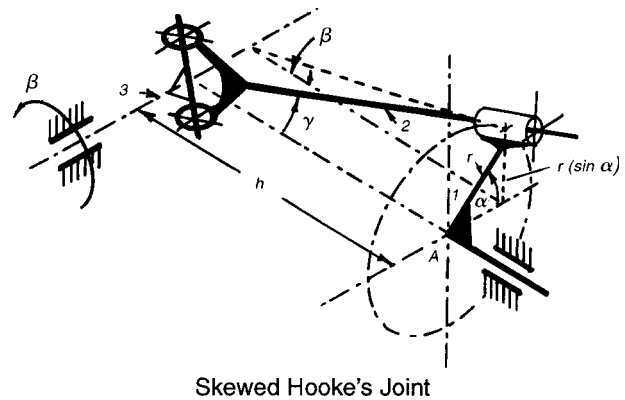
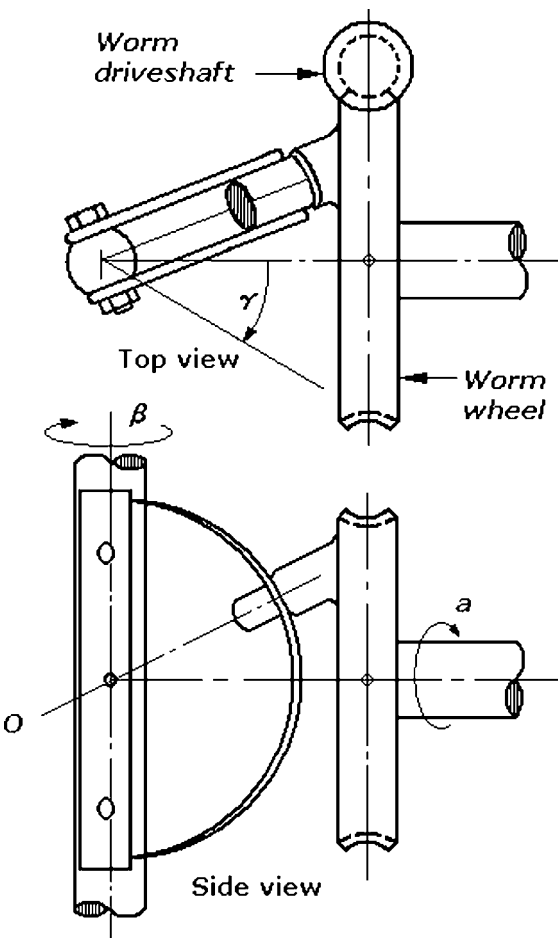
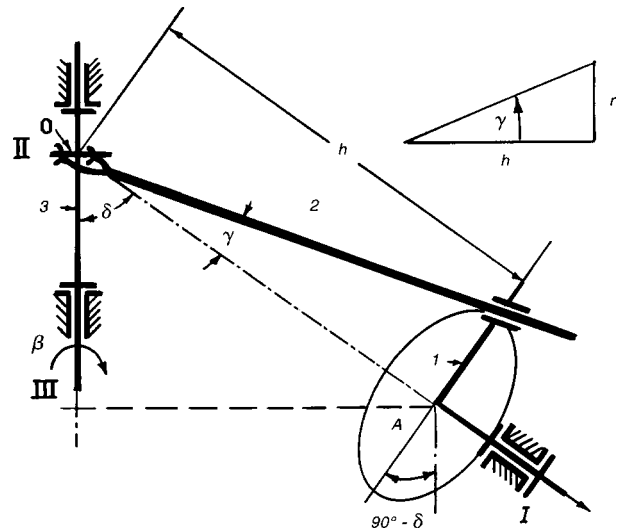
The two-dimensional slider crank is obtained from a four-bar linkage by making the oscillating arm infinitely long. By making an analogous change in the spherical crank, the spherical slider crank is obtained.

The uniform rotation of input shaft *I* is transferred into a nonuniform oscillating or rotating motion of output shaft *III*. These shafts intersect at an angle δ , corresponding to the frame link *4* of the spherical crank. Angle γ corresponds to the length of link *1*, and axis *II* is at right angle to axis *III*.

The output oscillates when γ is smaller than δ , but it rotates when γ is larger than δ .

The relation between input angle α and output angle β as designated in the skewed Hooke's joint is:

$$\tan \beta = \frac{(\tan \gamma)(\sin \alpha)}{\sin \delta + (\tan \gamma)(\cos \delta)(\cos \alpha)}$$



Skewed Hooke's Joint Drive

This variation of the spherical crank is specified where an almost linear relation is desired between the input and output angles for a large part of the motion cycle.

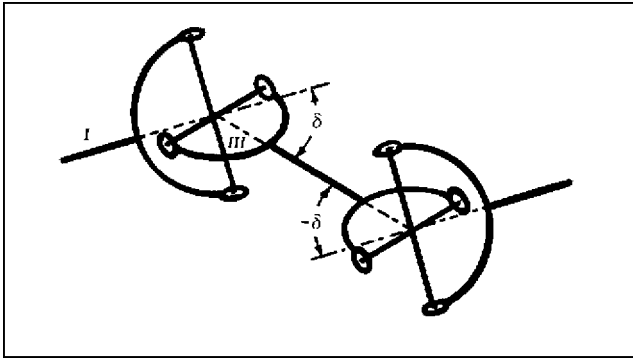
The equation defining the output in terms of the input can be obtained from the skewed Hooke's joint equation by making $\delta = 90^\circ$. Thus, $\sin \delta = 1$, $\cos \delta = 0$, and

$$\tan \beta = \tan \gamma \sin \alpha$$

The principle of the skewed Hooke's joint has been applied to the drive of a washing machine (see sketch).

Here, the driveshaft drives the worm wheel *I* which has a crank fashioned at an angle γ . The crank rides between two plates and causes the output shaft *III* to oscillate in accordance with the equation.

The dough-kneading drive is also based on the Hooke's joint, but it follows the path of link *2* to give a wobbling motion that kneads dough in the tank.



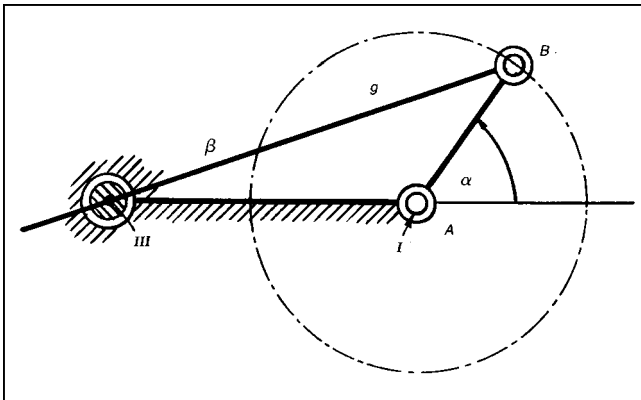
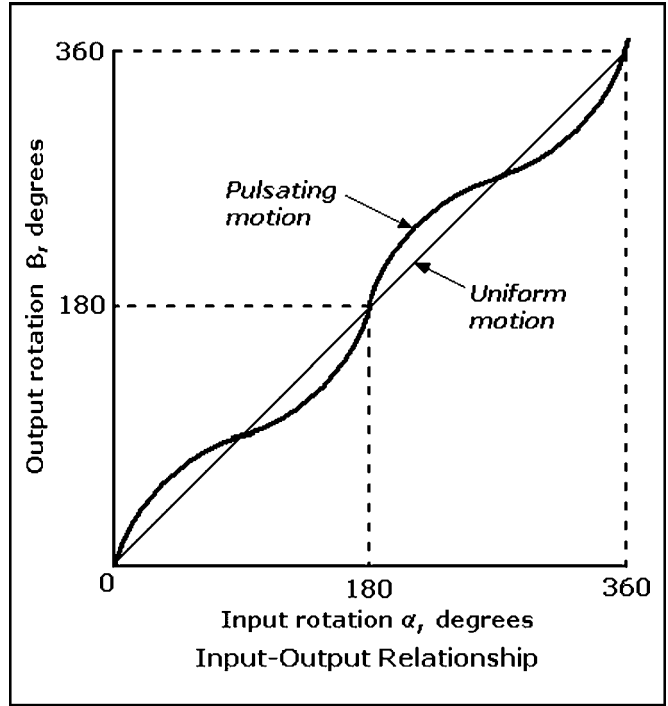
The Universal Joint Drive

The universal joint is a variation of the spherical-slide oscillator, but with angle $\gamma = 90^\circ$. This drive provides a totally rotating output and can be operated as a pair, as shown in the diagram.

The equation relating input with output for a single universal joint, where γ is the angle between the connecting link and shaft I, is:

$$\tan \beta = \tan \alpha \cos \delta$$

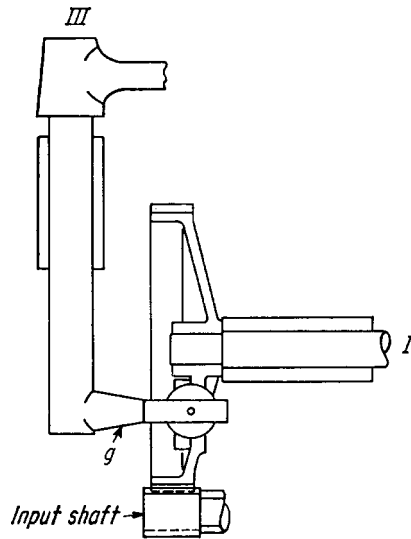
The output motion is pulsating (see curve) unless the joints are operated as pairs to provide a uniform motion.



The 3-D Crank Slide Drive

The three-dimensional crank slide is a variation of a plane crank slide (see sketch), with a ball point through which link g always slides, while a point B on link g describes a circle. A 3-D crank is obtained from this mechanism by shifting output shaft III so that it is not normal to the plane of the circle; another way to accomplish this is to make shafts I and III nonparallel.

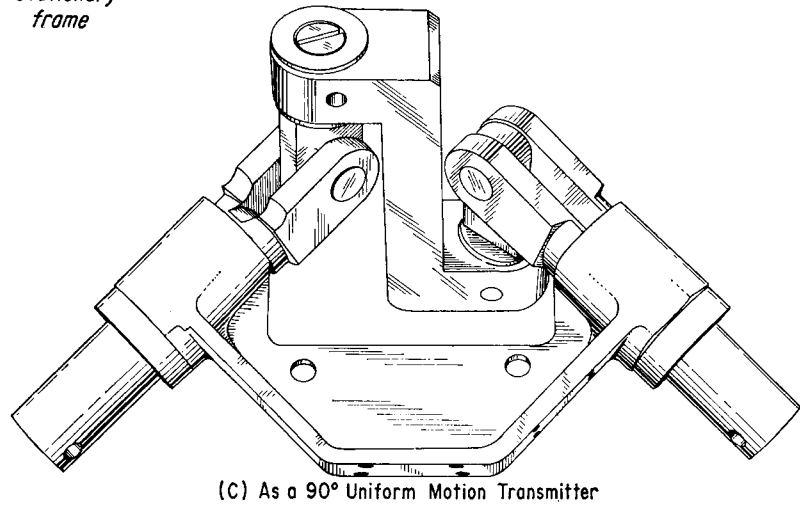
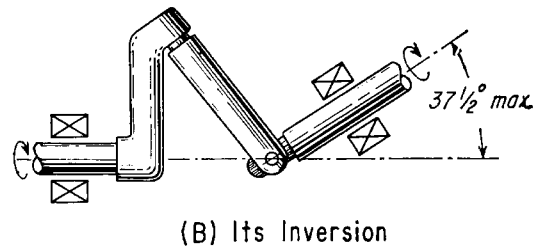
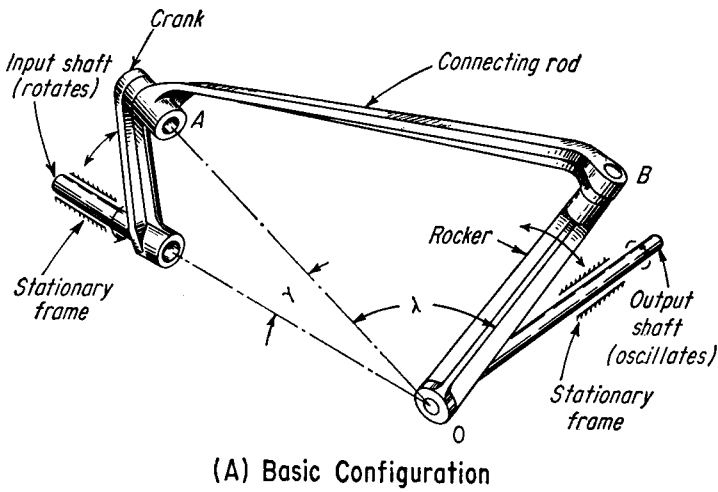
A practical variation of the 3-D crank slide is the agitator mechanism (see sketch). As input gear I rotates, link g



Agitator Mechanism

swivels around (and also lifts) shaft III . Hence, the vertical link has both an oscillating rotary motion and a sinusoidal harmonic translation in the direction of its axis of rotation. The link performs what is essentially a twisting motion in each cycle.

Three-Dimensional Linkage Drives (continued)



The Space Crank Drive

One of the more recent developments in 3-D linkages is the space crank shown in (A). It resembles the spherical crank, but has different output characteristics. The relationship between the input and output displacements is:

$$\cos \beta = (\tan \gamma)(\cos \alpha)(\sin \beta) - \frac{\cos \lambda}{\cos \gamma}$$

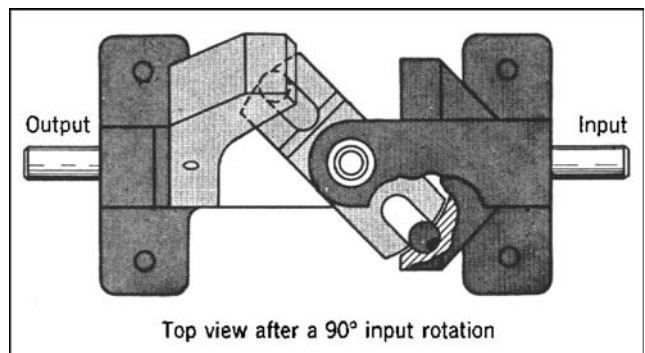
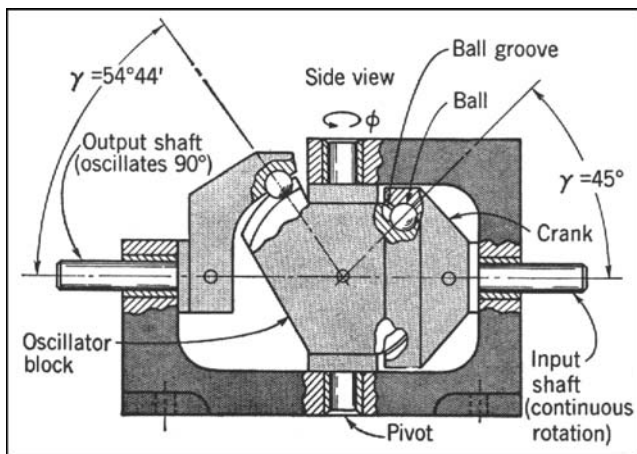
The velocity ratio is:

$$\frac{\omega_o}{\omega_i} = \frac{\tan \gamma \sin \alpha}{1 + \tan \gamma \cos \alpha \cot \beta}$$

where ω_o is the output velocity and ω_i is the constant input velocity.

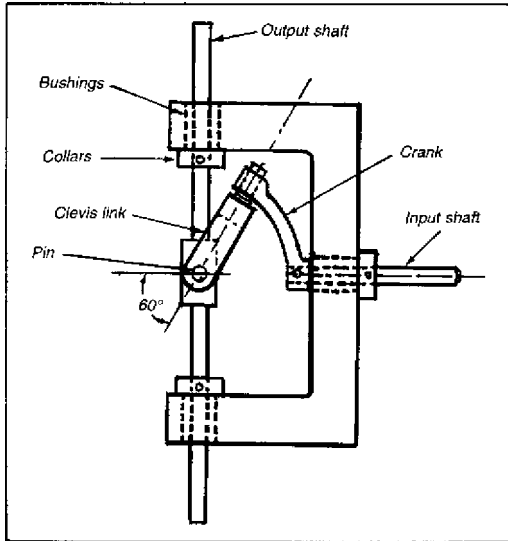
An inversion of the space crank is shown in (B). It can couple intersecting shafts, and permits either shaft to be driven with full rotations. Motion is transmitted up to 37 1/2° misalignment.

By combining two inversions (C), a method for transmitting an exact motion pattern around a 90° bend is obtained. This unit can also act as a coupler or, if the center link is replaced by a gear, it can drive two output shafts; in addition, it can transmit uniform motion around two bends.

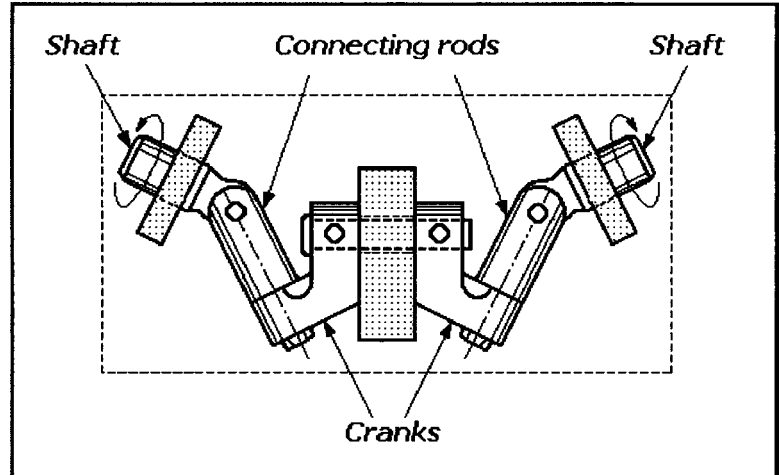


Steel balls riding within spherical grooves convert a continuous rotary input motion into an output that oscillates the shaft back and forth.

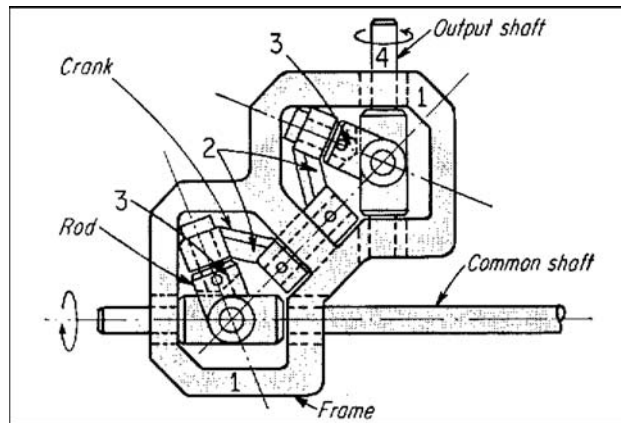
VARIATIONS OF THE SPACE CRANK



The oscillating motion is powered at right angles. The input shaft, in making full rotations, causes the output shaft to oscillate 120°.



A constant-speed-ratio universal is obtained by placing two "inversions" back-to-back. Motion is transmitted up to a 75° misalignment.



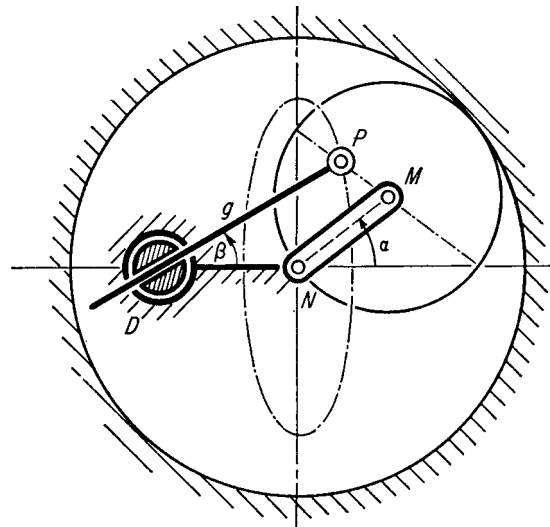
A right-angle limited-stroke drive transmits an exact motion pattern. A multiplicity of fittings can be operated from a common shaft.

The Elliptical Slide Drive

The output motion, β , of a spherical slide oscillator can be duplicated with a two-dimensional "elliptical slide." The mechanism has a link g that slides through a pivot point D and is fastened to a point P moving along an elliptical path. The ellipse can be generated by a Cardan drive, which is a planetary gear system whose planet gear has half the diameter of its internal gear. The center of the planet, point M , describes a circle; any point on its periphery describes a straight line, and any point in between, such as point P , describes an ellipse.

There are special relationships between the dimensions of the 3-D spherical slide and the 2-D elliptical slide: $\tan \gamma / \sin \delta = a/d$ and $\tan \gamma / \cot \delta = b/d$, where a is the major half-axis, b the minor half-axis of the ellipse, and d is the length of the fixed link DN . The minor axis lies along this link.

If point D is moved within the ellipse, a completely rotating output is obtained, corresponding to the rotating spherical crank slide.



THIRTEEN DIFFERENT TOGGLE LINKAGE APPLICATIONS

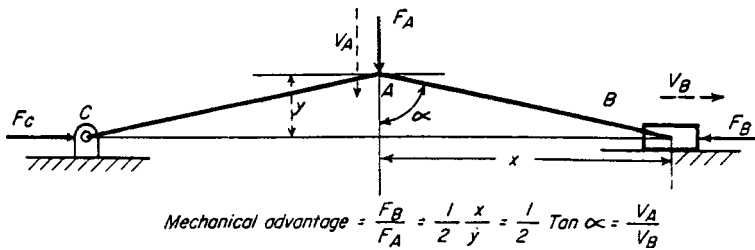


Fig. 1 Many mechanical linkages are based on the simple toggle that consists of two links which tend to line up in a straight line at one point in their motion. The mechanical advantage is the velocity ratio of the input point A with respect to the output point B: or V_A/V_B . As the angle is α approaches 90° , the links come into toggle, and the mechanical advantage and velocity ratio both approach infinity. However, frictional effects reduce the forces to much less than infinity, although they are still quite high.

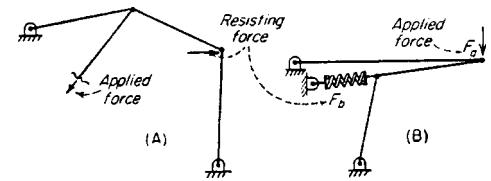


Fig. 2 Forces can be applied through other links, and need not be perpendicular to each other. (A) One toggle link can be attached to another link rather than to a fixed point or slider. (B) Two toggle links can come into toggle by lining up on top of each other rather than as an extension of each other. The resisting force can be a spring.

HIGH MECHANICAL ADVANTAGE

Fig. 3 In punch presses, large forces are needed at the lower end of the work stroke. However, little force is required during the remainder of the stroke. The crank and connecting rod come into toggle at the lower end of the punch stroke, giving a high mechanical advantage at exactly the time it is most needed.

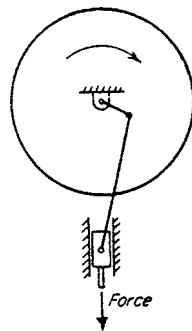


Fig. 4 A cold-heading rivet machine is designed to give each rivet two successive blows. Following the first blow (point 2) the hammer moves upward a short distance (to point 3). Following the second blow (at point 4), the hammer then moves upward a longer distance (to point 1) to provide clearance for moving the work-piece. Both strokes are produced by one revolution of the crank, and at the lowest point of each stroke (points 2 and 4) the links are in toggle.

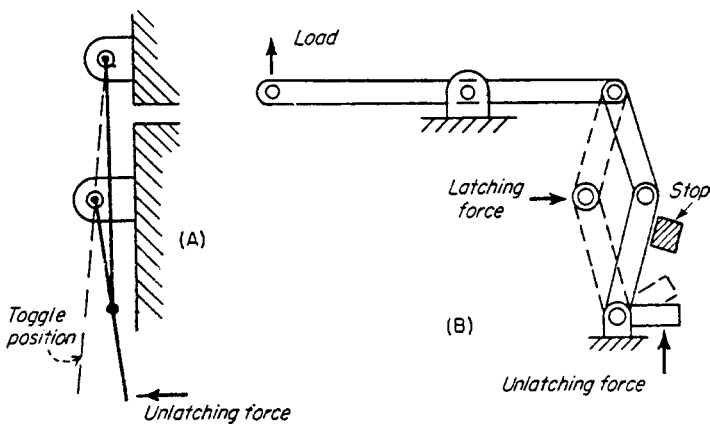
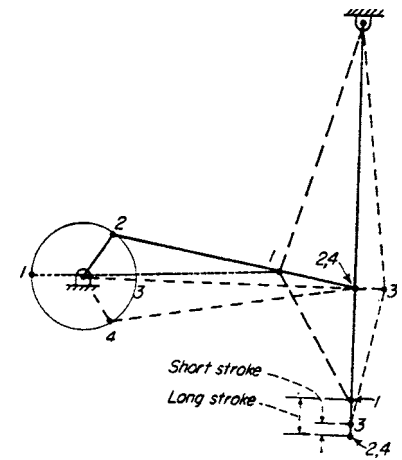


Fig. 5 Locking latches produce a high mechanical advantage when in the toggle portion of the stroke. A simple latch exerts a large force in the locked position (Fig. 5A). For positive locking, the closed position of latch is slightly beyond the toggle position. A small unlatching force opens the linkage (Fig. 5B).

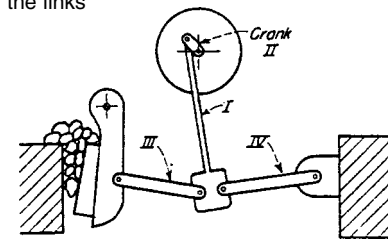
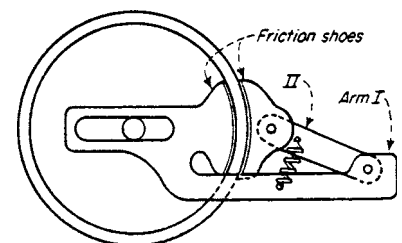


Fig. 6 A stone crusher has two toggle linkages in series to obtain a high mechanical advantage. When the vertical link I reaches the top of its stroke, it comes into toggle with the driving crank II; at the same time, link III comes into toggle and link IV. This multiplication results in a very large crushing force.

Fig. 7 A friction ratchet is mounted on a wheel; a light spring keeps the friction shoes in contact with the flange. This device permits clockwise motion of the arm I. However, reverse rotation causes friction to force link II into toggle with the shoes. This action greatly increases the locking pressure.



HIGH VELOCITY RATIO

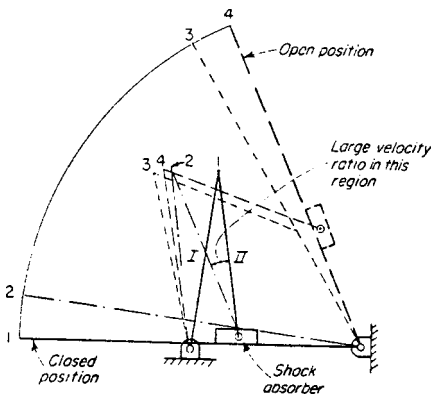


Fig. 8

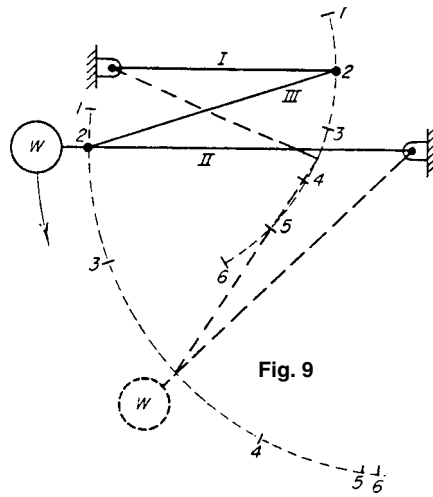


Fig. 9

Fig. 8 Door check linkage gives a high velocity ratio during the stroke. As the door swings closed, connecting link I comes into toggle with the shock absorber arm II, giving it a large angular velocity. The shock absorber is more effective in retarding motion near the closed position.

Fig. 9 An impact reducer is on some large circuit breakers. Crank I rotates at constant velocity while the lower crank moves slowly at the beginning and end of the stroke. It moves rapidly at the mid-stroke when arm II and link III are in toggle. The accelerated weight absorbs energy and returns it to the system when it slows down.

VARIABLE MECHANICAL ADVANTAGE

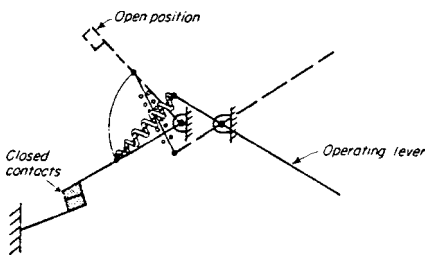


Fig. 10 A toaster switch has an increasing mechanical advantage to aid in compressing a spring. In the closed position, the spring holds the contacts closed and the operating lever in the down position. As the lever is moved upward, the spring is compressed and comes into toggle with both the contact arm and the lever. Little effort is required to move the links through the toggle position; beyond this point, the spring snaps the contacts open. A similar action occurs on closing.

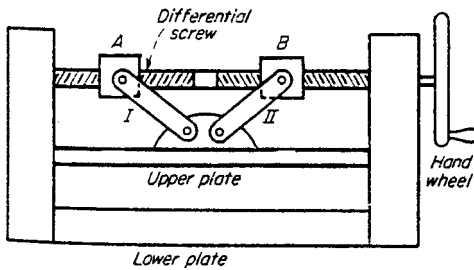


Fig. 11 A toggle press has an increasing mechanical advantage to counteract the resistance of the material being compressed. A rotating handwheel with a differential screw moves nuts A and B together, and links I and II are brought into toggle.

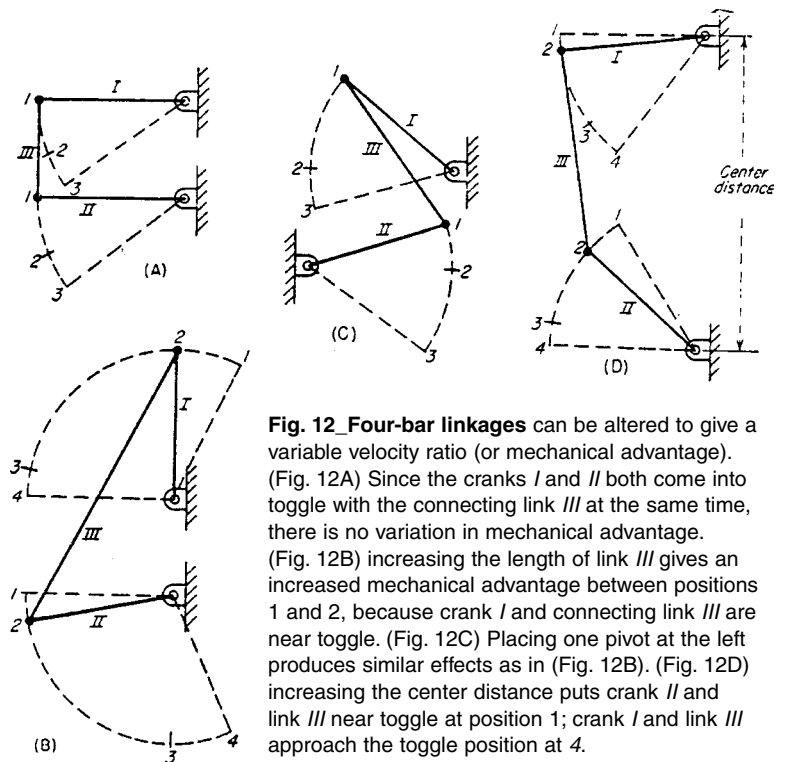


Fig. 12 Four-bar linkages can be altered to give a variable velocity ratio (or mechanical advantage). (Fig. 12A) Since the cranks I and II both come into toggle with the connecting link III at the same time, there is no variation in mechanical advantage. (Fig. 12B) increasing the length of link III gives an increased mechanical advantage between positions 1 and 2, because crank I and connecting link III are near toggle. (Fig. 12C) Placing one pivot at the left produces similar effects as in (Fig. 12B). (Fig. 12D) increasing the center distance puts crank II and link III near toggle at position 1; crank I and link III approach the toggle position at 4.

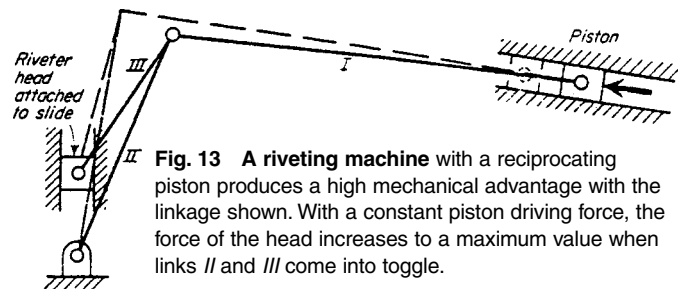


Fig. 13 A riveting machine with a reciprocating piston produces a high mechanical advantage with the linkage shown. With a constant piston driving force, the force of the head increases to a maximum value when links II and III come into toggle.

HINGED LINKS AND TORSION BUSHINGS

SOFT-START DRIVES

Centrifugal force automatically draws up the linkage legs, while the torsional resistance of the bushings opposes the deflection forces.

A spidery linkage system combined with a rubber torsion bushing system formed a power-transmission coupling. Developed by a British company, Twiflex Couplings Ltd., Twickenham, England, the device (drawing below) provides ultra-soft starting characteristics. In addition to the torsion system, it also depends on centrifugal force to draw up the linkage legs automatically, thus providing additional soft coupling at high speeds to absorb and isolate any torsional vibrations arising from the prime mover.

The TL coupling has been installed to couple marine main engines to gearbox-propeller systems. Here the coupling reduces propeller vibrations to negligible proportions even at high critical speeds. Other applications are also foreseen, including their use in diesel drives, machine tools, and off-the-road construction equipment. The coupling's range is from 100 hp to 4000 rpm to 20,000 hp at 400 rpm.

Articulating links. The key factor in the TL coupling, an improvement over an earlier Twiflex design, is the circular grouping of hinged linkages connecting the driving and driven coupling flanges. The forked or tangential links have resilient precompressed bonded-rubber bushings at the outer flange attachments, while the other pivots ride on bearings.

When torque is applied to the coupling, the linkages deflect in a positive or negative direction from the neutral position (drawings, below). Deflection is opposed by the torsional resistance of the rubber bushings at the outer pins. When the coupling is rotating, the masses of the linkage give rise to centrifugal forces that further oppose coupling deflection. Therefore, the working position of the linkages depends both on the applied torque and on the speed of the coupling's rotation.

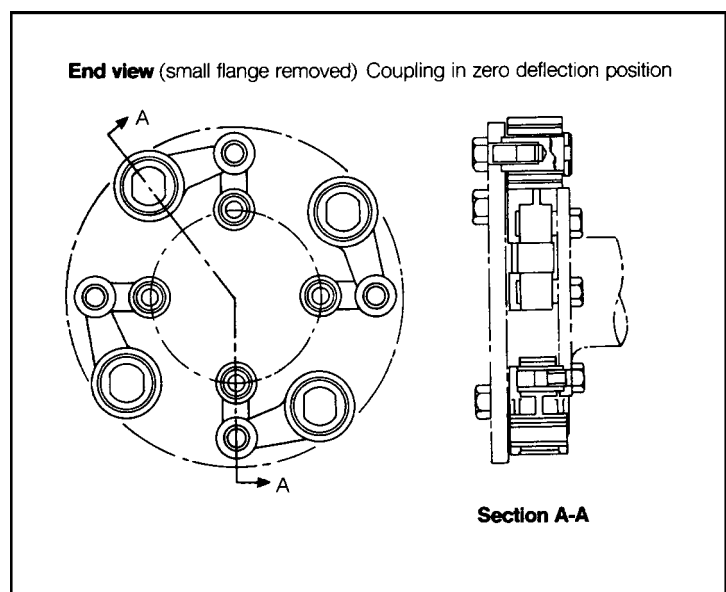
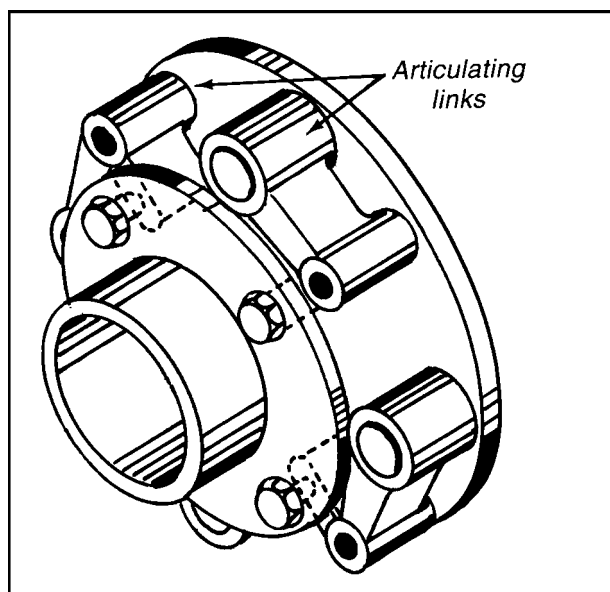
Tests of the coupling's torque/deflection characteristics under load have shown that the torsional stiffness of the coupling increases

progressively with speed and with torque when deflected in the positive direction. Although the geometry of the coupling is asymmetrical the torsional characteristics are similar for both directions of drive in the normal working range. Either half of the coupling can act as the driver for either direction of rotation.

The linkage configuration permits the coupling to be tailored to meet the exact stiffness requirements of individual systems or to provide ultra-low torsional stiffness at values substantially softer than other positive-drive couplings. These characteristics enable the Twiflex coupling to perform several tasks:

- It detunes the fundamental mode of torsional vibration in a powertransmission system. The coupling is especially soft at low speeds, which permits complete detuning of the system.
- It decouples the driven machinery from engine-excited torsional vibration. In a typical geared system, the major machine modes driven by the gearboxes are not excited if the ratio of coupling stiffness to transmitted torque is less than about 7:1—a ratio easily provide by the Twiflex coupling.
- It protects the prime mover from impulsive torques generated by driven machinery. Generator short circuits and other causes of impulsive torques are frequently of sufficient duration to cause high response torques in the main shafting.

Using the example of the TL 2307G coupling design—which is suitable for 10,000 hp at 525 rpm—the torsional stiffness at working points is largely determined by coupling geometry and is, therefore, affected to a minor extent by the variations in the properties of the rubber bushings. Moreover, the coupling can provide torsional-stiffness values that are accurate within 5.0%.



Articulating links of the new coupling (left) are arranged around the driving flanges. A four-link design (right) can handle torques from a 100-hp prime mover driving at 4000 rpm.

EIGHT LINKAGES FOR BAND CLUTCHES AND BRAKES

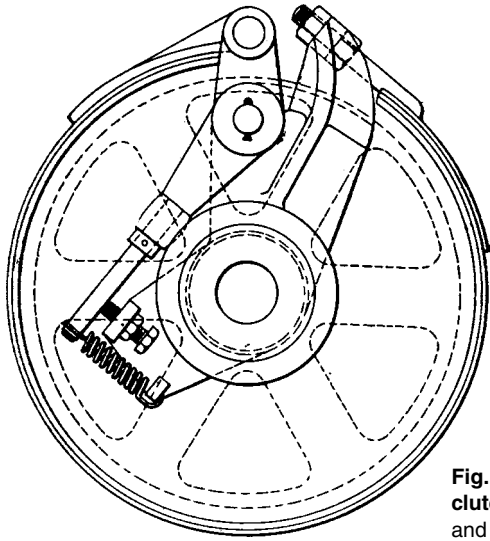


Fig. 1 An outside band clutch operated by a roller and cone.

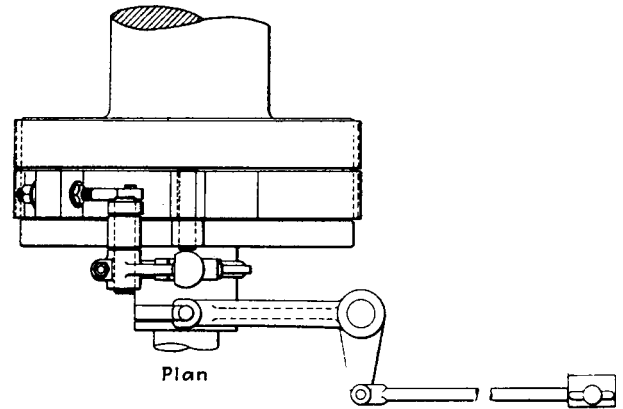


Fig. 3 An outside band clutch made as two full wrap bands with an intermediate equalizer.

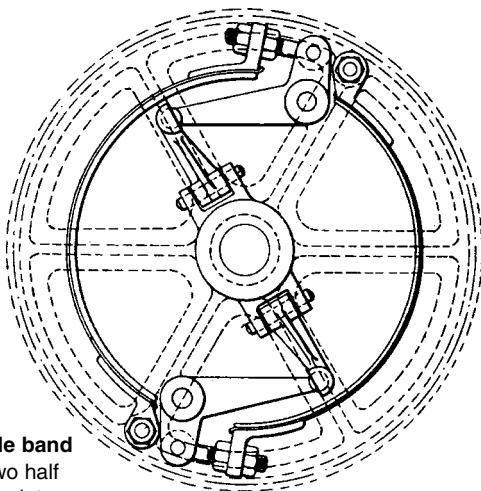


Fig. 2 An outside band clutch made as two half wrap bands with an intermediate equalizer.

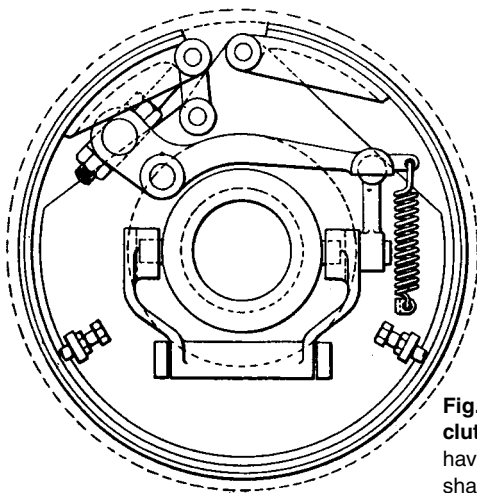
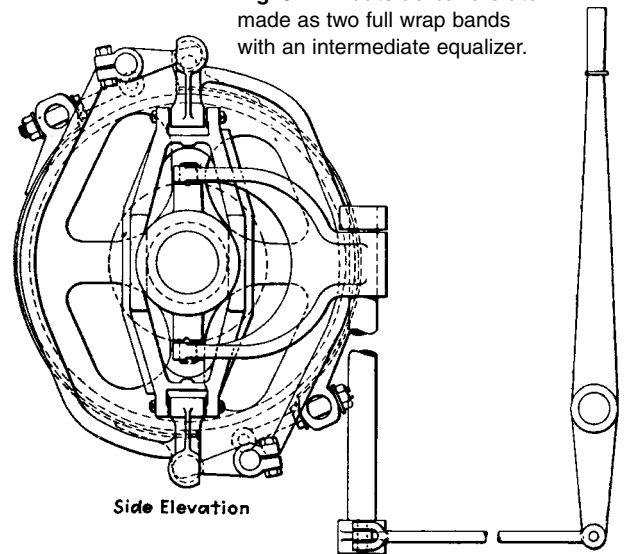


Fig. 4 An inside band clutch operated by a yoke having movement along the shaft.

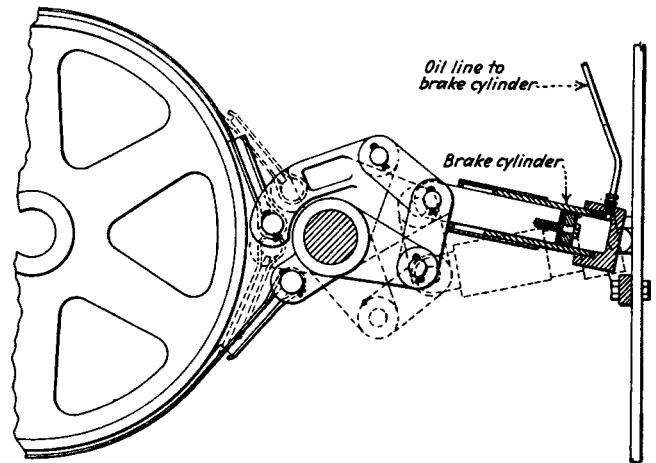


Fig. 5 A two-way acting band brake operated hydraulically.

Fig. 6 A hoist-drum band brake operated by a foot pedal.

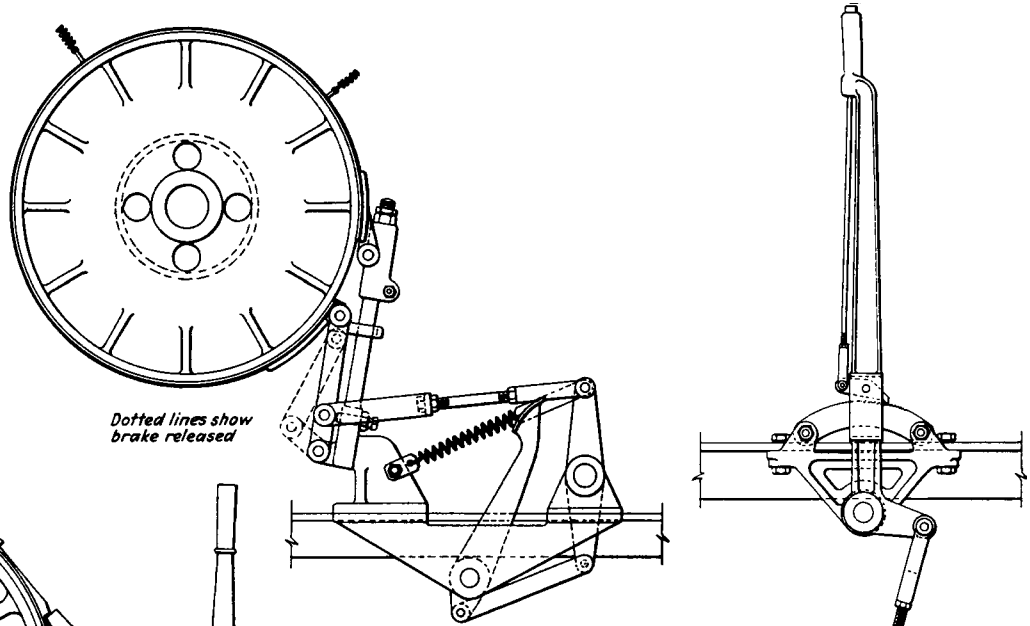


Fig. 7 A band brake with a single toggle action.

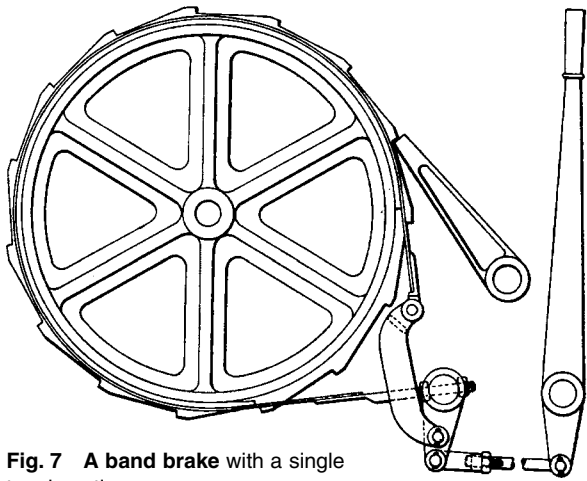
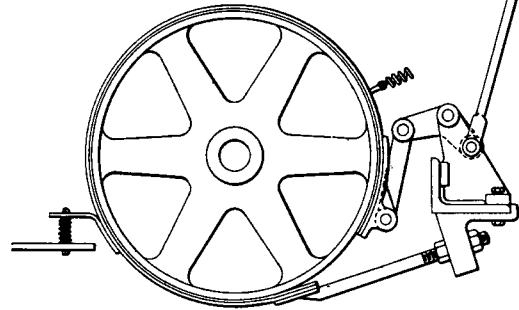


Fig. 8 A crawler-drive band brake operated by a ratchet lever.



DESIGN OF CRANK-AND-ROCKER LINKS FOR OPTIMUM FORCE TRANSMISSION

Four-bar linkages can be designed with a minimum of trial and error by a combination of tabular and iteration techniques.

The determination of optimum crank-and-rocker linkages has most effectively been performed on a computer because of the complexity of the equations and calculations involved. Thanks to the work done at Columbia University's Department of Mechanical and Nuclear Engineering, all you need now is a calculator and the computer-generated tables presented here. The computations were done by Mr. Meng-Sang Chew, at the university.

A crank-and-rocker linkage, $ABCD$, is shown in the first figure. The two extreme positions of the rocker are shown schematically in the second figure. Here ψ denotes the rocker swing angle and ϕ denotes the corresponding crank rotation, both measured counterclockwise from the extended dead-center position, AB_1C_1D .

The problem is to find the proportions of the crank-and-rocker linkage for a given rocker swing angle, ψ , a prescribed corresponding crank rotation, ϕ , and optimum force transmission. The latter is usually defined in terms of the transmission angle, m , the angle μ between coupler BC extended and rocker CD .

Considering static forces only, the closer the transmission angle is to 90° , the greater is the ratio of the driving component of the force exerted on the rocker to the component exerting bearing pressure on the rocker. The control of transmission-angle variation becomes especially important at high speeds and in heavy-duty applications.

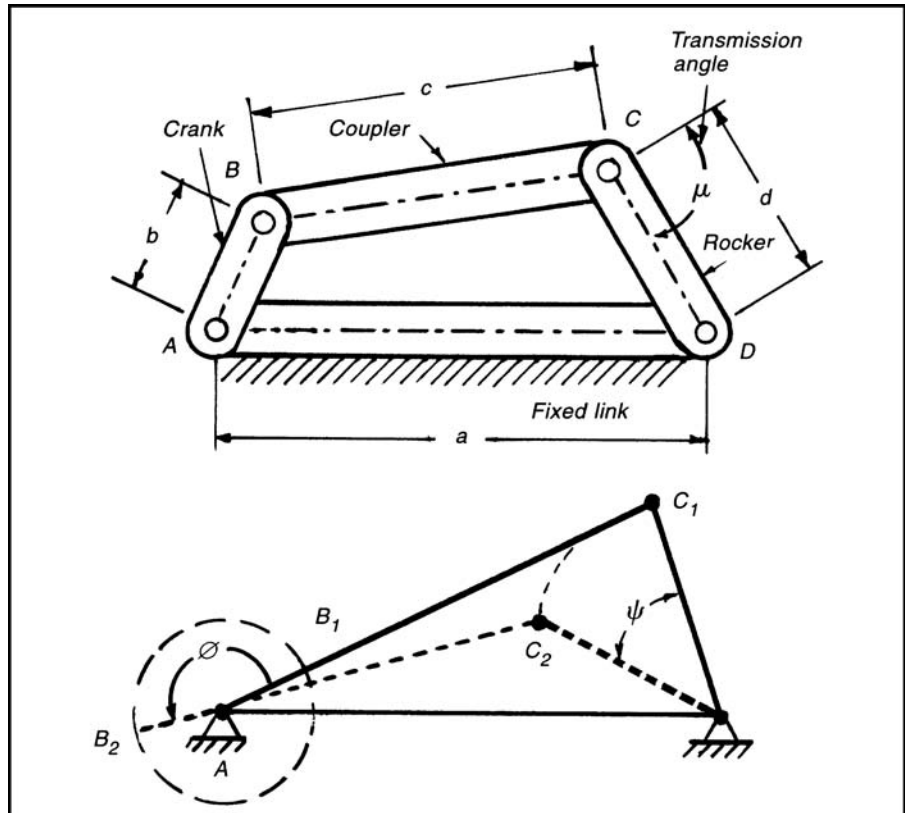
How to find the optimum. The steps in the determination of crank-and-rocker proportions for a given rocker swing angle, corresponding crank rotation, and optimum transmission, are:

- Select (ψ, ϕ) within the following range:

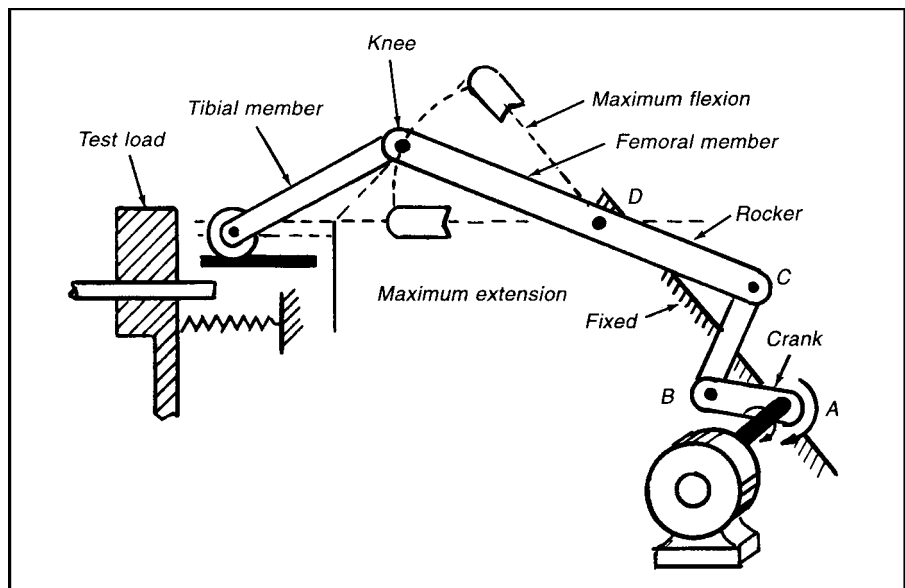
$$0^\circ < \psi < 180^\circ$$

$$(90^\circ + 1/2 \psi) < \phi < (270^\circ + 1/2 \psi)$$

- Calculate: $t = \tan 1/2 \phi$
 $u = \tan 1/2 (\phi - \psi)$
 $v = \tan 1/2 \psi$



The optimum solution for the classic four-bar crank-and-rocker mechanism problem can now be obtained with only the accompanying table and a calculator.



An example in this knee-joint tester designed and built by following the design and calculating procedures outlined in this article.

Designing Crank-and-Rocker Links (continued)

- Using the table, find the ratio λ_{opt} of coupler to crank length that minimizes the transmission-angle deviation from 90° . The most practical combinations of (ψ, ϕ) are included in the table. If the (ψ, ϕ) combination is not included, or if $\phi = 180^\circ$, go to next steps (a,b,c):
- (a) If $\phi = 180^\circ$ and (ψ, ϕ) fall outside the range given in the table, determine the arbitrary intermediate value Q from the equation: $Q^3 + 2Q^2 - t^2Q - (t^2/u^2)(1 + t^2) = 0$ where $(1/u^2 < Q < t^2)$. This is conveniently accomplished by numerical iteration:

Set
$$Q_1 = \frac{1}{2} \left(t^2 + \frac{1}{u^2} \right)$$

Calculate Q_2, Q_3, \dots from the recursion equation:

$$Q_{i+1} = \frac{2Q_i^2(Q_i + 1) + (t^2/u^2)(1 + t^2)}{Q_i(3Q_i + 4) - t^2}$$

Iterate until the ratio $[(Q_i + 1 - Q_i)/Q_i]$ is sufficiently small, so that you obtain the desired number of significant figures. Then:

$$\lambda_{opt} = t^2/Q$$

(b) If $\phi = 180^\circ$ and the determination of λ_{opt} requires interpolation between two entries in the table, let $Q_1 = t^2 \lambda^2$, where λ corresponds to the nearest entry in the table, and continue as in (a) above to determine Q and λ_{opt} . Usually one or two iterations will suffice.

(c) $\phi = 180^\circ$. In this case, $a^2 + b^2 = c^2 + d^2$; $\psi = 2 \sin^{-1}(b/d)$; and the maximum

deviation, Δ , of the transmission angle from 90° is equal to $\sin^{-1}(ab/cd)$.

- Determine linkage proportions as follows:

$$(a')^2 = \frac{u^2 + \lambda_{opt}^2}{1 + u^2}$$

$$(b')^2 = \frac{v^2}{1 + v^2}$$

$$(c')^2 = \frac{\lambda_{opt}^2 v^2}{1 + v^2}$$

$$(d')^2 = \frac{t^2 + \lambda_{opt}^2}{1 + t^2}$$

Then: $a = ka'$; $b = kb'$; $c = kc'$; $d = kd'$ where k is a scale factor, such that the length of any one link, usually the crank, is equal to a design value. The

Optimum values of lambda ratio for given ϕ and ψ

ϕ deg	ψ , deg									
	160	162	164	166	168	170	172	174	176	178
10	2.3532	2.4743	2.6166	2.7873	2.9978	3.2669	3.6284	4.1517	5.0119	6.8642
12	2.3298	2.4491	2.5891	2.7570	2.9636	3.2272	3.5804	4.0899	4.9224	6.6967
14	2.3064	2.4239	2.5617	2.7266	2.9293	3.1874	3.5324	4.0283	4.8342	6.5367
16	2.2831	2.3988	2.5344	2.6964	2.8953	3.1479	3.4848	3.9675	4.7482	6.3853
18	2.2600	2.3740	2.5073	2.6664	2.8615	3.1089	3.4380	3.9080	4.6650	6.2427
20	2.2372	2.3494	2.4805	2.6368	2.8282	3.0704	3.3920	3.8499	4.5848	6.1087
22	2.2145	2.3250	2.4540	2.6076	2.7954	3.0327	3.3470	3.7935	4.5077	5.9826
24	2.1922	2.3010	2.4279	2.5789	2.7631	2.9956	3.3030	3.7388	4.4338	5.8641
26	2.1701	2.2773	2.4022	2.5505	2.7314	2.9594	3.2602	3.6857	4.3628	5.7524
28	2.1483	2.2539	2.3768	2.5227	2.7004	2.9239	3.2185	3.6344	4.2948	5.6469
30	2.1268	2.2309	2.3519	2.4954	2.6699	2.8893	3.1779	3.5847	4.2295	5.5472
32	2.1056	2.2082	2.3273	2.4685	2.6401	2.8554	3.1384	3.5367	4.1668	5.4526
34	2.0846	2.1858	2.3032	2.4421	2.6108	2.8223	3.0999	3.4901	4.1066	5.3628
36	2.0640	2.1637	2.2794	2.4162	2.5821	2.7899	3.0624	3.4449	4.0486	5.2773
38	2.0436	2.1420	2.2560	2.3908	2.5540	2.7583	3.0259	3.4012	3.9927	5.1957
40	2.0234	2.1205	2.2330	2.3657	2.5264	2.7274	2.9903	3.3587	3.9388	5.1177
42	2.0035	2.0994	2.2103	2.3411	2.4994	2.6971	2.9556	3.3175	3.8868	5.0430
44	1.9839	2.0785	2.1879	2.3169	2.4728	2.6675	2.9217	3.2773	3.8364	4.9712
46	1.9644	2.0579	2.1659	2.2931	2.4468	2.6384	2.8886	3.2383	3.7877	4.9023
48	1.9452	2.0375	2.1441	2.2696	2.4211	2.6100	2.8563	3.2003	3.7404	4.8358
50	1.9262	2.0174	2.1227	2.2465	2.3959	2.5820	2.8246	3.1632	3.6945	4.7717

max deviation, Δ , of the transmission angle from 90° is:

$$\sin \Delta = \frac{|(a \pm b)^2 - c^2 - d^2|}{2cd}$$

$$0^\circ \leq \Delta \leq 90^\circ$$

+ sign if $\phi < 180^\circ$
- sign if $\phi > 180^\circ$

An actual example. A simulator for testing artificial knee joints, built by the Department of Orthopedic Surgery, Columbia University, under the direction of Dr. N. Eftekhari, is shown schematically. The drive includes an adjustable crank-and-rocker, ABCD. The rocker swing angle ranges from a maximum of about 48° to a minimum of about one-third of this value. The crank is 4 in. long and rotates at 150 rpm. The swing angle

adjustment is obtained by changing the length of the crank.

Find the proportions of the linkage, assuming optimum-transmission proportions for the maximum rocker swing angle, as this represents the most severe condition. For smaller swing angles, the maximum transmission-angle deviation from 90° will be less.

Crank rotation corresponding to 48° rocker swing is selected at approximately 170° . Using the table, find $\lambda_{opt} = 2.6100$. This gives $a' = 1.5382$, $b' = 0.40674$, $c' = 1.0616$, and $d' = 1.0218$.

For a 4 in. crank, $k = 4/0.40674 = 9.8343$ and $a = 15.127$ in., $b = 4$ in., $c = 10.440$ in., and $d = 10.049$ in., which is very close to the proportions used. The maximum deviation of the transmission angle from 90° is 47.98° .

This procedure applies not only for the transmission optimization of crank-and-rocker linkages, but also for other crank-and-rocker design. For example, if only the rocker swing angle and the corresponding crank rotation are prescribed, the ratio of coupler to crank length is arbitrary, and the equations can be used with any value of λ_2 within the range $(1, u^2t^2)$. The ratio λ can then be tailored to suit a variety of design requirements, such as size, bearing reactions, transmission-angle control, or combinations of these requirements.

The method also was used to design dead-center linkages for aircraft landing-gear retraction systems, and it can be applied to any four-bar linkage designs that meet the requirements discussed here.

Optimum values of lambda ratio for given ϕ and ψ

ϕ deg	ψ , deg									
	182	184	186	188	190	192	194	196	198	200
10	7.2086	5.3403	4.4560	3.9112	3.5318	3.2478	3.0245	2.8428	2.6911	2.5616
12	7.0369	5.2692	4.4227	3.8969	3.5282	3.2507	3.0317	2.8528	2.7030	2.5748
14	6.8646	5.1881	4.3795	3.8739	3.5174	3.2478	3.0341	2.8589	2.7117	2.5855
16	6.6971	5.1013	4.3287	3.8435	3.5000	3.2392	3.0317	2.8610	2.7171	2.5934
18	6.5371	5.0121	4.2726	3.8071	3.4768	3.2252	3.0245	2.8589	2.7189	2.5982
20	6.3857	4.9226	4.2131	3.7663	3.4487	3.2065	3.0129	2.8528	2.7171	2.5998
22	6.2431	4.8344	4.1518	3.7221	3.4167	3.1837	2.9972	2.8428	2.7117	2.5982
24	6.1090	4.7484	4.0900	3.6759	3.3818	3.1575	2.9780	2.8293	2.7030	2.5934
26	5.9830	4.6652	4.0284	3.6284	3.3447	3.1286	2.9558	2.8127	2.6911	2.5855
28	5.8644	4.5849	3.9676	3.5804	3.3062	3.0976	2.9311	2.7833	2.6763	2.5748
30	5.7527	4.5079	3.9080	3.5324	3.2669	3.0652	2.9045	2.7718	2.6592	2.5616
32	5.6472	4.4339	3.8500	3.4849	3.2272	3.0318	2.8764	2.7484	2.6399	2.5461
34	5.5475	4.3630	3.7936	3.4380	3.1875	2.9979	2.8473	2.7236	2.6190	2.5287
36	5.4529	4.2949	3.7388	3.3920	3.1480	2.9636	2.8175	2.6977	2.5967	2.5097
38	5.3631	4.2296	3.6858	3.3470	3.1089	2.9294	2.7873	2.6711	2.5734	2.4894
40	5.2776	4.1669	3.6345	3.3031	3.0705	2.8953	2.7570	2.6440	2.5492	2.4680
42	5.1960	4.1067	3.5848	3.2602	3.0327	2.8615	2.7266	2.6166	2.5246	2.4459
44	5.1180	4.0487	3.5367	3.2185	2.9956	2.8282	2.6964	2.5891	2.4996	2.4232
46	5.0432	3.9928	3.4901	3.1779	2.9594	2.7954	2.6665	2.5617	2.4744	2.4001
48	4.9715	3.9389	3.4450	3.1384	2.9239	2.7631	2.6369	2.5344	2.4491	2.3767
50	4.9025	3.8869	3.4012	3.0999	2.8893	2.7314	2.6076	2.5073	2.4239	2.3533

DESIGN OF FOUR-BAR LINKAGES FOR ANGULAR MOTION

How to use four-bar linkages to generate continuous or intermittent angular motions required by feeder mechanisms

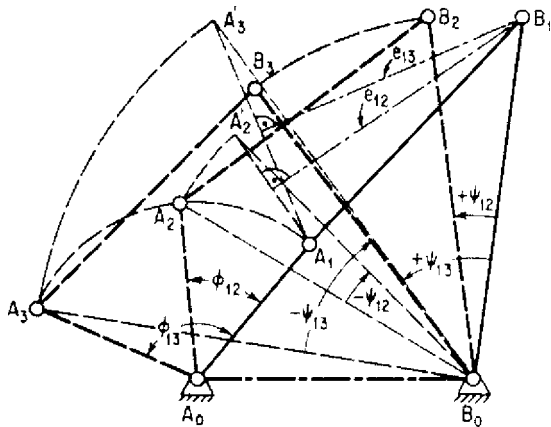


Fig. 1 Four-bar linkage synchronizes two angular movements, ϕ_{12} and ϕ_{13} , with ψ_{12} , and ψ_{13} .

In putting feeder mechanisms to work, it is often necessary to synchronize two sets of angular motions. A four-bar linkage offers one way. For example, in Fig. 1 two angular motions, ϕ_{12} and ϕ_{13} , must be synchronized with two others, ψ_{12} and ψ_{13} , about the given pivot points A_0 and B_0 and the given crank length A_0A_1 . This means that crank length B_0B_1 must be long enough so that the resulting four-bar linkage will coordinate angular motions ϕ_{12} and ϕ_{13} with ψ_{12} and ψ_{13} . The procedure is:

1. Obtain point A'_2 by revolving A_2 about B_0 through angle $-\psi_{12}$ but in the opposite direction.
2. Obtain point A'_3 similarly by revolving A_3 about B_0 through angle $-\psi_{13}$.
3. Draw lines $A_1A'_2$ and $A_1A'_3$ and the perpendicular bisectors of the lines which intersect at desired point B_1 .
4. The quadrilateral $A_0A_1B_1B_0$ represents the four-bar linkage that will produce the required relationship between the angles ϕ_{12} , ϕ_{13} , and ψ_{12} , ψ_{13} .

Three angles with four relative positions can be synchronized in a similar way. Figure 2 shows how to synchronize angles ψ_{12} , ψ_{13} , ψ_{14} with corresponding angles ϕ_{12} , ϕ_{13} , and ϕ_{14} , using freely chosen pivot points A_0 and B_0 . In this case, crank length A_0A_1 as well as B_0B_1 is to be determined, and the procedure is:

1. Locate pivot points A_0 and B_0 on a line that bisects angle $A_3A_0A_4$, the length A_0B_0 being arbitrary.
2. Measure off $1/2$ of angle $B_3B_0B_4$ and with this angle draw B_0A_4 which establishes crank length A_0A_1 at intersection of A_0A_4 . This also establishes points A_3 , A_2 and A_1 .
3. With B_0 as center and B_0B_4 as radius mark off angles $-\psi_{14}$, $-\psi_{13}$, $-\psi_{12}$, the negative sign indicating they are in opposite sense to ψ_{14} , ψ_{13} and ψ_{12} . This establishes points A'_2 , A'_3 and A'_4 , but here A'_3 and A'_4 coincide because of symmetry of A_3 and A_4 about A_0B_0 .
4. Draw lines $A_1A'_2$ and $A_1A'_4$, and the perpendicular bisectors of these lines, which intersect at the desired point B_1 .
5. The quadrilateral $A_0A_1B_1B_0$ represents the four-bar linkage that will produce the required relationship between the angles ϕ_{12} , ϕ_{13} , ϕ_{14} , and ψ_{12} , ψ_{13} , ψ_{14} .

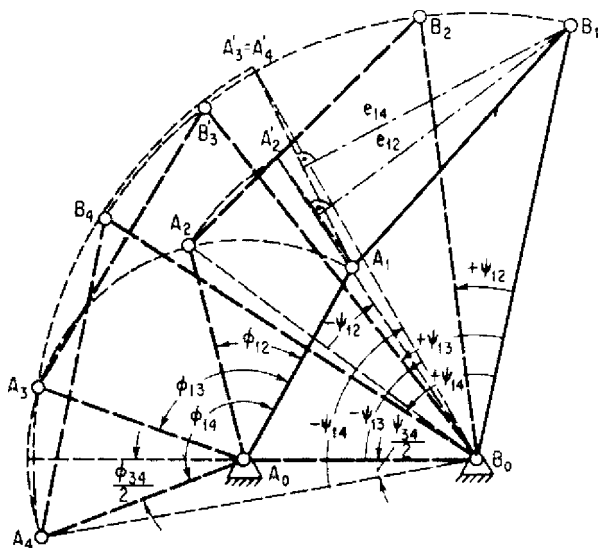


Fig. 2 Three angular positions, ϕ_{12} , ϕ_{13} , ϕ_{14} , are synchronized by four-bar linkage here with ψ_{12} , ψ_{13} , and ψ_{14} .

The illustrations show how these angles must be coordinated within the given space. In Fig. 3A, input angles of the crank must be coordinated with the output angles of the forked escapement. In Fig. 3B, input angles of the crank are coordinated with the output angles of the tilting hopper. In Fig. 3C, the input angles of the crank are coordinated with the output angles of the segment. In Fig. 3D, a box on a conveyor is tilted 90° by an output crank, which is actuated by an input crank through a coupler. Other mechanisms shown can also coordinate the input and output angles; some have dwell periods between the cycles, others give a linear output with dwell periods.

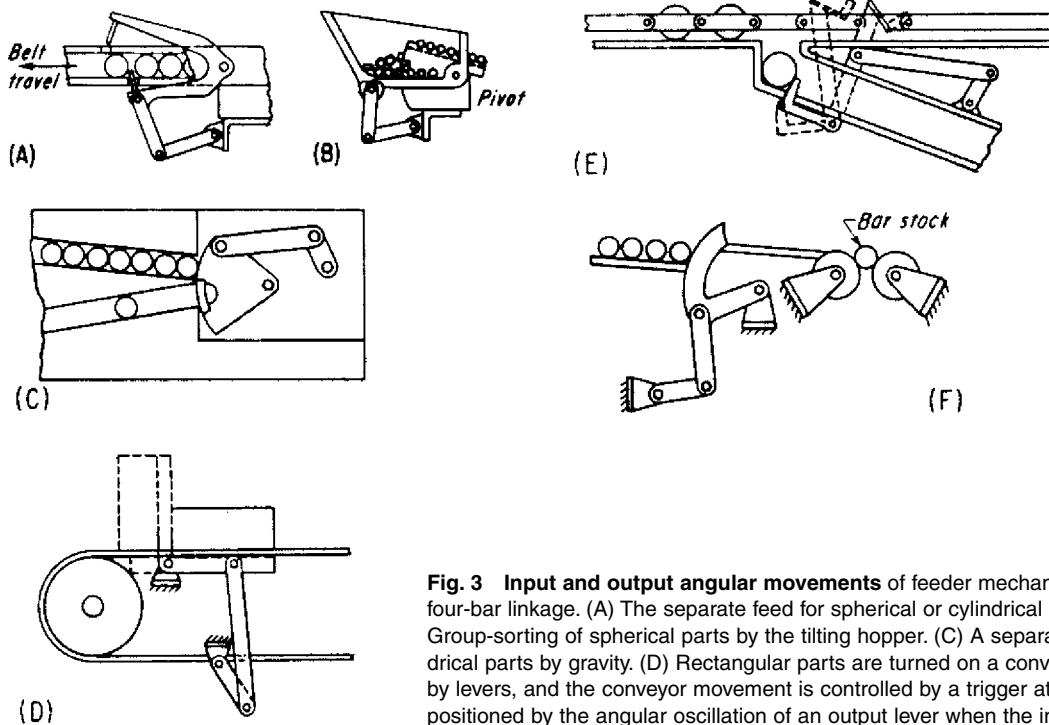


Fig. 3 Input and output angular movements of feeder mechanisms are synchronized by a four-bar linkage. (A) The separate feed for spherical or cylindrical parts on a conveyor. (B) Group-sorting of spherical parts by the tilting hopper. (C) A separate feed for spherical or cylindrical parts by gravity. (D) Rectangular parts are turned on a conveyor. (E) Parts are separated by levers, and the conveyor movement is controlled by a trigger at the right. (F) Bar stock is positioned by the angular oscillation of an output lever when the input crank is actuated.

MULTIBAR LINKAGES FOR CURVILINEAR MOTIONS

Four-bar linkages can be combined into six, eight, or more linkages for the feeder mechanisms in cameras, automatic lathes, farm machinery, and torch-cutting machines.

When feeder mechanisms require complex curvilinear motions, it might be necessary to use compound linkages rather than four links. However, four-bar linkages can be synthesized to produce curvilinear motions of various degrees of complexity, and all possibilities for four-bar linkages should be considered before selecting more complex linkages.

For example, a camera film-advancing mechanism, Fig. 1, has a simple four-bar linkage with a coupler point d , which generates a curvilinear and straight-line motion a resembling a D. Another more complex curvilinear motion, Fig. 2, is also generated by a coupler point E of a four-bar linkage, which controls an automatic profile cutter. Four-bar linkages can generate many different curvilinear motions, as in Fig. 3. Here the points of the coupler prongs, g_1 , g_2 , and g_3 on coupler b , and g_4 and g_5 on coupler e , are chosen so that their motions result in the desired progressive feeding of straw into a press.

A similar feeding and elevating device is shown in Fig. 4. The rotating device crank a moves coupler b and swinging lever c , which actuates the guiding arm f through the link e . The bar h carries the prone fingers g_1 through g_7 . They generate coupler curves a_1 through a_7 .

As another practical example, consider the torch-cutting machine in Fig. 5A designed to cut sheet metal along a curvilinear path a . Here the points A_0 and B_0 are fixed in the machine, and the lever A_0A_1 has an adjustable length to suit the different curvilinear paths a desired.

The length B_1B_1 is also fixed. The challenge is to find the length of the levers A_1B_1 and E_1B_1 in the four-bar linkage to give the desired path a , which is to be traced by the coupler point E on which the cutting torch is mounted.

The graphical solution for this problem, as shown in Fig. 5B, requires the selection of the points A_1 and E_4 so that the distances A_1E_1 to A_8E_8 are equal and the points E_1 to E_8 lie on the desired coupler curved a . In this case, only the points E_4 to E_8 represent the desired profile to be cut. The correct selection of points A_1 and E_1 depends upon making the following triangles congruent:

$$\begin{aligned}\Delta E_2A_2B_{01} &= \Delta E_1A_1B_{02} \\ \Delta E_3A_3B_{01} &= \Delta E_1A_1B_{03} \\ \Delta E_8A_8B_{01} &= \Delta E_1A_1B_{08}\end{aligned}$$

and so on until $E_8A_8B_{01} = E_1A_1B_{08}$. At the same time, all points A_1 to A_8 must lie on the arc having B_1 as center.

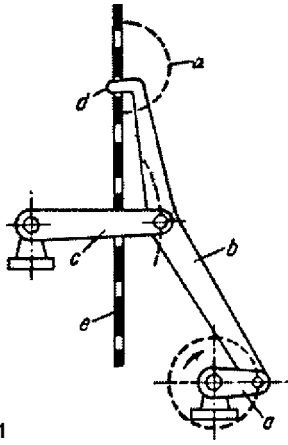


Fig. 1

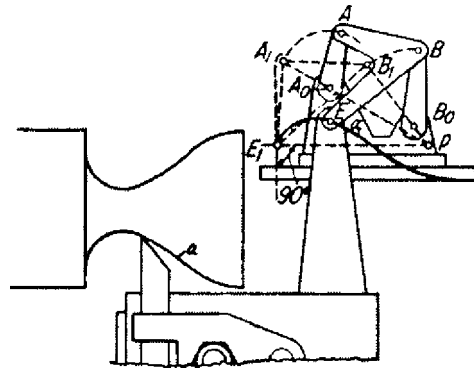


Fig. 2

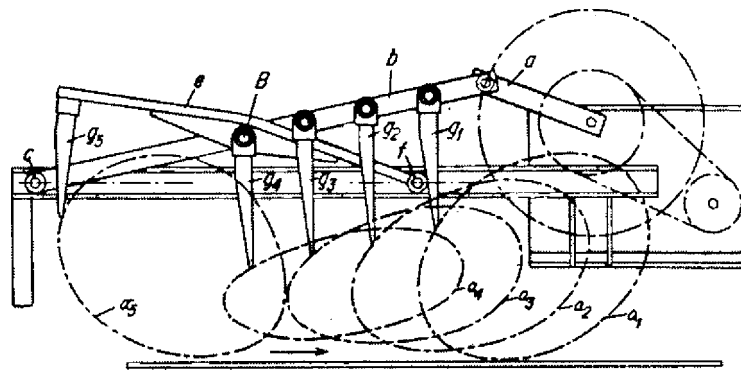


Fig. 3

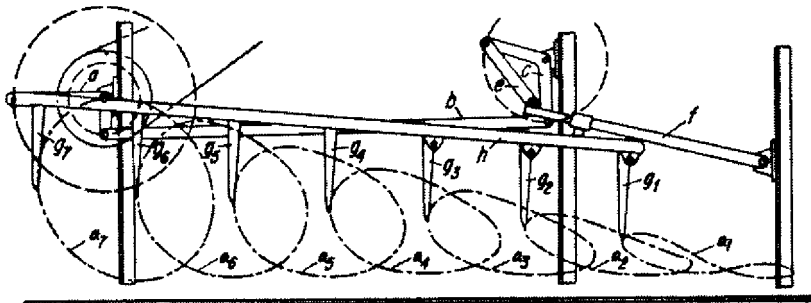
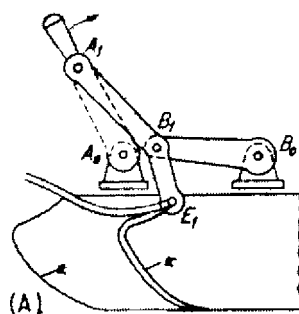
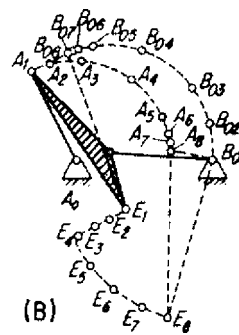


Fig. 4



(A)
Fig. 5



(B)

Synthesis of an Eight-Bar Linkage

Design a linkage with eight precision points, as shown in Fig. 6. In this mechanism the curvilinear motion of one four-bar linkage is coordinated with the angular oscillation of a second four-bar linkage. The first four-bar linkage consists of AA_0BB_0 with coupler point E which generates γ with eight precision points E_1 through E_8 and drives a second four-bar linkage HH_0GG_0 . Coupler point F generates curve δ with precision points F_1 through F_8 . The coupler points F_2, F_4, F_6, F_8 are coincident because straight links GG_0 and GH are in line with one another in these coupler positions. This is what permits HH_0 to oscillate, despite the continuous motion of the coupler point F . The coupler points F_1 coincident with F_5 , and F_3 coincident with F_7 , have been chosen so that F_1 is the center of a circle k_1 and F_3 is the center of a circle k_3 . These circles are tangent to coupler curve γ at E_1, E_5, E_3 , and E_7 , and they indicate the limiting positions of the second four-bar linkage HH_0GG_0 .

The limiting angular oscillation of HH_0 , which is one of the requirements of this mechanism, is represented by positions H_0H_1 and H_0H_3 . It oscillates four times for each revolution of the input crank AA_0 , and the positions H_1 to H_8 correspond to input crank positions A_1 to A_8 .

The synthesis of a compound linkage with dwell periods and coordinated intermittent motion is shown in Fig. 7. The four-bar linkage AA_0BB_0 generates an approximately triangular curve with coupler point E , which has six precision points E_1 through E_6 . A linkage that will do this is not unusual and can be readily proportioned from known methods of four-bar linkage synthesis. However, the linkage incorporates dwell periods that produce coordinated intermittent motion with a second four-bar linkage FF_0HB_0 . Here the tangent arcs k_{12}, k_{34} and k_{56} are drawn with EF as the radius from centers F_{12}, F_{34} and F_{56} .

These centers establish the circle with F_0 as the center and pivot point for the second four-bar linkage. Each tangent arc causes a dwell of the link FF_0 , while AA_0 rotates continuously. Thus, the link FF_0 , with three rest periods in one revolution, can produce intermittent curvilinear motion in the second four-bar linkage FF_0HB_0 . In laying out the center, F_0 must be selected so that the angle EFF_0 deviates only slightly from 90° because this will minimize the required torque that is to be applied at E . The length of B_0H can be customized, and the rest periods at H_{34}, H_{12} and H_{56} will correspond to the crank angles ϕ_{34}, ϕ_{12} and ϕ_{56} .

A compound linkage can also produce a 360° oscillating motion with a dwell period, as in Fig. 8. The two four-bar linkages are AA_0BB_0 and BB_0FF_0 , and the output coupler curve γ is traversed only through segment E_1, E_2 . The oscillating motion is produced by lever HH_0 , connected to the coupler point by EH . The fixed point H_0 is located within the loop of the coupler curve γ . The dwell occurs at point H_3 , which is the center of circular arc k tangent to the coupler curve γ during the desired dwell period. In this example, the dwell is made to occur in the middle of the 360° oscillation. The coincident positions H_1 and H_2 indicate the limiting positions of the link HH_0 , and they correspond to the positions E_1 and E_2 of the coupler point.

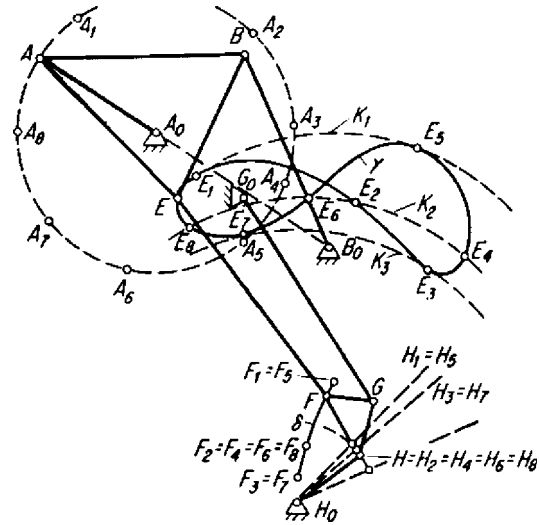


Fig. 6

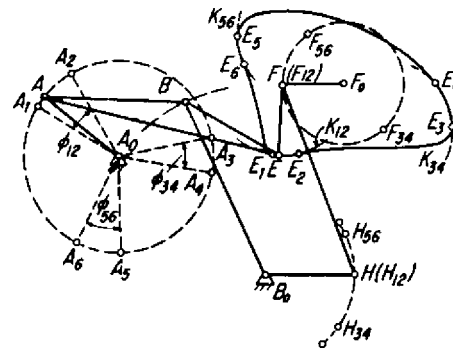


Fig. 7

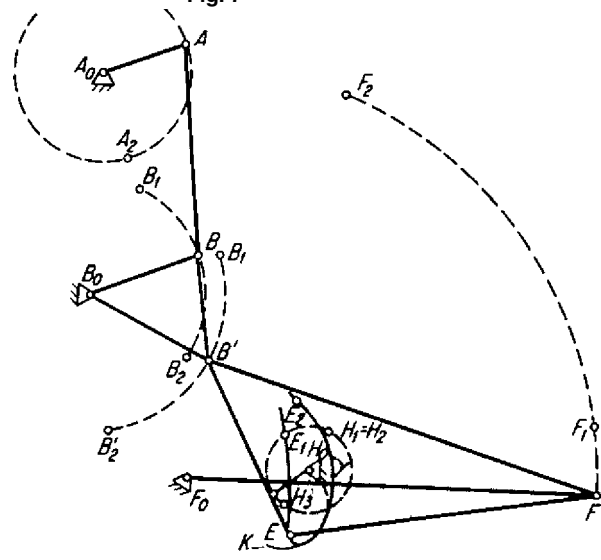


Fig. 8

ROBERTS' LAW HELPS TO DESIGN ALTERNATE FOUR-BAR LINKAGES

The three linkage examples

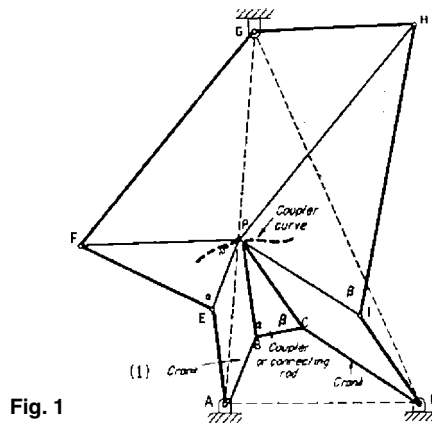


Fig. 1

When a four-bar linkage has been designed or selected from a catalog to produce a desired coupler curve, it is often found that one of the pivot points is inconveniently located or that the transmission angles are not suitable. (A coupler curve is produced by a point on the connecting rod joining the two cranks of the four-bar linkage). According to *Roberts' Law* there are at least two other four-bar linkages that will generate the same coupler curve. One of these linkages might be more suitable for the application.

Robert's Law states that the two alternate linkages are related to the first by a series of similar triangles. This leads to graphical solutions; three examples are shown. The first involves similar triangles, the second is a more convenient step-by-step method, and the third illustrates the solution of a special case where the coupler point lies along the connecting rod.

Method of Similar Triangles

Four-bar linkage $ABCD$ in Fig. 1 uses point P , which is actually an extension of the connecting rod BC , to produce desired curve. Point E is found by constructing EP parallel to AB , and EA parallel to PB . Then triangle EFP is constructed similar to triangle BPC . This calls for laying out angles α and β .

Point H is found in a similar way, and point G is located by drawing GH parallel to FP and GF parallel to HP .

The two alternate linkages to $ABCD$ are $GFEA$ and $GHID$. All use point P to produce the desired curve, and given any one of the three, the other two can be determined.

The Step-by-Step Method

With the similar-triangle method just described, slight errors in constructing the proper angles lead to large errors in link dimensions. The construction of angles can be avoided by laying off the link lengths along a straight line.

Thus, linkage $ABCD$ in Fig. 2 is laid off as a straight line from A to D in Fig. 3. Included in the transfers is point P . Points $EFGHI$ are quickly found by either extending the original lines or constructing parallel lines. Fig. 3, which now has all the correct dimensions of all the links, is placed under a sheet of tracing paper and, with the aid of a compass, links AB and CD are rotated

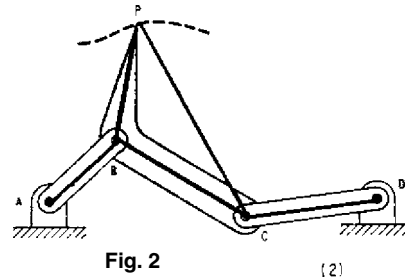


Fig. 2

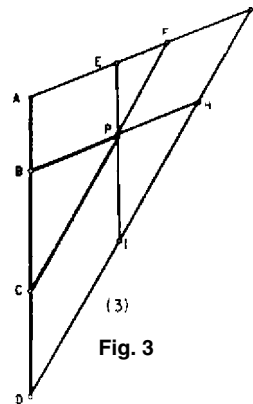


Fig. 3

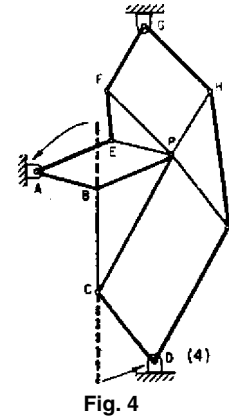


Fig. 4

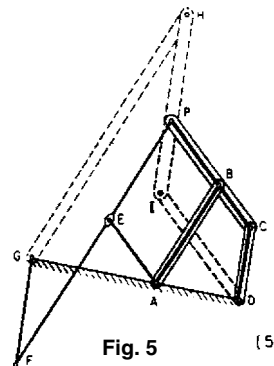


Fig. 5

Fig. 1 The method of similar triangles.

Fig. 2, 3, 4 A step-by-step method.

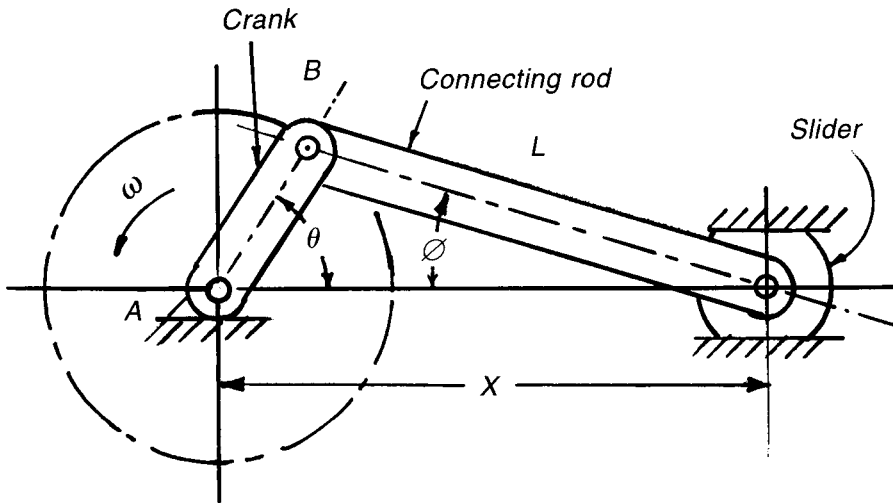
Fig. 5 This special case shows the simplicity of applying Roberts' Law.

(see Fig. 4) so that linkage $ABCD$ is identical to that in Fig. 2. Links PEF and PHI are rotated parallel to AB and CD , respectively. Completion of the parallelogram gives the two alternate linkages, $A EFG$ and $GHID$.

Special Case

It is not uncommon for the coupler point P to lie on a line through BC , as in Fig. 5. Links EA , EP and ID can be found quickly by constructing the appropriate parallel lines. Point G is located by using the proportion: $CB:BP = DA:AG$. Points H and F are then located by drawing lines parallel to AB and CD .

SLIDER-CRANK MECHANISM



The slider crank, an efficient mechanism for changing reciprocating motion to rotary motion, is widely used in engines, pumps, automatic machinery, and machine tools.

The equations developed here for finding these factors are in a more simplified form than is generally found in text books.

SYMBOLS

- L = length of connecting rod
- R = crank length; radius of crank circle
- x = distance from center of crankshaft A to wrist pin C
- x' = slider velocity (linear velocity of point C)
- x'' = slider acceleration
- θ = crank angle measured from dead center (when slider is fully extended)
- ϕ = angular position of connecting rod; $\phi = 0$ when $\theta = 0$
- ϕ' = connecting-rod angular velocity = $d\phi/dt$
- ϕ'' = connecting-rod angular acceleration = $d^2\phi/dt^2$
- ω = constant crank angle velocity

Displacement of slider

$$x = L \cos \phi + R \cos \theta$$

Also:

$$\cos \phi = \left[1 - \left(\frac{R}{L} \right)^2 \sin^2 \theta \right]^{1/2}$$

Angular velocity of the connecting rod

$$\phi' = \omega \left[\frac{(R/L) \cos \theta}{[1 - (R/L)^2 \sin^2 \theta]^{1/2}} \right]$$

Linear velocity of the piston

$$\frac{x'}{L} = -\omega \left[1 + \frac{\phi'}{\omega} \right] \left(\frac{R}{L} \right) \sin \theta$$

Angular acceleration of the connecting rod

$$\phi'' = \frac{\omega^2 (R/L) \sin \theta [(R/L)^2 - 1]}{[1 - (R/L)^2 \sin^2 \theta]^{3/2}}$$

Slider acceleration

$$\frac{x''}{L} = -\omega^2 \left(\frac{R}{L} \right) \left[\cos \theta + \frac{\phi''}{\omega^2} \sin \theta + \frac{\phi'}{\omega} \cos \theta \right]$$